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Modelling Time-varying Bond Risk Premia for Utilities Industry*

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Abstract

This paper offers an alternative method for modelling bond risk premia for a panel of corporate bond yields using a daily data set for 48 corporate bonds of utilities industry over four years. This is done by using proxies for default, liquidity and interest rate factors that we get employing Fama and MacBeth two step procedure. In the meanwhile, the investors' learning process is mimicked by the Kalman filter procedure that is introduced to capture the dynamics of bond risk premia that are driven by multiple factors. In particular, we show that with time varying risk premia, our model performs much better in explaining our panel of bond returns when compared with the famous Fama-MacBeth two step procedure and rolling regression procedure, that are commonly used in the finance literature due to its merits of simplicity and clarity.

Keywords: time-varying bond risk premia; utilities industry corporate bonds; Fama-Macbeth two step procedure; multivariate Kalman filter

JEL Code: C32, C38, G12

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1 Introduction

Significant amount of research has been published in finance literature related to the risk premia in equity markets, however, relatively little is known about risk premia in corporate bond markets. As in [Cochrane and Piazzesi \(2005\)](#), most of the literature illustrate results about time-varying risk premia in government bonds using term structure models. In addition to that, a lot of research has examined the time-series behaviour of betas, but the time-series behaviour of risk premium has received relatively little attention. One of the basic studies in that area is [Ferson and Harvey \(1991\)](#); they illustrate that the premia associated with interest rate risks capture predictability of the bond portfolio returns. [Fama and French \(1993\)](#) claim that a term premium and a default premium capture most of the variations in bond returns, where the expected value of the default premium is high when economic conditions are weak and default risks are high, and it is low when business conditions are strong.

On the other hand, recent empirical evidence suggests that corporate bonds earn an expected excess return over default-free government bonds, even after correcting for the likelihood of default. In order to explain this excess return, existing research examines tax and liquidity effects, and risk premia on systematic changes in credit spreads (if no default occurs). However, a complete empirical analysis that incorporates all proposed components is lacking within the framework of corporate bond yields and spreads that includes a default risk. There are some important studies in the literature that examines the common factors that explain the bond returns. [Knez et al. \(1994\)](#) propose a four factor model, where first two factors correspond to movements in the level and steepness of the term structure of money market rates, while other two factors account for differences in credit risk of the different money market instruments. On the other hand [Litterman and Scheinkman \(1991\)](#) use a principal components analysis, and find that US bond returns are mainly determined by three factors, which correspond to level, steepness, and curvature movements in the term structure.

Most of the literature on asset pricing theory focuses on two risk factors for pricing bond returns, default risk factor and term risk factor, as relevant risk factors for bonds. Accordingly, the factor loadings with respect to these factors are recommended as appropriate measures of risk. An alternative approach is to use bond characteristics, such as ratings, duration, maturity and convexity as measures of risk. In asset pricing theory, the factor loadings versus the characteristics

debate has been handled by [Daniel and Titman \(1997\)](#) and [Davis et al. \(2000\)](#). The basic point of these studies focus on measuring risk and mispricing. This is due to the fact that, the size and the value premium used in these studies do not have an unambiguous risk interpretation since they are constructed from observed security market anomalies. However, we can not apply the same mentality to corporate bonds since both factor loadings and characteristics are unambiguous proxies of risk. Consequently, the role of mispricing is limited to the issue of whether idiosyncratic risk implicit in ratings and duration might be priced in the corporate bond market. For the case of corporate bonds, factor loadings measure the systematic risk and the characteristics measure total risk and there is high correlation between those risk factors. The key question, therefore is whether combined usage of both types of risk factors will give an improvement for the explanation of cross sectional bond returns. In this paper we combined two risk measures to estimate our model.

There is a large theoretical literature on the pricing of corporate bonds. This literature distinguishes between two forms of models, i.e. structural and reduced form models. In structural models, a firm is assumed to default when the value of its liabilities exceeds the value of its assets, in which case bondholders assume control of the company in exchange for its residual value.¹ Reduced form models, by contrast, assume exogenous stochastic processes for the default probability and the recovery rate. These models can allow for premia to compensate investors for illiquidity and systematic credit risk. They can be fit econometrically to data on swap and corporate bond yields.² The added flexibility of the reduced-form approach allows default risk to play a somewhat greater role in the pricing of corporate bonds. Compared to the structural approach on the pricing of corporate bonds, this paper undertakes a less structured econometric analysis, examining which observable variables have explanatory power in explaining the corporate bond yields cross-sectionally and over time, where the time variation as an autoregressive process defines the dynamics of the factor loadings.

In this framework, the main contribution of this paper is twofold. First, we provide an empirical decomposition of corporate bond yields into several proposed determinants: interest rate risk premia, default risk premia and a proxy for liquidity factor. Second, this paper is, to the best of our knowledge, one of the few to estimate the time varying risk premium using multivariate Kalman filter in corporate bond research. In that framework, our estimation methodology is com-

¹See [Black and Scholes \(1973\)](#), [Merton \(1974\)](#) and [Ingersoll \(1977\)](#) for details.

²A paper by [Duffie and Singleton \(1997\)](#) is one of the many published.

posed of two steps. In the first step, we derive the beta estimates for our bond yields via famous Fama-MacBeth procedure, that is, given T periods of data, the cross sectional regressions for 48 corporate bonds are estimated using OLS for each t , $t = 1, \dots, T$. In the second step, instead of averaging the parameter estimates that we got from the first step to find the average slope coefficient, which is fixed, we use a multivariate Kalman filter for a time varying estimate of the average slope coefficient rather than getting a fixed one. In this way, we use an empirical methodology that takes investors' expectations of the factor loadings explicitly into account when estimating betas. In an environment, where the risk -factor loadings- change dramatically over time, investors may not know the exact riskiness of the assets that they are going to hold. Consequently, they need to form beliefs about the betas, and those beliefs are affected by the past levels of loadings.

To accomplish our estimation procedure, we use bond yield data instead of bond price data. This is due to the fact that, while relevant information regarding a firm's systematic risk is incorporated into both its stock and bond prices, the latter reveal key insights about investors' return expectations. The first thing to notice is that bond yields are calculated in the spirit of forward-looking internal rates of return. Therefore, bond yield is the expected return if the bond does not default and the yield does not change in the next period. Bond prices impound the probability in default and yield spreads contain the expected risk premium for taking default risk. Controlling for default risk, firms with higher systematic risk will have higher yield spreads; a relationship that holds period-by-period, cross sectionally.

The remainder of this paper is organised as follows. Section 2 outlines the details of utility industry bond data set, reports standard summary statistics and correlation structure over time and explains calculation of the duration and convexity measures that are employed as a proxy for the interest rate risk for the remaining parts of the paper. Section 3 illustrates the Fama-MacBeth cross-sectional regression results. This is motivated by the prevalence of Fama-MacBeth cross-sectional regressions in practitioner models of corporate bond prices. In section 4, we discuss the alternative modelling strategy using the Kalman filter and in section 5 we compare how good our model fits with respect to the rolling beta regressions that have been commonly used in the literature. Section 6 concludes.

2 Descriptive Statistics and Calculation of Duration and Convexity Measures

Our data set is a special data set partially restricted by availabilities, therefore, we will use 48 corporate bonds from the utilities industry (water, gas and electricity companies) that have been collected using DATASTREAM. All the corporate bonds are fixed coupon bonds of different maturities issued mostly in the UK and some in the US. All bonds are denominated in US Dollars. The data set used in this paper covers daily data of four years starting from 2002 and ending in 2005. The data set have daily yields, spreads and ratings during the period considered.

Insert Table B.1 about here

Table B.1 illustrates the cross sectional descriptive statistics of spread, yield and interest rate risk proxy variables for an average of four consecutive years. As can be seen from the descriptive statistics, the mean and volatility of convexity and duration increases over time, and although the mean levels decline for spread and yield, the volatility for the two variables also increase over time.

Insert Table B.2 about here

Table B.2 illustrates the cross sectional correlations of selected variables that we choose for the estimations used in the modelling process. The variables utilized as interest rate risk proxy have higher correlation with the yields compared to spreads. The correlation coefficient of both duration and convexity is around 0.50 with yields and the coefficient reduces to 0.15 with spreads. With the exception of issue size, all the variables have a positive correlation coefficient. Issue size is negatively correlated with all the variables. This is in line with our expectations, as the issue size proxies the debt burden and thus large issue size means higher probability of default. Therefore it increases the cost of debt.³ Duration and convexity has a correlation very close to one, therefore we are careful not to use them together as left hand side variables not to cause multicollinearity. The correlation structure between different variables does not change significantly over the period when the data is collected. However, if we examine Table B.2 closely, we observe that the correlation coefficient between nearly all the variables and spread increase slightly over the period that we examine.

³See Sengupta (1998) for details.

The sensitivity of bond prices to changes in market interest rates is obviously of great concern to investors. There are two basic observations with respect to the relationship between bond prices and yields. The first one is that, the bond prices and yields are inversely related: as yields increase, bond prices fall; as yields fall, bond prices rise. The second observation is that: an increase in a bond's yield to maturity results in a smaller price change than a decrease in yield of equal magnitude. This is due to the curvature of the true price-yield relationship. Curves with shapes such as that of the price-yield relationship are said to be convex; the curvature of the price yield curve is called the convexity of the bond.

Duration is a first order approximation for the impact of interest rates on bond prices. This is only an approximation that is less accurate for larger changes in bond yield. For larger changes in yield, duration will underestimate the increase in bond price when the yield falls and it overestimates the decline in price when the yield rises. To compensate for this underestimation we calculate convexity measure. Convexity allows us to improve the duration approximation for bond price changes. As a practical rule, we can easily argue that the bonds with higher convexity will exhibit higher curvature in the price-yield relationship.

Bonds' maturity measures the time to receipt of the final principle repayment and therefore, the length of time the bondholder is exposed to the risk that interest rates will increase and devalue the remaining cash flows. Although it is typically the case that, the longer a bonds' maturity, the more sensitive its price is to changes in interest rates, this relationship does not always hold. Maturity is an inadequate measure of the sensitivity of a bonds' price to changes in interest rates, because it ignores the effects of coupon payments and prepayment of principal. That is the basic reason why we use duration and convexity.

To explain the relationship between price changes, yields, and duration, we begin with the fundamental valuation formula. Let " C_t " represent the entire cash flow at time " t " generated by an asset that has yield " r ", and a maturity of " T " annual periods. In that case, the valuation formula is equal to:

$$P = \sum_{t=1}^T \frac{C_t}{(1+r)^t} \quad (1)$$

and, the modified duration equals minus the derivative of " P " with respect to " r ", divided by " P ". That is, we obtain the modified duration by differentiating the " P " with respect to " r " as follows:

$$\frac{\partial P}{\partial r} = - \sum_{t=1}^T \frac{tC_t}{(1+r)^t(1+r)} \quad (2)$$

$$\frac{\partial P}{\partial r} = - \frac{1}{(1+r)} \sum_{t=1}^T \frac{t \times C_t}{(1+r)^t}$$

and dividing by the price, “ P ” to obtain:

$$\begin{aligned} \frac{\partial P}{\partial r} \left(\frac{1}{P} \right) &= - \sum_{t=1}^T \frac{tC_t}{(1+r)^t(1+r)} \\ &= - \frac{1}{(1+r)} \sum_{t=1}^T \frac{tC_t}{(1+r)^t} \left(\frac{1}{P} \right) \end{aligned} \quad (3)$$

Ignoring the presence of the minus sign, this should be recognizable as the Macaulay duration divided by $(1+r)$.

The price response of a bond to changes in yield to maturity is consequently a function not only of the bond’s modified duration, but of its convexity as well. Whereas modified duration measures the sensitivity of bond prices to changes in yield to maturity, convexity measures the sensitivity of duration to changes in yield to maturity.

Considering the convexity for the bond data set, the formula for the convexity of a bond with a maturity of n years making annual coupon payments is:

$$Convexity = \frac{1}{P(1+y)^2} \sum_{t=1}^T \left[\frac{C_t}{(1+y)^t} (t^2 + t) \right] \quad (4)$$

where “ C_t ” is the cash flow paid to the bondholder at date t ; “ C_t ” represents either a coupon payment before maturity or final coupon payment before maturity or final coupon plus par value at the maturity date. We can quantify convexity as the rate of change of the slope of the price-yield curve, expressed as a fraction of the bond price. Convexity allows us to improve the duration approximation for bond price changes.

3 Fama-MacBeth Cross-Sectional Regression Analysis

In this part of the paper, we will introduce a cross-sectional regression method similar to one used by Fama and MacBeth (1973), in order to obtain default, liquidity and interest rate factors of 48 corporate bond yields of utilities sector, which then will be used as betas in the remaining section of the paper within the state space procedure.⁴ The standard Fama-MacBeth procedure is a two step procedure. Following the first step, for each single time period a cross sectional regression is performed using different proxies for the beta risk among the variables i.e. duration, convexity, coupon, issue size and rating. Then, in the second step, the final beta loading estimates are obtained as the average of the first step coefficient estimates. The general cross sectional regression, which is the first step of the procedure, for only one period can be written as:

$$y_i = \beta_{0,i} + \sum_j^k \beta_{1,i} X_{i,j} + \eta_i; j = 1, \dots, k; i = 1, \dots, n \quad (5)$$

where $X_{i,j}$ are used as explanatory variables (convexity, duration, coupon, issue size and rating) and $\beta_{0,i}$ and $\beta_{1,i}$ are the parameters to estimate. We estimate this equation for all the time periods and get the beta coefficients for duration, convexity, issue size and rating. The second step is to test if the average of the coefficients over time $\beta_{0,i}$ and $\beta_{1,i}$ is statistically significant from zero or not. Testing if coefficient $\beta_{0,i}$ is on average not different from zero would mean that there is no other common factor being able to explain the cross-sections of the returns.

Insert Table B.3 about here

Before illustrating the cross sectional results of the Fama-MacBeth regression, we can have a look at the expected signs of the variables that define the yield premium. Starting with the ratings, it is well documented that bond rating is one of the most significant determinant of bond premium and in the literature ratings are commonly used as a proxy for a bonds' default risk. Therefore,

⁴Instead of using Fama-Macbeth procedure, we could have extracted the betas via principal components analysis, but the drawback of using pure statistical models like principal components is that, statistical factors do not have immediate economic interpretation. However, as principal component analysis is a common procedure used in asset pricing literature, we illustrate a brief section related to this procedure in Appendix A.

we expect that bond rating is positively linked with yield premium.⁵ Duration can be used as a proxy for term or maturity risk of a bond, where we expect a negative sign between duration and yield premium. We also want to control for liquidity effects on corporate bonds relative to treasury bonds and as a proxy for the demand of liquidity we include the difference between the 30-day Euro-dollar and Treasury yields. Lastly, we include issue size in the regression as a proxy for cross sectional differences in corporate bond liquidity. Following [Elton et al. \(2004\)](#), we also include the coupon rate because bonds with higher coupons are taxed more throughout the life of the bond, making them less desirable compared to bonds with lower coupons.

Table [B.3](#) illustrates all the beta coefficients and they are highly significant. The R-square of all the Fama-MacBeth regressions change between 0.34 and 0.41. However, the significant value of the constant term in all the regressions implies that there might be some other risk factors which contain information that the factors that we use in the cross sectional equations cannot explain, because a statistically significant constant in the cross sectional regressions act as a misspecification test. A close examination of Table [B.3](#) show us that the signs of the coefficients are in line with our expectations. In the next section of the paper, we will use the betas that we extract from the Fama-MacBeth regressions of this section in order to estimate the loadings attached to each beta using a multivariate state space algorithm, and we will estimate the risk premia accordingly.

Insert Figure [C.1](#) about here

Figure [C.1](#) represents the beta series we employ to get the premia within the next section. We do not want to use the beta for duration and convexity in the same procedure as they have a correlation coefficient close to one, therefore we pick convexity because the variation in the data was higher, and convexity measure allows us to improve the duration approximation for bond price changes.

⁵Corporate bond ratings are issued by credit rating agencies such as Moody's and Standard and Poor's based on fundamental analysis (analysis of financial ratios with regard to profitability, liquidity, leverage, etc., management competence, growth opportunities, industry, competitive advantages and disadvantages) of the firm. Credit rating agencies group bonds into various risk classes that are meant to convey their estimate of the probabilities of default or delayed payments for an individual bond, and the recovery rate in case of default. Grouping bonds into broad risk categories of course ignores the differences between bonds within each group. Nevertheless, these ratings are commonly used because of their simplicity.

4 State Space Model and The Kalman Filter

Based on the assumption of normality, state space models are estimated numerically through a recursive algorithm known as the Kalman filter. A state space model is defined by a transition equation and a measurement equation. In the measurement equation we postulate the relationship between an observable vector, which consists of yields for our case and a state vector, while the transition equation describes the data generating process of the state variables, which consists of the beta variable for convexity, ratings and the issue size that we obtained from the Fama-MacBeth regressions that has been described in the previous part.

The linear Gaussian state space model can be defined in different ways. We will use the notation utilized in [Harvey \(1990\)](#) that is:

$$\begin{aligned}
 y_t &= \beta_t \alpha_t + d + \varepsilon_t & \varepsilon_t &\sim N(0, H_t), \\
 \alpha_{t+1} &= T_t \alpha_t + c + R_t \eta_t & \eta_t &\sim N(0, Q_t), \quad t = 1, \dots, n \\
 \alpha_1 &\sim N(a_1, P_1)
 \end{aligned} \tag{6}$$

where y_t is N -dimensional time series (yields in our case) and α_t is unobserved $m \times 1$ loadings vector, which is called the state vector. T_t is $m \times m$ matrix, c is $m \times 1$ vector for constants of transition equation, R_t is $m \times G$ matrix, β_t is $N \times m$ matrix that we obtained in the previous step, d is $N \times 1$ vector of constants for measurement equation. α_1 is the initial state where α_1 and P_1 are known.

The idea underlying the model is that the development of the system over time is determined by α_t according to the state equation, but because α_t cannot be observed directly we must base the analysis on observations y_t . The matrices β_t, T_t, R_t, H_t and Q_t are initially assumed to be known and the error terms ε_t and η_t are of $N \times 1$ and $G \times 1$ dimensions and they are assumed to be serially independent and independent of each other at all time points. The measurement equation error ε_t , and the transition equation error u_t are assumed to be Gaussian and to be uncorrelated at all lags, i.e. $E(\varepsilon_t u_t) = 0$ for all t .

The object here is to obtain the conditional distribution of α_{t+1} given Y_t for $t = 1, \dots, n$ where $Y_t = y_1, \dots, y_t$. We assume all the distributions are normal, conditional distributions of subsets of

variables given other subsets of variables are also normal. The required distribution is therefore determined by the knowledge of $\alpha_{t+1} = E(\alpha_{t+1}|Y_t)$ and $P_{t+1} = Var(\alpha_{t+1}|Y_t)$.

$\alpha_{t|t-1}$ and $P_{t|t-1}$ are the best estimators of α_t and P_t (respectively), based on the information available at time $t - 1$. v_t is the innovation process or the one-step forecast error of y_t given Y_{t-1} with covariance matrix $F_t = Var(v_t)$. [Harvey \(1990\)](#) shows that the direct recursion from $\alpha_{t+1|t}$ to $\alpha_{t|t-1}$ can be written as:

$$\alpha_{t+1|t} = (T_{t+1} - K_t\beta_t)a_{t|t-1} + K_t y_t + (c - K_t d) \quad (7)$$

where the gain matrix is:

$$K_t = T_{t+1}P_{t|t-1}\beta_t' F_t^{-1} \quad (8)$$

Then, we have

$$\alpha_{t+1|t} = T_{t+1}a_{t|t-1} + c + K_t(y_t - \beta_t a_{t|t-1} - d) \quad (9)$$

using the fact that the last term corresponds to the innovation process v_t , we have:

$$y_t = \beta_t a_{t|t-1} + d + v_t \quad (10)$$

$$\alpha_{t+1|t} = T_{t+1}a_{t|t-1} + c + K_t v_t \quad (11)$$

Let θ be the vector of parameters. The log-likelihood can be expressed in terms of the innovation process, which will be equal to:

$$l_t = LogL_t = -\frac{N}{2} \log 2\pi - \frac{1}{2} \log |F_t| - \frac{1}{2} v_t^T F_t^{-1} v_t \quad (12)$$

Given the transition and measurement equations above, we can write our measurement and transition equations as:

$$\begin{aligned}
y_t &= \alpha_{0,t}\beta_{Convexity,t} + \alpha_{1,t}\beta_{Ratings,t} + \alpha_{2,t}\beta_{Issue-size,t} + \varepsilon_t \\
\alpha_{0,t} &= \phi_0 + \phi_1\alpha_{0,t-1} + u_{t,0} \\
\alpha_{1,t} &= \phi_2 + u_{t,1} \\
\alpha_{2,t} &= \phi_3 + u_{t,2}
\end{aligned} \tag{13}$$

with:

$$\begin{aligned}
\varepsilon_t &\sim N(0, H_t) \\
H_t &= \sum_{\varepsilon_t} \\
\sum_{\varepsilon_t} &= \begin{bmatrix} \sigma_{\varepsilon,1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\varepsilon,48}^2 \end{bmatrix}
\end{aligned}$$

and

$$\begin{bmatrix} u_{t,0} \\ u_{t,1} \\ u_{t,2} \end{bmatrix} \sim N(0, \sum_u)$$

We suppose that

$$\sum_u = \begin{bmatrix} \sigma_{u,1}^2 & 0 & 0 \\ 0 & \sigma_{u,2}^2 & 0 \\ 0 & 0 & \sigma_{u,3}^2 \end{bmatrix} \tag{14}$$

The constant variances $\sigma_{\varepsilon,1}^2, \dots, \sigma_{\varepsilon,48}^2$ and $\sigma_{u,1}^2, \dots, \sigma_{u,3}^2$ and the state equation parameter ϕ_i are the hyperparameters of the system. A number of alternative specifications for the stochastic process of $\alpha_{i,t}$ can be derived by formulating different assumptions on ϕ_i . Two different specifications are used for the evolution of time varying loading (of convexity) in this paper, i.e. random walk (where ϕ_1 is one) and a stationary AR(1) process.

Insert Table B.4 about here

Table B.4 illustrates the estimated parameters using two different dynamics for the convexity-loading process. In both cases, we standardised the data and therefore $H_t = I_{48 \times 48}$. Standardizing the data results with a faster convergence as the number of hyperparameters that needed to be estimated decreases significantly. The first specification is our benchmark case that is illustrated in Equation 13, where the dynamic process of $\alpha_{0,t}$ is modelled as a stationary AR(1) process. The results of the maximum likelihood procedure for the benchmark case, i.e. an AR(1) process for the loading for convexity, takes place at Table B.4. The autoregressive coefficient for the loading is close to one, that points to a very high persistence for the dynamic process of the convexity-loading. We will use the benchmark model for to compare the result with static Fama-MacBeth cross sectional regressions and rolling regression in the next section.

Table B.4 also illustrates the results for the model where $\alpha_{0,t}$ in Equation 13 follows a random walk procedure (where ϕ_1 is one):

$$\alpha_{0,t} = \alpha_{0,t-1} + u_{t,0} \tag{15}$$

Table B.4 shows that the results of the estimation when we assume a random walk process for the dynamics of convexity loading, does not change significantly from the benchmark AR(1) model. Figure C.2 illustrates the evolution of parameters of Equation 13, i.e. $\beta_{0,t}$ that is the risk premium for convexity, $\beta_{1,t}$ risk premium for ratings and $\beta_{2,t}$, risk premium for the issue size.

Insert Figure C.2 about here

5 Pricing Errors of the Estimated Models

Rolling regression procedure is one of the most common procedures to check if the factor loadings for an asset are constant over time or not. It is possible that factor loadings of assets or portfolios are time-varying but the changes in factor loadings may not be very volatile. Fama and MacBeth (1973) assume that the betas change every year and estimate a rolling regression for betas using the past four year's data. Chen et al. (1986) also follow this approach, yet when estimating the betas

with rolling regression procedure they use five years. This procedure is named "annual rolling" in [Shanken \(1992\)](#), as the estimation period rolls once a year.

The performance of rolling regressions have been long discussed in the literature for testing the stability of factor loadings. In this respect, [Ebner and Neumann \(2005\)](#) evaluated a rolling regression, a random walk Kalman filter and a flexible least square model for individual German stocks. Their results support the later model as it considerably improves the accuracy of the beta estimators. [Fama and French \(1992\)](#) for instance, use only the rolling regression and reject the link between the beta and the return statistically.

We utilize the rolling regression model method with a time window of 60 observations. Then, we compare our results with constant Fama-MacBeth cross sectional model and a multivariate Kalman filter model, using the pricing errors that the models produce as the large pricing errors point to asset pricing anomalies.

Insert Table [B.5](#) about here

As can be seen from [Table B.5](#) mean of both rolling regression and the Kalman filter procedure is very low compared to famous Fama-Macbeth two step technique. [Figure C.3](#) is an illustration of the pricing errors that we get from estimating the benchmark model using multivariate Kalman filter. We plot the pricing errors of the benchmark model with the pricing errors that we extract using Fama-MacBeth cross sectional regressions and the rolling regressions. We observe that mean, median and standard deviation of pricing errors are considerably small when Kalman filter is used.

Insert Figure [C.3](#) about here

6 Conclusion

The aim of this paper is to derive the risk premia for corporate bonds of the utilities sector using multivariate Kalman filter. To do so we use 48 daily bond yields over four years. The modelling is done via getting betas for the common factors that define the bond yield process through Fama-Macbeth regressions using proxies for default, liquidity and interest rate factors. We use the data for ratings as a proxy for default, convexity measure as a proxy for interest rate and issue size as a proxy for liquidity. In the meanwhile, the investors' learning process is mimicked by the Kalman

filter procedure that is introduced to capture the dynamics of implied factor returns, i.e. risk premia that are driven by the common factors default, liquidity and interest rates. In particular, we show that with time varying implied factor returns, our model performs much better in explaining a panel of utilities bond yields when compared with other commonly used procedures. We compare different models with respect to pricing errors they produce as high and volatile pricing errors point to an asset pricing anomaly.

Comparison of the pricing errors show that the state space model with the dynamic implied factor returns perform much better than the Fama-MacBeth cross sectional regressions and the rolling regressions that are two of the most common estimation methods utilized in finance literature. In this way, it has been illustrated that there exists a clear improvement with the introduction of a dynamic structure for the implied factor returns. We can say that the static model of the Fama-MacBeth two step procedure cannot successfully mimic the investors' learning process, which leads to the difference between investors' true expectation of the implied factor returns and the constant factor return coefficients estimated by OLS.

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Appendix A Principal Component Analysis of Bond Yields

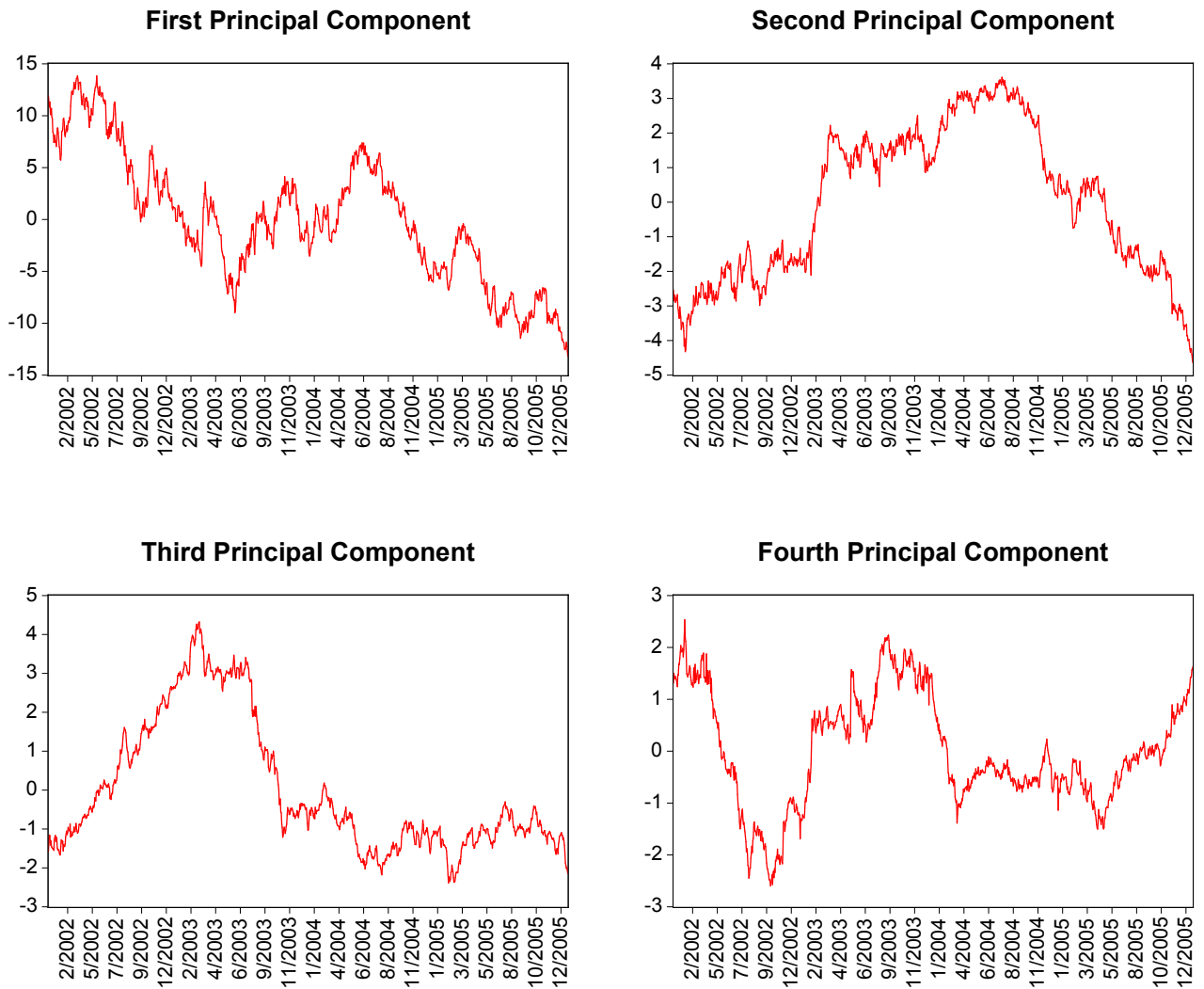
The capital asset pricing model (CAPM) suggests that the risk premium earned on equity is the product of the risk premium on the market portfolio and the beta of the stock. The CAPM is a single factor model, where the factor is the market risk premium and the loading on that factor equals the equity's beta. However, more general models of asset prices, such as the arbitrage pricing theory, suggest that multiple factor models should be more appropriate for modelling equity returns. Unfortunately, these more general models typically do not specify what those factors are and even how many factors are needed to price assets. Some approaches pre-specify macroeconomic variables (See [Chen et al. \(1986\)](#)) or proxies for fundamental variables (See [Fama and French \(1993\)](#)). Others extract the unspecified factors using a statistical approach such as factor analysis (See [Roll and Ross \(1980\)](#)) or principal components analysis (See [Connor and Korajczyk \(1986, 1993\)](#)). In this Appendix we apply principal components analysis to our bond yields. Principal component analysis is a dimension reduction technique that can be applied to a correlation matrix to determine the most important uncorrelated sources of variation. The objective is to determine how many factors are needed to adequately explain bond yields. Further, consideration of the factor loadings may give insights into the nature of the factors.

Most principal components studies in finance select a cut-off number in the range 0.8-1.0. If the eigenvalue for a component falls below this cut-off number, the factor is not considered significant in explaining returns. This method retains the specified number of factors or principal components that accounts for a specified percentage of the total variance, which in our case 97 percent for the first 4 principal components. Accordingly, our dataset have 4 common components as the eigenvalue declines to 0.43 after the fourth component. We can also use a scree graph, which plots the eigenvalues against their indices, in order to decide the number of components the data has within.

To determine the number of common components we use a criteria proposed by Bai and Ng ([2002](#)), which can be used to determine the number of factors/principal components for factor models of large dimensions. We carry out the test offered by Bai and Ng with the Matlab routine provided by the authors, however the results appear to be very sensitive to the maximum number

of factors that were pre-assigned. This sensitivity is closely related to the fact that the cross sectional dimension of our dataset is much smaller than the time-series dimension. Bai and Ng (2002) also discuss this sensitivity in their paper and state that the sensitivity increases for cross section size that is smaller than 50. Figure A.1 illustrates the time series behaviour of the four principal components that has been extracted from the dataset.

Figure A.1: First Four Principal Components of Utilities Sector Yields



Appendix B Tables

Table B.1: Cross Sectional Descriptive Statistics

Macaulay Duration				
	2002	2003	2004	2005
Mean	10.84	10.88	10.96	11.20
Median	11.96	11.95	12.00	12.29
Max.	15.01	15.96	16.27	17.29
Min.	4.36	4.37	4.38	4.38
Std. Dev.	3.05	3.10	3.14	3.34

Modified Duration				
	2002	2003	2004	2005
Mean	10.26	10.31	10.39	10.67
Median	11.30	11.38	11.39	11.75
Max.	14.22	14.89	15.40	16.45
Min.	4.20	4.24	4.27	4.28
Std. Dev.	2.87	2.93	2.96	3.17

Convexity				
	2002	2003	2004	2005
Mean	181.15	182.93	185.27	193.43
Median	202.61	203.05	203.54	207.14
Max.	347.11	434.53	458.11	506.88
Min.	24.39	24.81	25.12	25.14
Std. Dev.	95.55	98.98	100.72	108.53

Spread				
	2002	2003	2004	2005
Mean	127.51	94.92	94.55	90.71
Median	109.60	74.10	86.30	79.60
Max.	315.20	288.30	252.90	300.40
Min.	60.50	35.70	19.50	5.60
Std. Dev.	57.56	55.66	48.83	62.97

Yield				
	2002	2003	2004	2005
Mean	5.55	5.49	5.29	4.94
Median	5.56	5.49	5.37	4.86
Max.	7.21	7.63	7.11	7.11
Min.	3.88	3.12	2.57	2.55
Std. Dev.	0.63	0.71	0.79	0.79

Table B.2: Correlation Structure among Selected Variables

2002	Yield	Spread	Coupon	Convexity	Mac. Duration	Mod.Duration	Issue
Yield	1.00						
Spread	0.81	1.00					
Coupon	0.18	0.09	1.00				
Convexity	0.54	0.18	0.20	1.00			
Mac. Duration	0.50	0.16	0.13	0.99	1.00		
Mod.Duration	0.48	0.14	0.12	0.99	0.99	1.00	
Issue	-0.32	-0.18	-0.51	-0.17	-0.12	-0.12	1.00

2003	Yield	Spread	Coupon	Convexity	Mac. Duration	Mod.Duration	Issue
Yield	1.00						
Spread	0.73	1.00					
Coupon	0.32	0.24	1.00				
Convexity	0.50	0.10	0.19	1.00			
Mac. Duration	0.44	0.06	0.11	0.99	1.00		
Mod.Duration	0.42	0.05	0.11	0.99	0.99	1.00	
Issue	-0.52	-0.36	-0.51	-0.16	-0.11	-0.10	1.00

2004	Yield	Spread	Coupon	Convexity	Mac. Duration	Mod.Duration	Issue
Yield	1.00						
Spread	0.82	1.00					
Coupon	0.33	0.24	1.00				
Convexity	0.57	0.47	0.19	1.00			
Mac. Duration	0.52	0.44	0.11	0.99	1.00		
Mod.Duration	0.51	0.43	0.10	0.99	0.99	1.00	
Issue	-0.56	-0.46	-0.51	-0.16	-0.10	-0.09	1.00

2005	Yield	Spread	Coupon	Convexity	Mac. Duration	Mod.Duration	Issue
Yield	1.00						
Spread	0.82	1.00					
Coupon	0.28	0.17	1.00				
Convexity	0.38	0.57	0.19	1.00			
Mac. Duration	0.32	0.54	0.10	0.99	1.00		
Mod.Duration	0.30	0.52	0.10	0.99	0.99	1.00	
Issue	-0.53	-0.37	-0.51	-0.15	-0.09	-0.08	1.00

Table B.3: Parameter Estimates for the Fama-MacBeth Cross Sectional Regressions (Yield as Dependent Variable)

Equation 1	Beta	t-statistic	p-value
Convexity	1.33	108.90	0.000
Duration	-0.20	-55.40	0.000
Issue Size	-0.50	-52.80	0.000
Rating	0.07	70.30	0.000
Constant	6.77	54.60	0.000
R-square	0.40		

Equation 2	Beta	t-statistic	p-value
Convexity	1.58	99.69	0.000
Coupon	-0.07	-46.05	0.000
Duration	-0.26	-57.14	0.000
Issue Size	-0.58	-74.04	0.000
Rating	0.08	72.26	0.000
Constant	7.59	72.85	0.000
R-square	0.41		

Equation 3	Beta	t-statistic	p-value
Convexity	0.54	95.12	0.000
Issue Size	-0.59	-67.08	0.000
Rating	0.09	80.61	0.000
Constant	9.49	96.49	0.000
R-square	0.37		

Equation 4	Beta	t-statistic	p-value
Duration	0.13	91.74	0.000
Issue Size	-0.64	-71.32	0.000
Rating	0.10	86.79	0.000
Constant	11.46	110.86	0.000
R-square	0.34		

Table B.4: Parameter Estimates of the State Space Model

Mean Reverting Model AR(1)			
Value of Maximized	Log-Likelihood	Function: -53521.205	
	Estimates	Standard Error	p-value
Parameter 1 (ϕ_0)	0.005	0.007	0.420
Parameter 2 (ϕ_1)	0.994	0.004	0.000
Parameter 3 (ϕ_2)	-0.077	0.008	0.000
Parameter 4 (ϕ_3)	0.494	0.010	0.000
Parameter 5 ($\sigma_{u,1}^2$)	0.019	0.006	0.000
Parameter 6 ($\sigma_{u,2}^2$)	0.002	0.004	0.000
Parameter 7 ($\sigma_{u,3}^2$)	0.028	0.005	0.000

Random Walk Model			
Value of Maximized	Log-Likelihood	Function: -53522.114	
	Estimates	Standard Error	p-value
Parameter 1 (ϕ_0)	0.002	0.005	0.710
Parameter 2 (ϕ_2)	-0.077	0.008	0.000
Parameter 3 (ϕ_3)	0.494	0.010	0.000
Parameter 4 ($\sigma_{u,1}^2$)	0.018	0.006	0.000
Parameter 5 ($\sigma_{u,2}^2$)	0.002	0.004	0.000
Parameter 6 ($\sigma_{u,3}^2$)	0.027	0.005	0.000

Table B.5: Descriptive Statistics of The Pricing Errors

	Kalman Filter AR(1)	Rolling Regression	Fama-Macbeth
Mean	-0.001	0.000	0.055
Median	-0.002	0.164	0.074
Max.	0.443	1.536	0.611
Min.	-0.575	-2.826	-0.719
Std. Dev.	0.095	1.030	0.270

Appendix C Figures

Figure C.1: Fama-Macbeth Cross Sectional Betas

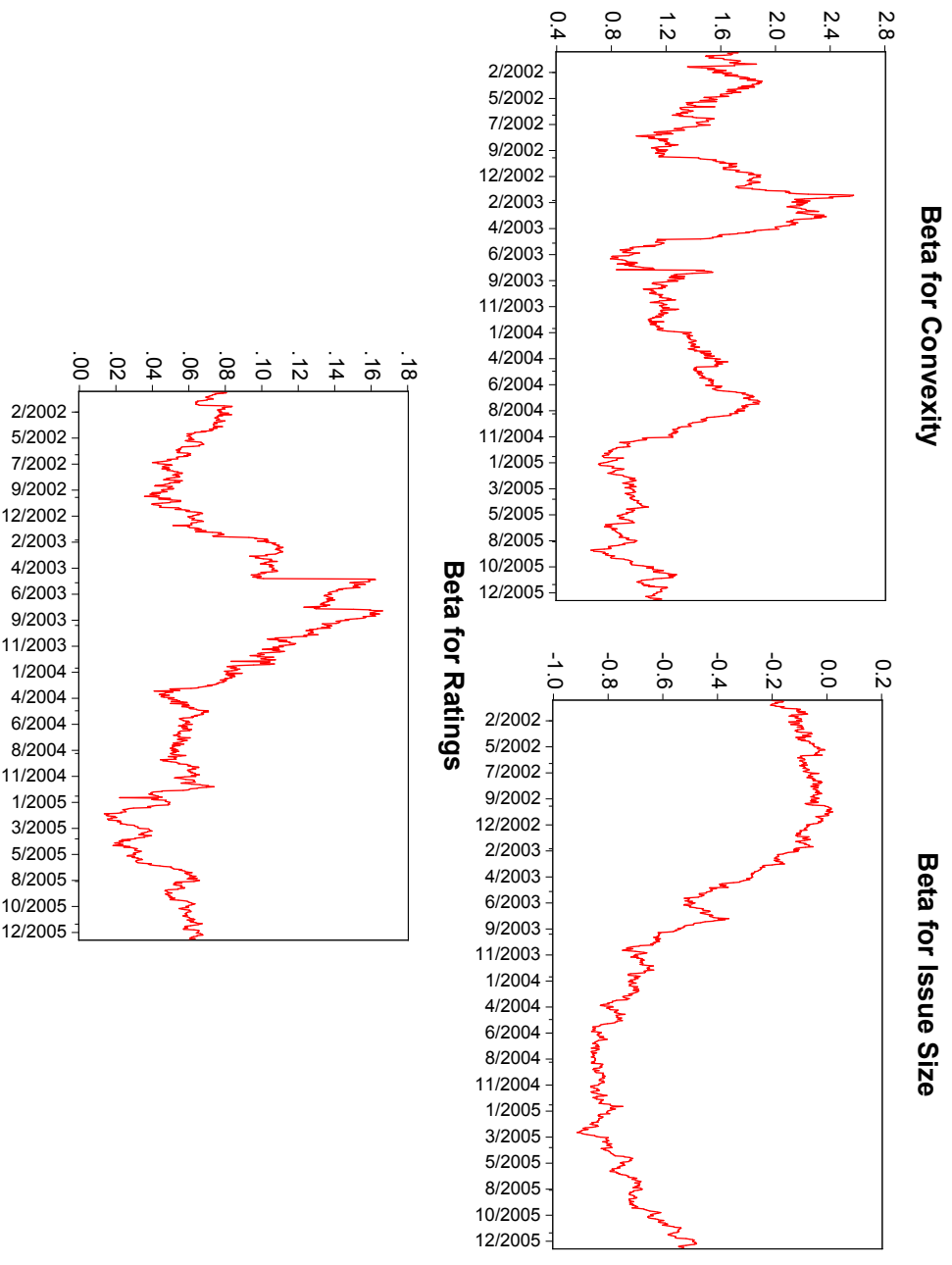


Figure C.2: Time-Varying Exposure to Convexity, Issue Size and Rating Factors

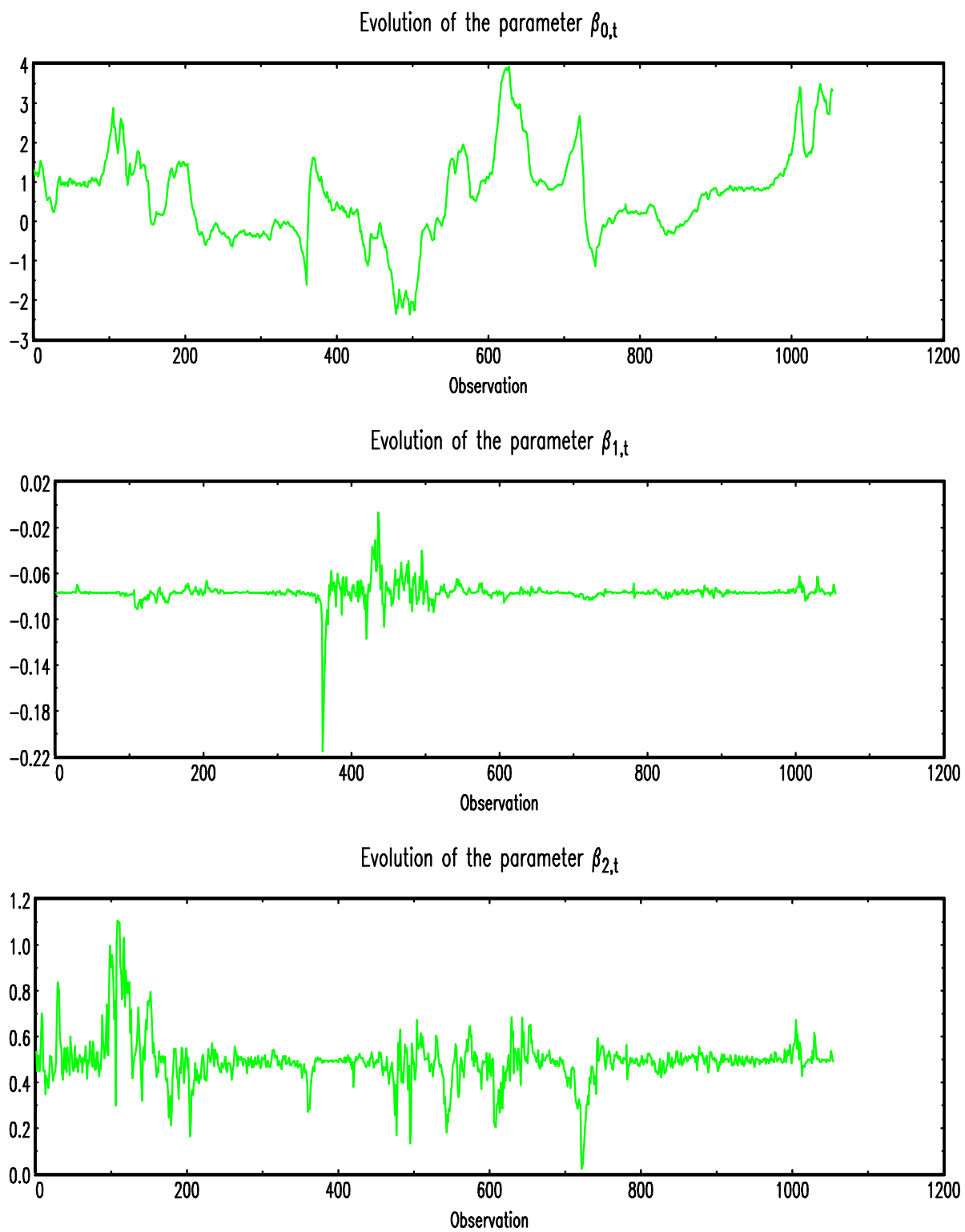
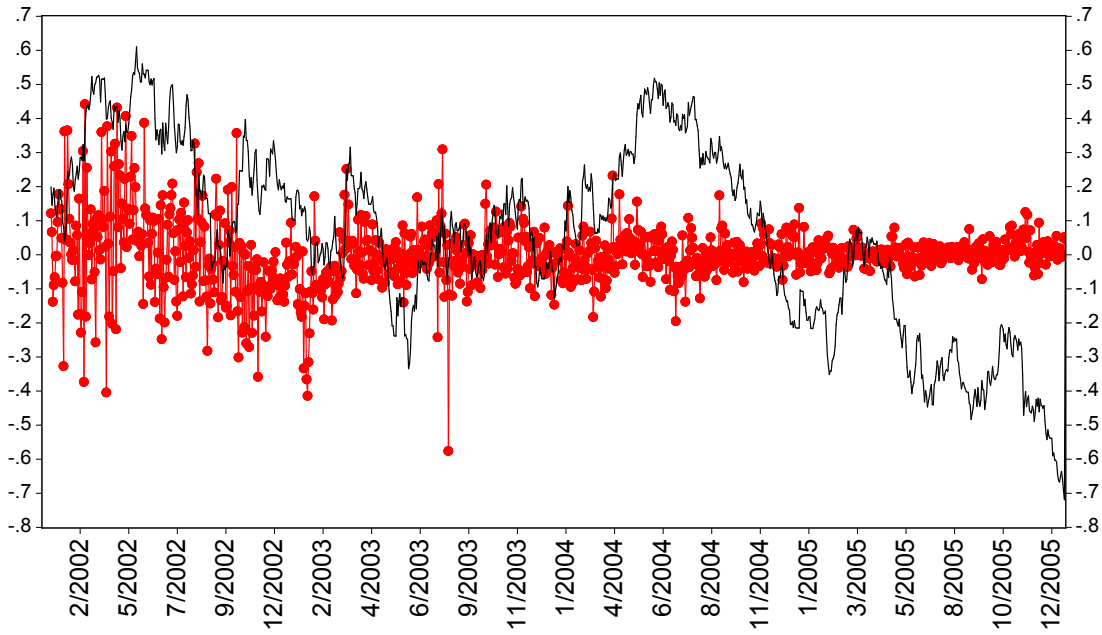
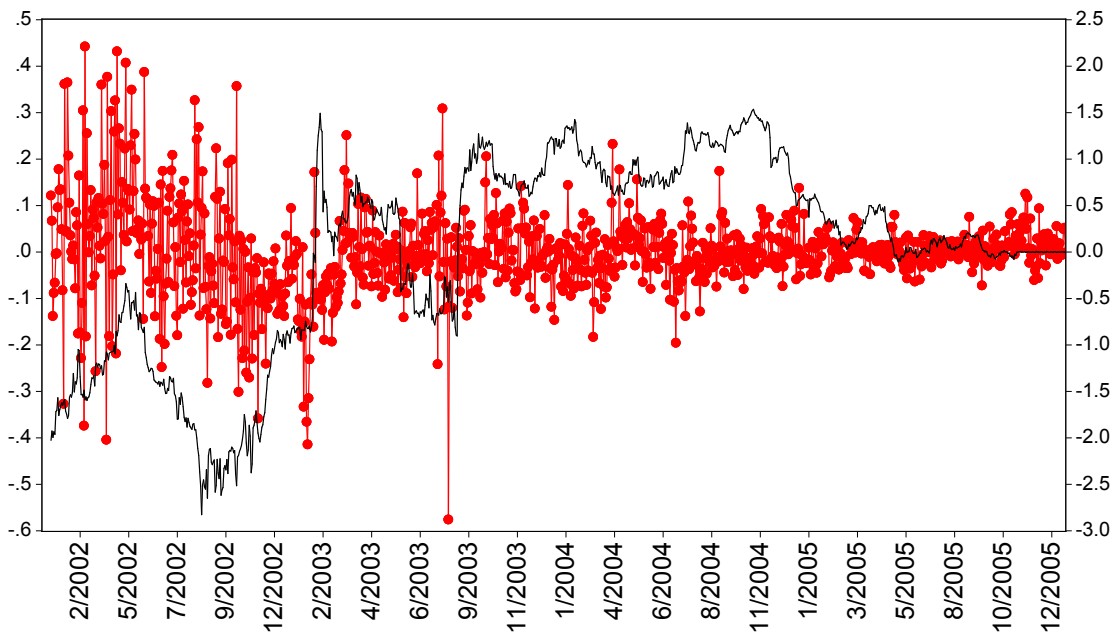


Figure C.3: Pricing Errors of Benchmark Model, Fama-MacBeth and Rolling Regression



—●— Kalman Filter AR(1) — Fama-MacBeth



—●— Kalman Filter AR(1) — Rolling Regression