Financial Spillovers Across Countries: Measuring shock transmissions.

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Abstract

We measure volatility spread among countries and summarize it into a volatility spillover index to provide a measurement of such interdependence. Our spillover index is based on the forecast error variance decomposition (FEVD) for a VAR model at h-step ahead forecast, and we construct it using both the orthogonalized FEVD and the generalized FEVD (GFEVD); both of them provide similar results, but the generalized version is easier to handle when a data set with more than 6 variables is involved and non theory in available to impose the restrictions needed by the orthogonal version; this is true since the GFEVD does not depend on the restrictions imposed by the Choleski decomposition. This fact makes it attractive when economic theory does not fit well with variables relationship. An R package for reproducing this chapter estimations is entirely developed.

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1 Introduction

In the last three decades, financial crises have been occurring with more regularity and according to Reinhart and Rogoff (2008) and Corsetti et al. (2001) recent crises are not so different from historical ones and they even show some similarities. One of the most important facts when crises occur is that "financial market volatility generally increases and spills over acros markets" (Diebold and Yilmaz, 2012), motivated by this consideration Diebold and Yilmaz (2009, 2012) introduce a new measure based on the well-known forecast error variance decomposition from vector autoregressions to summarize such a transmission of crisis in a single number easy to interpret and also they provide several tools as spillovers tables, directional spillovers and net spillover tables to track this measurement.

Diebold and Yilmaz (2009, 2012) methodology is not concerned about distinguishing contagion from interdependence, but it is concerned about providing a toolkit to measure the proportion of a crisis from one country that spills over another country or group of countries, this feature makes it useful when a policy-maker is willing to know what country (or group of countries) is more vulnerable when another country is hit by a crisis. One outstanding fact of this method is that it does not require a formal test for contagion for being able to provide a measurement of the spillover stemming from turmoil periods (it even works for stable periods).

In spite of the fact that spillover indexes do not represent a hypothesis test for contagion, there seems to be a pattern in the index that can be useful to anticipate a crisis, which can be due to contagion or interdependence. Such a pattern consists of a deeply decay before rising, this pattern is captured by the orthogonalized and the generalized index applied both for returns and volatility, if this pattern persists in all type of crises, then the dynamic spillover index could be helpful as a early-warning system to foresee a crisis as outlined in Diebold and Yilmaz (2012)

This Chapter is organized as follows: The econometric methodology and the form of the indexes are presented in section 2, empirical results such as orthogonalized and generalized spillover indexes for both, daily returns and intraday volatilities are in section 3. This chapter concludes with some comments in section 4.

2 The base model and the Spillover Index

2.1 The VAR(p) model and its MA(∞) representation

This section is devoted to review some notation and features regarding to Sims (1980) K-variables Vector Autoregressive model of order p generally referred to as VAR(p). As this model is the workhorse for the subsequent analysis we present some definitions and preliminaries concerning the VAR(p) which has the following matrix form:

$$\mathbf{y}_t = \mathbf{v} + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \ldots + \mathbf{A}_p\mathbf{y}_{t-p} + \varepsilon_t, \qquad t = 0, 1, \ldots, \tag{1}$$

where $\mathbf{y}_t = (y_{1t}, \ldots, y_{Kt})'$ is a $K \times 1$ random vector, the $\mathbf{A}_i$ are fixed $K \times K$ coefficients matrices, $\mathbf{v} = (v_1, \ldots, v_K)'$ is a fixed $K \times 1$ vector of intercept terms allowing for the possibility of the non-zero mean. Finally, $\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Kt})'$ is a K-dimensional *white noise* or *innovation process*. For the vector ε to be *white noise* the following conditions hold: $E(\varepsilon_t) = 0$, $E(\varepsilon_t, \varepsilon_t') = \Sigma_\varepsilon < \infty$ and $E(\varepsilon_t, \varepsilon_s') = 0, \quad$ for $t \neq s$.

In order to simplify the notation and make it more tractable, let us consider the simplest version of the VAR model by assuming $p = 1$ and $K = 2$, a bivariate VAR(1) model of the form:

$$\mathbf{y}_t = \mathbf{v} + \mathbf{A}_1\mathbf{y}_{t-1} + \varepsilon_t, \qquad t = 0, 1, \ldots \tag{2}$$

The model in (2) is said to be *stable* if all eigenvalues of $\mathbf{A}_1$ have modulus less than 1, which is equivalent to

$$\det(\mathbf{I_K} - \mathbf{A}_1 z) \neq 0 \qquad \text{for } |z| \leq 1. \tag{3}$$

Under the stability condition the process $\mathbf{y}_t$ in (2) is said to be invertible and has a Moving Average of infinity order (MA(∞)) representation[1]

[1] See Lutkepohl (1993) for further details on VAR models.

$$\mathbf{y}_t = \mu + \sum_{i=0}^{\infty} \mathbf{A}_1^i \varepsilon_{t-i}. \tag{4}$$

where $\mu := (\mathbf{I_K} - \mathbf{A_1}L)^{-1}\mathbf{v}$. Such a MA($\infty$) representation requires the VAR(1) to be stable in order to turns out in a sequence of matrix coefficients being absolutely summable, this ensures the MA(∞) process converges in quadratic mean and thus in probability to $\mathbf{y}_t$ (Lutkepohl, 1993). In the MA representation, the process $\mathbf{y}_t$ is expressed in terms of the past and present error vectors ε_t and the mean term μ which can be either zero or non-zero.

The MA representation in (4) can be re-written more compactly in terms of a polynomial in the lag operator,

$$\mathbf{y}_t = \mathbf{\Phi}(L)\varepsilon_t, \tag{5}$$

where μ is assumed to be zero, $\mathbf{\Phi}(L)$ is a polynomial[2] in the lag operator such that $\mathbf{\Phi}(L) := \sum_{i=0}^{\infty} \mathbf{A}_i L^i$ and L is the lag operator such that $L^j y_t = y_{t-j} \quad \forall j \in \mathbb{N}$.

The coefficients contained in $\mathbf{\Phi}$ are the impulse responses of the system. In other words, $\phi_{jk,i}$, the jk-the element of $\mathbf{\Phi}_i$ represents the reaction of the j-th variable of the system to a unit shock (forecast error) of variable k, i periods ago, provided of course, the effect is not contaminated by other shocks to the system (Lutkepohl, 1993).

In order to aviod such "contamination", let $\mathbf{\Sigma}_\varepsilon$ be the variance-covariance matrix of the reduced form residuals resulting from estimating a VAR(p) model with $E(\varepsilon_t, \varepsilon_s') \neq 0, \quad \text{for } t \neq s$, nevertheless as long as this matrix is positive definite symmetric matrix, it can be factorized as $\mathbf{\Sigma}_\varepsilon = \mathbf{PP'}$ where $\mathbf{P}$ is the lower triangular Choleski matrix[3] and $\mathbf{P'}$ is its correspond transpose, this is the so-called Choleski orthogonalization which prevents the "contamination" of variables by shocks coming from other variables in the system and also guarantees that $\mathbf{P}^{-1}\varepsilon_t$ is now a vector of orthogonalized (independent under normality assumption) innovations, therefore $E(\mathbf{P}^{-1}\varepsilon_t, \mathbf{P}^{-1}\varepsilon_s') = 0, \quad \text{for } t \neq s$ and in general $E(\mathbf{P}^{-1}\varepsilon_t, \mathbf{P}^{-1}\varepsilon_t') = \mathbf{I_K}$ holds. The Choleski factorization allows to re-write the process (5) as:

[2]Alternatively $\mathbf{\Phi}(L) := (\mathbf{I_K} - \mathbf{A}L)^{-1}$

[3]This factorization is order-dependent, which means that there is not only a unique $\mathbf{P}$ associated to a Σ_ε, but also there are $K!$ $\mathbf{P}$'s associated to Σ_ε each of them corresponding to each specific order of the variables.

$$\mathbf{y}_t = \mathbf{\Phi}(L)\mathbf{P}\mathbf{P^{-1}}\varepsilon_t \quad (6)$$

$$= \mathbf{\Theta}(L)u_t \quad (7)$$

Where $\mathbf{\Theta}(L) = \mathbf{\Phi}(L)\mathbf{P}$ and $\mathbf{u}_t = \mathbf{P}^{-1}\varepsilon_t$, being $\mathbf{P}$ the unique lower-triangular Choleski factor of the covariance matrix of ε_t for a given variable ordering. This transformation ensures $E(\mathbf{u}_t\mathbf{u}_t') = \mathbf{I}$ as mentioned above by imposing a recursive causal structure from the top variables to the bottom variables but not the other way around.

The advantage of represent a VAR(p) model as an MA(∞) model consists of its easiness to determine autocovariances and forecast error variance decomposition which is the target of the next section.

2.2 Orthogonalized Forecast Error Variance Decomposition

The MA(∞) representation (7) with orthogonal white noise is suitable to collect all the variances (for each variable k) when forecasting with the VAR and then properly account for by its contribution to the total variance produced by the whole system, that is variance decompositions allow us to split the forecast error variances of each variable into parts attributable to the various system shocks.

Relying on (7), the error of the optimal h-step ahead forecast is

$$\mathbf{y}_{t+h} - \mathbf{y}_t(h) = \sum_{i=0}^{h-1} \mathbf{\Theta}_i u_{t+h-i} \quad (8)$$

where $\mathbf{y}_{t+h}$ is the realization of the random vector at time $t+h$, whereas $\mathbf{y}_t(h)$ is the expectation of the process conditional on the information set available up to time t, denoted by $E(\mathbf{y}_{t+h}|\Im_t)$ and also frequently denoted by $\mathbf{y}_{t+h,t}$ which is a function of h.

Denoting the mn-element of $\mathbf{\Theta}_i$ by $\theta_{mn,i}$, the h-step forecast error of the j-th component of $\mathbf{y}_t$ is

$$y_{j,t+h} - y_{j,t}(h) = \sum_{i=0}^{h-1}(\theta_{j1,i}u_{1,t+h-i} + \ldots + \theta_{jK,i}u_{K,t+h-i}) \tag{9}$$

$$= \sum_{k=1}^{K}(\theta_{jk,0}u_{k,t+h} + \ldots + \theta_{jk,h-1}u_{k,t+1}). \tag{10}$$

Thus, the forecast error of the j-th component potentially consists of innovations of all other components of $\mathbf{y}_t$ as well. Of course, some of the $\theta_{mn,i}$ may be zero, due to the orthogonalization, so that the innovations of some components may not appear in (10). Note that, due to the orthogonalization, $u_{k,t}$ are uncorrelated and have variance one, hence the Mean Squared Error (MSE) associated to the prediction, $y_{j,t}(h)$ is

$$E(y_{j,t+h} - y_{j,t}(h))^2 = \sum_{k=1}^{K}(\theta_{jk,0}^2 + \ldots + \theta_{jk,h-1}^2). \tag{11}$$

Therefore

$$\theta_{jk,0}^2 + \theta_{jk,1}^2 + \ldots + \theta_{jk,h-1}^2 = \sum_{i=0}^{h-1}(e_j'\mathbf{\Theta}_i e_k)^2, \tag{12}$$

is sometimes interpreted as the contribution of innovations in variable k to the forecast error variance or MSE of the h-step ahead forecast of variable j (Lutkepohl, 1993). Here e_k is the k-th column of $\mathbf{I_K}$. Dividing (12) by

$$\text{MSE}\left[(y_{j,t}(h))\right] = \sum_{i=0}^{h-1}\sum_{k=1}^{K}\theta_{jk,i}^2,$$

gives the decomposition

$$\tilde{\alpha}_{jk,h}^{o} = \frac{\sum_{i=0}^{h-1}(e_j'\mathbf{\Theta}_i e_k)^2}{\text{MSE}\left[(y_{j,t}(h))\right]} = \frac{\sum_{i=0}^{h-1}(e_j'\mathbf{\Theta}_i e_k)^2}{\sum_{i=0}^{h-1}\sum_{k=1}^{K}\theta_{jk,i}^2} \tag{13}$$

which is the proportion of the h-step ahead forecast error variance of variable j accounted for by innovations in variable k. In this way the forecast error variance is decomposed into component accounted for by innovations in the different variables of the system. From (8) the h-step ahead MSE matrix is

$$\Sigma_y(h) = \mathrm{MSE} = [(y_t(h))] = \sum_{i=0}^{h-1} \Theta_i \Theta_i' = e_j' \Phi_i \Sigma_\epsilon \Phi_i' e_j \tag{14}$$

The diagonal elements of this matrix are the MSE of the y_{jt} variables which may be used in (13), consequently the full expression is

$$\tilde{\alpha}^o_{jk,h} = \frac{\sum\limits_{i=0}^{h-1} (e_j' \boldsymbol{\Theta}_i e_k)^2}{\sum\limits_{i=0}^{h-1} e_j' \Phi_i \Sigma_\epsilon \Phi_i' e_j} \tag{15}$$

So far, it is an easy matter to realize that forecast error variance decomposition answers the questions: What fraction of the h-step ahead error variance in forecasting y_j is due to shocks to y_k?

2.3 Generalized Forecast Error Variance Decomposition

As subsection 2.2 shows, the Orthogonalized Error Variance Decomposition (OFEVD) at h-step ahead forecast horizon lies on the structure of the impulse-response of the system, the Generalized Forecast Error Variance Decomposition (GFEVD), also lies on the same idea. The former decomposition needs an ordering-based orthogonalization procedure to ensure zero correlation between the errors and allows to claim *"ceteris paribus"* when analyzing economics relationships, whereas, the latter does not need such procedure, instead of controlling the impact of correlation among residuals, Generalized Impulse-Response Function (GIRF) follows the idea of nonlinear impulse response function and compute the mean impulse response function. When one variable is shocked, other variables also vary as is implied by the covariance which is not diagonal. GIRF computes the mean of the responses by integrating out all other shocks (Pesaran and Shin, 1998).

Using (5) and defining the GIRF as:

$$GI_y(h, \delta_j, \Im_{t-1}) = E(y_{t+h}|\varepsilon_{jt} = \delta_j, \Im_{t-1}) - E(y_{t+h}|\Im_{t-1}), \tag{16}$$

which means that instead of shocking all elements in ε, only the j-th element is shocked and the effect of other shocks is integrated out assuming an observed distribution of the errors. Assuming the errors follows a multivariate normal distribution, Koop et al. (1996) show

$$E(\varepsilon_t|\varepsilon_{jt} = \delta_j) = (\sigma_{1j}, \sigma_{2j}, \ldots, \sigma_{Kj})'\sigma_{jj}^{-1}\delta_j = \mathbf{\Sigma}_\varepsilon e_j \sigma_{jj}^{-1}\delta_j. \tag{17}$$

Hence, the $K \times 1$ vector of the unscaled GIRF of the effect of a shock in the j-th equation at time t on t_{t+h} is given by

$$\left(\frac{\mathbf{\Phi}_h \mathbf{\Sigma}_\varepsilon e_j}{\sqrt{\sigma_{jj}}}\right)\left(\frac{\delta_j}{\sqrt{\sigma_{jj}}}\right), \qquad h = 0, 1, 2, \ldots \tag{18}$$

And the scaled GIRF is obtained by setting $\delta_j = \sqrt{\sigma_{jj}}$

$$\psi_j^g(h) = \sigma_{jj}^{-\frac{1}{2}} \mathbf{\Phi}_h \mathbf{\Sigma}_\varepsilon e_j \qquad h = 0, 1, 2, \ldots, \tag{19}$$

which measures the effect of one standard error shock to the j-equation at time t on expected values of $\mathbf{y}$ at time $t + h$.

Finally, the GIRF can be used to define the GFEVD which has the same interpretation as the OFEVD, namely, is the proportion of the h-step ahead forecast error variance of variable j which is accounted for by the innovations in variable k in the VAR. Denoting the GFEVD by $\alpha_{jk,h}^g$ we have

$$\alpha_{jk,h}^g = \frac{\sigma_{jj}^{-1} \sum_{i=0}^{h-1} (e_j' \mathbf{\Phi}_i \Sigma_\epsilon e_k)^2}{e_j' \Phi_i \Sigma_\epsilon \Phi_i' e_j} \tag{20}$$

Note that by construction $\sum_{k=1}^K \tilde{\alpha}_{jk,h}^o = 1$ in (15) . However, due to the non-zero covariance between the original (non-orthogonalized) shocks, in general $\sum_{k=1}^K \alpha_{jk,h}^g \neq 1$ (Pesaran and Shin, 1998), but we can normalize $\alpha_{jk,h}^g$ by dividing it by the row sum and redefined as $\tilde{\alpha}_{jk,h}^g$ to be

$$\tilde{\alpha}^g_{jk,h} := \frac{\alpha^g_{jk,h}}{\sum\limits_{k=1}^{K} \alpha^g_{jk,h}}.$$

Note that, by construction, now $\sum_{k=1}^{K} \tilde{\alpha}^g_{jk,h} = 1$ and $\sum_{j,k=1}^{K} \tilde{\alpha}^g_{jk,h} = K$

2.4 Total Spillover Index

Diebold and Yilmaz (2009, 2012) introduced the *spillover index* or the *cross-variance shares index* to be the fractions of the h-step ahead error variances in forecasting y_j due to shocks to y_k for $j, k = 1, 2, \ldots, K$ and $j \neq k$ and *own variance shares* to be the fractions of the h-step ahead error variances in forecasting y_j due to shocks to y_k for $j = k$. To make this idea clearer, let us allocate all the elements of $\tilde{\alpha}^o_{jk,h}$ and $\tilde{\alpha}^g_{jk,h}$ into a matrix structure and denote them by Λ^o_h and Λ^g_h, respectively, where both matrices are of dimension $K \times K$,

$$\Lambda^i_h = \begin{pmatrix} \tilde{\alpha}^i_{11,h} & \tilde{\alpha}^i_{12,h} & \ldots & \tilde{\alpha}^i_{1K,h} \\ \tilde{\alpha}^i_{21,h} & \tilde{\alpha}^i_{22,h} & \ldots & \tilde{\alpha}^i_{2K,h} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\alpha}^i_{K1,h} & \tilde{\alpha}^i_{K2,h} & \ldots & \tilde{\alpha}^i_{KK,h} \end{pmatrix}, \qquad i = \text{o, g}. \tag{21}$$

Thus, the spillover index is the cross-variance shares obtained from (21) and it is denoted by S^i_h, the superscript i denotes we are referring to whether the orthogonalized ($i = \text{o}$) or the generalized ($i = \text{g}$) forecast error variance decomposition and h denotes the number of steps ahead of the forecast.

$$S^i_h = \frac{\sum\limits_{\substack{jk=1 \\ j\neq k}}^{K} \tilde{\alpha}^i_{jk,h}}{K} \times 100 \qquad i = \text{o, g}. \tag{22}$$

In order to look for the idea behind (22), let us consider the simplest case where $h = 1$ and $K = 2$, this means a spillover index based on a bivariate VAR with 1 step ahead forecast, furthermore, suppose we rely on the OFEVD (when $i = \text{o}$) and recall (13), therefore Λ^i_h boils out to Λ^o_1 , we have

$$\Lambda_1^o = \begin{pmatrix} \tilde{\alpha}_{11,1}^o & \tilde{\alpha}_{12,1}^o \\ \tilde{\alpha}_{21,1}^o & \tilde{\alpha}_{22,1}^o \end{pmatrix},$$

there are two possible spillovers in this simple example: y_{1t} shocks that affect the forecast error variance of y_{2t} with relative contribution $\tilde{\alpha}_{21,1}^o$ and y_{2t} shocks that affect the forecast error variance of y_{1t} with relative contribution $\tilde{\alpha}_{12,1}^o$, therefore, the *Spillover Index* is

$$S_1^o = \frac{\tilde{\alpha}_{12,1}^o + \tilde{\alpha}_{21,1}^o}{2} \times 100,$$

where $\tilde{\alpha}_{21,1}^o = \frac{\theta_{21,1}^2}{\theta_{21,1}^2 + \theta_{22,1}^2}$ and $\tilde{\alpha}_{12,1}^o = \frac{\theta_{12,1}^2}{\theta_{11,1}^2 + \theta_{12,1}^2}$ (see (13)) and 2 in the denominator follows from the fact that $\sum_{k=1}^2 \tilde{\alpha}_{jk,h}^o = 1$ by construction, therefore $\sum_{j,k=1}^2 \tilde{\alpha}_{jk,h}^o = 2$.

For obtaining the spillover index based on the GFEVD, the steps are the same. Consider we now have Λ_1^g with $\tilde{\alpha}_{jk,1}^g$ as its typical element, then spillover index is

$$S_1^g = \frac{\tilde{\alpha}_{12,1}^g + \tilde{\alpha}_{21,1}^g}{K} \times 100,$$

where $\tilde{\alpha}_{12,1}^g = \frac{\alpha_{12,1}^g}{\alpha_{11,1}^g + \alpha_{12,1}^g}$ and $\tilde{\alpha}_{21,1}^g = \frac{\alpha_{21,1}^g}{\alpha_{21,1}^g + \alpha_{22,1}^g}$ and $\tilde{\alpha}_{jk,1}^g$ is defined in (20).

It is worthy to highlight from (20), the spillover index has the same specification either for the OFEVD or GFEVD, the only difference between them is the way how $\tilde{\alpha}_{jk,h}$ is computed. Furthermore, the *total spillover* index measures the contribution of spillovers of shocks across financial markets to the total forecast error variance (Diebold and Yilmaz, 2012).

In spite of the fact that spillover index based either on the OFEVD or GFEVD has the same form, it is clear that the orthogonalized spillover requires the Choleski factorization which depends on the order of the variables in the VAR model, therefore, to make such a factorization we need to impose a causality restriction to identify the directionality of the shocks, this fact can be seen whether as an advantage or a disadvantage; it is an advantage when we have an economic theoretical framework to impose restrictions on the directionality of the shocks, if so, then Choleski factorization is the tool to handle and extract that directionality, hence we can claim about *directionality* and *causality* in terms of shocks. On the contrary, when such theoretical framework is absent, we are not able to claim neither directionality nor causality and identification through Choleski decomposition is not reachable anymore, nevertheless, the generalized spillover index overcome by providing the effects of shocks to variable k that affect variable j by integrating out all the effects as described above.

According to Diebold and Yilmaz (2012) the advantages of the GFEVD over the orthogonalized OFEVD are clear:

1. It allows to estimate a number of spillover alternatives at a lower computational cost, because we do not need to estimate $\mathbf{P}$ any more.
2. We will not require any theoretical restrictions for identifying the forecast error variance decomposition.
3. It enables us to provide a richer analysis due to the variety of volatility spillover indexes.
4. Directional spillovers and net spillovers are reachable now.
5. Volatility and return spillovers tables do make sense and are more informative than those ones based on OFEVD[4]

All these assertions, mentioned above, are inconclusive since GFEVD does not allow to identify directionality of the shocks; reduced form residuals are still correlated in the general framework of Pesaran and Shin (1998) making impossible to disentangle the idiosyncratic shock from common shocks in the system modeled by the VAR approach. A simple simulation exercise shows that the directionality of the spillover from country j to country k with $j \neq k$ under the GFEVD is not identified.

One alternative strategy to use when no theory is available to impose the restrictions in $\mathbf{P}$ is to compute all the $K!$ possible $\mathbf{P}$'s to cover all the possibilities and then take the mean from all Λ_h^o generated by this highly cost computational procedure, which yiels $\bar{\alpha}_{jk,h}^o$ as the typical element of $\bar{\Lambda}_h^o$; the other alternative is just estimate a certain number out of

[4] Those tables based on orthogonalized fevd do not provide information about directional patterns of transmission among variables.

the $K!$, instead of all $K!$ and again take the mean from all the new Λ_h^o generated, however this constitutes a methodological limitation (Diebold and Yilmaz, 2012).

It is worthy to point out that the so-called *"directional spillovers"* (Diebold and Yilmaz, 2012) are only attainable when the researcher have a theoretical framework for the Choleski decomposition. Once the researcher identifies the directionality and proceeds to apply the orthogonalization, then she already is able to claim *directionality* in the spillover spread, hence *directional spillovers* make sense, otherwise, when directionality is not reachable, neither directional spillovers are.

2.5 Directional and Net Spillovers

Directional spillovers measure the spillover received by country j from all other countries k,

$$S^o_{j\cdot,h} = \frac{\sum_{\substack{k=1 \\ k\neq j}}^{K} \tilde{\alpha}^o_{jk,h}}{K} \times 100$$

and the spillover transmitted by country j to all other countries k is

$$S^o_{\cdot j,h} = \frac{\sum_{\substack{k=1 \\ k\neq j}}^{K} \tilde{\alpha}^o_{kj,h}}{K} \times 100$$

One can think of the set of directional spillovers as providing a decomposition of the total spillovers to those coming from (or to) a particular source (Diebold and Yilmaz, 2012).

Note that directional spillovers require the identification of $\mathbf{P}$. Once the researcher is able to estimate the directional spillovers, she is also able to account for the *net spillovers*, namely the difference between the gross shocks transmitted *to* and those received *from* all other markets, formally

$$S^o_{j,h} = S^o_{\cdot j} - S^o_{j\cdot} \tag{23}$$

If we were to use either $\bar{\Lambda}^o_h$ or Λ^g_h in (23), then the resulting value would not be a *net* spillover index, since directionality is not identified, instead, we would replace the word *net* of the resulting value by *position of the k variable relative to the total mean spillover transmitted and received*, consequently, it will not be a *net spillover* anymore, it is a *mean relative net spillover* instead.

2.6 Spillovers table

To summarize all the types of spillovers previously presented, we provide an extended version of the matrix in (21) by appending *directional spillovers* and *total spillovers*, the new matrix is now renamed and it is called *Spillovers Table*.

Variable	1	2	...	K	C. from others
1	$\tilde{\alpha}^i_{11,h}$	$\tilde{\alpha}^i_{12,h}$	...	$\tilde{\alpha}^i_{1K,h}$	$\sum_{k=2}^K \tilde{\alpha}^i_{1k,h}$
2	$\tilde{\alpha}^i_{21,h}$	$\tilde{\alpha}^i_{22,h}$	...	$\tilde{\alpha}^i_{2K,h}$	$\sum_{\substack{k=1\\k\neq 2}}^K \tilde{\alpha}^i_{2k,h}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
K	$\tilde{\alpha}^i_{K1,h}$	$\tilde{\alpha}^i_{K2,h}$	...	$\tilde{\alpha}^i_{KK,h}$	
Contribution to others (Spillover)	$\sum_{j=2}^K \tilde{\alpha}_{j1,h}$	$\sum_{\substack{j=1\\j\neq 2}}^K \tilde{\alpha}_{j2,h}$	...	$\sum_{j=1}^{K-1} \tilde{\alpha}_{jK,h}$	$\frac{\sum_{\substack{jk=1\\j\neq k}}^K \tilde{\alpha}^i_{jk,h}}{K} \times 100$
Contribution to others including own	$\sum_{j=1}^K \tilde{\alpha}^i_{j1,h}$	$\sum_{j=1}^K \tilde{\alpha}^i_{j2,h}$	...	$\sum_{j=1}^K \tilde{\alpha}^i_{jK,h}$	$K \times 100$

Table 1: Spillover Table

The *Spillovers Table* has as its jk^{th} entry the estimated contribution *to* the forecast error variance of variable j coming *from* innovations to variable k. The off-diagonal column sums are the *Contributions to Others* or *Cross-variance shares* or *Spillovers*, while the row sums represent *Contributions from Others*, when these are totaled across variables then we have the numerator of the *Spillover Index*. Similarly, the columns sums or rows sums (including diagonal), when totaled across variables, give the denominator of the Spillover Index, which is 100 fold the number of variables ($100 \times K$).

Our objective is estimating Table 1 and based our analysis on it. In following sections we fill Table 1 with the estimated spillovers.

3 Empirical Results

Following Forbes and Rigobon (2002), stock market returns are calculated as two days rolling-average, this allows us to control for the fact that markets in different countries are not open during this same trading hours. For volatility we assume that is fixed within periods (in this case, days) but variable across periods, thus following Garman and Klass (1980) we use daily high, low, opening and closing prices to estimate daily volatility using (**??**).

Stock markets and countries analyzed in this chapter are the ones shown in **??**.

3.1 Static Spillovers

3.1.1 Returns

Here we provide a full-sample analysis of global stock market return spillovers based on both OFEVD and GFEVD. As part of this analysis, firstly, we present a single characterization of the full-sample spillovers providing a description in Table 2 over the sample period 17/6/2003 – 16/9/2009.

Table 2: Total spillover index at 10 step-ahead forecast horizon.

Index	Statistic	VAR(1)	VAR(6)	VAR(9)	VAR(10)
Orthogonalized	Min.	41.096	42.066	42.330	42.321
	Max.	45.111	45.201	45.491	45.433
	Range	4.016	3.135	3.161	3.112
	Mean	43.363	43.834	44.124	44.117
Generalized		54.192	54.818	54.733	54.795

Table 2 provides some *orthogonalized* and *generalized* spillover index results based upon different VAR specifications as far as the lag length is concerned and fixing $h = 10$. We estimate different VAR models as suggested by the selection criteria in Table 3: VAR(6), VAR(9) and VAR(10), additionally a VAR(1) is also estimated; under these circumstances we have no more information for using just one out of them and leave out the other ones.

Table 3: Lag length order selection criteria for returns.

Lag	AIC(p)	HQ(p)	SC(p)
1	-60.890	-60.838	-60.750
2	-61.669	-61.572	-61.409
3	-62.028	-61.887	-61.649
4	-62.244	-62.059	-61.745
5	-62.392	-62.162	-61.774
6	-62.512	-62.238	$-\mathbf{61.774}$
7	-62.598	-62.280	-61.741
8	-62.664	-62.301	-61.686
9	-62.721	$-\mathbf{62.314}$	-61.624
10	$-\mathbf{62.762}$	-62.311	-61.545

AIC(p): Akaike Information Criterion.

HQ(p): Hannan and Quinn Information Criterion.

SC(p): Schwarz Information Criterion.

Numbers in bold represents the minimum of each criteria.

The top panel of Table 2 contains a descriptive statistical summary about the orthogonalized spillover, while the generalized index is placed in the bottom of the table. Independently of the VAR model used, the orthogonalized spillover index is near 44% and the generalized rounds 54%.

In spite of the fact that VAR(1) is not chosen by any selection criterion, its results shown in Table 2 are slightly different from those provided by any other VAR suggested by the criteria, therefore our estimations and hence the subsequent analysis are based on the first order VAR, two main reasons support this selection:

1. VAR(1) results are not so different from other specifications, besides, a VAR(1) specification needs fewer parameters to be estimated than the other VAR models, hence it provides us with more degrees of freedom. Recalling that a VAR model with intercept requires the estimation of $K(1 + Kp)$ parameters, where K is the number of variables

and p is the lag length, we have 6 variables and for VAR(1) we need to estimate 42 parameters which is considerably less than 222 for a VAR(6) for example, not to say for a higher order VAR.

2. Orthononalized Spillover index gets stable more quickly when using a VAR(1) as it is shown in Figure 1. This aspect plays an important role when deciding how many steps-ahead to use when computing the spillover index. Furthermore, when all VAR get the stability, the difference between VAR(1) and other VARs is minimal.

As a simple empirical criterion for choosing how many steps-ahead (h) to use when estimating the spillover index is needed, then the criterion we use in order to pursue a reasonable h, consists of selecting an h at which the estimated spillover index experiments small variations, we refer to this situation as the *"stability"* of the index, so we are after an h such that the spillover index *gets stable*.

Figure 1 shows the behavior of several spillover indexes throughout different forecast horizons which spans from 1 up to and including 20 periods (days), we can note that all indexes get stable at different values of h. VAR(1) gets stable from ahead 7, VAR(6) shows an almost flat curve from ahead 8, both VAR(9) and VAR(10) are much slower to get stability.

In this context *stability* do not be confused with the *stability condition* (stationary condition for a VAR process), here what we meant with "a VAR gets stable at h ahead" is concerning with the limit of the index. When FEVD and hence the spillover index experiments small changes after h aheads then this VAR estimation reached its *'stability'* so the index associated to this VAR "gets stable". Following this definition we will use that step-ahead from which the spillover index does not change dramatically as a *good* choice for our analysis, this means that we should choose 7 step-aheads for VAR(1) in order to estimate the spillover index for returns when using the orthogonalized index and $h = 7$ when using the generalized spillover index. If we were to use VAR(6) then we would choose at least 8 aheads. Figure 1 shows the idea of what *'stability'* is in this context.

It is worthy to highlight the fact that when each VAR get stable, the value of the spillover index slightly differ from each other, therefore choosing that model with less number of parameters and which stability is not so different from the other ones is a good option.

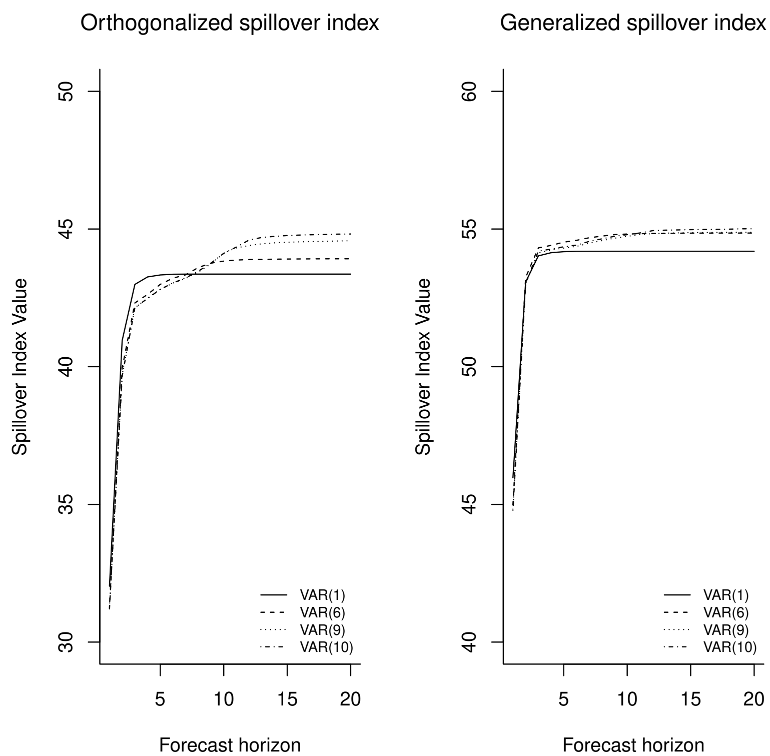

Figure 1: Spillover index for returns throughout different forecast horizons

Table 4: Mean spillover table based on OFEVD, 7 steps-ahead.

	US	UK	EU	BRA	JPN	AUS	C. from others
US	9.7449	1.8137	2.1251	2.3436	0.4610	0.1784	6.9218
UK	4.1073	6.0702	3.7913	1.6531	0.8352	0.2096	10.5965
EU	4.2722	3.7571	6.0975	1.5667	0.8075	0.1656	10.5691
BRA	3.1451	1.2359	1.2225	10.5489	0.4172	0.0970	6.1178
JPN	3.7667	1.5100	1.8193	1.5940	7.7372	0.2395	8.9295
AUS	0.0650	0.0654	0.0352	0.0302	0.0314	16.4394	0.2273
C. to others (spillover)	15.3564	8.3823	8.9933	7.1877	2.5522	0.8901	43.3619
C. to others including own	25.1013	14.4524	15.0909	17.7365	10.2894	17.3295	100.0000

Following Diebold and Yilmaz (2009) we also provide a full sample analysis of global stock market return spillovers by decomposing the Spillover index (*Contribution to others* in Table 4 and Table 5) into all the forecast error variance components for country j coming from country k, for all j and k. We report Spillover Indexes in the last column of the row named *C. to others (spillover)*. The jk-th entry in the table is the estimated contribution *to* the forecast error variance of country j coming from innovations to country k.

Note that static spillover tables shown in this section are the estimation of Table 1, though all spillover tables inhere are *standardized* by means of dividing all elements by K.

Paraphrasing Diebold and Yilmaz (2009), the Spillover table provides an *'input-output'* decomposition of the Spillover Index. We can learn from Spillover Table 4 that innovations to US are responsible, in mean, for 4.1073% of the error variance in forecasting 7-days-ahead UK returns. We can also see that the total spillover from US to other countries account for 15.3564%, meanwhile the spillover from other countries to US is 6.9218%, this evidences that the recent Global Financial Crisis triggered in US and spilled over the rest of countries. Results in Table 4 refer to the mean of the 720 orthogonalized spillover in returns.

One of the key results from Table 4 is the *Total Spillover Index* which accounts for the portion of the forecast error variance error coming from spillovers in returns, is 43.3619% for our full $2003 - 2009$ data sample.

Table 5: Spillover table based on GFEVD, 6 steps-ahead.

	US	UK	EU	BRA	JPN	AUS	C. from others
US	5.9757	1.7631	2.4799	5.9427	0.4610	0.0442	10.6910
UK	3.6858	3.6547	3.7081	4.8574	0.6966	0.0641	13.0119
EU	3.8191	2.9132	4.5829	4.6205	0.6790	0.0520	12.0838
BRA	2.6853	1.0724	1.3911	11.1502	0.3512	0.0164	5.5165
JPN	3.5090	1.7129	2.4189	4.8180	4.1165	0.0913	12.5501
AUS	0.0569	0.0662	0.0688	0.0567	0.0879	16.3301	0.3366
C. to others (spillover)	13.7561	7.5278	10.0668	20.2954	2.2757	0.2681	54.1900
C. to others including own	19.7318	11.1826	14.6497	31.4456	6.3922	16.5981	100.0000

Table 5 shows slightly different situation as its results are based on the general forecast error variance decomposition. In this table, some relevant changes take place, for example, US decreases its spillover from 15.3564% (according to Table 4) to 13.7561%, also UK suffers a reduction in its spillover, while Europe and Brazil experienced an increase. In this new scheme Brazil becomes the main contributor in terms of spillovers. We already expect these discrepancies on the indexes, because each of them is using a different structure of residuals for estimating the corresponding forecast error variance decomposition, as we mentioned before, the orthogonalized index is built upon uncorrelated errors since Choleski decomposition makes them to be independent (under normality), however due to the lack of theoretical background for imposing restrictions on the directionality of the shocks, we construct the spillover index by taking the mean of all the indexes calculated for all possible Choleski decomposition, which is not longer an index which directionality can be identified. For the case where we have generalized spillover index, from subsection 2.3 we know that the GFEVD is order invariant because it does not relies on any kind of orthogonalization, thus the residuals remains correlated and also identification of directionality is not possible. As a conclusion from this part we can say that using either the mean orthogonalized or the generalized spillover index, directionality is not possible to be established and the quantities inside the Spillover Tables should be used cautiously. Because directionality is not recognizable, we base all the analysis on the total spillover.

Just to mention the inaccuracy stemming from the lack of identifiability of the directionality in the spillover tables, the mean relative net spillover is presented in Table 6; when using the average orthogonalized spillover index we have that US, Brazil and Australia are net transmitters while the other countries are net receivers, in contrast, when using the generalized index, Australia is not longer a net transmitter, instead it happens to be a net receiver, while US and Brazil remain being net transmitters.

Net spillovers need one unique Choleski decomposition to be valid. When using taking mean of all possible decompositions, the *net* spillover becomes into *mean relative net spillover* as we pointed out in subsection 2.5.

Table 6: Net spillovers, returns.

	Orthogonalized Index				Generalized Index			
	To	From	Net	Net Transmiter?	To	From	Net	Net Transmiter?
US	15.3564	6.9218	8.4346	Yes	13.7561	10.6910	3.0651	Yes
UK	8.3823	10.5965	−2.2142	No	7.5278	13.0119	−5.4841	No
EU	8.9933	10.5691	−1.5758	No	10.0668	12.0838	−2.0170	No
BRA	7.1877	6.1178	1.0699	Yes	20.2954	5.5165	14.7789	Yes
JPN	2.5522	8.9295	−6.3773	No	2.2757	12.5501	−10.2744	No
AUS	0.8901	0.2273	0.6628	Yes	0.2681	0.3366	−0.0685	No

3.1.2 Volatility

In this section, the static volatility spillovers are analized, all the decision process about the lag length and the selection of h is undertaken as in the previous section. Volatility in this chapter is estimated using (**??**) which is found in Garman and Klass (1980).

For similar reasons as before, a VAR(1) is used to estimate the spillover for volatilities, Other alternatives to VAR(1), suggested by the selection criteria, are VAR(3), VAR(9) and VAR(10), see Table 7 and Figure 2. Here the difference between VAR(1) and VAR(3) are negligible and at the limit there are not big differences with VAR(9) or VAR(10) in terms of the value of the spillover index.

Using a VAR(1) and $h = 70$ as the best value for the forecasting horizon, Table 8 and Table 9, are estimated.

We learn from Table 8 that total volatility spillovers from US to others accounts for 17.63% (*C. to others (spillover)*) which is twice as big as total volatility spillovers from others to US (contributions from others) which only amounts about to 8.4966%. As intuitively was expected, volatility transmissions from US to the rest of the countries are much bigger than the transmissions from any other country to the rest of the stock markets, this result is plausible since US is the country where the GFC took place before to be spilled over the major stock markets.

Table 7: Lag length order selection criteria for intraday volatility.

Lag	AIC(p)	HQ(p)	SC(p)
1	-70.737	-70.685	-70.598
2	-70.802	-70.705	-70.542
3	-71.061	-70.920	**-70.682**
4	-71.095	-70.910	-70.596
5	-71.187	-70.957	-70.569
6	-71.219	-70.946	-70.482
7	-71.287	-70.969	-70.430
8	-71.343	-70.981	-70.367
9	-71.399	**-70.992**	-70.302
10	**-71.400**	-70.949	-70.184

AIC(p): Akaike Information Criterion.

HQ(p): Hannan and Quinn Information Criterion.

SC(p): Schwarz Information Criterion.

Numbers in bold represents the minimum of each criteria.

Table 8: Mean spillover table based on OFEVD, 70 steps-ahead.

	US	UK	EU	BRA	JPN	AUS	C. from others
US	8.1701	2.2711	1.5784	1.7334	0.7509	2.1628	8.4966
UK	4.6543	5.0116	2.3485	1.4564	0.7246	2.4713	11.6550
EU	4.7382	3.3597	4.6830	1.2482	0.7313	1.9063	11.9836
BRA	3.3817	1.5489	1.0369	8.7603	0.5944	1.3445	7.9063
JPN	3.3543	1.6729	1.4531	1.0216	7.8508	1.3139	8.8159
AUS	1.4978	1.4262	0.4834	0.4974	0.1787	12.5830	4.0836
C. to others (spillover)	17.6263	10.2788	6.9003	5.9570	2.9800	9.1987	52.9411
C. to others including own	25.7964	15.2905	11.5834	14.7174	10.8307	21.7817	100.0000

Now consider the total volatility spillover, which indicates that on average, 52.9411% percent of volatility forecast error variance in all 6 stock markets comes from spillovers in volatility.

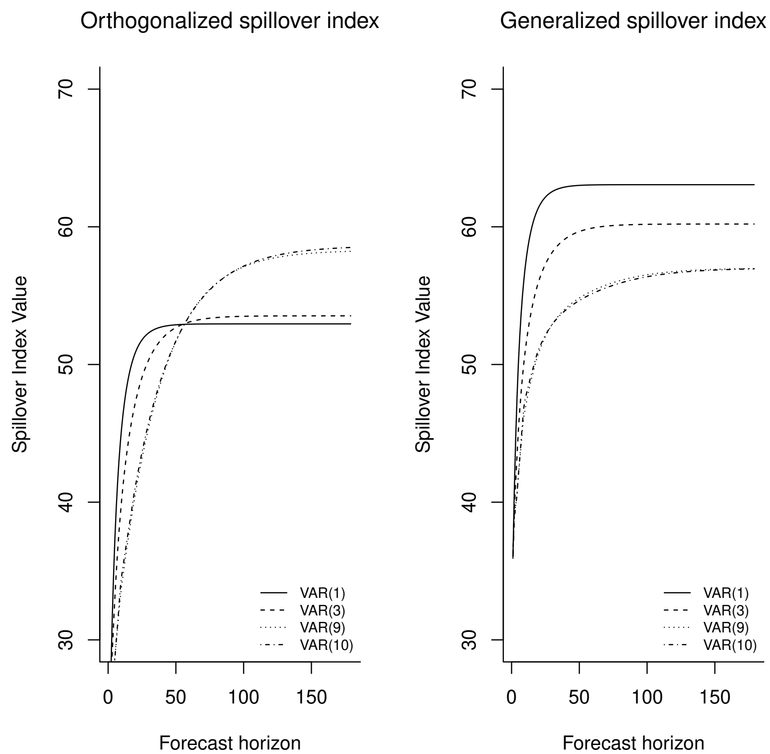

Figure 2: Spillover index for volatility throughout different forecast horizons

Table 9: Spillover table based on GFEVD, 70 steps-ahead.

	US	UK	EU	BRA	JPN	AUS	C. from others
US	5.7385	2.2865	2.1376	4.9569	0.7604	0.7868	10.9282
UK	3.9583	3.8317	2.8213	4.4119	0.7424	0.9011	12.8350
EU	4.0173	3.2173	4.0617	3.9409	0.7390	0.6905	12.6050
BRA	2.6178	1.3712	1.2441	10.5412	0.4959	0.3963	6.1254
JPN	3.4491	2.0636	2.1391	3.7252	4.7150	0.5747	11.9517
AUS	2.5025	1.9413	0.9690	2.7988	0.3958	8.0592	8.6075
C. to others (spillover)	16.5451	10.8800	9.3110	19.8338	3.1336	3.3493	63.0528
C. to others including own	22.2836	14.7116	13.3727	30.3750	7.8486	11.4085	100.0000

In Table 9, we see almost the same pattern exhibited in Table 8, nevertheless in the generalized version of the spillover for volatility the main contributor is Brazil followed by

Table 10: Net spillovers, volatility.

	Orthogonalized Index				Generalized Index			
	To	From	Net	Net Transmiter?	To	From	Net	Net Transmiter?
US	17.6263	8.4966	9.1297	Yes	16.5451	10.9282	5.6169	Yes
UK	10.2788	11.6550	−1.3762	No	10.8800	12.8350	−1.9550	No
EU	6.9003	11.9836	−5.0833	No	9.3110	12.6050	−3.2940	No
BRA	5.9570	7.9063	−1.9493	No	19.8338	6.1254	13.7084	Yes
JPN	2.9800	8.8159	−5.8359	No	3.1336	11.9517	−8.8181	No
AUS	9.1987	4.0836	5.1151	Yes	3.3493	8.6075	−5.2582	No

US while in the orthogonalized case, the main contributor is US followed by UK.

Here again, we show the 'net' spillover table where volatility exhibits the same pattern as returns. When using the orthogonalized spillover US and Australia are net transmitter and this result changes when using the generalized because in this case US remains being a net transmitter while Australia is not anymore and Brazil change position from being a net receiver to be a net transmitter.

3.2 Rolling sample analysis: Studying the dynamics of the spillovers

We prepare this section because several events might have taken place within our series as stock prices move from relative stable periods to turmoil ones, therefore with this financial market evolution, it is unlikely that prices remain constant over time so that any single fixed-parameter model would apply properly over the entire sample and gives rich information about its evolution.

Hence the full-sample spillover tables constructed earlier, although providing a useful summary of the average total spillover behavior, likely miss potentially important secular and cyclical movements in spillovers. To address this potential lose of dynamics, we now estimate spillover using 160-days[5] rolling windows which we examine graphically in the co-

[5]The width of the rolling windows does not affects the main findings. Diebold and Yilmaz (2009) performs an extensive set of robustness checking on this particular point showing that dynamic spillover index is

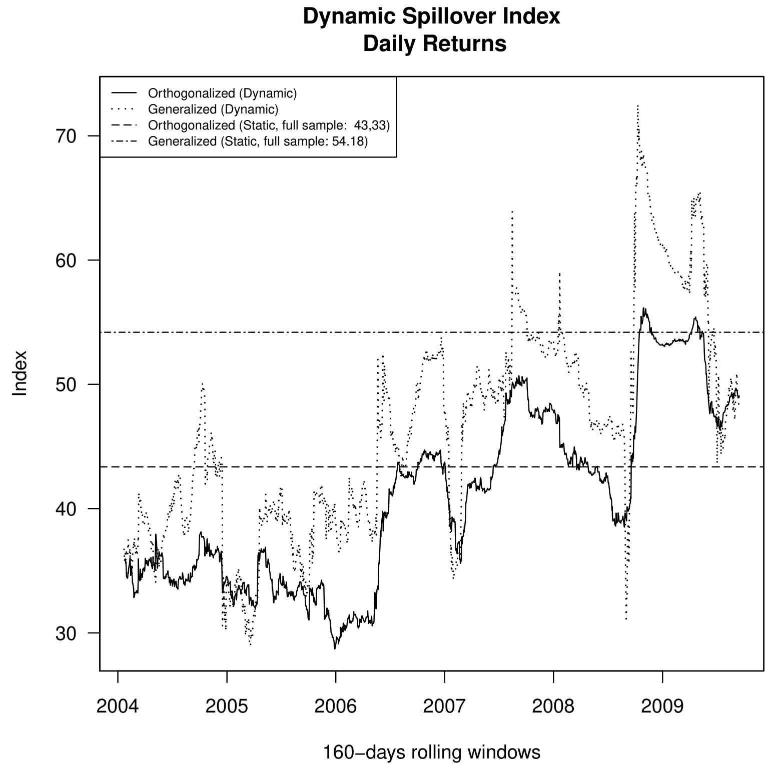

Figure 3: Dynamic spillovers for returns.

called total spillover plots (Diebold and Yilmaz, 2009, 2012). We provide results from both the orthogonalized and the generalized spillover index.

We can note, on October 2008, a increasing trend with a big jump capturing the Global Financial Crisis (GFC) triggered on August 4, 2008. The jumps previous to the biggest one clearly reflects how volatile the stock markets were during the Subprime Mortgage Crisis (hereafter: SMC) and this fact triggered the GFC. See daily dynamic plot in Figure 3.

Figure 4 shows the dynamic spillover index for volatilities using 160-days rolling windows. There are some common features between dynamic spillover in returns and dynamic spillover in volatilities, we see that both captures quite well the turbulence in late 2008, both have three main jumps corresponding to mid 2006, early 2007 and late 2008.

Figure 5 and Figure 6 shows the 'net' spillover dynamically. The dashed line at point

strongly robust to the size of the window.

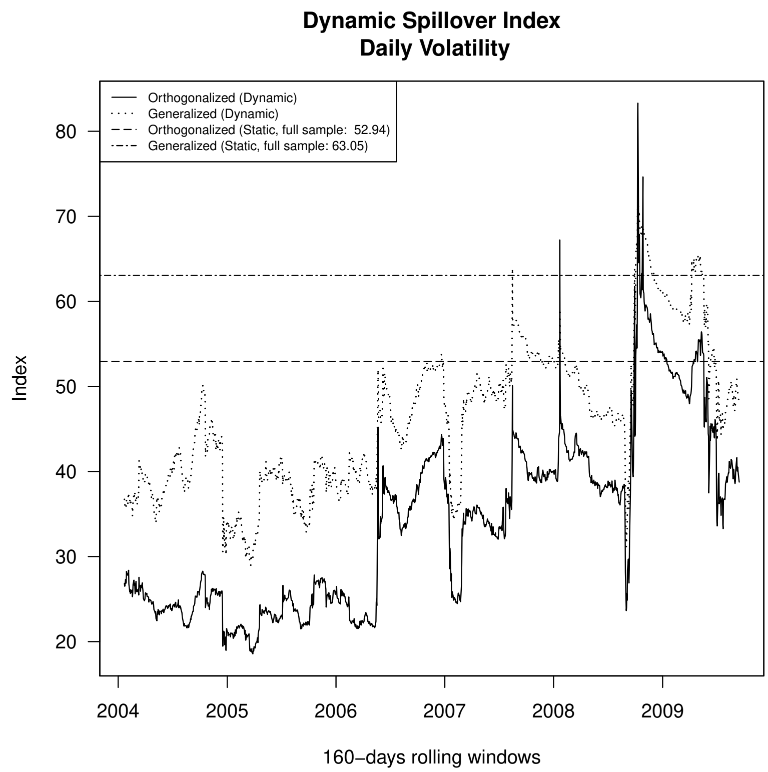

Figure 4: Dynamic spillovers for volatilities.

zero indicates that values above this line suggest the country is a net transmitter and values below indicate the country is a net receptor of shocks.

Figure 5 shows the US as net transmitter of shocks over the entire sample period while Brazil and Australia are net transmitters for most of the period, while UK and Europe are most of the time net receptor. Japan is always a net receptor for all period. Figure 6 shows very similar results except for US which behaves as a net receptor of shocks before 2008 and after the crisis in 2008 it becomes into a net transmitter and Brazil becomes into a net transmitter for all the period, the rest of countries behave the same as in Figure 5. It is important to note that the word net in this context should be use cautiously as neither in the (mean) orthogonalized nor in the generalized version of this section, directionality is identified.

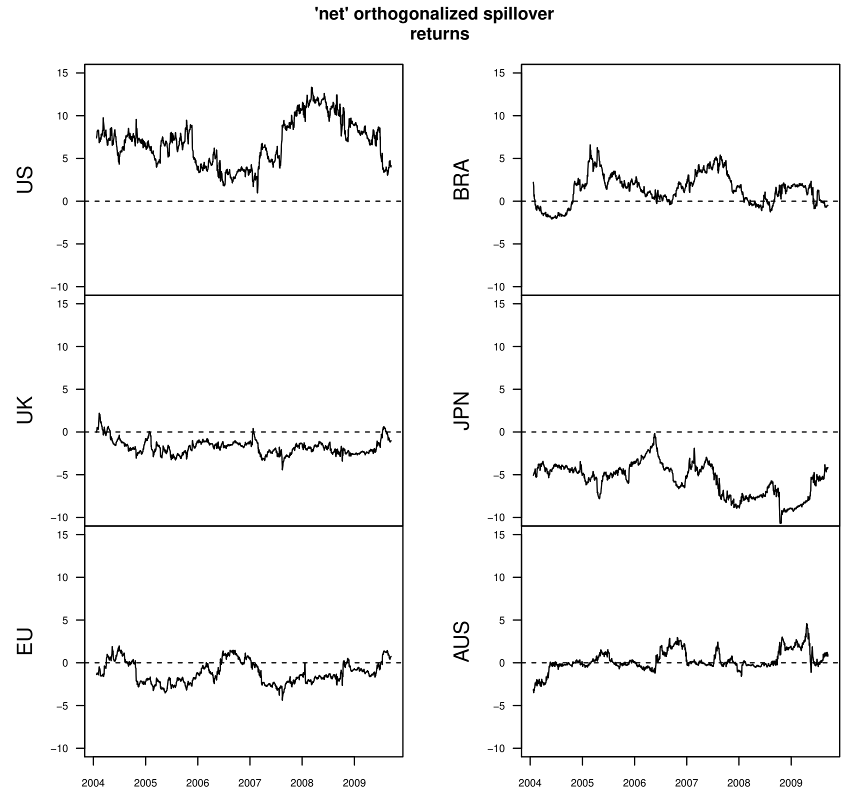

Figure 5: Dynamic orthogonalized ‘net’ spillovers for returns.

4 Conclusions

We utilize a spillover index to assess the proportion of variance that on average comes from spillover in other countries. Two versions of this spillover index are used in this work: the orthogonalized and the generalized version, where the former is based on the traditional forecast error variance decomposition using the Choleski orthogonalization, hence the order-dependence becomes a drawback; the latter is based on the generalized forecast error variance decomposition, which not depends on the ordering. It is worthy to note that the ordering dependence of the orthogonalized spillover index is a drawback when lack of a theoretical framework for imposing restrictions is involved, if we had such a theoretical background, then order dependence will not longer be a drawback, instead it would be an advantage since it

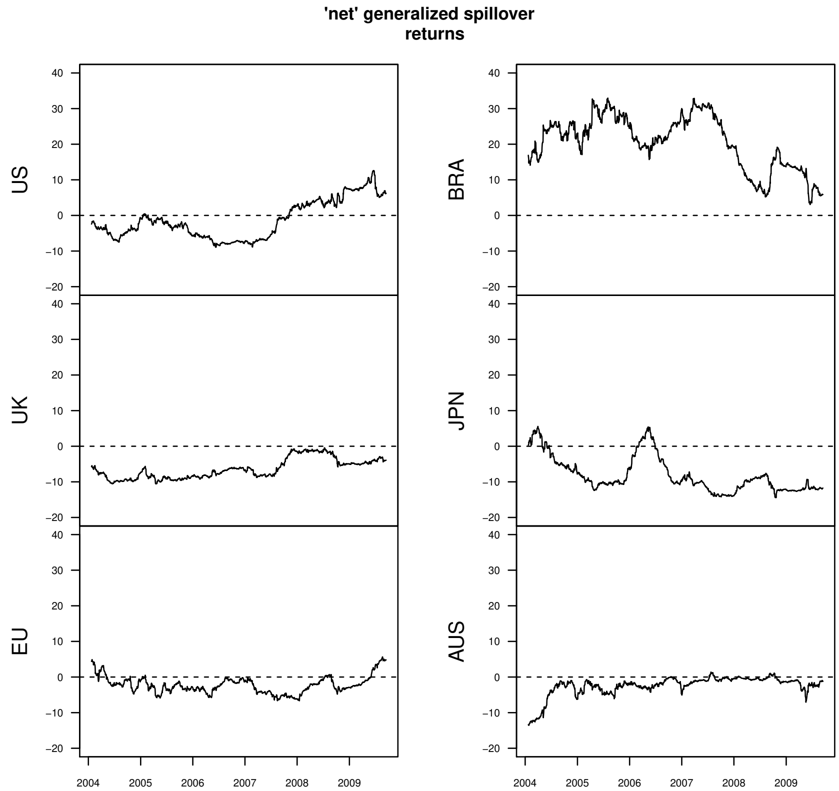

Figure 6: Dynamic generalized 'net' spillovers for returns.

will provide us with directionality, the spillover table would be meaningful and net spillovers indeed would account for net effects and the highly computational procedure will decreases dramatically.

Our empirical results suggest that around one-half of the total variance comes from spillovers in returns as well as in volatility.

Since the impossibility of identifying the shocks in the spillover tables, we consider that this procedure is useful to obtain total spillovers but not directional spillovers, therefore net spillovers are conclusive.

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