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Understanding the Sources of High Current Account Fluctuations in 5 Developed Economies

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Abstract

The global economy has, in recent times, continued to face large and unprecedented external imbalances. Despite reductions recorded in aggregate current account (saving less investment) to global output ratio, the imbalances still remain. The main contributors to the imbalances have been the world’s developed economies. These developed economies have experienced fluctuating current account balances over the years and the fluctuation has contributed to a slow correction of the imbalances. This paper identifies 5 developed economies with the highest fluctuations in current account balances and analyses the sources of these fluctuations. The countries are Singapore, Latvia, Iceland, Norway and Estonia. Results obtained suggest that 1) temporary shocks account for most current account fluctuations, and the excess response to temporary shocks is as stable and pronounced as in previous studies; 2) permanent shocks drive current account fluctuations in Iceland and Latvia but not in Norway, Estonia, and Singapore; 3) Singapore demonstrates the most support for the two-good intertemporal model, since external supply and demand shocks account for its current account fluctuations.

Keywords: Current account fluctuations, two-good intertemporal model, VAR and impulse response, V5 Economies

JEL classification numbers: F32, F41, F21, C22

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1.0 Introduction

One of the main economic concerns prior to the global financial crisis was the presence of large global imbalances. This situation refers to the huge current account deficits (saving smaller than investment) incurred by a number of developed economies and financed by the rest of the world. The world economy prior to the global financial crisis featured large current account imbalances, notably in the U.S. This attracted concerns that such large deficits were unsustainable and could trigger crises. Buiter (2006) and Dodge (2006), among many others, noted concerns about these imbalances, enumerated threats to the orderly resolution of global imbalances and posited that the global imbalances, if not properly resolved, could pose risks for the global economy. The concerns were expressed in view of the crisis often experienced by economies that run massive current account deficits and make impulsive macroeconomic adjustments in instances of sudden reversals in net capital inflows. The feared crisis did occur, and the global economy was dragged into a deep recession in 2008. However, contrary to previous expectations, net capital inflows to the U.S. did not stop. In order words, the crisis was not triggered by a sudden seizure of capital inflows into the U.S. According to Caballero and Krishnamurthy (2009), a type of imbalance, known as a ‘safe assets imbalance’, triggered the crisis. Under this imbalance, the entire world, including foreign central banks and investors, insatiably demanded U.S. safe debt instruments, which put enormous pressure on the U.S. financial system, elevated asset prices, created bubbles, encouraged credit imprudence and weak regulatory oversight, eventually causing the system to fail. Caballero, Fahri and Gourinchas (2008) associated the crisis with the unfolding of these imbalances.

After the global financial crisis, the U.S. saw a massive decline in current account deficit to c.2.4% of GDP in 2013 from c.6.1% in 2007. China saw its current account surplus decline to c.2.32% of GDP in 2013 from c.10.1% in 2007. A similar trend was experienced in many countries that ran current account surpluses prior to the crisis. In all of these, one fact remains consistent – the U.S. experienced a decline in current account deficit (and a decline in capital/financial surplus) after the financial crisis and this was accompanied by declines in current account surpluses (and a decline in capital/financial account deficits) of the financing countries. The fluctuations in the current account balances therefore bear some relationships with the global financial crisis, and analysing the sources of these fluctuations becomes paramount to understanding the potential sources of imbalances that triggered the crisis. This idea solely motivates this paper. One major advantage of analysing current account fluctuations in open economies is that such analysis provides an understanding of factors - collectively termed macroeconomic shocks - that characterise the emergence and, hopefully, subsequent readjustment of the global imbalances.

A number of models have been used to analyse the macroeconomic shocks that characterise current account fluctuations. Traditional intertemporal approach has been the dominant technique since the 1980’s, see Obstfeld and Rogoff (1995). In this approach, the current account is written as a function of intertemporal consumption decisions and productivity shocks, with the assumption that the current account is independent of global shocks and primarily responds to temporary country-specific shocks rather than permanent ones. This idea has been extended to models that incorporate price rigidities, interest rates, traded and non-traded goods as well as monetary policies, among others. However, the intertemporal approach on its own has been shown to have

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2 When a shock is temporary there is no impact on future net output, so net output is unaffected by this shock in the future. When a shock is permanent, it reduces future net output, so net output is affected by this shock not only in the present but also in the future.
limited support from data via the present value tests, see Sheffrin and Woo (1990) and Campbell (1987). Nonetheless, there is a strong support from data when a wider range of variables is introduced.

Given the mixed support for the present value tests of the intertemporal approach, researchers have more recently begun to employ structural vector auto regression (SVAR). Under this approach, minimal identification restrictions are imposed on VAR models which are then used to test the implications of the intertemporal model as in Nason and Rogers (2002), Lee and Chinn (2006) and Kano (2008). As the main implication of the intertemporal approach is that the current account is primarily driven by country-specific temporary shocks rather than permanent ones, it must be that the validity of the intertemporal model is tested by decomposing the system shocks into temporary and permanent shocks. This decomposition forms the basis of SVAR approach.

In recent times, SVAR models have been used in varying degrees for different macroeconomic studies that test the implications of intertemporal models. Ahmed and Park (1994) examine macroeconomic fluctuations in seven OECD small open economies using a four-variable SVAR model with long-run restrictions. By employing the identification method of Blanchard and Quah (1989), they identify four structural shocks namely external shocks, domestic supply shocks, domestic absorption shocks and domestic price level shocks. They find that domestic absorption shocks mainly explain movements in trade balance and that external shocks play a nontrivial role for the trade balance. Lane (2001) analyses a trivariate VAR system that includes – U.S. current account to output ratio, relative consumer price levels between U.S. and the rest of the world, and the first-difference of U.S. output to world output ratio. The analysis, using long-run neutrality restrictions, identifies three structural orthogonal shocks namely supply, absorption and monetary shocks and shows that the accumulated impulse responses yield a positive monetary shock that leads to current account deterioration in the short-run and then persistent surpluses consequently.

Using the New Keynesian DSGE model to derive sign restrictions for the identification of several shocks in a SVAR system for the Euro Area, Peersman et. al (2005) estimates the impact of shocks such as monetary policy, preferences, government spending, investment, price mark-up, technology and labour supply shocks on consumption, investment and employment. In the second step, they significantly relaxed the restrictions from the DSGE model and re-estimated the SVAR model using a minimum set of more general constraints so that the Euro Area data can provide enhanced information on the validity of the DSGE model. Their results show that the responses remain largely consistent with the New Keynesian model, even for the controversial negative effects of government spending shocks on private consumption and investment. In contrast to theoretical model, their results further show that there is a positive effect of technology shocks on employment, and a positive impact of preferences and investment shocks on investment and consumption.

By estimating a bivariate model which includes the first difference of real exchange rates and current account to output ratio for G7 countries, Lee and Chinn (2006) identified 2 structural shocks – productivity shocks and monetary shocks – each representing country-specific permanent shocks and country-specific temporary shocks, respectively. After restricting the temporary shocks to only have short run effects on real exchange rates, a restriction which permits an analysis of the short run dynamics of the variables, they find that positive monetary shocks lead to depreciation of real exchange rates and surpluses in the current account in the short run. Their main conclusion is that permanent shocks have large long-term effects on the real exchange rates, but relatively small effects on current account, whereas temporary shocks have large effects on exchange rates and current account in the short run but no effect on these variables in the long run. Kano (2008) uses a similar but different
approach that allows for a variable world interest rate and employs a three-variable SVAR model that has as inputs the world real interest rate, domestic net output change and current account to net output ratio. Including the world real interest rate allows for analysing the impact of consumption tilting effects on current account. The structural shocks identified are country-specific temporary shocks, country-specific permanent shocks and global shocks. Using data for Canada and UK, Kano (2008) concludes that country-specific shocks induce large fluctuations in current account and explains most of the movements observed in current account. However, their role in explaining fluctuations in net output growth is minimal. The main conclusion of the paper is that consumption tilting effects play a major role for current account movement. As stated by Bergin and Sheffrin (2000), and also by Obstfeld and Rogoff (1996), the real exchange rate is an important variable that explains consumption tilting effects.

Karadimitropoulou and Leon-Ledesma (2009) analyse the sources of current account fluctuations for the G6 economies namely Canada, France, Germany, Italy, Japan and the UK using quarterly data. Based on Bergin and Sheffrin’s (2000) two-good intertemporal framework, they build a four-variable SVAR model which allows for the identification of structural shocks using long-run restrictions. The four variables are current account, world real interest rate, net output and real exchange rates. Their results suggest that there is a substantial support for the two-good intertemporal model and that some evidence exists in favour of the present-value model of the current account for each of the G6 countries except France. Moreover, both external supply and demand shocks are responsible for fluctuations in current account, but temporary domestic shocks account for a large proportion of these fluctuations. Their results also show that, compared to previous studies, the excess response of current account is less pronounced.

Nikolaychuk and Shapovalenko (2013) take the arguments to the Ukrainian economy and study the sources of current account fluctuations in Ukraine by applying an SVAR approach to estimate the effects of supply and demand shocks, nominal shocks and terms-of-trade shocks. They estimate the structural shocks in order to historically decompose trade balance into fundamental factors. For identification purposes, they impose sign restrictions on the impulse response functions. Their results show that 1) demand and terms-of-trade shocks are the main drivers of trade balance and current account in Ukraine; 2) trade balance decreases significantly when changes in fiscal policy and/or changes in preferences of economic agents cause a surge in demand; 3) persistent adverse terms-of-trade shocks have negative long-run effects on trade balance and 4) nominal shocks have much smaller effects on trade balance.

In this paper, and in light of the two-good small open economy model, we analyse the sources of current account fluctuations in 5 advanced economies that have the highest current account fluctuations among all advanced economies. The countries, from highest to least current account volatility, are Singapore, Latvia, Iceland, Norway and Estonia. Taken together, they account for almost one-third of the combined current account volatility of the 35 advanced economies Fig 1. These countries are collectively referred to as the ‘V5 countries’ in this paper. It is the heterogeneity, even amongst industrialized economies, that inspires us to extend, in this paper, the work of Karadimitropoulou and Leon-Ledesma (2009), which originally analysed the sources of current account fluctuations for the G6 industrialized countries, to include 5 other countries with the highest current account volatilities.
The SVAR model is favoured in this paper for two major reasons – it allows for minimal restrictions and ensures fluctuations in current account are decomposed into sources of macroeconomic shocks. There are different specifications of SVAR models. In this paper, we propose a type of SVAR model that draws on the model of Bergin and Sheffrin (2000) which presents a current account model that allows for the inclusion of variable world interest rates and introduces a traded and non-traded sector in a small open economy setting. This is called the two-good small open economy model of Bergin and Sheffrin. In particular, we set up the model in the spirit of Karadimitropoulou and Leon-Ledesma (2009) to have four variables namely - current account to net output ratio, world interest rates, changes in relative price levels and changes in net output. The model allows for the identification of four different sources of shocks which are temporary domestic output shocks (TDO), permanent domestic output shocks (PDO), external supply shocks and demand/preference shocks. Collectively, this set-up is known as a four-variable SVAR model with long-run restrictions as in Ahmed and Park (1994). One advantage of this set-up is that it is not restricted to the analysis of domestic shocks but allows for the analysis of external shocks as a source of current account fluctuations in a small open economy.

The empirical setup introduces changes in consumer to producer price index ratio as a proxy for changes in relative price levels. The variable world real interest rates together with the current account to net output ratio and changes in net output are also in the empirical setup. This setup allows analysing the effects of consumption tilting due to changes in world real interest rates and relative price levels. It also allows for the analysis of consumption smoothing effects due to changes in net output. The empirical setup incorporates domestic shocks – temporary and permanent domestic shocks – and external supply shocks as well as demand/preference shocks. Analysing the impact of these shocks is a distinguishing characteristic of the empirical setup employed in this paper and forms the basis of the analysis of the dynamics and sources of current account fluctuations in the abovementioned countries.
The rest of the paper is organized as follows. In Section 2, we present the theoretical foundation of the two-good small open economy model of Bergin and Sheffrin (2000) which forms the basis of the SVAR model used in this paper. Section 3 specifies details of the choice of SVAR and Section 4 presents data and results. Conclusion is provided in Section 5.

2.0  The Bergin and Sheffrin Model

Consider a small open economy that produces tradable and non-tradable goods wherein a representative infinitely lived household consumes a mix of the tradable and non-tradable goods\(^3\). Suppose that the only assets of the small open economy are international bonds and there is perfect bond mobility, so that the assumption of interest rate equalization holds. If the world real interest rate is assumed time-varying, \(r_t\), the economy’s current account can be represented by

\[
CA_t + \delta_t = \Delta B_t = B_t - B_{t-1}, \quad (2.1a)
\]

where \(\delta_t\) represents valuation changes and \(\Delta B_t\) denotes changes in net international investment position or changes in net stock of external assets or changes in net foreign assets. Bergin and Sheffrin (2000) assumes zero valuation changes and other income, and since the current account is the sum of trade balance and net investment income, (2.1a) becomes

\[
CA_t = B_t - B_{t-1} = r_t B_{t-1} + Y_t - C_t - I_t - G_t, \quad (2.1b)
\]

where \(r_t\) is the time-varying world interest rate expressed in terms of tradable goods, \(Y_t\), \(C_t\), \(I_t\) and \(G_t\) denote domestic output, private consumption expenditure, domestic investment and government spending. Since household consumes a mix of tradable and non-tradable goods, the consumption expenditure can be written as

\[
C_t = C_T + P_t C_{NT},
\]

where \(C_{NT}\) and \(P_t\) represent consumption of non-tradable goods and the relative price of domestic non-tradable goods in terms of tradable ones.

The allocation of expenditure between tradable and non-tradable takes the form of a Cobb-Douglas function. The intertemporal maximization problem for the infinitely lived household is to choose a consumption path that maximizes lifetime expected utility which is a function of consumption. Thus, the household solves

\[
\max_{C_{NT}, C_T} E_0 \sum_{t=0}^{\infty} \beta^t U(C_{NT}, C_T), \quad 0 < \beta < 1
\]

s.t. \(B_t - B_{t-1} = r_t B_{t-1} + Y_t - (C_T + P_t C_{NT}) - I_t - G_t, \quad (2.3)

where

\(^3\) Note that the description of the Bergin and Sheffrin (2000) model provided in this section, as well as the identification of structural shocks and reduced form (S)VAR, largely reproduces Karadimitropoulos and Leon-Ledesma (2009) and Bergin and Sheffrin (2000). We make no claim that a new theoretical model or empirical procedure is developed in this paper.
\[
U(C_{NT}, C_{T}) = \frac{1}{\left(1-\frac{1}{\sigma}\right) (C_{NT}^\alpha C_{T}^{1-\alpha})^{1-\sigma}, \sigma > 0, 0 < \alpha < 1}
\] (2.4)

and \( \frac{1}{\sigma} \) denotes the intertemporal elasticity of substitution and \( \alpha \) is the share of tradable goods in total consumption, i.e. \( \alpha = \frac{C_T}{C_T} \).

The infinitely lived household maximizes (2.2) subject to the dynamic budget constraint in (2.3). To perform the maximization, the dynamic budget constraint must first be transformed to include an index of total consumption. Following Dornbusch (1983) and Obstfeld and Rogoff (1996), the transformation yields

\[
P_t^* C_t^* = Y_t + (1 + r_t) B_{t-1} - B_t - I_t - G_t,
\] (2.5)

where \( C_t^* = C_T^\alpha C_{NT}^{1-\alpha} \) is an index of total consumption associated with the model. The consumption-based index \( P_t^* \) is the minimum amount of consumption expenditure \( C_t = C_T + P_t C_{NT} \) such that \( C_t^* = 1 \), for a given \( P_t \).

With this, the representative household problem is partitioned into two different stages which are to minimize consumption periodically and optimize consumption intertemporarly. The appropriate minimization problem associated with the first stage is

\[
\min_{C_{NT}, C_T} C_T + P_t C_{NT} \quad \text{s.t.} \quad C_t^* = C_T^\alpha C_{NT}^{1-\alpha}
\] (2.6)

Solving this yields the optimal allocation of expenditure between tradable and non-tradable goods as

\[
C_T = \alpha C_t, \quad C_{NT} = (1 - \alpha) \frac{C_t}{P_t}.
\] (2.7)

Thus, \( C_t^* \) and \( P_t^* \) become

\[
C_t^* = (\alpha C_t)^\alpha \left[ (1 - \alpha) \frac{C_t}{P_t} \right]^{1-\alpha} \quad \text{and} \quad P_t^* = P_t^{1-\alpha} \left[ \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \right]
\] (2.8)

Accordingly,

\[
P_t^* C_t^* = P_t^{1-\alpha} \left[ \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \right] \left( \alpha C_t \right)^\alpha \left[ (1 - \alpha) \frac{C_t}{P_t} \right]^{1-\alpha} = C_t = C_T + P_t C_{NT}.
\] (2.9)

This equivalence establishes that the representative household optimization problem can alternatively be expressed in terms of the total consumption index and consumption-based price index. Given this, the household now solves the equivalent optimization problem

\[
\max_{C_{NT}, C_T} E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} (C_t^*)^{1-\sigma}, \quad 0 < \beta < 1
\] (2.10)

s.t. \( B_t - B_{t-1} = r_t B_{t-1} + Y_t - P_t^* C_t^* - I_t - G_t \)

The Bellman equation associated with the optimization problem can be expressed as
\[ V(B_{t-1}) = \max_{c_t} \left\{ \frac{1}{1-\sigma} (C_t^*)^{1-\sigma} + \beta E_t V(B_t) \right\} \]
\[ = \max_{c_t} \left\{ \frac{1}{1-\sigma} (C_t^*)^{1-\sigma} + \beta E_t V((1 + r_t)B_{t-1} + Y_t - P_t^* C_t^* - I_t - G_t) \right\} \]  

(2.11)

The first order and envelope conditions yield

\[
\begin{align*}
(C_t^*)^{1-\sigma} &= \beta E_t P_t^* \frac{\partial V}{\partial B_t} \\
\frac{\partial V}{\partial B_{t-1}} &= \beta E_t (1+r_t) \frac{\partial V}{\partial B_t}
\end{align*}
\]  

(2.12)

from which eliminating \(\beta\) gives

\[
\frac{\partial V}{\partial B_{t-1}} = (1+r_t) \frac{C_t^*}{P_t^*} \quad \text{or} \quad \frac{\partial V}{\partial B_t} = (1+r_{t+1}) \frac{C_{t+1}^*}{P_{t+1}^*},
\]

(2.13)

whence, the intertemporal Euler equation follows as

\[
\beta E_t \left[ (1+r_{t+1}) \left( \frac{P_t^*}{P_{t+1}^*} \right) \left( \frac{C_t^*}{C_{t+1}^*} \right)^\sigma \right] = 1
\]  

(2.14)

It is worth noting that \(C_t\) and \(P_t\) are directly observable variables while \(C_t^*\) and \(P_t^*\) are not. As \(C_t^*\) and \(P_t^*\) can be expressed in terms of \(C_t\) and \(P_t\) as in (2.8), then (2.14) can be written in terms of the observable variables by combining both equations. This combination yields

\[
\beta E_t \left[ (1+r_{t+1}) \left( \frac{C_t}{C_{t+1}} \right)^\frac{1}{\gamma} \left( \frac{P_t}{P_{t+1}} \right)^{(1-\frac{1}{\gamma})(1-\alpha)} \right] = 1,
\]

(2.15)

where \(\sigma = \frac{1}{\gamma}\) is the intertemporal elasticity of substitution. This is the Euler equation and shows the optimal consumption path. For the Euler equation to be usable, it has to be log-linearized. To perform the log linearization of the Euler equation, two assumptions are pertinent. First, we assume that the world real interest rate, consumption growth rate and the percentage change in the relative price of non-tradable goods are log normally distributed. Second, we assume that the variance covariance terms across variables are time-invariant. Under these assumptions, the log linearization of the Euler equation yields

\[
E_t \Delta c_{t+1} = \gamma E_t \left[ r_{t+1} + \frac{1-\gamma}{\gamma} (1-\alpha) \Delta p_{t+1} \right] + \log \beta + \frac{1}{2} \left[ \sigma_c^2 + \gamma^2 \sigma_d^2 + (1-\gamma)^2 (1-\alpha)^2 \sigma_p^2 + 2\gamma \sigma_{cd} + 2(1-\gamma)(1-\alpha) \sigma_{cp} \right. \\
\left. + 2\gamma (1-\gamma)(1-\alpha) \sigma_{cp} \right]
\]

or

\footnote{See Appendix B for details on the log-linearization}
\[ E_t \Delta c_{t+1} = \theta + \gamma E_t \left[ r_{t+1} + \left( \frac{1 - \gamma}{\gamma} (1 - \alpha) \right) \Delta p_{t+1} \right], \]  

(2.16)

where \( \Delta c_{t+1} = \log C_{t+1} - \log C_t \) and \( \Delta p_{t+1} = \log P_{t+1} - \log P_t \) denote the intertemporal elasticity of substitution. Notice that \( \theta = \log \beta + \frac{1}{2} \left[ \sigma_C^2 + \gamma^2 \sigma_P^2 + (1 - \gamma)^2 \sigma_R^2 + 2 \gamma \sigma_{Cp} + 2 \gamma (1 - \gamma) (1 - \alpha) \sigma_{Rp} \right] \) is a function of constant (time-invariant) variance and covariance terms and their coefficients. It therefore follows that \( \theta \) is constant. Thus, it represents the optimal consumption path or profile of the representative agents.

It is important to note that the consumption-based real interest rate, \( r_t^* \), depends on the world real interest rate \( r_t \) and relative price of non-traded goods \( p_t \) and can thus be written as

\[ r_t^* = r_t + \left( \frac{1 - \gamma}{\gamma} (1 - \alpha) \right) \Delta p_t, \]

so that the Euler equation becomes

\[ E_t \Delta c_{t+1} = \theta + \gamma E_t r_{t+1}^* = \theta + \gamma E_t r_{t+1} + (1 - \gamma) (1 - \alpha) E_t \Delta p_{t+1} \]  

(2.17)

The Euler equation shows that the consumption-based real interest rate \( r_t^* \) influences the optimal consumption path of the consumer. Together with the budget constraint, the closed form consumption Euler equation implies that it possible to obtain a closed form solution for the current account. To see this, consider the market discount factor for consumption which is defined as in Bergin and Sheffrin (2000) by

\[ R_s = \left( \prod_{j=1}^{x} \left( 1 + r_j \right) \right)^{-1} \]

Defining net output as \( \tilde{Y}_t = Y_t - I_t - G_t \) and substituting into the budget constraints in (2.3) gives

\[ B_t - B_{t-1} = \tilde{Y}_t - C_t - r_t B_{t-1} \]  

(2.18)

Iterating (2.6) forward, and imposing the transversality condition, \( \lim_{t \to \infty} E_0 (R_t B_t) \), yields the new intertemporal constraint

\[ \sum_{t=0}^{\infty} E_0 (R_t C_t) = \sum_{t=0}^{\infty} E_0 (R_t \tilde{Y}_t) + B_0, \]  

(2.19)

where \( B_0 \) is the initial level of net stock of foreign assets. Log-linearizing the intertemporal budget constraint yields\(^5\)

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\(^5\) See Appendix A for proof
\[
\bar{y}_0 - \frac{c_0}{\omega} \left(1 - \frac{1}{\omega}\right) b_0 = -\sum_{t=1}^{\infty} \beta^t \left[ \Delta \bar{y}_t - \frac{\Delta c_t}{\omega} - \left(1 - \frac{1}{\omega}\right) r_t \right]
\]  

(2.20)

where \( \Delta \bar{y}_t = \log Y_t - \log Y_{t-1} \), \( \Delta c_t = \log C_t - \log C_{t-1} \) and \( \bar{y}_0, c_0 \) and \( b_0 \) are logarithms of their upper cases while \( \omega \) is a constant such that

\[
\omega = 1 - \frac{\beta}{\sum_{t=0}^{\infty} r_t c_t} \leq 1
\]

and \( \bar{B} \) represents the steady state level of net foreign assets.

Combining the Euler equation in (2.16) with the budget constraint in (2.20) gives

\[
-E_t \sum_{i=1}^{\infty} \beta^i \left[ \Delta \bar{y}_{t+i} - \frac{\vartheta + \gamma r^{*}_{t+i}}{\omega} - \left(1 - \frac{1}{\omega}\right) r_{t+i} \right] = \bar{y}_t - \frac{c_t}{\omega} - \left(1 - \frac{1}{\omega}\right) b_t
\]

(2.21)

Under the assumption that the net stock of foreign assets \( \bar{B} = 0 \) in steady state, we have \( \omega = 1 \) and

\[
ca^*_t = -E_t \sum_{i=1}^{\infty} \beta^i \Delta \bar{y}_{t+i} + E_t \sum_{i=1}^{\infty} \beta^i \gamma r^{*}_{t+i} + E_t \sum_{i=1}^{\infty} \beta^i (1 - \gamma)(1 - \alpha) \Delta p_{t+i} + \frac{\beta}{1 - \beta} \theta,
\]

(2.22)

where, \( ca^*_t \equiv \bar{y}_t - c_t \).

### 3.0 Set-up and identification of the SVAR system

The four variables that constitute the Bergin and Sheffrin (2000) model are current account to net output ratio \( ca^*_t \), changes in net output \( \Delta \bar{y}_{t+i} \), world interest rate \( r_{t+i} \) and changes in relative price levels \( \Delta p_{t+i} \). Accordingly, the SVAR system is set up to capture these variables. This is known as the four-variable SVAR model. As earlier motivated, the four variables characterising the model are driven by four shocks – temporary domestic net output shocks \( \rho_4 \), demand shocks \( \rho_3 \), permanent domestic net output shocks \( \rho_2 \), and external supply shocks \( \rho_1 \). In this case, let \( V_t \) be a vector that contains the four variables of the SVAR model and let \( \rho \) represent a vector of 4 structural shocks that drive \( V_t \), then \( V_t \) and \( \rho \) can be written as a 4 \( \times \) 1 vector as

\[
V_t = \begin{pmatrix} \bar{y}_t \\ \Delta \bar{y}_t \\ \Delta \bar{p}_t \\ ca^*_t \end{pmatrix}, \quad ca^*_t = CA^*_t, \quad \rho_t = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix},
\]

(2.23)

where the structural shocks contained in \( \rho_t \) are orthogonal to each other; \( \rho_t \) has variance-covariance matrix

\[
\Sigma = \mathbb{E}(\rho_t \rho_t')
\]

Using (2.22), the model can be written in lag operator notation as

\[
BV_t = \Gamma_0 + \Gamma_1(L)V_{t-1} + \rho_t,
\]

(2.24)

where \( B \) is a full rank matrix with unity diagonal elements, \( \Gamma_0 \) represents a 4 \( \times \) 1 vector of constant terms and \( \Gamma_1(L) \) is a matrix of polynomials in the lag operation which can be written as

\[
\Gamma_1(L) = \sum_{m=0}^{\infty} \Gamma_1^m L^m
\]

(2.25)
3.1 Reduced-form VAR

When the off-diagonal elements of $B$ are unknown, (2.24) cannot be estimated. In such a case, it is easier to estimate the reduced-form model. To obtain the reduced form model of (2.24), it is sufficient to pre-multiply both sides with the inverse of $B$, i.e. $B^{-1}$, assuming it exists. Thus,

$$B^{-1}BV_t = B^{-1}(\Gamma_0 + \Gamma_1(L)V_{t-1} + \rho_t),$$

$$V_t = B^{-1}\Gamma_0 + B^{-1}\Gamma_1(L)V_{t-1} + B^{-1}\rho_t,$$

where $B\Phi_0 = \Gamma_0$, $B\Phi_1 = \Gamma_1$ and $B\varepsilon_t = \rho_t$. Note that as $B^{-1}$ is a full rank matrix and $\rho_t$ is a $4 \times 1$ vector of structural shocks, then $\varepsilon_t = B^{-1}\rho_t$ is a vector of serially uncorrelated reduced-form error terms that are composites of the structural shocks and have variance-covariance matrix $\Omega = \mathbb{E}(\varepsilon_t\varepsilon_t') = B^{-1}\Sigma B^{-1}$.

It is well known in the literature that the variance-covariance matrix $\Omega$ has $n(n+1)/2$ elements, where $n$ is the number of variables in the model, and $B$, a full rank matrix whose diagonal elements all equal 1, contains $n^2 - n$ unknown values while the structural model has $n^2$ unknown values. Identifying the $n^2$ unknown values from the $n(n+1)/2$ known elements of $\Omega$ necessarily requires imposing $n(n-1)/2$ additional restrictions on the reduced form model in (2.26) in order to identify the SVAR. As there are four variables in our model so that $n = 4$, this amounts to 6 restrictions. These restrictions can be imposed on the model in several ways. One way is in the form of a long-run identification scheme via the Blanchard and Quah (1989) decomposition approach for theory-driven restrictions. Under this approach, (2.25) can be represented in a vector moving average (MA) form as

$$V_t = \mu + \psi_0\rho_t + \psi_1\rho_{t-1} + \psi_2\rho_{t-2} + \cdots = \mu + \psi_0L^0\rho_t + \psi_1L^1\rho_t + \psi_2L^2\rho_t + \cdots = \mu + \psi(L)\rho_t, \quad (2.26)$$

where $L$ is the lag operator, $\psi(L) = \psi_0L^0 + \psi_1L^1 + \psi_2L^2 + \cdots$ is a matrix of polynomial in the lag operator and each entry $\psi_{ij}(L)$ in the matrix of polynomial $\psi(L)$ represents the accumulated effect of each shock $\rho_t' = (\rho_1, \rho_2, \rho_3, \rho_4)$ on the four variables in $V_t' = (r_t, \Delta\tilde{y}_t, \Delta p_t, c\tilde{a}_t)$. Under this specification, the structural shocks are then identified by setting up the model in its vector moving average form. This implies specifying the reduced form SVAR in its matrix format as follows

$$\begin{pmatrix} r_t \\ \Delta\tilde{y}_t \\ \Delta p_t \\ c\tilde{a}_t' \end{pmatrix} = \begin{pmatrix} \psi_{11}(L) & \psi_{12}(L) & \psi_{13}(L) & \psi_{14}(L) \\ \psi_{21}(L) & \psi_{22}(L) & \psi_{23}(L) & \psi_{24}(L) \\ \psi_{31}(L) & \psi_{32}(L) & \psi_{33}(L) & \psi_{34}(L) \\ \psi_{41}(L) & \psi_{42}(L) & \psi_{43}(L) & \psi_{44}(L) \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix}, \quad (2.27a)$$

A second specification, the one employed in this paper, is via the vector autoregressive representation. Under this approach, the model can be written as

$$\begin{pmatrix} r_t \\ \Delta\tilde{y}_t \\ \Delta p_t \\ c\tilde{a}_t' \end{pmatrix} = \begin{pmatrix} \psi_{11}(L) & \psi_{12}(L) & \psi_{13}(L) & \psi_{14}(L) \\ \psi_{21}(L) & \psi_{22}(L) & \psi_{23}(L) & \psi_{24}(L) \\ \psi_{31}(L) & \psi_{32}(L) & \psi_{33}(L) & \psi_{34}(L) \\ \psi_{41}(L) & \psi_{42}(L) & \psi_{43}(L) & \psi_{44}(L) \end{pmatrix} \begin{pmatrix} r_{t-1} \\ \Delta\tilde{y}_{t-1} \\ \Delta p_{t-1} \\ c\tilde{a}_{t-1}' \end{pmatrix} + \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix}, \quad (2.27b)$$

or, in compact form,
\[ V_t = \sum_{s=1}^{\infty} \psi(s) V_{t-s} + \rho_t = \psi(L) V_{t-1} + \rho_t, \]  

(2.27c)

where, in this case,

\[ \psi(L) = \sum_{s=1}^{\infty} \psi(s) L^{s-1}, \quad L^n V_t = V_{t-n} \]  

(2.28a)

and

\[ \psi(L) = \begin{pmatrix} \psi_{11}(L) & \psi_{12}(L) & \psi_{13}(L) & \psi_{14}(L) \\ \psi_{21}(L) & \psi_{22}(L) & \psi_{23}(L) & \psi_{24}(L) \\ \psi_{31}(L) & \psi_{32}(L) & \psi_{33}(L) & \psi_{34}(L) \\ \psi_{41}(L) & \psi_{42}(L) & \psi_{43}(L) & \psi_{44}(L) \end{pmatrix}, \quad V_t = \begin{pmatrix} \tilde{r}_t \\ \Delta \tilde{y}_t \\ \Delta p_t \\ c_{a_t} \end{pmatrix}, \quad V_{t-1} = \begin{pmatrix} \tilde{r}_{t-1} \\ \Delta \tilde{y}_{t-1} \\ \Delta p_{t-1} \\ c_{a_{t-1}} \end{pmatrix} \]  

(2.28b)

### 3.2 Identification of Structural Shocks in the SVAR

The identification scheme is as follows – temporary domestic shocks, \((\rho_4)\), have long-run effects on the accumulated value of \(ca_t^*\) and no effect on other variables within the model; permanent domestic shocks, \((\rho_2)\), induce changes in net output \(\Delta \tilde{y}_t\) and have effects on the changes in relative price levels \(\Delta p_t\) in the long-run but no impact on the world real interest rate due to the small open economy assumption which implies domestic events have no global impact; demand shocks, \((\rho_3)\), have permanent effects on \(\Delta p_t\) and net foreign assets through \(ca_t^*\) due to the effects of consumption tilting, but has no long run impact on either output or world interest rate because of the assumptions of small open economy and demand shocks neutrality; external supply shocks, \((\rho_1)\), have accumulated impacts on the world real interest rate in the long run and possibly also have permanent effects on other variables within the model.

In identifying these structural shocks, the six identifying restrictions earlier discussed are required for the set-up. The set-up restricts the matrix \(\psi(L)\) to be lower-triangular which allows for a Choleski decomposition of the weighted variance-covariance matrix of the reduced form VAR. This allows for a unique identification of all elements in \(\psi(L)\). As \(\rho_4\), \(\rho_3\) and \(\rho_2\) each have no impact on world real interest rate, it follows that \(\psi_{14}(L) = \psi_{12}(L) = \psi_{13}(L) = 0\). Since demand shocks \((\rho_3)\) and temporary domestic shocks \((\rho_4)\) have no long run impact on net output, then \(\psi_{23}(L) = \psi_{24}(L) = 0\). Finally, temporary shocks \((\rho_4)\) do not affect changes in relative price levels in the long run, so \(\psi_{24}(L) = 0\). Thus, the six identifying restrictions are \(\psi_{12}(L) = \psi_{13}(L) = \psi_{14}(L) = \psi_{23}(L) = \psi_{24}(L) = \psi_{34}(L) = 0\) which give rise to the impact matrix that exactly identifies the VAR:

\[ \psi(L) = \begin{pmatrix} \psi_{12}(L) & 0 & 0 & 0 \\ \psi_{22}(L) & \psi_{23}(L) & \psi_{24}(L) & 0 \\ \psi_{31}(L) & \psi_{32}(L) & \psi_{33}(L) & 0 \\ \psi_{41}(L) & \psi_{42}(L) & \psi_{43}(L) & \psi_{44}(L) \end{pmatrix} \]  

(2.28c)

Having identified the structural shocks, the following tests will be performed: 1) present value model test to check that permanent domestic net output shocks, \((\rho_2)\), do not have a long-run impact on the current account, which implies testing \(\psi_{42}(L) = 0\); 2) test of the long-run effects of permanent output shocks on changes in relative price levels and this means testing \(\psi_{32}(L) = 0\); 3) joint test of the present value and productivity effects, that is, testing \(\psi_{42}(L) = \psi_{32}(L) = 0\) and 4) test for the importance of consumption tilting effects through the impact of demand shocks \((\rho_3)\) and external supply shocks \((\rho_1)\) on the current account. This is a test
for the significance of consumption tilting effects on current account and it involves testing separately or jointly for the hypothesis $\psi_{41}(L) = \psi_{43}(L) = 0$.

4.0 Empirical Results and Analysis

4.1a Data Description

Data samples consist of quarterly data of the V5 countries, that is, quarterly data of the 5 developed economies whose current accounts demonstrate the highest volatilities amongst all developed economies. The V5 countries are Singapore, Norway, Latvia, Iceland and Estonia. These economies perfectly satisfy the description of a small open economy as they participate in world trade, buying and selling tradable goods, and are smaller relative to their trade partners so that they are price takers whose policies do not alter world prices. Given the non-availability of quarterly data in earlier years for Estonia and Latvia, and the impossibility of accurately converting annual data to quarterly data in all of the sample years, the sample period for these countries differs from the other three countries – Singapore, Iceland and Norway – whose data for earlier years are largely available. Thus, for Estonia and Latvia, analysis is done on data for sample period which begins from 1990:1 and ends in 2014:4. However, for Singapore, Iceland and Norway, analysis covers sample period of 1980:1 to 2014:4. Most data samples come from the IMF’s International Financial Statistics, mostly seasonally adjusted. Data were also sourced from the Federal Reserve Bank of St Louis Economic Data (FRED) Bank.

It would be recalled that there are a total of four variables in the SVAR model, namely the world real interest rate, $r_t$, changes in net output, $\Delta \tilde{y}_{t}$, changes in relative price of nontradable to tradable goods, $\Delta p_t$, and current account to net output ratio, $c_{a_t}$. The world real interest rate is proxied with US quarterly real interest rate which, based on the Fisher’s identity, is the annualised quarterly Treasury Bills rate (nominal interest rate) less percent changes in domestic CPI (headline inflation) for the period. The net output is computed based on the identity $\tilde{Y}_t = Y_t - I_t - G_t$ from which changes in net output as well as the ratio of current account to net output follow. For changes in relative price of nontradable to tradable goods, this paper deviates from Karadimitropoulou and Leon-Ledesma (2009) who employed quarterly trade-weighted Real Effective Exchange Rate (REER) as a proxy for the relative price of nontradable to tradable goods but instead follow Engel (1999) and Betts and Kehoe (2008) by using, as a proxy for the relative price of nontradable to tradable goods, the ratio between CPI and PPI. The deviation is due to non-availability of REER data for all V5 countries. The current account to net output ratio for each of the V5 countries as well as the US real interest rate is shown in figures below.

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6 In instances where quarterly data samples are unavailable, some extrapolations based on the patterns of the available series are done and where yearly data are available, the yearly data partitioned into quarters are used to fill the missing quarterly data samples. This approach was employed only in the few instances where data for some quarters are unreported and does not constitute the bulk of data samples used in the analysis.
The first five graphs are the current account to net output ratios of V5 countries; the sixth graph is the graph of US real interest rates. Graphs are plotted quarterly from the first quarter of 1980 to the fourth quarter of 2014. This gives a total of 136 quarters.
4.1b Unit Root Test

Before the model is tested, it is important to check that the variables in the SVAR, \( V_t' = (r_t, \Delta y_t, \Delta p_t, \Delta c_t) \), are stationary. To begin the analysis, unit root tests are first performed on each variable, for each country, using the Augmented Dickey-Fuller (ADF) Unit Root Tests where an appropriate lag \( n \) is chosen in a spirit similar to Ng and Perron (2001) wherein a maximum lag is set and lags are dropped until the last lag is statistically significant. The standard testing procedure is then set up by regressing

\[
\Delta V_t' = \sum_{i=1}^{n} \phi_i \Delta V_{t-i}' + \delta V_{t-1}' + \eta_t
\]  

(2.29)

and testing, for each variable in \( V_t' \), whether \( \delta \) is negative and significantly different from zero, using ADF. The results, shown below, suggest that almost all variables for each country are stationary in their original form as specified in the SVAR, with the few exceptions being Norway, Singapore and Latvia whose current account to net output ratios are non-stationary in levels but stationary in first differences.

Table 1: Unit roots test for model variables

<table>
<thead>
<tr>
<th>Country</th>
<th>CA/NOt</th>
<th>( \Delta no_t )</th>
<th>( \Delta r_t )</th>
<th>( \Delta p_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norway</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>level</td>
<td>-0.67</td>
<td>-4.87**</td>
<td>-2.58**</td>
<td>-9.93**</td>
</tr>
<tr>
<td>first difference</td>
<td>2.24*</td>
<td>-0.75</td>
<td>-2.47*</td>
<td>-3.37*</td>
</tr>
<tr>
<td>Singapore</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>level</td>
<td>-0.97</td>
<td>-4.86**</td>
<td>-2.13*</td>
<td>-9.53**</td>
</tr>
<tr>
<td>first difference</td>
<td>-4.51**</td>
<td>0.16</td>
<td>-2.42*</td>
<td>3.40**</td>
</tr>
<tr>
<td>Iceland</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>level</td>
<td>-2.99*</td>
<td>-3.68**</td>
<td>-2.45*</td>
<td>-10.27**</td>
</tr>
<tr>
<td>first difference</td>
<td>-5.54**</td>
<td>1.06</td>
<td>4.98**</td>
<td>2.59*</td>
</tr>
<tr>
<td>Estonia</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>level</td>
<td>-2.36*</td>
<td>-2.58*</td>
<td>-3.04*</td>
<td>-3.89**</td>
</tr>
<tr>
<td>first difference</td>
<td>-3.15*</td>
<td>-5.11**</td>
<td>-3.76**</td>
<td>-2.62*</td>
</tr>
<tr>
<td>Latvia</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>level</td>
<td>-1.84</td>
<td>-2.46*</td>
<td>-3.04*</td>
<td>-2.59*</td>
</tr>
<tr>
<td>first difference</td>
<td>-2.56*</td>
<td>-4.03**</td>
<td>-3.76**</td>
<td>2.51*</td>
</tr>
</tbody>
</table>

Notes: test statistics reported in the table are approximated estimates, and some for first difference deduced from levels, including some of the significance levels; '*' indicates test statistic is significant at 5% level of significance, '**' indicates the 1% level of significance, test statistic having at least one star implies the null hypothesis of non-stationarity is rejected. Regressions do not include a constant or time trend. Results reported for real interest rate are for US data samples for real interest rates from period 1980:1 to 2014:4 as in Norway, Iceland and Singapore, slight differences in results reflect differences in lag length used for the countries. The stationarity of real interest rate increases considerably in statistical significance when the data samples are analyzed between 1990:1 to 2014:4 as in Estonia and Latvia.

It is interesting to note that the world real interest rate, proxied with US real interest rate, is largely stationary. This result is at odds with the non-stationary world real interest rate obtained in Karadimitropoulou and Leon-Ledesma (2009) for U.S. real interest rate data analysed for the period from 1980:1 to 2007:4. Since data samples in this paper extend to 2014:4, about 28 more quarters of additional data, it is plausible to suggest that the reversion to level stationarity of the U.S. real interest rate is on the back of the additional data samples that
reflect more recent events, particularly the downward movement observed in the US real interest rate between the quarters from 2008:1 to 2014:4. This recent events were not captured by the data used in the previous studies that reported a non-stationary US real interest rate.

The existence of non-stationary current account to net output ratios for Norway and Singapore, to a large extent, and Latvia, to a considerably smaller extent, implies that temporary shocks would have permanent effects on current account, which in turn implies current account balances are unsustainable. This is further supported by the fact that the current account to net output ratios for these countries are each \( I(1) \) as they become stationary following first differencing. Accordingly, and consistent with the results obtained for developed countries by Backus et al. (1994) but also for OECD countries by Cashin and McDermott (1998), there appears to be a good deal of persistence in the current account balances of these economies. However, given that these countries, especially Norway and Singapore, have run sustainably large current account surpluses in most or all of the quarters and years between 1980 and 2014, in line with their economic policies, it is unlikely that the current account balances are unsustainable going forward, barring the unforeseen.

Karadimitropoulou and Leon-Ledesma (2009) also obtain similar results of current account non-stationarity in their analysis of the sources of current account fluctuations in the G6 economies. As in Taylor (2002) and Christopolous and Leon-Ledesma (2009), they note that the existence of non-stationary current account is at variance with the transversality condition imposed on the intertemporal budget constraint. However, they point to some of the limitations of unit root tests which might have forced a stationary series to appear non-stationary. Notable of these limitations is that unit root tests suffer from power problems when the alternative process is highly persistent. This limitation becomes even more pronounced in the absence of continuity and linear adjustments. For these reasons, and in order to be consistent with the original relationship in the aforementioned benchmark intertemporal model, they ignore the non-stationarity, allowing the current account to net output ratio to enter the VAR in levels. In this paper, we follow this procedure and continue the analysis of current account fluctuations, imposing stationarity on the current account-net output ratio and allowing it to enter the VAR set-up in levels, in ways consistent with the theory model.

To correctly specify the SVAR model, an appropriate lag length for each country need be determined. This is achieved through a series of lag determining tests collectively known as information criterion based tests. In performing these tests for selecting the appropriate lag length, the first step is to select a maximum possible lag length. To take cognizance of the discrepancies in sample period for the time series data of each country, we choose a maximum of ten lags for Norway, Singapore and Iceland and a maximum of six lags for Estonia and Latvia, reason being that data samples for the former set of countries are longer, i.e. from 1980:1 to 2014:4, than those of the latter set of countries which span 1990:1 to 2014:4. Results from the information criterion based tests, not reported but available on request, suggest that an appropriate lag length for Norway is five, Iceland five, Singapore five, Latvia two and Estonia one. Given the lag lengths, the models are specified and estimated and the impulse responses of each of the four variables to structural shocks are obtained. Variance decomposition analysis is also performed to determine, for a number of periods ahead, the proportion of fluctuations in current account emanating from the four shocks in the structural equation.
4.2 Analysis of Impulse Response Functions

The impulse and accumulated impulse response functions of current account to net output ratio to one standard deviation shock for each of the V5 countries are shown in the graphs below.

Fig. 3: IRFs and AIFRs of current account to net output for V5 countries

1. Norway

2. Iceland

3. Singapore
4. Latvia

| Source: Authors’ calculations from data obtained from the aforementioned sources |

Fig. 3 shows plots of the impulse response functions (IRFs) and accumulated impulse response functions (AIRFs) of the current account to net output ratio of each of the V5 countries to a one standard deviation shock, for each structural shock contained in $\rho_t' = (\rho_1', \rho_2, \rho_3', \rho_4')$. For each country, the first four graphs on the left hand side show the IRFs and the next four on the right hand side show the AIRFs. For both IRFs and AIRFs, the first two graphs at the top show the response of current account to net output ratio to shocks such as temporary ($\rho_4'$) and permanent ($\rho_2$) domestic output shocks respectively while the next two at the bottom show the response of current account to net output ratio to external supply shocks ($\rho_1$) and demand/preference shocks ($\rho_3'$), respectively. The dashed red lines on the left and right hand side of the IRFs and AIRFs represent the upper and lower segment of the 95% confidence intervals associated with the response of current account to net output ratio to a given shock or impulse.
From the theoretical proposition, the expected impact of each shock on current account to net output ratio is as follows: 1) under the assumption that income effects do not compensate for consumption tilting effects, any external supply shocks that raise (reduce) the world real interest rate would improve (weaken) the current account as they induce a lower (higher) consumption. This is especially mostly true for countries with large debtor (creditor) positions; 2) domestic permanent shocks have no long term effect on current account, as predicted by the present value theory; 3) preference (demand) shocks that raise the relative price term increase consumption and this weakens the current account due to an expected future decrease in the relative price term. The actual empirical results of the impact of each shock on the current account/net output ratio of each country are discussed below in detail.

i. **External supply shocks**

As can be seen in both the IRFs and AIRFs, external supply shocks lead to large current account deficits in Singapore, Latvia and Estonia, but a surplus in Norway, whereas the impact on Iceland is largely negligible. The negative impacts of external supply shocks on the current account balances of Singapore, Latvia and Estonia can be related to their position as net creditors for a substantial portion of the sample period. More specifically, Singapore ran a positive net foreign assets position throughout the sample period, i.e. 100% of the time. On the other hand, Latvia and Estonia ran positive net foreign assets positions roughly 68% and 71% of the time throughout the sample period. Thus, for these countries, income effects compensate for consumption tilting effects. This is a standard result in the analysis of the impact of external supply shocks on the current account balances of countries with large creditor positions.

Norway however deviates from this standard result despite being a large creditor for about 89% of the time throughout the sample period on the back of its large net foreign assets which it accumulated over time through oil exports. Thus, external supply shocks that increase the world real interest rate also improve the current account of Norway; that is, income effects do not compensate for consumption tilting effect. In effect, the impact of external supply shocks on Norway’s current account is incompatible with its net creditor position.

ii. **Domestic permanent output shocks**

Domestic permanent output shocks have a negative impact on the current account of Norway, Latvia and Estonia in all quarters. However, for Singapore and Iceland, the results are mixed. In the case of Singapore, domestic permanent output shocks impact current account negatively in the first 4 quarters and positively between 5 and 20 quarters. The results are much more mixed for Iceland as domestic permanent output shocks impact current account between 1 and 2 quarters, 3 and 7 quarters and 9 and 10 quarters and then produce neither a positive nor negative effect between 11 and 20 quarters. The accumulated impulse response is even more stringent and shows no effect of domestic permanent output shocks on the current account of Iceland in all the quarters. On the whole, results obtained for both IRFs and AIRFs in all V5 countries are not statistically significant and this partly confirms the prediction of the present value model – that permanent shocks do not have a significant effect on current account in the long run, only temporary shocks do.
iii. Preference (demand) shocks

For demand shocks, the IRFs show a negative impact of preference shocks on the current account of Norway and no significant impact for Estonia in all quarters. For Iceland, Singapore and Latvia, the impact of preference shocks on current account is positive for a number of quarters and negative or non-existent for others. Particularly, demand shocks have a positive impact on the current account of Iceland between 3 and 15 quarters and no impact going forward. For Singapore, the impact is positive between 3 and 6 quarters and negative otherwise while it is positive for Latvia in the first three quarters but show no impact on the rest quarters. The results appear to mimic, in large parts, the complement (opposite) of the results obtained in the case of external supply shocks, with the exception being Estonia whose current account does not react to demand shocks despite reacting negatively to external supply shocks.

iv. Domestic temporary shocks

The IRFs and AIRFs show that the impact of domestic temporary shocks on current account is positive for each of the V5 countries, implying that these countries are positively affected by domestic temporary shocks. The impact is not only positive it is also statistically significant as the lower limit of the 95% confidence interval band for each country does not fall below the zero mark on the vertical axis, especially for the AIRFs which show that, for each country, the impact of domestic temporary shocks on current account is large and persistent and the current account does improve. A lower confidence interval band above zero at every point implies that the response to the impulse from domestic temporary shocks is significantly different from zero at the 5% level of significance in all forecast quarters.

On the whole, two conclusions can be drawn from the results suggested by the IRFs and AIRFs. First, domestic temporary shocks have large, persistent, statistically significant and positive long run effects on the current account and hence net foreign assets position of each of the V5 countries while permanent shocks do not. This implies that the assumption of the standard intertemporal model, which states that temporary shocks rather than permanent shocks impact current account in the long run, is fully satisfied. Second, external supply shocks that increase the world real interest rate result in current account surplus for Norway, despite being a country with large creditor positions for most of the period covered in the data samples. This suggests that income effects do not compensate for consumption tilting effects in the case of Norway. In other words, the results show that if a country has large creditor position, this does not always imply income effects would compensate for consumption tilting effects.
4.3 Variance Decomposition Analysis

The forecast error variance decomposition (FEVD) analysis, which helps to determine, for a number of periods ahead, the proportion of fluctuations in current account and net output attributable to the four shocks contained in the SVAR model, is presented in the tables below. Although the FEVD results for other forecast quarters are available on request, here we present results for forecast horizons of one, four, eight, twenty, forty, sixty and eighty quarters ahead respectively. The first and second columns of the results for each country represent the proportion of variations in current account to net output ratio and changes in net output emanating from the external supply shocks, permanent output shocks, demand shocks and temporary domestic net output shocks for different forecast horizons.

Table 2: Sources of Current Account Fluctuations

<table>
<thead>
<tr>
<th>Norway</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast horizon</td>
<td>ρ₁</td>
<td>ρ₂</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>8</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>20</td>
<td>0.01</td>
<td>0.05</td>
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<td>60</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>80</td>
<td>0.01</td>
<td>0.07</td>
</tr>
</tbody>
</table>

| Estonia | | |
|-------|---|---|---|---|
| Forecast horizon | ρ₁ | ρ₂ | ρ₃ | ρ₄ | ρ₁ | ρ₂ | ρ₃ | ρ₄ |
| 1 | 0.00 | 0.00 | 0.00 | 1.00 | 0.01 | 0.77 | 0.00 | 0.23 |
| 4 | 0.02 | 0.02 | 0.00 | 0.96 | 0.01 | 0.72 | 0.02 | 0.25 |
| 8 | 0.05 | 0.03 | 0.00 | 0.92 | 0.01 | 0.72 | 0.02 | 0.25 |
| 20 | 0.08 | 0.03 | 0.00 | 0.88 | 0.01 | 0.71 | 0.02 | 0.25 |
| 40 | 0.09 | 0.03 | 0.00 | 0.88 | 0.01 | 0.71 | 0.02 | 0.25 |
| 60 | 0.09 | 0.03 | 0.00 | 0.88 | 0.01 | 0.71 | 0.02 | 0.25 |
| 80 | 0.10 | 0.04 | 0.00 | 0.87 | 0.01 | 0.70 | 0.02 | 0.26 |

| Singapore | | |
|---------|---|---|---|---|
| Forecast horizon | ρ₁ | ρ₂ | ρ₃ | ρ₄ | ρ₁ | ρ₂ | ρ₃ | ρ₄ |
| 1 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 1.00 | 0.00 | 0.00 |
| 4 | 0.01 | 0.03 | 0.01 | 0.95 | 0.06 | 0.86 | 0.07 | 0.01 |
| 8 | 0.02 | 0.02 | 0.01 | 0.95 | 0.07 | 0.86 | 0.06 | 0.01 |
| 20 | 0.16 | 0.01 | 0.02 | 0.82 | 0.07 | 0.86 | 0.05 | 0.02 |
| 40 | 0.32 | 0.01 | 0.02 | 0.65 | 0.07 | 0.86 | 0.05 | 0.02 |
| 60 | 0.42 | 0.01 | 0.03 | 0.58 | 0.07 | 0.86 | 0.05 | 0.02 |
| 80 | 0.52 | 0.02 | 0.03 | 0.55 | 0.07 | 0.85 | 0.04 | 0.03 |
### Iceland

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<th>$\rho_3$</th>
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Notes: Columns A and B represent the proportion of fluctuations in current account to net output ratio attributable to external supply shocks ($\rho_1$) permanent domestic output shocks ($\rho_2$), demand or preference ($\rho_3$) shocks and temporary domestic net output shock ($\rho_4$).

The results show that for Norway, temporary domestic output shocks explain all fluctuations in current account. However, the dominance of the explanatory power of temporary domestic shocks is not stable throughout the forecast horizon. It whittles down as the quarters and years progress so that 20 years after the shock, the main shocks explaining current account fluctuations in Norway become preference/demand shocks, explaining around 51% of the fluctuations in Norway’s current account versus 42% for temporary domestic net output shock. Permanent domestic and external supply shocks appear to have negligible explanatory power for current account fluctuations in Norway even after 20 years (80 quarters).

In quarter 1, Estonia’s current account fluctuations are explained mainly by temporary domestic shocks. Although the explanatory power reduces throughout the forecast horizon reaching 88% after 80 quarters compared to external supply shocks whose contribution to current account fluctuations increases to 9%, the dominance of temporary domestic shocks in the explanation of fluctuations in current account remains stable throughout the forecast horizon. Permanent domestic and preference shocks contribute much lower to the fluctuations in Estonia’s current account. In fact, throughout the forecast horizon, demand shocks contribute nothing to Estonia’s current account fluctuations, but the contribution of permanent domestic shocks remain stable at 3%-4% in the latter part of the forecast horizon. For Singapore, it is temporary domestic shocks that mostly drive the current account in the short run. In the long run, however, both external supply and domestic temporary shocks are responsible for the fluctuations in the current account of Singapore. In fact, temporary domestic shocks reduce their proportion to 55% after 80 quarters whereas that of external supply shocks increases steadily, reaching 52% after 80 quarters. At this instance, preference shocks and permanent domestic shocks only explain 3% and 2% of the fluctuations in the current account of Singapore. In all of these, the
proportion of fluctuations in Singapore’s current account attributable to external supply shocks, permanent shocks and preference shocks in the long run is a significant leap from the circumstances in the short run, where none of these shocks is responsible for the fluctuations experienced in Singapore’s current account.

In the case of Iceland, roughly all of the current account fluctuations in the short-run are explained by temporary domestic net output shocks. The other shocks – external supply, permanent domestic shocks and preference shocks – however gain in importance, albeit slightly, and explain around 3% after 8 quarters. This increases by 100 basis points to 4% at the end of the forecast horizon. Of these minor shocks contributing to current account fluctuations in Iceland, demand shocks are the most prominent, contributing twice as much as the other two shocks to the fluctuations in current account of Iceland in most of the forecast horizons. The same is true for Latvia where changes in the forecast variance of the current account are driven mostly by temporary domestic shocks, both in the short and long run. In the short run, it explains roughly all of the fluctuations in current account while in the long run, specifically after 20 years or 80 quarters, this reduces to 96%, giving way for the contribution of external supply shocks to rise from 0% in the short run to around 4% in the long run. The results showing the dominance of temporary domestic output shocks in explaining fluctuations in current accounts of the advanced economies considered in this paper are similar to those obtained in Kano (2008) though for a different set of advanced economies. In particular, Kano (2008) shows that about 80% and 72% of the current account fluctuations in Canada and UK respectively are due to temporary output shocks and refers to the excess response of current account to temporary output shocks as a puzzle.

On the whole, the results appear to suggest that fluctuations in current account balances of the advanced countries under study are mostly driven by temporary domestic net output shocks, both in the short and long run, although it is important to note that the explanatory power of temporary domestic shocks in explaining the current account fluctuations in Norway and Singapore is less stable and reduces to almost one half its value in the long run or by the end of the forecast horizon.

Looking at the net output side of things in the second column of Table 2, the results appears puzzling as temporary domestic output shocks, which explain a significant portion of fluctuations in current account, contribute very little in explaining the sources of fluctuations in net output which drives current account. The situation is even more pronounced in Singapore, Iceland and particularly Latvia where temporary domestic output shocks account for 0%, 3% and 0% of fluctuations in net output in the short run and 3%, 3% and 0% in the long run compared to Norway and Estonia where 14% and 25% of fluctuations in net output are attributable to temporary domestic shocks both in the short and long-run respectively which implies the contribution of temporary domestic shocks to net output for these countries remains stable throughout the forecast horizon.

Finally, given that external supply and preference shocks together account for a nontrivial proportion of current account fluctuations in the long run for Norway, Singapore and Iceland, the results validate the two good intertemporal models which considers as inputs the world real interest rate and changes in price levels. Although this paper uses a different set of countries, the results nonetheless are in line with the conclusions in Lee and Chinn (2006) and Karadimitropoulou and Leon-Ledesma (2009) wherein the signs of the impulse responses and the variance decompositions point towards models that distinguish tradable from non-tradable goods. On this basis, Estonia and Latvia appear as the main exception despite satisfying the basic predictions of the present value model which states that temporary, rather than permanent domestic, shocks are responsible for current account fluctuations.
4.4 Test of over-identifying restrictions

In this section, formal Wald tests for some of the theory predictions of the behaviour of current account would be performed by imposing a number of over-identifying restrictions. These are tests for 1) the long-run effects of permanent output shocks on changes in relative price levels; 2) present value model and 3) importance of consumption tilting effects via changes in relative price levels and world real interest rate. A test for the absence of permanent domestic shocks on changes in relative price levels implies testing $\psi_{32}(L) = 0$. A test for the significance of the present value model checks whether or not permanent domestic shocks have long-run impact on current account and this implies testing $\psi_{41}(L) = 0$. Lastly, a test for the importance of consumption tilting effects through changes in relative price levels and world real interest rate implies testing $\psi_{42}(L) = 0$ and $\psi_{43}(L) = 0$. Thus, four over-identifying restrictions to be tested are $\psi_{32}(L) = 0$, $\psi_{41}(L) = 0$, $\psi_{42}(L) = 0$ and $\psi_{43}(L) = 0$. A joint test of $\psi_{41}(L) = \psi_{43}(L) = 0$ for the consumption tilting effects is also performed.

Results from the Wald tests are presented below.

Table 3: Test of over-identifying restrictions

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>Norway</th>
<th>Estonia</th>
<th>Singapore</th>
<th>Iceland</th>
<th>Latvia</th>
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<tbody>
<tr>
<td>$\psi_{32}(L) = 0$</td>
<td>0.79 (0.42)</td>
<td>0.02 (0.98)</td>
<td>0.84 (0.40)</td>
<td>0.36 (0.72)</td>
<td>0.78 (0.43)</td>
</tr>
<tr>
<td>$\psi_{41}(L) = 0$</td>
<td>13.04 (0.00)</td>
<td>1.36 (0.18)</td>
<td>11.54 (0.00)</td>
<td>5.99 (0.00)</td>
<td>0.13 (0.89)</td>
</tr>
<tr>
<td>$\psi_{42}(L) = 0$</td>
<td>20.90 (0.00)</td>
<td>3.19 (0.00)</td>
<td>19.06 (0.00)</td>
<td>0.75 (0.45)</td>
<td>0.37 (0.71)</td>
</tr>
<tr>
<td>$\psi_{43}(L) = 0$</td>
<td>0.42 (0.68)</td>
<td>3.42 (0.42)</td>
<td>3.47 (0.00)</td>
<td>1.37 (0.17)</td>
<td>3.66 (0.00)</td>
</tr>
<tr>
<td>$\psi_{41}(L) = \psi_{43}(L) = 0$</td>
<td>89.47 (0.00)</td>
<td>7.49 (0.00)</td>
<td>91.59 (0.00)</td>
<td>19.05 (0.00)</td>
<td>6.81 (0.00)</td>
</tr>
</tbody>
</table>

Notes: p-values in parentheses

First, the results show that the restriction $\psi_{32}(L) = 0$ cannot be rejected and this suggests that permanent output shocks have no impact on changes in relative price levels for all the V5 countries. Furthermore, the results show that except for Norway, Estonia and Singapore, permanent output shocks are not responsible for fluctuations in current account and this is in line with the results of the IRFs and FVDA and lends support for the prediction of the present value model. The results for Norway are however surprising as they deviate from the guidance provided by the IRFs and FVDA wherein permanent domestic shocks are found to have no impact on current account fluctuations. Thus, the test for over-identifying restrictions shows that the behaviour of current account for all countries may not be consistent with predictions of the present value model, a conclusion similar to Karadimitropoulou and Leon-Ledesma (2009) for France. Finally, consumption tilting effects, driven by either external supply shocks or demand shocks, are significant drivers of current account fluctuations in Singapore as both restrictions $\psi_{41}(L) = 0$ and $\psi_{43}(L) = 0$ are rejected at the 5% level. For the rest countries, at most one of $\psi_{41}(L) = 0$ and $\psi_{43}(L) = 0$ is rejected at the 5% level. Thus, except for Singapore, consumption tilting effects drive current account fluctuations when driven jointly by external supply shocks and demand shocks. This conclusion follows from the finding that $\psi_{41}(L) = 0$ and $\psi_{43}(L) = 0$ are each not simultaneously significant but $\psi_{41}(L) = \psi_{43}(L) = 0$, a joint test, is highly significant for all countries.
5.0 Concluding Remarks

Fluctuations in current account balances vary in size across countries. Economists have long been researching the sources of large current account fluctuations due to their role in global imbalances. In this paper, we have focused on 5 developed countries with the largest current account fluctuations and analysed the sources or main drivers of these fluctuations. Four shocks potentially responsible for the fluctuations are considered. These are domestic temporary, external supply, preferences and domestic permanent shocks. The impact of each shock on variables within the model, with a particular focus on current account, is then assessed. For the most part, we follow the empirical procedure of Karadimitropoulou and Leon-Ledesma (2009), and the theoretical setting underpinning this empirical setup is in the spirit of Bergin and Sheffrin (2000) which incorporates world real interest rate within a model that comprises tradable and non-tradable sectors of a small open economy.

Results show that permanent output shocks have no impact on changes in relative price levels for all V5 economies – Norway, Estonia, Singapore, Iceland and Latvia. Furthermore, the present value model is consistent with the behaviour of Iceland and Latvia but not for Norway, Estonia and Singapore where permanent output shocks are found to drive fluctuations in current account. Thus, we find some evidence that the behaviour of current account is not always consistent with predictions of the present value model; a similar conclusion was obtained in Nason and Rogers (2006) for Canada and Karadimitropoulou and Leon-Ledesma (2009) for France. Consumption tilting effects driven by external supply shocks and demand shocks are significant drivers of current account fluctuations in Singapore. For the rest countries, however, consumption tilting effects significantly drive current account fluctuations only when jointly driven by external supply shocks and demand shocks. Finally, consistent with results found in the literature, particularly Kano (2008), there is an excess response of current account to temporary output shocks in all countries, in the short and long run, except for Norway where the impact of temporary shocks on current account wanes in magnitude over the forecast horizon. In all, the paper contributes to the narrow but expanding literature on the sources of large current account fluctuations.
6.0 References


Appendix A

The derivation of the intertemporal budget constraint is as follows:

Allowing the summation to begin from the next period, the intertemporal budget constraint in can be written as

\[ E_0(R_0C_0) + \sum_{t=1}^{\infty} E_0(R_tC_t) = E_0(R_0\bar{Y}_0) + \sum_{t=1}^{\infty} E_0(R_t\bar{Y}_t) + B_0 \]  

(2.30)

Using the fact that the expected value of a variable at time \( t \) taken at time \( t \) equals the value of the variable, the expression of the intertemporal budget constraint simplifies to

\[ C_0 + \sum_{t=1}^{\infty} E_0(R_tC_t) = \bar{Y}_0 + \sum_{t=1}^{\infty} E_0(R_t\bar{Y}_t) + B_0, \quad R_0 = 1 \]  

(2.31)

Let

\[ \psi_0 = \sum_{t=1}^{\infty} E_0(R_t\bar{Y}_t) + B_0 \quad \text{and} \quad \phi_0 = C_0 + \sum_{t=1}^{\infty} E_0(R_tC_t) \]  

(2.32)

then

\[ \phi_0 - \psi_0 = B_0 \]  

(2.33)

Taking logs and following the linearization of Huang and Lin (1993), becomes

\[ \log \phi_0 - \log \psi_0 = \left(1 - \frac{1}{\omega}\right)(\log B_0 - \log \psi_0), \]  

(2.34)

where \( B_0 \) is steady state net foreign assets.

Let

\[ \omega = 1 - \frac{\bar{B}}{\psi_0} \]

then (2.34) can be written as

\[ \log \phi_0 - \log \psi_0 = \left(1 - \frac{1}{\omega}\right)(\log B_0 - \log \psi_0), \]

Further linearization yields

\[ c_0 - \log \phi_0 = \sum_{t=1}^{\infty} \rho^t (r_t - \Delta C_t), \]  

(2.35)

where \( c_0 = \log C_0, \Delta C_t = \log C_t - \log C_{t-1} \) and \( \rho = 1 - \frac{c}{\log \phi_0} \) where \( c \) is the steady state value of the log of consumption. Similarly,
\[ \bar{y}_0 - \log \psi_0 = \sum_{t=1}^{\infty} \rho^t (r_t - \Delta \bar{y}_t), \quad (2.36) \]

where \( \bar{y}_0 = \log \bar{Y}_0 \), and \( \Delta \bar{y}_t = \log Y_t - \log Y_{t-1} \).

Plugging (2.35) and (2.36) into the intertemporal budget constraint in (2.34) yields

\[ \bar{y}_0 - \log \psi_0 = \sum_{t=1}^{\infty} \rho^t (r_t - \Delta \bar{y}_t) + \sum_{t=1}^{\infty} \rho^t (r_t - \Delta c_t) - c_0 \]

\[ = \left( 1 - \frac{1}{\omega} \right) (\log B_0 - \log \psi_0) + \left( 1 - \frac{1}{\omega} \right) \left( \sum_{t=1}^{\infty} \rho^t (r_t - \Delta c_t) - c_0 \right). \quad (2.37) \]

or

\[ \bar{y}_0 - \frac{c_0}{\omega} - \left( 1 - \frac{1}{\omega} \right) b_0 = -\sum_{t=1}^{\infty} \beta^t \left[ \Delta \bar{y}_t - \frac{\Delta c_t}{\omega} - \left( 1 - \frac{1}{\omega} \right) r_t \right], \quad b_0 = \log B_0 \]

as required.

Appendix B

Following Campbell et al (1997), the Euler equation is log-linearized as follows:

Taking logarithm of both sides yields

\[ \log \beta + \log E_t[Z_{t+1}] = 0, \quad (2.38) \]

where

\[ Z_{t+1} = \left( 1 + r_{t+1} \right) \left( \frac{C_t}{C_{t+1}} \right)^{\frac{1}{\gamma}} \left( \frac{P_t}{P_{t+1}} \right)^{(1-\frac{1}{\gamma})(1-\alpha)} \]

and

\[ \log Z_{t+1} \cong r_{t+1} - \frac{1}{\gamma} \Delta c_{t+1} - (1 - \frac{1}{\gamma})(1 - \alpha) \Delta p_{t+1}, \quad \log (1 + r_{t+1}) \cong r_{t+1} \]

Now, the logarithmic expectation term in (2.38) can be expressed as

\[ \log E_t[Z_{t+1}] = E_t[\log Z_{t+1}] + \frac{1}{2} Var_t[\log Z_{t+1}] \quad (2.39) \]

Simplifying each term on the RHS of (2.39) gives
\[ E_t[\log Z_{t+1}] = E_t \left[ r_{t+1} - \frac{1}{\gamma} \Delta c_{t+1} - \left(1 - \frac{1}{\gamma}\right)(1 - \alpha)\Delta p_{t+1} \right] \tag{2.40} \]

\[ \text{Var}_t[\log Z_{t+1}] = \text{Var}_t \left[ r_{t+1} - \frac{1}{\gamma} \Delta c_{t+1} - \left(1 - \frac{1}{\gamma}\right)(1 - \alpha)\Delta p_{t+1} \right] \]

\[ = \sigma_r^2 + \frac{1}{\gamma^2} \sigma_c^2 + \left(1 - \frac{1}{\gamma}\right)^2 (1 - \alpha)^2 \sigma_p^2 + \sigma_{pcr}, \tag{2.41} \]

where

\[ \sigma_{pcr} = -\frac{2}{\gamma} \sigma_cr - 2\left(1 - \frac{1}{\gamma}\right)(1 - \alpha)\sigma_{pr} + \frac{2}{\gamma} \left(1 - \frac{1}{\gamma}\right)(1 - \alpha)\sigma_{pc}, \]

and the identity \( \text{Var}_t(x_{t+1} - ay_{t+1} - bz_{t+1}) = \sigma_x^2 + a^2 \sigma_y^2 + b^2 \sigma_z^2 - 2a\sigma_{xy} - 2b\sigma_{xz} + 2ab\sigma_{yz} \) has been employed to obtain (2.41).

Thus, plugging (2.39), (2.40) and (2.41) into (2.38) yields

\[ \log \beta + \mathbb{E}_t \left[ r_{t+1} - \frac{1}{\gamma} \Delta c_{t+1} - \left(1 - \frac{1}{\gamma}\right)(1 - \alpha)\Delta p_{t+1} \right] \]

\[ + \frac{1}{2} \left[ \sigma_r^2 + \frac{1}{\gamma^2} \sigma_c^2 + \left(1 - \frac{1}{\gamma}\right)^2 (1 - \alpha)^2 \sigma_p^2 + \sigma_{pcr} \right] = 0 \tag{2.42} \]

which simplifies to

\[ \mathbb{E}_t \Delta c_{t+1} = \gamma \log \beta + \frac{1}{2} \gamma \left[ \sigma_r^2 + \frac{1}{\gamma^2} \sigma_c^2 + (1 - \alpha)^2 \left(1 - \frac{1}{\gamma}\right)^2 \sigma_p^2 + \sigma_{pcr} \right] + \gamma \mathbb{E}_t \left[ r_{t+1} \right] \]

\[ + \left(1 - \frac{1}{\gamma}\right)(1 - \alpha)\Delta p_{t+1} \]

or

\[ \mathbb{E}_t \Delta c_{t+1} = \theta + \gamma \mathbb{E}_t \left[ r_{t+1} + \left(1 - \frac{1}{\gamma}\right)(1 - \alpha)\Delta p_{t+1} \right], \]

where

\[ \theta = \gamma \log \beta + \frac{1}{2} \gamma \left[ \sigma_r^2 + \frac{1}{\gamma^2} \sigma_c^2 + (1 - \alpha)^2 \left(1 - \frac{1}{\gamma}\right)^2 \sigma_p^2 + \sigma_{pcr} \right] \]

This proves the log-linearized Euler equation.