The Welfare Cost of Banking Regulation

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Abstract

The Basel Accords promote the adoption of capital adequacy requirements to increase the banking sector's stability. Unfortunately, this type of regulation can hamper economic growth by shifting banks' portfolios from more productive risky investment projects toward less productive but safer projects. This paper introduces banking regulation in an overlapping-generations model and studies how it affects economic growth, banking sector stability, and welfare. In this model, a banking crisis is the outcome of a productivity shock, and banking regulation is modeled as a constraint on the maximal share of banks' portfolios that can be allocated to risky assets. This model allows us to evaluate quantitatively the key trade-off, inherent in this type of regulation, between ensuring banking stability and fostering economic growth. The model implies an optimal level of regulation that prevents crises but at the same time is detrimental to growth. We find that the overall effect of optimal regulation on social welfare is positive when productivity shocks are sufficiently high and economic agents are sufficiently risk-averse.

Keywords: Overlapping Generations, Competitive Equilibrium, Economic Growth, Banking Regulation.

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1 Introduction

As pointed out by Freixas and Rochet (1997), the usual justification for banking regulation is to increase banking system stability. Specifically, the Basel committee established a list of "best practices" for the regulation and supervision of banks. This has been adopted by many countries in the belief that it will improve the stability of their banking systems and promote financial development. This accord has three pillars, the most important being capital adequacy requirements, which aim to provide incentives for banks to hold less risky portfolios.[1] Unfortunately, this regulation can hamper economic growth by shifting banks' portfolios from more productive risky investment projects toward less productive safe projects.

There is now a fair amount of theoretical and empirical work on the effects of capital adequacy requirements on the stability of the banking system. Some studies of these requirements, as implemented under the Basel I Accord, argue that they can end up by increasing the fragility of the banking system (see, e.g., Kim and Santomero (1984) and Blum (1999)), but others argue that they may be effective in improving banking system stability (see, e.g., Dewatripont and Tirole (1994), Berger, Herring, and Szegö (1995), Freixas and Rochet (1997), Gale (2004)). An empirical assessment of this issue by Barth, Caprio, and Levine (2004) shows that the link between capital requirements and stability was not robust under the experience of the Basel I Accord. The Basel II Accord attempts to account for that by improving the assessment of the risk-weighted assets uses to compute the capital adequacy ratio. We assume in this paper that this improvement makes capital adequacy requirements effective for banking system stability. There are also a number of papers on the optimality of capital adequacy requirements (see, e.g., Hellman, Murdock, and Stiglitz (2000) Allen and Gale (2003), Gale (2003, 2004), which is an extension of Allen and Gale (2004), and Gale and Özgür (2004)).

These welfare assessment ignore the fact that changes to banks' portfolio composition have a significant impact on growth, since they are structural shifts, i.e., moving capital from risky assets toward safe investments. The main issue of regula-

[1] As pointed out by Bank for International Settlements (2003), the new Basel Accord consists of three pillars: (1) minimum capital requirement, (2) supervisory review of capital adequacy, and (3) public disclosure.

tion, when studying its impact on welfare at the macroeconomic level, is to assess the trade-off between ensuring stability and promoting economic growth. In fact, when a regulatory scheme is effective, it improves welfare because it reduces the probability of banking crisis, but at the same time it hampers growth—therefore, it can then be welfare reducing.

This paper aims at providing a framework to study this trade-off. It is an overlapping-generations model in which banks served as financial intermediaries and banking regulation is modeled as a constraint on banks' portfolios. In fact, Dewatripont and Tirole (1994), and Gale (2004), argue that equity capital reduces incentives for excessive risk taking. Consequently, banks hold less risky portfolios. Therefore, capital requirements and portfolio restrictions end up having the same effect on the riskiness of banks' portfolios: They reduce the amount of the risky assets a bank can hold.

Our model is built in a general equilibrium framework. In the setup, each young individual has access to two types of Cobb-Douglas production technology: a risky, highly productive technology and a risk-free, less productive one. The outcome of the risky production process is stochastic and i.i.d. These technologies serve to produce two intermediate goods, which are used to produce a final good via a CES technology. When young, individuals are entrepreneurs, while elders become lenders. Not having an initial endowment of capital, the entrepreneur borrows from the lender through a competitive banking sector. We assume that banks can observe the state of nature, but lenders cannot. This provides a rationale for the existence of banks. These banks transfer resources from elders to entrepreneurs by borrowing from the former at the equilibrium rental rate and lending to entrepreneurs using optimal lending contracts.

We derive many interesting results from this model. First, we show that when productivity shocks are idiosyncratic, the competitive economy can achieve the first-best allocation. We then verify that regulation hampers growth and maintains the economy at a lower level of production than that of the unregulated banking economy. Second, in the presence of an unanticipated aggregate productivity shock, the introduction of capital adequacy requirements has a positive effect on banking stability. In fact, in this case, bankruptcy can occur in the unregulated economy, but adequate banking regulation can eliminate this bankruptcy outcome by providing more available resources when the shock occurs.

Regulation affects social welfare through four channels. The first channel is its effect on the proportion of entrepreneurs involved in the risky project: We will refer to this as the *weight channel*. The second and the third channels are its effects on risky and risk-free entrepreneurs' incomes. We will refer to them as *type 1 and type 2 income channels*, respectively. The last channel is the effect of regulation on interest, we will refer to it as the *interest channel*. Some of these channels are related to the stabilization effect of regulation while others are related to the growth effect. The magnitude of the shock, and the behavior of individuals toward uncertainty, are key determinants of the importance of the stabilization effect of regulation.

We find that the overall impact of the optimal level of regulation on social welfare depends critically on the magnitude of the productivity shock, its probability, and whether economic agents are sufficiently risk-averse. In fact, the stabilization effect deriving from tighter regulation dominates the growth effect in these cases.

The rest of the paper is organized as follows. The model is described in section 2. In the third section, we investigate the effect of regulation on growth in the basic model. In section 4, we study the welfare implications of regulation in an economy with an aggregate, unanticipated shock. Section 5 provides a quantitative assessment, and section 6 considers extensions to the basic framework. Concluding remarks are contained in section 7.

2 Model

In this section we consider a simple extension to the standard OLG model, in which banks serve as financial intermediaries and banking regulation is modeled as a constraint on banks' portfolios. This model is a suitable framework for investigating the effects of banking regulation on key macroeconomic variables and for assessing its social welfare implications.

2.1 Preferences and Endowments

The economy consists of a continuum of banks, firms, and individuals. Individuals live for two periods. When young, an individual is called an entrepreneur, and when old becomes a lender. The population is constant and normalized to one. Each individual of generation $t \geq 1$ is endowed with two types of technology when young, but can implement only one, and no technology when old. Each member of generation t has preferences over consumption streams given by

$$U(c_t^y, c_t^o) = E[u(c_t^y) + \beta u(c_t^o)], \tag{1}$$

where c_t^y and c_t^o are the consumptions of a young respectively of a old of generation t, and u is strictly increasing, strictly concave, twice continuously differentiable and satisfies Inada's conditions, and β is a time-preference parameter. Each member of the initial old generation is endowed with an equal share of the aggregate capital stock k_0 and enjoys only last period consumption i.e., $U(c_0^o) = u(c_0^o)$.

2.2 Production and Investment

There are two types of technology, a high-return risky technology $y_{1t} = z_t f(k_{1t})$, and a low-return safe technology $y_{2t} = f(k_{2t})$, where k denotes physical capital, z_t is an independent and identically distributed random variable with discrete probability distribution $Prob(z_t = z_j) = \pi_j$, with $j \in \{h, l\}$ and $z_h \geq z_l$,. We assume that the mean of z_t is $\overline{z}$ and that it is greater than one.[2] Let us denote π_h by π. These technologies serve to produce two intermediate goods. We assume that f is C^2 and satisfies $f(0) = 0$, $f' > 0$, $f'' < 0$, $\lim\limits_{k \longrightarrow 0} f'(k) = \infty$, and $\lim\limits_{k \longrightarrow \infty} f'(k) = 0$. The assumption $f'' < 0$ is one way of providing a positive revenue to entrepreneurs. The random variable z_t determines the quality of the risky investment.

There are a large number of competitive firms, which produce the final good using these two intermediate goods as inputs according to the production function

$$Y_t = F(Y_{1t}, Y_{2t}) = [\gamma Y_{1t}^{\sigma} + (1-\gamma) Y_{2t}^{\sigma}]^{\frac{1}{\sigma}}\ , \tag{2}$$

where Y_{1t} denotes the risky input and Y_{2t} denotes the risk-free input at time t. Let us recall that γ is the distribution parameter. It helps to explain the relative factor shares, so it is in $[0, 1]$. σ, in $(-\infty, 1]$, is the substitution parameter—it helps in the derivation of the elasticity of substitution. Assuming a CES production function for

[2] This is one way of making the risky technology more productive than the risk-free technology.

the final good is one way of taking into account the fact that, in any economy in which one sector receives a shock, other sectors may also be in trouble.

Capital is durable, and is the only way for young agents to save. One unit of consumption placed into investment in period t yields one unit of capital in period $t+1$.

2.3 Banks

We assume that there is free entry into banking activity. This leads to a competitive banking sector. Therefore, some banks will be specialized in the risky technology and others in the risk-free one. In fact, if we suppose that this is not the case, then banks can remove resources from one type of entrepreneur and give them to others. In this case, a new bank can enter the market, specialize in the technology of the "exploited" entrepreneurs, provide a greater amount of transfer to them, and thus capture the entire market and make a positive profit.[3]

The old generation invests in the bank that promises to pay the highest interest rate. This drives all banks to promise the same interest rate to each lender. Banks behave as follows. They collect savings from the old cohort (with a promise to give them some level of consumption good in the next period) and lend to entrepreneurs.

Before presenting the problem of a bank formed in period t, we introduce some notation. p_1 is the price of the risky intermediate good, while p_2 is the price of the risk-free intermediate good. Lending contracts are set according to the type of technology: $(k_{1t}, \tau_{1t}(z_t))$ for the risky technology and (k_{2t}, τ_{2t}) for the risk-free, where τ_{it} is the transfer provided to an entrepreneur implementing technology i at time t. This may be a function of the idiosyncratic shock if the entrepreneur implements the risky investment. The optimal contract for those operating the risky technology $(k_{1t}, \tau_{1t}(z_t))$ solves the following optimization problem:

$$\max_{(k_{1t}, \tau_{1t}(z_t))} E_t \left[v(\tau_{1t}(z_t), r_{t+1}) \right] \tag{3}$$

[3] We can also obtain this result by assuming that each type of project requires specialized evaluation and monitoring. These evaluations can only be performed via two types of technology with a large fixed cost. Because of this fixed cost, each bank can have access only to one type of evaluation technology, these technologies are not accessible to individuals.

subject to the zero-profit constraint,

$$\pi\tau_{1t}(z_h) + (1-\pi)\tau_{1t}(z_l) + r_t k_{1t} = p_1 \overline{z} f(k_{1t}),$$

where $v(\tau_{it}, r_{t+1})$ is the indirect utility function of each individual and is given by

$$v(\tau_{it}(z_t), r_{t+1}) = u(\tau_{it}(z_t) - s(\tau_{it}(z_t), r_{t+1})) + \beta E_t [u((1+r_{t+1}) s(\tau_{it}(z_t), r_{t+1}))], \quad (4)$$

where the optimal savings function $s(\tau_{it}(z_t), r_{t+1})$ is given by

$$s(\tau_{it}(z_t), r_{t+1}) = \arg\max_s \{u[\tau_{it}(z_t) - s] + \beta E_t (u[(1+r_{t+1})s])\}. \quad (5)$$

Before describing the objective function, we describe the constraint. It states that entrepreneurs' transfers plus the interest payment received by lenders is equal to banks' resources, which are the nominal value of inputs produced by entrepreneurs, i.e., the quantity produced times the price of each unit of input. The objective function describes the expected utility of an individual implementing the risky technology at time t.

The optimal contract for those operating the risk-free technology (k_{2t}, τ_{2t}) solves the optimization problem:

$$\max_{(k_{2t}, \tau_{2t})} v(\tau_{2t}, r_{t+1}) \quad (6)$$

subject to the zero-profit constraint,

$$\tau_{2t} + r_t k_{2t} = p_2 f(k_{2t}).$$

This problem can be interpreted in the same way as the one above.

2.4 Individuals

At time t, each entrepreneur chooses between two types of technology. Then it borrows from banks an amount of capital according to the type of technology chosen. It produces intermediate goods and gives them to banks. Banks sell the intermediate goods to firms producing the consumption good. After production takes place, lenders receive the interest payment and their capital. They sell their capital and obtain the consumption good. Therefore, each old agent has $(1 + r_t)$ units of consumption good for each unit of capital owned at the beginning of the period. They consume all their goods and exit the economy. The entrepreneur receives a transfer and consumes and saves according to the transfer and the anticipated interest rate. Figure 1 describes the timing of events for an individual born at time t.

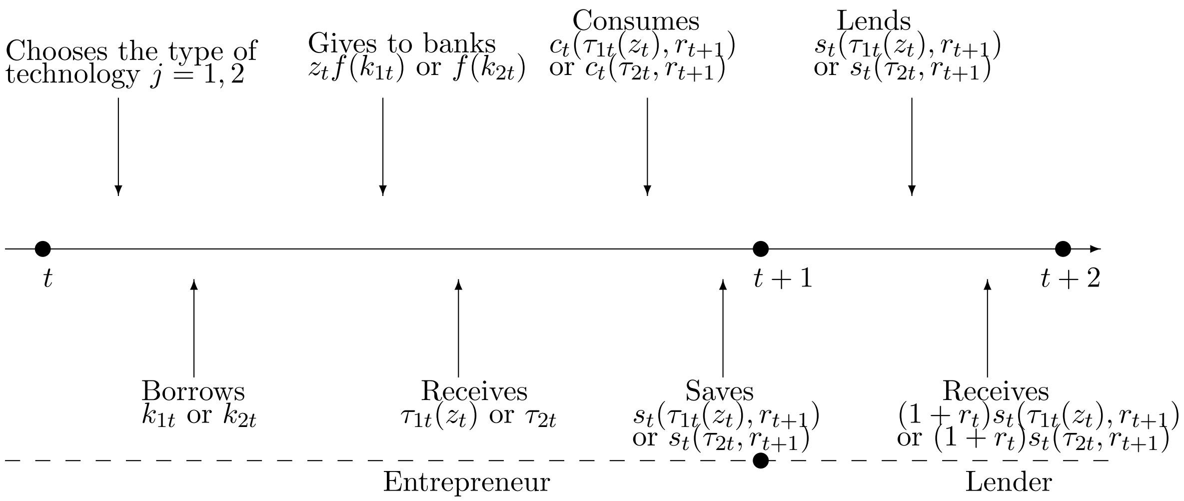


Figure 1. Timing of events for an individual born at time t

We resolve this problem recursively using indirect utility. To simplify derivation of our model we make some further assumptions. We assume that u is a power utility function of the form

$$u(c) = \frac{c^{1-\rho} - 1}{1 - \rho}. \tag{7}$$

With this assumption, we obtain (as in Castro, Clementi, and MacDonald (2004)) that $v(\tau_{it}(z_t), r_{t+1})$ is strictly increasing, strictly concave, and a linear translation of a log-separable function of $\tau_{it}(z_t)$.

In the remainder of the paper, we assume that final goods and input markets open at any time t. As a benchmark, we investigate the properties of banking regulation in the simple model given above.

3 Economy without Banking Crisis

In the above model, productivity shocks are idiosyncratic, so at the aggregate level there is no uncertainty. Although there is no market failure that can provide a rationale for bank regulation, it has been introduced in order to assess its effects on growth. In the remainder of this section we characterize the evolution of this economy in an unregulated banking environment, and then explore how the paths of variables such as capital and output change in response to the introduction of regulation.

3.1 Unregulated Banking

Before characterizing the economy, let us define a competitive equilibrium.

Definition 1. *Given k_0 units of capital in period $t = 0$, a sequential market equilibrium is defined by the consumption level of the initial old generation c_0^o, the consumption allocation for entrepreneurs who choose the risky technology (hereafter type 1 entrepreneurs) $\{c_{1t}^y(z_t), c_{1t}^o(z_t)\}_{t=0}^{\infty}$, the consumption allocation for those who choose the risk-free technology (hereafter type 2 entrepreneurs) $\{c_{2t}^y, c_{2t}^o\}_{t=0}^{\infty}$, aggregate capital $\{k_{t+1}\}_{t=0}^{\infty}$, the proportion of the type 1 entrepreneurs $\{n_t\}_{t=0}^{\infty}$, contracts $\{(k_{1t}, \tau_{1t}(z_t))\}_{t=0}^{\infty}$, for those operating the risky technology, and $\{(k_{2t}, \tau_{2t})\}_{t=0}^{\infty}$ for type 2 entrepreneurs, allocation $\{Y_t, Y_{1t}, Y_{2t}\}_{t=0}^{\infty}$ for firms, and sequences of prices $\{r_t, p_{1t}, p_{2t}\}_{t=0}^{\infty}$, such that for all $t \geq 0$:*

1. *consumers optimize, i.e., $c_0 = k_0(1 + r_0)$, for $t > 0$ and for $i = 1, 2$, $c_{it}^y(z_t) = \tau_{it}(z_t) - s(\tau_{it}(z_t), r_{t+1})$* and *$c_{it}^o(z_t) = (1 + r_{t+1})s(\tau_{it}(z_t), r_{t+1})$;*

2. *contracts are optimal, i.e., they solve the banks' problem;*

3. *ex ante, entrepreneurs are indifferent between technologies, i.e.,*

$$E\left[v(\tau_{1t}(z_t), r_{t+1})\right] = v(\tau_{2t}, r_{t+1});$$

4. *firms optimize, i.e., $\{Y_t, Y_{1t}, Y_{2t}\}_{t=0}^{\infty}$ solves the firms' problem ;*

5. *aggregate capital stock equals supply, i.e.,*

$$n_{t+1}k_{1t+1} + (1 - n_{t+1})k_{2t+1} = n_t\left[\pi s_{1t}(z_h) + (1 - \pi)s_{1t}(z_l)\right] + (1 - n_t)s_{2t};$$

6. *the risky input market clears, i.e., $Y_{1t} = n_t\bar{z}f(k_{1t})$;*

7. *the risk-free input market clears, i.e., $Y_{2t} = (1 - n_t)f(k_{2t})$.*

We now characterize the portfolio of investments in this economy.

The concavity of the instantaneous utility function drives banks dealing with type 1 entrepreneurs to provide them with risk-free contracts. This result holds for the rest of this section, so all economic variables are determined with certainty and we will omit z_t in front of variables.

Before providing the equilibrium values of the key endogenous variables, we first find the input demands. The demands for inputs are derived from the firms' problem and satisfy $p_{it} = \frac{\partial F(Y_{1t}, Y_{2t})}{\partial Y_{it}}$ for $i = 1, 2$. We can now characterize optimal contracts.

Lemma 1. *Optimal contracts offered by banks to entrepreneurs are*

$$\left(k_{1t} = f'^{-1}\left(\frac{r_t}{\overline{z}p_{1t}}\right); \tau_{1t} = \overline{z}p_{1t}\left[f\left(k_{1t}\right) - f'\left(k_{1t}\right)k_{1t}\right]\right) \tag{8}$$

to type 1 entrepreneurs, and

$$\left(k_{2t} = f'^{-1}\left(\frac{r_t}{p_{2t}}\right); \tau_{2t} = p_{2t}\left[f\left(k_{2t}\right) - f'\left(k_{2t}\right)k_{2t}\right]\right) \tag{9}$$

to type 2 entrepreneurs.

Proof. This follows directly from the First Order Conditions (FOCs) of problems (3) and (6). For details, see appendix A. ■

As expected, the optimal contracts show that the demand for capital for the risky technology is a decreasing function of the interest rate, but an increasing function of average productivity and the price of the risky intermediate good. The same results hold for the demand for capital for the risk-free technology. The only difference is that the latter is not a function of productivity. The transfer is simply the remuneration of entrepreneurship. To obtain a closed-form solution, we assume until the end of this paper that inputs are produced with a Cobb-Douglas production function, i.e., $f(x) = x^{\alpha}$ with $\alpha < 1$.[4]

Lemma 2. *At equilibrium, in any period t,*

(i) *each entrepreneur receives the same level of capital regardless of the type of technology implemented, i.e.*

$$k_{1t} = k_{2t}; \tag{10}$$

[4]Assuming that $\alpha < 1$ allows us to fulfill the condition $f" < 0$.

(ii) *the proportion of the type 1 entrepreneurs is a constant given by*

$$n^* = \left[1 + \left(\frac{1-\gamma}{\gamma\bar{z}^{\sigma}}\right)^{\frac{1}{1-\sigma}}\right]^{-1}. \tag{11}$$

Proof. These results are obtained using the values for optimal contracts provided by lemma 1, the indifference between technologies condition of entrepreneurs, and the market clearing conditions for intermediate goods. Details are provided in appendix A. ■

This lemma shows that the share of banks' portfolios used to produce the risky input in the entire economy is time invariant, so we omit t on n_t in the rest of this subsection. It also shows that this share increases with productivity, the distribution parameter γ, and with the substitution parameter σ.[5] When the substitution parameter increases, the share of bank portfolios allocated to the risky technology increases, and when this tends to 1, (i.e., the elasticity of substitution is equal to infinity), this share tends to 1. When $\sigma < 1$, i.e., the elasticity of substitution of inputs in the final good's production technology is different from infinity, n^* is strictly less than one. This is an interesting result, because empirically in economies without capital adequacy requirements or asset holding restrictions, the amount of safe assets held by banks is strictly positive.[6] We assume for the rest of this paper that $\sigma \in (0, 1)$.

Direct calculations show that prices (p_{1t} and p_{2t}) are time invariant. In fact, they are simply a function of n, which is constant. This result was expected, because input prices are a function of their relative scarcity and their complementarity in the production process. This result holds in the rest of this section.

Finally, by replacing n with its equilibrium value (n^*) in the final good production function, we obtain that the economy evolves exactly as a standard OLG economy of

[5] When $\sigma = 0$, (case of the Cobb-Douglas technology, i.e., $F(Y_1, Y_2) = Y_1^{\gamma} Y_2^{1-\gamma}$) $n^* = \gamma$. In this case, n is just equal to the share of input 1 in the production process. It is then not a function of the inputs'productivity. When $\sigma = -\infty$, (case of the Leontief technology, i.e., $F(Y_1, Y_2) = \min(Y_1, Y_2)$) $n^* = \frac{1}{2}$.

[6] As pointed out by Alexander (2004), in the 1970s and early 1980s, most countries did not have minimum capital requirements for banks.

capital accumulation, with

$$
\begin{aligned}
Y_t &= \phi(\overline{z})\, k_t^{\alpha}, \qquad (12)\\
\text{where } \phi(\overline{z}) &= \left[(\gamma\overline{z}^{\sigma})^{\frac{1+\sigma}{1-\sigma}} + (1-\gamma)^{\frac{1+\sigma}{1-\sigma}}\right]\left[(\gamma\overline{z}^{\sigma})^{\frac{1}{1-\sigma}} + (1-\gamma)^{\frac{1}{1-\sigma}}\right]^{-\sigma}.
\end{aligned}
$$

It is obvious that the portfolio composition of banks in competitive equilibrium is efficient. In fact, this competitive equilibrium yields the same level of transfer, the same level of capital per entrepreneur, and also a deterministic interest rate for the old cohort. It is then like a competitive equilibrium with a representative agent in a deterministic environment. There is no way to have market failure, which could provide a rationale for a planner interventing to achieve a better portfolio of assets. Besides, the Balasko-Shell (1980) criterion for optimality is met (i.e., the indifference curves have neither flat parts nor kinks, aggregate endowments are uniformly bounded from above, and the infinite sum of t-period gross interest rates diverges), thus dynamic inefficiency of the OLG model is impossible in our model.

3.2 Regulated Banking

Since the competitive equilibrium portfolio of banks is efficient, regulation cannot be welfare improving. But what is its amplitude and its effect on the evolution of some major macroeconomic indicators?[7] To assess those effects, let us first define the new competitive equilibrium.

Definition 2. *Given k_0 units of capital in period $t = 0$, a sequential market equilibrium is defined by the consumption level of the initial old generation c_0^o, the consumption allocation for type 1 entrepreneurs $\{\widehat{c}_{1t}^y(z_t), \widehat{c}_{1t}^o(z_t)\}_{t=0}^{\infty}$, the consumption allocation for the type 2 entrepreneurs dealing with the risky bank $\{\widehat{c}_{2t}^y, \widehat{c}_{2t}^o\}_{t=0}^{\infty}$, the consumption allocation for the type 2 entrepreneurs dealing with the risk-free bank $\{c_{2t}^y, c_{2t}^o\}_{t=0}^{\infty}$, aggregate capital $\{k_{t+1}\}_{t=0}^{\infty}$, the proportion of the type 1 entrepreneurs in the risky bank $\{\widehat{n}_t\}_{t=0}^{\infty}$, the proportion of entrepreneurs who choose the risky bank $\{m_t\}_{t=0}^{\infty}$, the contracts $\left\{(\widehat{k}_{1t}, \widehat{\tau}_{1t}(z_t))\right\}_{t=0}^{\infty}$, for those*

[7] Bernanke and Gertler (1985) state that most of the original regulation was imposed on macroeconomic grounds. Therefore, to assess the welfare cost of regulation one needs to study its effect on macroeconomic variables.

operating the risky technology, $\left\{(\widehat{k}_{2t}, \widehat{\tau}_{2t})\right\}_{t=0}^{\infty}$ *for entrepreneurs implementing the risk-free technology in the risky bank,* $\{(k_{2t}, \tau_{2t})\}_{t=0}^{\infty}$ *for those operating the risk-free technology in the risk-free bank, allocation* $\{Y_t, Y_{1t}, Y_{2t}\}_{t=0}^{\infty}$ *for firms, and sequences of prices* $\{r_t, p_{1t}, p_{2t}\}_{t=0}^{\infty}$ *, such that for all* $t \geq 0$ *:*

1. *consumers optimize, i.e.,* $c_0 = k_0(1 + r_0)$*, for* $t > 0$ $c_{2t}^y = \tau_{2t} - s(\tau_{2t}, r_{t+1})$ and $c_{2t}^o = (1 + r_{t+1})s(\tau_{2t}, r_{t+1})$*, and for* $i = 1, 2$ *,* $\widehat{c}_{it}^y(z_t) = \widehat{\tau}_{it}(z_t) - s(\widehat{\tau}_{it}(z_t), r_{t+1})$ and $\widehat{c}_{it}^o(z_t) = (1 + r_{t+1})s(\widehat{\tau}_{it}(z_t), r_{t+1})$;

2. *contracts are optimal, i.e., they solve the banks' problem*;

3. *ex ante, entrepreneurs operating the risk-free technology are indifferent between banks, i.e.,* $v(\widehat{\tau}_{2t}, r_{t+1}) = v(\tau_{2t}, r_{t+1})$;

4. *ex ante, entrepreneurs in the risky bank are indifferent between technologies, i.e.,*
$$E\left[v(\widehat{\tau}_{1t}(z_t), r_{t+1})\right] = v(\widehat{\tau}_{2t}, r_{t+1});$$

5. *firms optimize, i.e.,* $\{Y_t, Y_{1t}, Y_{2t}\}_{t=0}^{\infty}$ *solves their problem*;

6. *aggregate capital stock equals supply, i.e.,*
$$\begin{aligned} & m_{t+1}\widehat{n}_{t+1}\widehat{k}_{1t+1} + m_{t+1}(1 - \widehat{n}_{t+1})\widehat{k}_{2t+1} + (1 - m_{t+1})k_{2t+1} \\ = \; & m_t\widehat{n}_t\left[\pi\widehat{s}_{1t}(z_h) + (1 - \pi)\widehat{s}_{1t}(z_l)\right] + m_t(1 - \widehat{n}_t)\widehat{s}_{2t} + (1 - m_t)s_{2t}; \end{aligned}$$

7. *the risky input market clears, i.e.,* $Y_{1t} = m_t\widehat{n}_t\overline{z}f(\widehat{k}_{1t})$;

8. *the risk-free input market clears, i.e.,* $Y_{2t} = m_t(1 - \widehat{n}_t)f(\widehat{k}_{2t}) + (1 - m_t)f(k_{2t})$.

Before characterizing the portfolio of investments in this economy, let us define the new bank's problem. The regulated bank's problem is unchanged for those implementing the risk-free technology, but it is impossible for a bank to be specialized in the risky technology. Therefore, the formerly risky bank will now deal with both types of entrepreneurs. Since, as we stated in the previous subsection, banks provide risk-free contracts to entrepreneurs, we will not use the expected indirect utility function, will determine optimal contracts for entrepreneurs by solving the following problem,

$$\max_{\left(\widehat{k}_{1t},\widehat{\tau}_{1t},\widehat{k}_{2t},\widehat{\tau}_{2t}\right)} v(\widehat{\tau}_{1t}, r_{t+1}) \tag{13}$$

subject to,

$$\begin{aligned} \widehat{n}_t\widehat{\tau}_{1t} + (1-\widehat{n}_t)\widehat{\tau}_{2t} + r_t\left(\widehat{n}_t\widehat{k}_{1t} + (1-\widehat{n}_t)\widehat{k}_{2t}\right) &= \widehat{n}_t\overline{z}p_{1t}\widehat{k}_{1t}^{\alpha} + (1-\widehat{n}_t)p_{2t}\widehat{k}_{2t}^{\alpha}, & (14)\\ v(\widehat{\tau}_{2t}, r_{t+1}) &\geq v(\tau_{2t}, r_{t+1}), & (15)\\ \frac{\widehat{n}_t\widehat{k}_{1t}}{\widehat{n}_t\widehat{k}_{1t} + (1-\widehat{n}_t)\widehat{k}_{2t}} &\geq \theta. & (16)\end{aligned}$$

Let us describe the objective function and then the constraints. The objective function is the indirect utility function of an entrepreneur implementing the risky technology. In fact, banks specialized in risky projects only value the welfare of type 1 entrepreneurs. Equation (14) is the zero-profit condition for intermediaries, while inequality (15) is the participation constraint for type 2 entrepreneurs. Inequality (16) is the regulatory constraint, which states that banks' portfolios cannot have more than a given proportion of capital allocated to the risky technology.[8] In fact, in the presence of regulation, there is an additional constraint set for banks specialized in the risky technology. They are forced to provide at least a given share $(1-\theta)$ of their portfolio to entrepreneurs operating the risk-free technology.

We now characterize this new equilibrium. It depends on the value of θ. In fact, we have two different types of adjustment depending on the interval to which θ belongs.

Case of $\theta \in (n^*, 1)$

In this case, the equilibrium allocation of capital per entrepreneur satisfies the following property : $\widehat{k}_{1t} = \widehat{k}_{2t} = k_{2t}$. Let us consider the following solution: The proportion of type 1 entrepreneurs in the risky bank is $\widehat{n}_t = \theta$, while the proportion of people in the risky bank is $m_t = \frac{n^*}{\theta}$. This solution yields the same capital, transfer and interest rate to entrepreneurs as the unregulated economy solution. In fact, the introduction of regulation drives entrepreneurs to move only from the risk-free bank to the risky bank. They move until the transfer in the risky bank equals that in

[8] We do not omit t on $\widehat{n}_t$ in the above problem because it is a new one and we cannot say at this point if $\widehat{n}_t$ is an independent function of t.

the risk-free bank. This will be achieved with no deterioration in welfare until this adjustment is no longer possible. Since, the maximum proportion of entrepreneurs in the risky bank cannot exceed 1, from $m_t = \frac{n^*}{\theta}$, we obtain that this way of adjustment is possible only if $\theta \geq n^*$.

Case of $\theta \in (0, n^*)$

In this case, banks and entrepreneurs cannot adjust to obtain the first-best solution. The following lemma provides the optimal contracts of the regulated risky banks.

Lemma 3. *Optimal contracts proposed by the regulated, risky banks are,*

$$\left(\widehat{k}_{1t} = \theta(1-\widehat{n}_t)\left[\frac{\alpha B_t}{r_t}\right]^{\frac{1}{1-\alpha}}; \widehat{\tau}_{1t} = (1-\alpha)\frac{(1-\widehat{n}_t)}{\widehat{n}_t}\left[\widehat{n}_t B_t^{\frac{1}{1-\alpha}} - p_{2t}^{\frac{1}{1-\alpha}}\right]\left[\frac{\alpha}{r_t}\right]^{\frac{\alpha}{1-\alpha}}\right)$$

for entrepreneurs using the risky technology, and

$$\left(\widehat{k}_{2t} = \widehat{n}_t(1-\theta)\left[\frac{\alpha B_t}{r_t}\right]^{\frac{1}{1-\alpha}}; \widehat{\tau}_{2t} = (1-\alpha)p_{2t}\left[\frac{\alpha p_{2t}}{r_t}\right]^{\frac{\alpha}{1-\alpha}}\right)$$

for entrepreneurs using the risk-free technology. Where

$$B_t = \overline{z}p_{1t}\theta^\alpha(1-\widehat{n}_t)^{\alpha-1} + p_{2t}(1-\theta)^\alpha \widehat{n}_t^{\alpha-1}. \tag{17}$$

Proof. This follows from the FOCs of problems (13) and (6). The details are available in appendix B. ■

Lemma 4. *At equilibrium, in any period t,*

(i) *the proportion of risky input producers (n_t) is time invariant;*

(ii) *the ratio of risky input to risk-free input, $\frac{Y_{1t}}{Y_{2t}}$ denoted by Φ_t, is time invariant.*

Proof. These results are obtained using the values of the optimal contracts, the indifference between technologies condition of entrepreneurs, and the market clearing conditions for intermediate goods. See appendix B for details. ■

Intuitively, the proportion of entrepreneurs implementing the risky technology and the ratio of risky to risk-free inputs depend on the final good technology and on the regulation coefficient. Since they are fixed, n_t is time invariant. This lemma also implies that input prices are time invariant, so we will omit t in the price notation.

Lemma 5. *Y_{1t} and Φ_t are increasing and continuous functions of θ, while Y_{2t} is a decreasing function of θ.*

Proof. This result is obtained by using the values of the optimal contracts, the indifference between technologies condition of entrepreneurs, and the market clearing conditions for intermediate goods. The details are available in appendix B. ■

The amount of input produced by risk-free entrepreneurs increases with the amount of capital invested; we can refer to it as the volume effect, while the risk-free input price decreases since it is abundant. Since $\sigma > 0$, the volume effect dominates the price effect and therefore the anticipated transfer to type 2 entrepreneurs is an increasing function of θ. Thus, entrepreneurs produce more risk-free input. The demand for risk-free inputs by firms remains unchanged after regulation, because it is function of the final good's technology. However, as we have proven, its supply changes after regulation. In fact, with regulation, more capital is available for the risk-free technology. This shifts the supply curve of capital to the right, thus reducing the price of the risk-free input. The price reduction has a negative effect on supply (general-equilibrium effect), but this effect is not dominant when $\sigma > 0$. We assume for the remainder of this paper that we are under this condition.

3.3 The Effect of Regulation on Growth

This subsection investigates the implications of banking regulation on output and growth .

Lemma 6 *Given the capital supply, the equilibrium aggregate output increases with θ.*

Proof. To prove this, we differentiate the expression for aggregate production Y_t with respect to θ, using the results in lemma 4 and 5. Details are available in appendix C. ■

Intuitively, when the supply of capital is given, regulation has a negative effect on production of the risky input and a positive effect on production of the risk-free input. Regulation thus has two opposite effects on aggregate output. This shifts the optimal composition of inputs to the left on the transformation frontier (TF).

Since the isoquants of production curves are convex, the new equilibrium will be on an isoquant with a lower level of production. This is illustrated in figure 2. Output decreases from v_1 to v_2.

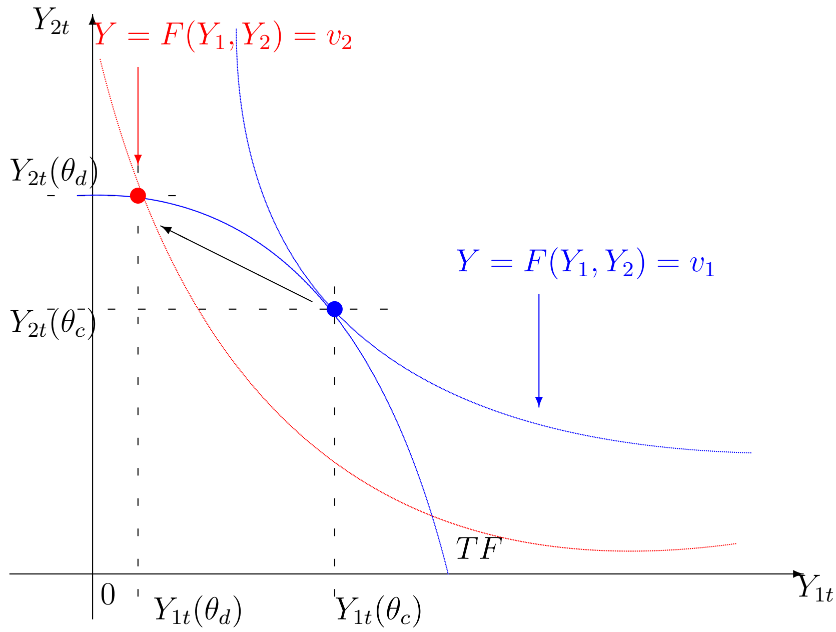


Figure 2. Effect of regulation on output.

Proposition 1. *When the instantaneous utility function is logarithmic, growth is an increasing function of* θ.

Proof. The idea underlying this proof is to differentiate the expression for growth with respect to θ and verify that it is positive. We split this proof into two steps. The first step provides an expression for growth as a function of θ, the second finds its derivative with respect to θ and verifies under which conditions it is positive. Details are available in appendix C. ∎

This result also holds in any situation in which regulation has a negative impact on the level of savings. Let us show that intuitively, by comparing the dynamics of two economies differing only in terms of θ. Let θ_c and θ_d be the maximum shares of the portfolio a bank is allowed to invest in the risky technology in economy c, respectively economy d, and suppose $\theta_c > \theta_d$. Let us start at time $t = 0$. Since the initial capital stock is given, the supply of capital by the old generation is completely inelastic at k_0. The impact of the regulation is reflected in different demand curves for aggregate capital. It results in a lower interest rate r_0 in the economy with θ_d, and the transfer received by entrepreneurs τ_0 is lower.

Intuitively, since the amount of capital invested in the production of the risky input is fixed and lower in economy d, and given that the demand for capital increases with productivity, demand is lower and supply is unchanged, so the interest rate will adjust, i.e., r_0 is lower. On the other hand, the production of the risky input will be lower while the production of the risk-free input will be higher. lemma 6 shows that production is lower, i.e., $Y_0(\theta_d) < Y_0(\theta_c)$. It has the same effect on prices, that is $p_2(\theta_d) < p_2(\theta_c)$. Regulation has two opposite effects on τ_{20}, since τ_{20} is an increasing function of p_{20} and a decreasing function of r_0. The price effect is always dominated, because r_0 is proportional to p_{20} and its weight is less than p_{20}'s weight in the expression for τ_{20}.

Capital demand increases with θ. Therefore, economy d's demand curve is to the left of the economy c's demand curve. A lower value of τ_0 implies a shift in the capital supply curve to the left at $t = 1$ if the substitution effect deriving from the lower interest rate dominates the income effect. In this case, $k_1(\theta_d) < k_1(\theta_c)$. This is always the case when $\rho \leq 1$.

Finally, this means that for any pair of regulation parameters (θ_c, θ_d) with $\theta_c > \theta_d$, $\tau_0(\theta_c) > \tau_0(\theta_d)$. It will also be the case that $k_1(\theta_d) < k_1(\theta_c)$, $r_1(\theta_d) < r_1(\theta_c)$, and $\tau_1(\theta_c) > \tau_1(\theta_d)$. By repeating the same argument at any period t, we conclude that capital accumulation is higher in the economy with less regulation.

The result can be generalized when the income effect dominates the substitution effect, but in a way that causes the slope of the supply curve to be lower (in absolute value) than the slope of the demand curve. In this case, for all $t \geq 1$, $k_t(\theta_d) < k_t(\theta_c)$. Therefore, even when $\rho > 1$, we can still have the same result.

4 The Economy with Banking Crises

To introduce a possibility of banking failure, we use the competitive equilibrium results from section 3. Then we allow the occurrence of an unanticipated state ϖ, in which aggregate productivity in the risky sector is lower than the banking system's ability to meet its promise to lenders, and show that this can provide a rationale for regulation. We then compute the optimal level of regulation and study its welfare implications.

4.1 Characterization of Equilibrium

With $\rho \geq 1$, it is obvious that at any time t an individual must consume a positive amount of the final good. Thus, each bank must provide a positive transfer to each entrepreneur. This is not only a modeling assumption. In fact, Halac and Schmukler (2004) document many ways that borrowers are bailed out in the resolution of crises: (1) when bank loans are transferred to the central bank or an asset management company (making it relatively easy for borrowers to default on their debts), (2) when governments provide debt relief programs, (3) or when the central bank establishes a preferential exchange rate for foreign-currency denominated debt, i.e., the central bank sells dollars to debtors at a subsidized exchange rate.[9] To achieve this, let us assume in the rest of this section that banks must provide at least a minimum transfer to each entrepreneur and denote by $\underline{\tau}$ this minimum. We assume that the occurrence of the aggregate productivity shock is very unlikely. Therefore, it is unanticipated. Following Allen and Gale (2000), the state ϖ $(z_t = z_w)$ will occur with probability zero. We also assume that this occurs only at the steady state. Each bank is required to meet its promise to pay a given interest rate to lenders if it can at least pay the minimum transfer to each entrepreneur dealing with it.

We now describe the equilibrium at time t in state ϖ. At t, a bank can find itself in one of these two situations: It can be *solvent*, that is it provides the promised interest rate to lenders, or it can be *bankrupt*, that is it cannot pay the promised interest rate to lenders. These definitions are motivated by the assumption that lenders are very often the ones who are protected in case of banking crisis. In many countries, governments operate a deposit insurance fund that guarantees lenders' deposits.[10] Governments also stand ready to provide support to banks when they face difficulties, or banks can be taken over by the government, which then guarantees that depositors will receive all their deposits.

[9] The transfer to debtors tends to be large because borrowers often take advantage of the bailout and stop paying their debts, regardless of their capacity to pay.

[10] In the United States of America, the Federal Deposit Insurance Corporation (FDIC) pays depositors the first \$100,000 they deposited in the bank no matter what happens to the bank.

Lemma 7. *If the minimum transfer is strictly positive in any unregulated banking economy, there exists a positive number $\underline{z}$ such that if $z_w < \underline{z}$, any risky bank goes bankrupt when ϖ occurs.*

Proof. We use the resource constraint of a bank specialized in the risky technology to show that it cannot fulfill its promise to lenders. The complete proof is available in appendix D. ■

When the state of nature is $z_t = \overline{z}$, the economy continues to work as in section 3. But if the special state ϖ occurs, the risky bank in an unregulated economy cannot both pay the promised interest to its lenders and provide the minimum transfer to those implementing the risky technology. We assume in the rest of this section that $z_w < \underline{z}$. Therefore, when the unexpected state of nature occurs, any bank specialized in the risky technology goes bankrupt. In this case, the risky bank provides to entrepreneurs the minimum transfer and to lenders an equal share of the remaining resources. We will refer to this as the *bankruptcy rule.*

4.2 The Effect of Regulation on Banking Crises

As in the previous section, banking regulation forces risky banks to finance a positive proportion of the risk-free input's production. In the following proposition we show that there is an adequate value of θ (the coefficient set by the regulation) such that when the unanticipated state occurs, the regulated risky bank is able to pay the promised interest rate to lenders and still be able to provide more than the minimum transfer to entrepreneurs.

Proposition 2. *There is a non empty set $S \subset [0, n^*]$ containing an open interval of real numbers such that, if $\theta \in S$, the risky banks can always fulfill their promises toward lenders.*

Proof. This proof is based on the zero profit constraint. We show that under regulation, banks dealing with entrepreneurs implementing the risky technology have enough resources to provide at least the minimum transfer to entrepreneurs and pay the promised interest to lenders. The complete proof is available in appendix D. ■

It is hard to prove that S is an interval, but for all examples we computed numerically we obtained that it is an interval. We thus assume until the end of this section that S is an interval which has as upper bound $\underline{\theta}$. When $\theta < \underline{\theta}$, if $z_t = z_w$, the risky bank has enough resources to pay lenders and type 2 entrepreneurs and provide more than the minimum transfer to type 1 entrepreneurs. It is then possible to set a regulation coefficient such that a banking crisis cannot occur in this economy. But as we saw in the previous section, it can have a negative impact on economic development and growth. The next step is a welfare assessment of regulation.

4.3 Welfare Analysis of Regulation

We now turn to study the welfare implications of regulation. First, let us define the welfare notion we will use. Since the shock is unanticipated, the appropriate welfare notion is realized welfare. It was introduced by Starr (1973) and has been proven to have the best properties for policy analysis. Let's $W_t(c(z_t), c(z_{t+1}))$ be the Von Neumann-Morgenstern social welfare function per generation, which depends upon individuals' realized utility. It represents the realized welfare of generation t individuals and is defined by

$$\begin{aligned} W_t(c(z_t), c(z_{t+1})) &= n_t v^r(\tau_{1t}(z_t), r_{t+1}(z_{t+1})) + (1-n_t) v^r(\tau_{2t}(z_t), r_{t+1}(z_{t+1})), \ \textit{with} \qquad (18) \\ v^r(\tau(z_t), r_{t+1}(z_{t+1})) &= u\left[\tau(z_t) - s\left(\tau(z_t), r_{t+1}(\overline{z})\right)\right] + \beta\left(u\left[(1+r_{t+1}(z_{t+1})) s\left(\tau(z_t), r_{t+1}(\overline{z})\right)\right]\right). \end{aligned}$$

In the case in which the productivity shock occurs, the welfare implications of regulation on generations living at that time is the result of a trade-off between the growth effect and the stabilization effect described by the following figure.

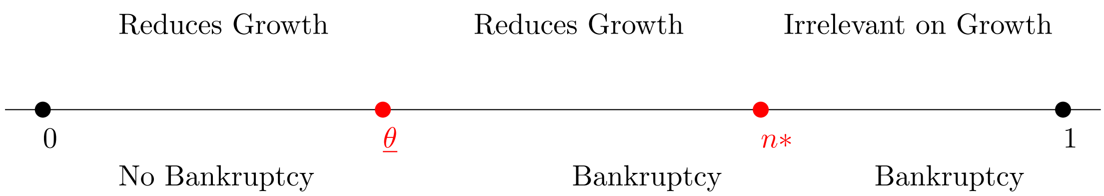


Figure 3. Effect of regulation on growth and banking Stability

With a regulation coefficient lower than $\underline{\theta}$, regulation helps to protect banks from bankruptcy, but it reduces the expected output, which translates into lower growth. We assume in the remainder of this section that $\theta \leq \underline{\theta}$, i.e., regulation helps the banking system gain stability—it cannot go bankrupt even when the unexpected state of nature occurs. Regulation affects social welfare through four channels:

(1) the weight channel, which is its effect through the proportion of type 1 entrepreneurs (n_t): Regulation can reduce n_t, diminishing the number of individuals exposed to crises in the economy. It is then welfare improving in case of a crisis, but welfare reducing in its absence.

(2) the type 1 *revenue channel,* which is its effect through the transfer received by type 1 entrepreneurs (τ_{1t}): Regulation increases τ_{1t} in case of a crisis, since it exceeds the minimum. The type 1 revenue channel is then welfare improving in the case of a crisis and welfare reducing otherwise.

(3) the type 2 *revenue channel,* which is its effect through the transfer received by type 2 entrepreneurs (τ_{2t}): Regulation reduces τ_{2t} in any case, so it is welfare reducing. In fact, the steady state transfer to type 2 entrepreneurs is low in a regulated economy, even in crisis periods, compared to in the unregulated economy.

(4) the interest channel, which is its effect through the interest rate, r_t: Regulation reduces the productivity of capital, resulting in a lower interest rate. This can reduce the savings rate, or the amount saved, thus diminishing the amount of consumption when individuals are old. Therefore, in the absence of a crisis, this channel is welfare reducing. However, in the case of a crisis, regulation helps banks to provide the promised interest rate. It is then welfare improving.

The type 2 revenue channel is related to the growth effect, while the interest rate channel is related to the stabilization effect. Two others channels, the type 1 revenue channel and the weight channel, account for both effects.

Let us assume that, at $t = t_1$, the economy is at the steady state and the shock occurs. Even for generation t, the overall social effect of regulation is ambiguous. Regulation is welfare improving only if the stabilization effect dominates the growth effect. There are two generations living in a crisis period. In fact, if the productivity shock occurs at t_1, the old (generation $t_1 - 1$) will be affected through the interest channel. When there is a crisis at t_1, the old who are dealing with the risky bank cannot obtain the promised interest rate. Thus, the crisis affects the ex post interest rate negatively.

We assume for the rest of this section that we are in situations in which regulation is welfare improving for generations living in a crisis period. In this case, it is obvious that for generations living in a crisis period, the optimal regulation is less than $\underline{\theta}$, i.e., there is an appropriate level of capital adequacy requirements that is welfare improving.[11]

But there are many generations in the economy, and the above analysis has shown that the portfolio composition of banks at time t affects future generations through its effects on the dynamics of the capital stock. If t is an ex ante crisis period, regulation is welfare reducing as we saw in section 3. In fact, in an ex ante crisis period, regulation affects welfare through two channels—the revenue channels and the interest channel. In fact, the type 1 revenue channel is exactly the type 2 revenue channel and the weight channel is irrelevant since type 1 entrepreneurs have exactly the same welfare as type 2. It follows from section 3 that the revenue channel and the interest channel are welfare reducing. Therefore, the regulation is welfare reducing for the generation living before a crisis.

What about generations living after the crisis? At $t_1+i;\ i \geq 1$, individuals obtain the same transfer and the same interest rate regardless of the type of technology they implement. Therefore, the weight channel is irrelevant. After the crisis, in many cases there is more aggregate capital in the regulated than in the unregulated economy. But the implications for welfare are complex and depend on the technology's parameters. After some periods, the economy returns to the steady state and then regulation has a negative impact on the welfare of generations living in those periods.

To take into account the welfare of future generations, we define a social welfare measure. Unfortunately, as pointed out by Ennis and Keister (2003), there is no clear criterion for aggregating utilities across generations. Following them, we take a simple approach and define the realized social welfare function by

[11] This result helps to provide a rationale for the Barth, Caprio and Levine (2003) empirical result. If the regulation coefficient is inappropriate i.e., $\theta \in (\underline{\theta}, n^*)$ it will end up with a negative effect on financial and economic development.

$$
\begin{aligned}
W(c(z^t)) &= \sum_{t=0}^{\infty} \delta^t W_t(c(z_t), c(z_{t+1})), \qquad (19) \\
with\ 0 &< \delta < 1.
\end{aligned}
$$

To assess the welfare cost of banking regulation, we follow Lucas (1988) to define it as the additional proportion Ω of consumption that a representative agent should pay the planner to ensure implementation of the regulation. If this proportion is positive, then regulation is welfare improving; if it is negative, then it is welfare reducing. We refer to Ω as the relative welfare gain from regulation. Ω is then the solution of the following equation:

$$
W((1+\Omega)c(z^t)) = W(\widehat{c}(z^t)). \qquad (20)
$$

Due to the complexity of the problem and the number of channels, it is not possible to provide an analytical assessment of the effect of regulation on aggregate social welfare. Thus, we will conduct a numerical assessment.

5 Numerical Analysis

We conduct a quantitative assessment of our model by simulating it with calibrated parameters from the US economy. Let us first calibrate the model to fit the observed data.

5.1 Calibration

The aim of calibration is to match the proportion of investment in the risky sector and also the relative productivity of that sector. Some parameters are taken in the literature as a priori information, others are estimated.

A priori information. We take, as is usual in the literature, the power utility parameter $\rho = 1.5$, and the share of capital in the production of the inputs, $\alpha = 0.34$.

Estimated and calibrated parameters. Table 1 provides the estimated average life expectancy, the annual interest rate, and the proportion of high-tech production in total exports from these economies over the period 1960–2000.[12]

[12] This seems the best proxy for the importance of the higher productivity sector in an economy.

Table 1. Data, average (1960–2004)

Country	Life expectancy (years)	Interest rate (%)	High-Tech (%) of Exp. (%)
USA	74	4.1	33

Source: WDI (2006)

Given the fact that people typically start to work at age 16, while in our model individuals begin working at birth, we remove 16 years from the life expectancy to obtain the life span of an individual. We obtain 58 years, so we assume that a period represents 29 years. It follows from the annual interest rate of 4.1 per cent that a period interest rate is $r = 2.2$.

To calibrate productivity in the higher productivity sector, we use a proxy for its return. We assume that high-tech is usually financed through the stock market. From stock market data, the long run annual rate is estimated as 6.8 per cent. This yields a return of $R = 5.7$ over a period. A proxy for the return in other sectors is the average real interest rate. Since we normalized the productivity of other sectors to one, we have $\overline{z} = \frac{R}{r} = 2.5$. We calibrate the lower productivity to a major crisis period, such as the episode in 2001 when the NASDAQ index lost more than 1/3 of its value. This also means that, over a period, the return in the high-tech sector is approximately the same as the return in the other sector. This allows us to set $z_w = 0.8$.

The intergenerational discount rate is $\beta = 0.3$. It is equivalent to an annual discount factor of 0.96 which is set to match the steady state interest rate of 2.2. We calibrate σ so that the effect of a shock on prices is less than the productivity effect, precisely $\sigma = 0.9$. We calibrate γ to obtain the proportion $n^* = 0.33$, i.e., we solve $n^* = \left[1 + \left(\frac{1-\gamma}{\gamma \overline{z}^{\sigma}}\right)^{\frac{1}{1-\sigma}}\right]^{-1}$, and we obtain $\gamma = 0.3$.

We now need to provide a value for the minimum transfer to type 1 entrepreneurs in case of a crisis. We use as a proxy the revenue that the creditor retains in case of bankruptcy. It follows from Richardson and Troost (2006) that, during the great depression, more that 50 per cent of loans were not recovered. During the 1980s and 1990s, Mason (2005) documents that the maximum rate of loan recoveries was close to 75 per cent. We thus take $\underline{\tau} = 0.25\tau$ as a proxy for the minimum transfer received by entrepreneurs. Finally, we assume that social and individual discount factors are the same. Table 2 summarizes the calibrated parameters.

Table 2. Benchmark Parameter Values

Symbol	Value	Description
Preferences		
β	0.30	individual discount factor
δ	0.30	social discount factor
ρ	1.50	coefficient of relative risk aversion
Technology		
α	0.34	capital's share of income
γ	0.30	distribution parameter
σ	0.70	substitution parameter
$\overline{z}$	2.50	anticipated productivity
z_w	0.80	unanticipated productivity shock
Bankruptcy rules		
$\underline{\tau}$	0.25	minimum transfer to entrepreneurs
Period		
t	29	number of years in a period

5.2 Results

Using the above parameters, we obtain that the optimal level of regulation, which can prevent the banking crisis and be welfare improving for generations living in a crisis period, is $\theta^* = 0.3$, corresponding to a reduction of 10 per cent in the level of investment devoted to the risky technology. Figure 4 in appendix E provides several charts on the dynamics of an economy with and without regulation using the above parameters.

Table 3 provides a social welfare assessment as a function of the arrival time of the productivity shock and the relative risk-aversion coefficient of individuals.

Table 3. Relative Welfare Gain (%)

$T/*$	1	2	3	4	5
ρ					
1.5	0.2	−8.4	−10.8	−11.6	−11.8
2.5	13.4	−3.7	−9.4	−11.1	−11.7
3.5	37.0	7.2	−5.5	−9.9	−11.3
4.5	65.3	27.0	4.2	−6.3	−10.1
5.5	90.4	51.4	21.5	2.6	−6.7
6.5	101.0	74.0	42.5	18.0	1.7

$/*$ is the number of periods before the shock occurs
Source: Simulation results

The benchmark simulation shows that the relative welfare gain from regulation is a decreasing function of the time of the crisis. More specifically, if the crisis occurs at the beginning of the steady state, the relative gain from regulation is close to 0.2 per cent. This relative gain declines to a negative value if the shock occurs later. It also increases with the power utility function parameter, ρ. In fact, an increase in ρ improves the stabilization effect of regulation.

We conduct another assessment assuming that the regulator is not aware of the time of the productivity shock. For that purpose, we assume that the likelihood of a shock occuring at any time in the steady state is constant. The results are presented in figure 5 in appendix E. The benchmark simulation shows that the relative welfare gain of regulation is a decreasing function of ρ. More specifically, when the relative risk aversion coefficient is lower than 4.7, the stabilization effect of regulation is dominated by the growth effect and therefore the regulation is not needed. But when the coefficient is greater than 4.7, the stabilization effect is dominant. Specifically, when $\rho = 5.5$, the relative welfare gain from regulation is evaluated at 15 per cent. However, when $\rho < 4.7$, the regulation is welfare reducing: e.g., when $\rho = 1.5$, the relative welfare gain is evaluated at -13 per cent—it is then a cost. The result that the welfare gain of regulation increases with the risk-aversion coefficient is robust to changes to some parameters of the model.

The first parameter which may be relevant, but which has not be calibrated, is the discount factor of the regulator. Let us assume now that the regulator discounts the future more than individuals (this has sometimes been viewed as a rationale for regulating by the regulator). Suppose that the time preference for the planner is 0.98 per year, which corresponds to 0.55 for a period. The qualitative results do not change, but quantitatively the risk-aversion coefficient is now lower than before. When $\rho = 4$, the welfare improvement is up to 7.5 per cent. As before, when $\rho = 1.5$, the welfare gain is evaluated at –11 per cent.

Another parameter of interest is the minimum transfer received by entrepreneurs ($\underline{\tau}$). Let us assume that entrepreneurs receive less; for example, suppose $\underline{\tau}= 0.23\tau$. The result of the simulation is that a decrease in the minimum transfer to entrepreneurs induces a greater welfare improvement from regulation. In fact, when entrepreneurs receive less, they save less, so the stock of capital in an unregulated economy is lower when $\underline{\tau}= 0.23\tau$ than when $\underline{\tau}= 0.25\tau$. This raises the importance of bankruptcy rules or liquidation rules in the welfare-gain analysis of regulation.

6 Discussion

In the above development we have not taken into account the fact that banking crises often have associated costs. Four main costs are highlighted by Hoeslcher and Quintyn (2003). Three of these costs are fiscal, so are irrelevant when we are studying the economy without modeling government, but the macroeconomic cost attributable to the fact that bankruptcy can impair the intermediation function of banks is relevant to our analysis.[13] Taking this into account increases the welfare gain of regulation on generations living after the crisis. In fact, after the crisis, banks specialized in the risky technology can suffer under-financing, so risky investments will be lower than usual. This can lead to a transitional or permanent structural change in the magnitude of inputs into the final good production process. In any case, it will reduce the growth effect of unregulated banking in the post-crisis period—thereby enhancing the welfare effect of regulation. This does not change the qualitative result obtain previously. It extends the maximum period of time during which the shock can occur and regulation continues to be welfare improving, and increases the relative welfare improvement in all periods.

[13]According to Bernanke and Gertler (1989), and Mishkin (2000), a banking crisis reduces the amount of financial intermediation undertaken by banks and therefore leads to a decline in investment and aggregate economic activity.

Also, in the previous developments, two key assumptions explain why the economy is subject to banking crises: the productivity shock and the fact that entrepreneurs must receive a minimum transfer in any case. A third assumption presented above is the fact that the shock is unanticipated. Although we have not provided an assessment of the case in which the shock occurs with a positive probability, we believe we can obtain the same results without this assumption. In fact, under the second assumption, type 1 entrepreneurs have a kind of insurance in the case of a banking crisis, therefore their expected utility is higher than the effective expected utility. Since banks maximize only the expected utility of entrepreneurs, they will end up with more risky portfolios, and thus be subject to banking crises.

7 Conclusion and Policy Implications

In the first part of this paper we introduced banking regulation in the familiar two-period OLG model of capital accumulation, in which technological shocks are idiosyncratic. The level of regulation is measured by capital adequacy requirements—the main quantitative component of Basel Accords. In this environment, our model produces several interesting implications. First, the portfolio of banks in competitive equilibrium is efficient. Second, banking regulation is detrimental to economic growth. In fact, it constrains banks to adjust their portfolio of investments towards safer, less productive assets. This structural change reduces output and also individuals' incomes. It then results in decreased savings and, therefore, investment.

In the second part we introduced an unanticipated sectorial shock, equivalent to overall lower productivity in the risky sector. We found that the economy will be subject to banking crises. In this event, there is an optimal capital adequacy requirement coefficient that can prevent crises. Although it is generally welfare improving for generations living in the crisis period, it is generally welfare reducing for populations living outside of this period.

We calibrated the model to reflect an economy such as the United States. We found that it is socially optimal to regulate when the regulator thinks that a shock will occur soon. This shows that, even when banking crises are due to real productivity shocks and impose no extra cost, there still exists a rationale for regulation when the magnitude of the productivity shock is sufficiently large and the likelihood of the shock is high. When there is no information available on the likelihood of shocks, regulation is welfare improving only with a greater level of risk aversion—levels that are higher than the usual acceptable level of risk aversion for the US economy. We also found that parameters on the bankruptcy rule, preferences, and technologies have a significant effect on the welfare improvements attributable to regulation.

Some policy implications can be drawn from this paper. First, since the welfare gain is a function of when the shock occurs, it is important for regulators to predict this time with a great degree of accuracy and raise capital requirements only when they believe that a crisis is imminent. Therefore, we advocate for a time variant regulation scheme. Second, since bankruptcy rules matter and are country variant, we advocate for country-variant regulation.

8 Appendix

8.1 Appendix A

Proof of Lemma 1

τ_{1t} is obtained from the risky bank's problem, and since banks provide a risk-free transfer to entrepreneurs, this problem is now set as:

$$\max_{(k_{1t},\tau_{1t})} v(\tau_{1t}, r_{t+1})$$

subject to the zero-profit constraint $\tau_{1t} + r_t k_{1t} = p_{1t}\overline{z}f(k_{1t})$.

Also, τ_{2t} is obtained from the risk-free bank's problem:

$$\max_{(k_{2t},\tau_{2t})} v(\tau_{2t}, r_{t+1})$$

subject to the zero-profit constraint $\tau_{2t} + r_t k_{2t} = p_{2t}f(k_{2t})$.

From the zero-profit conditions, transfers are given by $\tau_{1t} = p_{1t}\overline{z}f(k_{1t}) - r_t k_{1t}$ and $\tau_{2t} = p_{2t}f(k_{2t}) - r_t k_{2t}$. Then, by strict monotonicity, banks will simply choose capital to maximize transfers. The optimal capital levels derived from the bank's problem are

$$(k_{1t}) \quad : \quad \overline{z}p_{1t}f'(k_{1t}) = r_t \tag{21}$$

$$(k_{2t}) \quad : \quad p_{2t}f'(k_{2t}) = r_t \tag{22}$$

From (21), we have $k_{1t} = f'^{-1}\left(\frac{r_t}{\overline{z}p_{1t}}\right)$, and from (22), $k_{2t} = f'^{-1}\left(\frac{r_t}{p_{2t}}\right)$ Finally, substituting r_t by its value yields

$$\begin{aligned}\tau_{1t} &= \overline{z}p_{1t}\left[f(k_{1t}) - f'(k_{1t})k_{1t}\right] \\ \tau_{2t} &= p_{2t}\left[f(k_{2t}) - f'(k_{2t})k_{2t}\right].\end{aligned}$$

Proof of Lemma 2

(i) With assumption 2, at equilibrium each entrepreneur receives the same level of capital at any time t regardless the technology implemented. In fact, from lemma 1, $r_t = \overline{z}p_{1t}f'(k_{1t}) = p_{2t}f'(k_{2t})$, which implies the following relationship between input prices:

$$\frac{\overline{z}p_{1t}}{p_{2t}} = \frac{f'(k_{2t})}{f'(k_{1t})}. \tag{23}$$

On the other hand, the monotonicity of $v(\tau_t, r_{t+1})$ in its first argument yields that the indifference condition between technologies is given by $\tau_{1t} = \tau_{2t}$. Substituting (23) in this indifferent condition yields $\frac{f'(k_{2t})}{f'(k_{1t})} = \frac{f(k_{2t})-f'(k_{2t})k_{2t}}{f(k_{1t})-f'(k_{1t})k_{1t}}$. Given assumption 2, the above equation is equivalent to $\left[\frac{k_{1t}}{k_{2t}}\right]^{\alpha-1} = \left[\frac{k_{1t}}{k_{2t}}\right]^{\alpha}$. This implies that

$$k_{1t} = k_{2t}.$$

(ii) From $k_{1t} = k_{2t}$, equation (23) yields $\frac{\overline{z}p_{1t}}{p_{2t}} = 1$. But this is just a relation between prices. To obtain n_t, we must go further and provide an expression for prices as a function of n_t. For that purpose, we use the market clearing conditions for intermediate goods; i.e., $Y_{1t} = n_t \overline{z} k_{1t}^{\alpha}$, and $Y_{2t} = (1-n_t)k_{2t}^{\alpha}$. We recall that $p_{1t} = F_{1t}$, $p_{2t} = F_{2t}$, and $F(Y_{1t}, Y_{2t}) = [\gamma Y_{1t}^{\sigma} + (1-\gamma)Y_{2t}^{\sigma}]^{\frac{1}{\sigma}}$. Therefore,

$$\frac{\overline{z}p_{1t}}{p_{2t}} = \frac{\overline{z}F_{1t}}{F_{2t}} = \frac{\overline{z}^{\sigma}\gamma}{1-\gamma}\left(\frac{n_t}{1-n_t}\right)^{\sigma-1}. \tag{24}$$

Substituting the above equality in $\frac{\overline{z}p_{1t}}{p_{2t}} = 1$ yields $n_t = \left[1 + \left(\frac{1-\gamma}{\gamma\overline{z}^{\sigma}}\right)^{\frac{1}{1-\sigma}}\right]^{-1}$.

8.1.1 Appendix B

Proof of Lemma 3

The bank provides capital for both types of technology. The optimal capital supply must satisfy the regulatory constraint with equality. The regulatory constraint can then be reset as

$$\widehat{k}_{2t} = \frac{n_t(1-\theta)}{\theta(1-n_t)}\widehat{k}_{1t}. \tag{25}$$

Therefore, to obtain the optimal capital offered by the bank for each type of contract, we simply need to maximize the objective function according to $\widehat{k}_{1t}$. Furthermore, we have seen that the indirect utility function is a strictly increasing function of its first argument, given the zero-profit constraint and the free entry assumption for any type of bank in the economy, the bank will provide $\tau_{2t} = \widehat{\tau}_{2t}$ to type 1 entrepreneurs.

Given that there is no uncertainty and that the indirect utility of individuals is an increasing function of the transfer, the optimal choice of capital for the risky technology will be one that maximizes the amount of transfer provided to entrepreneurs.

i.e., $\widehat{k}_{1t} \equiv \arg\max_k \{\tau_{1t}(k)\}$. Where $\tau_{1t}(k)$ is obtained by substituting $\widehat{k}_{2t}$ and $\widehat{\tau}_{2t}$ with their expressions in the zero-profit condition. Then,

$$\widehat{\tau}_{1t} = \frac{B_t}{\theta^\alpha (1-n_t)^{\alpha-1}} \widehat{k}_{1t}^\alpha - \frac{r_t}{\theta} \widehat{k}_{1t} - \frac{(1-n_t)}{n_t} \widehat{\tau}_{2t}. \tag{26}$$

From the FOC, capital demand for the risky technology is given by,

$$\widehat{k}_{1t} = \theta(1-n_t) \left[\frac{\alpha B_t}{r_t}\right]^{\frac{1}{1-\alpha}}. \tag{27}$$

Given (25), and replacing $\widehat{k}_{1t}$ by its value in (27), we obtain $\widehat{k}_{2t} = (1-\theta) n_t \left[\frac{\alpha B_t}{r_t}\right]^{\frac{1}{1-\alpha}}$. Substituting for $\widehat{k}_{1t}$ and $\widehat{\tau}_{2t}$ in the zero-profit condition yields,

$$\widehat{\tau}_{1t} = (1-\alpha) \frac{(1-n_t)}{n_t} \left[n_t B_t^{\frac{1}{1-\alpha}} - p_{2t}^{\frac{1}{1-\alpha}}\right] \left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}}. \tag{28}$$

$\widehat{\tau}_{2t}$ is obtained from the participation constraint $\tau_{2t} = \widehat{\tau}_{2t}$ and from lemma 1 (in case of assumption 2).

Proof of Lemma 4

(i) The proof when $\theta \in (n^*, 1)$ is straightforward. We now investigate when $\theta \in (0, n^*)$.

The equilibrium proportion of entrepreneurs using the risky technology in the bank is obtained from the indifference between technologies condition, $\widehat{\tau}_{1t} = \widehat{\tau}_{2t}$. Using the optimal transfers given by lemma 3, this condition is equivalent to

$$[(1-\widehat{n}_t)\widehat{n}_t]^{1-\alpha} B_t = p_{2t}. \tag{29}$$

To complete the determination of $\widehat{n}_t$, we must determine p_{2t} and B_t.

1. Computation of p_{2t} :

From the market clearing conditions, we have $Y_{1t} = m_t \widehat{n}_t \overline{z} \widehat{k}_{1t}^\alpha$; and $Y_{2t} = m_t(1-\widehat{n}_t)\widehat{k}_{2t}^\alpha + (1-m_t)k_{2t}^\alpha$. In this case we know that $m_t = 1$, so $\widehat{n}_t = n_t$. Substituting for $\widehat{k}_{1t}$ and $\widehat{k}_{2t}$ in the above equations yields

$$Y_{1t} = n_t \overline{z} \theta^\alpha (1-n_t)^\alpha \left[\frac{\alpha B_t}{r_t}\right]^{\frac{\alpha}{1-\alpha}}, \text{ and} \tag{30}$$

$$Y_{2t} = (1-n_t) n_t^\alpha (1-\theta)^\alpha \left[\frac{\alpha B_t}{r_t}\right]^{\frac{\alpha}{1-\alpha}}. \tag{31}$$

Let us recall that, in the proof of lemma 2, we found that $\frac{\overline{z}p_{1t}}{p_{2t}} = \frac{\overline{z}\gamma}{1-\gamma}\left(\frac{Y_{1t}}{Y_{2t}}\right)^{\sigma-1}$. Substituting Y_1 and Y_2 in the above expression yields,

$$p_{2t} = p_{1t}\frac{(1-\gamma)}{\gamma}\left[\frac{n_t\overline{z}\theta^\alpha(1-n_t)^\alpha}{(1-n_t)n_t^\alpha(1-\theta)^\alpha}\right]^{1-\sigma} \tag{32}$$

2. Computation of B_t.

We know from (17) that

$$B_t = \overline{z}p_{1t}\theta^\alpha(1-n_t)^{\alpha-1} + p_{2t}(1-\theta)^\alpha n_t^{\alpha-1}.$$

We will express this as function of p_{2t}. From (32) we have $p_{1t} = \frac{\gamma p_{2t}}{1-\gamma}\left[\frac{n_t^{1-\alpha}\theta^\alpha\overline{z}}{(1-n_t)^{1-\alpha}(1-\theta)^\alpha}\right]^{\sigma-1}$. Substituting p_{1t} in the expression of B_t yields

$$B_t = \left[\frac{\gamma}{1-\gamma}\frac{\overline{z}\theta^\alpha}{(1-n_t)^{1-\alpha}}\left(\frac{n_t^{1-\alpha}\theta^\alpha\overline{z}}{(1-n_t)^{1-\alpha}(1-\theta)^\alpha}\right)^{\sigma-1} + (1-\theta)^\alpha n_t^{\alpha-1}\right]p_{2t}. \tag{33}$$

We now substitute the above expression of B_t into (29) and obtain

$$\frac{\gamma\overline{z}^\sigma\theta^{\alpha\sigma}}{1-\gamma}n_t^{(1-\alpha)\sigma}(1-n_t)^{(1-\alpha)(1-\sigma)}(1-\theta)^{\alpha(1-\sigma)} = 1-(1-\theta)^\alpha(1-n_t)^{1-\alpha}. \tag{34}$$

This is also equivalent to $G(n_t) = 0$ where

$$G(x) = \frac{\gamma\overline{z}^\sigma\theta^{\alpha\sigma}}{1-\gamma}x^{(1-\alpha)\sigma}(1-x)^{(1-\alpha)(1-\sigma)}(1-\theta)^{\alpha(1-\sigma)} - 1 + (1-\theta)^\alpha(1-x)^{1-\alpha}.$$

Since no term in $G(.)$ depends on t , $\widetilde{n_t}$ (which is the solution to $G(n_t) = 0$) is independent of t , therefore it will be denoted $\widetilde{n}$.

(ii) The ratio of the aggregate risky input to the aggregate risk-free input is

$$\Phi_t = \frac{Y_{1t}}{Y_{2t}}. \tag{35}$$

Substituting Y_{1t} and Y_{2t} by their respective values from (30, resp. 31) yields $\Phi_t = \overline{z}\left(\frac{n_t}{1-n_t}\right)^{1-\alpha}\left(\frac{\theta}{1-\theta}\right)^\alpha$. Since n_t is time invariant, it follows that Φ_t is time invariant.

Proof of Lemma 5

(1) We use the logarithmic transformation to study the monotonicity of Y_{1t} with respect to θ. We obtain

$$\frac{\partial\log(Y_{1t})}{\partial\theta} = \frac{1-\alpha}{n}\frac{\partial n}{\partial\theta} + \frac{\alpha}{\theta}.$$

It follows that

$$\frac{\partial \log(Y_{1t})}{\partial \theta} > 0 \Leftrightarrow \frac{\partial n}{\partial \theta} > -\frac{\alpha n}{\theta(1-\alpha)}. \tag{36}$$

(2) We use the logarithmic transformation to study the monotonicity of Φ with respect to θ. The transformation is equivalent to

$$\log(\Phi) = \log(\overline{z}) + (1-\alpha)\log(n) - (1-\alpha)\log(1-n) + \alpha\log(\theta) - \alpha\log(1-\theta).$$

This implies that

$$\frac{\partial \log(\Phi)}{\partial \theta} = \frac{(1-\alpha)}{n(1-n)}\frac{\partial n}{\partial \theta} + \frac{\alpha}{\theta(1-\theta)},$$

and it follows that

$$\frac{\partial \log(\Phi)}{\partial \theta} \geq 0 \Longleftrightarrow \frac{\partial n}{\partial \theta} \geq -\frac{\alpha n(1-n)}{\theta(1-\theta)(1-\alpha)}.$$

(3) Following the same method we obtain that

$$\frac{\partial \log(Y_{2t})}{\partial \theta} = -\frac{(1-\alpha)}{1-n}\frac{\partial n}{\partial \theta} - \frac{\alpha}{1-\theta}.$$

which implies

$$\frac{\partial \log(Y_{2t})}{\partial \theta} < 0 \Leftrightarrow \frac{\partial n}{\partial \theta} > -\frac{\alpha(1-n)}{(1-\theta)(1-\alpha)}. \tag{37}$$

(1), (2), and (3) are verified if

$$\frac{\partial n}{\partial \theta} > -\frac{\alpha}{(1-\alpha)}\left[\min\left\{\frac{(1-n)}{(1-\theta)}, \frac{n(1-n)}{\theta(1-\theta)}, \frac{n}{\theta}\right\}\right].$$

It follows that

$$\frac{\partial n}{\partial \theta} > \left\{\begin{array}{cc} -\frac{\alpha}{(1-\alpha)}\frac{n}{\theta} & if \quad n < \theta \\ -\frac{\alpha}{(1-\alpha)}\frac{(1-n)}{(1-\theta)} & if \quad not \end{array}\right. .$$

So, we now need to compute $\frac{\partial n}{\partial \theta}$ to complete this proof.

We differentiate the logarithm of (34) with respect to θ and obtain

$$\begin{aligned} \frac{\partial n}{\partial \theta} &= \frac{\alpha n(1-n)}{(1-\alpha)\theta(1-\theta)}\frac{[-\theta + \sigma(1-(1-\theta)^{\alpha}(1-n)^{1-\alpha})]}{[n - \sigma(1-(1-\theta)^{\alpha}(1-n)^{1-\alpha})]} \\ &= -\frac{\alpha n(1-n)}{(1-\alpha)\theta(1-\theta)}\frac{[\theta - \sigma C]}{[n - \sigma C]}, \\ with\ C &= 1-(1-\theta)^{\alpha}(1-n)^{1-\alpha}. \end{aligned} \tag{38}$$

It follows from direct calculations that the lemma holds under this condition

$$\left\{\begin{array}{cc} \frac{[\theta-\sigma C]}{[n-\sigma C]} > \frac{(1-\theta)}{(1-n)} & if \quad n < \theta \\ \frac{[\theta-\sigma C]}{[n-\sigma C]} < \frac{\theta}{n} & if \quad not \end{array}\right. .$$

- When $n < \theta$, $\frac{[\theta - \sigma C]}{[n - \sigma C]} > \frac{(1-\theta)}{(1-n)}$ implies $\sigma < \frac{1}{C}$ which is obvious since $C < 1$;
- When $n < \theta$, it is obvious that $\frac{[\theta - \sigma C]}{[n - \sigma C]} < \frac{\theta}{n}$.

It follows that, in any case, this lemma holds unconditionally.

8.2 Appendix C

Proof of Lemma 6

The idea of this proof is to differentiate the expression for aggregate production (Y_t) with respect to θ and verify that it is a positive quantity. We can split this proof into three steps. The first step provides an expression for aggregate production as a function of θ, the second provides the derivative of Y_t with respect to θ, and the third verifies under which conditions this is a positive quantity. We assume in this proof that k_t is given .

Step 1. the accurate expression of Y_t.

Let us start with the aggregate production expression

$$Y_t = [\gamma Y_{1t}^{\sigma} + (1-\gamma) Y_{2t}^{\sigma}]^{\frac{1}{\sigma}} .$$

In the case of regulation, we have found that $Y_{1t} = n^{1-\alpha}\theta^{\alpha}\overline{z}k_t^{\alpha}$ and $Y_{2t} = (1-n)^{1-\alpha}(1-\theta)^{\alpha}k_t^{\alpha}$.

Substituting Y_{1t} and Y_{2t} into $Y_t = [\gamma Y_{1t}^{\sigma} + (1-\gamma) Y_{2t}^{\sigma}]^{\frac{1}{\sigma}}$ yields

$$Y_t = \left[\gamma \left(n^{1-\alpha}\theta^{\alpha}\overline{z}\right)^{\sigma} + (1-\gamma)\left((1-n)^{1-\alpha}(1-\theta)^{\alpha}\right)^{\sigma}\right]^{\frac{1}{\sigma}} k_t^{\alpha}. \tag{39a}$$

But n is a function of θ. We now use this fact to simplify the above expression for Y_t. From (34), we have that

$$\gamma \left(n^{1-\alpha}\theta^{\alpha}\overline{z}\right)^{\sigma} = \frac{(1-\gamma)\left[1-(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right]}{\left[(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right]^{(1-\sigma)}}.$$

Substituting the above expression into (39a) yields,

$$Y_t = (1-\gamma)^{\frac{1}{\sigma}}\left[(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right]^{\frac{(\sigma-1)}{\sigma}} k_t^{\alpha}. \tag{40}$$

Step 2. Derivative of Y_t.

It is appropriate, given the above expression for Y_t, to use logarithmic differentiation:

$$\frac{\partial log(Y_t)}{\partial\theta} = \frac{(1-\sigma)}{\sigma}\left[\frac{\alpha}{(1-\theta)} + \frac{(1-\alpha)\frac{\partial n}{\partial\theta}}{(1-n)}\right]. \tag{41}$$

Step 3. Discussion.

The sign of the above derivative is positive if $\frac{\partial n}{\partial\theta} \geq -\frac{\alpha}{(1-\alpha)}\frac{(1-n)}{(1-\theta)}$; which is exactly condition (37). It follows then from the proof of lemma 5 that this is always the case. Therefore, $\frac{\partial log(Y_t)}{\partial\theta} \geq 0$.

Proof of Proposition 1

The idea of this proof is to differentiate the expression for growth with respect to θ and verify that it is positive. We will split this proof into two steps. The first step provides an expression for growth as a function of θ, the second provides the derivative of economic growth with respect to θ, and verifies under which conditions this is positive. We assume in this proof that k_t is given.

Step 1. Expression for growth as a function of θ

We start with $Y_t = [\gamma Y_{1t}^{\sigma} + (1-\gamma)Y_{2t}^{\sigma}]^{\frac{1}{\sigma}}$, and obtain, as in the proof of lemma 5, that $Y_t = (1-\gamma)^{\frac{1}{\sigma}}\left[(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right]^{\frac{(\sigma-1)}{\sigma}} k_t^{\alpha}$. Therefore, $\frac{Y_{t+1}}{Y_t} = \left(\frac{k_{t+1}}{k_t}\right)^{\alpha}$. From the definition of equilibrium we have $k_{t+1} = s_t$, but

$$\begin{aligned} s_t(\tau_t, r_{t+1}) &= b(r_{t+1}))\tau_t & (42)\\ \text{with } b(r_{t+1}) &= \frac{1}{1+\left[\beta\left(1+r_{t+1}\right)^{1-\rho}\right]^{\frac{-1}{\rho}}}. & (43)\end{aligned}$$

Since, at equilibrium, $\tau_t = \tau_{2t}$, from lemma 3, $\tau_t = (1-\alpha)p_{2t}\left[\frac{\alpha p_{2t}}{r_t}\right]^{\frac{\alpha}{1-\alpha}}$. Besides, $k_t = n\widehat{k}_{1t} + (1-n)\widehat{k}_{2t}$. Using the expressions for $\widehat{k}_{1t}$ and $\widehat{k}_{2t}$ provided by lemma 3, we obtain

$$k_t = n(1-n)\left[\frac{\alpha B}{r_t}\right]^{\frac{1}{1-\alpha}}. \tag{44}$$

Furthermore, the indifference between technologies condition of entrepreneurs yields

$$n(1-n)B^{\frac{1}{1-\alpha}} = p_2^{\frac{1}{1-\alpha}}. \tag{45}$$

Substituting (45) into (44) yields $k_t = \left[\frac{\alpha p_2}{r_t}\right]^{\frac{1}{1-\alpha}}$. We observe that $\tau_t = (1-\alpha)p_2 k_t^{\alpha}$. Then $k_{t+1} = b(r_{t+1})(1-\alpha)p_2 k_t^{\alpha}$, which implies that the growth rate of capital is given by

$$\frac{k_{t+1}}{k_t} = (1-\alpha)b(r_{t+1})p_2 k_t^{\alpha-1}. \tag{46}$$

With the logarithmic utility function $b(r_{t+1}) = \frac{\beta}{1+\beta}$, so (46) is equivalent to

$$\frac{k_{t+1}}{k_t} = \frac{(1-\alpha)\beta}{1+\beta} p_2 k_t^{\alpha-1}.$$

Step 2. Differentiating growth with respect with θ.

Since, at t, k_t is given, $\frac{\partial\left[\frac{k_{t+1}}{k_t}\right]}{\partial\theta}$ has the sign of $\frac{\partial p_2}{\partial\theta}$. We will now focus on p_2.

We obtain from direct calculation that

$$p_2 = (1-\gamma)\left[\frac{Y_t}{Y_{2t}}\right]^{1-\sigma}.$$

Substituting Y_t and Y_{2t} by their values in the above expression yields

$$p_2 = (1-\gamma)^{\frac{1}{\sigma}}\left[(1-\theta)^{\alpha}(1-n)^{1-\alpha}\right]^{\frac{\sigma-1}{\sigma}} = \frac{Y_t}{k_t^{\alpha}}.$$

Therefore, $\frac{\partial p_2}{\partial\theta}$ has the sign of $\frac{\partial Y_t}{\partial\theta}$. Under the conditions of lemma 6, $\frac{\partial Y_t}{\partial\theta}$ is always positive.

8.3 Appendix D

Proof of Lemma 7

The idea underlying this proof is to use the resource constraint of a bank specialized in the risky technology to show that it cannot fulfill its promise to lenders. At the steady state, the promised interest rate is given by $r = \alpha p_2 k^{\alpha-1}$, it has a constant value, the minimum transfer to entrepreneurs is a positive number $\underline{\tau}$, and τ_{1t} has a constant positive value. Banks cannot meet their promises toward lenders when the unexpected state of nature occurs if resources are less than the promised interest (rk) plus the minimum transfer. i.e.,

$$p_1(z_w)z_w k^{\alpha} < \underline{\tau} + rk. \tag{47}$$

Since $\underline{\tau} > 0$, there exists a positive number κ such that $\underline{\tau} = \kappa\tau_1$, where $\tau_1 = (1-\alpha)p_1(\overline{z})\overline{z}k^{\alpha}$ and $rk = \alpha p_2(\overline{z})k^{\alpha} = \alpha p_1(\overline{z})\overline{z}k^{\alpha}$. Substituting r and $\underline{\tau}$ by their values in (47) yields the following price-ratio inequality,

$$\frac{p_1(z_w)z_w}{p_1(\overline{z})\overline{z}} < \kappa(1-\alpha) + \alpha.$$

Furthermore, the price of the risky intermediate good is given by $p_1 = \gamma Y_1^{\sigma-1}\left[\gamma Y_1^{\sigma} + (1-\gamma)Y_2^{\sigma}\right]^{\frac{1-\sigma}{\sigma}}$; with $Y_1 = nz_w k^{\alpha}$ and $Y_2 = (1-n)k^{\alpha}$. Substituting Y_1 and Y_2 in the above expression for $p_1(z_t)$ yields,

$$p_1(z_t) = \gamma\left(nz_t\right)^{\sigma-1}\left[\gamma\left(nz_t\right)^{\sigma} + (1-\gamma)(1-n)^{\sigma}\right]^{\frac{1-\sigma}{\sigma}}.$$

So the price ratio can be rewritten as,

$$\frac{p_1(z_w)z_w}{p_1(\overline{z})\overline{z}} = \left(\frac{z_w}{\overline{z}}\right)^{\sigma}\left(\frac{\gamma\left(nz_w\right)^{\sigma} + (1-\gamma)(1-n)^{\sigma}}{\gamma\left(n\overline{z}\right)^{\sigma} + (1-\gamma)(1-n)^{\sigma}}\right)^{\frac{1-\sigma}{\sigma}}.$$

It follows that (47) is now equivalent to

$$\left(\frac{z_w}{\overline{z}}\right)^{\sigma}\left(\frac{\gamma\left(nz_w\right)^{\sigma} + (1-\gamma)(1-n)^{\sigma}}{\gamma\left(n\overline{z}\right)^{\sigma} + (1-\gamma)(1-n)^{\sigma}}\right)^{\frac{1-\sigma}{\sigma}} < \kappa(1-\alpha) + \alpha. \tag{48}$$

We obtain from $n = \left[1 + \left(\frac{1-\gamma}{\gamma\overline{z}^{\sigma}}\right)^{\frac{1}{1-\sigma}}\right]^{-1}$ that $\gamma n^{\sigma-1}\overline{z}^{\sigma} = (1-\gamma)(1-n)^{\sigma-1}$. It then follows by direct calculations that (48) is equivalent to

$$z_w^{\sigma}((nz_w^{\sigma} + \overline{z}^{\sigma}(1-n)))^{\frac{1-\sigma}{\sigma}} < (\kappa(1-\alpha)+\alpha)\overline{z}.$$

Since $z_w < \overline{z}$, when $\sigma > 0$, we have $z_w^{\sigma} < nz_w^{\sigma} + \overline{z}^{\sigma}(1-n) < \overline{z}^{\sigma}$. Therefore,

$$z_w^{\sigma}((nz_w^{\sigma} + \overline{z}^{\sigma}(1-n)))^{\frac{1-\sigma}{\sigma}} < z_w^{\sigma}\overline{z}^{1-\sigma}.$$

If $z_w^{\sigma}\overline{z}^{1-\sigma} < (\kappa(1-\alpha)+\alpha)\overline{z}$, a bank specialized in the risky technology will fail to fulfill its promise. This condition is equivalent to $z_w < (\kappa(1-\alpha)+\alpha)^{\frac{1}{\sigma}}\overline{z}$. We can then take $\underline{z} = \frac{(\kappa(1-\alpha)+\alpha)^{\frac{1}{\sigma}}\overline{z}}{1+\epsilon}$ where ϵ is any small positive number.

Proof of Proposition 2

This proof is based on the zero profit constraint. We show that under regulation, banks dealing with the type 1 entrepreneurs have enough resources to provide at least the minimum transfer to entrepreneurs and pay the promised interest to lenders.

When the aggregate shock occurs, the total resources of the regulated risky bank is given by $p_1(z_w)nz_w\widehat{k}_1^{\alpha} + p_2(z_w)(1-n)\widehat{k}_1^{\alpha} \equiv Y^r(z_w)$. From the expressions for $\widehat{k}_1$ and $\widehat{k}_2$ given by lemma 3 and direct calculations, we obtain $\widehat{k}_1 = \frac{\theta}{n}k_2$ and $\widehat{k}_2 = \frac{(1-\theta)}{(1-n)}k_2$. Therefore, $n\widehat{k}_1 + (1-n)\widehat{k}_2 = k_2$. The overall interest promised to lenders,

$r(n\widehat{k}_1+(1-n)\widehat{k}_2)$ is then equal to $\alpha p_2(\overline{z})k_2^\alpha$, while the promised transfers are $\tau_2\left(\overline{z}\right)=\tau_1\left(\overline{z}\right)=(1-\alpha)p_2(\overline{z})k_2^\alpha$.

We need to make explicit the expressions for $p_2(\overline{z})$ and $p_1(z_w)nz_w\widehat{k}_1^\alpha+p_2(z_w)(1-n)\widehat{k}_1^\alpha$ in order to use them in the zero profit constraint analysis. Direct calculations yield

$$\begin{aligned} p_2(\overline{z}) &= (1-\gamma)\left[(1-n)^{1-\alpha}(1-\theta)^\alpha k_2^\alpha\right]^{\sigma-1}Y(\overline{z})^{1-\sigma}, \text{ and} \\ Y^r(z_w) &= \left[\gamma\left(n^{1-\alpha}\theta^\alpha z_w\right)^\sigma+(1-\gamma)\left((1-n)^{1-\alpha}(1-\theta)^\alpha\right)^\sigma\right]^{\frac{1}{\sigma}}k_2^\alpha. \end{aligned}$$

Therefore, saying that when the state ϖ occurs the promised transfers and interests will be less than the available resources (i.e., $rk_2<Y^r(z_w)-(1-n)\tau_2-n\kappa\tau_1$), is equivalent to the following inequality,

$$\alpha+(1-\alpha)\left[(1-n)+n\kappa\right]<\frac{Y^r(z_w)}{p_2(\overline{z})k_2^\alpha}. \tag{49}$$

The explicit form of the right-hand side of the above inequality is,

$$\frac{Y^r(z_w)}{p_2(\overline{z})k_2^\alpha}=\frac{\left[(1-n)^{1-\alpha}(1-\theta)^\alpha\right]^{1-\sigma}\left[\gamma\left(n^{1-\alpha}\theta^\alpha z_w\right)^\sigma+(1-\gamma)\left((1-n)^{1-\alpha}(1-\theta)^\alpha\right)^\sigma\right]^{\frac{1}{\sigma}}}{(1-\gamma)\left[\left[\gamma\left(n^{1-\alpha}\theta^\alpha\overline{z}\right)^\sigma+(1-\gamma)\left((1-n)^{1-\alpha}(1-\theta)^\alpha\right)^\sigma\right]^{\frac{1}{\sigma}}\right]^{1-\sigma}}.$$

We now use functional analysis to obtain a set of regulation coefficients under which no banking crisis can occur. For that purpose, we use inequality (49) to define G, a continuous function of θ, as follows:

$$G(\theta)=\frac{(1-\gamma)\left[\alpha+(1-\alpha)\left[(1-n)+n\kappa\right]\right]}{\left[(1-n)^{1-\alpha}(1-\theta)^\alpha\right]^{1-\sigma}}-\left[\frac{\gamma\left(n^{1-\alpha}\theta^\alpha z_w\right)^\sigma+(1-\gamma)\left((1-n)^{1-\alpha}(1-\theta)^\alpha\right)^\sigma}{\left[\gamma\left(n^{1-\alpha}\theta^\alpha\overline{z}\right)^\sigma+(1-\gamma)\left((1-n)^{1-\alpha}(1-\theta)^\alpha\right)^\sigma\right]^{1-\sigma}}\right]^{\frac{1}{\sigma}}. \tag{50}$$

We also recall that in the proof of lemma 4 (in appendix B), n solves

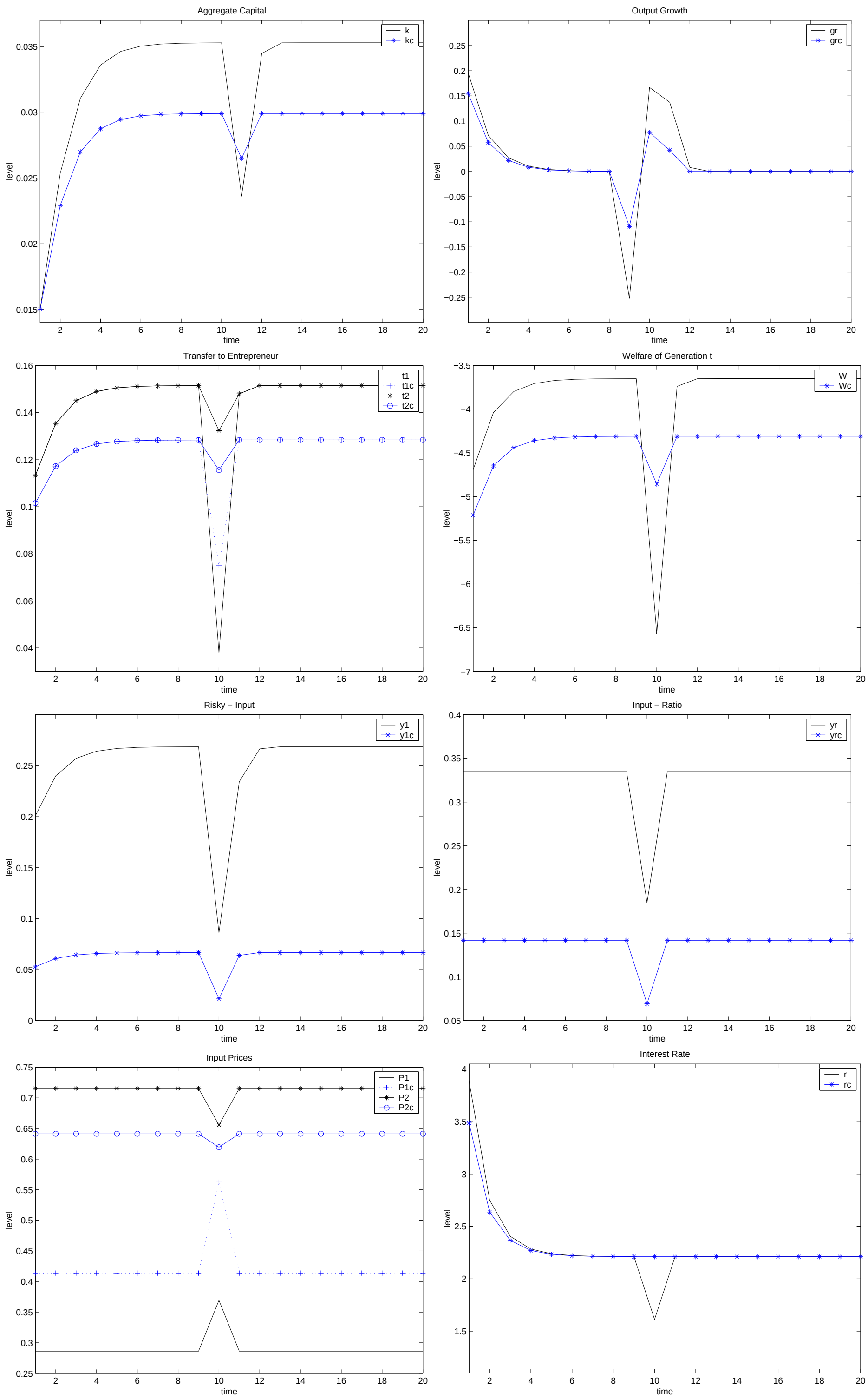
$$\frac{\gamma\overline{z}^\sigma\theta^{\alpha\sigma}}{1-\gamma}n^{(1-\alpha)\sigma}(1-n)^{(1-\alpha)(1-\sigma)}\left(1-\theta\right)^{\alpha(1-\sigma)}=1-(1-\theta)^\alpha(1-n)^{1-\alpha}. \tag{51}$$

Using equations (50) and (51) we obtain that $G(0)=-\gamma$ and $G(n^*)>0$. Since G is a continuous function of θ, there exists at least one θ_0 such that $G(\theta_0)=0$. Let us denote by $S\equiv\{\theta\in[0,n^*]/G(\theta)\leq 0\}$ and by $\underline{\theta}$the minimum of θ such that $G(\theta)=0$, then $(0,\underline{\theta})$ is an open interval included in S.

Given the fact that there is no explicit result for n, it is very hard to prove that G is a monotonic function of θ, but for all examples we computed numerically we obtained that G is monotonic. Thus, we assume until the end of this section that G is monotonic.

8.4 Appendix E

Figure 4. Comparative Dynamic of a Regulated and an Unregulated Banking Economies


Aggregate Capital
k
kc
Output Growth
gr
grc
Transfer to Entrepreneur
t1
t1c
t2
t2c
Welfare of Generation t
W
Wc
Risky − Input
y1
y1c
Input − Ratio
yr
yrc
Input Prices
P1
P1c
P2
P2c
Interest Rate
r
rc
level
time

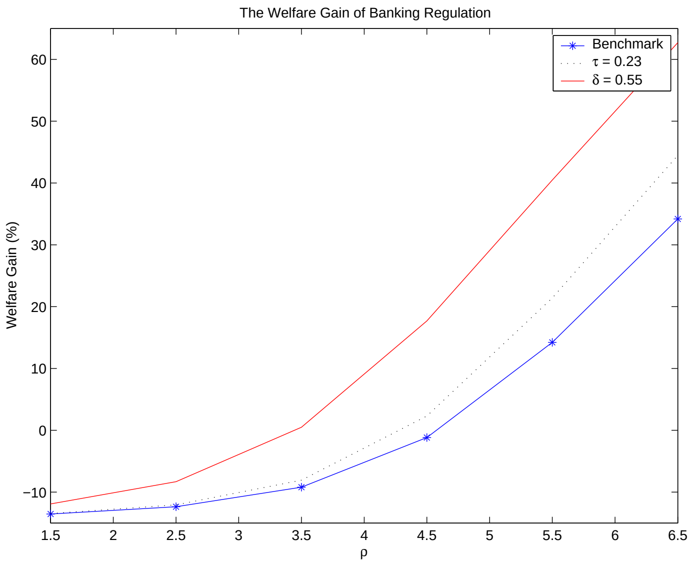


Figure 5. Relative Welfare-Gain

References

[1] Alexander, Samuel K. 2004. "Why Banks Hold Capital in Excess of Regulatory Requirements: The Role of Market Discipline, " Journal of International Banking Regulation, 6: 6-9.

[2] Allen, Franklin, and Douglas Gale. 2000. "Financial contagion," Journal of Political Economy, 108: 1-33.

[3] Allen, Franklin, and Douglas Gale. 2003. "Capital Adequacy Regulation: In Search of a Rationale. " In: Arnott,R., Greenwald, B., Kanbur, R., Nalebuff, B., (Eds), Economics for an Imperfect World: Essays in Honor of Joseph Stiglitz. Cambridge, MA: MIT Press.

[4] Allen, Franklin, and Douglas Gale. 2004. "Financial Intermediaries and Markets," Econometrica, 72: 1023-1061.

[5] Balasko, Yves, and Karl Shell. 1980. "The Overlapping-Generations Model I: the Case of Pure Exchange Without Money," Journal of Economic Theory, 23: 281-306.

[6] Barth, James R., Gerard Caprio, and Ross Levine. 2001. "Bank Regulation and Supervision: What Works Best? " The World Bank Working Paper

[7] Berger, Allen N., Richard J. Herring, Giorgio P. Szegö. 1995. "The Role of Capital in Financial Services Industry: Causes, Consequences, and Implications for the Future," Journal of Banking and Finance, 23: 135-194.

[8] Bernanke, Ben, and Mark Gertler. 1985. "Banking and Macroeconomic Equilibrium." In: Barnnett, W.A., Kenneth J. S. (Eds.), Cambridge, Cambridge University Press: 89-111.

[9] Bernanke, Ben, and Mark Gertler. 1989. "Agency Costs, Net Worth, and Business Fluctuations," American Economic Review, 79: 14-31

[10] Bank, for International Settlements. 2003. "Overview of the New Basel Capital Accord," Basel Committee on Banking Supervision of Bank for International Settlements.

[11] Blum, Jürg. 1999. "Do Capital Adequacy Requirements Reduce Risks in Banking?" Journal of Banking and Finance, 23: 755-771.

[12] Castro, Rui, Gian Luca Clementi, and Glenn MacDonald. 2004. "Investor Protection, Optimal Incentives, and Economic Growth," The Quarterly Journal of Economics, 119: 1131-1175.

[13] Dewatripont, Mathias, and Jean Tirole. 1994. The Prudential Regulation of Banks, Cambridge, Mass.: MIT Press.

[14] Ennis, Huberto M., and Todd Keister. 2003. "Economic Growth, Liquidity, and Banks Runs," Journal of Economic Theory, 109: 220-245.

[15] Freixas, Xavier, and Jean C. Rochet. 1997. Microeconomics of Banking, Cambridge, Mass.: MIT Press.

[16] Gale, Douglas. 2004. "Financial Regulation in a changing Environment. " In: Thomas Courchene and Edwin Neave (Eds.) Franing Financial Structure in an

Information Environment. Kingston, Ontario: John Deutsh Institute for the Study of Economic Policy, Queen's University.

[17] Gale, Douglas. 2004. "Notes on Optimal Capital Regulation," Working Paper, Department of Economics New York University.

[18] Gale, Douglas, and Martin Hellwig. 1985. "Incentive-Compatible Debt Contracts : the One-Period Problem," The Review of Economic Studies, 52: 647-663.

[19] Gale, Douglas, and Onur Özgür. 2005. "Are Bank Capital Ratios Too High or Too Low? Risk Aversion, Incomplete Markets, and Optimal Capital Structure," Journal of European Economic Association.

[20] Halac, Marina and Sergio L. Schmukler. 2004. "Distributional Effects of Crises: The Financial Channel," Economia, Fall 2004.

[21] Hellman, Thomas F., Kevin C. Murdock, and Joseph E. Stiglitz. 2000. "Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?" American Economic Review, 90: 147-165

[22] Hoelscher, David S., and Marc Quintyn. 2003. "Managing Systemic Banking Crises," Occasional Paper of the International Monetary Fund, 224.

[23] Kim, Daesik, and Anthony M. Santomero. 1988. "Risk in Banking and Capital Regulation," Journal of Finance, 43: 1219-1233

[24] Lucas, Robert E. Jr. 1987. Models of Business Cycles, Yrjo Jahnson Lectures Series. London: Blackwell

[25] Mason, Joseph R. 2005. "A Real Options Approach to Bankruptcy Costs: Evidence from Failed Commercials Banks During the 1990's," Journal of Business, 78: 1523-1554.

[26] Mishkin, Frederic S. 2000. "Prudential Supervision : Why is it Important and What are the Issues," NBER Working Paper Series, 7926.

[27] Richardson, Gary, and William Troost. 2006. "Monetary Intervention Mitigated Banking Panics During the Great Depression: Quasi-Experimental Evidence from the Federal Reserves District Border in Mississippi, 1929 to 1933," NBER Working Paper Series, 12591.

[28] Starr, Ross M. 1973. "Optimal Production and Allocation under Uncertainty," The Quarterly Journal of Economics, 87: 81-95.

[29] World Bank. 2006, The World Development Indicators. Washington, DC: World Bank,