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3 January 2017

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January 2, 2017

Abstract

This paper extends the theory between Kappa ratio and stochastic dominance (SD) and risk-seeking SD (RSD) by establishing several relationships between first- and higher-order risk measures and (higher-order) SD and RSD. We first show the sufficient relationship between the $(n+1)$-order SD and the n-order Kappa ratio. We then find that, in general, the necessary relationship between SD/RSD and the Kappa ratio cannot be established. Thereafter, we find that when the variables being compared belong to the same location-scale family or the same linear combination of location-scale families, we can get the necessary relationships between the $(n+1)$-order SD with the n-order Kappa ratio when we impose some conditions on the means. Our findings enable academics and practitioners to draw better decision in their analysis.

[*]Corresponding author: Wing-Keung Wong; Email: wong@asia.edu.tw. The second author would like to thank Robert B. Miller and Howard E. Thompson for their continuous guidance and encouragement. This research has been partially supported by grants from the Fundamental Research Funds for the Central Universities, China Postdoctoral Science Foundation (2016M600951), Humanities and Social Sciences Planning Fund of Ministry of Education (15YJA910004), Natural Science Foundation of Zhejiang Province (LY15A010006), Asia University, Lingnan University, and the Research Grants Council of Hong Kong, and Ministry of Science and Technology, R.O.C.

KEYWORDS: Stochastic Dominance, Kappa ratio, Omega Ratio, Sortino ratio, mean-risk analysis, risk aversion, risk seeking

JEL Classification: C0, D81, G10

1 Introduction

There are two methods to compare assets performance. One is the mean-risk (MR) analysis and the other is to the stochastic dominance (SD) approach. Readers may read Markowitz (1952a), Sharpe (1966), Leung and Wong (2008), Wong and Ma (2008), Bai *et al.* (2009, 2012) and the references therein for the MR approach, read Hanoch and Levy (1969) and many others for the SD approach for risk averters, and read Hammond (1974), Stoyan (1983), Wong and Li (1999), Li and Wong (1999), Levy (2015), and many others for the risk-seeking SD (RSD).

Is MR model consistent with SD rule? Markowitz (1952b) defines a mean-variance (MV or mean-standard deviation) rule for risk averters and Wong (2007) defines a MV rule for risk seekers. Wong (2007) further establishes consistence of the MV rules with second-order SD (SSD) and second-order RSD (SRSD) rules under some conditions. Ogryczak and Ruszczyński (1999) show that under some conditions the standard semi-deviation and absolute semi-deviation make the mean-risk model consistent with the second-order SD (SSD). Ogryczak and Ruszczyński (2002) establish the equivalence between TVaR and the SSD. In addition, Leitner (2005) shows that AV@R as a profile of risk measures is equivalent to the SSD under certain conditions. Ma and Wong (2010) establish the equivalence between SSD and the C-VaR criteria.

So far, in the literature, academics have studied some relationships between mean-risk models and the second-order SD. Recently, Niu, *et al.* (2016) establish the consistency of a risk measure with respect to first-order SD. Is there any relationship between higher-order risk measure and (higher-order) SD? This paper bridges the gap in the literature to study the issue. We extend the theory between Kappa ratio and stochastic dominance (SD) by establishing relation between higher-order risk measure and (higher-order) SD. We first show the sufficient relationship between the $(n+1)$-order SD and the n-order Kappa ratio. We then find that, in general, the necessary relationship between SD/RSD and the Kappa ratio cannot be established. Thereafter, we find that when the variables being compared belong to the same location-scale family or the same linear combination of location-scale families, we can get the necessary relationships between the $(n+1)$-order SD with the n-order Kappa ratio when we impose some conditions on the means. Our findings enable academics and practitioners to draw better decision in their analysis.

2 Definitions and Notations

We first define risk-averse and risk-seeking investors as follows: For any integer j, $U_j = \{u : (-1)^i u^{(i)} \leq 0\ ,\ i = 1, \cdots, j\}$ and $U_j^R = \{u : u^{(i)} \geq 0\ ,\ i = 1, \cdots, j\}$ are sets of utility functions in which $u^{(i)}$ is the i^{th} derivative of the utility function u. We call investors the j-order risk averters and the j-order risk seekers if their utility functions $u \in U_j$ and U_j^R, respectively. We note that the theory can be easily extended to non-differentiable utility functions. Readers may refer to Wong and Ma (2008) and the references therein for more information.

For any integer j, we now define the j-order integral, $F_Z^{(j)}$, and the j-order reverse integral, $F_Z^{(j)R}$, of Z to be

$$F_Z^{(j)}(\eta) = \int_{-\infty}^{\eta} F_Z^{(j-1)}(\xi) d\xi \quad \text{and} \quad F_Z^{(j)R}(\eta) = \int_{\eta}^{\infty} F_Z^{(j-1)R}(\xi) d\xi\ , \tag{2.1}$$

respectively, with $F_Z^{(0)R} = F_Z^{(0)} = f_Z$ to be the probability density function (pdf) of Z for $Z = X, Y$. When $j = 1$, $F_Z^{(1)} = F_Z$ is the cumulative distribution function (cdf) of Z.

Following the definition of stochastic dominance (SD), see, for example, Hanoch and Levy (1969), prospect X first-order stochastically dominates prospect Y, denoted by

$$X \succeq_{FSD} Y \quad \text{if and only if} \quad F_X^{(1)}(\eta) \leq F_Y^{(1)}(\eta) \quad \text{for any} \quad \eta \in R, \tag{2.2}$$

and prospect X n-order stochastically dominates prospect Y, denoted by

$$X \succeq_{nSD} Y \ \text{ if and only if } \ F_X^{(n)}(\eta) \leq F_Y^{(n)}(\eta) \text{ for any } \eta \in R, \text{ and } \ F_X^{(k)}(\infty) \leq F_Y^{(k)}(\infty) \tag{2.3}$$

with $2 \leq k \leq n$. Here, FSD and nSD stands for first- and n-order stochastic dominance. For $n = 2$, 2SD can also be written as SSD (second-order SD).

We also follow Li and Wong (1999), Wong and Li (1999), Wong (2007), Levy (2015), Guo and Wong (2016), and others to define risk-seeking stochastic dominance (RSD)[1] for risk seekers.

[1]Levy (2015) denotes it as RSSD while we denote it as RSD.

Prospect X second-order risk-seeking stochastically dominates prospect Y, denoted by

$$X \succeq_{SRSD} Y \quad \text{if and only if} \quad F_X^{(2)R}(\eta) \geq F_Y^{(2)R}(\eta) \quad \text{for any} \quad \eta \in R. \tag{2.4}$$

Here, SRSD or 2RSD denotes second-order RSD.

We note that

$$\text{if} \;\; X \succeq_{nSD} Y \;\; \text{or} \;\; X \succeq_{nRSD} Y \quad \text{for any} \;\; n \geq 1 \;, \quad \text{then} \quad \mu_X \geq \mu_Y \;. \tag{2.5}$$

This property will be used in the proofs of the theorems we developed in our paper.

3 The Theory

Shadwick and Keating (2002) first introduce Omega Ratio, $\Omega_X(\eta)$. We rewrite it as follows:

$$\Omega_X(\eta) = \frac{\int_\eta^\infty (1 - F_X(\xi))d\xi}{\int_{-\infty}^\eta F_X(\xi)d\xi} = \frac{F_X^{(2)}(\eta) - (\eta - \mu_X)}{F_X^{(2)}(\eta)} = 1 + \frac{\mu_X - \eta}{F_X^{(2)}(\eta)}. \tag{3.1}$$

Kaplan and Knowles (2004) first develop Kappa ratio:

$$K_X^{(n)}(\eta) = \frac{\mu_X - \eta}{(E[(\eta - X)_+^n])^{1/n}}. \tag{3.2}$$

In this paper, we will develop properties for the $(n+1)$-order SD with the Kappa ratio with subscript n. As far as we know, our paper is the first paper establishing the relationships between high-order risk measure with high-order SD in details. Thus, we call the Kappa ratio in Equation (3.2) the n-order Kappa ratio. Since

$$F_X^{(n+1)}(\eta) = \int_{-\infty}^\eta F_X^{(n)}(x)dx = \frac{1}{n!}E[(\eta - X)_+^n],$$

the n-order Kappa ratio can be expressed as:

$$K_X^{(n)}(\eta) = \frac{\mu_X - \eta}{(n!F_X^{(n+1)}(\eta))^{1/n}}. \tag{3.3}$$

We first extend Darsinos and Satchell (2004) by developing the following result to state the sufficient relationship between $(n+1)$-order SD with the n-order Kappa ratio:

Theorem 3.1 *For any two returns X and Y with means, μ_X and μ_Y, and n-order Kappa ratios, $K_X^{(n)}(\eta)$ and $K_Y^{(n)}(\eta)$, respectively, and for any $n \geq 1$, if $X \succeq_{(n+1)SD} Y$, then $K_X^{(n)}(\eta) \geq K_Y^{(n)}(\eta)$ for any $\eta \leq \mu_X$.*

Here, we give a short proof for Theorem 3.1: for any $n \geq 1$, if $X \succeq_{(n+1)SD} Y$, we have $F_X^{(n+1)}(\eta) \leq F_Y^{(n+1)}(\eta)$ and $\mu_X \geq \mu_Y$. For $\eta \leq \mu_X$, it follows that $\mu_X - \eta \geq 0$. Then, we get:

$$\frac{\mu_X - \eta}{(n!F_X^{(n+1)}(\eta))^{1/n}} \geq \frac{\mu_X - \eta}{(n!F_Y^{(n+1)}(\eta))^{1/n}} = \frac{\mu_Y - \eta}{(n!F_Y^{(n+1)}(\eta))^{1/n}} + \frac{\mu_X - \mu_Y}{(n!F_Y^{(n+1)}(\eta))^{1/n}} \geq \frac{\mu_Y - \eta}{(n!F_Y^{(n+1)}(\eta))^{1/n}}, \tag{3.4}$$

and thus, the assertions in Theorem 3.1 holds. □

We note that the first-order Kappa ratio can represent the Omega ratio because when $n = 1$, $K_X^{(1)}(\eta) = \Omega_X(\eta) - 1$. Thus, from Theorem 3.1, we obtain the following corollary:

Corollary 3.1 *For any two returns X and Y with means, μ_X and μ_Y, and Omega ratios, $\Omega_X(\eta)$ and $\Omega_Y(\eta)$, respectively, if $X \succeq_{2SD} Y$, then $\Omega_X(\eta) \geq \Omega_Y(\eta)$ for any $\eta \leq \mu_X$.*

We note that Guo, *et al.* (2016) have established Corollary 3.1. We also note that the second-order Kappa ratio can represent the Sortino ratio because when $n = 2$, $K_X^{(2)}(\eta) = S_X(\eta)$, in which $S_X(\eta)$ is the Sortino ratio (Sortino and van der Meer, 1991) of X. Thus, from Theorem 3.1, we obtain the following corollary:

Corollary 3.2 *For any two returns X and Y with means, μ_X and μ_Y, and Sortino ratios, $S_X(\eta)$ and $S_Y(\eta)$, respectively, if $X \succeq_{3SD} Y$, then $S_X(\eta) \geq S_Y(\eta)$ for any $\eta \leq \mu_X$.*

In sum, we find that the preference of second-order stochastic dominance implies the preference of the corresponding Omega ratios and the preference of third-order stochastic dominance implies the preference of the corresponding Sortino ratios only when the return is less than the mean of the higher-return asset.

We note that Darsinos and Satchell (2004) have proved that $(n+1)$-order SD "implies" (n-order) Kappa dominance. For example, they show that second-order SD "implies" Omega dominance while third-order SD "implies" Sortino dominance. However, in their analysis, they have not taken into consideration the sign of the term $\mu_X - \eta$. For $\eta > \mu_X$, the dominance relationship cannot be asserted. Actually, for $\eta > \mu_X$, $\mu_X - \eta < 0$, and thus, we have

$$\frac{\mu_X - \eta}{(n!F_X^{(n+1)}(\eta))^{1/n}} \leq \frac{\mu_X - \eta}{(n!F_Y^{(n+1)}(\eta))^{1/n}} = \frac{\mu_Y - \eta}{(n!F_Y^{(n+1)}(\eta))^{1/n}} + \frac{\mu_X - \mu_Y}{(n!F_Y^{(n+1)}(\eta))^{1/n}} \geq \frac{\mu_Y - \eta}{(n!F_Y^{(n+1)}(\eta))^{1/n}}.$$

Consequently, we cannot determine the sign of $K_{n,X}(\eta) - K_{n,Y}(\eta)$. Guo, *et al.* (2016) have given the explicit description of the case when $n = 1$. When $n = 2$ Corollary 3.2 gives the condition $\eta \leq \mu_X$. With this condition, the third-order SD does imply the Sortino dominance.

We turn to study the necessary relationship between SD and Kappa ratio. We first obtain the following property:

Property 3.1 *In general, the necessary relationship between* $(n+1)$*-order SD with the* n*-order Kappa ratio cannot be established.*

However, in some special cases, for example, in a special family of distributions like the location-scale family, we can get the necessary relationship between the SD and RSD with the first- and higher-order Kappa ratio as shown in the following theorem:

Theorem 3.2 *For any two returns* X *and* Y *that belong to the same location-scale family or same linear combination of location-scale families with means,* μ_X *and* μ_Y*, and* n*-order Kappa ratios,* $K_X^{(n)}(\eta)$ *and* $K_Y^{(n)}(\eta)$*, respectively, we have*

1. *if* $\mu_X > \mu_Y$ *and*
   (a) *if there exists at least one* η *satisfying* $\eta \geq \mu_X$ *such that* $K_X^{(n)}(\eta) \leq K_Y^{(n)}(\eta)$ *for* $n = 1, 2$*, then* $E\left[u(X)\right] \geq E\left[u(Y)\right]$ *for any risk-averse investor with utility function* $u \in U_k$ *for any* $k \geq 2$*; and*
   (b) *if there exists at least one* η *satisfying* $\mu_Y \geq \eta$ *such that* $K_X^{(n)}(\eta) \leq K_Y^{(n)}(\eta)$ *for any* $n \geq 1$*, then* $E\left[u(X)\right] \geq E\left[u(Y)\right]$ *for any risk-seeking investor with utility function* $u \in U_k^R$ *for any* $k \geq 2$*; and*
2. *if* $\mu_X = \mu_Y = \mu$ *and*
   (a) *if there exists at least one* η *satisfying* $\mu \geq \eta$ *such that* $K_X^{(n)}(\eta) \geq K_Y^{(n)}(\eta)$ *for any* $n \geq 1$*, then* $E\left[u(X)\right] \geq E\left[u(Y)\right]$ *for any risk-averse investor with utility function* $u \in U_k$ *for any* $k \geq 2$*; and*
   (b) *if there exists at least one* η *satisfying* $\eta \geq \mu$ *such that* $K_X^{(n)}(\eta) \geq K_Y^{(n)}(\eta)$ *for* $n = 1, 2$*, then* $E\left[u(X)\right] \geq E\left[u(Y)\right]$ *for any risk-seeking investor with utility function* $u \in U_k^R$ *for any* $k \geq 2$*.*

4 Concluding Remarks

This paper extends the theory between Kappa ratio and stochastic dominance (SD) and risk-seeking SD (RSD) by establishing several relationships between first- and higher-order risk measures and (higher-order) SD and RSD. We first show the sufficient relationship between the $(n+1)$-order SD and the n-order Kappa ratio. We then find that, in general, the necessary relationship between SD/RSD and the Kappa ratio cannot be established. Thereafter, we find that when the variables being compared belong to the same location-scale family or the same linear combination of location-scale families, we can get the necessary relationships between the $(n+1)$-order SD with the n-order Kappa ratio when we impose some conditions on the means.

Some academics and practitioners only use the MR approach and some only use the SD approach in their analysis. For example, Bai, *et al.* (20013) apply the MR approach to compare the performance of Commodity Trading Advisors while Fong, *et al.* (2005) only apply the SD test to examine the momentum effect in stock returns, Fong, *et al.* (2008) apply SD test to study the preference of different types of investors on internet stocks verses "old economy" stocks, and Chan, *et al.* (2012) apply the SD approach to examine the efficiency of the UK covered warrants market. Nonetheless, many academics and practitioners have been using both MR and SD approaches to analyze some important financial and economic issues. For example, applying both MR and SD approaches, Hoang, *et al.* (2015) find that, in general, risk-averse investors prefer not to include gold while risk-seeking investors prefer to include it in their stock–bond portfolios, especially in crisis periods while Clark, *et al.* (2016) find that risk averters prefer spot to futures, risk seekers prefer futures to spot. Investors with S-shaped utility functions prefer spot (futures) to futures (spot) when markets move upward (downward), and investors with reverse S-shaped utility functions prefer futures (spot) to spot (futures) when markets move upward (downward). Nonetheless, most, if not all the studies that applying both MR and SD to analyze real data examine only second-order MR and second-order SD.

There are some work using higher-order SD relationships in real analysis. For example, Gasbarro, *et al.* (2007) find third order SD preference among iShares while Vinod (2004) apply the fourth-order stochastic dominance to compare mutual funds. We note that recently Bai, *et al.* (2015) develop SD test for both risk averters and risk seekers up to the third order. Once could easily extend their work to develop high-order SD test. There are also some work suggesting to use higher-order moments in real analysis. For example, Beedles (1979), Levy (1969), and many others suggest that investors prefer positive third moment. Brockett and Garven (1998) examine the relationship between risk, return, skewness, and utility-based preferences and show that ceteris paribus analysis of preferences and moments, as occasionally used in the literature, is impossible since equality of higher-order central moments implies the total equality of the distributions involved. However, Brockett and Kahane (1992) consider choice between individual projects and show that when the choice set includes arbitrary distributions, then any assumed relationship between expected utility theory and general moment preferences for individual decision makers is theoretically unsound. In particular, a risk averse investor with any common utility function may, when choosing between two positive return opportunities, prefer the project simultaneously having a lower mean, higher variance, and lower positive skewness. Thus, we can conclude that higher-order SD and higher-order moments are important in real analysis. This could imply that high-order mean-risk measures should be useful in real-data analysis but as far as we know, there is very few work studying the issue if there is any. This paper bridges the gap to study the relationship higher-order risk measures and (higher-order) SD and RSD should be useful to academics and practitioners in their analysis and assist them to draw better decision in their analysis.

References

Bai, Z.D., Hui, Y.C., Wong, W.K., Zitikis, R., 2012, Evaluating Prospect Performance: Making a Case for a Non-Asymptotic UMPU Test, Journal of Financial Econometrics 10(4), 703-732.

Bai, Z.D., Lui, H.X., Wong, W.K., (2009), Enhancement of the Applicability of Markowitz's Portfolio Optimization by Utilizing Random Matrix Theory, Mathematical Finance, 19(4), 639-667.

Bai, Z.D., Li, H., McAleer, M., Wong, W.K. (2015). Stochastic dominance statistics for risk averters and risk seekers: an analysis of stock preferences for USA and China. Quantitative Finance 15(5), 889-900.

Bai, Z.D., Hui, Y.C., Wong, W.K., Zitikis, R., 2012, Evaluating Prospect Performance: Making a Case for a Non-Asymptotic UMPU Test, Journal of Financial Econometrics 10(4), 703-732.

Bai, Z.D., Phoon, K.F., Wang, K.Y., Wong, W.K., 2013, The Performance of Commodity Trading Advisors: A Mean-Variance-Ratio Test Approach, North American Journal of Economics and Finance, 25, 188-201.

Beedles, W.L. 1979. Return, Dispersion, and Skewness: Synthesis and Investment Strategy, Journal of Financial Research 2, 71-80.

Brockett, P.L., Garven, J.R., 1998, A Reexamination of the Relationship Between Preferences and Moment Orderings by Rational Risk-Averse Investors, GENEVA Papers on Risk and Insurance Theory, 23(2), 127-137.

Brockett, P.L., Kahane, Y. 1992, Risk, Return, Skewness and Preference, Management Science, 38(6), 851-866.

Chan, C.Y., de Peretti, C., Qiao, Z., Wong, W.K., 2012, Empirical test of the efficiency of the UK covered warrants market: Stochastic dominance and likelihood ratio test approach, Journal of Empirical Finance 19(1), 162-174.

Clark, E., Qiao, Z., Wong, W.K. 2016, Theories of Risk: Testing Investor Behaviour on the Taiwan Stock and Stock Index Futures Markets, Economic Inquiry 54(2), 907-924.

Darsinos, T., Satchell, S., 2004. Generalizing universal performance measures. Risk (June), 80-84.

Fong, W.M., Wong, W.K., Lean, H.H., (2005), International momentum strategies: a stochastic dominance, Journal of Financial Markets, 8, 89-109.

Fong, W.M., Wong, W.K., Lean, H.H., (2005), International momentum strategies: a stochastic dominance, Journal of Financial Markets, 8, 89-109.

Gasbarro, D., Wong, W.K., Zumwalt, J.K. (2007), Stochastic Dominance Analysis of iShares, European Journal of Finance 13, 89-101.

Guo, B. and Xiao, Y. G. 2016. A note on why doesn't the choice of performance measure matter? Finance Research Letters 16, 248-254.

Guo, X., Jiang, X.J., Wong, W.K. 2016. A note on Stochastic Dominance and the Omega Ratio. Social Science Research Network Working Paper Series 2827058

Guo, X., Wong, W.K. (2016). Multivariate Stochastic Dominance for Risk Averters and Risk Seekers. RAIRO - Operations Research 50(3), 575-586.

Hammond, J. S. (1974). Simplifying the choice between uncertain prospects where preference is nonlinear, Management Science 20(7), 1047-1072.

Hanoch G., Levy, H. (1969). The Efficiency Analysis of Choices Involving Risk. Review of Economic studies 36, 335-346.

Hoang, T.H.V., Wong, W.K., Zhu, Z.Z., 2015, Is gold different for risk-averse and risk-seeking investors? An empirical analysis of the Shanghai Gold Exchange, Economic Modelling 50, 200-211.

Kaplan, P.D. , Knowles, J.A. , 2004. Kappa: A generalized downside risk-adjusted performance measure. J. Perform. Meas. 8(3), 42-54.

Leitner, J. (2005). A short note on second-order stochastic dominance preserving coherent risk measures. Mathematical Finance 15(4), 649-651.

Leung, P.L., Wong, W.K., 2008, On testing the equality of the multiple Sharpe Ratios, with application on the evaluation of iShares, Journal of Risk, 10(3), 1-16.

Levy, H. (2015). Stochastic Dominance: Investment Decision Making Under Uncertainty. Third Edition, Springer, New York.

Li, C.K., Wong, W.K., 1999, Extension of stochastic dominance theory to random variables, RAIRO - Operations Research 33(4), 509-524.

Ma, C., Wong, W.K. (2010) Stochastic Dominance and Risk Measure: A Decision-Theoretic Foundation for VaR and C-VaR, European Journal of Operational Research 207(2), 927-935.

Markowitz, H.M. (1952a). Portfolio Selection, Journal of Finance. **7** 77-91.

Markowitz, H.M. (1952b). The utility of wealth, Journal of Political Economy 60, 151-156.

Niu, C.Z., Wong, W.K., Zhu, L.X. 2016. First Stochastic Dominance and Risk Measurement, MPRA Paper 75027, University Library of Munich, Germany.

Ogryczak, W., Ruszczyński, R. (1999). From stochastic dominance to mean-risk models: semideviations as risk measures. European Journal of Operational Research 116, 33-50.

Ogryczak, W., Ruszczyński, A. (2002). Dual stochastic dominance and related mean-risk models. SIAM Journal of Optimization 13, 60-78.

Schuhmacher, F., Eling, M. (2012). A decision-theoretic foundation for reward-to-risk performance measures. Journal of Banking Finance 36(7), 2077-2082.

Shadwick, W.F., Keating, C. , 2002. A universal performance measure. J. Perform. Meas. 6(3), 59-84.

Sharpe, W.F., 1966, Mutual Funds Performance. Journal of Business 39, 119-138.

Sortino, F.A., van der Meer, R., 1991. Downside risk. Journal of Portfolio Management 17, 27-31.

Sriboonchitta, S., Wong, W.K., Dhompongsa, S., Nguyen, H.T. 2009. SD and Applications to Finance, Risk & Economics, Taylor and Francis Group, USA.

Stoyan, D., Daley, D.J. (1983). Comparison methods for queues and other stochastic models, John Wiley & Sons, New York, USA.

Vinod, H.D., 2004. Ranking mutual funds using unconventional utility theory and stochastic dominance, Journal of Empirical Finance, 11(3), 353-377.

Wong, W.K. 2007. Stochastic dominance and mean-variance measures of profit and loss for business planning and investment, European Journal of Operational Research 182, 829-843.

Wong, W.K., Li, C.K., 1999. A Note on Convex Stochastic Dominance Theory, Economics Letters 62, 293-300.

Wong, W.K., Ma, C. (2008). Preferences over Location-scale Family, Economic Theory 37, 119-146.