Are the social security benefits of pensions or child-care policies best financed by a consumption tax?

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Are the social security benefits of pensions or child-care policies best financed by a consumption tax?*

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Abstract

Our paper sets an endogenous fertility model and examines how tax revenues derived from a consumption tax should be used for social security benefits such as pension and child-care policies. An additional pension financed by a consumption tax can achieve Pareto-improving allocations. Child allowances and an education subsidy decrease the older generation's utility because of tax burdens and the lack of additional benefit. Even if child allowances can raise the share of young people in society and some future generation's utility, that future generation's utility decreases because of a decrease in income growth. However, with certain parametric conditions, an education subsidy can raise every generation's utility, except for that of the older generation, because of the increase in income growth.

JEL Classification: H55, I20, J13, H20

Keywords: Endogenous fertility, Human capital, Child allowance, Education subsidy, Pension

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1 Introduction

This paper sets an endogenous fertility model with human capital accumulation. Subsequently, an examination is made of how child-care policies financed by a consumption tax affect fertility, educational investment for children, and each generation's welfare. Financing a social security system based on a consumption tax is a policy that is considered in some developed countries with an aging society with fewer children. In Japan, the national tax burden for basic pension benefit is raised. Then the consumption tax rate is raised.[1] However, if an aging society is progressing, then the share of older people among the total population is increasing. The social security burden per capita has risen drastically. To alleviate this situation, the government should raise the future generation's population size. The increase in the social security burden will ease if the progress of an aging society halts.

[Insert Fig.1 around here.]

In France and Sweden, child-care policies are actively provided. Because of active child-care policies, fertility rates in France and Sweden are maintained at an appropriately high level. In contrast, the fertility rate in Japan remains at a low level because child-care policies are negatively provided compared with France and Sweden and because child-care service is insufficiently supported. Child care support policies to decrease child-care costs are necessary to increase fertility. Our paper presents consideration of educational investment for children.

[Insert Fig.2 around here.]

Expenditures for educational investment are important to explain why fertility decreases. Our paper presents consideration of child-care policies of two types: one for a child allowance that is provided proportionally for the number of children in a family and the other for a subsidy for investment in education. In France and Sweden, subsidies for education investment are actively provided, too. In these countries, education costs that households pay are low.

[1] In Japan, the consumption tax rate is pulled up to 8% and finally reaches to 10% in 2015 in governmental schedules. It is discussed how the government should use revenue with an increase in consumption tax rate for social security benefit.

Moreover, the cost of having children is low. Therefore, a decrease in the number of children stops. However, negative child-care policies exist, such as high education costs that households pay and insufficiently provided child-care services. Therefore, a rapidly aging society with fewer children is progressing.

Our paper presents a derivation demonstrating that child allowances do not always increase fertility because of decreased income growth. Child allowances reduce education investment for children. Then human capital accumulation is prevented and income growth decreases. However, a subsidy for education investment can raise the rates of fertility and income growth. An increase in income growth would provide sufficient tax revenues to provide social security benefits. Therefore, a subsidy for educational investment should be adopted to resolve problems of an aging society. Nevertheless, the government must consider social welfare when providing the policies. With child-care policies financed by a consumption tax, the older generation's welfare necessarily decreases even if young and future generations' welfare increases. In such a case, Pareto-improving allocations are not achieved by child-care policies because child allowances reduce income growth and decrease future generations' welfare. However, if the government provides an additional pension benefit financed by a consumption tax, then Pareto improving allocations are achieved. Therefore, considering social welfare, an additional pension is best. In addition, giving a subsidy for education investment is useful to stop an aging society.

The remainder of the paper is presented as follows. Section 2 shows related literature. The model economy is set in section 3, followed by examination of how additional pension and child-care policies affect welfare, fertility, and income growth in section 4. Section 5 discusses another means to finance for an additional pension. The final section concludes our paper.

2 Related Literature

Sleebos (2003) reports child-care policies and fertility in different countries. Intuitively, one might expect that active child-care policies raise the fertility rate. That inference is supported by reports from Laroque and Salanie (2005), showing that fertility is affected by a financial incentive such as a child allowance. Many investigators have set an endogenous fertility model and have examined whether child allowances can raise fertility or not: Oshio (2001), van Groezen, Leers and Meijdam (2003), Yasuoka (2006), van Groezen and Meijdam (2008), Fanti and Gori (2009), Yasuoka and Goto (2011).

Education cost is an important consideration related to how fertility is determined. Many reports in the literature present consideration of how fertility and the quality of children as education investment are determined in terms of the quality and quantity of a child model: Becker, Murphy and Tamura (1990), Tamura (1994), Wigniolle (2002), de la Croix and Doepke (2003, 2004) and Yakita (2010). Especially, Zhang (1995, 1997), Zhang and Casagrande (1998), and Yasuoka and Miyake (2014) set an endogenous fertility with human capital accumulation and examine how child-care policies such as child allowances and a subsidy for education affect fertility and income growth rates.

Optimal pension, tax, and child-care policies have been examined by some studies. Zhang and Zhang (2007) and Hirazawa and Yakita (2009) derive an optimal pension policy in an endogenous fertility model. Van Groezen, Leers and Meijdam (2003), van Groezen and Meijdam (2008) and Yasuoka and Goto (2011) derive optimal pensions and child allowances. Cremer, Gahvari and Pestieau (2011) examine optimal child allowances and education subsidy to hold first best allocations. Our paper considers Pareto-improving allocations with consumption tax and examines whether each generation's welfare is pulled up by the policies or not.

Galor and Weil (1996) derive that fertility is negatively correlated with income because of the opportunity cost of having children. However, this negative relation changes to a positive one, as shown by Day (2012). Child-care services are regarded as bringing about a positive relation (Apps and Lees (2004), Ferrero and Iza (2004), Yasuoka and Miyake (2010) and Day (2012)).

If one considers children as one kind of investment to produce children care in a later period, then a pension system negatively affects the fertility rate (Nishimura and Zhang (1992), Zhang and Zhang (1998), Wigger (1999), Zhang and Zhang (2004) and Oshio and Yasuoka (2009)). However, if one considers children not as an investment, then the pension benefit positively affects fertility, as derived in many papers. Pension benefits financed by a consumption tax are examined by Lin and Tian (2003). Our paper presents an examination not only of pension reform but also of child care policies financed by a consumption tax.

Our paper is closely related to Peters (1995) and Meier and Wrede (2010). Peters (1995) sets an endogenous fertility and education investment model with pay-as-you-go pension and examines the tax policy and subsidy policy for children to achieve welfare maximizing solution. Meier and Wrede (2010) consider the pension model and derive child care policy to raise the welfare by internalizing externality for pension benefit.

Our paper also considers how the child care policies and pension policy affect the fertility, pension benefit and welfare. However, our paper is different from these two quoted ones in the following points. Peters (1995) examines the welfare analysis at the steady state and derives the optimal subsidy policy to achieve the welfare maximizing allocations. On the other hand, our paper examines how the child allowance and the subsidy for the education investment affect the fertility and the human capital of children. Moreover, our paper examines whether the policies can bring about Pareto improving or not. Meier and Wrede (2010) consider the pension incentive policy as the policy to raise the fertility and the education investment and derives how the pension incentive policy affects the fertility and the education investment. In addition, first best solution and second best solution are derived. Our paper considers the consumption tax to finance the tax revenue. Peters (1995) and Meier and Wrede (2010) consider the income tax and lump-sum tax. However, in an aging society, the consumption tax should be considered because of intergenerational inequality.

Our paper considers three policies to raise the pension benefit: the policy of a direct increase in pension benefit, the policy of an indirect increase in pension benefit by the child allowance and the subsidy for the education investment. We compare three policies in terms of social welfare, which is not considered by Peters (1995) and Meier and Wrede (2010).

3 The Model

The model economy consists of a two-period (young and old) overlapping-generations model. For these analyses, we assume a small open economy. Agents of three types exist: households, firms, and a government.

3.1 Households

Households experience two periods: young and old. During the young period, each household supplies labor inelastically to earn labor income. Households are concerned about the quantity of children, the quality of children (which depends on educational investment), and consumption during the young and old period. Households must save to consume during the old period. In addition to household behavior, the government levies a labor income tax to provide pension benefits. Moreover, the government levies a consumption tax to provide an additional pension benefit, a child allowance, and a subsidy for education investment. Pension benefits are provided for older people, but a child allowance and an educational subsidy are provided for younger people. The household's lifetime budget constraint is therefore shown as

$$(1+\tau_c)c_{1t} + \frac{(1+\tau_c)c_{2t+1}}{1+r_{t+1}} + (1-x_t)e_t n_t + (z_t - q_t)n_t = (1-\tau)w_t h_t + \frac{p_{t+1}}{1+r_{t+1}}. \tag{1}$$

Therein, c_{1t} and c_{2t+1} respectively denote consumption during the young period and during the old period. n_t represents the number of children. In addition, r_{t+1} represents an interest rate, w_t and h_t denote the wage rate per unit of effective labor and human capital, e_t shows education investment, τ and τ_c respectively represent the labor income tax rate and the consumption tax rate ($0 < \tau < 1$ and $0 < \tau_c$ are assumed.), and z_t denotes the child-care cost. The government provides q_t unit of child allowance for a child. A child allowance q_t is assumed $q_t < z_t$. Here, x_t denotes the subsidy rate for educational investment ($0 < x_t < 1$ is assumed). Older people can receive pension benefit p_{t+1}.

A household's utility function u_t is given as follows [2]

$$u_t = \alpha \ln n_t h_{t+1} + \beta \ln c_{1t} + (1-\alpha-\beta) \ln c_{2t+1},\ 0 < \alpha, 0 < \beta, \alpha+\beta < 1. \tag{2}$$

The children's human capital h_{t+1} is assumed according to the following equation.

$$h_{t+1} = \gamma e_t^{\epsilon} h_t^{1-\epsilon},\ 0 < \gamma,\ 0 < \epsilon < 1 \tag{3}$$

[2] Glomm and Ravikumar (1992) assumed that parents care about educational investment for their children. However, de la Croix (2003) and others assumed that parents care about children's future income h_{t+1} instead of e_t. As shown by one analysis (3) and another by de la Croix and Doepke (2003), children's future income depends on educational investment. Therefore, the assumption of a utility function (2) here is nearly equivalent to that in a model described by de la Croix and Doepke (2003).

A household chooses consumption during young and old life c_{1t}, c_{2t+1} and chooses educational investment for children (quality of children) e_t and fertility (quantity of children) n_t to maximize lifetime utility (2) subject to the lifetime budget constraint (1) and human capital accumulation (3). The first-order condition derives the following equations.

$$c_{1t} = \frac{\beta}{1+\tau_c}\left((1-\tau)wh_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{4}$$

$$c_{2t+1} = \frac{(1+r)(1-\alpha-\beta)}{1+\tau_c}\left((1-\tau)wh_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{5}$$

$$n_t = \frac{\alpha(1-\epsilon)}{z_t - q_t}\left((1-\tau)wh_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{6}$$

$$e_t = \frac{z_t - q_t}{1-x_t}\frac{\epsilon}{1-\epsilon}. \tag{7}$$

3.2 Firms

Our paper assumes firms of two types. One firm produces final goods. Final goods are produced by a production function $y = f(k)$, $f' > 0$ and $f'' < 0$. y and k respectively denote the final goods per labor input and the capital per labor input. These analyses assume a small open economy. Then, considering that a competitive market and a world interest rate are given, an interest rate r and w are determined using a world interest rate.

Next, we consider the child-care service market. The aggregate child-care service Y_t is produced by $Y_t = \rho L_t^c$, and $\rho > 0$ and L_t^c denote the labor input for child-care services.[3] Denoting w_t^c as the wage rate of child-care service, the profit function π_t is

$$\pi_t = z_t \rho L_t^c - w_t^c L_t^c. \tag{8}$$

Profit maximization reduces to $z_t = \frac{w_t^c}{\rho}$. Assuming free labor mobility between the final goods sector and child-care service sector, the wage rate w_t^c is given as $w_t^c = wh_t$. Thereby, we obtain $z_t = \hat{z}wh_t$, where $\hat{z} = \frac{1}{\rho}$.

3.3 Government

The government provides a pension benefit for older people. The pension benefit is financed by taxation for a wage income that younger people gains, i.e., the pension system in our paper is

[3] This function is assumed by Yasuoka and Miyake (2010) and by Day (2012).

a pay-as-you-go system. Then, the pension benefit is shown as

$$p_{t+1} = \tau w n_t h_{t+1} + \bar{p}_{t+1}. \tag{9}$$

$\bar{p}_{t+1}$ denotes an additional pension benefit financed by a consumption tax. Moreover, the government provides an additional pension benefit, a child allowance and a subsidy for education investment, which are financed by a consumption tax. The budget constraint is show as

$$\tau_c \left(c_{1t} + \frac{c_{2t}}{n_{t-1}} \right) = q_t n_t + \frac{\bar{p}_t}{n_{t-1}} + x_t e_t n_t. \tag{10}$$

4 Policy Effect

This section presents examination of whether an additional pension, child allowance, or education subsidy can increase each generation's utility or not. These social security benefits are financed by a consumption tax.

4.1 Additional Pension Benefit

The government provides an additional pension benefit financed by a consumption tax. An additional pension benefit is assumed by $\bar{p}_t = \hat{p} w h_t$. Then, considering $q_t = x_t = 0$, the government budget constraint for any t period is shown as $\frac{\bar{p}_t}{n_{t-1}} = \tau_c \left(c_{1t} + \frac{c_{2t}}{n_{t-1}} \right)$, that is,

$$\frac{\hat{p} w h_t}{n_{t-1}} = \frac{\tau_c}{1+\tau_c} \left(\beta(1+g) + \frac{(1+r)(1-\alpha-\beta)}{n_{t-1}} \right) \left((1-\tau) w h_{t-1} + \frac{p_t}{1+r} \right), \tag{11}$$

where $1+g = \frac{h_{t+1}}{h_t}$. Completely differentiating by $\hat{p}$ and τ_c at the approximation of $\tau_c = 0$, we obtain the following equation:

$$\frac{d\hat{p}}{d\tau_c} = \left(\beta n + \frac{(1+r)(1-\alpha-\beta)}{1+g} \right) \left(1 - \tau + \frac{\tau n}{1+r} \right). \tag{12}$$

Because of a positive sign of $\frac{d\hat{p}}{d\tau_c}$, an increase in consumption tax raises an additional pension benefit. An additional pension raises fertility because an increase in pension benefits raises the household's lifetime income.

$$\frac{dn}{d\hat{p}} = \frac{\alpha(1-\epsilon)}{1+r} \frac{1}{\hat{z} - \frac{\alpha(1-\epsilon)\tau(1+g)}{1+r}} = \frac{n(1+g)}{(1+r)(1-\tau)} \tag{13}$$

Investment in education is not changed by an additional pension. An income growth rate given by the human capital growth rate is shown as

$$1+g=\gamma\left(\frac{\epsilon w\hat{z}}{1-\epsilon}\right)^{\epsilon}. \tag{14}$$

We derive the condition by which an additional pension benefit can bring about Pareto-improving allocations for every generation. First, we examine how an additional pension benefit affects the young and future generation's utility. Substituting (4)–(6) and (14) into (2), we obtain the young generation's utility as

$$u_t^y=\ln n+\alpha\ln(1+g)-(1-\alpha)\ln(1+\tau_c)+\ln h_t+\beta\ln\frac{\beta\hat{z}w}{\alpha(1-\epsilon)}+(1-\alpha-\beta)\ln\frac{(1+r)(1-\alpha-\beta)\hat{z}w}{\alpha(1-\epsilon)}. \tag{15}$$

We define u_t^y as the utility accruing to the young generation at t period and u_t^o as the utility accruing to the older generation at t period. In addition, u_{t+j} $(1\leq j)$ is defined as the utility accruing to a young generation at $t+j$. We consider u_{t+j} as the future generation's utility. Completely differentiating by u_t^y, τ_c and n at approximation of $\tau_c=0$ for given h_t, we obtain $\frac{du_t^y}{d\tau_c}$ and the young generation's utility increases as long as the following inequality holds.

$$\frac{du_t^y}{d\tau_c}=\frac{1}{n}\frac{dn}{d\tau_c}-(1-\alpha)>0. \tag{16}$$

Although the consumption tax reduces consumption in each period and the utility, an additional pension benefit can raise the level of utility. Considering (12), (13), and (16), we obtain the condition of raising the younger generation's utility.

$$n>\frac{\alpha(1-\alpha)(1-\epsilon)(1-\tau)(1+r)}{\hat{z}\left(\beta n+\frac{(1+r)(1-\alpha-\beta)}{1+g}\right)}. \tag{17}$$

Defining n_p^y as fertility n to hold $n=\frac{\alpha(1-\alpha)(1-\epsilon)(1-\tau)(1+r)}{\hat{z}\left(\beta n+\frac{(1+r)(1-\alpha-\beta)}{1+g}\right)}$, $n_p^y<n$ signifies an increase in the young generation's utility, as shown in Fig. 3. L and R denote the left-hand-side and the right-hand-side of (20). This condition is adopted for any future generation.

[Insert Fig.3 around here.]

Secondly, we examine how an additional pension affects the older generation's utility. Being different from younger people, a consumption tax does not exist in the younger period and consumption is levied for consumption in the older period. Then, the older generation's utility is shown as

$$u_t^o = \ln n - (1-\alpha-\beta)\ln(1+\tau_c) + (1-\alpha)\ln h_{t-1} + \beta \ln \frac{\beta w \hat{z}}{\alpha(1-\epsilon)} + (1-\alpha-\beta)\ln \frac{(1+r)(1-\alpha-\beta)\hat{z}w}{\alpha(1-\epsilon)}. \tag{18}$$

Completely differentiating by u_t^o, τ_c and n at approximation of $\tau_c = 0$ for given h_{t-1}, we obtain $\frac{du_t^o}{d\tau_c}$. Then the older generation's utility increases as long as the following inequality holds.

$$\frac{du_t^o}{d\tau_c} = \frac{1}{n}\frac{dn}{d\tau_c} - (1-\alpha-\beta) > 0 \tag{19}$$

An additional pension raises the older generation's utility if the positive effect of an additional pension on the utility is greater than the negative effect by a consumption tax. Considering (12), (13), and (19), we obtain the condition to raise the younger generation's utility.

$$n > \frac{\alpha(1-\alpha-\beta)(1-\epsilon)(1-\tau)(1+r)}{\hat{z}\left(\beta n + \frac{(1+r)(1-\alpha-\beta)}{1+g}\right)}. \tag{20}$$

Defining $n_p^o \equiv \frac{\alpha(1-\alpha-\beta)(1-\epsilon)(1-\tau)(1+r)}{\hat{z}\left(\beta n + \frac{(1+r)(1-\alpha-\beta)}{1+g}\right)}$, $n_p^o < n$ means an increase in the older generation's utility. Moreover, we find $n_p^o < n_p^y$. Consequently, the following proposition is established.

Proposition 1 With $n_p^y < n$, an additional pension benefit financed by a consumption tax can raise every generation's utility, i.e., Pareto improvement is brought about.

With $n_p^o < n < n_p^y$, an additional pension increases the older generation's utility and decreases the younger generation and future generations' utility. If the fertility rate is higher, an additional pension financed by a consumption tax is larger, as shown by (12). Therefore, given high fertility, every generation's utility increases and Pareto improving allocations are achieved. For the reason that we obtain $n_p^o < n_p^y$, the older generation does not pay a consumption tax for the younger period. Because the older generation's tax burden is not heavy, compared with the young and future generations, the older generation's utility increases even if the younger and future generations' utility decreases.

In developed countries, an aging society is progressing. Because of a numerical decrease in younger generations, the social security benefit financed by a consumption tax increases. Even if a consumption tax raises the pension benefit, the welfare level falls as long as the fertility rate is low.

4.2 Child Allowance

This subsection presents an examination of the effects on welfare of a child allowance financed by a consumption tax. Child allowances financed by a consumption tax reduce the older generation's utility at t period if the government levies a consumption tax at t period. Child allowances raise fertility at $t+j$ period. Therefore, for the older generation at t period, only the consumption tax is levied. Their utility decreases and no Pareto improving allocation can be achieved.

For the younger generation at t period and future generations, child allowances can raise fertility. Considering (3) and (6), income growth and fertility with child allowances are shown as

$$\begin{aligned} 1+g &= \gamma\left(\frac{\epsilon w(\hat{z}-\hat{q})}{1-\epsilon}\right)^{\epsilon}, \\ n &= \frac{\alpha(1-\epsilon)(1-\tau)}{\hat{z}-\hat{q}\frac{\tau\alpha(1-\epsilon)(1+g)}{1+r}}. \end{aligned} \tag{21}$$

Child allowances are assumed by $q_{t+j} = \hat{q}wh_{t+j}, (0 \leq j)$. Then, considering $\hat{p} = x_t = 0$, the government budget constraint with child allowances $q_{t+j}n = \tau_c\left(c_{1t+j} + \frac{c_{2t+j}}{n}\right)$ is shown as

$$\hat{q}wh_{t+j}n = \frac{\tau_c}{1+\tau_c}\left(\beta(1+g) + \frac{(1+r)(1-\alpha-\beta)}{n}\right)\left((1-\tau)wh_{t+j-1} + \frac{\tau nwh_{t+j}}{1+r}\right). \tag{22}$$

Completely differentiating (21)–(23) with $n, 1+g, \tau_c, \hat{q}$ at the approximation of $\tau_c = 0$, we find the effect of child allowances on fertility and income growth, as demonstrated below.

$$\frac{dn}{d\hat{q}} = \frac{n\left(1-\frac{\tau\alpha\epsilon(1-\epsilon)(1+g)}{\hat{z}(1+r)}\right)}{\hat{z}-\frac{\tau\alpha(1-\epsilon)(1+g)}{1+r}}, \tag{23}$$

$$\frac{dg}{d\hat{q}} = -\frac{\epsilon(1+g)}{\hat{z}} < 0 \tag{24}$$

Child allowances decrease the income growth rate because households decrease education investment that is more expensive than the cost of increasing the number of children. The fertility rate does not always increase. With $\tau < \frac{\hat{z}(1+r)}{\alpha\epsilon(1-\epsilon)(1+g)}$, we obtain $\frac{dn}{d\hat{q}} > 0$. $\frac{d\hat{q}}{d\tau_c}$ is given as

$$\frac{d\hat{q}}{d\tau_c} = \frac{\hat{z}}{\alpha(1-\epsilon)}\left(\beta + \frac{(1+r)(1-\alpha-\beta)}{(1+g)n}\right). \tag{25}$$

The younger generation or future generation's utility is shown as follows:

$$u_{t+j} = \ln n + (j+\alpha)\ln(1+g) + (1-\alpha)\ln(\hat{z}-\hat{q}) - (1-\alpha)\ln(1+\tau_c) + \ln h_t + \beta \ln \frac{\beta w}{\alpha(1-\epsilon)} + (1-\alpha-\beta)\ln \frac{(1+r)(1-\alpha-\beta)w}{\alpha(1-\epsilon)}. \tag{26}$$

Therein, u_{t+j} at $j=0$ means u_t^y, which is the younger generation's utility at t period. With $1 \leq j$, u_{t+j} denotes the future generation's utility. Completely differentiating (27) by u_{t+j},n,g, τ_c, and $\hat{q}$ at the approximation of $\tau_c = 0$, we obtain $\frac{du_{t+j}}{d\tau_c}$ as

$$\frac{du_{t+j}}{d\tau_c} = \left(\frac{1}{n}\frac{dn}{d\hat{q}} + \frac{j+\alpha}{1+g}\frac{dg}{d\hat{q}} - \frac{1-\alpha}{\hat{z}}\right)\frac{d\hat{q}}{d\tau_c} > 1-\alpha. \tag{27}$$

With $\frac{du_{t+j}}{d\tau_c} > 0$, child allowances financed by a consumption tax raise the utility. The condition is shown as

$$\frac{n\hat{z}\left(1 - \frac{\tau\alpha\epsilon(1-\epsilon)(1+g)}{\hat{z}(1+r)}\right)}{(1-\epsilon)(1-\tau)} > \frac{\alpha(1-\alpha)(1-\epsilon)}{\beta + \frac{(1+r)(1-\alpha-\beta)}{(1+g)n}} + (1-\alpha) + \epsilon(j+\alpha). \tag{28}$$

We try explaining how each generation's utility changes. We assume that $\tau < \frac{\hat{z}(1+r)}{\alpha\epsilon(1-\epsilon)(1+g)}$ as a positive sign of the left-hand-side of (28). The left-hand-side and the right-hand-side show an increase effect and a decrease effect on utility by child allowances. The first term of the right-hand-side shows a decrease in income growth by child allowances. The second term shows a decrease in $\frac{c_{1t+j}}{n_{t+j}}$ and $\frac{c_{2t+j+1}}{n_{t+j}}$. The third term shows a consumption tax burden. Fertility to hold inequality (28) is depicted in Fig. 4.

[Insert Fig.4 around here.]

In these analyses, L_q and R_q respectively denote the left-hand-side and the right-hand-side of inequality (28). Defining n_q as fertility to equalize (28), then $n_q < n$ brings about an increase in the utility of j generation. n_q increases with j. Then, even if the younger generation and some future generation's utility increase by virtue of low n_q, more future generation's utility might decrease because of high n_q. Consequently, $n_q < n$ holds only to a slight degree.

With $\tau > \frac{\hat{z}(1+r)}{\alpha\epsilon(1-\epsilon)(1+g)}$, no fertility exists for inequality (28). Then, every generation's utility decreases. The following proposition is established.

Proposition 2 Child allowances financed by a consumption tax always reduce the older generation's utility. With $n_q < n$ and $\tau < \frac{\hat{z}(1+r)}{\alpha\epsilon(1-\epsilon)(1+g)}$, j generation's utility increases. However, because n_q increases with j, the future generation's utility decreases more because $n_q < n$ is only slightly held. However, $\tau > \frac{\hat{z}(1+r)}{\alpha\epsilon(1-\epsilon)(1+g)}$ lowers every generation's utility.

Child allowances increase fertility and decrease income growth. A decrease in income growth reduces the young and future generations' utility. For future generations, a decrease effect of income growth on utility becomes large. Then, an increase in the fertility rate pulls up the utility directly and indirectly via the pension benefit. However, as long as fertility with child allowances is constant over time, the negative effect prevails. With $\tau > \frac{\hat{z}(1+r)}{\alpha\epsilon(1-\epsilon)(1+g)}$, both fertility and an income growth decrease because of a decrease in pension benefits. Then, it goes without saying that the utility necessarily decreases.

4.3 Subsidy for Education

This subsection presents an examination of how the subsidy for pension benefits affects each generation's utility. Similarly with child allowances, the older generation's utility in t period decreases because it has only a consumption tax burden with no subsidy benefit. Therefore, this policy does not bring about Pareto-improving allocations because the older generation's utility necessarily decreases. Setting $x_t = x$ and considering $\hat{p} = \hat{q} = 0$, then the government budget constraint is shown as

$$xe_t n_t = \frac{\tau_c}{1+\tau_c}\left(\beta(1+g) + \frac{(1+r)(1-\alpha-\beta)}{n}\right)\left((1-\tau)wh_{t+j-1} + \frac{n\tau w h_{t+j}}{1+r}\right). \tag{29}$$

Fertility and the income growth rate are

$$n = \frac{\alpha(1-\epsilon)(1-\tau)}{\hat{z} - \frac{\tau\alpha(1-\epsilon)(1+g)}{1+r}}, \tag{30}$$

$$1+g = \gamma\left(\frac{\epsilon w \hat{z}}{(1-\epsilon)(1-x)}\right)^{\epsilon}. \tag{31}$$

With complete differentiation (29)–(31) done with n, g, x, τ_c at the approximation of $x = 0$, we obtain the following equations:

$$\frac{dn}{dx} = \frac{\frac{n\tau\alpha\epsilon(1-\epsilon)(1+g)}{1+r}}{\hat{z} - \frac{\tau\alpha(1-\epsilon)(1+g)}{1+r}} > 0, \tag{32}$$

$$\frac{dg}{dx} = \epsilon(1+g) > 0. \tag{33}$$

We obtain $\frac{dx}{d\tau_c}$ as

$$\frac{dx}{d\tau_c} = \frac{\beta + \frac{(1+r)(1-\alpha-\beta)}{(1+g)n}}{\alpha\epsilon} > 0. \tag{34}$$

The young generation or future generation's utility is shown as

$$\begin{aligned} u_{t+j} &= \ln n + (j+\alpha)\ln(1+g) - (1-\alpha)\ln(1+\tau_c) \\ &\quad + \ln h_t + \beta \ln \frac{\beta \hat{z} w}{\alpha(1-\epsilon)} + (1-\alpha-\beta)\ln \frac{(1+r)(1-\alpha-\beta)\hat{z}w}{\alpha(1-\epsilon)}. \end{aligned} \tag{35}$$

With $j = 0$, u_t denotes the younger generation's utility at t period. With $1 \leq j$, u_{t+j} denotes the future generation's utility. Completely differentiating (36) by u_{t+j}, n, g, x and τ_c for given h_t, we obtain $\frac{du_{t+j}}{d\tau_c}$as

$$\frac{du_{t+j}}{dx} = \left(\frac{1}{n}\frac{dn}{dx} + \frac{j+\alpha}{1+g}\frac{dg}{dx}\right)\frac{dx}{d\tau_c} - (1-\alpha). \tag{36}$$

The condition to have $\frac{du_{t+j}}{dx} > 0$ is

$$\frac{n\tau(1+g)}{(1+r)(1-\tau)} + (j+\alpha) > \frac{\alpha(1-\alpha)}{\beta + \frac{(1+r)(1-\alpha-\beta)}{(1+g)n}}. \tag{37}$$

The left-hand-side of (37) shows an increase effect on the utility. An educational subsidy increases education investment and income growth. An increase in income growth raises fertility because of an increase in pension and household income. The right-hand-side of (37) shows a decreased effect on utility because a consumption tax exists. The utility increases by an educational subsidy if this inequality holds.

We try explaining fertility to hold inequality (37) with Fig. 5.

[Insert Fig.5 around here.]

L_x and R_x denote the left-hand-side and the right-hand-side of (37), respectively. As shown in Fig. 5, if fertility exists between n^* and n^{**}, an education subsidy can not raise the utility. Then, the negative effect of a consumption tax burden on the utility is large. Why can $n < n^*$ and $n > n^{**}$ raise the utility? $n > n^{**}$ signifies that fertility is large and that the pension benefit increased by income growth increases more. $n < n^*$ signifies that the fertility rate is low.

However, because of high child care costs, $\hat{z}$ lowers the fertility rate. However, with high $\hat{z}$, an additional education subsidy is magnified. Then, the effect of an increase in pension benefits is large and the utility is raised.

An educational subsidy always increases the utility if L is depicted as a dashed line. The left-hand-side increases with j because of income growth. Then, even if the younger generation's and some future generations' utility decrease, the future generation's utility increases. Our paper shows that every generation's utility, except for that of the older generation, increases because of an education subsidy. Considering (37) at $j = 0$, we consider the following quadratic equations:

$$\frac{(1+g)\tau\beta}{(1+r)(1-\tau)}n^2 + \left(\frac{\tau}{1-\tau} - \alpha\right)(1-\alpha-\beta)n + \frac{\alpha(1+r)(1-\alpha-\beta)}{1+g} = 0. \tag{38}$$

Then, the following inequality holds, and every generation's utility except for that of the old generation, increases.

$$\left(\frac{\tau}{1-\tau} - \alpha\right)^2 (1-\alpha-\beta) < \frac{4\alpha\beta\tau}{1-\tau} \tag{39}$$

Then, the following proposition is established.

Proposition 3 Apart from child allowances, an education subsidy can raise every generation's utility except for the older generation if (39) holds.

Both child allowances and an education subsidy are provided only for young and future generations. However, child allowances necessarily decrease future generations' utility because of a decrease in income growth. In contrast, an education subsidy can raise every generation's utility except for that of the older generation. Therefore, an education subsidy should be selected in terms of welfare.

5 Discussion

Child allowances and an education subsidy financed by a consumption tax decrease the older generation's utility. Therefore, the Pareto-improving allocations are achieved only by an additional pension financed by consumption if the fertility rate is higher than n_p^y. We consider an increase in τ as another means to finance additional pension benefits. Defining $\hat{\tau}$ as the contribution rate before providing an additional pension and completely differentiating by u_t^y, n, and τ at the approximation of $\tau = \hat{\tau}$, we obtain $\frac{du_t^y}{d\tau} = \frac{1}{n}\frac{dn}{d\tau}$. $\frac{dn}{d\tau}$ as

$$\frac{dn}{d\tau} = \frac{\alpha(1-\epsilon)}{\hat{z}} \frac{\frac{n(1+g)}{1+r} - 1}{1 - \frac{\alpha(1-\epsilon)\tau(1+g)}{\hat{z}}}. \tag{40}$$

With $n > \frac{1+r}{1+g}$, the young generation's utility increases. This is Aaron condition.[4] Then, the future generation's utility increases, too. The older generation's utility always increases because of a lack of burden for an additional pension, as is known generally. We obtain both $\frac{1+r}{1+g} > n_t^y$ and $\frac{1+r}{1+g} < n_t^y$ based on the parametric condition. With $n_t^y < n < \frac{1+g}{1+r}$, Pareto improving allocations are brought about by an increase in the contribution rate or labor income tax, not a consumption tax. Even if an aging society is progressing and it is considered important for financing by a consumption tax, an increase in the contribution rate or labor income tax rate is desirable because such a policy can provide Pareto improvement.

In this case, an increase in τ raise the lifetime income and welfare. However, as shown by (16) and (19), an increase in pension financed by the consumption tax can not raise the welfare even if $n > \frac{1+r}{1+g}$. This result shows that the labor income taxation for only young generation is different from the consumption tax for not only young generation but also old generation.

6 Conclusions

Our paper presents an examination of how additional tax revenues derived from a consumption tax should be provided for social security benefits such as pension and child care policies. An additional pension can raise every generation's utility and can achieve Pareto-improving allocations. However, child-care policies such as child allowances and an education subsidy can raise every generation's utility, except for the older generation. Moreover, even if child allowances raise the young and some future generation's utility, future generations' utility decreases because of a decrease in income growth. However, with a parametric condition, an education subsidy can always raise every generation's utility except for that of the older generation because such a subsidy can increase income growth. Results of our paper demonstrate that an additional

[4] Adema, Van Groezen and Meijdam (2009) explains Aaron condition.

pension benefit financed by a consumption tax should be provided because Pareto-improving allocations are brought about. Even if the government wants to provide child-care policies, an education subsidy should be provided because child allowances decrease the future generations' utility.

Finally, we compare an additional tax financed by a consumption tax with that financed by a labor income tax. Although a consumption tax should be used to finance social security benefits in an aging society, an additional pension financed by a consumption tax can not achieve Pareto improving allocations if the fertility rate is low. Then, an additional pension financed by a labor tax can achieve Pareto improvement. Therefore, the government should consider which tax should be used for an additional pension.

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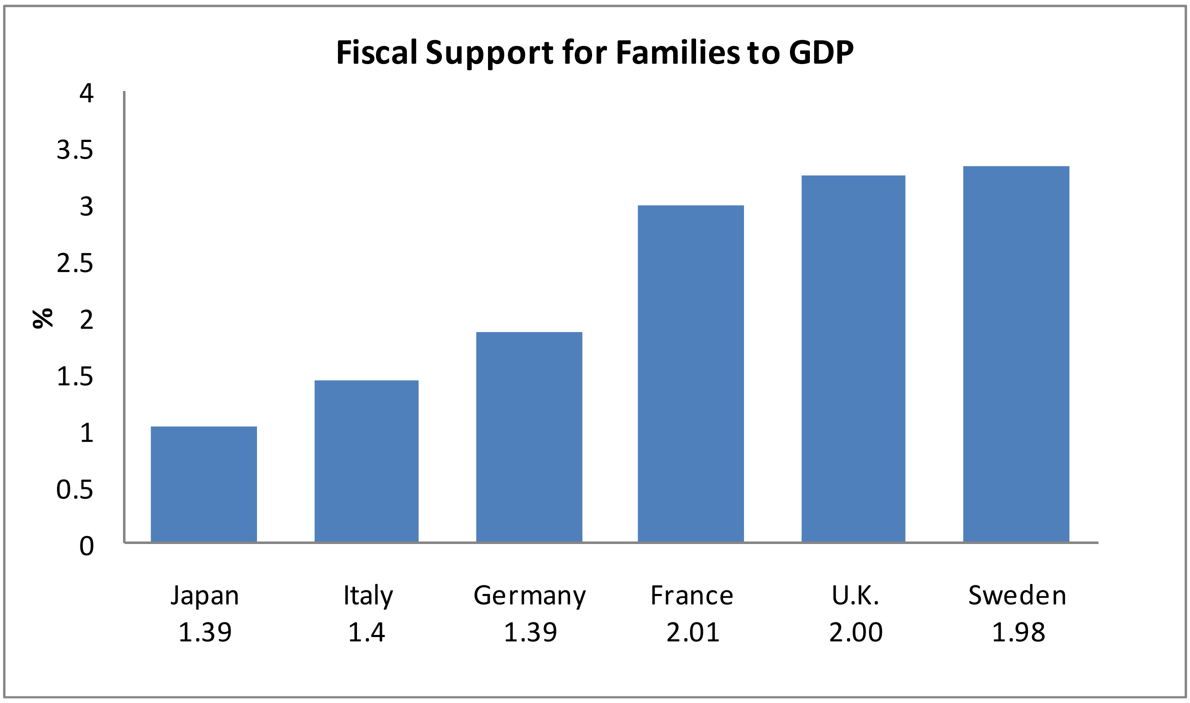

Fig. 1: Fertility (below the country) and Fiscal Support for Family (share of Gross Domestic Product) (Data: OECD Social Expenditure Database (November 2008), A 2012 Declining Birthrate White Paper (2012), Demographic Yearbook (UN) and Vital Statistics in Japan (Ministry of Health, Labour and Welfare (in Japan).) Data of Fiscal Support for Families are those of 2007. Fiscal Support for Families includes benefits in kind (day-care/home help and other benefits in kind) and cash benefits (family allowance, maternity and parental leave and other cash benefits). Data of the total Fertility Rate are those of 2010.)

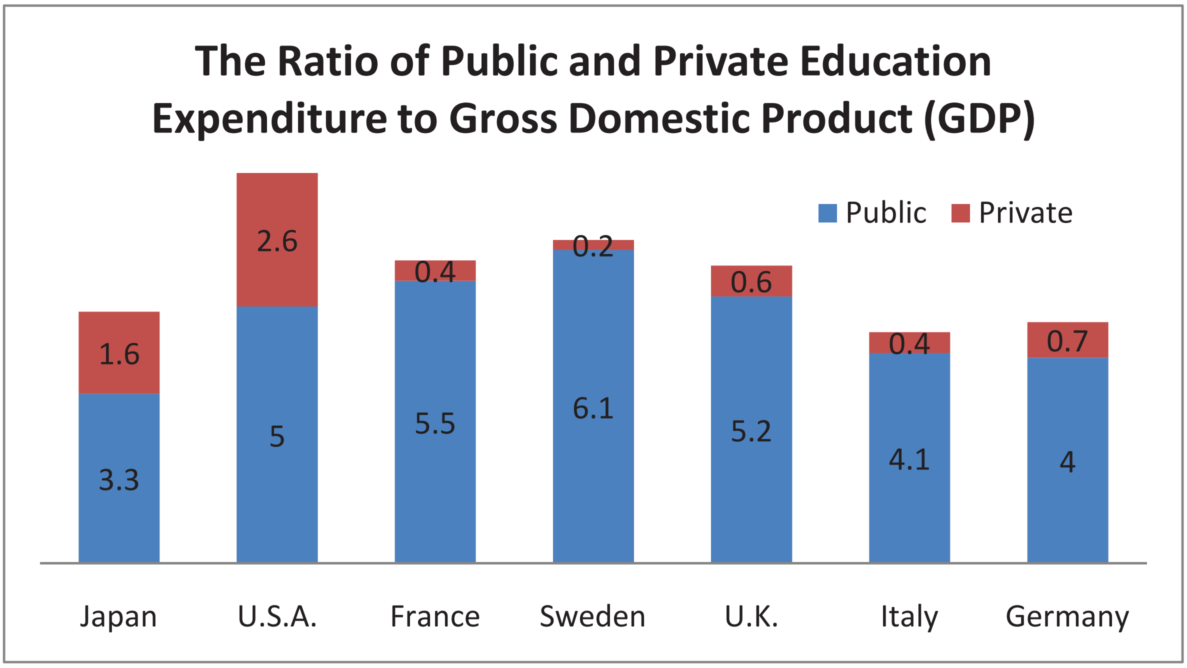

Fig. 2: Ratio of Public and Private Education Expenditure to Gross Domestic Product (GDP).

(Data: OECD (2010) Education at a Glance 2010. Data years are 2007. Fiscal Support for Families includes in-kind benefits (day care, home help and other in-kind benefits) and cash benefits (family allowance, maternity and parental leave, and other cash benefits).)

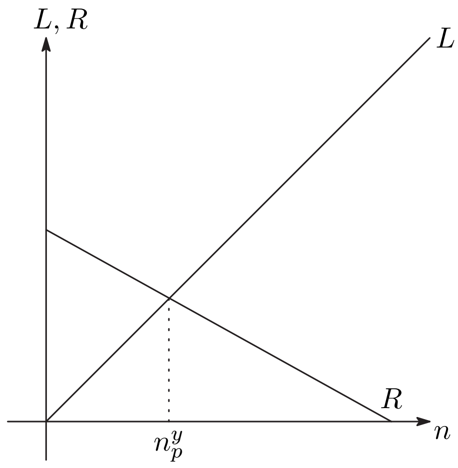

Fig. 3 Fertility to hold Eq. (20).

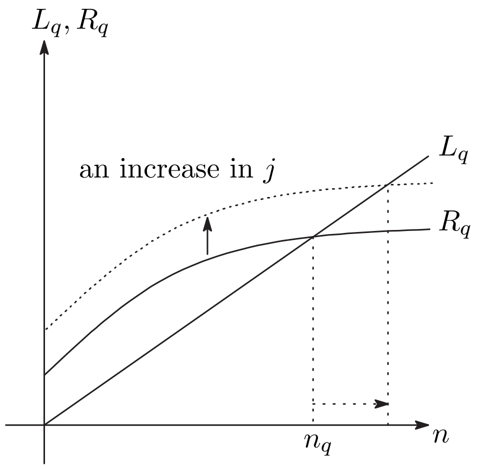

Fig. 4 Fertility to hold Eq. (28).

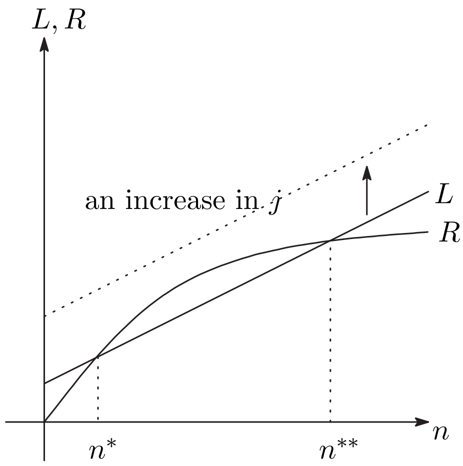

Fig. 5 Fertility to hold Eq. (37).