Welfare Analysis of Cournot and Bertrand Competition With(out) Investment in R D

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Abstract

I consider the model of a differentiated duopoly with process R&D when goods are either substitute, complements or independent. I propose a non-cooperative two-stage game with two firms producing differentiated goods. In the first stage, firms decide their technologies and in the second stage, they compete in quantities or prices. I evaluate the social welfare within a framework of Cournot and Bertrand competition models with or without investment in research and development. I prove that the Cournot price can be lower than Bertrand price when the R&D technology is relatively inefficient; thus, Cournot market structure can generate larger consumer's surplus and welfare.

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1 Introduction

The comparison of Cournot and Bertrand results in a static oligopoly setting have extensively been studied in the literature. This paper focuses on welfare under Cournot and Bertrand competition in differentiated oligopolies. From the literature, it is known that Bertrand competition yields lower prices and profits and higher consumer surplus and welfare than Cournot competition (e.g; [2],[8],[9],[10] and [14]). Accordingly, exploiting cost asymmetries including some other factors like research and development (R & D) or endogenous privatization; some researchers like [1], [4], [5] and [6] have constructed models where at least one of the latter conclusions fails to hold. One important scope of the regulator in the economy is to get a better model that maximizes social welfare (sum of total surplus net of total cost and other negatives externalities). Welfare comparison within Cournot and Bertrand models can solve a social planner's problem who wish to choose a better model in market competition to improve the well-being of individuals in a society.

In this paper, I consider a non-cooperative, two-stage game with two firms producing differentiated goods. In the first stage, firms independently decide their R&D (no spillovers effects) investment that determines production technology and, in the second stage, they compete in quantities or prices. The linear demands are derived from the utility maximizing problem of the representative consumer. This paper generalizes the work of [6] in the sense that R&D technology function in their paper is a specific case of this model. The main works of this paper can be summarized as follows:

1. Firstly, I compare the welfare under both models when firms are not investing in research and development and my results are consistent with the literature (e.g, [10]). As my first contribution, I formally show that Cournot and Bertrand competition models equilibria coincide if and only if the products are independent. Then, I compare the welfare when products are either independent, substitutes or complements.

2. Secondly, taking into account the investment in research and development makes the welfare comparison under both models more interesting. After presenting the equilibrium outcomes of each model, I discuss the well-being of consumers under the two possibilities offered to the firms, investing or not in R&D. At the end, a discussion is given to evaluate the effects of R&D size on different equilibrium outcomes. These comparative statics lead to a condition on which social welfare is better under Cournot model relative to Bertrand model. All the results found in this framework can be classified among situations where the pioneering findings of [10] do not necessarily hold. As my second contribution, I prove that Cournot price can be lower than

Bertrand price when R & D technology is relatively inefficient; thus Cournot market structure can generate larger consumer's surplus and welfare.

The remainder of the paper proceeds as follows. Section 2 provides a brief literature review of comparing welfare through Cournot and Bertrand models using R&D process. Section 3.1 and 3.2 describe the equilibriums along with its implications on the welfare within a framework of Cournot and Bertrand models. Section 4 examines the comparative statics. Section 5 concludes. The main proofs are collected in the appendix.

2 Literature review

In a competitive market, firms often compete against each other by investing in research and development in order to improve product quality (in the case of product R&D) and/or to reduce production cost (in case of process R&D). The important difference between process and product R&D as explained by [13] is that the product R&D directly affects gross consumer surplus. This is because product R&D raises product quality, and quality enters directly into each consumer's utility function. Process R&D affects gross consumer surplus only indirectly through a reduction in marginal cost and a consequent increase in output.

The traditional results (e.g; [2],[8],[9],[10], and [14]) mentioned above about the efficient equilibrium outcomes under Bertrand model relative to Cournot model are generally obtained under the assumption that firms face the same demand and cost structures. In a dynamic environment, if the R&D investments differ across the Bertrand and Cournot models, the post-innovation demand and cost structures will also differ, even though they were identical before the investment. The question which follows is whether the traditional results are affected in any way. Given the above difference between process and product R&D, it is unclear whether the results from models with process R&D carry over to the case of product R&D.

Qiu([9]) focused on cost-reducing R&D with spillover effects and reevaluated the relative efficiency of Cournot and Bertrand equilibria. He considered a non-cooperative, two stage game with two firms producing differentiated goods. In the first stage, which is called R&D stage, each firm independently sets out cost-reducing R&D. In the second stage called market stage, both firms produce and compete in prices or quantities. He found that "Cournot competition induces more R&D effort than Bertrand competition. The price is low and output is larger in Bertrand than in Cournot model. The traditional efficiency result holds if either R&D productivity is low, or spillovers are weak, or products are very different. However, if R&D productivity is high, spillovers are strong, and goods are close substitutes, then Bertrand equilibrium is less efficient than the Cournot equilibrium ". The latter challenged the traditional efficiency result.

Symeonidis [13] studied a variant of the standard linear demand model. He focused on product R&D and used the quality-augmented linear structure proposed by [11], [12] and introduced a cost structure that allows for R&D spillovers. He found almost the same results as [9]: "Indeed, R&D expenditure, prices and firm's net profits are always higher under Cournot competition than under Bertrand competition. Output, consumer surplus and total welfare are higher under price competition than under quantity competition if either R&D spillovers are weak or products are sufficiently differentiated. Furthermore, if R&D spillovers are strong and products are not too differentiated, then output, consumer surplus and total welfare are lower in the Bertrand case than in the Cournot case ".

Motta [7] analyzed two versions of a vertical product differentiation model, one with fixed and the other with variable costs of quality. The case of fixed cost of quality improvement may be thought of as a situation where firms should engage in R&D and advertising activities to improve quality. He found that economy is better off when firms compete on prices (with fixed costs of quality, not only consumer but also producer surplus is higher under price competition). Compare to the model of Symeonidis ([13]), the study of Motta ([7]) did not allow for R&D spillovers.

Delbono and Denicolo ([3]) compared the equilibrium R&D investment under price and quantity competition in a symmetric and homogenous oligopoly. They found that the welfare comparison is generally ambiguous in the context of a homogenous product duopoly with process R&D in the form of patent race. They have shown that: "although the R&D investment is greater under price competition (in fact, it is even higher than the social optimal level), social welfare net of R&D cost, may be greater under Cournot competition ".

This paper focuses on differentiated duopoly with process R&D with no spillovers when goods are either substitutes, complements or independent. The R&D function considered in this setting is the generalized form of the one used by [6] in a non-cooperative two-stage game with two firms producing differentiated goods. In the first stage, firms decide their technologies and in the second stage, they compete in quantities or prices. I am comparing the social welfare (sum of net consumer surplus and total producer surplus) within a framework of Cournot and Bertrand competition models with or without investment in R&D. The results are consistent with those cited above (traditional results) and with those found by [6] for the specific case of R&D. At the equilibrium, Cournot firms invest a larger amount on R&D than the Bertrand firms. Contrary to the traditional results, Bertrand price can be higher than Cournot price. It is proven that this phenomenon happens in the case where the R&D technology is relatively inefficient; thus Cournot market structure can generate larger consumer's surplus and welfare.

3 The model

I consider a non-cooperative, two-stage game with two firms producing differentiated goods. In the first-stage, firms independently decide their R&D investment that determines production technology. In the second stage, firms simultaneously and non-cooperatively decide their quantity or prices depending on whether there is Cournot competition or Bertrand competition. I am using backward induction to solve the game, that is, first solve the second stage problem and then the first stage, taking the behavior of firms at the second stage. Assuming that the R&D technology is given by :

$$R(c_i) = r\frac{c_i^{-e}}{e} \tag{1}$$

Where $e \geq 1$, r is a positive parameter, c_i the firms i's marginal cost of production. It shows that the higher the value of r, the higher is the R&D cost, the lower the efficiency.
Following [10], I assume that the representative consumer's preferences are described by the utility function:

$$U(q_1, q_2) = a(q_1 + q_2) - b(q_1^2 + 2\theta q_1 q_2 + q_2^2)/2 + m, \tag{2}$$

with q_i is the quantity of firm's production, $a > 0$, $b \in [0, 1]$, m is the numeraire good (composite good) and $\theta \in [0, 1[$ is a positive parameter. To derive the demand function, we need to solve the following utility maximization problem:

$$\begin{gathered} \max_{q_1, q_2} \quad \left\{U(q_1, q_2) = a(q_1 + q_2) - b(q_1^2 + 2\theta q_1 q_2 + q_2^2)/2 + m\right\} \\ s.t\ I = p_1 q_1 + p_2 q_2 + m \end{gathered} \tag{3}$$

where I is the consumer's income. Rearranging the constraint of maximizing problem (3) by writing $m = I - p_1 q_1 - p_2 q_2$ and substituting into utility function gives the following equivalent maximization problem:

$$\max_{q_1, q_2} \quad \left\{U(q_1, q_2) = a(q_1 + q_2) - b(q_1^2 + 2\theta q_1 q_2 + q_2^2)/2 + I - p_1 q_1 - p_2 q_2\right\}\ . \tag{4}$$

The first order condition for maximization problem is given by:

$$\frac{\partial U(q_i, q_j)}{\partial q_i} = a - b(q_i + \theta q_j) - p_i = 0;\ \ i, j = 1, 2\ i \neq j. \tag{5}$$

Solving (5) for p_i gives :

$$p_i = a - b(q_i + \theta q_j),\ i, j = 1, 2\ i \neq j. \tag{6}$$

Solving (6) for q_i yields :

$$q_i = \frac{a}{b(1+\theta)} - \frac{1}{b(1-\theta^2)}p_i + \frac{\theta}{b(1-\theta^2)}p_j;\ i, j = 1, 2\ i \neq j. \tag{7}$$

More generally, the demand function for good i derived from this type of utility function can be written in the form:

$$q_i = \alpha - \beta p_i + \gamma p_j \quad i, j = 1, 2; i \neq j. \tag{8}$$

I assume that $\beta > \gamma$ and $\alpha > (\beta - \gamma)c$. In this specific case, I have :

$$\alpha = \frac{a}{b(1+\theta)},\ \beta = \frac{1}{b(1-\theta^2)},\ \gamma = \frac{\theta}{b(1-\theta^2)}. \tag{9}$$

These notations will be useful later. In what follows, I calculate the optimal output and prices under Cournot competition model and Bertrand model with or without investment in R&D. In each competition model, I derive the optimal social welfare and make some comparisons.

3.1 Cournot and Bertrand competition without investment in R&D

3.1.1 Cournot model without investment in R & D

In this set up, $R(c_i) = 0$, so that there is a one short game where firms maximize simultaneously their profits, each firm taking the quantity of other firm as given. From the fact that the demand function is given by equation (6), the inverse demand function for each firm is given as follows:

$$p_i = \frac{\alpha}{\beta - \gamma} - \frac{\gamma}{\beta^2 - \gamma^2}q_j - \frac{\beta}{\beta^2 - \gamma^2}q_i,\ i \neq j. \tag{10}$$

For simplification, I will denote

$$\omega = \frac{\alpha}{\beta - \gamma},\ \phi = \frac{\gamma}{\beta^2 - \gamma^2},\ \lambda = \frac{\beta}{\beta^2 - \gamma^2}. \tag{11}$$

The values of ω, λ, ϕ are positive and the inverse demand function can be rewritten as follows:

$$p_i = \omega - \phi q_j - \lambda q_i,\ i \neq j. \tag{12}$$

I assume that firms have the same marginal cost i.e $c_i = c_j = c$ and $c > a$, so the following lemma provides the Cournot equilibrium price, quantity and profit for each firm.

Lemma 3.1. *The Cournot equilibrium* (q^c, p^c, π^c) *is given by* $q^c = \frac{a-c}{b(2+\theta)}$, $p^c = \frac{a+c(1+\theta)}{2+\theta}$ *and* $\pi^c = \frac{1}{b}[\frac{a-c}{2+\theta}]^2$.

From Lemma 3.1, the equilibrium price is higher than the marginal cost ($p^c - c = \frac{a-c}{2+\theta} > 0$) and each firm makes positive profit. The equilibrium consumer surplus $CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2$, total producer profit ($\Pi = \pi_1 + \pi_2$), and welfare (W = total surplus) in Cournot competition are given as follows :

$$CS^c = \frac{(1+\theta)}{b}(\frac{a-c}{2+\theta})^2 + m \ ; \ \Pi^c = \frac{2}{b}(\frac{a-c}{2+\theta})^2 \ ; \ W^c = CS^c + \Pi^c = \frac{(3+\theta)}{b}(\frac{a-c}{2+\theta})^2 + m \quad (13)$$

3.1.2 Bertrand model without investment in R&D

In this framework, I still have $R(c) = 0$, and firms solve simultaneously their usual profit-maximization problem, in order to obtain their optimal prices, each firm taking the price of other firm as given. The fact that I assumed the same constant marginal cost implies that, at equilibrium, firms produce the same quantity, sell at the same price and earn the same positive profit. The following lemma generalizes the Bertrand equilibriums quantity, price and profit.

Lemma 3.2. *The Bertrand equilibrium* (q^B, p^B, π^B) *is given by :*
$p^B = \frac{a(1-\theta)+c}{2-\theta}$, $q^B = \frac{a-c}{b(1+\theta)(2-\theta)}$ *and* $\pi^B = \frac{1-\theta}{b(1+\theta)}[\frac{a-c}{2-\theta}]^2$.

According to lemma 3.2, $p^B - c = \frac{(1-\theta)(a-c)}{2-\theta}$, $a > c$ and $\theta < 1$, it follows that each firm sells products at the price above marginal cost and earns a positive economic profit. The equilibrium consumer surplus CS^B, the total producer profit Π^B, and welfare (W^B = total surplus) in Bertrand competition are given by:

$$CS^B = \frac{1}{b(1+\theta)}(\frac{a-c}{2-\theta})^2 + m \ ; \ \Pi^B = \frac{2(1-\theta)}{b(1+\theta)}(\frac{a-c}{2-\theta})^2 \ ; \ W^B = CS^B + \Pi^B = \frac{(3-2\theta)}{b(1+\theta)}(\frac{a-c}{2-\theta})^2 + m \quad (14)$$

The results (Lemmas 3.1 and 3.2) found above clearly show that the equilibrium at the market is different in both Cournot and Bertrand models. In fact, in Cournot competition model, firms find the optimal quantities to maximize their profits rather than solving for the optimal prices as in the Bertrand competition model. Hence, social welfare is different according to each market equilibrium. What happens to these different levels of social welfare if both equilibrium outcomes coincide? We might be interested in looking under which model, society is better off, Cournot or Bertrand competition? In what follows, I provide one condition for which the equilibrium outcomes coincide and I study the consequences of this condition on social welfare.

Theorem 3.1. *The Cournot equilibrium and the Bertrand equilibrium coincide if and only if the goods are independent, i.e the parameter* θ *in the linear demand function (6) is zero.*

The proof of this theorem is provided in the appendix. The result of Theorem 3.1 is consistent with the traditional results found by [10] that at the equilibrium market, prices are lower and outputs are higher under Bertrand competition compared to Cournot competition for substitutes or complements goods. Moreover, profits are larger, equal or smaller in Cournot than in Bertrand competition, depending on whether the goods are substitutes, independent, or complements. Here, with goods that are substitutes, I find that the highest profit is achieved at the Cournot equilibrium.

Notice that if the parameter $\theta = 0$, I can rewrite the demand function as follows: $q_i = \frac{1}{b}(a - p_i)$ and $q_j = \frac{1}{b}(a - p_j)$. The demand function in this case are independent. We have two monopolies firms in the market which produce the same quantity $q^M = \frac{a-c}{2b}$ of different products, sell them at the same monopoly price $p^M = \frac{a-c}{2}$ and earn the same profit $\pi^M = \frac{1}{b}\frac{(a-c)^2}{4}$. At this equilibrium, the consumer surplus is $SC^M = \frac{1}{b}(\frac{a-c}{2})^2 + m$, the total producer surplus $\Pi^M = \frac{2}{b}(\frac{a-c}{2})^2$ and the social welfare $W^M = \frac{3}{b}(\frac{a-c}{2})^2 + m$.

Firms earn higher profit under monopoly competition than Bertrand or Cournot competition. Less competition in the market leads to higher prices, lower production and eventually to lower consumer surplus. Society is better off under Bertrand competition or Cournot competition compared to monopoly market. It can be proved that $CS^M < CS^B$, $CS^M < CS^c$, $\Pi^M > \Pi^B$, $\Pi^M > \Pi^c$, $W^M < W^B$ and $W^M < W^c$. The following proposition generalizes the welfare comparison between Cournot and Bertrand models without investment in research and development.

Proposition 3.1.

$$CS^B - CS^c = \frac{(a-c)^2}{b}\frac{\theta^2[2+\theta+(1+\theta)(2-\theta)]}{(1+\theta)(4-\theta^2)^2}, \tag{15}$$

$$\Pi^c - \Pi^B = \frac{4(a-c)^2}{b}\frac{\theta^3}{(1+\theta)(4-\theta^2)^2}, \tag{16}$$

and

$$W^B - W^c = -\frac{(a-c)^2}{b(1+\theta)(4-\theta^2)^2}\theta^2(\theta^2+2\theta-4). \tag{17}$$

Since $\theta \in [0, 1[$ and $a > c$, using the equations (15), (16) and (17), I have $CS^B \geq CS^c$, $\Pi^c \geq \Pi^B$ and $W^B \geq W^c$. Lower prices and higher quantities are always better in welfare terms. Consumer surplus is decreasing and convex as a function of prices. Therefore, in term of consumer surplus, the Bertrand equilibrium dominates the Cournot equilibrium as proved in equation (15). Given that the goods are substitutes, low prices mean lower profit which implies that Cournot's total producer surplus is higher than Bertrand's total producer surplus as shown by equation (16). The same results were found by [10], that consumer surplus and total surplus $U(q_1, q_2)$ are larger in

Bertrand equilibrium than in Cournot competition except when the goods are independent. In this latter case, consumer surplus and total surplus are equal under both competition models. Singh and Vives ([10]) have shown that the converse of this assertion is true if goods are complements, because in order to increase profits, firms have to lower prices from the Cournot levels to gain the market share. From equation (17), the Bertrand equilibrium is more efficient than Cournot equilibrium.

3.2 Cournot and Bertrand competition with investment in R&D

3.2.1 Cournot model with investment in R&D

In what follows, I use backward induction to solve the subgame perfect equilibrium of the two-stage game described at the beginning of section 3. Let π_i^{ci} denote firm i's market profit at stage 2, then: $\pi_i^{ci} = (p_i - c_i)q_i - r\frac{c_i^{-e}}{e}$, where "$ci$ " means Cournot with investment in R&D. At this stage, firms choose simultaneously their optimal level of production taking the marginal cost as given. So, firm i solves the following maximization problem:

$$\begin{gathered}\max_{q_i} \quad \left\{\pi_i^{ci}(q_i, q_j) = (p_i - c_i)q_i - r\frac{c_i^{-e}}{e}\right\} \\ s.t\ p_i = a - bq_i - b\theta q_j\end{gathered} \tag{18}$$

In the first stage, firms choose simultaneously their optimal level of R&D cost considering the optimal level of production at the second stage. The following proposition describes the equilibrium conditions of the game.

Proposition 3.2. *The conditions describing the two-stage Cournot equilibrium are summarized in the following equations:*

$$\frac{4}{4-\theta^2}q^{ci} = r\widehat{c}^{-e-1} \tag{19}$$

$$q^{ci} = \frac{a - \widehat{c}}{b(2+\theta)} \tag{20}$$

$$p^{ci} = \frac{a + \widehat{c}(1+\theta)}{2+\theta} \tag{21}$$

$$\pi^{ci} = \frac{1}{b}[\frac{a-\widehat{c}}{2+\theta}]^2 - r\frac{\widehat{c}^{-e}}{e}, \ e \geq 1 \tag{22}$$

The proof of this proposition is provided in appendix. From this proposition, the equilibrium in symmetric Cournot competition with investment in R&D are as follows: consumer surplus $CS^{ci} = \frac{(1+\theta)}{b}(\frac{a-\widehat{c}}{2+\theta})^2 + m$, the total producer profit $\Pi^{ci} = \frac{2}{b}(\frac{a-\widehat{c}}{2+\theta})^2 - 2r\frac{\widehat{c}^{-e}}{e}$, and welfare $W^{ci} = \frac{(3+\theta)}{b}(\frac{a-\widehat{c}}{2+\theta})^2 + m - 2r\frac{\widehat{c}^{-e}}{e}$, where $\widehat{c}$ is the optimal level of R&D cost chosen by firms at the first stage.

Using equation (14) in the Cournot model in section 2, the welfare comparison under symmetric Cournot model with and without investment in R&D are given by:

$$\begin{cases} CS^{ci} - CS^{c} & = & \frac{1+\theta}{b(2+\theta)^2}(c-\widehat{c})(2a-c-\widehat{c}) \\ \Pi^{ci} - \Pi^{c} & = & \frac{2}{b(2+\theta)^2}(c-\widehat{c})(2a-c-\widehat{c}) - 2r\frac{\widehat{c}^{-e}}{e} \\ W^{ci} - W^{c} & = & \frac{3+\theta}{b(2+\theta)^2}(c-\widehat{c})(2a-c-\widehat{c}) - 2r\frac{\widehat{c}^{-e}}{e} \end{cases} \tag{23}$$

Remark 3.1. *It follows that* $CS^{ci} \gtreqless CS^{c}$ *if and only if* $c \gtreqless \widehat{c}$*;* $\Pi^{ci} < \Pi^{c}$ *and* $W^{ci} < W^{c}$ *if* $\widehat{c} \geq c$*. If* $\widehat{c} < c$*, the sign of* $\Pi^{ci} - \Pi^{c}$ *and* $W^{ci} - W^{c}$ *are ambiguous. A clear comparison of welfare can be provide when full information about the size of R and D cost* $R(\widehat{c})$ *can be estimated. I may expect to have higher level of welfare in case where* $\widehat{c} < c$ *because by investing more in R&D, firms lower their marginal cost, produce more, and this can ultimately improves social welfare.*

3.2.2 Bertrand competition with investment in R&D

In this section, the demand curve is given by equation (7) or (8). I still use backward induction to solve the R&D problem maximization as in the previous section. At stage 2, firms choose simultaneously prices to maximize profits taken the R&D marginal cost as given. Using equation (6), the firm i's profit maximization problem at stage 2 is set up as follows:

$$\max_{p_i} \left\{ \pi_i^{Bi}(p_i, p_j, c_i, c_j) = (p_i - c_i)q_i = (p_i - c_i)(\alpha - \beta p_i + \gamma p_j) - r\frac{c_i^{-e}}{e} \right\}, \tag{24}$$

where "Bi" means Bertrand competition with investment in R&D.
In the first stage, firms choose simultaneously their optimal level of R&D cost, considering the optimal level of production at the second stage. I have the following result:

Proposition 3.3. *The conditions describing this two-stage Bertrand equilibrium are characterized in the following equations:*

$$-\frac{\theta^2}{b(1-\theta^2)(4-\theta^2)}(p^{Bi} - \widetilde{c}) + \frac{a - p^{Bi}}{b(1+\theta)} = r\widetilde{c}^{-e-1} \tag{25}$$

$$p^{Bi} = \frac{a(1-\theta) + \widetilde{c}}{2-\theta} \tag{26}$$

$$q^{Bi} = \frac{a - \widetilde{c}}{b(1+\theta)(2-\theta)} \tag{27}$$

$$\pi^{Bi} = \frac{1-\theta}{b(1+\theta)}[\frac{a-\widetilde{c}}{2-\theta}]^2 - r\frac{\widetilde{c}^{-e}}{e}, \ e \geq 1 \tag{28}$$

The proof of this proposition is provided in the appendix. The equilibrium in symmetric Bertrand competition with investment in R&D are as follows: consumer surplus $CS^{Bi} = \frac{1}{b(1+\theta)}(\frac{a-\widetilde{c}}{2-\theta})^2 + m$, total producer profit $\Pi^{Bi} = \frac{2(1-\theta)}{b(1+\theta)}(\frac{a-\widetilde{c}}{2-\theta})^2 - 2r\frac{\widetilde{c}-e}{e}$, and welfare $W^{Bi} = \frac{3-2\theta}{b(1+\theta)}(\frac{a-\widetilde{c}}{2-\theta})^2 + m - 2r\frac{\widetilde{c}-e}{e}$, where $\widetilde{c}$ is the optimal level of R&D cost chosen by firms at the first stage.

It is important to mention that the welfare comparison that I did in the previous section can be done here as well. The better off situation depends on the gap $(c - \widetilde{c})$ between the different marginal cost and /or the size of R and D investment.

Assuming that firms compete under Cournot competition, and firm i invests enough in the R&D in the first stage. It turns out that this investment in R&D reduces its marginal cost at the second stage and increases its output. Given that the quantities are strategic substitutes, the quantity produced by firm j is reduced, which increases firm i's profit. Given that prices are strategic complements, I have the reverse result under Bertrand competition model.

Proposition 3.4. *In equilibrium, Cournot firms have larger incentive to invest in R and D, i.e* $\widehat{c} < \widetilde{c}$.

The proof of this result is given in the appendix. This proposition is consistent with the ones shown by Kabiraj and Roy ([6]) for a specific case R&D technology (when $e = 1$).
In the next section, I study the effect of the size of R&D technology on marginal costs, prices and social welfare at equilibrium.

4 Comparative statics

The following comparative static result shows that, as the investment in research and development becomes more and more inefficient, marginal cost under each Cournot or Bertrand model increases, but it increases more under Bertrand competition. The following lemma shows the variation of optimal technology $\widetilde{c}$ and $\widehat{c}$ with the size r of R&D. We recall that these technologies depend on the parameter r. Higher r implies that the R&D investment becomes more inefficient. The following propositions also generalize the results found by [6]. Proposition 4.1 evaluates the effect of R&D technology size on the equilibrium marginal costs of both models and Proposition 4.2 provides the effect of investment in R&D technology on equilibrium prices.

Proposition 4.1. $\frac{d\widehat{c}}{dr} > 0$, $\frac{d\widetilde{c}}{dr} > 0$ *and* $\frac{d\widehat{c}}{dr} < \frac{d\widetilde{c}}{dr}$.

Proposition 4.2. $\frac{dp^{ci}}{dr} > 0$, $\frac{dp^{Bi}}{dr} > 0$ *and* $\frac{dp^{Bi}}{dr} \geq \frac{dp^{ci}}{dr}$

The proofs of these propositions are provided in the appendix. The latter result (Proposition 4.2) describes the shape of curves $p^{ci}(r)$ and $p^{Bi}(r)$. It shows that both curves are upward sloping with $p^{Bi}(r)$ having a greater slope than $p^{ci}(r)$.

From Proposition 4.2, I can derive the following corollary about the comparison of prices under Cournot and Bertrand model with investment in R&D.

Corollary 4.1. *There exists r^* such that, $p^{Bi} > p^{ci}$ if and only if $r > r^*$.*

This corollary tells us that, if the R&D technology is inefficient (higher r), then Cournot price will be lower than Bertrand price and vice versa. Large r increases marginal cost (see proposition 4.1), but it increases more under Bertrand competition, leading to higher prices relative to Cournot prices. This result is consistent with the results found by Qiu([9]) and Kabiraj and Roy ([6]). They have proved that when R&D technology is more efficient (lower r), the Cournot prices are greater than Bertrand prices at equilibrium. When r goes up, firms invest more in R&D under the Cournot model, then the marginal cost increases at a lower rate than under Bertrand model which lead to lower prices. This latter result is still consistent in this general setting.

One interesting implication of these results is that, inefficient R&D technology will generate larger consumer's surplus and social welfare under the Cournot competition model. But, it is also important to mention that consumers' surpluses under both models decrease with the size of R&D technology. We can show that $\frac{dCS^{ci}}{dr} = \frac{-2(1+\theta)}{b(2+\theta)^2}\frac{d\widehat{c}}{dr}(a-\widehat{c})$ and $\frac{dCS^{Bi}}{dr} = \frac{-2}{b(1+\theta)(2-\theta)^2}\frac{d\widetilde{c}}{dr}(a-\widetilde{c})$. The usefulness of these latter expressions are still being studied.

5 Conclusion

The aim of this paper was to compare Cournot and Bertrand models on efficiency of results in term of social welfare. The important result that challenged the traditional result on efficiency of Bertrand equilibrium outcome is that at the equilibrium, not only the Cournot firms invest a larger amount on R&D than the Bertrand firms, but Bertrand price can be higher than Cournot price. I prove that this occurs when the R&D technology is relatively inefficient; thus a Cournot market structure can generate larger consumer's surplus and total welfare. In this paper, all payoff functions and costs are parametric and there are only two firms. One eminent project is to generalize this research by allowing a competition between a large number of firms, and by using non-parametric functions. I expect that this general case can provide something like a necessary and/or sufficient condition that could provide more practical guidance. Moreover, I did not report the effects of R&D

size on producer surplus or total welfare because they are still being studied. Some future research will be based on evaluating these effects on one side, and comparing Cournot and Bertrand models on other separate issues from R&D.

References

[1] Arghya, G. and Manipushpak, M., 2008,‘Comparing Bertrand and Cournot Outcomes in The Presence of Public Firms’, *Working paper*, 16.

[2] Cheng, L., 1985, ‘Comparing Bertrand and Cournot equilibria: a geometric approach’, *Rand Journal of Economics*, 16, pp. 146-152.

[3] Delbono, F. and Denicolo, V., 1990, ‘R & D Investment In a Symmetric And Homogeneous Oligopoly, Bertrand vs Cournot’, *International Journal of Industrial Organization*, 8, pp. 297-313.

[4] Hackner, J., 2000, ‘A Note on Price and Quantity Competition in Differentiated Oligopolies’, *Journal of Economic Theory*, 93, pp. 233-239.

[5] Hsu, J. and Wang, X. H., 2005, ‘On Welfare under Cournot and Bertrand Competition in Differentiated Oligopolies’, *Review of Industrial Organization*, 27, pp. 185-191.

[6] Kabiraj, T. and Roy, S., 2002, ‘Cournot and Bertrand Prices In A Model Of Differentiated Duopoly With R and D’, *Working paper.*

[7] Motta, M., 1993, ‘Endogenous Quality Choice Price vs Quantity Competition’, *The Journal of Industrial Economics*, 41, 2, pp. 113-131.

[8] Okuguchi, K.,1987, ‘Equilibrium Prices in the Bertrand and Cournot Oligopolies’, *Journal of Economic Theory*, 42, pp. 128-139.

[9] Qiu, L. D., 1997, ‘On the dynamic efficiency of Bertrand and Cournot equilibria’, *Journal of Economic Theory*, 75, pp. 213-229.

[10] Singh, N. and X. Vives, 1994, ‘Price and quantity competition in a differentiated duopoly’, *Rand Journal of Economics* , 15, pp. 546-554.

[11] Sutton, J., 1997, ‘One Smart Agent’, *RAND Journal of Economics* 28, pp. 605-628.

[12] Sutton, J., 1998, 'Technology and Market Structure', *M.I.T Press*, MA.

[13] Symeonidis, G., 2003, 'Comparing Cournot and Bertrand equilibria in a differentiated duopoly with product R&D', *International Journal of Industrial Organization*, 21, pp. 39-45.

[14] Vives, X., 1985, 'On the efficiency of Bertrand and Cournot equilibria with product differentiation', *Journal of Economic Theory*, 36, pp. 166-175.

6 Appendix

Proof of lemma 3.1:

The marginal cost of each firm is c, then firm i's profit - maximization problem is set up as:

$$\begin{array}{c}\max\limits_{q_i} \quad \{\pi_i(q_i, q_j) = (p_i - c)q_i\} \\ s.t \; p_i = \omega - \phi q_j - \lambda q_i\end{array} \quad (A1).$$

The first order condition for maximization problem is given by:

$$\frac{\partial \pi_i}{\partial q_i} = \omega - \phi q_j - 2\lambda q_i - c = 0 \; (A2),$$

and the second derivative of profit is given by:

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -2\lambda.$$

In order to maximize the profit, the second derivative should be negative. Since, the sign of λ is positive, I conclude that the second condition is satisfied. Solving equation (A2) for q_i gives :

$$q_i = \frac{\omega - \phi q_j - c}{2\lambda},$$

and using the fact that by symmetry $q_i^c = q_j^c = q^c$, I obtain

$$2\lambda q^c + \phi q^c = \omega - c,$$

and solving for q^c gives :

$$q^c = \frac{\omega - c}{2\lambda + \phi} \; (A3).$$

Now, I obtain the equilibrium price for each firm by substituting the equilibrium quantity (A3) in the inverse demand function in equation (12). Furthermore, by symmetry $p_i^c = p_j^c = p^c$, then $p^c = \omega - (\phi + \lambda)q^c$. It follows that:

$$\begin{aligned} p^c &= \omega - (\phi + \lambda)\frac{(\omega - c)}{2\lambda + \phi)} \\ &= \frac{(2\lambda + \phi)\omega - (\phi + \lambda)(\omega - c)}{2\lambda + \phi} \\ &= \frac{(2\lambda + \phi)\omega - (\phi + \lambda)\omega + c(\phi + \lambda)}{2\lambda + \phi} \\ &= \frac{\omega(2\lambda + \phi - \phi - \lambda) + c(\phi + \lambda)}{2\lambda + \phi} \\ &= \frac{\lambda\omega + c\phi + c\lambda}{2\lambda + \phi} \\ p^c &= \frac{\lambda\omega + c(\phi + \lambda)}{2\lambda + \phi} \; (A4). \end{aligned}$$

Therefore,

$$
\begin{aligned}
p^c - c &= \frac{\lambda\omega + c(\phi+\lambda)}{2\lambda+\phi} - c \\
&= \frac{\lambda\omega + (\phi+\lambda-2\lambda-\phi)c}{2\lambda+\phi} \\
&= \frac{\lambda(\omega-c)}{2\lambda+\phi}.
\end{aligned}
$$

As above, firms have the same profit $\pi^c = (p^c - c)q^c$. Substituting for the value of q^c and p^c in the profit function gives :

$$
\pi^c = \frac{\lambda(\omega-c)}{2\lambda+\phi} \times \frac{\omega-c}{2\lambda+\phi} = \lambda(\frac{\omega-c}{2\lambda+\phi})^2.
$$

Since I denoted $\omega = \frac{\alpha}{\beta-\gamma}, \phi = \frac{\gamma}{\beta^2-\gamma^2}, \lambda = \frac{\beta}{\beta^2-\gamma^2}$ in (11), I want to simplify the expressions of q^c, p^c and π^c in terms of α, β and γ.

1. I have $q^c = \frac{\omega-c}{2\lambda+\phi}$ from (18).
   - $\omega - c = \frac{\alpha}{\beta-\gamma} - c = \frac{\alpha-c(\beta-\gamma)}{\beta-\gamma}$;
   - $2\lambda+\phi = \frac{2\beta}{\beta^2-\gamma^2} + \frac{\gamma}{\beta^2-\gamma^2} = \frac{2\beta+\gamma}{\beta^2-\gamma^2} = \frac{2\beta+\gamma}{(\beta-\gamma)(\beta+\gamma)}$;

   It follows that $q^c = \frac{\omega-c}{2\lambda+\phi} = \frac{\alpha-c[\beta-\gamma]}{(\beta-\gamma)} \times \frac{(\beta-\gamma)(\beta+\gamma)}{2\beta+\gamma}$, by simplifying the factor $(\beta-\gamma)$, I get $q^c = \frac{[\alpha-c(\beta-\gamma)](\beta+\gamma)}{2\beta+\gamma}$ $(A5)$.

2. I have $p^c = \frac{\lambda\omega+c(\phi+\lambda)}{2\lambda+\phi}$ from $(A4)$.
   - $\phi+\lambda = \frac{\beta+\gamma}{(\beta+\gamma)(\beta-\gamma)} = \frac{1}{\beta-\gamma}$;
   - $\lambda\omega = \frac{\alpha\beta}{(\beta-\gamma)(\beta^2-\gamma^2)}$, so $\lambda\omega + c(\phi+\lambda) = \frac{\alpha\beta}{(\beta-\gamma)(\beta^2-\gamma^2)} + \frac{c}{\beta-\gamma} = \frac{\alpha\beta+c(\beta^2-\gamma^2)}{(\beta-\gamma)(\beta^2-\gamma^2)}$;

   Then, $p^c = \frac{\lambda\omega+c(\phi+\lambda)}{2\lambda+\phi} = \frac{\alpha\beta+c[\beta^2-\gamma^2]}{(\beta-\gamma)(\beta^2-\gamma^2)} \times \frac{(\beta^2-\gamma^2)}{2\beta+\gamma}$, and by simplifying the factor $(\beta^2-\gamma^2)$, I obtain $p^c = \frac{\alpha\beta+c[\beta^2-\gamma^2]}{(\beta-\gamma)(2\beta+\gamma)}$ $(A6)$.

3. I have $\pi^c = \lambda(\frac{\omega-c}{2\lambda+\phi})^2$ from (19).
   - $\frac{\omega-c}{2\lambda+\phi} = \frac{\alpha-c(\beta-\gamma)}{\beta-\gamma} \times \frac{(\beta-\gamma)(\beta+\gamma)}{2\beta+\gamma} = \frac{[\alpha-c(\beta-\gamma)](\beta+\gamma)}{2\beta+\gamma}$;
   - $\lambda(\frac{\omega-c}{2\lambda+\phi})^2 = \frac{\beta}{(\beta-\gamma)(\beta+\gamma)} \times (\beta+\gamma)(\beta+\gamma)[\frac{\alpha-c(\beta-\gamma)}{2\beta+\gamma}]^2$.

   By simplifying the factor $(\beta+\gamma)$ in the latter expression, I obtain $\pi^c = \frac{\beta(\beta+\gamma)}{\beta-\gamma}[\frac{\alpha-c(\beta-\gamma)}{2\beta+\gamma}]^2$ $(A7)$.

From equation (9), substituting the expressions of parameters α, β and γ in terms of parameters a, b, θ in the expressions $(A5)$, $(A6)$ and $(A7)$, I obtain $\beta+\gamma = \frac{1}{b(1-\theta)}$; $2\beta+\gamma = \frac{2+\theta}{b(1-\theta^2)}$ and $\alpha - (\beta-\gamma)c = \frac{a-c}{b(1+\theta)}$. Therefore, I deduce that the Cournot equilibrium quantity is $q^c = \frac{a-c}{b(2+\theta)}$, the Cournot equilibrium $p^c = \frac{a+c(1+\theta)}{2+\theta}$ and the positive Cournot equilibrium profit $\pi^c = \frac{1}{b}[\frac{a-c}{2+\theta}]^2$.

Proof of lemma 3.2:

The firm i's profit-maximization problem is:

$$\max_{p_i} \{\pi_i(p_i, p_j) = (p_i - c)q_i = (p_i - c)(\alpha - \beta p_i + \gamma p_j)\} \ (A8).$$

The first order condition of problem (A8) is given by:

$$\frac{\partial \pi}{\partial p_i} = \alpha - \beta p_i + \gamma p_j - \beta(p_i - c) = 0 \ (A9),$$

and the second order condition is:

$$\frac{\partial^2 \pi}{\partial p_i^2} = -2\beta < 0 \ since \ \beta > 0.$$

Solving for equation (A9) with respect to p_i gives :

$$2\beta p_i = \alpha + \gamma p_j + \beta c \ (A10).$$

By symmetry, the equilibrium price for each firm is such that $p_i^B = p_j^B = p^B$. Then, using these latter equalities and substituting in the equilibrium price's equation lead to $2\beta p^B = \alpha + \gamma p^B + \beta c$, rearranging terms and solving the latter equation for price p^B gives $p^B = \frac{\alpha+\beta c}{2\beta-\gamma}$ $(A11)$.
Now, I find the quantity of each firm at the equilibrium. By symmetry, $q_i^B = q_j^B = q^B$, then substituting the equilibrium price in demand function (8) yields:

$$\begin{aligned}
q^B &= \alpha - (\beta - \gamma)p^B \\
&= \alpha - (\beta - \gamma)\frac{\alpha + \beta c}{2\beta - \gamma} \\
&= \alpha - \frac{(\beta - \gamma)(\alpha + \beta c)}{2\beta - \gamma} \\
&= \frac{\alpha(2\beta - \gamma) - (\beta - \gamma)(\alpha + \beta c)}{2\beta - \gamma} \\
&= \frac{\alpha(2\beta - \gamma) - \alpha(\beta - \gamma) - \beta c(\beta - \gamma)}{2\beta - \gamma} \\
&= \frac{\alpha(2\beta - \gamma - \beta + \gamma) - \beta c(\beta - \gamma)}{2\beta - \gamma} \\
&= \frac{\alpha\beta - \beta c(\beta - \gamma)}{2\beta - \gamma} \\
q^B &= \frac{\beta[\alpha - (\beta - \gamma)c]}{2\beta - \gamma} \ (A12).
\end{aligned}$$

Symmetry assumption implies that $\pi_i^B = \pi_j^B = \pi^B$. Given that $\pi^B = (p^B - c)q^B$ $(A13)$, I use the above expressions of p^B and q^B to obtain the complete form of π^B. Using equation $(A11)$, I get:

$$\begin{aligned} p^B - c &= \frac{\alpha + \beta c}{2\beta - \gamma} - c \\ &= \frac{\alpha + \beta c - 2\beta c + \gamma c}{2\beta - \gamma} \\ &= \frac{\alpha - \beta c + \gamma c}{2\beta - \gamma} \\ p^B - c &= \frac{\alpha - (\beta - \gamma)c}{2\beta - \gamma} \ (A14). \end{aligned}$$

From equation $(A14)$, since $\alpha > (\beta - \gamma)$, I conclude that $p^c > c$. Substituting equations $(A12)$ and $(A14)$ in equations $(A13)$ above gives $\pi^B = [\frac{\alpha-(\beta-\gamma)c}{2\beta-\gamma}] \times [\frac{\beta[\alpha-(\beta-\gamma)c]}{2\beta-\gamma}]$, and then $\pi^B = \beta[\frac{\alpha-(\beta-\gamma)c}{2\beta-\gamma}]^2$ $(A15)$.
Since $\alpha = \frac{a}{b(1+\theta)}$, $\beta = \frac{1}{b(1-\theta^2)}$, $\gamma = \frac{\theta}{b(1-\theta^2)}$, it follows that :

$$\alpha + \beta c = \frac{a(1-\theta) + c}{b(1-\theta^2)},\ 2\beta - \gamma = \frac{2-\theta}{b(1-\theta^2)} \ and\ \alpha - (\beta - \gamma)c = \frac{a-c}{b(1+\theta)}.$$

Therefore, I conclude that $p^B = \frac{a(1-\theta)+c}{2-\theta}$, $q^B = \frac{a-c}{b(1+\theta)(2-\theta)}$ and $\pi^B = \frac{1-\theta}{b(1+\theta)}[\frac{a-c}{2-\theta}]^2$.

Proof of theorem 3.1:

The Cournot equilibrium (q^c, p^c, π^c) is given by :

$$p^c = \frac{a + c(1+\theta)}{2+\theta}, q^c = \frac{a-c}{b(2+\theta)}, \pi^c = \frac{(a-c)^2}{b(2+\theta)^2}.$$

and the Bertrand equilibrium (q^B, p^B, π^B) is given by :

$$p^B = \frac{a(1-\theta)+c}{2-\theta}, q^B = \frac{b(1+\theta)}{(2-\theta)}, \pi^B = \frac{1-\theta}{b(1+\theta)} \frac{(a-c)^2}{(2-\theta)^2}.$$

I also have:

$$p^c - p^B = \frac{a-c}{4-\theta^2}\theta^2$$

$$q^B - q^c = \frac{a-c}{b(1+\theta)(4-\theta^2)}\theta^2$$

and

$$\pi^c - \pi^B = \frac{2(a-c)^2}{b(1+\theta)(4-\theta^2)^2}\theta^3$$

From these equations, I show that $p^c = p^B$, $q^c = q^B$ and $\pi^c = \pi^B$ if and only if the parameter θ is equal zero. Moreover, given that $a > c$, if $\theta > 0$, I have $p^c > p^B$, $q^B > q^c$ and $\pi^c > \pi^B$.

Proof of proposition 3.2:

Let π_i^{ci} denote firm i's market profit at stage 2, then: $\pi_i^{ci} = (p_i - c_i)q_i - r\frac{c_i^{-e}}{e}$. At this stage, firms choose simultaneously their optimal level of production taking the marginal cost as given. So, firm i solves the following maximization problem:

$$\max_{q_i} \quad \left\{ \pi_i^{ci}(q_i, q_j) = (p_i - c_i)q_i - r\frac{c_i^{-e}}{e} \right\}$$
$$s.t \; p_i = a - bq_i - b\theta q_j$$

The first order condition for maximization problem is given by:

$$\frac{\partial \pi_i^{ci}}{\partial q_i} \quad = \quad a - b\theta q_j - 2bq_i - c_i = 0; i \neq j; \; i, j = 1, 2 \; (A16);$$

and the second derivative of profit is given by:

$$\frac{\partial^2 \pi_i^{ci}}{\partial q_i^2} = -2b < 0.$$

From equation (A16), I have the following system: $\begin{cases} 2bq_i & + & b\theta q_j & = a - c_i \\ b\theta q_i & + & 2bq_j & = a - c_j \end{cases}$.

Solving this system for q_i and q_j gives :

$$q_i^{ci} = \frac{a}{b(2+\theta)} + \frac{2c_i - \theta c_j}{b(\theta^2 - 4)} \; (A17)$$

$$q_j^{ci} = \frac{a}{b(2+\theta)} + \frac{2c_j - \theta c_i}{b(\theta^2 - 4)} \; (A18)$$

The firm i's profit at stage 1 is given by: $\pi_i^{ci}(q_i^{ci}, q_j^{ci}, c_i, c_j) = (p_i^{ci} - c_i)q_i^{ci} - r\frac{c_i^{-e}}{e}$ which can be rewritten as $\pi_i^{ci}(q_i^{ci}, q_j^{ci}, c_i, c_j) = (a - bq_i^{ci} - b\theta q_j^{ci} - c_i)q_i^{ci} - r\frac{c_i^{-e}}{e}$.

At this stage, firms choose simultaneously their optimal level of R&D cost considering the optimal level of production at the next stage. Then firm i's profit maximization problem is given by:

$$\max_{c_i} \quad \left\{ \pi_i^{ci}(q_i^{ci}, q_j^{ci}, c_i, c_j) = (a - bq_i^{ci} - b\theta q_j^{ci} - c_i)q_i^{ci} - r\frac{c_i^{-e}}{e} \right\}$$
$$s.t \; (A17) \; and \; (A18)$$

The first order condition for maximization problem is given by:

$$\frac{d\pi_i^{ci}}{dc_i} = \frac{\partial \pi_i^{ci}}{\partial q_i^{ci}} \times \frac{\partial q_i^{ci}}{\partial c_i} + \frac{\partial \pi_i^{ci}}{\partial q_j^{ci}} \times \frac{\partial q_j^{ci}}{\partial c_i} + \frac{\partial \pi_i^{ci}}{\partial c_i} = \frac{4}{\theta^2 - 4} q_i^{ci} + rc_i^{-e-1} = 0,$$

and the second derivative of profit is given by:

$$\frac{d^2 \pi_i^{ci}}{\partial c_i^2} = \frac{8}{b(\theta^2 - 4)^2} - (e+1)rc_i^{-e-2}.$$

I assume that the second derivative is satisfied. Let also assume the symmetric equilibrium at first stage, then $c_i^{ci} = c_j^{ci} = \widehat{c} < a$, then $q_i^{ci} = q^{ci} = q_j^{ci}$ and $p_i^{ci} = p^{ci} = p_j^{ci}$. Using the same algebras as in the section 2, I obtain the required conditions.

Proof of proposition 3.3:

Firm i's profit maximization problem at stage 2 is set up as:

$$\max_{p_i}\left\{\pi_i^{Bi}(p_i,p_j,c_i,c_j)=(p_i-c_i)q_i=(p_i-c_i)(\alpha-\beta p_i+\gamma p_j)-r\frac{c_i^{-e}}{e}\right\}$$

The first order condition of problem (31) is given by:

$$\frac{\partial \pi}{\partial p_i} = \alpha-2\beta p_i+\gamma p_j+\beta c_i=0,\ i\neq j,\ i,j\in\{1,2\}\ (A19),$$

and the second order condition is:

$$\frac{\partial^2\pi}{\partial p_i^2}=-2\beta<0\ (since\ \beta>0).$$

From equation (A19), I retrieve the following system: $\begin{cases} 2\beta p_i - \gamma p_j = \alpha+\beta c_i \\ -\gamma p_i + 2\beta p_j = \alpha+\beta c_j \end{cases}$.

Solving this system for p_i and p_j give s:

$$p_i^{Bi} = \frac{\alpha}{2\beta-\gamma}+\frac{2\beta^2c_i+\beta\gamma c_j}{4\beta^2-\gamma^2}\ (A20)$$

$$p_j^{Bi} = \frac{\alpha}{2\beta-\gamma}+\frac{2\beta^2c_j+\beta\gamma c_i}{4\beta^2-\gamma^2}\ (A21)$$

The firm i's profit at stage 1 is given by:

$$\pi_i^{Bi}(p_i^{Bi},p_j^{Bi},c_i,c_j)=(p_i^{Bi}-c_i)q_i^{ci}-r\frac{c_i^{-e}}{e},$$

or

$$\pi_i^{Bi}(p_i^{Bi},p_j^{Bi},c_i,c_j)=(p_i^{Bi}-c_i)(\alpha-\beta p_i^{Bi}+\gamma p_j^{Bi})-r\frac{c_i^{-e}}{e}.$$

At this stage, firms choose simultaneously their optimal level of R&D cost considering the optimal level of price at the next stage. Thus, firm i solves the following profit maximization problem:

$$\max_{c_i}\ \left\{\pi_i^{ci}(q_i^{ci},q_j^{ci},c_i,c_j)=(p_i^{Bi}-c_i)(\alpha-\beta p_i^{Bi}+\gamma p_j^{Bi})-r\frac{c_i^{-e}}{e}\right\}$$
$$s.t\ (A20)\ and\ (A21)$$

The first order condition for maximization problem is given by:

$$\frac{d\pi_i^{Bi}}{dc_i}=\frac{\partial\pi_i^{Bi}}{\partial p_i^{Bi}}\times\frac{\partial p_i^{Bi}}{\partial c_i}+\frac{\partial\pi_i^{Bi}}{\partial p_j^{Bi}}\times\frac{\partial p_j^{Bi}}{\partial c_i}+\frac{\partial \pi_i^{ci}}{\partial c_i}=\frac{\gamma^2\beta}{4\beta^2-\gamma^2}(p_i^{Bi}-c_i)-(\alpha-\beta p_i^{Bi}+\gamma p_j^{Bi})+rc_i^{-e-1}=0,$$

and the second derivative of profit is given by:

$$\frac{d^2\pi_i^{Bi}}{dc_i^2} = 2\beta[\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}]^2 - (e+1)rc_i^{-e-2}.$$

I assume that the second derivative is satisfied. Let also assume the symmetric equilibrium at first stage, then $c_i^{Bi} = c_j^{Bi} = \widetilde{c} < a$, then $p_i^{Bi} = p^{Bi} = p_j^{Bi}$ and $q_i^{Bi} = q^{Bi} = q_j^{Bi}$. After doing some algebras and substituting α, β and γ by their expressions in terms of a, b and θ, I obtain the equilibrium conditions.

Proof of proposition 3.4:

By symmetry of equilibrium for any c satisfying (25) and after doing some algebras, I obtain:

$$\frac{d\pi^{Bi}}{dc} = \frac{(2\theta^2-4)(a-c)}{b(1+\theta)(2-\theta)(4-\theta^2)} + rc^{-e-1}$$

By first order condition, I have $\frac{d\pi^{Bi}}{dc}|_{c=\widetilde{c}} = 0$. Also, $\frac{d\pi^{Bi}}{dc}|_{c=\widehat{c}} = \frac{2\theta(a-\widehat{c})(\theta^2+\theta+2)}{b(4-\theta^2)^2(1+\theta)}$ which is positive. By again using symmetry of equilibrium for any c satisfying equation (19), I have:

$$\frac{d\pi^{ci}}{dc} = \frac{-4(a-c)}{b(4-\theta^2)(2+\theta)} + rc^{-e-1}$$

By the first order condition, $\frac{d\pi^{ci}}{dc}|_{c=\widehat{c}} = 0$ and $\frac{d\pi^{ci}}{dc}|_{c=\widetilde{c}} = -\frac{2\theta^3(a-\widetilde{c})}{b(4-\theta^2)^2(1+\theta)}$ which is negative. It follows that firm have more incentive to invest in R&D under Cournot equilibrium rather than Bertrand equilibrium, and then $\widehat{c} < \widetilde{c}$.

Proof of proposition 4.1:

Case 1: $\frac{d\widehat{c}}{dr} > 0$.

From equations (19) and (20), I obtain the following equation:

$$\frac{4}{b(2+\theta)(4-\theta^2)}(a-\widehat{c})\widehat{c}^{e+1} = r \ (A22).$$

Differentiating equation (A22) with respect to r and solving for $\frac{d\widehat{c}}{dr}$ gives :

$$\frac{d\widehat{c}}{dr} = \frac{b(2+\theta)(4-\theta^2)}{4} \times \frac{\widehat{c}^{-e}}{[(e+1)(a-\widehat{c})-\widehat{c}]} \ (A23).$$

Let denote $C(\theta) = \frac{b(2+\theta)(4-\theta^2)}{4}$. Given that $C(\theta)$ is positive, I just need to show that $(e+1)(a-\widehat{c})-\widehat{c}$ is also positive. To prove that, I use the second order condition from proposition 3.2 (stage 1 maximization problem). From there, I have the second order condition: $\frac{\partial^2\pi_i^{ci}}{\partial c_i^2} = \frac{4}{\theta^2-4}\frac{\partial q_i^{ci}}{\partial c_i} - (e+1)r\frac{c_i^{-e-1}}{c_i} < 0$. Given that I assume symmetry equilibrium, $c_i = \widehat{c}$ and $q_i^{ci} = q^{ci}$; from equation (20), I have $\frac{\partial q_i^{ci}}{\partial c_i} = \frac{\partial q^{ci}}{\partial \widehat{c}} = \frac{-1}{b(2+\theta)}$. Substituting $\frac{\partial q_i^{ci}}{\partial c_i}$ and r from equation (A22) in the second order condition, and simplify, I get $\frac{\partial^2 \pi^{ci}}{\partial \widehat{c}^2} = \frac{4}{b(4-\theta^2)(2+\theta)}[\frac{\widehat{c}-(e+1)(a-\widehat{c})}{\widehat{c}}]$. Since $\frac{\partial^2 \pi^{ci}}{\partial \widehat{c}^2} < 0$, then $\widehat{c} < (e+1)(a-\widehat{c})$ i.e $(e+1)(a-\widehat{c}) - \widehat{c} > 0$ and $\frac{d\widehat{c}}{dr} > 0$.

Case 2: $\frac{d\widetilde{c}}{dr} > 0$.

Using equation (26) from Bertrand equilibrium, I have $\frac{\partial p^{Bi}}{\partial r} = \frac{1}{2-\theta}\frac{d\widetilde{c}}{dr}$. Differentiating equation (25) with respect to r and solving for $\frac{d\widetilde{c}}{dr}$ yields:

$$\frac{d\widetilde{c}}{dr} = \frac{\widetilde{c}^{-e}}{\frac{\theta^2(1-\theta)}{b(1-\theta^2)(4-\theta^2)(2-\theta)}\widetilde{c} - \frac{\widetilde{c}}{b(1+\theta)(2-\theta)} + (e+1)r\widetilde{c}^{-e-1}} = \frac{\widetilde{c}^{-e}}{B}.$$

Now, I show that $B > 0$. I use equations (25), (26), the second order condition derived from proposition 3.3 (stage 1 maximization problem) and symmetric equilibrium. Using (26), I get $p^{Bi} - \widetilde{c} = \frac{(1-\theta)(a-\widetilde{c})}{2-\theta}$ and $a - p^{Bi} = \frac{a-\widetilde{c}}{2-\theta}$. Substituting these latter expressions into equation (25) lead to the following expression:

$$r\widetilde{c}^{-e-1} = \frac{-\theta^2}{b(1-\theta^2)(4-\theta^2)}\frac{(1-\theta)(a-\widetilde{c})}{2-\theta} + \frac{a-\widetilde{c}}{b(2-\theta)(1+\theta)} \quad (A23).$$

Substituting $r\widetilde{c}^{-e-1}$ from equation (A23) into expression B and doing some algebras gives:

$$B = \frac{2(2-\theta^2)}{b(1+\theta)(4-\theta^2)(2-\theta)}[(e+1)(a-\widetilde{c}) - \widetilde{c}].$$

The second order condition for proposition 3.3 can be rewritten as follows:
$\frac{\partial^2 \pi_i^{Bi}}{\partial c_i^2} = \frac{\theta^2}{b(1-\theta^2)(4-\theta^2)}[\frac{\partial p_i^{Bi}}{\partial c_i} - 1] + \frac{1}{b(1-\theta^2)}\frac{\partial p_i^{Bi}}{\partial c_i} - \frac{\theta}{b(1-\theta^2)}\frac{\partial p_j^{Bi}}{\partial c_i} - r(e+1)\widetilde{c}_i^{-e-2} < 0$. By symmetric equilibrium, I have $p_i^{Bi} = p^{Bi}$, $\widetilde{c}_i = \widetilde{c}$ for $i \in \{1,2\}$ and $\frac{\partial p^{Bi}}{\partial \widetilde{c}} = \frac{1}{2-\theta}$. Using these latter information, the second order condition can be rewritten as:
$\frac{\partial^2 \pi^{Bi}}{\partial \widetilde{c}^2} = -\frac{\theta^2(1-\theta)\widetilde{c}}{b(1-\theta^2)(4-\theta^2)(2-\theta)} + \frac{\widetilde{c}}{b(1+\theta)(2-\theta)} - r(e+1)\widetilde{c}^{-e-1} < 0$. Substituting equation (A23) into this latter second order condition lead to $[\frac{\theta^2(1-\theta)}{b(1-\theta^2)(4-\theta^2)(2-\theta)} - \frac{1}{b(1+\theta)(2-\theta)}](e+1)(a-\widetilde{c}) - \widetilde{c}) < 0$ i.e $-\frac{2(2-\theta^2)}{b(1+\theta)(4-\theta^2)(2-\theta)}[(e+1)(a-\widetilde{c}) - \widetilde{c}] < 0$ or $-B < 0$. It follows that $B > 0$, then $\frac{d\widetilde{c}}{dr} > 0$.
Note that B positive means $(e+1)(a-\widetilde{c}) - \widetilde{c}$ is also positive, given that $\frac{2(2-\theta^2)}{b(1+\theta)(4-\theta^2)(2-\theta)} > 0$. Finally, the expression $\frac{d\widetilde{c}}{dr}$ can be rewritten as follows:

$$\frac{d\widetilde{c}}{dr} = \frac{b(1+\theta)(4-\theta^2)(2-\theta)}{2(2-\theta^2)} \times \frac{\widetilde{c}^{-e}}{[(e+1)(a-\widetilde{c}) - \widetilde{c}]}.$$

Let denote $B(\theta) = \frac{b(1+\theta)(4-\theta^2)(2-\theta)}{2(2-\theta^2)}$.

Case 3: $\frac{d\widetilde{c}}{dr} > \frac{d\widehat{c}}{dr}$.

From some expressions in cases 1 and 2, I write:

$$\frac{d\widetilde{c}}{dr} / \frac{d\widehat{c}}{dr} = \frac{B(\theta)}{C(\theta)} \times (\frac{\widehat{c}}{\widetilde{c}})^{e+1} \times \frac{(e+1)[\frac{a}{\widehat{c}} - 1]}{(e+1)[\frac{a}{\widetilde{c}} - 1]}.$$

I have $\frac{B(\theta)}{C(\theta)} = \frac{2(1+\theta)(2-\theta)}{(2+\theta)(2-\theta^2)} > 1$ and $\frac{(e+1)[\frac{a}{\widehat{c}}-1]}{(e+1)[\frac{a}{\widetilde{c}}-1]} > 1$ since $\widetilde{c} > \widehat{c}$. Assuming that the ratio of marginal cost is close to one which means that the term $(\frac{\widehat{c}}{\widetilde{c}})^{e+1}$ is not so low to reduce the product of terms on the right side of the ratio of marginal effect of R&D size on marginal cost far to one, I conclude that $\frac{d\widetilde{c}}{dr}/\frac{d\widehat{c}}{dr} > 1$ or $\frac{d\widetilde{c}}{dr} > \frac{d\widehat{c}}{dr}$. Without this assumption, I may also have $\frac{d\widetilde{c}}{dr} < \frac{d\widehat{c}}{dr}$ in case where Bertrand firms invest too much in R&D for instance, or in case where the ratio of marginal cost is too low.

Proof of proposition 4.2:

From equation (21), I have $p^{ci} = \frac{a+\widehat{c}(1+\theta)}{2+\theta)}$, so $\frac{dp^{ci}}{dr} = \frac{1+\theta}{2+\theta}\frac{d\widehat{c}}{dr}$. By proposition 4.1, $\frac{d\widehat{c}}{dr} > 0$, then $\frac{dp^{ci}}{dr} > 0$. From equation (26), $p^{Bi} = \frac{a(1-\theta)+\widetilde{c}}{2-\theta}$, so $\frac{dp^{Bi}}{dr} = \frac{1}{2-\theta}\frac{d\widetilde{c}}{dr}$. Using again proposition 4.1, $\frac{dp^{Bi}}{dr} > 0$ since $\frac{d\widetilde{c}}{dr} > 0$. I have $\frac{dp^{Bi}}{dr} - \frac{dp^{ci}}{dr} = \frac{1}{2-\theta}\frac{d\widetilde{c}}{dr} - \frac{1+\theta}{2+\theta}\frac{d\widehat{c}}{dr}$. By doing some algebras, $\frac{1}{2-\theta} - \frac{1+\theta}{2+\theta} = \frac{\theta^2}{4-\theta^2}$ which is non negative. Since, I have shown that $\frac{d\widetilde{c}}{dr} > \frac{d\widehat{c}}{dr}$, hence $\frac{dp^{Bi}}{dr} - \frac{dp^{ci}}{dr} \geq 0$.