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Abstract

In economically developed countries, aging societies with fewer children are progressing. Increased longevity has necessitated postponement of the working retirement age. Our paper presents an examination of how subsidies for an elderly labor supply affect the elderly labor supply. Our paper presents derivation that this subsidy raises the elderly labor supply. Then, the wage rate of younger laborers can be increased because of complementarity between the younger labor supply and older labor supply. This effect is explained as an externality. By virtue of the externality effect, a subsidy to facilitate the elderly labor supply should be provided in support of social welfare.

Keywords: Aging society, Elderly labor, Subsidy

JEL Classifications: J14, J26, H20, H55

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1. Introduction

A low birthrate and an aging society are progressing rapidly in Japan. The ratio of older people to the total population in 2016 has reached 27%.[1] Japan and other economically developed countries are suffering from the effects of the aging situation.

An aging society with fewer children presents several problems, one of which is the social security system. Social security benefits such as pensions and medical care are supported mainly by payments made by younger people. The total fertility rate in 2015 was 1.46.[2] Although the total fertility rate has been increasing in recent years, a rate of 2.07 must be maintained to keep the population size in Japan. It goes without saying that the population size in Japan is decreasing.

The government in Japan cut pension benefits as a pension reform measure. Concretely, the start age of pension benefits was raised to 65 years. Some OECD countries such as Germany and the U.S.A. have increased the starting age of pension benefits because the government is concerned about pension funding shortages.

From Fig. 1, which presents the employment rate of older workers of population aged 55–64, it is apparent that the older people actively supply labor in many OECD countries. Especially, 55–64 aged people in Japan work actively. The labor participation rate of older people older than 65 years is about 20%.[3]

[Insert Fig. 1 around here.]

The government of Japan facilitates an elderly labor supply. The law “Concerning Stabilization of Employment of elderly” demands an increase of the retirement age, the abolition of mandatory retirement, and continued employment for firms. Moreover, based on the employment insurance act in Japan, the elderly labor supply is provided a subsidy if the wage rate of elderly labor is less than a certain level. The government promotes an increase in the elderly labor supply with these acts.

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[1]Data: Statistics Japan.
[2]Data: Ministry of Health, Labour and Welfare, Japan.
[3]Data: OECD Statistics.

Our paper presents an examination showing that the subsidy for an elderly labor supply affects the elderly labor supply. A model is set in which final goods are produced by inputting capital stock and labor provided by younger people and older people. Our paper presents a derivation demonstrating that this subsidy increases the elderly labor supply. Then, the wage rate of younger labor supply can be raised because of complementarity between the younger labor supply and older labor supply. This effect is explained as an externality. By virtue of the externality effect, a subsidy to facilitate the elderly labor supply is expected to be provided for social welfare.

Some analyses have specifically addressed the elderly labor supply. Feldstein (1977) and Hu (1979) set the endogenous retirement and examine a partial equilibrium model. Hu (1979) examines how public pensions affect the retirement age. Matsuyama (2008) sets an Overlapping Generations Model (OLG) and examines a retirement decision. Matsuyama (2008) shows that an increase in savings hastens the retirement age because an increase in savings will engender an increase in capital, which increases the next generation's wage income and income growth. Aísa, Pueyo and Sanso (2012) derive that an increase in longevity raises the participation rate of labor supply by older people. Kangasharju (2007) examines empirically that the subsidy for elderly labor can raise the labor supply of older people. For instance, the subsidy policy is more efficient than a policy of direct income transfer to older people, as demonstrated through empirical research by Acs and Toder (2007).

If the retirement age is late and older people can obtain sufficient income to live, financial resources of a pay-as-you-go pension are affordable because the government can cut the pension benefit. Butrica, Smith, and Steuerle (2006) demonstrate that retirement age extension has a positive effect on incomes and pensions. However, Lachance (2007) derives a negative correlation between retirement age and social welfare. Mizuno and Yakita (2013) consider heterogeneity among individuals and derive that the timing of retirement affects the total fertility rate. Cocco and Gomes (2012) study longevity risk effects on individual saving and retirement decisions. Christian, Keuschnigg, and Keuschnigg (2010) examine the impact for retirement behavior of policy such as lengthening the calculation period and moving from wage to price indexation of benefits in pension system. Echevarria and Iza (2006) elucidate the relation between life expectancy and the growth rate of income per capita. Higher life expectancy lowers per capita GDP growth because the share of the active population decreases and the burden for social security increases.

Is the elderly labor supply substitutive for a supply of young laborers? As demonstrated by Ohta (2007), this is either substitutive or complementary. However, substitution is observed in the construction industry. In addition, the substitution is observed in a part time job, too, which does not mean that the substitution is generally observed in all industries and all work. OECD average data show that both young labor participation and old labor participation increase together.[4] These results show partial complementarity between young and old workers.

Some reasons can explain the complementarity. For example, intergenerational skill inheritance by young workers from old workers raises the productivity of young workers. In the field of human capital theory, Glomm and Ravikumar (1992), who conducted fundamental studies related to human capital, considers that human capital as labor productivity depends not only on educational investment but also on the level of human capital of the prior generation. Therefore, complementarity between young and old workers can be considered. Our paper presents consideration of complementarity between young and old workers and derives how the subsidy for elderly labor affects the labor force participation of older people.

The remainder of this paper includes the following. Section 2 sets the model and derives the equilibrium in section 3. Section 4 presents an examination of how the policy affects the elderly labor supply. The optimal subsidy policy to maximize social welfare is derived as explained in section 5. The final section concludes the research described in this paper.

2. The Model

[4]Data: OECD Data "Labour force participation rate." In the OECD average, the labor participation of 25–54 years is 80.5% in 2001, which rises to 81.59% in 2015. The labor participation of 55–64 years is 50.45% in 2001, which rises to 61.08% in 2015.

This model economy has agents of three types: households, firms, and government. The individuals of households live in two periods: a young period and old period. Our paper presents consideration of an overlapping generations model. Then the younger people and older people coexist in any t period.

2.1 Household

Households derive utility from consumption in young and old periods and leisure in the old period. Our paper assumes the following log utility function u_t, shown as

$$u_t = \alpha ln c_{1t} + \beta ln c_{2t+1} + (1-\alpha-\beta) ln l_{t+1}, 0 < \alpha, 0 < \beta, \alpha + \beta < 1, \tag{1}$$

where c_{1t} and c_{2t+1} respectively denote consumption in the young period and in the old period. Also, l_{t+1} denotes leisure time in the old period.

Households in the young period provide the labor supply inelastically to obtain a wage. The younger people must pay a tax and allocate disposable income to consumption in the young period and saving to consume in the old period. Then, the budget constraint in the young period is shown as

$$c_{1t} + s_t = (1-\tau) w_{1t} - T_t, \tag{2}$$

where s_t denotes savings. w_{1t} represents the wage rate of younger people. τ and T respectively stand for the income tax rate and lump-sum taxation. A lump-sum tax is used to finance the employment policy for older people. We assume $0 < \tau < 1$ and $0 < T_t$.

In the old period, the older people have a unit of time and enjoy leisure time and provide a labor supply. If the older people enjoy leisure time l_{t+1}, then the older people provides labor time $1 - l_{t+1}$.[5] The government levies the income tax for older people to finance pension benefits. However, the government provides a subsidy for the labor supply of older people to facilitate the labor supply. Older people can obtain the labor income, capital income, and pension benefits. These incomes are consumed during the old period. No bequest occurs, by assumption. Then, the budget constraint during the old period is shown as

[5] We consider the labor time $1 - l_{t+1}$ as retirement timing. If the labor time $1 - l_{t+1}$ is small, then we consider that retirement year old is earlier.

$$c_{2t+1} = (1 + r_{t+1})s_t + p_{t+1} + (1 - \varepsilon)(1 + \sigma)w_{2t+1}(1 - l_{t+1}). \tag{3}$$

In that equation, $1 + r_{t+1}$ denotes the interest rate. ε and σ respectively denote the income tax rate for older people and the subsidy rate for the labor supply of older people. w_{2t+1} denotes the wage rate of older people. p_{t+1} stands for the pension benefit. Income tax rate ε is levied to finance pension benefits. The pension system has aspects of inter-generation and intra-generation income redistribution.[6]

Considering (2) and (3), the lifetime budget constraint is obtainable as

$$c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = (1 - \tau)w_{1t} - T_t + \frac{(1 - \varepsilon)(1 + \sigma)w_{2t+1}(1 - l_{t+1})}{1 + r_{t+1}} + \frac{p_{t+1}}{1 + r_{t+1}}. \tag{4}$$

The household's optimal allocations to maximize the utility function (1) subject to the budget constraint (4) are derived as

$$c_{1t} = \alpha I, \tag{5}$$

$$c_{2t+1} = \beta(1 + r_{t+1})I, \tag{6}$$

$$l_{t+1} = \frac{(1 - \alpha - \beta)(1 + r_{t+1})}{(1 - \varepsilon)(1 + \sigma)w_{2t+1}} I, \tag{7}$$

where $\mathrm{I} = (1 - \tau)w_{1t} - T_t + \frac{(1-\varepsilon)(1+\sigma)w_{2t+1}}{1+r_{t+1}} + \frac{p_{t+1}}{1+r_{t+1}}$.

2.2 Firm

The firm produces the final goods by inputting capital stock and labor. The production function is assumed as[7]

$$Y_t = K_t^{\theta}\left(\gamma L_t^{\rho} + \delta L_{t-1}^{\rho}\right)^{\frac{1-\theta}{\rho}}, 0 < \theta < 1, \rho < 1. \tag{8}$$

Therein, Y_t stands for the output of final goods. K_t denotes the capital stock. L_t and L_{t-1} respectively represent the labor input of younger people and that of older people.

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[6]In Japan, older workers younger than seventy years old must pay premiums for a pension as the rule of public pensions.

[7]With $\rho = 1$, we obtain the production function $Y_t = K_t^{\theta}(\gamma L_t + \delta L_{t-1})^{1-\theta}$. This production function is considered as a perfectly substitutive form. Then, the wage rates of young workers and old workers are constant in a small open economy. With $\rho = 0$, we obtain the product function $Y_t = K_t^{\theta} L_t^{\gamma(1-\theta)} L_{t-1}^{(1-\gamma)(1-\theta)}$. Then, the wage rates of young workers and old workers depend on the labor supply of older people and the population growth rate, as shown by (11) and (12). Our paper presents consideration of different cases including a substitutive case $(0 < \rho < 1)$ and a complementary case $(-\infty < \rho < 0)$, except for perfect substitution and perfect complementarity.

This production function, which is formed as a Cobb–Douglas function between K_t and $(\gamma L_t^{\rho} + \delta L_{t-1}^{\rho})^{\frac{1}{\rho}}$, is formed as a Constant Elasticity of Substitution (CES) function between L_t and L_{t-1}. Our paper presents consideration of the substitution level of younger laborers and older laborers to examine details of how the labor demand for older people affects the demand for younger people. Setting profit function π_t as $\pi_t = Y_t - (1 + r_t)K_t - w_{1t}L_t - w_{2t}L_{t-1}$ and assuming perfect competition and full capital stock depreciation in a period, profit maximization for capital stock K_t produces the following equation.

$$1 + r_{t+1} = \theta k_t^{\theta-1} \tag{9}$$

In that equation, the following variable is used.

$$k_t = \frac{K_t}{(\gamma L_t^{\rho} + \delta L_{t-1}^{\rho})^{\frac{1}{\rho}}} \tag{10}$$

Profit maximization of the labor input of younger people L_t derives the following equation.

$$w_{1t} = \gamma(1-\theta)k_t^{\theta}\left(\gamma + \delta\left(\frac{1-l_t}{n}\right)^{\rho}\right)^{\frac{1}{\rho}-1} \tag{11}$$

In that equation, n signifies the gross population growth rate. Assuming N_t and N_{t-1} as population size of young generation and old generation, respectively, the gross population growth rate is $\mathrm{n} = \frac{N_t}{N_{t-1}}$.[8] The labor input of older people changes the marginal productivity of the labor of younger people and wage rate w_{1t}. Profit maximization of the labor input of older people L_{t-1} derives the following equation.

$$w_{2t} = \delta(1-\theta)k_t^{\theta}\left(\gamma\left(\frac{n}{1-l_t}\right)^{\rho} + \delta\right)^{\frac{1}{\rho}-1} \tag{12}$$

An increase in the population of younger people or a decrease in labor input of older people raises the marginal productivity of labor of older people and wage rate w_{2t}. Considering (11) and (12), the wage rate ratio is derived as

$$\frac{w_{1t}}{w_{2t}} = \frac{\gamma}{\delta}\left(\frac{1-l_t}{n}\right)^{1-\rho}. \tag{13}$$

[8]Then, we obtain $L_t = N_t$ because younger people provide labor inelastically. However, older people provide $1 - l_t$ of labor time. We obtain $L_{t-1} = (1 - l_t)N_{t-1}$.

An increase in the population growth rate reduces the wage ratio $\frac{w_{1t}}{w_{2t}}$ because of an increase in labor supply by younger people.

2.3. Government

The government provides a subsidy for the labor supply by older people and a pension benefit for older people. The subsidy for the labor supply by older people is financed by lump-sum tax T_t from younger people. With a balanced budget, the following equation is obtained.

$$T_t = \frac{\sigma w_{2t}(1 - l_t)}{n}. \tag{14}$$

Therein, T_t is determined to hold the balanced budget (14).

The government provides the pension benefit for older people, which is financed by labor income taxation for younger and older people. This pension system is partially a pay-as-you-go pension because the revenue from younger people is given to older people. However, this pension is financed by older people, too: this pension system includes intergenerational and intra-generational income transfer. With a balanced budget, the following equation is obtained.

$$p_t = n\tau w_{1t} + \varepsilon w_{2t} \tag{15}$$

3. Equilibrium

This section presents derivation of the equilibrium of the model economy. For these analyses, we assume a small open economy. Then, the interest rate is given by the fixed world interest rate. Considering (9), the capital-effective labor ratio k_t given by (10) is fixed, too. w_{1t} and w_{2t} depend on l_t.

Considering (7) and (15), leisure time l_{t+1} is shown as

$$l_{t+1} = \frac{(1+r)(1-\alpha-\beta)}{(1-\varepsilon)(1+\sigma)w_{2t+1}}\left((1-\tau)w_{1t} - T_t + \frac{(1-\varepsilon)(1+\sigma)w_{2t+1}}{1+r} + \frac{n\tau w_{1t+1} + \varepsilon w_{2t+1}}{1+r}\right), \tag{16}$$

where T_t is given by (14). Therefore, given l_{t-1}, we obtain l_t and the wage rate w_{1t}, w_{2t} are obtained, too. The equilibrium is specified by the dynamic equation of l_t (16).

We consider the steady state equilibrium with $l_{t+1} = l_t = l$. Now, we assume the steady state with $T = \sigma = 0$. Then, considering (11)–(13) and (16), the leisure time at steady state l is given to satisfy the following equation.

$$(1-\varepsilon)l = (1+r)(1-\alpha-\beta)\frac{\gamma}{\delta}\left(\frac{1-l}{n}\right)^{1-\rho}\left(1+\frac{n-(1+r)}{1+r}\tau\right) + (1-\alpha-\beta) \tag{17}$$

Defining $L = (1-\varepsilon)l$ and $R = (1+r)(1-\alpha-\beta)\frac{\gamma}{\delta}\left(\frac{1-l}{n}\right)^{1-\rho}\left(1+\frac{n-(1+r)}{1+r}\tau\right) + (1-\alpha-\beta)$, we can obtain the unique steady state because there exists an intersect, as shown in Fig. 2.[9]

[Insert Fig. 2 around here.]

We obtain $\frac{dl_{t+1}}{dl_t}$ as shown below.

$$\frac{dl_{t+1}}{dl_t} = -\frac{\dfrac{(1+r)(1-\alpha-\beta)\gamma(1-\rho)(1-\tau)\left(\frac{n}{1-l}\right)^{1-2\rho}}{(1-\varepsilon)(1-l)\left(\gamma+\delta\left(\frac{1-l}{n}\right)^{\rho}\right)}}{1+\dfrac{\gamma(1+r)(1-\alpha-\beta)(1-\rho)}{\delta(1-\varepsilon)(1-l)}\left(\dfrac{\gamma(1-\tau)\left(\frac{n}{1-l}\right)^{1+\rho}}{\gamma+\delta\left(\frac{1-l}{n}\right)^{\rho}}+\dfrac{n\tau}{1+r}\left(\frac{1-l}{n}\right)^{1-\rho}\right)}. \tag{18}$$

If $\left|\frac{dl_{t+1}}{dl_t}\right| < 1$, then the steady state equilibrium is stable.

4. Policy and labor supply by older people

This section presents a description of how the subsidy for labor supplied by older people affects the wage and labor supply of older people.

Subsidy for elderly labor supply

First, we examine whether the subsidy for elderly labor supply raises the elderly labor supply or not. For simplicity, we consider the model economy without pension benefit, that is, $\tau = \varepsilon = 0$. Then, the following equation is obtained. The leisure time is given to satisfy this equation.

[9] We assume $\varepsilon < 1-\alpha-\beta$ to have the intersect of R and L.

$$\mathrm{l} = (1+r)(1-\alpha-\beta)\left(\frac{1}{1+\sigma}\frac{\gamma}{\delta}\left(\frac{1-l}{n}\right)^{1-\rho} - \frac{1}{n}\frac{\sigma}{1+\sigma}(1-l) + \frac{1}{1+r}\right) \quad (19)$$

Total differentiation by l and σ yields $\frac{dl}{d\sigma}$, shown as

$$\frac{dl}{d\sigma} = -\frac{\frac{(1+r)(1-\alpha-\beta)}{(1+\sigma)^2}\left(\frac{\gamma}{\delta}\left(\frac{1-l}{n}\right)^{1-\rho} + \frac{1-l}{n}\right)}{1+\frac{(1+r)(1-\alpha-\beta)}{n}\left(\frac{1-\rho}{1+\sigma}\frac{\gamma}{\delta}\left(\frac{1-l}{n}\right)^{-\rho} - \frac{\sigma}{1+\sigma}\right)}. \quad (20)$$

The sign of (20) is ambiguous because the sign of the denominator of (20) is ambiguous. If $\sigma = 0$, then we obtain $\frac{dl}{d\sigma} < 0$. However, graphical analysis derives $\frac{dl}{d\sigma} < 0$ for any σ. Defining $L_\sigma = \mathrm{l}$ and $R_\sigma = (1+r)(1-\alpha-\beta)\left(\frac{1}{1+\sigma}\frac{\gamma}{\delta}\left(\frac{1-l}{n}\right)^{1-\rho} - \frac{\sigma}{1+\sigma}(1-l) + \frac{1}{1+r}\right)$, one finds that an increase in σ reduces leisure time and raises the labor supply by elderly people.

[Insert Fig. 3 around here.]

These results are intuitive. An increase in subsidies for elderly labor supply increases the opportunity cost of leisure. Then, the elderly people increase the labor supply. In addition, the tax burden for subsidy policy reduces the household lifetime income and leisure, as a normal good. The elderly people raise the labor supply. Thereby, the following proposition is established.

Proposition 1

An increase in subsidies for labor supplied by older people increases the labor supply of older people.

Wage inequality at the steady state is derived by (13), as

$$\frac{w_1}{w_2} = \frac{\gamma}{\delta}\left(\frac{1-l}{n}\right)^{1-\rho}. \quad (21)$$

As shown by (21), an increase in labor supply by elderly people raises wage inequality $\frac{w_1}{w_2}$. The wage rate of younger people w_1 increases in a relative sense. This result demonstrates that the labor supplies from younger people and older people are mutually complementary. By virtue of an increase in labor supply by older people, the marginal labor productivity of younger people is raised.

An aging society with fewer children

Second, we examine how the existence of fewer children affects the labor supply by older people and intergenerational wage inequality. Concretely, we consider the decrease in the population growth rate. Considering (19), a decrease in the population growth rate makes the R_σ curve above shift in the model without pension and subsidy $\tau = \varepsilon = \sigma = 0$ because we wish to examine the direct effect of elderly labor supply, as shown in Fig. 4.

[Insert Fig. 4 around here.]

As shown in Fig. 4, a decrease in population growth reduces the labor supplied by older people. Now, with total differentiation of (17), we can confirm the result using the following calculation:

$$\frac{dl}{dn} = -\frac{\frac{\gamma(1-l)(1+r)(1-\alpha-\beta)(1-\rho)}{\delta n^2}\left(\frac{1-\rho}{n}\right)^{-\rho}}{1+\frac{\gamma(1+r)(1-\alpha-\beta)(1-\rho)}{\delta n}\left(\frac{1-\rho}{n}\right)^{-\rho}} < 0. \tag{22}$$

From total differentiation of (21) and (22), we obtain

$$\frac{d\frac{w_1}{w_2}}{dn} = \frac{1}{(1+r)(1-\alpha-\beta)}\frac{dl}{dn} < 0, \tag{23}$$

because of $\frac{dl}{dn} < 0$: the relative lack of children raises wage inequality. Then, the following proposition can be established.

Proposition 2

A decrease in the population growth rate reduces the labor supplied by older people and has the effect of increasing intergenerational wage inequality.

A decrease in the population growth rate means that the labor supplied by younger people is slight compared with the labor supplied by older people. Then, the marginal productivity of older labor supply decreases. The wage rate decreases because of complementarity between the younger labor supply and older labor supply. A decrease in the wage rate of older people decreases the opportunity cost of having leisure. Then older people increase leisure time instead of decreasing labor time.

Pension Policy

Third, we examine how the pension policy affects the labor supply by the older people. Considering (17), the effect of an increase in pension contribution rate τ on the elderly labor supply is determined, depending on $n-(1+r)$, as shown in Fig. 5. An increase in pension contribution rate ε increases leisure time and decreases the labor supply by older people, as shown in Fig. 6.

[Insert Fig. 5 around here.]

[Insert Fig. 6 around here.]

We can confirm the effect of the contribution rate of pension on the leisure time with total differentiation of (17), too.

$$\frac{dl}{d\tau}=\frac{\frac{\gamma(1-\alpha-\beta)\left(n-(1+r)\right)}{\delta}\left(\frac{1-l}{n}\right)^{1-\rho}}{1-\varepsilon+\frac{\gamma(1-\rho)(1+r)(1-\alpha-\beta)}{\delta n}\left(\frac{1-l}{n}\right)^{-\rho}\left(1+\frac{n-(1+r)}{1+r}\tau\right)}, \tag{24}$$

$$\frac{dl}{d\varepsilon}=\frac{l}{1-\varepsilon+\frac{\gamma(1-\rho)(1+r)(1-\alpha-\beta)}{\delta n}\left(\frac{1-l}{n}\right)^{-\rho}\left(1+\frac{n-(1+r)}{1+r}\tau\right)}. \tag{25}$$

Then, the following proposition can be established.

Proposition 3

An increase in pension benefits with an increase in the contribution rate for older people decreases the labor supplied by older people. An increase in the contribution rate for younger people decreases the elderly labor supply if $n>1+r$.

An increase in pension benefit raises the household lifetime income if $n>1+r$.

Then, an increase in income increases leisure time because leisure is a normal good. Moreover, an increase in the contribution rate reduces the disposable wage rate that a household can obtain, which reduces the opportunity cost of having leisure and leisure time increase.

5. Welfare

This section presents consideration of optimal subsidy policy for the elderly labor supply. We set a social welfare function to examine optimal policy. As an assumption by van Groezen, Leers and Meijdam (2003), we assume the following social welfare function, as

$$W = \sum_{s=t}^{\infty} \eta^{s-t} (\alpha lnc_{1s-1} + \beta lnc_{2s} + (1-\alpha-\beta) lnl_s), (0 < \eta < 1), \tag{26}$$

where η denotes the discount factor for each generation's utility.

We consider resource constraints in each period. Considering the income per young person as $y_t = \frac{Y_t}{L_t}$, we obtain

$$y_t = k^{\theta} \left(\gamma + \delta \left(\frac{1-l_t}{n} \right)^{\rho} \right)^{\frac{1}{\rho}}. \tag{27}$$

Then, the resource constraint in each period is derived as

$$k^{\theta} \left(\gamma + \delta \left(\frac{1-l_t}{n} \right)^{\rho} \right)^{\frac{1}{\rho}} + (nd_{t+1} - d_t) - rd_t = c_{1t} + \frac{c_{2t}}{n} + nk - k. \tag{28}$$

Therein, d_t represents the debt for foreign countries. Considering (26) and (28), we set the Lagrange equation as shown below.

$$\begin{aligned} L = \sum_{s=t}^{\infty} \eta^{s-t} (\alpha lnc_{1s-1} + \beta lnc_{2s} + (1-\alpha-\beta) lnl_s) \\ + \sum_{s=t}^{\infty} \lambda_s \left(k^{\theta} \left(\gamma + \delta \left(\frac{1-l_s}{n} \right)^{\rho} \right)^{\frac{1}{\rho}} + (nd_{s+1} - d_s) - rd_t - c_{1s} - \frac{c_{2s}}{n} \right). \end{aligned} \tag{29}$$

Then, the following allocations are derived to solve the maximization problem.

$$\frac{c_{1t}}{c_{2t}} = \frac{\alpha\eta}{\beta n}, \tag{30}$$

$$\frac{c_{1t+1}}{c_{1t}} = \frac{\eta(1+r)}{n}, \tag{31}$$

$$\frac{c_{2t+1}}{c_{1t}} = \frac{\beta(1+r)}{\alpha}, \tag{32}$$

$$\frac{c_{1t}}{l_{t+1}} = \frac{\alpha}{1-\alpha-\beta}\frac{1}{1+r}\frac{w_{1t+1}}{\gamma(1-\theta)}\left(\frac{1-l_{t+1}}{n}\right)^{\rho-1}, \tag{33}$$

With (5) and (7), we obtain $\frac{c_{1t}}{l_{t+1}}$ at the decentralized economy. Compared with (33) at the steady state, we obtain optimal subsidy rate σ as

$$\sigma = \frac{1}{(1-\theta)} - 1. \tag{34}$$

Then, the following proposition can be established.

Proposition 4

The subsidy for elderly labor supply should be provided to maximize social welfare.

This model economy contains externality in the labor market. If the younger or older labor supply increases, then the marginal productivity of the older or younger labor supply rises. However, the household does not consider the externality effect. Therefore, a subsidy policy is needed.

6. Conclusions

Our paper sets a model in which both younger people and older people supply labor. An examination is presented of whether the subsidy for the elderly labor supply facilitates the elderly labor supply or not. This subsidy raises the elderly labor supply and is expected to be provided for social welfare.

In economically developed countries, an aging society with fewer children is processing. Then, the retirement age is postponed. A trend of working during the old period is expected to be necessary to obtain sufficient income to spend during the old period. In addition, the government can cut pension benefits if older people supply labor actively. This result solves an increase in the burden for social security costs.

Moreover, by virtue of an increase in elderly labor supply, the wage rate of younger people can be raised because the marginal productivity of younger labor increases with the elderly labor supply.

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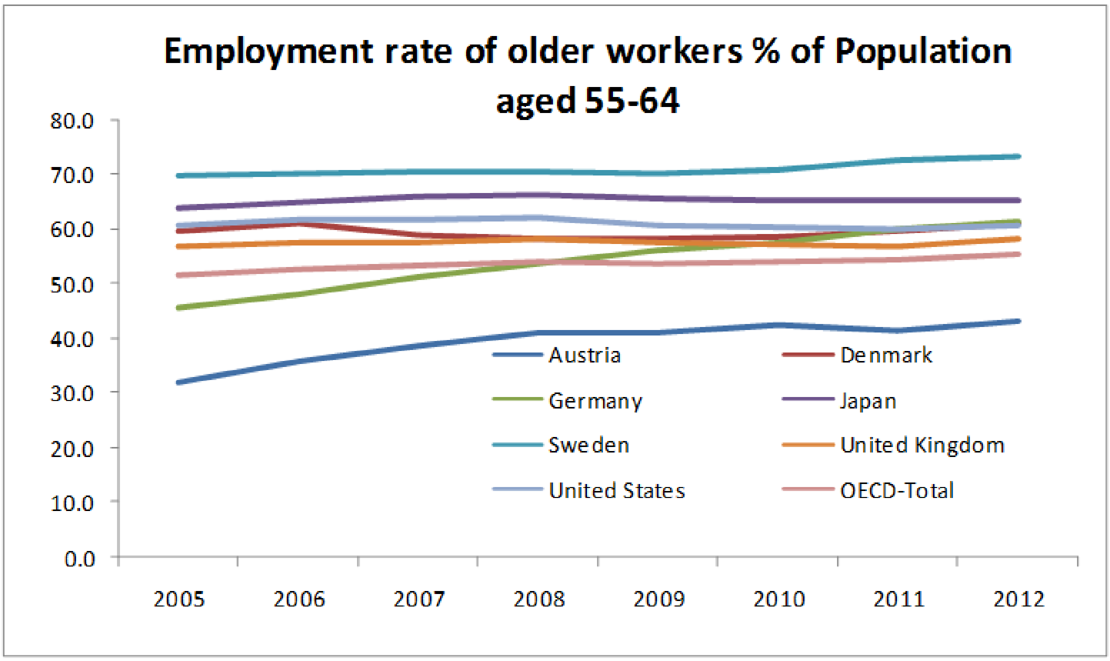

Fig. 1 Employment rate of older workers % of population aged 55–64 (Data: OECD Statistics).

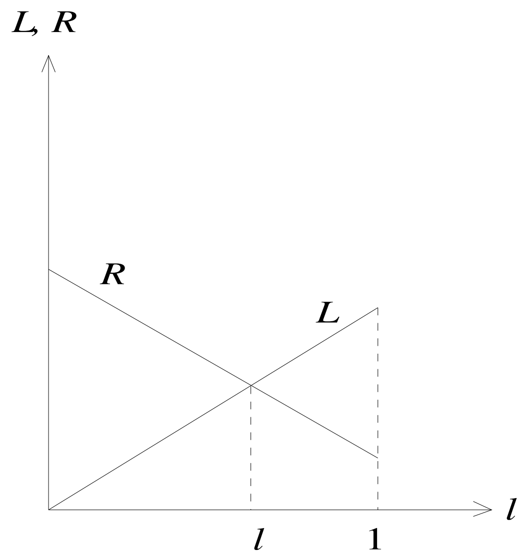

Fig. 2 Leisure time at the steady state.

R is given by the straight line at $\rho = 0$. If $\rho \neq 0$, then R is given by the curve.

However, the decreasing function at l does not change.

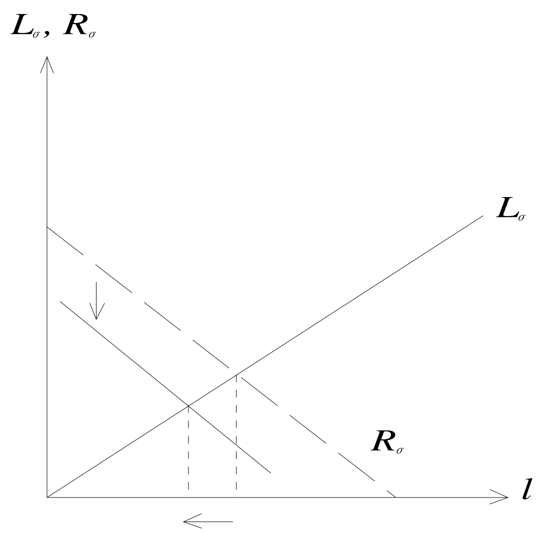

Fig. 3 Leisure time and subsidy for elderly labor supply.

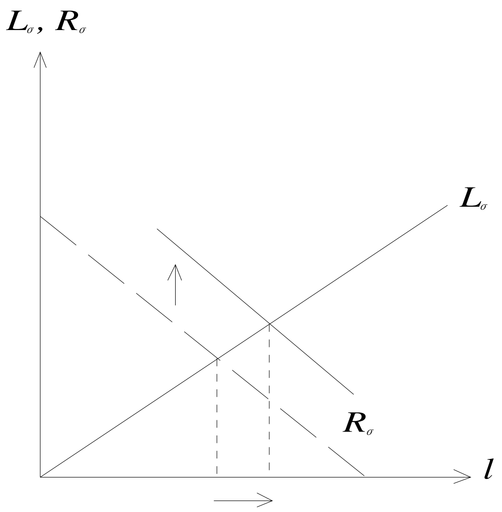

Fig. 4 Fewer children and leisure time.

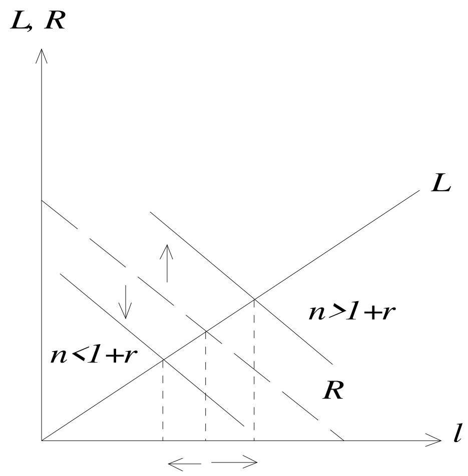

Fig. 5 Increase in contribution rate τ.

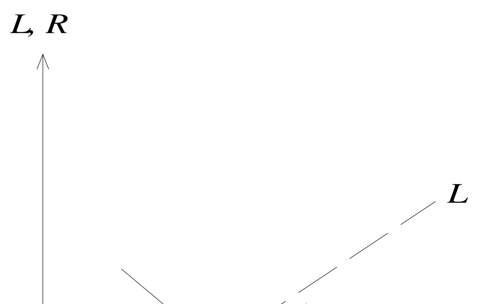

Fig. 6 Increase in contribution rate ε.