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Batabyal, Amitrajeet and Nijkamp, Peter

Department of Economics, Rochester Institute of Technology, Tinbergen Institute

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by

Amitrajeet A. Batabyal[2]

and

Peter Nijkamp[3]

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[2] Department of Economics, Rochester Institute of Technology, 92 Lomb Memorial Drive, Rochester, NY 14623-5604, USA. Internet aabgsh@rit.edu

[3] Department of Spatial Economics, VU University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands and A. Mickiewicz University, Poznan, Poland. Internet p.nijkamp@vu.nl

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Abstract

We study whether pessimism and optimism about the future by forward looking agents provides a rationale for social security. We first distinguish between an agent's true and pessimistic preferences and then analyze whether this agent's level of saving depends on the pessimism parameter (π) and how welfare measured by the agent's true preferences depends on π. Next, we examine whether it is possible for pessimism to increase the agent's true utility and then show that this kind of pessimism does not provide a rationale for social security. Moving on to optimism, we study the need for a pay-as-you-go (PAYG) social security system when the agent is optimistic about the generosity of the PAYG system. This optimism is modeled with a parameter (ω). In this setting, we first study the impact that an increase in ω has on the agent's saving and then examine whether this agent's welfare increases or decreases in ω. Finally, we show that this kind of optimism does not justify the presence of the PAYG social security system.

Keywords: Optimism, Overlapping Generations, Pessimism, Saving, Social Security

JEL Codes: H55, R28

1. Introduction

1.1. Overview

The question of whether and how best to provide social security (pensions)[4] to retired citizens is one that has greatly concerned governments in different regions of the world. As noted by Hindriks and Myles (2013, p. 775), this concern arises in part because "the issues that are raised with pensions are the potential transfers of resources between generations and the effect on savings behavior in the economy." In addition, retired persons in many developed countries—such as the American Association of Retired Persons (AARP) in the United States—have formed lobbying groups that actively seek to promote their welfare. Given the practical significance of social security, economists have analyzed the provision of pensions and the reform of pension systems from a variety of perspectives. Therefore, there is now a vast literature on this subject. We now briefly summarize the more recent literature to give the reader a sense for some of the issues that researchers have grappled with.[5]

Focusing on the Netherlands, Van Sonsbeek (2010) studies how the ageing of the population affects the sustainability of public finances. He first shows that under certain conditions, the pension costs of the state rise less slowly than the number of pensioners and then examines the budgetary and labor participation effects of five pension reform policies. Raab (2011) studies the pay-as-you go (PAYG) pension system in Austria and demonstrates that the *behavioral* responses of pensioners to financial incentives are not only large but also differentiated by gender.

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The term "social security" is often used to refer to pensions and to the provision of other benefits such as unemployment and health benefits. Even so, we shall use the terms "social security" and "pensions" interchangeably in this paper.

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See Kotlikoff (1987), Feldstein and Leibman (2002), Sabelhaus and Topoleski (2007), Hindriks and Myles (2013, pp. 775-807), and the many references in these sources for a detailed perspective on the literature on social security.

Behaghel and Blau (2012) study one way to reform social security in the United States (US) by varying the so called full retirement age. The key finding emanating from their study is that retirement behavior in the US is characterized by the presence of *framing* effects. Boersch-Supan (2013) concentrates on the challenges faced by the governments of nations with ageing populations and points to the salience of what he calls "negative *behavioral* effects" in pension design. Specifically, he contends that such effects can be minimized by debunking false beliefs about ageing and, at the same time, providing better information about the chances and the challenges associated with the process of ageing.

It is well known in the US that claiming social security benefits before reaching the full retirement age can permanently lower benefits. As such, Knoll and Olsen (2014) show that a delay in claiming benefits leads to larger increases in both monthly and lifetime benefits than what would be possible with an incentive scheme that alters the early retirement reductions. Reyers *et al.* (2015) use South African data and show that *behavioral* factors play an important role in predicting retirement behavior. Specifically, these researchers contend that behavioral factors related to bounded rationality and computational complexity in the decision-making environment influence the decision to either access or not access accumulated retirement funds when changing jobs.

1.2. A key lesson and this paper's contribution

An important lesson emerging from the above review of the recent literature on pensions is that *behavioral* factors have a substantial impact on decision-making by agents. In a recent paper, Hollanders (2015) draws on the prior research of Cronqvist and Thaler (2004), Benartzi and Thaler (2007), and Willis (2011) and credibly supports this perspective. He convincingly argues that because choices by agents are affected by framing, because agents do not have much financial knowledge, and because agents do not choose actively, discussions about social security and social security reform by policymakers ought to be informed by the findings of behavioral economics.

Having said this, it should be clear to the reader that a variety of behavioral factors can influence decision-making by agents in the context of social security. Therefore, to fix ideas and to point to the contribution of this paper, we focus on the notion of *myopia*. As pointed out initially by Feldstein (1985) and more recently by Andersen and Bhattacharya (2011), myopia or shortsightedness in decision-making by forward looking agents can provide a rationale for the existence of social security. The logic behind this result is as follows. Left to their own devices and because they are shortsighted, a substantial number of agents will save inadequately. Therefore, in such a situation, a paternalistic government can increase the welfare of such shortsighted agents by instituting a PAYG pension system.

We adapt the theoretical framework in Andersen and Bhattacharya (2011) and in Hindriks and Myles (2013, pp. 781-807) and use a simple, two-period overlapping generations model—on which more below in section 2—to study two types of deviations from perfect foresight that have, to the best of our knowledge, not been studied adequately in the extant literature. The first type of deviation we study is the myopia or shortsightedness that leads to the undervaluation of future consumption. We refer to this type of deviation as *pessimism*. The logic here is that young agents do not adequately think about the twin facts that they will *age* and that their *health* will decline in the future because such thinking makes them pessimistic about their future. The second type of deviation we analyze is the overvaluation of the benefits from an existing PAYG pension system and we refer to this type of deviation as *optimism*. In this case, the logic is that young agents think they will be richer in the future—possibly because of ongoing economic growth—and hence they are unrealistically optimistic about the generosity of the PAYG pension system.

The remainder of this paper is organized as follows. Section 2 delineates the two-period overlapping generations theoretical framework. Section 3 analyzes the case of pessimism in detail and then answers the question as to whether pessimism provides a rationale for social security. Section 4 examines the case of optimism at length and then sheds light on the question as to whether optimism provides a justification for PAYG social security. Section 5 concludes and then offers two suggestions for extending the research described in this paper.

2. The Theoretical Framework

Our framework involves working with a standard overlapping generations model. Since this model is well known, in what follows, we shall be brief.[6] Consider an economy in which time is discrete and the length of the unit time interval is equal to the time between the birth of one generation and the birth of the next generation. At the beginning of each period a new generation of young agents is born. Each agent lives for two time periods. Generation t consists of the set of agents who are born at the beginning of period t. The population in this economy at any time t consists of young and old agents.

Each agent in our economy works only during the first time period of his or her life and inelastically supplies one unit of labor. We can think of this unit of labor as the agent's endowment. Given this interpretation, the total quantity of labor in our economy is equal to the total number of young agents. In the second period of their lives, agents are retired and hence they supply no labor. Retired agents live off the savings they accumulated when young and working. The income earned

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This model has been used routinely in the macroeconomics and public finance literatures. For textbook expositions of the model, see Hindriks and Myles (2013, pp. 753-773) and Romer (2012, pp. 77-100).

by a young agent in the first period of his or her life is divided between consumption and savings. Second period consumption or consumption while retired is equal to savings plus the interest earned on these savings. All agents are identical with the single exception that they are born at different points in time.

For an agent born in time period t, c_t^t denotes consumption when young and c_t^{t+1} denotes consumption when old and retired. We assume that this agent incurs no disutility from the supply of the single unit of labor when young. In addition, for an agent born in period t, let s_t denote his or her savings and let i_t denote the income received from the supply of the single unit of labor. Therefore, this agent's budget constraint can be written as

$$c_t^t + s_t = i_t. \tag{1}$$

Let r_{t+1} denote the interest rate at which savings grow in our economy. This tells us that the value of consumption to an agent when (s)he is retired is given by

$$(1 + r_{t+1})s_t = c_t^{t+1}. \tag{2}$$

Now, combining equations (1) and (2), the life-cycle budget constraint for our agent born in period t is given by

$$c_t^t + \frac{c_t^{t+1}}{1 + r_{t+1}} = i_t. \tag{3}$$

With this background in place, we are now in a position to study the phenomenon of pessimism about the future and whether pessimism of this sort provides a rationale for social security.

3. Pessimism

3.1. Preliminaries

Consider an agent in our economy whose *true* preferences can be described by a utility function that has the following Cobb-Douglas form

$$U^T = \{c_t^t\}^{\beta}\{c_t^{t+1}\}^{1-\beta}, \tag{4}$$

where the parameter $\beta \in (0,1)$. As discussed in section 1.2, this agent does not have perfect foresight but displays myopia or shortsightedness about the future, i.e., his or her life as a retired person. This myopia or deviation from perfect foresight leads our agent, when young, to undervalue his or her consumption when old. We refer to this type of deviation as *pessimism*.

Why is this agent pessimistic about the future? We consider two reasons. First, we model the idea that our young agent either does not or cannot adequately think about the fact that (s)he will *age* and hence get old one day . To this end, let $\alpha \in (0,1)$ denote the ageing parameter. Second, we also account for the notion that our young agent, once again, either does not or cannot satisfactorily think about the fact that his or her *health* will decline when (s)he is old and retired. We denote the health parameter by $\gamma \in (0,1)$. The twin facts of ageing and health decline make our agent pessimistic about the future and this explains why our agent undervalues his or her future consumption.

The next question concerns the precise manner in which the ageing and the health concerns lead our agent to undervalue his or her future consumption. We consider two cases. In the first case, these twin concerns lead to *additive* discounting of second period consumption c_t^{t+1} so that this consumption effectively becomes $\pi^a c_t^{t+1}$ where the additive pessimism parameter $\pi^a = \alpha + \gamma$ and we need $\pi^a \in (0,1)$. In the second case, the above two concerns lead to *multiplicative* discounting of second period consumption c_t^{t+1} so that this consumption effectively becomes $\pi^m c_t^{t+1}$ where the multiplicative pessimism parameter $\pi^m = \alpha\gamma$ and it is clear that $\pi^m \in (0,1)$.

Now, as it turns out, our subsequent qualitative results are unaffected by whether the discounting of second period consumption is additive or multiplicative. Therefore, in the remainder of this section, we dispense with the a and the m superscripts on the pessimism parameter and work with a single pessimism parameter $\pi \in (0,1)$. This means that the discounted or undervalued second period consumption is given by πc_t^{t+1}. Given this undervaluation of second period consumption, we can write our agent's *pessimistic* preferences with a utility function that also has the Cobb-Douglas form and is given by

$$U^P = \{c_t^t\}^{\beta} \{\pi c_t^{t+1}\}^{1-\beta}. \tag{5}$$

Our task now is to determine how the agent's level of saving s_t is impacted by the pessimism parameter π.

3.2. Pessimism and saving

To determine this impact, we solve our agent's constrained utility maximization problem, recognizing that the appropriate utility function is the one depicting this agent's pessimistic preferences. Specifically, this agent solves

$$\max_{\{c_t^t, c_t^{t+1}\}} \{c_t^t\}^{\beta} \{\pi c_t^{t+1}\}^{1-\beta} \tag{6}$$

subject to the budget and value of consumption constraints given in equations (1) and (2) respectively. We shall set the value of i_t in equation (1) equal to unity and the reader should note that this is without any loss of generality. The Lagrangian function for the above maximization problem can be written as

$$\mathcal{L} = (c_t^{\,t})^{\beta}(s_t)^{1-\beta}(\pi + \pi r_{t+1})^{1-\beta} + \lambda(1 - c_t^{\,t} - s_t), \tag{7}$$

where λ is the Lagrange multiplier on the budget constraint given in equation (1).

The first order necessary conditions for an optimum are

$$\frac{\partial \mathcal{L}}{\partial c_t^{\,t}} = \beta(c_t^{\,t})^{\beta-1}(s_t)^{1-\beta}(\pi + \pi r_{t+1})^{1-\beta} - \lambda = 0, \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial s_t} = (c_t^{\,t})^{\beta}(1-\beta)(s_t)^{-\beta}(\pi + \pi r_{t+1})^{1-\beta} - \lambda = 0, \tag{9}$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 1 - c_t^{\,t} - s_t = 0. \tag{10}$$

Manipulating equations (8), (9), and (10), we get

$$c_t^t = \beta \text{ and } s_t = 1 - \beta. \tag{11}$$

Inspecting equation (11), it is clear that the level of saving when young or s_t is independent of the pessimism parameter π. Therefore, we conclude that pessimism about the future does *not* impact our agent's level of saving. We now analyze the way in which this agent's level of welfare measured by his or her true preferences is affected by the pessimism parameter π.

3.3. Pessimism and welfare

Inspecting equation (4), we see that our agent's true preferences are independent of the pessimism parameter π. From this it clearly follows that pessimism about the future has *no* effect on this agent's level of welfare as measured by his or her true preferences.

Now suppose that the economy under study consists of a total number of A agents and that these agents all act in accordance with their pessimistic preferences. Further, suppose that the equilibrium interest rate in our economy is given by $r_{t+1} = j - ks_t$ where s_t is the aggregate level of savings in our economy and (j,k) are positive constants. Can pessimism about the future ever increase the true utilities of all the agents in this economy? We answer this question next.

3.4. Pessimism and true utilities

We begin by reiterating that the pessimism about the future that we are studying in this paper does not alter the saving decision of an individual agent. Therefore, it follows that the aggregate level of savings in our economy is also *unaffected* by pessimism. Looked at a little differently, in solving their individual utility maximization problems, agents treat the interest rate as exogenous to their decision-making. As such, each agent selects his or her own level of saving without considering how the aggregate level of savings influences the equilibrium interest rate.

Now, from our analysis thus far in sections 3.1 to 3.3, we can write the representative agent's true preferences as

$$U^{T} = \beta^{\beta}\{(1-\beta)(1+r_{t+1})\}^{1-\beta}. \tag{12}$$

We also know that the equilibrium interest rate is given by $r_{t+1} = j - ks_t$ and that the aggregate level of savings equals $As_t = A(1-\beta)$. Substituting these last two expressions in equation (12) and then simplifying, we get

$$U^{T} = \beta^{\beta}[(1-\beta)\{1+j-kA(1-\beta)\}]^{1-\beta}. \tag{13}$$

Inspecting equation (13), it is clear that the true utilities of the A agents in our economy do not depend on the pessimism parameter π. Therefore, it follows that pessimism about the future can *never* increase the true utilities of these agents. Our final task in this section is to answer the following important question. Does pessimism about the future provide a rationale for social security?

3.5. Pessimism and social security

We begin our answer to the above question by first pointing to two salient results from the literature. The relatively early literature on this topic, beginning with the seminal work of Feldstein (1985), has shown that myopia alone can provide a rationale for a PAYG social security system but the provision of this kind of social security reduces and can even crowd out private saving. More recently, Andersen and Bhattacharya (2011, p. 137) have demonstrated that with myopia, "if private saving is positive...the optimal public pension is zero."

Now, from the discussion in section 3.4 we know that pessimism about the future does not affect the representative agent's saving decision, the aggregate level of savings in our economy, and it can also never raise the true utilities of any of the agents in our economy. We also know that the aggregate level of savings equals $As_t = A(1-\beta) > 0.$ These results together provide additional support for the finding in Andersen and Bhattacharya (2011) mentioned in the preceding paragraph and tell us that pessimism about the future does *not* provide a rationale for social security in the economy under consideration. We now proceed to analyze the phenomenon of optimism and whether optimism of a particular sort provides a justification for social security.

4. Optimism

4.1. Preliminaries

Consider an economy of the sort studied above in section 3. Once again, consider an agent whose true (non-optimistic) preferences can be described by a utility function that has the following natural logarithmic form

$$U = \log_e\{c_t^t\} + \log_e\{c_t^{t+1}\}. \tag{14}$$

We suppose that a PAYG pension system is presently in place in this economy. However, in contrast with the pessimism displayed by the agents in section 3, now, possibly because of economic growth, agents believe that they will be richer in the future and therefore they are *optimistic* about this future.

We model this optimism by supposing that agents *overvalue* the benefit from the existing PAYG pension system and believe that the actual pension ζ and the social security tax τ are linked by the linear functional relationship

$$\zeta = (1+\omega)\tau, \tag{15}$$

where the optimism parameter $\omega > 0.$[7] As in section 3, we abstract away from population growth and

[7] The case we are studying is the opposite of the "pension pessimism" case first studied by Feldstein (1985).

hence the true value of the pension $\zeta = \tau$. With the PAYG pension system in place, the budget constraint for an agent displaying no optimism is given by

$$c_t^{\,t} + \frac{c_t^{\,t+1}}{1+r_{t+1}} = i_t + \frac{\zeta}{1+r_{t+1}} - \tau, \tag{16}$$

where the left-hand-side (LHS) denotes consumption over the agent's lifetime and the right-hand-side (RHS) denotes net income over the same lifetime. Given this setting, our task now is to determine the impact that an increase in the optimism parameter ω has on savings in the economy under study.

4.2. Optimism and saving

To ascertain this impact, we solve our agent's constrained utility maximization problem. Specifically, this agent solves

$$\max_{\{c_t^{\,t}, c_t^{\,t+1}\}} \log_e\{c_t^{\,t}\} + \log_e\{c_t^{\,t+1}\}, \tag{17}$$

subject to the budget constraint given in equation (16). The Lagrangian function for this maximization problem is

$$\mathfrak{L} = \log_e\{c_t^{\,t}\} + \log_e\{c_t^{\,t+1}\} + \lambda\left[i_t + \frac{\zeta}{1+r_{t+1}} - \tau - c_t^{\,t} - \frac{c_t^{\,t+1}}{1+r_{t+1}}\right], \tag{18}$$

where λ is the Lagrange multiplier on the budget constraint given in equation (16).

The first order necessary conditions for an optimum are

$$\frac{\partial \mathcal{L}}{\partial c_t^t} = \frac{1}{c_t^t} - \lambda = 0, \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial c_t^{t+1}} = \frac{1}{c_t^{t+1}} - \frac{\lambda}{1 + r_{t+1}} = 0, \tag{20}$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda} = i_t + \frac{\zeta}{1 + r_{t+1}} - \tau - c_t^t - \frac{c_t^{t+1}}{1 + r_{t+1}} = 0. \tag{21}$$

Manipulating equations (19) and (20), we get

$$\frac{c_t^{t+1}}{c_t^t} = 1 + r_{t+1}. \tag{22}$$

Using equations (16) and (22), we get

$$c_t^{\,t}=\frac{1}{2}\{i_t+\frac{\zeta}{1+r_{t+1}}-\tau\}. \tag{23}$$

Because $s_t=i_t-c_t^{\,t}-\tau$, using equation (23), it follows that

$$s_t=i_t-c_t^{\,t}-\tau=\frac{1}{2}\{i_t-\frac{\zeta}{1+r_{t+1}}-\tau\}. \tag{24}$$

From the discussion in section 4.1, it is clear that the actual value of the pension $\zeta=\tau$. However, optimistic agents believe that the value of the pension is given by equation (15). Therefore, the saving decision of optimistic agents is given by substituting from equation (15) into equation (24). This gives us

$$s_t=\frac{1}{2}\{i_t-\frac{(1+\omega)\tau}{1+r_{t+1}}-\tau\}. \tag{25}$$

We can now determine the impact of an increase in the optimism parameter ω on savings by partially differentiating equation (25) with respect to ω. This gives us

$$\frac{\partial s_t}{\partial\omega}=-\frac{1}{2}\frac{\tau}{1+r_{t+1}}<0. \tag{26}$$

The sign of the partial derivative in (26) tells us that, consistent with our intuition, when agents in our economy become more optimistic about the future, they end up saving *less* for this same future. What happens to the welfare of an agent when (s)he becomes more optimistic—ω increases—about the future? We now answer this question.

4.3. Optimism and welfare

Our answer involves ascertaining the consumption trajectory and the lifetime utility of an agent who expects to receive a pension $\zeta=(1+\omega)\tau$ but, in fact, receives only $\zeta=\tau$. We know that our agent's non-optimistic consumption in the first period of his or her life is given by equation (23). Therefore, substituting from equation (15) into (23), the optimistic agent's first period consumption is given by

$$c_t^{\,t}=\frac{1}{2}\{i_t+\frac{(1+\omega)\tau}{1+r_{t+1}}-\tau\}. \tag{27}$$

Similarly, this agent's consumption when old and retired is given by

$$c_t^{t+1}=(1+r_{t+1})s_t+\zeta=\frac{1}{2}\{(i_t-\tau)(1+r_{t+1})-(1+\omega)\tau\}+\tau. \tag{28}$$

Having obtained analytic expressions for our agent's first and second period consumption, let us now partially differentiate this agent's utility function. This gives us

$$\frac{\partial U}{\partial \omega}=\frac{\partial U}{\partial c_t^{\,t}}\frac{\partial c_t^{\,t}}{\partial \omega}+\frac{\partial U}{\partial c_t^{\,t+1}}\frac{\partial c_t^{\,t+1}}{\partial \omega}. \tag{29}$$

Straightforward differentiation tells us that $(\partial U/\partial c_t^{\,t})(\partial c_t^{\,t}/\partial\omega)=(1/2)(1/c_t^{\,t})\{\tau/(1+r_{t+1})\}$ and that $(\partial U/\partial c_t^{\,t+1})(\partial c_t^{\,t+1}/\partial\omega)=-(1/2)(\tau/c_t^{\,t+1})$. Substituting these last two expressions in the RHS of equation (29) and recalling that $c_t^{\,t}(1+r_{t+1})/c_t^{\,t+1}>1$ for an optimistic agent, we get

$$\frac{\partial U}{\partial \omega}=\frac{1}{2}\frac{1}{c_t^{\,t}}\frac{\tau}{1+r_{t+1}}\{1-\frac{c_t^{\,t}}{c_t^{\,t+1}}(1+r_{t+1})\}<0. \tag{30}$$

In words, the result in (30) tells us that an increase in optimism by our agent or, alternately, an increase in the overvaluation of the pension system's payout *lowers* this agent's welfare. This result arises because an increase in the optimism parameter ω unrealistically raises the gap between the amount that our agent will actually receive from the PAYG pension system and the amount that (s)he thinks (s)he will receive. Our final task in this section is to shed light on the question about whether optimism about the future or the overvaluation of the second period payout from the existing PAYG social security system provides a justification for the continued existence of this system.

4.4. Optimism and social security

The welfare of the optimistic agent when there is no social security program in place is given by

$$U^{no} = \log_e\{\frac{1}{2}i_t\} + \log_e\{\frac{1}{2}i_t(1+r_{t+1})\}. \tag{31}$$

Using equations (27) and (28), the welfare of this same agent in the presence of the PAYG social security system is given by

$$U^{yes} = \log_e[\frac{1}{2}\{i_t - \tau + \frac{(1+\omega)\tau}{1+r_{t+1}}\}] + \log_e[\frac{1}{2}\{(i_t-\tau)(1+r_{t+1}) - (1+\omega)\tau\} + \tau]. \tag{32}$$

It is clear that the PAYG pension system ought to be retained in the economy under study if it raises the welfare of an agent in this economy. For this to happen, we must have $U^{yes} - U^{no} > 0$. Using equations (31) and (32), this difference is given by

$$U^{yes} - U^{no} = \log_e[\frac{1}{2}\{i_t - \tau + \frac{(1+\omega)\tau}{1+r_{t+1}}\}] + \log_e[\frac{1}{2}\{(i_t-\tau)(1+r_{t+1}) - (1+\omega)\tau\} + \tau] -$$

$$[\log_e\{\frac{1}{2}i_t\} + \log_e\{\frac{1}{2}i_t(1+r_{t+1})\}]. \tag{33}$$

After several steps of algebra, equation (33) can be simplified to

$$U^{yes} - U^{no} = \log_e [\frac{\{(i_t - \tau)(1 + r_{t+1}) + \tau\}^2 - (\omega\tau)^2}{\{i_t(1 + r_{t+1})\}^2}]. \quad (34)$$

Using the properties of the logarithm function and equation (34), we can tell that the difference $U^{yes} - U^{no}$ is positive when

$$\frac{\{(i_t - \tau)(1 + r_{t+1}) + \tau\}^2 - (\omega\tau)^2}{\{i_t(1 + r_{t+1})\}^2} > 1. \quad (35)$$

The inequality in (35) can be simplified to

$$-\tau r_{t+1}\{2i_t + (2i_t - \tau) r_{t+1}\} > (\omega\tau)^2. \quad (36)$$

Now, from equation (24) we know that $s_t = i_t - c_t^t - \tau$ and therefore $s_t + c_t^t = i_t - \tau$. Clearly, $s_t + c_t^t > 0$ and this tells us that $i_t - \tau > 0$ and hence that $2i_t - \tau > 0$. Using this last inequality, it is obvious that the LHS of the inequality in (36) is negative. However, the RHS of the inequality in (36) is positive. From these last two results, it follows that the inequality in (36) is never satisfied. Therefore, the difference $U^{yes} - U^{no}$ is *never* positive and this allows us to conclude that when our agent displays optimism of the sort analyzed in this section, the PAYG social security system ought *not* to be retained. This completes our analysis of pessimism and optimism by forward looking agents and the need for social security.

5. Conclusions

In this paper, we studied whether pessimism and optimism about the future by forward looking agents provided a rationale for social security. We first distinguished between an agent's true and pessimistic preferences and then analyzed whether this agent's level of saving depended on the pessimism parameter (π) and how welfare measured by the agent's true preferences depended on π. Next, we examined whether it was possible for pessimism to increase the agent's true utility and then showed that this kind of pessimism did *not* provide a rationale for social security. Moving on to optimism, we studied the need for a PAYG social security system when the agent was optimistic about the future generosity of the PAYG system. This optimism was modeled with a parameter (ω). In this setting, we first studied the impact that an increase in ω had on the agent's saving and then examined whether this agent's welfare increased or decreased in ω. Finally, we showed that this kind of optimism did *not* justify the presence of the PAYG social security system.

The analysis in this paper can be extended in a number of directions. In what follows, we suggest two possible extensions of this paper's research. First, it would be useful to analyze the rationale for social security question when this question is set in a context in which pessimism about the future affects the saving rate of individual agents in an economy. Second, it would be helpful to introduce population growth into the model and then study whether optimism of the sort analyzed in this paper provides a rationale either for a PAYG pension system or a fully funded social security system. Studies of pessimism and optimism and the provision of social security that incorporate these aspects of the problem into the analysis will provide additional insights into an intergenerational resource transfer problem that has significant economic and social ramifications.

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