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31 December 2016

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Capital theory and the associated with it price effects resulting from changes in the distributive variables hold centre stage when it comes to the internal consistency of both classical and neoclassical theories of value. The article briefly reviews the literature and then focuses on the detected skew eigenvalue distribution of the vertically integrated technical coefficients matrices of actual economies. The findings prompt the use of the Schur triangularization theorem for the construction even of a single industry from the input-output structure of the entire economy. Such a hyper-basic industry, in combination with hyper-non-basic industries, embodies properties that may capture the behaviour of the entire economic system. Thus, we can derive some meaningful results consistent with the available empirical evidence.

Key words: Capital theory, Eigenvalue distribution, Production prices, Hyper-basic industry, Effective rank

JEL classifications: B21, B51, C67, D46, D57

1. Introduction

One of the enduring puzzling and still not from the fully resolved issues in economic theory is the effects of changes in income distribution on commodity prices and factors of production. We know that Ricardo ([1821] 1951, pp. 30-43) was from the first to formulate the question and to argue that a definitive answer can be only obtained with the possession of an "invariable measure of value". That is, a commodity whose value would, under all technological and distributional circumstances, remain the same and using this as the *numéraire* commodity, we could identify the source of changes in the prices of all other commodities. Ricardo devoted in vain his entire intellectual life to defining either analytically or practically such a standard of value, which would remain invariant to both changes in income distribution and production conditions. Marx ([1894] 1959, Chap. 11) also faced a similar problem and proposed a solution on the basis of the difference of an industry's capital-intensity from the economy-wide average capital-intensity.

The advent of neoclassical theory at the end of the nineteenth century defined relative prices as indexes of relative scarcity. As a consequence, the prices of factors of production were theorized to move monotonically in the upward or downward direction with changes in income distribution. However, the determination of the price of a unit of capital in a way which would be consistent with the premises of the neoclassical theory was very hard to pinpoint. Robinson (1953) inspired by Piero Sraffa's teaching and writings exposed the inconsistencies in the neoclassical theorization of capital as a 'factor of production'. Subsequently, Sraffa (1960) changed fundamentally the established ideas on the relations between commodity prices and income distribution.

The underlying idea in Sraffa's (1960) analysis is that an industry's capital-intensity depends on changes in income distribution which may initiate complex movements in relative prices that may even alternate the characterization of an industry from capital to labour intensive and *vice versa*. Consequently, the old classical rule according to which the change in relative prices is strictly related to the capital-intensity of the industry relative to others or some kind of invariable average does not in general hold. Furthermore, Sraffa showed that it is possible that a capital-intensive technique may be chosen for both low and high rates of profit, a result that runs contrary to the neoclassical theory of scarcity prices. Under these circumstances, the determination of a well-behaved demand for capital schedule is in question, and if such a core schedule is questioned, then the presence of interdependency rules out the possibility of confidently determining the remaining important demand and supply schedules. The consequences for neoclassical analysis are thus quite upsetting (Tsoulfidis, 2010, p. 207).

The remainder of the paper is organized as follows: Section 2 briefly reviews the relevant literature on the issues at hand. Section 3 refers to a spectral decomposition of the price-wage-profit system and explicates the possible monotonic movement of the price-profit rate curves. More specifically, it shows that, if the rank of the vertically integrated technical coefficients matrix is low, then the behaviour of the entire economic system may be described by just a few, or even a single, 'hyper-basic' industries without any significant loss in economic information. Section 4 deals with the spectral properties of actual input-output structures mainly of the US economy as well as a number of diverse economies that are found to display spectral characteristics quite similar to those of the US economy. Finally, Section 5 summarizes and makes some concluding remarks with notes for further future research efforts.

2. Literature Review

The capital controversies of the 1960s have shown that long-period commodity prices do not necessarily display monotonic paths with respect to changes in income distribution; as a consequence, the rate of profit (or interest) could not be taken as a consistent index of the relative scarcity of capital. These theoretical findings, however, were not corroborated by analogous empirical evidence, either because such research was extremely difficult to pursue at that time, or for the reason that, if a theory is found logically inconsistent, then there is no any pressing reason to test it empirically.

In each case, the controversy during the 1960s and 1970s was carried out theoretically without any conduct with actual economic data, as this can be judged by the numerical examples that they were used on both sides of the debate. Thus, Samuelson's (1962) parable of a one-commodity world, with the straight lines wage-profit rate (WPR) curves and the associated with these well-behaved supply of capital schedule, was sharply contrasted to the multi-commodity-world of his critiques with WPR curves characterized by any number of curvatures and shapes suggesting switching from one technique and reswitching to other techniques even with slight changes in distributive variables. These 'exotic' shapes of the WPR curves indicated that the price-profit rate (PPR) curves displaying extremes and inflection points rendering untenable the neoclassical theorization of prices as scarcity indexes and that the capital-intensity could not be defined in any uncontroversial way. The theoretical findings were more in favour of the Sraffa-inspired critique as Samuelson (1966), the leading figure from the neoclassical camp, admitted. The same is true with Robert M. Solow and Charles E. Ferguson, while the list could be extended to include Lucas (1988) who opined that the debate was won from the Cambridge UK side and so did Mas Colell (1989) by noting that the relationship between capital-intensity and profit rate could take 'any' possible shape. It seems, nevertheless, that the neoclassical school gradually lost interest in the capital theory debates, while the newer generations of neoclassical economists rarely refer to these issues and continue using various forms of 'production functions' as if there was no problem with the theory they are based on.

In the meantime, the production price-wage-profit rate system of actual economies (but, *ex hypothesis*, linear, closed and single-product) has been examined in a relatively large number of studies. From Sekerka et al. (1970), Krelle (1977) and Shaikh (1984, 1998) onwards, the key stylized findings in these empirical studies are that:

(i). The vectors of vertically integrated labour coefficients, or labour values, and 'actual production prices' are close to each other, as judged by alternative measures of deviation.[1] The estimated deviations are not too sensitive to the type of measure used for their evaluation (Mariolis and Tsoulfidis, 2010).

(ii). The 'actual profit rate' is usually no greater than 50% of its maximum feasible value and, most of the time, is in the range of 30% to 40%. Therefore, the polynomial approximation (Steedman, 1999) of the actual production prices, expressed in terms of Sraffa's (1960, Chaps. 4-5) Standard commodity (SSC), through 'dated quantities of embodied labour' requires the inclusion of just a few terms.

(iii). Non-monotonic PPR curves, expressed in terms of SSC, are not only relatively rare (i.e. not significantly more than 20% of the tested cases) but also have no more than one extreme point. Cases of reversal in the direction of deviation between production prices and labour values ('price-labour value reversals') are rarer. In fact, the price-movement is, more often than not, governed by the 'capital-intensity effect', i.e. by the difference between the industry's vertically integrated capital-intensity and the capital-intensity of the Sraffian Standard system (SSS), where the latter equals the reciprocal of the maximum feasible value of the profit rate. However, this 'traditional flavour' condition *can* be modified by the 'price effect', i.e. the revaluation of the industry's vertically integrated capital, which depends on the entire economic system and, therefore, is not predictable at the level of any single industry (Sraffa 1960, pp. 14-15; Pasinetti 1977, pp. 82-84; Mariolis et al. 2015). Empirical evidence associated with quite diverse economies, and spanning different time periods, showed that the capital-intensity effect overshadows the price effect, although there are cases where the latter effect is strong enough that it can supersede the former giving rise to extrema and 'price-labour value reversals' (also see Tsoulfidis and Mariolis, 2007, Tsoulfidis, 2008, Mariolis and Tsoulfidis, 2009). It then follows that the idea of

[1] The terms 'actual production prices' and 'actual profit rate' are used to signify production prices and profit rate that correspond to the 'actual' real wage rate. The latter is estimated on the basis of the available input-output data.

representing the PPR curves through linear or, *a fortiori*, quadratic approximations is absolutely justifiable and empirically powerful (Bienenfeld, 1988; Shaikh, 2012; Iliadi et al., 2014).

(iv). Although the actual economies deviate considerably from the Ricardo-Marx-Dmitriev-Samuelson 'equal value compositions of capital' case, the WPR curves are near-linear, i.e. the correlation coefficients between the distributive variables tend to be above 99%, and their second derivatives change sign no more than once or, very rarely, twice, irrespective of the *numéraire* chosen (Leontief, 1985; Ochoa, 1989; Petrović, 1991; Han and Schefold, 2006). All these stylized findings imply that, although the actual economies *cannot* be analyzed on the basis of 'neoclassical parables', the role of price-feedback effects is actually of limited quantitative significance.

In the late 2000s the relevant research took a new direction on the basis of the modern classical theory of value corollaries and the spectral representation (or, in more general terms, the 'modern state variable representation') of linear systems.[2] It has been particularly pointed out that the functional expressions of the price-wage-profit rate relationships admit lower and upper norm bounds, while their monotonicity could be connected to the characteristic value distribution of the matrix of vertically integrated technical coefficients and, therefore, to the 'effective rank' of this matrix. Since nothing can be said a priori about this crucial factor in real-world economies, the examination of actual input-output data became absolutely necessary. Thus, it has been well-ascertained that, across countries and over time, the moduli of the eigenvalues as well as the singular values of actual economies follow exponentially decaying trends. Moreover, when the capital stock matrices are taken into account, they are characterized by a nearly 'L-shaped' pattern. Namely, in the latter, more realistic case, the decay of the characteristic values is remarkably faster (see Mariolis and Tsoulfidis, 2016b). This new stylized fact implies that only a few eigenvalues really matter for the observed shapes of the P-WPR curves, which is another way to say that these curves tend to be similar to those of low-dimensional systems.[3] In effect, it seems that similarity transformations that result in only a few industries

[2] See Schefold (2008, 2013a), Mariolis and Tsoulfidis (2009, 2011, 2014). For further analytical investigations, see Schefold (2013b, 2016), Mariolis and Tsoulfidis (2016a), Shaikh (2016, Chap. 9).

[3] Bienenfeld (1988, p. 255) has already shown that, in the extreme case where the non-dominant eigenvalues equal zero, the production prices in terms of SSC are strictly linear functions of the profit rate, and Shaikh (1998) has noted that "[a] large disparity between first and second eigenvalues is another possible source of linearity." (p. 244; also see p. 250, note 9).

extract the essential features contained in the original-actual system and provide the basis for constructing reliable approximations of the observed relationships.

In a nutshell, this recently developed research line suggests not the irrelevance of Sraffian analysis but a new logic approach for (i) revealing the essential properties of the static and dynamic behaviour of a linear production system as a whole; (ii) determining the extent to which these properties deviate from those predicted by the traditional theories of value; and (iii) deriving meaningful theoretical results consistent with the available empirical evidence.

3. Spectral Decomposition of the Price System

Let us suppose a linear circulating capital model of production described by the irreducible $n \times n$ matrix of direct technical coefficients, **A**, whose Perron-Frobenius eigenvalue is less than one, and the surplus produced is distributed between profits and wages. Let $\mathbf{l}$ be the $1 \times n$ vector of direct labour coefficients, w the uniform money wage paid *ex post*, and r the economy-wide profit rate.[4] On the basis of these assumptions we can write the vector of production prices, $\mathbf{p}$, as follows

$$\mathbf{p} = w\mathbf{l} + (1+r)\mathbf{pA} \tag{1}$$

After rearrangement, equation (1) becomes

$$\mathbf{p} = w\mathbf{v} + r\mathbf{pH}$$

or

$$\mathbf{p} = w\mathbf{v} + \rho\mathbf{pJ} \tag{2}$$

where $\mathbf{v} \equiv \mathbf{l}[\mathbf{I}-\mathbf{A}]^{-1}$ denotes the vector of vertically integrated labour coefficients, or labour values, and $\mathbf{H} \equiv \mathbf{A}[\mathbf{I}-\mathbf{A}]^{-1}$ the vertically integrated technical coefficients matrix. Moreover, $\rho \equiv rR^{-1}$, $0 \le \rho \le 1$, denotes the relative profit rate, which equals the share of profits in the SSS, and $R \equiv \lambda_{\mathbf{A}1}^{-1} - 1 = \lambda_{\mathbf{H}1}^{-1}$ the maximum profit rate, i.e. the profit rate corresponding to [$w = 0$, $\mathbf{p} > \mathbf{0}$], which equals the ratio of the net product to the means of production in the SSS (see Sraffa, 1960, pp. 21-23). Finally, $\mathbf{J} \equiv R\mathbf{H}$ denotes the normalized vertically integrated technical coefficients matrix,

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[4] The transpose of a $1 \times n$ vector $\mathbf{y} \equiv [y_j]$ is denoted by $\mathbf{y}^{\mathrm{T}}$. Furthermore, $\lambda_{\mathbf{A}1}$ denotes the Perron-Frobenius eigenvalue of a semi-positive $\mathbf{A}$, and $(\mathbf{x}_{\mathbf{A}1}^{\mathrm{T}}, \mathbf{y}_{\mathbf{A}1})$ the corresponding eigenvectors, while $\lambda_{\mathbf{A}k}$, $k = 2,...,n$ and $|\lambda_{\mathbf{A}2}| \ge |\lambda_{\mathbf{A}3}| \ge ... \ge |\lambda_{\mathbf{A}n}|$, denote the non-dominant eigenvalues, and $(\mathbf{x}_{\mathbf{A}k}^{\mathrm{T}}, \mathbf{y}_{\mathbf{A}k})$ the corresponding eigenvectors. Finally, $\mathbf{I}$ denotes the $n \times n$ identity matrix.

$\lambda_{\mathbf{J}1} = R\lambda_{\mathbf{H}1} = 1$, and the moduli of the normalized eigenvalues of system (2) are less than those of system (1), i.e. $|\lambda_{\mathbf{J}k}| < |\lambda_{\mathbf{A}k}| \lambda_{\mathbf{A}1}^{-1}$ holds for all k (see, e.g. Mariolis and Tsoulfidis, 2014, pp. 213-214).

If SSC is chosen as the *numéraire*, i.e. $\mathbf{pz}^{\mathrm{T}} = 1$, where $\mathbf{z}^{\mathrm{T}} = [\mathbf{I} - \mathbf{A}]\mathbf{x}_{\mathbf{A}1}^{\mathrm{T}}$ and $\mathbf{lx}_{\mathbf{A}1}^{\mathrm{T}} = 1$, then the WPR curve is the following linear relation

$$w = 1 - \rho$$

and, if $\rho < 1$,

$$\mathbf{p} = (1-\rho)\mathbf{v}[\mathbf{I} - \rho\mathbf{J}]^{-1} \tag{3}$$

It then follows that $w(0) = 1$, $\mathbf{p}(0) = \mathbf{v}$, $w(1) = 0$, $\mathbf{p}(1) = (\mathbf{y}_{\mathbf{J}1}\mathbf{z}^{\mathrm{T}})^{-1}\mathbf{y}_{\mathbf{J}1}$ and equation (3) gives the production prices as functions of ρ.[5]

The *empirical* results usually give near linear WPR curves as well as virtual quasi-linear price movements; these findings are explained by the shape of the distribution of eigenvalues. More specifically, the eigenvalues of the matrix **J** follow a rectangular hyperbola-like distribution in the case of circulating capital, and a nearly L-shaped form in the case of fixed capital stock. In other words, the stylized facts show that the non-dominant eigenvalues of matrix **J** are by far lower than the Perron-Frobenius eigenvalue. The large gap between the second and dominant eigenvalues of the matrices **J** allow pretty accurate approximations of the PRP trajectories through low order spectral approximations. Such a possibility is implicit in Bródy (1997) and appears for the first time in Bienenfeld (1988), Shaikh (1998), Schefold (2007) and Mariolis and Tsoulfidis (2009). Thus, although the actual matrices **J** that we are dealing with appear to have full rank, the particular distribution of eigenvalues give rise to an effective rank *much lower* than the actual rank. It then follows that even an effective rank (or dimensionality) of one is sufficient to reproduce the virtual PRP and WRP trajectories.

Such findings indicate that the *a priori* vague shapes of the WPR and PPR curves tend to be explained by the particular positions of the system matrix eigenvalues. Consequently, the real paradox, in the sense of knowledge vacuum, is

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[5] If wages are paid *ex ante*, then the WPR curve is non-linear, i.e. $w = (1+R\rho)^{-1}(1-\rho)$, and ρ is no greater than the share of profits in the SSS; however, equation (3) holds true. In the case of fixed capital *à la* Leontief (1953)-Bródy (1970), $\mathbf{H}$ should be replaced by $\mathbf{K}[\mathbf{I}-\mathbf{A}]^{-1}$, where $\mathbf{K}$ denotes the matrix of capital stock coefficients.

not the 'paradoxes in capital theory' but the rectangular hyperbola-like distribution of the eigenvalues to which we turn our attention. Thus, we decompose matrix $\mathbf{J}$ to its 'spectral representation' (see, e.g. Meyer 2001, 517-518):

$$\mathbf{J} = (\mathbf{y}_{\mathbf{J}1}\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}})^{-1}\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}}\mathbf{y}_{\mathbf{J}1} + \sum_{k=2}^{n} \lambda_{\mathbf{J}k}(\mathbf{y}_{\mathbf{J}k}\mathbf{x}_{\mathbf{J}k}^{\mathrm{T}})^{-1}\mathbf{x}_{\mathbf{J}k}^{\mathrm{T}}\mathbf{y}_{\mathbf{J}k} \tag{4}$$

Now if there are strong quasi-linear dependencies amongst the technical conditions of production in all the vertically integrated industries, then $rank[\mathbf{J}] \approx 1$, or $\left|\lambda_{\mathbf{J}k}\right| \approx 0$ for all k, and, therefore, $\mathbf{J} \approx \mathbf{J}^{\mathrm{A}} \equiv (\mathbf{y}_{\mathbf{J}1}\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}})^{-1}\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}}\mathbf{y}_{\mathbf{J}1}$. Hence, from equation (3) it follows that

$$\mathbf{p} \approx \mathbf{p}^{\mathrm{A}} \equiv (1-\rho)\mathbf{p}(0)[\mathbf{I} - \rho\mathbf{J}^{\mathrm{A}}]^{-1}$$

or, by applying the Sherman-Morrison formula,[6]

$$\mathbf{p} \approx \mathbf{p}^{\mathrm{A}} = \mathbf{p}(0) + \rho(\mathbf{p}(1) - \mathbf{p}(0)) \tag{5}$$

This eigenvalue decomposition *rank-one* approximation for the price vector has the following properties:

(i). It is linear and exact at the extreme, economically significant, values of ρ.

(ii). Its accuracy is directly related to the magnitudes of $\left|\lambda_{\mathbf{J}k}\right|^{-1}$.

(iii). When $rank[\mathbf{J}] = 1$, i.e. $\mathbf{J} = \mathbf{J}^{\mathrm{A}}$, it becomes exact for all ρ. Moreover, by the Schur triangularization theorem (see, e.g. Meyer, 2001, pp. 508-509) it follows that $\mathbf{J}^{\mathrm{A}}$ can be transformed, via a semi-positive similarity matrix $\mathbf{T}$, into

$$\tilde{\mathbf{J}} \equiv \mathbf{T}^{-1}\mathbf{J}^{\mathrm{A}}\mathbf{T} = \begin{bmatrix} 1 & \tilde{\mathbf{J}}_{12} \\ \mathbf{0}_{(n-1)\times 1} & \mathbf{0}_{(n-1)\times(n-1)} \end{bmatrix} \tag{6}$$

where the first column of $\mathbf{T}$ is $\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}}$ (the remaining columns are arbitrary), and $\tilde{\mathbf{J}}_{12}$ is a $1\times(n-1)$ positive vector. If, for instance,

$$\mathbf{T} = [\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}}, \mathbf{e}_2^{\mathrm{T}}, ..., \mathbf{e}_n^{\mathrm{T}}] \tag{6a}$$

then

$$\tilde{\mathbf{J}}_{12} = (\mathbf{y}_{\mathbf{J}1}\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}})^{-1}[y_{2\mathbf{J}1}, y_{3\mathbf{J}1}, ..., y_{n\mathbf{J}1}] \tag{6b}$$

That is, the original system is economically equivalent to an $n\times n$ Samuelson-Hicks-Spaventa, or 'corn-tractor', system, even if $\mathbf{J}$ is irreducible (Mariolis, 2013, 2015).

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[6] Let $\boldsymbol{\chi}$, $\boldsymbol{\psi}$ be arbitrary $n-$vectors. Then $\det[\mathbf{I} - \boldsymbol{\chi}^{\mathrm{T}}\boldsymbol{\psi}] = 1 - \boldsymbol{\psi}\boldsymbol{\chi}^{\mathrm{T}}$ and, iff $\boldsymbol{\psi}\boldsymbol{\chi}^{\mathrm{T}} \neq 1$, $[\mathbf{I} - \boldsymbol{\chi}^{\mathrm{T}}\boldsymbol{\psi}]^{-1} = \mathbf{I} + (1 - \boldsymbol{\psi}\boldsymbol{\chi}^{\mathrm{T}})^{-1}\boldsymbol{\chi}^{\mathrm{T}}\boldsymbol{\psi}$ (see, e.g. Meyer 2001, p. 124).

Thus, the first row in the transformed matrix $\tilde{\mathbf{J}}$ represents an industry which can be characterized as 'hyper-basic', and the price system (2) is transformed to

$$\boldsymbol{\pi} = w\boldsymbol{\omega} + \rho\boldsymbol{\pi}\tilde{\mathbf{J}} \tag{7}$$

where $\boldsymbol{\pi} \equiv \mathbf{pT}$, $\boldsymbol{\omega} \equiv \mathbf{vT}$, $\pi_1 = \mathbf{px}_{\mathbf{J}1}^{\mathrm{T}}$ and $\omega_1 = \mathbf{vx}_{\mathbf{J}1}^{\mathrm{T}}$. The first equation in system (7) corresponds to the 'tractor industry', which is no more than the SSS, while the remaining equations correspond to non-uniquely determined 'corn-industries'.

It goes without saying that equation (4) provides the basis for constructing higher-rank approximations.[7]

4. Stylized Facts

In what follows we give empirical content to some of our theoretical findings starting off with the distribution of eigenvalues for a number of major economies. We restrict ourselves to a single year provided that the exponential-like configuration of eigenvalues is pretty much the same for all the economies that have been tested so far (Mariolis and Tsoulfidis, 2016a, Chaps. 5-6, 2016b). We use data from the WIOD (http://www.wiod.org), where the number of industries is not different across countries and the data are compiled with the same methods and they are expressed in dollars thereby facilitating inter-country comparisons (also see Timmer et al., 2015).

Table 1 reports the moduli of the eigenvalues of $\mathbf{J}$ (sorted in descending order) and three metrics of distribution of moduli of the subdominant eigenvalues, namely, (i) the arithmetic mean, *AM*, that assigns equal weight to all moduli; (ii) the geometric mean, *GM*, which assigns more weight to lower moduli, and, therefore, is more appropriate for detecting the central tendency of an exponential set of numbers; (iii) the so-called spectral flatness, *SF*, defined as the ratio of the geometric mean to the arithmetic mean, and shows how spiky or flat is the distribution at hand.

[7] For their relationships with Bienenfeld's (1988) and Steedman's (1999) polynomial approximations, see Mariolis and Tsoulfidis (2016a, Chap. 5).

Table 1: Distribution of the moduli of eigenvalues, USA, China, Germany, India, Australia, Brazil and France, 2011[8]

Eigenvalues Ranking	USA	CHN	DEU	IND	AUS	JPN	BRZ	FRC	Average
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.488	0.409	0.526	0.408	0.359	0.472	0.379	0.437	0.435
3	0.488	0.316	0.399	0.408	0.289	0.472	0.359	0.333	0.383
4	0.429	0.291	0.399	0.264	0.240	0.422	0.311	0.292	0.331
5	0.293	0.247	0.348	0.245	0.212	0.406	0.254	0.257	0.283
6	0.293	0.221	0.260	0.232	0.184	0.272	0.254	0.224	0.243
7	0.235	0.220	0.218	0.232	0.184	0.251	0.230	0.224	0.224
8	0.207	0.220	0.218	0.167	0.154	0.230	0.230	0.221	0.206
9	0.207	0.208	0.209	0.122	0.136	0.214	0.216	0.221	0.192
10	0.147	0.194	0.209	0.122	0.136	0.155	0.216	0.193	0.172
11	0.117	0.099	0.180	0.122	0.115	0.155	0.174	0.184	0.143
12	0.106	0.084	0.180	0.122	0.110	0.121	0.162	0.157	0.130
13	0.105	0.084	0.171	0.096	0.104	0.121	0.103	0.152	0.117
14	0.105	0.067	0.171	0.096	0.104	0.076	0.103	0.152	0.109
15	0.085	0.058	0.150	0.085	0.088	0.063	0.097	0.122	0.093
16	0.085	0.039	0.124	0.063	0.088	0.063	0.050	0.122	0.079
17	0.077	0.039	0.112	0.054	0.067	0.049	0.050	0.109	0.070
18	0.077	0.035	0.112	0.054	0.067	0.037	0.024	0.082	0.061
19	0.051	0.035	0.093	0.028	0.058	0.037	0.018	0.075	0.051
20	0.051	0.025	0.093	0.026	0.045	0.036	0.016	0.068	0.046
21	0.049	0.019	0.091	0.026	0.043	0.036	0.016	0.068	0.044
22	0.049	0.014	0.069	0.014	0.043	0.027	0.014	0.056	0.036
23	0.049	0.014	0.042	0.014	0.039	0.017	0.014	0.042	0.029
24	0.032	0.012	0.034	0.014	0.039	0.017	0.003	0.035	0.023
25	0.032	0.012	0.034	0.009	0.022	0.016	0.003	0.032	0.020
26	0.024	0.010	0.031	0.009	0.020	0.016	0.000	0.024	0.017
27	0.024	0.010	0.031	0.005	0.013	0.012	0.000	0.024	0.015
28	0.023	0.005	0.030	0.001	0.013	0.012	0.000	0.019	0.013
29	0.017	0.005	0.030	0.001	0.011	0.012	0.000	0.008	0.010
30	0.008	0.004	0.025	0.001	0.005	0.009	0.000	0.008	0.008
31	0.005	0.004	0.015	0.000	0.002	0.008	0.000	0.008	0.005
32	0.003	0.003	0.007	0.000	0.002	0.006	0.000	0.002	0.003
33	0.001	0.000	0.005	0.000	0.000	0.006	0.000	0.002	0.002
34	0.001	-	0.005	0.000	0.000	0.000	0.000	0.000	0.001
AM	0.146	0.121	0.165	0.119	0.117	0.143	0.126	0.146	0.135
GM	0.058	0.022	0.085	0.012	0.024	0.048	0.001	0.060	0.052
SF	0.397	0.179	0.517	0.099	0.201	0.339	0.006	0.415	0.382

[8] The last eigenvalues for India, Brazil and also Australia are indistinguishable from zero and this is the reason for their too small geometric means. China's industry 19 (see the Appendix for the nomenclature of industries) contains no data, and thus the number of eigenvalues of this country is 33. As a result, we do not include China in the estimation of the average because of heterogeneity; obviously, by including China or not, the exponential trend of eigenvalues does not change in any appreciable way.

We also report (in the last column of Table 1) the average of all these countries because we find that the differences in the distribution of eigenvalues are minimal and so one exponential function fits them all alike (see Figure 1). In fact, this fit is extremely good and could become nearly perfect, had we eliminated one obvious outlier, that is, the dominant eigenvalue.

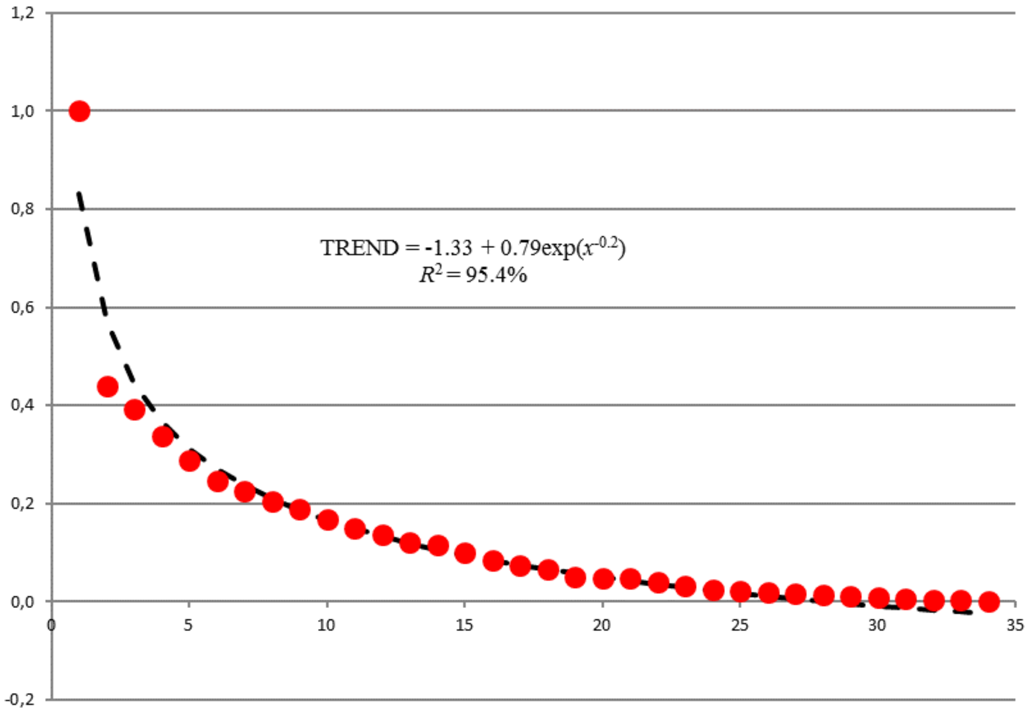


Figure 1. Eigenvalue distribution; average of eight diverse economies

Another aspect of the distribution of eigenvalues of utmost importance is the relation of the second to the first eigenvalue, that is, the so-called spectral ratio. As we know from Bródy's (1997) conjecture, a higher spectral ratio implies a slower attainment of equilibrium in a perturbed economic system and *vice versa*. Namely, once the economy is out of equilibrium it takes longer time for the vectors of prices and/or outputs to attain their equilibrium paths.

Bródy (1997) argued that the larger the system, the faster the convergence because the spectral ratio would diminish. This might be true in case we have randomly generated matrices expanding to infinite size, but in actual economies, it has been repeatedly shown that Bródy's conjecture is not supported by the currently available large size input-output tables (Mariolis and Tsoulfidis, 2011, 2014, 2016a;

Gurgul and Wójtowicz, 2015). Pires and Shaikh (2015) and Shaikh (2016, Chap. 9) have shown that the higher disaggregated the input-output data of the economy (and therefore the higher the dimensions of the matrix of input-output coefficients) the higher the spectral ratio. As a matter of fact in experimenting with US input-output data of the year 2002 they find that the spectral ratio starts from low values and increases in a step wise fashion with the size of the matrix up until the attainment of a certain size past of which the spectral ratio becomes insensitive to further disaggregation.

In what follows, we bring some additional evidence based on recently released data from the US economy that cast doubt on Bródy's conjecture. Thus, in Figure 2 we display the spectral ratios for matrices of different sizes spanning a quite long time period, i.e. from 1995 to 2014. We observe that the spectral ratio increases as we move from lower to higher dimensions input-output matrices, while over the years the same dimension matrices display rising trends indicating increasing instability in the sense of Bródy. Especially, on the basis of the above graph, we can infer that the spectral ratio increased in the US economy in the early 2000s as well as during the years before the Great Recession. These findings prompt further research in this direction so as such a speculation may find more empirical support if not a rigorous theoretical explanation.

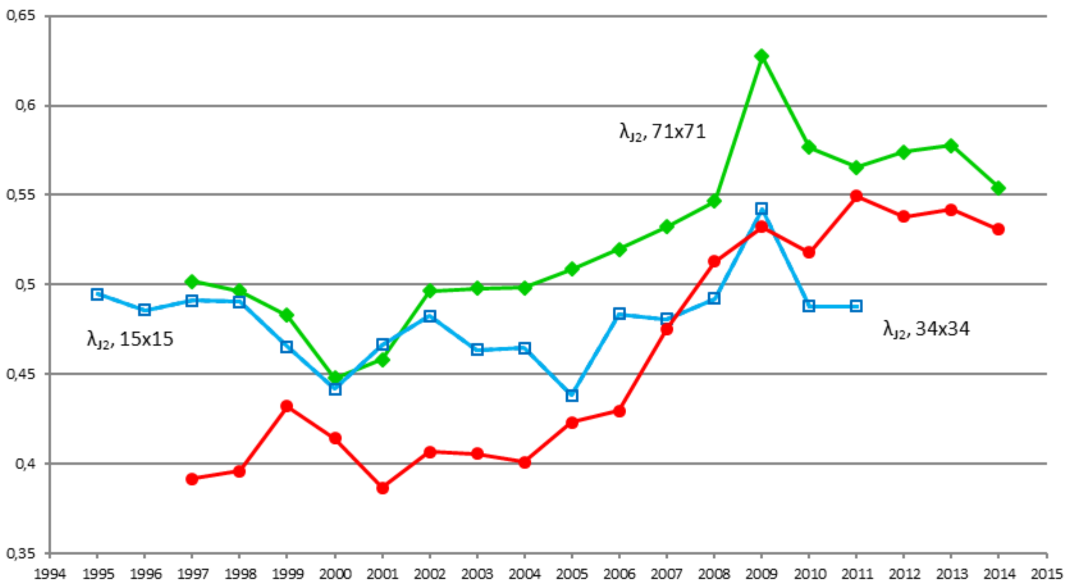


Figure 2. The evolution of the spectral ratio of the normalized vertically integrated technical coefficients matrices; USA 1995-2014

In the two-panel Figure 3, we display the eigenvalues of the matrix **J** spanning the period 1997-2014 and the dimensions are 15 industries. The left panel refers to the period 1997-2005 and the right panel refers to the period 2006-2014. Clearly, the two panels convey pretty much the same picture. In particular, the second eigenvalue remains below the 0.40 borderline with the only exception of the year 2009, where the second eigenvalue is equal to 0.41.

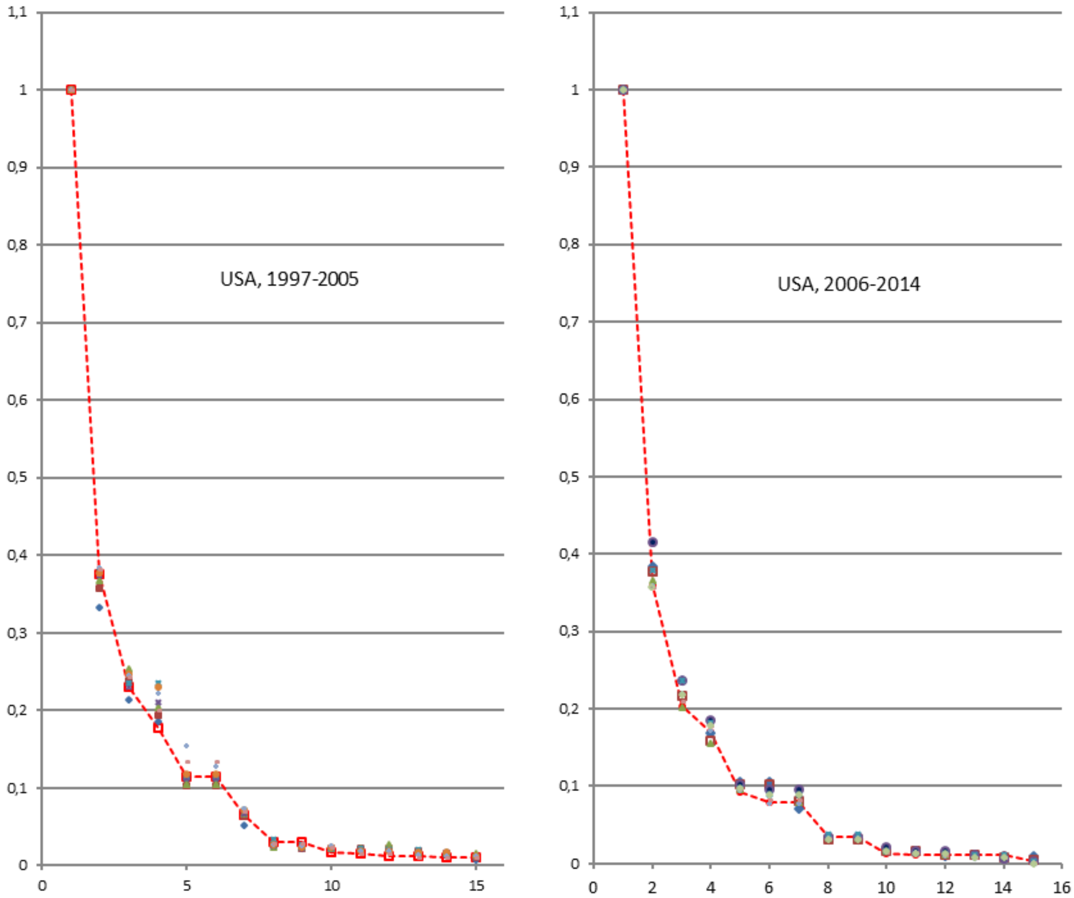
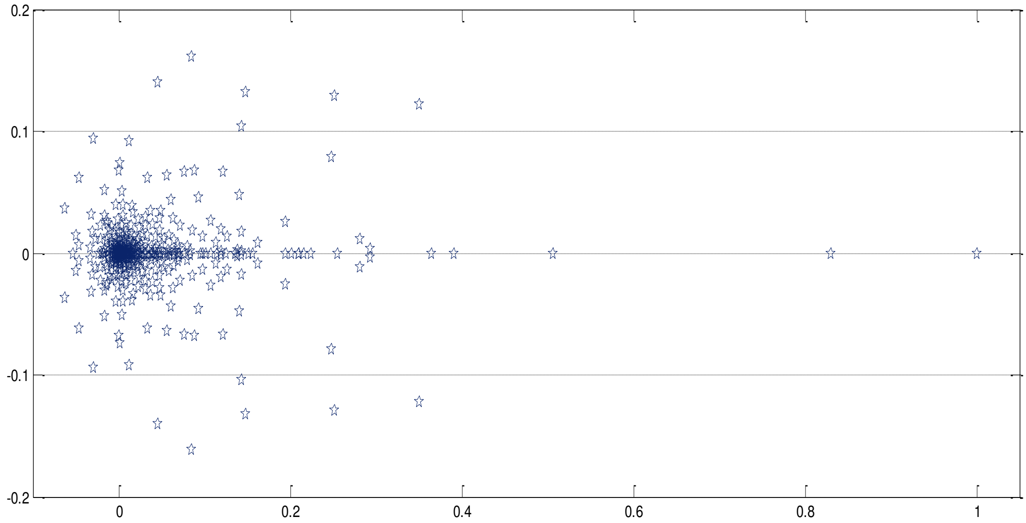


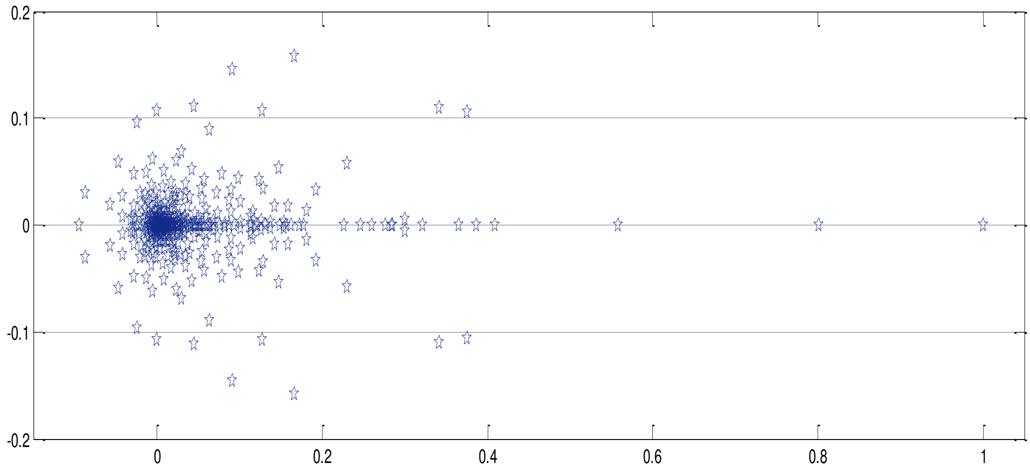
Figure 3. Eigenvalue distributions; USA, 1997-2005 and 2006-1014, 15 industries

The picture remains the same for the much larger in dimensions input-output tables, which have been recently published by the BEA. In the three-parts Figure 4, we display on the top graph the distribution of the eigenvalues of the year 1997 with 488 industries. The middle graph refers to input-output data of the year 2002 with 426 industries.[9] Finally, the last graph is the recently released data of the input-output table of the year 2007 with 389 industries (source: http://www.bea.gov).

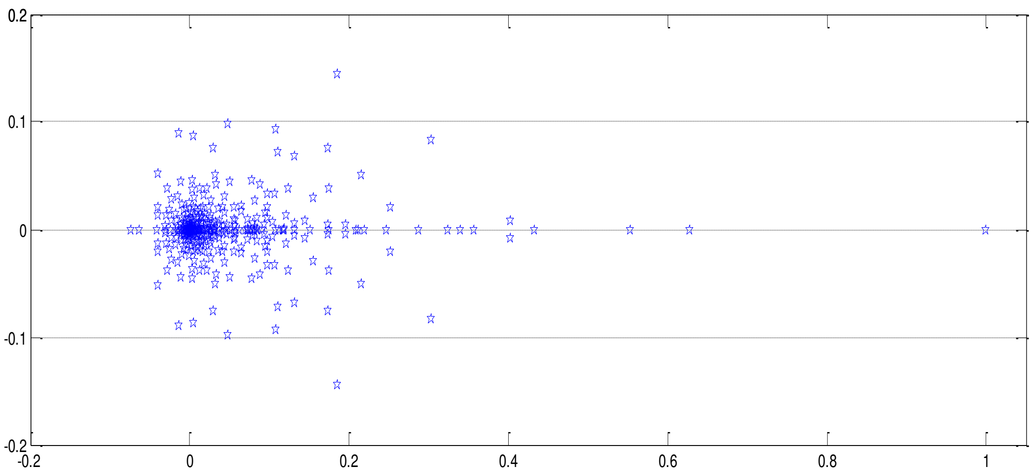
[9] For details on the matrices of the years 1997 and 2002, see Mariolis and Tsoulfidis (2014).



(a) Year 1997; 488 industries



(b) Year 2002; 426 industries



(c) 2007; 389 industries

Figure 4. Eigenvalue distributions of the 'large' matrices; USA, years 1997, 2002 and 2007

In the stylized facts are included the rising of the second and also third eigenvalues with the size of the matrix. For example, in the US economy of the year 2007 (389 industries), the second eigenvalue is 0.61 and the third eigenvalue is 0.58. It is important to note that for the same year the 71 x 71 dimensions matrix gave the second eigenvalue equal to 0.53 and the third eigenvalue equal to 0.47, while the old BEA series of input-output tables of 65 x 65 dimensions gave quite similar numbers: 0.54 for the second and 0.45 for the third eigenvalues. The 34 x 34 dimensions matrix gave the second eigenvalue 0.48 and the third 0.34. Finally, the 15 x 15 industries input-output table was associated with the *smallest* of all second and third eigenvalues of 0.38 and 0.22, respectively. The rest of the eigenvalues constellate at values near zero and the evidence so far has shown that they do not play any significant role in the observed shapes of PPR or WPR curves of the system. From the findings of the very large dimensions input-output tables of the years 1997, 2002 and 2007 we may conclude that the second eigenvalue increases, as we move from lower to higher dimensions input-output tables rendering Bródy's conjecture untenable for actual economies. It is also interesting to note that the imaginary parts of eigenvalues appear only for the very small eigenvalues and the imaginary part when it exists is usually smaller than the real part. In fact, the maximum imaginary part in 2007 does not exceed 0.15 and approximately the same holds for the years 1997 and 2002. This is equivalent to saying that the complex eigenvalues play no perceptible role in the question at hand (however, they may be crucial in other topics; see, e.g. Rodousakis, 2011).

These results along with other similar evidence that we have collected from our research so far prompt the use of smaller dimensions input-output tables for reasons of simplicity and clarity of presentation. It goes without saying that the larger dimensions input-output tables provide a more detailed picture but they do not lead to any different conclusions. Thus, our focus is on the data of the last available input-output table of the US economy in the year 2014 and on the basis of data conveyed we make operational the discussion about the virtual price trajectories in the face of changes in the distributive variable, as this is described by equation (3), provided that for the other years the results are not very different and, therefore, the conclusions are essentially the same. These 15 trajectories of price-labour value ratios are displayed on the top part in the four-panels Figure 5. The capital-intensities are displayed right below the price graphs in the same Figure (for the nomenclature of the 15 industries see Table A-2 in the Appendix).

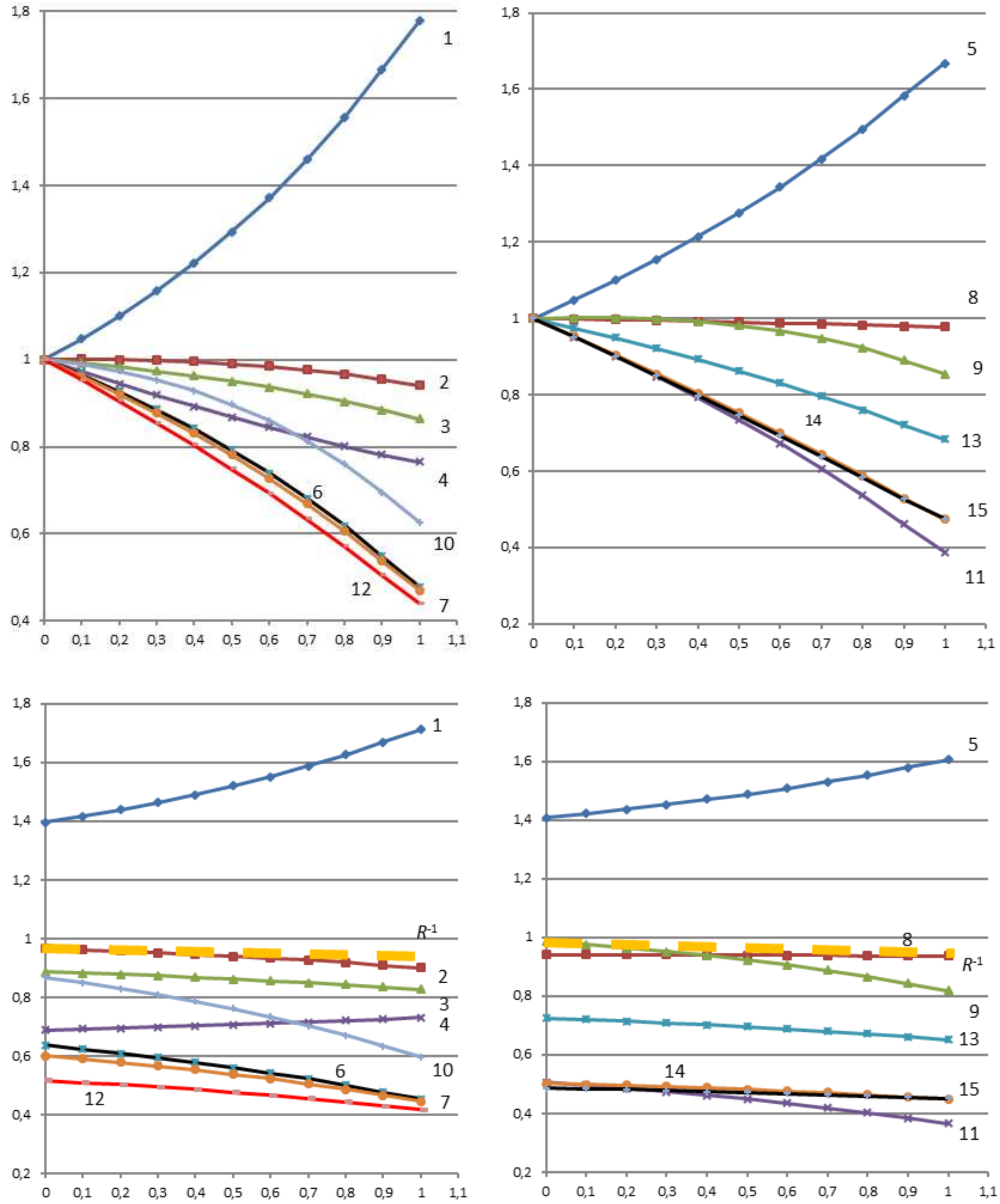


Figure 5. Price-labour value ratios and capital-intensities in the face of changes in the relative profit rate; USA, year 2014, 15 industries

Again in the top two panels in Figure 5, at first sight, the trajectories of prices appear that pretty much move monotonically. On closer examination, however, we find in two out of 15 industries there is 'price-labor value reversals', that is, the industries 2 and 9 cross the line of equality between production prices and labour values. The trajectories of the two prices (l.h.s. panel) and capital intensities (r.h.s. panel) of industries 2 and 9 are repeated in magnification in Figure 6 below. We restricted the relative profit rate up to 30% for reasons of visual clarity.

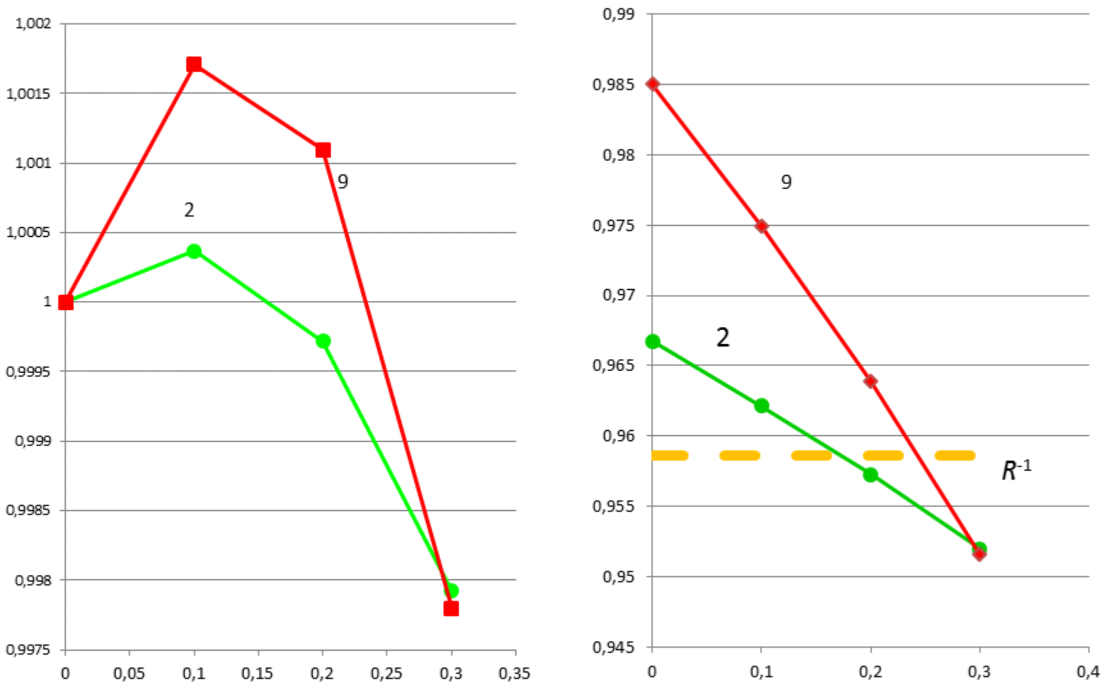


Figure 6. Price-labour value ratios and capital-intensities of vertically integrated industries 2 and 9; USA, year 2014, 15 industries

The results that we derive are in line with our past studies and quite similar to those derived by other researchers. We observe in this particular Figure 6 that the extreme points appear for a 'small' relative rate of profit (10%) and that the line of price-value equality is crossed at a relative profit rate in the range of 15% to 20% (industry 2) or 20% to 25% (industry 9). In both cases, the deviations from the line of price-labour value equality are infinitesimally small, less than 0.17% and 0.15%, respectively and remain extremely small for reasonable relative profit rates, i.e. for empirically estimated values of the share of profits in the net value added (which is usually in the range of 30% to 40%). This means that the capital-intensities of these industries were not so far from the reciprocal of the Standard ratio, R^{-1}, which is equal to 0.959.

These near linear PPR curves prompt us to further study the properties of the structure of the system in line with our previous efforts that a single or a few hyper-basic industries can give rise to very similar results with respect to the movement of prices. For this reason we use the matrix $\mathbf{J}^{\mathrm{A}}$ created from the only positive, and therefore, economically meaningful left and right-hand eigenvectors of matrix $\mathbf{J}$ such that $\mathbf{J} \approx \mathbf{J}^{\mathrm{A}} \equiv (\mathbf{y}_{\mathbf{J}1}\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}})^{-1}\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}}\mathbf{y}_{\mathbf{J}1}$ and we replace it in the price equation (see equation (5)). Since our approximation is linear, the error is maximized at intermediate values of ρ. In our case, the maximal error in the negative direction is 10.28% at ρ = 60%. On the positive direction, the maximal error is 8.87% at ρ = 60%, and the average absolute deviation, without the extreme values of ρ, is 2.67% (see Figure 7).

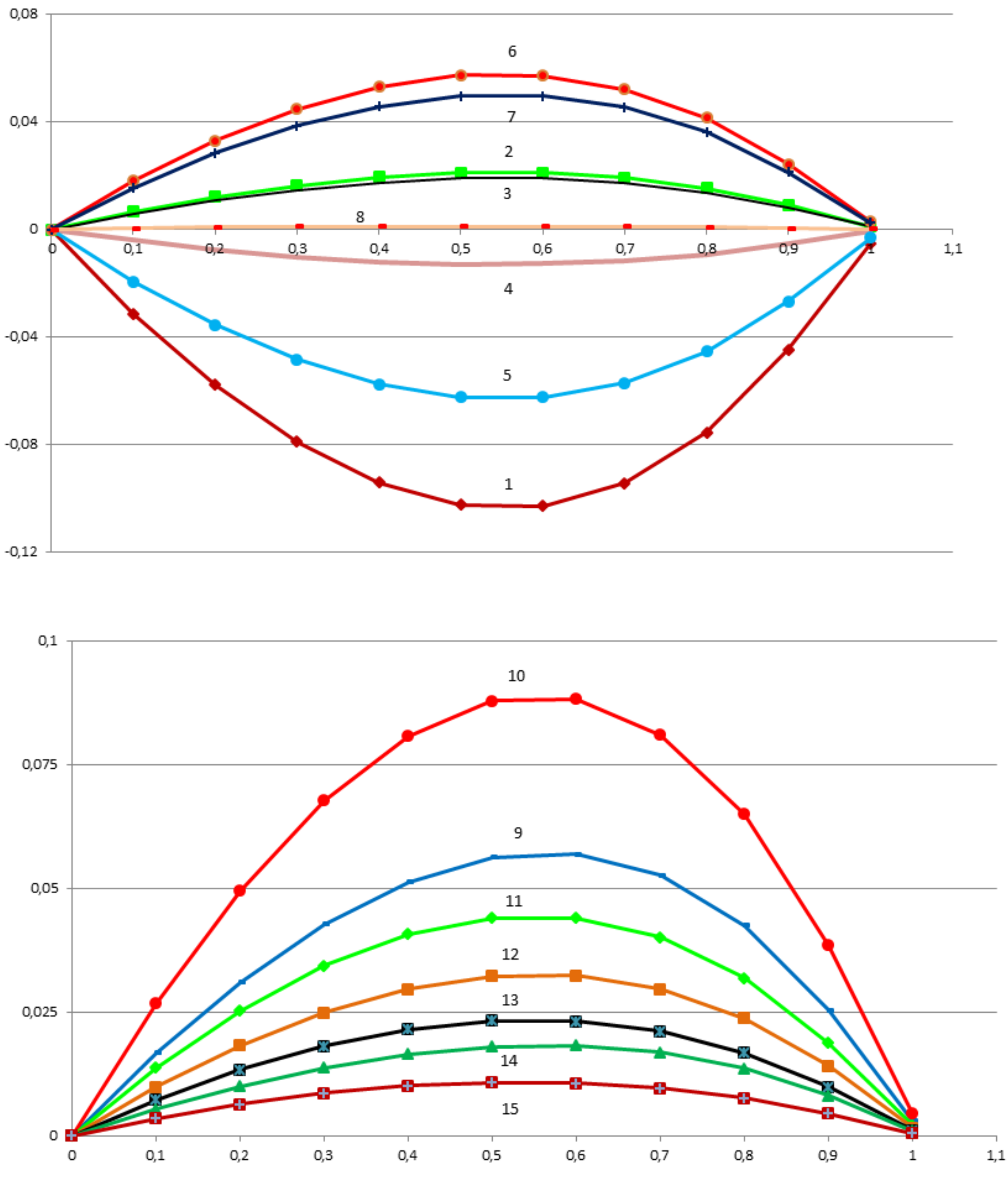


Figure 7. Errors in the linear approximation of prices as functions of the relative profit rate

Having established that $\mathbf{J}^{\mathrm{A}} \equiv (\mathbf{y}_{\mathbf{J}1}\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}})^{-1}\mathbf{x}_{\mathbf{J}1}^{\mathrm{T}}\mathbf{y}_{\mathbf{J}1}$ is a good approximation of $\mathbf{J}$, in the sense that both matrices give rise to price trajectories close to each other, we apply the Schur triangularization theorem to matrix $\mathbf{J}^{\mathrm{A}}$ (see equations (6) and (6a,b)). It then follows that the first row of the semi-positive and rank-1 matrix $\mathbf{T}^{-1}\mathbf{J}^{\mathrm{A}}\mathbf{T}$ is:

1	0.139	0.136	0.230	0.385
0.114	0.123	0.266	0.182	0.097
0.127	0.151	0.191	0.147	0.158

In a similar first row we arrive by applying the Schur triangularization theorem to the original matrix $\mathbf{J}$. This row is:

1	0.124	0.121	0.204	0.343
0.101	0.110	0.237	0.162	0.086
0.113	0.134	0.170	0.131	0.140

We observe that the 'mean absolute percentage deviation' between these two rows is 11.5%, whereas the normalized $d-$distance' (Steedman and Tomkins, 1998; Mariolis and Soklis 2010, p. 94) is 2.3%. It can, therefore, be concluded that the transformed approximate rank-1 matrix $\mathbf{T}^{-1}\mathbf{J}^{\mathrm{A}}\mathbf{T}$ fared extremely well in extracting the essential features embedded in the original-actual system. It goes without saying that due to the nearly L-shaped form of the eigenvalue distribution, such an extraction would be, in general, much more powerful in the presence of fixed capital stock.

5. Concluding Remarks

The input-output data of many diverse economies suggested that the non-dominant eigenvalues of the normalized vertically integrated technical coefficients matrices concentrate at very low values and this means that actual systems can be adequately described by a single or just a few non-Sraffian Standard systems. It follows, therefore, that the price-wage-profit rate relationship tends to be monotonic and the evidence so far suggests that its approximation through low-order formulae (ranging from linear to quadratic) give pretty accurate approximations, which can be improved only marginally by the inclusion of higher order terms.

A salient feature of our analysis is the tendency towards uniformity in the eigenvalue distribution across countries and over time. Such a typical finding could be viewed as a manifestation of technological characteristics embedded in the structure of actual economies, whose further exploration may become the focus of future research efforts. In similar fashion, the movement of the spectral ratio may shed more light onto the stability of the whole economic system. The theoretical and empirical findings indicate that many of issues still lie hidden underneath the surface of capital theory debates and may be discovered through a combination of proper economic theory and use of data derived from the structure of actual economies. In such a direction, it appears that, although a lot is lost by one-commodity world postulations, there is room for using n-by-n corn-tractor models as surrogates for actual single-product systems and that little is gained by considering higher dimensions. In this sense, less is more.

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Appendix

The 34 industries that we utilized in the estimation of the moduli of eigenvalues of the eight countries of our research are listed in Table A-1. The 15 industries input-output structure of the USA is reported in Table A2. For the estimation procedures we refer to Mariolis and Tsoulfidis (2016a) and the literature therein.

Table A-1: Nomenclature of Industries, WIOD base

1	Agriculture, Hunting, Forestry and Fishing	18	Construction
2	Mining and Quarrying	19	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel
3	Food, Beverages and Tobacco	20	Wholesale Trade and Commission Trade, Except of Motor Vehicles & Motorcycles
4	Textiles and Textile Products	21	Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods
5	Leather, Leather and Footwear	22	Hotels and Restaurants
6	Wood and Products of Wood and Cork	23	Inland Transport
7	Pulp, Paper, Paper, Printing and Publishing	24	Water Transport
8	Coke, Refined Petroleum and Nuclear Fuel	25	Air Transport
9	Chemicals and Chemical Products	26	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
10	Rubber and Plastics	27	Post and Telecommunications
11	Other Non-Metallic Mineral	28	Financial Intermediation
12	Basic Metals and Fabricated Metal	29	Real Estate Activities
13	Machinery, Nec	30	Renting of Machines and Equipment and Other Business Activities
14	Electrical and Optical Equipment	31	Public Administration and Defence; Compulsory Social Security
15	Transport Equipment	32	Education
16	Manufacturing, Nec; Recycling	33	Health and Social Work
17	Electricity, Gas and Water Supply	34	Other Community, Social and Personal Services

Table A-2: Nomenclature of 15 Industries, USA economy

1	Agriculture, forestry, fishing, and hunting
2	Mining
3	Utilities
4	Construction
5	Manufacturing
6	Wholesale trade
7	Retail trade
8	Transportation and warehousing
9	Information
10	Finance, insurance, real estate, rental, and leasing
11	Professional and business services
12	Educational services, health care, and social assistance
13	Arts, entertainment, recreation, accommodation, and food services
14	Other services, except government
15	Government