The Narrow and the Broad Approach to Evolutionary Modeling in Economics

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Abstract

Some models in evolutionary economics rely on direct analogies to genetic evolution: Assuming a population of firms with routines, technologies and strategies on which forces of diversity generation and selection act. This narrow conception can build upon previous findings from evolutionary biology. Broader concepts of evolution allow either many or just one adaptive entity instead of necessarily requiring a population. Thus, an institution or a society can also be understood as the evolutionary entity. Both the narrow and the broad approach have been extensively used in the literature, albeit in different literature traditions. The paper gives an overview over the conception and the development of both approachs to evolutionary modeling and argues that a generalization is needed to realize the full potential of evolutionary modeling.

1 History of Evolutionary Theories in Economics

Darwin's theory of evolution shook the very foundation of Western[1] science, philosophy, and culture. Suddenly, there were endless possibilities for new explanations of the world, of nature, of reality; the wise plan of the creator was not required any more and neither were the moral imperatives or those of rationality. Of course, these endless possibilities did also not preclude some very narrow and vile interpretations, typically termed social-Darwinist. Nevertheless and notwithstanding those, the power of the new approach yielded many small scientific revolutions over the next years, decades, and centuries. The most direct of those is, of course, the one in biology. But there have been applications in mathematics, in evolutionary game theory [Maynard Smith and Price, 1973] and optimization [Storn and Price, 1997], in anthropology [McElreath et al., 2005], in psychology [Cosmides and Tooby, 2013], and also in economics.

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[1]It of course also affected thought and philosophy in other cultures but this appears less prominent as all non-Western cultures were at the time more or less uprooted by direct imposition or inadvertent encroachment of Western ideas, values and customs.

One of the first scholars to call for an evolutionary approach in economics was likely Thorstein Veblen. He notes both the lack of adaptiveness of economics as a field of scholarship itself (which he contrasts with psychology and anthropology) and the ignorance in economics towards any realistic conception of human decision making beyond that of "a lightning calculator of pleasures and pains" [Veblen, 1898]. He insisted that human nature and psychology should be taken into account; that these were the dominant forces shaping the development of human society. Institutions, commonly recognized rules of behavior, would force a much larger possibility space of theoretically thinkable behavioral patterns into a much narrower shape that would be partly ceremonial, partly progressive (instrumental) and, most of all, constantly changing. More formal-mathematical aspects were introduced by later generations of institutional-evolutionary economists. Veblen's concept of *cumulative causation* was later extended to *circular cumulative causation* by Gunnar Myrdal [Myrdal, 1974] - mathematically a system with self-reinforcing processes caused by positive feedback mechanisms. Myrdal located such mechanisms in the development of countries or in the lack thereof, either being influenced by the social envirmonment, public institutions, the education system, etc. Such institutional interaction systems are, in a way, evolutionary, as they outline a partly directed development that favors some patterns over others, that propagates some behaviors or institutions while dissuading people from following others.

From the 20th century on, specific formal models that included dynamic sytems with positive and negative feedback mechanisms emerged in economics. In particular, such models were applied to the business cycle phenomenon [Goodwin, 1967, Kaldor, 1940] and financial economics [Grasselli and Costa Lima, 2012, Keen, 1995, Minsky, 1980], while Bush [Bush, 1987] and others extended the Veblenian theory of institutions to development patterns of ceremonial and instrumental institutions with later moves to develop formal models [Elsner, 2012, Heinrich and Schwardt, 2013, Villena and Villena, 2004].

In another branch of evolutionary economics, Schumpeter [Schumpeter, 1943] argued in favor of interpreting the economy as a system that periodically undergoes cycles of "creative destruction". Less advanced or dysfunctional enterprises are destroyed, which causes a general economic downturn but makes room for the comprehensive development of more modern industries. Schumpeter did, however, also note that the driving force behind this is not some selection of the intrinsically better but rather a continuous technological development that, if successful, temporarily grants higher profit margins but requires huge investments. As such, it is a continuous and not necessarily directed process; Schumpeter understood both the importance of government in providing the environment for such development and the fact that large firms have much better means to engage in risky innovation than small ones. A formal model matching the Schumpeterian framework was added in the 1970s in the form of agent-based models by Nelson and Winter [Nelson and Winter, 1974, Nelson et al., 1976] and essentially fueled the rise of modern mathematical evolutionary economics. On the one hand, a large number of other evolutionary models working along similar lines was published in the successive years[2], on the other, methodological developments from the fields

[2] E.g., [Silverberg et al., 1988]; for an overview, see Safarzynska and van den Bergh [Safarzyńska and van den Bergh, 2010]

of complex systems, game theory, dynamic systems, network theory and others were adapted into economics. Though this reception of new methods, most notably driven by the Santa Fe Institute for Complex Systems, was not only due to the rise of modern evolutionary economics, it certainly played a part.

It should be noted that Schumpeter [Schumpeter, 1943] was much more literal in his concept's analogy to biological evolution. Actual entities - enterprises - would compete, would adapt to their current social and economic environment, would mutate with innovations, would diversify by adopting different technologies and would finally be subjected to selection and would prevail if sufficiently well-adapted or otherwise be eliminated by creative destruction. Note that in evolutionary economic models, the evolutionary vehicle is typically not the human being running the firm but the firm itself; the replicators are typically technologies, strategies, routines, or behaviors.[3] The analogy that is not present is that of the genotype-phenotype distinction in genetic evolution.

And indeed, this has more recently been a major cause for concern in evolutionary institutional economics. Various scholars warned that analogies between genetic and cultural or economic evolution should not be taken literally [Cordes, 2006, Wäckerle, 2013, Witt, 2005]. There is neither a close equivalent of the gene (generally the genotype-phenotype distinction) nor of reproduction with recombination or inheritance of recessive traits without phenotypic manifestation; many other examples could be found.[4] Further, humans are able to reflect and modify their behaviors and strategies as well as the technological and organizational basis of firms they are in charge of. They can consciously and purposefully interfere if the evolutionary performance of their firm does not meet their expectations and they will have a good chance of success. Finally, they are, even in modern economic life, constrained by what genetic evolution has prepared them for in the very different context of small bands of nomadic hunter-gatherers; this may have significant psychological consequences, especially for collaboration in groups.

2 Stability and Dissipative Structures: The Broad Approach to Evolutionary Modeling

Social systems tend to develop in time without ever coming to an equilibrium-like stable state and while directed trends and circular patterns have been identified, most social systems tend to exhibit more complex forms of behavior. Nevertheless, in order to understand their behavior, simplified models must be constructed and one of the predominant forms of modeling in economics is the search for and identification and characteriziation of (global) equilibria or optima. For some systems, this provides a good benchmark.In light of the mentioned complex patterns doubts may be cast, however,

[3] For a critique of the use of the concept of replicators in non-genetic context, see Cordes [Cordes, 2006].

[4] It is, of course, possible to find roughly similar concepts in tacit knowledge or routines (for the genotype, as was argued to be the case in Nelson-Winter type models [Heinrich, 2016]), in creative recombination of technologies or strategies (for recombining reproduction), in unused aspects of technologies (for genetic carriers), but none of these analogies have the rigidity of the system in genetic evolution.

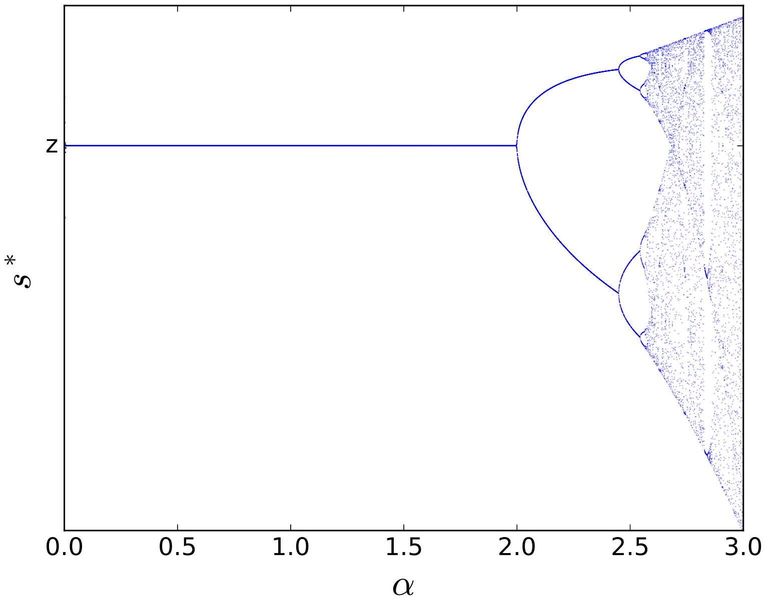

Figure 1: Bifurcation diagram of the discrete growth process.

on how close general equilibrium models may represent economic reality. This calls for less restrictive approaches to modeling that nevertheless are mathematically able to generate stability - because if they were not able to do that, their predictions would not only be exceedingly difficult to verify they would also not be particularly useful.[5] They would, in the course of the development predicted by the model, be subject to constant turbulent change. Good candidate mechanisms for providing stability may be found in non-linear feedback loops, which, in turn are conveniently modeled in dynamical systems.

Consider as an example a simple discrete growth process acting on a variable s with a capacity boundary z and with initial[6] growth rate α,[7]

$$s_{t+1} = s_t + \alpha s_t \left(1 - \frac{s_t}{z}\right). \tag{1}$$

An economic example could be a discrete (e.g., annual or seasonal) reinvestment cycle, for instance in a particular agricultural sector, with the capacity boundary introdured by resource availability or limited demand. Both the discrete system and the continuous

[5]This may sound like a Popperian take at the philosophy of science. However, it is not claimed here, that theories in general must be testable; it is only claimed that testable theories are required in order to make predictions which, in turn, is self-evident as the preditions could be used to test those theories. In fact, it is not difficult to imagine theories that would be true (close to reality) but not testable or at least not testable with currently available means. However, philosophy of science lies beyond the scope of the present work.

[6]That is, the limit of the growth rate for $s_t \to 0$, $\alpha = \lim_{s_t \to 0} \frac{s_{t+1} - s_t}{s_t}$. The growth rate declines as the variable approaches the capacity boundary.

[7]Note that this corresponds to the continuous system $\frac{ds}{dt} = \alpha s \left(1 - \frac{s}{z}\right)$.

system (as given in footnote 7) have two equilibriaat $s^* = 0$ and $s^* = z$ the first of which is not stable. An equilibrium approach would hence derive this equilibrium set and its stability conditions and proceed to argue that since there is exactly one stable equilibrium,[8] the system must converge to this equilibrium at some point. While this is generally true for the continuous system, the discrete system as considered here undergoes a phase transition at $\alpha = 2$; for initial growth rates larger than that, the system will instead converge to a cyclic attractor oscillating between 2 (or 4, 8, 16, etc.) points before entering a chaotic regime for ca. $\alpha > 2.8$; cf. the bifurcation diagram in figure 1.[9] In the cyclic regime, the equilibrium set, while existing and computable, does not help much in describing the development of the system. And while this development is still analytically computable for such a simple system, it is easy to see that the difficulty of that increases superlinearily with increasing dimensionality, and also with nonlinearity, etc. Consider for instance a system with further small disturbances in the (investment) cycle[10]

Hence, as an equilibrium approach will not help in this case, which options do we have instead?

As pointed out by Gregoire Nicolis and Ilya Prigogine [Nicolis and Prigogine, 1977], a broader mathematical concept of stability compared to the strict definition of an *attractor* may be defined: that of a *dissipative structure*. A dissipative structure may have a *basin of attraction*, but no strict convergence towards a well-defined central attractor is required. Nicolis and Prigogine proceed to prove the characteristics of dissipative structures by showing that the entropy of dynamic systems is maximized in the vicinity of attractors and dissipative structures, hence providing that with the second law of thermodynamics, the system will converge there.

The concepts still hold for stochastically disturbed dynamic systems as long as the stochastic influence is small compared to the dynamic forces of the corresponding analytic system.

Further, so far, our system was a closed system. It is, however, easy to extend this to an open system by including a parameter (take the parameter α in the growth process considered above as a simple example) that may change exogenously. The parameter may hence induce the system in certain cases to undergo phase transitions (as with the critical value $\alpha = 2$), allowing new dissipative structures to emerge, changing existing dissipative structures or destroying dissipative structures. The system will through its own dynamic process adapt to this, a process described as evolution.

It may be appropriate to describe this adaptation process as evolution, though many characteristics of genetic evolution, the concept this analogy alludes to, are absent from such a dynamic system. Genetic evolution is characterized by a population of

[8] I.e., the basin of attraction is $G = (0, \infty]$, it comprises the entire permissible phase space.

[9] Also note that the system is an instance of a (modified) logistic map which has extensively been researched since the 1970s, see e.g. May [May, 1976] or Li and Yorke [Li and Yorke, 1975].

[10] E.g., $s_{t+1} = s_t + \alpha s_t \left(1 - \frac{s_t}{z}\right) + \beta s_t \left(1 - \left(1 - \left(\frac{s_t}{z}\right)^2\right)\right)$ where β is small compared to α but where the last term becomes a dominant source of turbulence in the vicinity of the fixed point.

replicators,[11] structures of non-trivial order, that are able to reproduce themselves, to keep their physical form functional and to multiply (i.e., they are natural von Neumann machines). They are able to inherit and pass on a codified set of specifications, written in a base-4 alphabet, their genome (representing their genotype), which determines their phenotype and behavior. The evolutionary process introduces variation by recombination of genotypes on the one hand and by creating imperfect copies (mutation) on the other. The differential reproductive success of different replicators, if not the direct elimination of less well-adapted ones, introduces selection to the process. The envirmonment governing the selection process may change continuously and require a continuous adaptation process. The degree of adaptation, hence the expected performace in the evolutionary process, may be captured in an (artificial) auxiliary variable, the fitness[12], allowing convenient mathematical formalizations of replicator dynamics (see section A.5).

While in the above dynamic system there is no genotype-phenotype distinction and the population size is limited to one, there is a continuous process of adaptation which sustains the continued existence of a non-trivial structure, in this case the dissipative structure, which may be seen as a simple replicator.

The concept of dissipative structures is quite universal, capturing a large number of systems both in economics and other fields, basically any not artificially controlled ordered (hence self-organizing) system. It may therefore be rather universally applicable but also fairly general and not impressively powerful as a method. This is perhaps why recent decades have seen numerous assertions that economics is about self-organizing or adaptive or evolutionary systems [Elsner, 2012, Gräbner and Kapeller, 2015, Kauffman, 1993, Kirman, 1997, Farmer and Lo, 1999, Farmer, 2002] as well as numerous candidate models but on the other hand very little in terms of a powerful alternative to equilibrium models.

3 Populations and Selection: The Narrow Approach to Evolutionary Modeling

Many of the more specific models took a narrower interpretation of the analogy, attempting to identify true populations of replicators in economics that undergo an evolutionary process fairly close to the genetic one with evolving memes (or routines, etc) [Dawkins, 1976, Hodgson, 2002] or evolving firms with innovation-imitation dynamics for technological change [Nelson and Winter, 1974, Safarzyńska and van den Bergh, 2010, Silverberg et al., 1988]. Such models are able to directly represent a wide range of classes of conscious, decision-making entities such as firms, individuals, government agencies, etc. They can get close to very intuitive and very descriptive models of economic reality and have been studied with exceedingly promising results in the con-

[11] The term replicator may be confusing as it is used both for the members of the population on which evolution acts and for the mathematical process that describes it (written with a replicator equation, see section A.5.); the two are, of course different concepts and should not be confused.

[12] Fitness is typically denoted f; in the example below in section A.5, the growth rate α, being the only characteristic distinguishing the performance of the subpolutations, takes the place of the fitness.

text of agent-based simulation (see section A.6). It is tempting to apply ideas from genetics to these systems as this would provide an opportunity to benefit from decades of well-funded research with well-established, well-tested and widely accepted results in evolutionary biology, something that evolutionary economics simply does not have.[13] Replicator dynamics and evolutionary game theory can be adapted to economics and the estimation of mutation rates, population sizes and quasi-genetic information could be used to characterize the evolutionary process further. It should be noted though, that in this case it is not the agent, representing a firm or individual, that takes the place or the replicator but rather the technology, strategy or routine used in the firm. Scholars have rightly criticized this practice of drawing very close analogies (see Cordes [Cordes, 2006], cf. [Watkins, 2010, Witt, 2005]) with some evolutionary economists instead advocating the more detailed investigation of the interaction of biological and anthropological constraints (resulting from earlier, slow, genetic evolution) with cultural evolution and economic systems [Cordes, 2006, Witt, 2005]. In some limited cases, it may nevertheless be possible to argue in favor of Dawkinsian meme-like replicators, perhaps in the case of techniques in traditional craftsmanship, in the case of skills in pre-modern societies, or in the case of routines in large dynamic industries in more modern times.[14] Beyond that, some of the results of evolutionary biology may also be applicable to the wider understanding of evolution as dissipative systems (see above), but the general unresolved problem in drawing this analogy remains how to measure the size of evolutionary information in the abscence of an actual genome. Information is doubtlessly present and it should also be possibly to quantify it, perhaps in terms of information theoretic methods, but it would be much less straightforward than in genetics (cf. [Heinrich, 2016]).

4 Conclusion

Especially in recent years, stunning regularities have been discovered with methods from complexity scoience in industrial dynamics (firm size distributions), technological change, financial economics, and production networks [Farmer and Lafond, 2016, Hidalgo et al., 2007, Heinrich and Dai, 2016]. They may serve as a reminder that economics is a field of structural emergence and pattern evolution; a delicate equilibrium is not an apt metaphor for economic systems, an evolving pattern is.

While the narrow and the broad approach to evolutionary modeling in economics have been advanced by very different literature traditions, it may be time to unify them again. It may be time to apply the advanced tools that were developed in

[13] In his keynote to the ECCS 2014 conference in Barcelona, Doyne Farmer compared the use of agent-based simulation in economics to the use of simulation in meteorology concluding that huge amounts of both financial investments and labour would have to be undertaken before evolutionary, institutional and complexity economics could be expected to match equilibrium economics even though its approach and methods are quite promising. Given the lack of acceptence of evolutionary economics in the mainstream of the profession, this is not exceedingly likely to happen in the foreseeable future.

[14] In all these cases, routines and tacit knowledge are the determining factors of the system; the organizational entities are relatively small and tacit knowledge, being first and foremostly a characteristic of the respective individuals, can move and diffuse between them.

mathematical biology to a broader concept. It may be time to reap the fruits that new techniques like agent-based modeling and increasing computation power have made available. Recognizing that there are indeed vast differences between genetic evolution and evolution in economic systems, we must turn to a broader concept of evolution. Then, we can hope to develop models that represent economic reality and complexity adequately, that may be able to discover the mechanisms and regularities that drive economics.

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A Methods for Evolutionary Modeling

As a broader approach, beyond isolated equilibria, evolutionary modeling must capture processes, developments in time, it must take them into account, it must describe them explicitly. *Dynamic systems*, typically modeled as systems of difference or differential equations, are generally the mathematical concept of choice for this task. This method is fairly general; while it has applications in many other areas of study as well, notable subfields for evolutionary modeling include chaos theory, *bifurcation theory* and *replicator dynamics*. Dynamic systems can be combined with network theory approaches and *stochastic processes*. Modeling in social and economic systems does, however, also call for explicit representations of entities (i.e. firms, individuals, etc.) and interactions among the same. While this is perfectly feasible in a dynamic system, these *agent-based models* will result in dynamic systems of massive scales, with state variables and development equations for each and any of the entities which, in the best case act as if they were independent decision makers. Such a complex model will, in turn, make deriving analytical solutions difficult at best and more often impractical and unlikely to be successful. A more powerful approach in terms of limited computation capacity is *simulation*, in this case agent-based simulation. Compared to analytical approaches, simulation is able to retain massively larger amounts of complexity of the real system under investigation in the simulation model though the simulation results will always be approximations, never exact and exhaustive solutions.

This section will give very brief overviews over each of these methods.

A.1 Dynamic Systems

A dynamic systems is characterized by an n-dimensional vector of state variables X and a function $F(X)$ defining its development. There are two major approaches: The development equations may be difference equations - in which case the time development is defined over finite timesteps and the dynamic system is a system of difference equations,

$$X_{t+1} = F(X_t), \tag{2}$$

or the development equations may be differential equations - in which case the system is time-continuous with infinitesimal time steps and the dynamic system is a system of differential equations,

$$\frac{\nabla X}{dt} = \widetilde{F}(X). \tag{3}$$

Note that $F(X)$ and $\widetilde{F}(X)$ are functions over vectors with the same number of elements (n) as the state vector X,

$$\begin{pmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{pmatrix} = \begin{pmatrix} F_1(x_1, x_2, ..., x_n) \\ F_2(x_1, x_2, ..., x_n) \\ ... \\ F_n(x_1, x_2, ..., x_n) \end{pmatrix}. \tag{4}$$

The set of equilibria of the respective systems is conveniently obtained by letting the dynamic change be $= 0$ in all dimensions, i.e.

$$X_{t+1} = X_t \tag{5}$$

or

$$\frac{\nabla X}{dt} = 0 \tag{6}$$

respectively.

All elements F_1, F_2, ... may be different and define as many different and potentially interdependent dynamic developments as the vector has elements. The dynamic development of the system as a whole can be characterized locally by the eigenvalues λ of the linearized Jacobian of $F(X)$ (or $\widetilde{F}(X)$),

$$\mathcal{J} = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \cdots & & \frac{\partial F_2}{\partial x_n} \\ \cdots & & & \cdots \\ \frac{\partial F_n}{\partial x_1} & \cdots & & \frac{\partial F_n}{\partial x_n} \end{pmatrix}, \tag{7}$$

defined with corresponding eigenvectors v as

$$\lambda v = \mathcal{J} v \tag{8}$$

where the elements of the Jacobian have to be linearized in order to yield scalars for the n eigenvalues λ_1, λ_2, ..., λ_n. It can be shown that each one of the eigenvalues characterizes an independent dynamic motion. Further, for differential equation systems, this motion is contracting if the real component of λ is negative,

$$Re(\lambda) < 0; \tag{9}$$

for difference equation systems, it is contracting if the modulus of λ is smaller than 1,

$$|\lambda| < 1. \tag{10}$$

This being the case, the global dynamic development is dominated by the eigenvalue with the largest real component (for differential equation systems) or the one with the largest modulus (for difference equation systems) respectively; these are conveniently refered to as the dominant eigenvalues. An equilibrium (fixed point) will be stable if the condition holds for the dominant eigenvalue (that is, for all eigenvalues) in the vicinity of the fixed point, since in this case all dynamics would contract towards it. More mathematical details including the relation of the present considerations to general functional solutions of dynamic systems are discussed in Heinrich [Heinrich, 2013, chapter 2] or in the standard books on the subject [Gandolfo, 1997, Grosche et al., 1995]. It is common to compare equivalent systems in time-continuous and time-discrete version by letting

$$\widetilde{F}(X) = F(X) - X, \tag{11}$$

hence

$$\frac{\nabla X}{dt} = \widetilde{F}(X) = F(X) - X. \tag{12}$$

It should be noted that while the two systems, $\frac{\nabla X}{dt} = \widetilde{F}(X)$ and $X_{t+1} = F(X_t)$ have the same equilibrium set, i.e. the same set of fixed points, the dynamic properties including the stability of the fixed points may diverge radically as shown in section 2.

When considering more complex concepts like stochastically disturbed dynamic systems, the above equilibrium concept (equations 6 and 5) becomes meaningless as it also includes unstable equilibria. A more useful concept would be that of attractors, regions in the phase space $\Re^n$ that absorb dynamic developments from a specific (local or global) environment, its basin of attraction. Let an attractor be a non-seperable set of points B within the phase space of a dynamic system with an environment $U : |U - B| \leq \varepsilon$ within which trajectories converge to B,

$$|F(X) - B| < |X - B| \qquad \forall X \in U. \tag{13}$$

Stable fixed points fulfill this condition, but other structures do so as well. Attractors may be cyclic with a finite cycle lengths, in the easiest case oscillating (for difference equation systems) or revolving (for differential equation systems) around an unstable equilibrium (limit cycle attractors); more complicated attractors may have internal chaotic dynamics (strange attractors).

A.2 Dissipative Structures

The technical definition of an attractor (fixed point atractor or otherwise) is a set of points B in an analytic (non-stochastic) dynamic system $X_{t+1} = F(X_t)$ (or $\frac{\nabla X}{dt} = F(X) - X$) with an environment $U : |U - B| \leq \varepsilon$ within which the dynamic strictly approaches the attractor B,[15]

$$|F(X) - B| < |X - B| \qquad \forall X \in U.$$

As pointed out by Nicolis and Prigogine [Nicolis and Prigogine, 1977], a different, less strict, concept of stability may be defined, that of a dissipative structure. Let $\widetilde{B}$ be a set of points in an analytic dynamic system with an environmnet $\widetilde{U} : |\widetilde{U} - \widetilde{B}| \leq \eta$ such that the system will not leave the environment once it is entered,

$$|F(X) - \widetilde{B}| > \eta \qquad \forall X \in \widetilde{U}.$$

A second environment with distance $\widetilde{\varepsilon}$ around $\widetilde{U}$ with trajectories strictly conveging to $\widetilde{U}$ may be defined as with an attractor, i.e. $\widetilde{U}$ may have a basin of attraction. Nicolis and Prigogine [Nicolis and Prigogine, 1977] proceed to prove the characteristics of dissipative structures by showing that the entropy of dynamic systems is maximized in the vicinity of attractors and dissipative structures, hence providing that with the second law of thermodynamics, the system will converge there.

A.3 Bifurcation Theory

In linear dynamic systems, the Jacobian matrix only contains scalars. Consequently, no linarization for specific locations in the phase space is necessary and the dynamics are

[15] See equation 13 in section A.1 below.

globally constant; the system can also not contain any more complex structures than a single stable or unstable fixed point. For nonlinear dynamic systems, this is not the case any more; as explained above, dynamic properties only hold locally and there may be different attarctors with different properties. What is more, the structure of systems may be discontinuous. The simplest case of this is a set of parameters, exogenous variables, in which the system undergoes discontinuities, also called bifurcations or catastrophes. Fixed points may appear, disappear or change their properties with small or infinitesimal changes in the parameters and so may other attractors.[16]

Let $\frac{\nabla X}{dt} = F(A, X)$ be an n-dimensional dynamic system[17] with n state variables X and and m control parameters A; let the structure of the equations F have discontinuities for certain points $\widetilde{Z}$ but be continuous otherwise. For a some values of $\widetilde{Z}$ in some points of the system's phase space, the convergence properties of the system will change, namely when its dominant eigenvalue becomes 0,

$$Z: \quad \lambda_i(A, X) \leq 0 \quad \forall i \quad \wedge \quad \exists \lambda_i(A, X) = 0 \quad \forall (A, X) \in Z. \tag{14}$$

The resulting structure may be $n + m$-dimensional, $Z \in \Re^{n+m}$. The equilibrium condition

$$\frac{\nabla X}{dt} = F(A, X) = 0$$

yields an also $n + m$-dimensional equilibrium surface

$$V: \quad F(A, X) = 0 \quad \forall X \in V.$$

$V \in \Re^{n+m}$ may be called a surface instead of a (disparate) set as with structural continuity in (most of) A, the fixed points will not also behave continuously and be adjacent to each other in A though not in X. Now consider the intersection of V and Z, $V \cap Z$; this is the system's bifurcation set; the set of points in which the structure of the system's fixed points changes. It is comprised of fixed points of marginal stability. If the functions $\lambda_i(A, X)$ are continuous and the point (A, X) does not happen to be an extreme point of the dominant λ_i, the fixed points become stable and unstable in different adjacent areas in the control space A or cease to exist entirely. The bifurcation set $V \cap Z$ may still have dimension $n + m$ but for simple cases, it may also collapse into the control space $\Re^m$. As an example, consider the growth process in equation 1 with $n = 1$ and $m = 1$ and control parameter α as depicted in figure 1.

A.4 Stochastic Disturbances and Markov Chains

Any dynamic system can be extended into a stochastic dynamic system by adding a (typically small and unbiased) random term drawn from a specific distribution as already mentioned above. The properties of the system including equilibrium set and stability properties will, in this case, remain the same. Of course, it is also possible to

[16]For more thorough introductions, see [Thom, 1975, Nicolis and Prigogine, 1977, Woodcock and Davis, 1978, Rosser, 2000].

[17]While this is a differential equation system, the derivation for difference equation systems is equivalent and straightforward.

integrate larger or biased stochastic processes in dynamic systems; they would interfere with the properties of the system.

A particularly interesting and useful example, discrete-time Markov chains, involves dynamic changes (in discrete time) in a probability distribution over a finite or infinite set of discrete states. The dimensionality of the system would be given by the size of the set as every state requires a development equation.

Call the frequency vector corresponding to the state vector at time t p_t and the frequencies of individual states $p_{t,1}$, $p_{t,2}$, ... $p_{t,n}$,

$$p_t = \begin{pmatrix} p_{t,1} \\ p_{t,2} \\ \dots \\ p_{t,n} \end{pmatrix} \tag{15}$$

For any two states i and j, there are transition probabilities[18] $pr_{ji}(x_{t+1} = i|x_t = j)$ and $pr_{ij}(x_{t+1} = j|x_t = i)$ which are constant for time-homogenous Markov chains (the case most often considered). A transition matrix J could be constructed out of these probabilities

$$J = \begin{pmatrix} pr_{11} & pr_{12} & \dots & pr_{1n} \\ pr_{21} & \dots & & \dots \\ \dots & & & \dots \\ pr_{n1} & \dots & & pr_{nn} \end{pmatrix} \tag{16}$$

which would simultaneously be the Jacobian of a dynamic system

$$p_{t+1} = Jp_t. \tag{17}$$

Markov chains like these may be studied like any other dynamic system with stable steady state distributions (stable fixed points)

$$p_{t+1} = p_t$$

being of particular importance. Note, however, that Markov chains may be infinite, i.e., they may have a (countably or uncountably) infinite number of states; steady state distributions may still exist and may still be computed. An example from evolutionary is the study of the distribution of firm sizes [Heinrich and Dai, 2016, Cordes et al., 2015]; firm sizes in terms of the number of employes are discrete, can thus be captured in Markov chains, but there are infinitely many possible firm sizes. The example will be taken up again below in section B.

A.5 Replicator Dynamics

Another source of interest in systems like the ones considered above (section A.4) from evolutionary perspective is the study of population shares of subpopulations in

[18] Here, x denotes a random variable passing through the Markov chain.

evolutionary systems. Consider a number of subpopulations of a population with absolute sizes s

$$s = \begin{pmatrix} s_1 \\ s_2 \\ \dots \\ s_n \end{pmatrix}. \tag{18}$$

Let the initial[19] growth rates of subpopulation i be α_i and let the population be subject to a capacity boundary z

$$\frac{ds_i}{dt} = \alpha_i s_i \left(1 - \frac{\sum_j s_j}{z}\right). \tag{19}$$

Call the average initial growth rate of the population $\overline{\alpha}$

$$\overline{\alpha} = \frac{\sum_j \alpha_j s_j}{\sum_j s_j} \tag{20}$$

Naturally, total population growth is

$$\frac{d\left(\sum_j s_j\right)}{dt} = \left(\sum_j \alpha_j s_j\right)\left(1 - \frac{\sum_j s_j}{z}\right) \tag{21}$$

with population shares

$$p_i = \frac{s_i}{\sum_j s_j}. \tag{22}$$

Consequently,

$$\begin{aligned}
\frac{dp_i}{dt} &= \frac{\frac{ds_i}{dt}\sum_j s_j - \frac{d\left(\sum_j s_j\right)}{dt} s_i}{\sum_j^2 s_j} \\
&= \frac{\alpha_i s_i \left(1 - \frac{\sum_j s_j}{z}\right)\sum_j s_j - \left(\sum_j \alpha_j s_j\right)\left(1 - \frac{\sum_j s_j}{z}\right) s_i}{\sum_j^2 s_j} \\
&= \alpha_i p_i \left(1 - \frac{\sum_j s_j}{z}\right) - \frac{\left(\sum_j \alpha_j s_j\right)}{\sum_j s_j}\left(1 - \frac{\sum_j s_j}{z}\right) p_i \\
&= \alpha_i p_i \left(1 - \frac{\sum_j s_j}{z}\right) - \overline{\alpha}\left(1 - \frac{\sum_j s_j}{z}\right) p_i \\
&= p_i \left(1 - \frac{\sum_j s_j}{z}\right)(\alpha_i - \overline{\alpha}).
\end{aligned} \tag{23}$$

Hence, the development of the subpopulation's share depends on the current size of that share, p_i, on how close the population is to the capacity boundary, $\left(1 - \frac{\sum_j s_j}{z}\right)$, and on the relative performance of subpopulation i compared to the population average $(\alpha_i - \overline{\alpha})$. The latter term consequently is the auxiliary variable for the subpopulation's evolutionary potential, its fitness. It does not necessarily have to be the growth rate itself, any number of other quantities or combinations thereof could be considered for it. It also does not have to be constant in the population size s_i or in the population

[19] That is, for $\lim_{s_i \rightarrow 0}$

share p_i (or even in time). This allows for richer dynamics beyond the ”survival of the fittest”, including advantages of scale (”survival of the first”), or ”survival of all” to use Nowak’s [Nowak, 2006] terms. It is also possible to consider replicator dynamic systems without capacity boundary, i.e. as if the capacity boundary were infinite, for which case the term $\left(1 - \frac{\sum_j s_j}{z}\right)$ becomes $= 1$ and vanishes. This would essentially abstract from the absolute population sizes and reduce the systems to a pure population share dynamic without any loss of generality in the behavior of the population size (which may be growing, constant, or declining).

Replicator dynamic systems following these lines are frequently considered in evolutionary modeling both in economics and in other fields. For more thorough introductions, see [Taylor and Jonker, 1978, Schuster and Sigmund, 1983, Nowak, 2006].

A.6 Agent-Based Modeling

So far, all the considered methods and systems exclusively operate on an aggregated layer - as if individuals would not exist or as if their actions and interactions were of negligible importance, easily subsumed into aggregated quantities. It is not clear if and in which cases this holds for real systems; in fact, recent advances in evolutionary economics [Nelson and Winter, 1974, Nelson and Winter, 1982, Silverberg et al., 1988, Conlisk, 1989, Kwasnicki, 1996], network theory [Newman et al., 2001, Pyka and Saviotti, 2002, Newman, 2003, Boccaletti et al., 2006, Uchida and Shirayama, 2008, Delre et al., 2007, Lee et al., 2006, Vitali and Battiston, 2013, Stephen and Toubia, 2009, Battiston et al., 2007, Barash et al., 2012, Schweitzer et al., 2009, Iori et al., 2008] and in complex systems [Nicolis and Prigogine, 1977, Foley, 1998, Kirman, 1997, Lux and Marchesi, 1999, Mandelbrot and Hudson, 2004, Farmer and Lillo, 2004, Taleb and Tapiero, 2010, Taleb, 2009, Mandelbrot, 2009, Elsner, 2013] suggest it typically does not. At the very least, it would be necessary to show that the micro level can safely be neglected without changing the macro-behavior of the system. For some systems, aspects and models, this is likely true, for many others, it is not.

Considering the layer of individual agents explicitly allows to realistically model decision making heuristics while taking findings from anthropology, psychology and behavioral economics [Fehr et al., 2002, Henrich, 2004, Cordes, 2006, Kahneman, 2011] into account. It further allows to model interactions directly, thereby allowing for feedbacks on other interactions. Also, the network structure between agents and its effect on interaction patterns and aggregated level outcomes can be studied.

Agent-based modeling started with the advent of game theory, psychology and behavioral theories in economics [Cyert et al., 1959, Smith, 1962]; its success, however, began with Nelson and Winter’s [Nelson and Winter, 1974, Nelson and Winter, 1982] formalization of Schumpeter’s theory of innovation. Many of the subsequent models in evolutionary economics and complexity economics including the ones cited above, in the first paragraph of this section, make use of agent-based modeling.

Formally, agent-based modeling comes down to describing decision making and interaction patterns of agents in individual terms. This does not seem radical at all as it has been common practice in neoclassical rational optimization. The departure from the neoclassical theory comes when optimization is replaced by a "boundedly rational" or heuristical approach, when heterogenity among agents is allowed and especially when no attempt is made at aggregating individual decisions into some collective demand/supply pattern of collective action plan. Instead, local interaction patterns can be studied with methods from game theory; competition between strategies or firms can be modeled using replicator dynamics. Though this certainly involves aggregation again,[20] some results for the aggregated or global level can be obtained by considering feedback loops in individual decision making: Do for instance decisions of the first movers compell everyone else to follow suite? This is the case in models of network externalities as studied by Arthur and others [Arthur et al., 1982, Arthur et al., 1987, Arthur, 1989, David, 1985, Carrillo Hermosilla, 2006]. Generalized urn schemes and other methods have successfully been employed in such systems [Arthur et al., 1982, Arthur et al., 1987, Dosi et al., 1994, Dosi and Kaniovski, 1994].

Agent-based modeling, itself of tremendous importance in evolutionary economics, is typically used in conjunction with simulation which is considered in the following section.

A.7 Simulation and Agent-Based Simulation

Though feedback mechanisms are also of crucial and sometimes surprising importance in other systems (such as systemic risk [Battiston et al., 2007, Delli Gatti et al., 2008, Taleb and Tapiero, 2010]), in most cases analytical methods fail to perform sufficiently well without utilization of a tool that can cope with higher levels of complexity; i.e. simulation, or rather agent-based simulation [Pyka and Fagiolo, 2005, Gräbner, 2015, Elsner et al., 2015].

Simulation is not an exact method; instead of analytically deriving fixed points and functional solutions, the development of the systems is modeled and observed for a set of selected initial values. As phase spaces of dynamical systems may be non-discrete and infinite in several (or many) dimensions, it is impossible to include every possible set of initial conditions. Instead, depending on the nature of the simulation study, either the focus is placed on a specific region that perhaps corresponds to empirical observations or the phase space is sweeped by selecting a set of initial values that is uniformly distributed across the phase space. The first approach would deliver a forcast for the development for the selected initial condistions and would mostly be used in predictive simulations following the KIDS approach (see below). The second, generally employed in explorative simulations, would give a comprehensive picture of dynamic movements in the phase space; the denser the phase space is covered, the more reliable the resulting picture will be.

Simulation can be and is used in many contexts: Mathematical quantities or objects that are currently impossible, too complicated or just too costly to compute exactly

[20] Sometimes, however, the effect of employing different aggregarion mechanisms is also studied.

can be approximated with Monte Carlo simulations; rich and large agent-based models that would involve hundreds or thousands of equations are conveniently simulated with the help of computer technology[21]; and control theory also involves simulation of less complicates systems to rule out mistakes in the analytical derivation of projects before these projects are realized - both in engineering, production planning, and with numerous applications in economic policy. The historically earliest simulations were virtually all Monte Carlo simulations; they are still common and valued in theoretical sciences for their efficiency - In statistics, one common approach is also known as 'bootstrapping' (of distributions or quantities that are otherwise unknown[22] Agent-based simulations became more common with the advent of behavioral economics [Cyert et al., 1959, Smith, 1962] and especially with Nelson and Winter's famous simulation studies in the 1970s [Nelson and Winter, 1974, Nelson et al., 1976].

As discussed in more detail in Pyka and Fagiolo [Pyka and Fagiolo, 2005] or Elsner et al. [Elsner et al., 2015, chapter 9], there are two major approaches to agent-based simulation: On the one hand, there is the KIDS approach - Keep it Descriptive, Stupid - which is designed to include as much of the real system under investigation as possible in order to generate accurate predictions. On the other, there is the KISS approach - Keep it Simple, Stupid - which attempts to isolate and understand crucial mechanisms responsible for observed phenomena or stylized facts. The second approach is not so much predictive but rather explorative, testing different configurations and sweeping their phase space until all plausible explanations for the phenomena are discovered. In practice, resource constraints and practicality often force the investigation to take hybrid approaches.

The standard procedure for conducting simulations can be summarized in eight steps

1. The system under investigation is selected,
2. it is algorithmicized,
3. to then be transformed into a computer program.
4. The program is then set up by selecting the set of initial values for which
5. the simulation is then run.
6. Further, the simulation has to be validated - ensuring that it indeed simulates the selected system under investigation -
7. and its results verified to ensure reliability and reproducibility for good scientific practices.

[21]Note that this number includes development variables for properties of the agents, e.g. their capital stock, that may in many cases be similar or identical for many agents.

[22]E.g., the plausibility of a fit of a certain sample to a specific distribution may be assessed with a Kolmogorov-Smirnov-test: A large number of artificial (simulated) samples of equal size are drawn from the fitted distribution and their goodnesses of fit are compared to the original sample, the fit is only considered plausible if the original sample performs better than a certain share, typically 10% of the simulations.

8. Finally, conclusions for the real system under investigation are drawn.

There are, however, several challenges.

First of all, as simulation is not an exact method, it should be avoided whenever possible. If simulation is indeed used, a good justification must be offered. This may involve that the system is sufficiently well-understood to validate the plausibility of the simulation results; in agent-based modeling, the complexity of the system under investigation typically plays a role, as it would be impractical to compute analytical results in systems of hundreds or thousands of equations. Further, the simulation must be documented sufficiently well to allow other researchers to replicate it.[23]

Second, as simulations are essentially artificial reconstructions of finite time-iterations of the system, the simulation model has to be discrete. For many systems in reality, this is neither the case nor an immediately plausible approximation; reality is continuous. Many simulation models choose algorithmic specifications instead of being modeled after mathematical equations, but the problem remains. To illustrate it, consider a continuous (i.e., differential equation) system

$$\frac{\nabla X}{dt} = F(X).$$

The immediate discrete equivalent is

$$X_{t+1} = F(X) + X$$

but in this case the implicit assumption is made that time steps of a length $t = 1$ are used. As shown above in section 2, this does not necessarily generate the same dynamic behavior since the dynamic change is essentially linearized at point X and stretched to cover a time distance $t = 1$. In areas of localized changes between dynamic patterns (such as around attractors, at the tipping points between basins of attraction, etc.) the coarse-grainedness of the discrete dynamic may accidentally put the system into a different basin of attraction of effect other considerable changes. The obvious solution is to choose a more fine-grained dynamic, a shorter time step size, e.g. $t = \frac{1}{100}$,

$$X_{t+1} = \frac{F(X)}{100} + X.$$

There are, however, obvious limits as this also increases the computation requirements by a factor of 100. Of course, the problem of discreteness does not only exist for the time steps but also for values that need to be rounded, objects, etc.

Third, agent-based models typically involve stochastic components, which, in turn, almost always makes results of single simulation runs non-reproducible. What researchers therefore work with is averaging across massive numbers of simulation runs; the exact sample size for this varies depending on the simulation and depends on when the emerging distribution becomes verifiable. Of course, aggregations across many simulation runs

[23] Replication is an emerging field of its own [Edmonds and Hales, 2003, Zimmermann, 2015]; though simulation studies are a major aspect here, the field is, of course, also relevant for econometrics etc.

are much less illustrative and also typically reduce the results to the quantity for which the distribution or statistical property is computed which may hide other important aspects of the simulation.

Fourth, there are aspects of decision making in real systems that can not be represented adequately, either because of insufficient understanding of these aspects or because of ontic constraints; e.g. true uncertainty can not be represented (though decision making under true uncertainty can still be approximated with an adequate theory of it).

And finally, simulation studies, particularly agent-based simulations are subject to model selection problems just like other methods in quantitative economics. If a sufficiently large number of sufficiently complex simulation models is considered, the researcher is bound to eventually find one that just by chance seems to fit the empirical pattern the researcher attempts to explain. The standard solution for this is, of course, to make the exact course of the investigation transparent, to specify the designs to be considered beforehand, and to offer descriptive interpretations and justifications. Note that this argument holds for the selection of the functional form of the model, not for the sweeping of the parameter space in search of dynamics that reproduce empirical facts as the latter would be akin to fitting an econometric model, not to the selection of the model specification.

While it remains obviously necessary to deal with these challenges case by case, agent-based simulation has - for good reasons as explaned above - come to be one of the most important tools in evolutionary economics with an abundance of studies published in the literature [Nelson and Winter, 1974, Nelson et al., 1976, Nelson and Winter, 1982, Silverberg et al., 1988, Conlisk, 1989, Kwasnicki, 1996, Albin and Foley, 1992, Silverberg and Lehnert, 1994, Gilbert et al., 2001, Delli Gatti et al., 2005, Lux, 2006, Silverberg and Verspagen, 2007, Battiston et al., 2007, Saviotti and Pyka, 2008, Uchida and Shirayama, 2008, Saviotti and Pyka, 2013, Sugiarto et al., 2015, Lengnick et al., 2013, Mueller et al., 2015].

B Self-Organizing Systems

After the brief overview over common methods in evolutionary economics in section A, this section will continue to discuss the role of evolutionary modeling in evonomics.

As mentioned in section above, the strongest argument for evolutionary modeling is that reality itself in many fields appears self-organizing, non-static and evolutionary - in the wide sense as described above: There are dissipative structures that remain stable for considerable amounts of time though they are subject to changes and to processes of adaptation. The argument was put forward that this is what enables us to purposefully interact with our environment at all, to understand it and to make predictions about its future development, both in terms of scientific forecasting and in terms of every-day interaction with our environment. The present section will elaborate on examples for self-organization, especially but not exclusively those relevant to economic systems.

There are many famous and well-studied examples for self-organizing processes in natural sciences. Prigogine and his collaborators [Nicolis and Prigogine, 1977, Prigogine and Stengers, 1984] studied the classical example of self-sustained chemical reactions. This may be extended into biochemistry: There are self-reproducing molecules [Lehn, 2002] and self-reproducing proteine-folding patterns [Halfmann and Lindquist, 2010]; such structures are hypothesized to be connected to the origines of life. The same self-reproducing or at least self-sustaining abilities must be attributed to ecosystems; they also tend to remain stable over periods of time.[24] Further, certain quantities tend to follow very specific and stable distributions; consider the sizes of (unmanaged) dense forrest areas on the one hand and of forrest fires on the other hand; they have been argued to reliably follow power laws [Bak et al., 1990].[25] In fact, while the distribution does change and may vary and deviate considerably from the ideal power law, the greater vulnerability of larger forrests to fires restores the original distribution. The same is true for frequencies of words in natural languages, city sizes, as well as a number of other quantities [Newman, 2005]; there are also examples in economics (see below). Finally, as the list draws closer to social and economic systems, human societies should also be mentioned. As Georgescu-Roegen and others [Georgescu-Roegen, 1975b, Georgescu-Roegen, 1975a, Daly, 1968] vehemently argued, humanity will not indefinitely be able to continue to houshold with the planet's resources as it is now; nevertheless, the state is remarkable: In the paleolithic, the ecological carrying capacity of Earth amounted to perhaps one or a few million humans; humanity managed - without centralized planning - to extend this 7000-fold to 7 billion by means of domestication of plants and animals for food and energy, more efficient organization of labor, and populating even remote and inhospitable corners the planet.

On the definitions of the closely related terms complexity, emergence, and self-organization, no scientific consensus has emerged yet. There is a large number of candidate definitions, semantic and syntactic ones; see Elsner et al. [Elsner et al., 2015, chapter 11] for an overview. However, some aspects are typically considered under this label, including, among other things, highly skewed distributions in general and power law distributions in particular. This is because such distributions are, other than, say, Gaussians, rather unlikely to appear at random. The central limit theorem provides that sums over sufficiently many independent elements of well-behaved distributions (distributions with existing, finite moments) will converge to distributions of the Gaussian family. In Gaussians, the probabilities of observations in the tails (i.e. very large or very small observations) are comparatively small. Traditional approaches tend to rely on this; so much so, that it came as a major surprise when it emerged that many empirically measured quantities are not Gaussian but skewed and heavy-tailed (with a much larger probabilities for tail observations). Attempts to model this include distributions of the lognormal family

$$pr(x) = x^{-1} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\log x - \mu)^2}{2\sigma^2}}, \tag{24}$$

[24] Of course, there are exceptions, catastrophic failures and changes; consider the collapse of pre-cambrian (ediacaran) ecosystems as an example [Erwin et al., 2011]; there are several more mass extinction events in the palaeobiological records.

[25] Note that later research found for this particular case that the extents of forrest fires follow another heavy-tailed distribution [Newman, 2005].

of the exponential family

$$pr(x) = \lambda e^{-\lambda x} \tag{25}$$

and of the power law family

$$pr(x) = Cx^{-\alpha}. \tag{26}$$

While the prevalence of lognormal distributions in modeling may well be due to the fact that they can easily be transformed into Gaussians[26] for further computations, any stable heavy tailed distribution including lognormals are more difficult to explain. At the very least, there must be some non-trivial dynamics involved that drive the divergence of the tail observations.

To consider a few economic examples:

The firm size distribution has been analyzed to be power law distributed [Ijiri and Simon, 1977, Gabaix, 1999, Axtell, 2001, Delli Gatti et al., 2005, Luttmer, 2007, Gaffeo et al., 2003, Zhang et al., 2009, Coad, 2010, Dahui et al., 2006, Heinrich and Dai, 2016] though there are some more careful voices that characterize the distribution just as heavy-tailed and raise doubts that it actually follows a power law [Dosi, 2007, Bottazzi and Secchi, 2006, Bottazzi et al., 2007, Dosi et al., 2015]. What nobody disputes, however, is that self-organizing processes are at the core of the observed pattern whichever distribution it actually adheres to. Some scholars tried to explain it as being dominated by other power law distributed quantities [Helpman et al., 2004] but this only shifts the explanation into another domain without avoiding it.

The study of the firm size distribution is also one of the major applications of Markov chain dynamic systems in evolutionary economics [Cordes et al., 2015, Heinrich and Dai, 2016]. Consider a Yule process of interconnected firms of size b; firms may with a certain probability expand their business - represented here as the firm growing by 1 and adding a new supplier, a firm of initial size 1; if the growing firm is already a supplier of another firm, the growth causes synergies for that firm and it will also grow by 1. This model is extremely stylized and strictly hierarchical (which is not the case for firm supply networks in reality) but it may serve to illustrate the present point. Note that sizes b are in this model always identical to the size of the supplier network. The resulting model (following [Newman, 2005]) is a Markov chain of firm sizes b and corresponding absolute frequencies $f(b)$ and relative frequencies $p(b)$; n is the total number of firms. Each iteration adds a firm of size 1, hence Markov state $f(1)$ will with certainty grow by 1; for all other Markov states $f(b > 1)$ growth has a probability of $\frac{b-1}{n} f(b)$ (the probability that any single element of size $b-1$ grows to size b multiplied by the number of elements $b-1$, $f(b-1)$). At the same time, all Markov states face the possibility that one element grows to size $b+1$ in which case Markov state b would decrease in size as it lost an element. The dynamic system is

$$\begin{aligned} f_{n+1}(1) &= f_n(1) - \tfrac{1}{n} f_n(1) + 1 \\ f_{t+1}(b) &= f_n(b) - \tfrac{b}{n} f_n(b) + \tfrac{b-1}{n} f_n(b-1) \qquad \forall b > 1 \end{aligned} \tag{27}$$

[26]The logarithm of a lognormal distribution is a Gaussian normal distribution.

The steady state conditions are

$$\begin{array}{ll} f(1) & = f(1) - \frac{1}{n} f(1) + 1 \\ f(b) & = f(b) - \frac{b}{n} f(b) + \frac{b-1}{n} f(b-1) \qquad \forall b > 1 \end{array} \tag{28}$$

or, written with relative frequencies using $p_n(b) = \frac{f_n(b)}{n}$

$$\begin{array}{ll} (n+1)p(1) & = np(1) - p(1) + 1 \\ (n+1)p(b) & = np(b) - bp(b) + (b-1)p(b-1) \qquad \forall b > 1 \end{array} \tag{29}$$

It follows that $p(1) = 1/2$ and (where $\sim$ gives the approximation for the tail, i.e. for large b)

$$\begin{array}{ll} p(b) = \frac{b-1}{b+1} p(b-1) = \frac{2(b-1)!}{(b+1)!} p(1) & = \frac{2(b-1)!}{(b-1)!b(b+1)} p(1) \\ & = \frac{2}{b^2+b} p(1) = \frac{2}{b^2+b} \sim b^{-2}. \end{array} \tag{30}$$

which is the definition of a power law tail with exponent $\alpha = 2$.

Spacial agglomerations too are a very asymmetric example of economic systems. Economists have long been aware of this and of the fact that it changes economic dynamics [Marshall, 1890, Krugman, 1994, Krugman, 1997, Saxenian, 1996]; at least for urban agglomerations, it is established that they follow power law distributions [Newman, 2005], but it is safe to say that spacial and regional industrial organization is highly skewed and heavy tailed with large industrial agglomerations forming the tails events.

Another instance of highly skewed patterns in economics is industry structure; the prevalence of oligopolistic industry structures prompted the not only the very early development of theories of oligopolistic competition [Cournot, 1838] but also led Schumpeter [Schumpeter, 1943] to theorize about their role in innovation. The formalization of Schumpeter's theory by Nelson and Winter [Nelson and Winter, 1974, Nelson and Winter, 1982] provided an explanation for the origin of the asymmetry: firms that by chance gain a technological advantage are able to grow and will also be able to invest more in future research; Nelson and Winter's model leads to a fairly quick oligopolization of an initially perfectly evenly distributed industry structure [Heinrich, 2013]. This is potentially amplified by the presence of increasing returns [Kaldor, 1940, Lorenz, 1987] and by network externalities which likely play a prominent role in innovation and technological change [David, 1985, Arthur, 1989, Heinrich, 2013, Heinrich, 2014b]. Further effects that may interfere are compatibility dynamics, technological niches and deliberate management of the same [Heinrich, 2013, Heinrich, 2015]. It has been emphasized previously that open standards may help avoid negative fallout of the skewed industry structure [Gallaway and Kinnear, 2002, Gallaway and Kinnear, 2004, Simcoe, 2006, West, 2007, Heinrich, 2013].

Heavy tailed and power law distributions have also been found for financial market data series, specifically both logarithmic returns of assets and their trading volume are power law distributed [Lux and Marchesi, 1999, Mandelbrot, 2001, Gabaix et al., 2003, Farmer and Lillo, 2004, Lux, 2006, Gabaix et al., 2007, Sornette, 2009, Heinrich, 2014a]. It is as yet not clear how these patterns, first investigated by Mandelbrot in the 1960s [Mandelbrot and Hudson, 2004], emerge; Lux [Lux, 2006] provides an overview over the available approaches and explanations. While universally stable distributions were earlier hypothesized with an exponent of $\alpha \approx 3$ [Gabaix et al., 2003], work by the present author [Heinrich, 2014a] indicates that this is probably not the case and the exponents are probably subject to change. In financial systems, heavy tails have especially the consequence of additional difficulties in the prediction of development and associated risk of assets, of fragility and under certain conditions of systemic risk if prices behave unexpectedly and too many financial actors fail [Delli Gatti et al., 2005, Battiston et al., 2007, Taleb and Tapiero, 2010]. In a world that is dominated by financial sector dynamics - today more than ever - this may not only lead to damage to the real economic sector, it will also interact with the asymmetric industry structure considered above: large clearing houses already speculate with basic commodities, but the game gradually becomes available to the average financial market participant by way of exchange traded funds (ETFs) and other instruments.

Another solid regularity far from equilibria in economic systems are business cycles, investment cycles, growth cycles etc. The circular pattern has been observed and investigated for some time [Goodwin, 1967, Freeman and Perez, 1988, de Groot, 2006, Diebolt and Doliger, 2006, de Groot and Franses, 2008, Bormotov, 2009] with the different theories on their origin being put forward ranging from a statistical result of natural or artificial moving averages [Slutzky, 1927] to predator prey systems [Goodwin, 1967] to pro-cyclical investment patterns in financial markets [Minsky, 1980, Keen, 1995, Stockhammer and Michell, 2014] to increasing returns [Kaldor, 1940, Lorenz, 1987] and finally to rather irregular growth wavelets caused by radical innovations [Silverberg and Lehnert, 1994, Chen, 2005, Heinrich, 2013].

Further examples of self-organization in economics include regularities in technological progress and hence long-term economic development (Wright's law [Nagy et al., 2013, Farmer and Lafond, 2016]) and the emergence and development of institutions and the institutional structure [Veblen, 1899, Waller, 1982, Bush, 1987]. Developing formal models in these fields have recently increasingly attracted attention, namely by Nagy and Farmer and collaborators [Nagy et al., 2013, Farmer and Lafond, 2016] in the case of Wright's law and by a number of scholars in the case of institutional emergence [Villena and Villena, 2004, Elsner, 2012, Heinrich and Schwardt, 2013].

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