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# Malice in the Rubinstein bargaining game

# Brishti Guha<sup>1</sup>

### Abstract

This is the first paper to incorporate malice into the Rubinstein alternating offers bargaining game. Initially, I examine outcomes with one-sided malice, allowing one of the bargaining players to be malicious in the sense that he obtains a positive payoff in every period in which the other player does not obtain any piece of the pie. This "malice payoff" is independent of whether the malicious player himself obtains the pie or not. I identify a unique SPNE of the bargaining game, and find that malice confers bargaining advantage; if the respondent is malicious, this can in some cases completely erode and even reverse first mover advantage. I then examine two-sided malice. I find that the proposer's share when both players are malicious may either increase or decrease relative to the traditional Rubinstein shares. Even with two-sided malice, the proposer may end up with a lower share than the respondent. The results remain qualitatively similar under an alternative "continuous" formulation of malice. I contrast them with the case of envious preferences.

Keywords: Malice, Rubinstein alternating offers game, disagreement.

# JEL Classification: C72.

#### 1. Introduction

Malice – broadly defined as deriving utility when someone else is harmed or deprived, even though this harm or deprivation need not translate into a direct economic benefit to oneself – has been experimentally shown to be an important factor motivating many individuals. The experimental literature on malice includes, among others, Beckman et al (2002), Bosman and van Winden (2002), Bosman et al (2006), Albert and Mertins (2008), Zizzo and Oswald (2001), Abbink and Sadrieh (2008), and Abbink and Herrmann (2011). Abbink and Herrmann (2011), for instance, use a one-shot "joy of destruction" game to show that 10-25% of their subjects destroyed others' endowments without any economic gain to themselves. Zizzo and Oswald

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(2001) find, in a "money burning" game, that two-thirds of their subjects were so eager to destroy others' earnings that they were actually willing to pay for the privilege of doing so. Bosman and van Winden (2002) find, in a "power to take" experiment, that 21% of their subjects destroy their own earnings when told that a portion of these earnings would later go to another subject, while Beckman et al find that 50% of their experimental subjects oppose Pareto improvements that make others better off without making them any worse off.

In addition to the laboratory, malice has also been recognized as an important motivating factor in the courtroom; legal systems throughout history have acknowledged the possibility that lawsuits may be driven by a malicious desire to cause financial and reputational harm to the defendant, and have incorporated countermeasures to discourage this (Guha 2016).

In the present paper I apply malice to the Rubinstein alternating offers bargaining game. Rubinstein (1982) provides a simple and elegant solution to the problem of two players bargaining over a fixed pie where delay is costly to both parties (both players discount the future). He shows that even allowing for alternating offers bargaining extending infinitely far into the future, there is a unique, sub-game perfect solution that involves a determinate split of the pie in the very first period.

In Rubinstein's model, both players unambiguously dislike delay, and it is this that drives the bargaining solution. However, now consider the impact of allowing one of the players to be malicious. As before, delay is costly in the sense that both players discount the future. Also, as before, the pie is fixed in size and each player derives utility from a larger share of the pie. Now, however, the malicious player obtains a positive payoff whenever the other player does not obtain a share of the pie – regardless of whether he himself obtains the pie, or not – a malice payoff. I find that, with one-sided malice, the presence of malice confers a bargaining advantage, increasing the malicious player's share relative to the traditional model. Intuitively, this is because periods of disagreement are valuable to the malicious player but not to the non-malicious one. I also find that if the malicious player is the respondent, this can completely erode and even reverse first mover advantage, so that the non-malicious proposer ends up getting a lower share of the pie than the respondent. The subgame perfect Nash equilibrium of the bargaining game is shown to be unique.

Next, I examine the case of two-sided malice. I find that the proposer's share may either increase or decrease relative to the traditional model. If both players are equally malicious, the shares become more equal than under the traditional model. However, even with two-sided malice, it is possible for first mover advantage to be completely eroded and indeed, reversed, yielding the proposer an equilibrium share of less than half. The results remain qualitatively robust to an alternative formulation of malice. Under this alternative formulation, I allow a malicious player to obtain malice utility not only when the opponent does not obtain the pie at all, but also allow malice payoffs to be decreasing in the opponent's eventual share (increasing in one's own).

While Rubinstein's paper has given rise to a rich game theoretic literature, none of it, to my knowledge, discusses malice. Here, I confine myself to discussing the contributions most relevant to my paper. Avery and Zemsky (1994) looks at the possibility of "money burning", where one of the players may seek to improve his bargaining position by threatening to "burn money" – that is, take an action to reduce the size of the pie – in a subsequent period. They show that provided this threat is credible, the ability to burn money gives rise to multiple equilibria, including an equilibrium where one player always burns money, and one where he never does so. Money burning, however, used in this sense, differs fundamentally from the malice motive that I model in this paper. While burning or not burning money is a strategic choice, malice is an intrinsic component of utility that obtains whenever the other player does not obtain a share of the pie. Thus, while malice would be served even in an outcome where bargaining fails, because the other player does not obtain the pie, this would not happen in a money burning model. While the sole objective of burning money would be to improve bargaining power, malice gives the malicious player an independent payoff (from malice utility) if the bargain fails or is delayed.

A paper that is highly relevant to my model is Kohler (2013), which introduces envy into the alternating offers framework. Section 2.4 of my paper contains a detailed comparison with Kohler's paper, in terms of concepts, results, and their interpretation. Therefore, I confine myself to a brief discussion here. Envy refers to one-sided inequality aversion. Envious players thus dislike not reaching an agreement just as much as non-envious players do: while malicious players always obtain utility (from malice) in any period of disagreement, unlike non-malicious players. Moreover, envy is discontinuously triggered when the opponent's share exceeds half; this is not the case with malice. These differences underlie the differences in results – for example, in Kohler's paper, the proposer's share always exceeds half (even with an envy-free proposer and an envious respondent) and only converges to half in the special case of equal envy and equal discount factors. However, in my model, the proposer's share may dip below half, even if malice is two-sided.

The rest of the paper is organized as follows. In Section 2 I set up the model and study equilibrium outcomes under (i) one-sided malice, (ii) two-sided malice, and (iii) an alternative "continuous" formulation of malice. Section 2.4 contains a discussion, in particular comparing malice with Kohler's (2013) paper on envy. Section 3 concludes.

# 2. Alternating offers bargaining with a malicious player

#### 2.1 The basic model with one-sided malice

Consider an infinite horizon alternating offers bargaining game in which two players bargain over a pie of size 1, and have a common discount factor of  $\delta$ . However, one player is malicious. In addition to her eventual share of the pie, she also obtains a malice payoff of  $\kappa$  in every period in which the other player does not obtain any part of the pie. Thus, the utility of a malicious player who obtains a share *z* in the T<sup>th</sup> period of bargaining, is

$$u^{M}(z,T) = \delta^{T-1}z + (1+\delta+..\delta^{T-2})\kappa = \delta^{T-1}z + \frac{(1-\delta^{T-1})\kappa}{1-\delta}, \text{ for } z < 1,$$
$$u^{M}(1,T) = \delta^{T-1}(1+\kappa) + (1+\delta+..\delta^{T-2})\kappa = \delta^{T-1} + \frac{(1-\delta^{T})\kappa}{1-\delta}, \text{ for } z = 1.$$
(1)

Here, the superscript M denotes malice. The malice parameter  $\kappa$  is common knowledge. Equation (1) shows that the malicious player gets utility from malice in the periods before she gets a share, simply because the other player has not obtained a share either. In addition, if the final allocation results in the malicious player getting the entire pie, she also gets malice utility in period T, as the other player does not obtain any part of the pie. The non-malicious player's utility is simply

$$u(1-z,T) = \delta^{T-1}(1-z)$$
(2)

To start with, we focus on the case where the malicious player is the second mover.

**Proposition 1.** In an infinite horizon alternating offers game with a malicious respondent, immediate agreement on the split  $\left[\frac{1-\delta-\kappa}{1-\delta^2}, \frac{\delta(1-\delta)+\kappa}{1-\delta^2}\right] = [x_{1*}, 1-x_{1*}]$  can be supported as a SPNE provided  $\kappa < \kappa^* = 1-\delta$ .

**Proof:** Let time be indexed by t, starting at t=1; the non-malicious player is the proposer in odd periods, while the malicious player is the proposer in even periods. The proposed SPNE can be supported by the following strategies:

- (i) The non-malicious player proposes  $[x_1*, 1-x_1*]$  in each odd period.
- (ii) In each odd period, the malicious respondent accepts any split that gives her at least 1-x<sub>1\*.</sub> She refuses a smaller offer and counter-offers [1-y\*,y\*] in each even period where  $y^* = \frac{1-\delta+\kappa\delta}{1-\delta^2}$ .
- (iii) In any even period, the non-malicious player accepts any split that gives him at least 1-y\*, refusing a smaller share and counter-offering in the next period as specified in (i).

Initially, we check that it is subgame perfect for player 1 to act as in (i) given that the rest of the equilibrium strategies hold. In any odd period, if he offers the malicious player  $1-x_{1*}$ , this is accepted by (ii), so he obtains  $x_{1*}$ . He will not offer the malicious player more, as that would also be accepted (by (ii)), thus lowering player 1's own payoff. If he offers the malicious player less, this is refused (by (ii)) so that player 1 then obtains the payoff from the malicious player's counter-offer, discounted by one period:  $\delta(1-y^*)$ . Now given  $\delta < 1$ , we have  $\frac{1-\delta-\kappa}{1-\delta^2} > \frac{\delta^2(1-\delta-\kappa)}{1-\delta^2}$ , or  $x_{1*} > \delta(1-y^*)$ . Thus, player 1 will not offer the malicious player less, either, verifying that he offers exactly  $[x_{1*}, 1-x_{1*}]$ .

Next, we check that it is subgame perfect for player 1 to act as specified in (iii), given that the rest of the equilibrium strategies hold. If in any even period t, the non-malicious player refuses a split that gives him at least  $1-y^* = \frac{\delta(1-\delta-\kappa)}{1-\delta^2}$ , he gets to propose in the next period t+1, and this proposal is accepted; however that payoff is discounted, so that in period t terms, he obtains  $\delta x_{1*}=1-y^*$ . Thus, he is indifferent between acceptance and rejection, and

gains nothing by refusing a split that gives him 1-y\*. At the same time, he refuses a smaller split as he can get  $\delta x_{1*}=1-y^{*}$  by doing so, making the threat of refusal credible.

Next, we check the malicious player, player 2's behavior in odd periods, holding the rest of the equilibrium strategies fixed. By refusing  $1-x_{1*}$  (or a larger offer) in any odd period t, she obtains a malice utility of  $\kappa$  by depriving the non-malicious player of any share in that period; her counter-offer in period t+1 is accepted (by (iii)), giving her y\*, discounted by one period, so that player 2 can get  $\delta y^{*+\kappa}$  in period t terms. However,  $\delta y^{*+\kappa} = \frac{\delta(1-\delta)+\kappa}{1-\delta^2} = 1-x_{1*}$  and therefore the malicious player is indifferent between acceptance and refusal. She therefore gains nothing by refusing the offer 1- $x_{1*}$ . Moreover, she always refuses a smaller offer as by doing so, she can obtain  $\delta y^{*+\kappa} = 1-x_{1*}$ .

We now check the malicious player's behavior in even periods. Given (iii), offering player 1 1-y\* in any even period t is accepted, giving her y\*. She has no incentive to offer more, as that would also be accepted, lowering her own payoff. If she offers less, her offer is refused, from (iii), so that she obtains  $\delta(1-x_{1*})+\kappa$  in period t terms (she obtains malice utility by delaying agreement one period, and in t+1 she accepts player 1's equilibrium proposal, to which the discount factor is applied). Now  $\delta(1-x_{1*})+\kappa = \delta[\delta y^*+\kappa]+\kappa < y^*$  as long as  $y^* > \frac{(1+\delta)\kappa}{1-\delta^2}$  which is always the case given  $\kappa < \kappa^*$ . Thus, she has no incentive to deviate from her equilibrium offer in even periods.

Finally, we can verify that given  $\kappa < \kappa^*$ , (i) the equilibrium payoffs from bargaining are positive for both players, and that (ii) the malicious player's discounted payoff from never participating in bargaining,  $\frac{\kappa}{1-\delta}$ , is less than  $1-x_{1^*}$ . Thus, she participates actively in bargaining. **QED** 

*Remark 1:* The malicious player's share in the pie in Proposition 1 depends positively on  $\kappa$  and  $\delta$ . The presence of malice lowers the non-malicious proposer's share relative to what he obtains in a Rubinstein game without malice.  $(1/(1 + \delta))$ .

Intuitively, high malice as well as patience make it credible for the malicious player to delay agreement longer, ensuring a higher share as the price of immediate agreement.

**Observation 1.** For a non-empty range of values for the malice parameter -  $\kappa \in (\frac{(1-\delta)^2}{2}, 1-\delta)$ , the malicious respondent obtains a greater share of the pie than the non-malicious proposer, so that first mover advantage is completely eroded and reversed.

What is the outcome when the malicious party is the proposer? Intuitively, if bargaining were to fail in period 1 of the game in Proposition 1, we would move into an infinite horizon alternating offers game with the malicious party as proposer. We therefore have

**Corollary 1.** In an infinite horizon alternating offers game with a malicious proposer, immediate agreement on the split  $\left[\frac{1-\delta+\kappa\delta}{1-\delta^2}, \frac{\delta(1-\delta-\kappa)}{1-\delta^2}\right] = [y^*, 1-y^*]$  can be supported as a SPNE provided  $\kappa < \kappa^* = 1-\delta$ .

The result follows from the fact that the proposed SPNE is in accordance with the equilibrium behavior specified in the malicious respondent game of Proposition 1 of which the malicious proposer game is a subgame. The malicious proposer's share increases in his malice parameter. Next, we show that the SPNE in Proposition 1 is unique for the parameter range in which Proposition 1 holds.

# **Proposition 2:** *The SPNE in Proposition 1 is unique for* $\kappa < \kappa^*$ *.*

**Proof:** Suppose not. Let  $m_1$  and  $M_1$  denote the minimum and the maximum share respectively that player 1 obtains in any SPNE; correspondingly, let  $m_2$  and  $M_2$  denote the minimum and maximum SPNE shares for player 2. Now, if player 2 refuses player 1's offer, the minimum payoff that she can obtain in the second period is  $m_2$ , which is discounted; to this must be added  $\kappa$ , which she obtains by depriving her opponent of a share in the pie for one period. Thus, she must be given at least  $\delta m_2 + \kappa$  in period 1. We therefore have

$$M_1 \le 1 - \kappa - \delta m_2 \tag{3}$$

Using similar logic, we see that player 2 cannot be given more than  $\delta M_2 + \kappa$  in period 1, yielding

$$m_1 \ge 1 - \kappa - \delta M_2 \tag{4}$$

(3) and (4) together imply

$$M_1 - m_1 \le \delta(M_2 - m_2) \tag{5}$$

Similarly, the non-malicious player cannot be given less than  $\delta m_1$  or more than  $\delta M_1$ , so we have

$$M_2 \le 1 - \delta m_1 \tag{6}$$

$$m_2 \ge 1 - \delta M_1 \tag{7}$$

From (6) and (7),

$$M_2 - m_2 \le \delta(M_1 - m_1) \tag{8}$$

(5) and (8) together imply

$$M_1 - m_1 \le \delta^2 (M_1 - m_1)$$

Given  $\delta < 1$ , this implies M<sub>1</sub>=m<sub>1</sub>, which, from (8), implies M<sub>2</sub>=m<sub>2</sub>. Thus the SPNE is unique for the given parameters. **QED** 

### 2.2 Making malice two-sided

The preceding sub-section shows that when malice is one-sided, the malicious bargainer has an advantage and obtains a larger share of the pie than in the traditional Rubinstein model – regardless of whether she is the respondent or the proposer to start with. If the malicious player moves second, the presence of malice helps erode first mover advantage; while if the malicious player moves first, malice enhances her first mover advantage. We now look at what happens when both bargainers are malicious. Let the second mover's utility be depicted by (1), and now denote the first mover's malice parameter by  $\mu$ . Then, the first mover's utility is

$$u^{1}(z,T) = \delta^{T-1}z + (1 + \delta + \dots \delta^{T-2})\mu = \delta^{T-1}z + \frac{(1 - \delta^{T-1})\mu}{1 - \delta}, \text{ for } z < 1,$$

$$u^{1}(1,T) = \delta^{T-1}(1+\mu) + (1+\delta+..\delta^{T-2})\mu = \delta^{T-1} + \frac{(1-\delta^{T})\mu}{1-\delta}, \text{ for } z = 1.$$
(9)

**Proposition 3.** In an infinite horizon alternating offers game with two-sided malice, immediate agreement on the split  $\left[\frac{1-\delta-\kappa+\delta\mu}{1-\delta^2}, \frac{\delta(1-\delta)+\kappa-\delta\mu}{1-\delta^2}\right] = [x^*, 1-x^*]$  can be supported as a SPNE provided  $\kappa+\mu < 1-\delta$ .

**Proof.** The proposed SPNE can be supported by the following strategies:

- (i) Player 1 with malice parameter  $\mu$  proposes [x\*,1-x\*] in each odd period.
- (ii) In each odd period, player 2 (with malice parameter  $\kappa$ ) accepts any split that gives her at least 1-x\*. She refuses a smaller offer and counter-offers [1-y\*,y\*] in each even period where  $y^* = \frac{1 \delta + \kappa \delta \mu}{1 \delta^2}$ .
- (iii) In any even period, player 1 accepts any split that gives him at least 1-y\*, refusing a smaller share and counter-offering in the next period as specified in (i).

We check that it is subgame perfect for player 1 to act as in (i) given that the rest of the equilibrium strategies hold. In any odd period, if he offers 1-x\*, this is accepted by (ii), so he obtains x\*. He has no incentive to offer more, as that would also be accepted. If he offers less, this is refused (by (ii)). In this event, player 1 then obtains a malice utility of  $\mu$  by depriving player 2 of a share for one period; he then obtains the payoff from the malicious player's counter-offer, discounted by one period. Thus his payoff if he offers less than x\* in an odd period is  $\delta(1-y^*)+\mu = \frac{\delta^2(1-\delta-\kappa)+(1+\delta-\delta^2)\mu}{1-\delta^2}$  which is less than x\* given  $\kappa+\mu < 1-\delta$ . Thus, player 1 will not offer less, either, verifying that he offers exactly [x\*, 1-x\*].

Next, we check that it is subgame perfect for player 1 to act as specified in (iii), given that the rest of the equilibrium strategies hold. If in any even period t, he refuses a split that gives him at least  $1-y^* = \frac{\delta(1-\delta-\kappa)+\mu}{1-\delta^2}$ , he obtains a malice utility  $\mu$  by depriving player 2 of any share in that period, and gets to propose in the next period t+1. This proposal is accepted; so that in period t terms, he obtains  $\delta x^* + \mu = 1-y^*$ . Thus, he is indifferent between acceptance and rejection, and gains nothing by refusing a split that gives him  $1-y^*$ . At the same time, he refuses a smaller split as he can get  $\delta x^* + \mu = 1-y^*$  by doing so, making the threat of refusal credible.

The rest of the proof mimics that of Proposition 1. QED

**Observation 2.** The presence of two-sided malice can either increase or decrease the proposer's share relative to the traditional Rubinstein shares.

**Proof.** It is evident that the proposer's share increases relative to the traditional share if and only if  $\mu > \kappa/\delta$ . We may check that this inequality is consistent with the restriction under which Proposition 3 holds, namely that  $\mu < 1-\delta - \kappa$ , as long as  $\kappa < \delta(1-\delta)/(1+\delta)$ . Thus, it is possible for first

mover advantage in bargaining to increase with two-sided malice, if the respondent is not very malicious. However, it can be easily perceived that if the proposer is only slightly more malicious than the respondent – so that  $\kappa/\delta > \mu > \kappa$  – or if the two are equally malicious (with  $\kappa=\mu$ ), or if the respondent is more malicious, then two-sided malice reduces the proposer's share relative to the traditional Rubinstein division. *QED* 

**Observation 3.** If  $\mu < \min[\frac{1-\delta}{2}, \frac{\kappa}{\delta}, 1-\delta-\kappa]$ , then with two-sided malice, the proposer may obtain a lower share than the respondent, so that first mover advantage is completely eroded and reversed.

**Proof.** Inspection of the equilibrium shares shows that the proposer obtains a smaller share than the respondent if and only if we have  $(1 - \delta)^2 < 2(\kappa - \delta\mu)$ . This is feasible only if  $\mu < \kappa/\delta$  so that the RHS of the inequality is positive. Moreover it can be checked that this inequality is compatible with the restriction under which Proposition 3 holds (namely that  $\mu < 1-\delta-\kappa$ ) as long as  $\mu < (1-\delta)/2$ . *QED* 

Thus, even with two-sided malice, first mover advantage may be completely eroded and reversed.

#### 2.3 An alternative continuous formulation of malice

In our current formulation of malice, there is a discontinuity in a malicious player's utility at z=1. Since the malicious player's malice is served only when the other player does not obtain any part of the pie, the malicious player does not obtain malice utility in the period of agreement (provided the agreement does not give her the entire pie). An alternative way of modeling malice allows for the malicious player to obtain a "continuous" malice utility of  $z\kappa$ ,  $z\leq 1$ , when her share in the pie is z (since this also lowers the other player's share by z). Formally, we can have

$$u^{M}(z,T) = \delta^{T-1}z(1+\kappa) + (1+\delta+..\delta^{T-2})\kappa = \delta^{T-1}z(1+\kappa) + \frac{(1-\delta^{T-1})\kappa}{1-\delta}$$
(10)

We may check that  $u^{M}(1,T) = \delta^{T-1}(1+\kappa) + \frac{(1-\delta^{T-1})\kappa}{1-\delta} = \delta^{T-1} + \frac{(1-\delta^{T})\kappa}{1-\delta}$ . Thus, there is no discontinuity in this formulation.

It turns out that all our previous results are qualitatively unaffected by this alternative formulation of malice.

**Proposition 4.** In an infinite horizon alternating offers game with a malicious respondent and "continuous" malice, immediate agreement on the split  $\left[\frac{1-\delta-\frac{\kappa}{1+\kappa}}{1-\delta^2}, \frac{\delta(1-\delta)+\frac{\kappa}{1+\kappa}}{1-\delta^2}\right] = [x_{1*}, 1-x_{1*}]$  can be supported as a SPNE provided  $\kappa/(1+\kappa) < 1-\delta$  or  $\kappa < 1-\delta/\delta$ .

**Proof.** The proposed SPNE can be supported by the following strategies:

- (i) The non-malicious player proposes  $[x_1*, 1-x_1*]$  in each odd period.
- (ii) In each odd period, the malicious respondent accepts any split that gives her at least 1-x<sub>1\*.</sub> She refuses a smaller offer and counter-offers [1-y\*,y\*] in each even period where  $y^* = \frac{1 \delta + \frac{\delta \kappa}{1 + \kappa}}{1 \delta^2}$ .
- (iii) In any even period, the non-malicious player accepts any split that gives him at least 1-y\*, refusing a smaller share and counter-offering in the next period as specified in (i).

We check the behavior of the malicious player, player 2's behavior in odd periods, holding the rest of the equilibrium strategies fixed. By accepting 1-x<sub>1</sub>\* in any odd period t, she obtains a period t utility of  $(1 - x_1)(1 + \kappa)$ , inclusive of malice. However, if she refuses, she obtains a malice utility of  $\kappa$  by depriving the non-malicious player of any share in that period; her counter-offer in period t+1 is accepted (by (iii)), giving her a utility of y\*(1+ $\kappa$ ), discounted by one period, so that player 2 can get  $\delta y^*(1+\kappa)+\kappa$  in period t terms. However,  $\delta y^*(1+\kappa)+\kappa = \frac{\delta(1-\delta)+\frac{\kappa}{1+\kappa}}{1-\delta^2} = (1-x_1)(1+\kappa)$  and therefore the malicious player is indifferent between acceptance and refusal. She therefore gains nothing by refusing the offer 1-x<sub>1</sub>\*. Moreover, she always refuses a smaller offer as by doing so, she can obtain  $\delta y^*(1+\kappa)+\kappa = (1-x_1*)(1+\kappa)$ .

We now check the malicious player's behavior in even periods. Given (iii), offering player 1 1-y\* in any even period t is accepted, giving her y\* and a malice-inclusive period t utility of  $y^*(1+\kappa)$ . She has no incentive to offer more, as that would also be accepted, lowering her own payoff. If she offers less, her offer is refused, from (iii), so that she obtains

 $\delta(1-x_{1*})(1+\kappa)+\kappa$  in period t terms (she obtains malice utility by delaying agreement one period, and in t+1 she accepts player 1's equilibrium proposal, to which the discount factor is applied). Now  $\delta(1-x_{1*})(1+\kappa)+\kappa < y^*(1+\kappa)=(1-\delta x_{1*})(1+\kappa)$  as long as  $\kappa<(1-\delta)/\delta$ . Thus, she has no incentive to deviate from her equilibrium offer in even periods.

The rest of the steps mimic the proof of Proposition 1. QED

Two-sided malice can easily be accommodated in the continuous malice framework, and the results remain qualitatively similar.

**Proposition 5.** In an infinite horizon alternating offers game with "continuous" two-sided malice, immediate agreement on the split  $\left[\frac{1-\delta-\frac{\kappa}{1+\kappa}+\delta\frac{\mu}{1+\mu}}{1-\delta^2},\frac{\delta(1-\delta)+\frac{\kappa}{1+\kappa}-\delta\frac{\mu}{1+\mu}}{1-\delta^2}\right] = [x^*,1-x^*]$  can be supported as a SPNE provided  $\frac{\kappa}{1+\kappa} + \frac{\mu}{1+\mu} < 1-\delta$ .

The proof is available on request.

From Proposition 5, it is clear that even in the alternative formulation, two-sided malice may cause the proposer's share to either increase or decrease relative to the traditional Rubinstein shares, that for equally malicious parties, the shares become more equal, and that the first mover's advantage may be completely eroded and even reversed (specifically, the proposer gets a smaller share than the respondent if  $\frac{\kappa}{1+\kappa} - \delta \frac{\mu}{1+\mu} > \frac{(1-\delta)^2}{2}$ ) as in the model of section 2.2.

### 2.4 Discussion: a comparison with envy

Kohler (2013) models an infinite horizon alternating offers bargaining game between "envious" players. These players care about their own share in the pie but incur disutility if their opponent gets a larger share than they do. This disutility is due to envy (one-sided inequality aversion) and its magnitude depends on an envy parameter. Envious preferences differ from malice in some key respects. First, a malicious player(s) obtains a malice payoff even in periods when no bargain results, since the opponent does not obtain a share of the pie in these periods. This is regardless of the fact that she herself does not obtain the pie or any share of it either. This factor is absent in the case of envious preferences. Thus, under malice, players have an additional incentive to hold out and delay agreement. Secondly, a malicious player obtains malicious

satisfaction if the opponent does not obtain a share of the pie, and in the continuous formulation of section 2.3, obtains malicious satisfaction which is continuously decreasing in the opponent's share (increasing in her own share). However, unlike in the case of envy, there is no discontinuous trigger at the point where the opponent's share exceeds her own.

Our results also differ from Kohler's. Kohler finds (assuming both players are envious) that the proposer's share always exceeds half<sup>2</sup>, converging to half in the special case of equal envy parameters and equal discount factors. However, we find that with two-sided malice (both in the discontinuous and continuous malice formulations) that first mover advantage can even be reversed. It is possible for the proposer to settle for less than half the pie, even if she (as well as the respondent) is malicious. Intuitively, the difference in results stems from the fact that malicious preferences, unlike envious ones, do not get discontinuously triggered at the point  $z = \frac{1}{2}$ . As indicated in the preceding paragraph, a difference in malice also translates into a greater willingness to postpone reaching the bargain, as malice utility is obtained in each period until the pie is actually divided. Thus, if the respondent is more malicious than the proposer, the former has a correspondingly greater willingness to delay agreement, translating into a reduction in the share that the proposer is willing to settle for.

# **3.** Conclusion

This is the first paper to introduce malice into the Rubinstein alternating offers bargaining game. I label "malice" as a utility that the malicious party gets when her opponent does not obtain a share in the pie (regardless of whether she herself gets a share). In the continuous formulation of malice, I allow malice payoff to be continuously decreasing in the opponent's share (continuously increasing in one's own). I derive a unique subgame perfect Nash equilibrium of the alternating offers bargaining game with (i) one-sided malice, (ii) two-sided malice, and (iii) "continuous" malice. In general, malice confers a bargaining advantage; under one-sided malice, therefore, a malicious proposer's share increases relative to her share in the traditional Rubinstein model, while if the malicious party is the respondent, this tends to reduce the first-

 $<sup>^{2}</sup>$  This is so even if the proposer is envy-free and the respondent is envious. Contrast this case with Observation 1 of our model.

mover advantage of the traditional model and may completely erode and even reverse it. With two-sided malice, the proposer's share may either increase or decrease relative to the traditional model. If both players are equally malicious, the shares become more equal than under the traditional model. However, it is possible for first mover advantage to be completely eroded and reversed even in the case of two-sided malice, so that the proposer can get less than half the pie. This contrasts sharply with the case of envious preferences.

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