Competitive Search Equilibrium in the Credit Market under Asymmetric Information and Limited Commitment

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Abstract

This paper develops a model of a competitive search credit market under hidden information and limited commitment. Using the model, it provides a theoretical account that links time delays and costs in financial intermediation as well as lack of collateral to the distribution of credit supply and interest rate spreads. The link sheds light on and explains the possibility of pure credit rationing due to the credit frictions. This paper also demonstrates the possibility of contract dispersion among homogeneous borrowers.

Keywords : credit frictions, competitive search, contract, market tightness
JEL classification : E51, E43, D82, G21

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1 Introduction

It has been long noticed that asymmetric information and limited commitment are key frictions, which should be considered seriously to understand credit market performance. Thus many papers have investigated the intensive margin in credit supply, i.e., the terms of loan agreements, through the lens of the literature on contracting under asymmetric information. However they have paid little attention to the extensive margin, i.e., the probability that a potential borrower gets a loan. In existing works based on contract theory, if necessary, the extensive margin has been modeled by a randomized contracting strategy together with a contractual term that indicates the probability with which the other terms are offered. This scheme introducing the artificial term into the intensive margin might be considered as a reduced-form approach to modelling the extensive margin because it is not explicit about costs and benefits of extending credit supply.

This paper develops a model of a competitive search credit market under hidden information and limited commitment. This model is explicit about the extensive margin while using the standard bilateral contracting framework for the intensive margin, and hence it allows us to see how two margins jointly channel the influences of the credit frictions. More specifically, introducing the friction in competitive search theory, this paper extends the work by Besanko and Thakor (1987) on a competitive credit market under hidden information. The equilibrium concept proposed by Guerrieri, Shimer, and Wright (2010) is used for this.

In the model of this paper, lending is carried out through banks, which are principals that design credit policies, or mechanisms offering credit contracts. Each bank can serve many borrowers at once, but it costs to increase the maximum number of borrowers it can serve. In order to introduce this capacity constraint associated with bilateral contracting, the notion of vacancy in search-theoretic labor market models is adopted, and bilateral matching of a borrower and a vacancy is assumed. This bilateral matching itself does not necessarily mean a search friction. However here the credit market is assumed to have a search friction, which makes both sides of the market left unmatched, though the sources of the friction might be different from those in a typical decentralized market. Banks with vacancies publicly and credibly announce their credit policies, advertising the vacancies. They wait for applicants to visit because they cannot locate potential borrowers. A potential borrowers observes all the policies available and choose for which one to apply. Because it takes time and resources to access and process loan application, he cannot approach more than a few banks, or a fraction of the vacancies, within a given period of time. This imperfect matching technology leads to the search friction in spirit same to Moen (1997).

In the competitive search credit market, banks compete with each other, and free entry of vacancies leads to zero profits as in the large literature on competitive credit market and competitive banking. Though this might be an extreme assumption, it affords decisive advantages in tractability to consider an equilibrium in which banks take possible responses of other banks into consideration. It might be noticeable that a competitive search equilibrium can be considered as a possible solution to the Rothschild and Stiglitz (1976) nonexistence problem. Moreover the assumption allows us to focus on the roles for asymmetric information because a competitive search market under full information yields the first best allocation. Once a search friction exists, any other market structure, for example, a typical search market with bargaining, generically has an inefficient allocation even under full information. Thus it looks essential to investigate a competitive search equilibrium allocation first before investigating allocations under alternative market structures.

Results of this paper exhibit the possibility of endogenous credit rationing in the extensive margin, i.e., pure Type II credit rationing. This does not indicate the possibility that a potential borrowers fails to receive credit simply due to the search friction, or an unmodeled characteristic. Given imperfect matching technology and capacity expansion costs, banks in the competitive search market would supply the socially optimal levels of credit if there were no informational frictions. However, under hidden information, the supply of credit in it may be not efficient because the terms of a contract themselves affect the riskiness of the loan by sorting potential borrowers. A key finding is the possibility that potential borrowers are rationed because banks charge interests more than the efficient levels for the purpose of screening. This is a feature distinguished from the results of existing works on a competitive credit market under hidden information because here credit rationing is not due to lack of screening but rather a result of screening.[1]

The results also demonstrate the possibility that unproductive banking as well as lack of collateral makes the credit market tight, extending interest rate spreads. This possibility might be important for understanding the occurrence of a credit crunch. For example, when banks have more difficulties in finding qualified borrowers, the tightness of credit market may be endogenously amplified. The possibility also helps to understand small enterprises' limited access to bank finance, simply assuming that the entire credit market consists of many submarkets separated by observable characteristics of firms.[2] For example, it explains why a credit market in which firms with less collateralable wealth participate is more tight and has larger interest rate spreads.

Another key finding is the possibility of contract dispersion among homogenous borrowers of the same type. This possibility arises under a standard environment, which would yield no dispersion if there were no search frictions, or if information were symmetric. This sheds light on and explains the possibility that sunspots affect the distribution of credit supply as well as the possibility that imperfect banking enhances interest rate dispersion.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 defines equilibrium and introduces a way to characterize it. Section 4 characterizes equilibrium and investigates the results. Section 4 gives concluding remarks. All proofs of the lemmas and the propositions are in the Appendix.

[1] Though some of Stiglitz-Weiss papers (e.g., Stiglitz and Weiss, 1981) demonstrates possibility of pure credit rationing under hidden information, this rationing is a pooling or semi-pooling equilibrium phenomenon. As pointed out by Bester (1985), the possibility is due to lack of screening, and it disappears if banks have a screening device that yields a separating equilibrium. Introducing the search friction into a standard environment, this paper provides an account for a separating equilibrium with pure credit rationing.

[2] There are many empirical studies about this limited access, in which main sources of banks' reluctance to extend credit to small enterprises has been discussed (e.g., Green, 2003). Their reluctance is considered as mainly associated with high administrative costs of small-scale lending, asymmetric information, small firms' lack of collateral, the high risk perception attributed to small enterprises, and the underdeveloped financial system. This paper provides a model that allows to see how these sources work together.

2 Model

The model incorporates the friction in competitive search theory into a standard one-period credit market model with hidden information. For easier comparison, the standard part follows Besanko and Thakor (1987), hereafter BT. There is a continuum of potential borrowers with total mass normalized to one, and a large number of ex ante homogenous banks compete in the supply of loans. Both borrowers and banks are risk neutral.

Each borrower owns a project and has a type $\delta \in (0,1]$. If a type δ borrower invests an initial outlay of q, his project yields a gross return of $f(q,\delta)$ with probability δ and zero with probability $1-\delta$. The function $f : \mathbb{R}_+ \times [0,1] \mapsto \mathbb{R}_+$ is twice continuously differentiable, $f_q > 0$, and $f_{qq} < 0$ with $f(0,\delta) = 0$, $\lim_{q\to 0} f_q(q,\delta) = \infty$ and $\lim_{q\to\infty} f_q(q,\delta) = 0$ for every δ. It is also assumed that $f_\delta > 0$ and $f_{q\delta} > 0$. The distribution of borrower types is concentrated on $D \equiv [\delta_0, \delta_I]$ with a continuous probability density function $g : D \mapsto \mathbb{R}_{++} \equiv (0,\infty)$. In addition to income that can be earned by investment, all the borrowers have a deterministic and identical end-of-period wealth κ.[3] However, this wealth is illiquid, and hence a borrower who wants to finance investment must approach a bank for a loan. All of f, g and κ are common knowledge, but each borrower's type is his private information.[4]

Banks participate in the market by creating *vacancies* and posting a *credit policy* for each. A credit policy is a mechanism of which execution determines what *credit contract* is offered, where a credit contract is a vector $(q,x,k) \in \mathbb{R}_+^2 \times [0,\kappa]$ that specifies the loan size q, the repayment x, and the collateral requirement k. The posting of a policy means that the bank publicly announces and commits to the policy. The capacity of each vacancy is one borrower, and there is a sunk cost $\gamma \in \mathbb{R}_{++}$ associated with the creation of a vacancy.[5]

[3] As discussed in the introduction, heterogeneity in collateralable wealth could be easily introduced by assuming that the entire credit market consists of many submarket separated by observable characteristics of potential borrowers. These submarkets would operate independently under the model environment.

[4] It is also assumed that there is no way in the model for any information about the type of a borrower to be credibly revealed, except possibly by self-selection. This is to focus on the role of borrowers' directed search in information transfer through banks' screening. Borrowers' signaling in the model directs banks' search, and this effect can conflict with its role for information transfer (see Delacroix and Shi, 2007).

[5] The vacancy creation requires time and resources as in search-theoretic labor market models. It might have two kinds of costs: one is to enhance capacity, employing labor and capital for additional loan processing,

Notice that γ represents the constant marginal cost of bank's capacity expansion. Taking as given the probability that each vacancy is filled with a borrower, banks can create vacancies as many as they wants. Banks finance their loans from loanable deposit funds, of which supply is perfectly elastic at the exogenous gross interest rate $\rho \in \mathbb{R}_{++}$. Thus the *interest rate spread* in lending with credit contract (q, x, k) is $x/q - \rho$.

Each borrower can see all the credit policies posted, and then he chooses for which one to apply or decides not to participate in the market.[6] The set of vacancies with a certain policy and the set of borrowers applying for the policy form a submarket. Matching is bilateral, and the mass of matches in each submarket is determined by a standard constant returns to scale matching technology that captures a search friction.[7] If the vacancy-applicant ratio, or market tightness, in a submarket is $\theta \in \bar{\mathbb{R}}_+ \equiv [0, \infty]$, an applicant to the policy matches with a vacancy with probability $\alpha(\theta)$, and a vacancy is filled with an applicant with probability $\beta(\theta) \equiv \alpha(\theta)/\theta$. The function $\alpha : \bar{\mathbb{R}}_+ \mapsto [0, 1]$ is twice continuously differentiable, $\alpha' > 0$, $\alpha'' < 0$, $\alpha(0) = 0$, and $\alpha(\infty) = 1$. It is also assumed that $\beta(0) = \alpha'(0) = 1$. If a vacancy is filled with an applicant, its credit policy is executed, and the pair of the bank and the borrower enter into the resulted contract.[8]

If a type δ borrower obtains a contract $c = (q, x, k)$, his expected surplus is

$$u(c, \delta) \equiv \delta [f(q, \delta) - x] - (1 - \delta) \min \{x, k\}.$$

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and the other is to search for an additional borrower, for example, by making advertisement.

[6] If a borrower chooses a credit policy for which to apply, he approaches banks searching for a vacancy with the policy. Here it is assumed that each borrower approaches banks for only one policy although he is allowed to randomize his choice in case that he is indifferent about which one to apply for. Anyhow this restriction is not essential to the results. As noted by Moen (1997), we can allow the case where borrowers search for vacancies with different policies by introducing their search intensity for each policy and adjusting their contribution to the matching in each submarket.

[7] If there were no friction other than the capacity constraint associated with bilateral matching technology, the short side of the market would be assured of matching. However here it is assumed that there exists a search friction that makes both sides left unmatched simultaneously. As discussed in the introduction, the sources of this friction can be time delays and costs due to the completion and processing of applications as well as those due to the construction of relationship with banks and unmodeled heterogeneities.

[8] After the mechanism has been executed, the bank always offers the resulted contract because of a law or reputation effects. Then the borrower always accepts the contract in equilibrium although he could reject it. He does not try to negotiate it since his bargaining power is assumed to be too small to improve it.

For banks, the expected surplus from entering into the contract with a type δ borrower is

$$v(c,\delta) \equiv \delta x + (1-\delta)\min\{x,k\} - \rho q.$$

Notice that here limited commitment is assumed: borrowers may choose to default when their project failed. Banks attempt to overcome this possibility with the collateral requirement.[9] Implementing the contract with a type δ borrower generates total expected surplus

$$S(q,\delta) \equiv \delta f(q,\delta) - \rho q.$$

Define $q^\star : D \mapsto \mathbb{R}_{++}$ and $S^\star : D \mapsto \mathbb{R}_{++}$ such that

$$\delta f_q(q^\star(\delta),\delta) = \rho,\ \ S^\star(\delta) = S(q^\star(\delta),\delta),\ \ \forall\delta.$$

For all δ, $q^\star(\delta) = \arg\max_{q\in\mathbb{R}_+} S(q,\delta)$ and $S^\star(\delta) = \max_{q\in\mathbb{R}_+} S(q,\delta)$. Assume that

$$\delta S^\star(\delta) + (1-\delta)\min\{S^\star(\delta), \kappa - \rho q^\star(\delta)\} > \gamma,\ \ \forall\delta \tag{1}$$

to ensure the existence of mutually beneficial contract for every type.[10]

Define $\bar{q} : D \mapsto \mathbb{R}_{++}$ such that $S(\bar{q}(\delta),\delta) = 0$ for every δ. For each δ, $S(\bar{q}(\delta),\delta) > 0$ if and only if $0 < q < \bar{q}(\delta)$. In addition, both $\bar{q}$ and $S^\star$ are strictly increasing. Thus there is no loss of generality in restricting the set of feasible credit contracts to

$$C \equiv [0,\bar{q}(\delta_I)] \times [0,S^\star(\delta_I)] \times [0,\kappa].$$

Moreover, by the revelation principle, banks can without loss restrict themselves to incentive-compatible direct mechanisms in searching for an optimal policy. Hence there is no loss of generality in restricting the set of feasible credit policies to

$$\mathcal{C} \equiv \{\mathring{c} : D \mapsto C \mid u(\mathring{c}(\delta),\delta) \geq u(\mathring{c}(\delta'),\delta),\ \forall\delta,\delta' \in D\}.$$

[9] The actual realized return on a borrower's project is his private information that is not verifiable to banks. However, as in BT, banks are able to notice whether his project succeeded or not. For justification, one can assume that borrowers cannot abscond with any wealth at the end of period, though they default whenever optimal. Then in equilibrium no borrower defaults when his project succeeded.

[10] The definition of equilibrium in the next section does not require all the types to satisfy this condition. Thus, relaxing the assumption, one can investigate the effect of a change in the model parameters on the cutoff type of market participation. However, like BT, pursuing this is not the subject of this paper.

A credit policy specifies that, if a vacancy is filled with a borrower, the borrower truthfully announces his type δ and the contract $\mathring{c}(\delta)$ is implemented.

For each credit policy $\mathring{c} \in \mathcal{C}$, let $\vartheta^p(\mathring{c})$ indicate the vacancy-applicant ratio in the submarket for the policy. Given beliefs about $\vartheta^p : \mathcal{C} \mapsto \bar{\mathbb{R}}_+$, the expected surplus for a type δ borrower applying to a policy $\mathring{c} \in \mathcal{C}$ is

$$U^p(\mathring{c}, \delta; \vartheta^p) \equiv \alpha(\vartheta^p(\mathring{c}))\, u(\mathring{c}(\delta), \delta).$$

Let $\Phi^p(\mathring{c}, \cdot)$ denote the probability measure that represents the distribution of the types of applicants to policy $\mathring{c}$. The measure is defined on the Borel set of D, denoted by $\mathcal{B}_D$, and for any $D_0 \in \mathcal{B}_D$, $\Phi^p(\mathring{c}, D_0)$ indicates the share of applicants to the policy whose type is $\delta \in D_0$. Given beliefs about $\vartheta^p : \mathcal{C} \mapsto \bar{\mathbb{R}}_+$ and $\Phi^p : \mathcal{C} \times \mathcal{B}_D \mapsto [0,1]$, the expected profit from creating a vacancy offering a policy $\mathring{c} \in \mathcal{C}$ is

$$\pi^p(\mathring{c}; \vartheta^p, \Phi^p) \equiv \beta(\vartheta^p(\mathring{c})) \int v(\mathring{c}(\delta), \delta)\, \Phi^p(\mathring{c}, d\delta) - \gamma.$$

Notice that both ϑ^p and Φ^p are defined on the set of all revelation policies. Though most of them are not posted in equilibrium, but it is still necessary to define beliefs about the vacancy-applicant ratio and the types of applicants to those policies if they were posted.

The timing of events is as follows. At Stage 0, nature draws a type for each borrower, and borrowers learn their own types. At Stage 1, banks create vacancies and post credit policies under their beliefs about ϑ^p and Φ^p. At Stage 2, every borrower observes what banks post and then applies to at most one policy under his beliefs about ϑ^p. At Stage 3, borrowers and vacancies in each submarket are matched according to the matching process. At Stage 4, each borrower matched with a vacancy announces his type, and the pair implement the contract for the type. An *outcome* in the model is $\mathcal{G} : D \times \mathcal{B}_C \mapsto \mathbb{R}_+$, where $\mathcal{G}(\delta, \cdot)$ is the measure that specifies how many type δ borrowers obtain each contract: $\mathcal{G}(\delta, C_0)$ indicates total mass of contracts in C_0 obtained by type δ, or equivalently, the mass of type δ borrowers obtain some contract in C_0.

3 Equilibrium

The equilibrium concept used here is competitive search equilibrium under asymmetric information introduced by Guerrieri, Shimer, and Wright (2010), hereafter GSW. To focus on equilibrium outcomes, following GSW, let a competitive search equilibrium be defined in a restricted model. It will be shown that, in terms of outcomes, an equilibrium in this model is equivalent to a competitive search equilibrium in the unrestricted model.

In the restricted model, banks post a single contract rather than a policy for each vacancy they create, and each borrower applies to a single contract he likes or does not participate in the market. For each credit contract $c \in C$, let $\vartheta(c)$ indicate the the vacancy-applicant ratio at the contract. Given beliefs about $\vartheta : C \mapsto \bar{\mathbb{R}}_+$, the expected surplus for a type δ borrower applying to a contract $c \in C$ is

$$U(c, \delta; \vartheta) \equiv \alpha(\vartheta(c))\, u(c, \delta).$$

Put $U(\varnothing, \delta; \vartheta) = 0$ for all δ, letting the null contract $\varnothing$ represent the outside option for borrowers not to participate in the market. For each contract $c \in C$, let $\Phi(c, D_0)$ indicate the share of applicants to the contract whose type is $\delta \in D_0$, and let $\phi(c, \cdot)$ be the probability density function associated with $\Phi(c, \cdot)$, defined by the Radon-Nikodym derivative of $\Phi(c, \cdot)$ with respect to the Lebesgue measure. Given beliefs about $\vartheta : C \mapsto \bar{\mathbb{R}}_+$ and $\Phi : C \times \mathcal{B}_D \mapsto [0, 1]$, the expected profit from posting a contract $c \in C$ is

$$\pi(c; \vartheta, \Phi) \equiv \beta(\vartheta(c)) \int v(c, \delta)\, \Phi(c, d\delta) - \gamma.$$

Put $\beta(\vartheta(\varnothing))\, v(\varnothing, \delta) = \gamma$ for all δ, letting the null contract also represent the outside option for banks not to create a vacancy. Let $\Psi : \mathcal{B}_C \mapsto \mathbb{R}_+$ be the measure that specifies how many each contract banks post: $\Psi(C_0)$ indicates the mass of contracts in C_0 posted by all banks. Its support C_Ψ represents the set of contracts actually posted.

Definition 1. A competitive search equilibrium is a list of functions $\{\Psi, \vartheta, \Phi, \bar{U}\}$, where Ψ is a measure on $\mathcal{B}_C$ with support C_Ψ, $\vartheta : C \mapsto \bar{\mathbb{R}}_+$, $\Phi(c, \cdot)$ is a probability measure on $\mathcal{B}_D$ for every $c \in C$, and $\bar{U} : D \mapsto \mathbb{R}_+$, that satisfies the following conditions:

(i) For every $(c,\delta) \in C \times D$, $U(c,\delta;\vartheta) \leq \bar{U}(\delta) \equiv \max_{\hat{c}\in C_\Psi \cup \{\varnothing\}} U(\hat{c},\delta;\vartheta)$, with equality and $u(c,\delta) \geq 0$ if $\vartheta(c) < \infty$ and $\phi(c,\delta) > 0$,

(ii) For every $c \in C$, $\pi(c;\vartheta,\Phi) \leq 0$, with equality if $c \in C_\Psi$

(iii) For every $\delta \in D$, $\int_{C_\Psi} [\phi(c,\delta)/\vartheta(c)]\,\Psi(dc) \leq g(\delta)$, with equality if $\bar{U}(\delta) > 0$.

Definition 2. An allocation is a list of functions $\{\Psi, \tilde{\vartheta}, \tilde{\Phi}, \bar{U}\}$, where Ψ is a measure on $\mathcal{B}_C$ with support C_Ψ, $\tilde{\vartheta} : C_\Psi \mapsto \bar{\mathbb{R}}_+$, $\tilde{\Phi}(c,\cdot)$ is a probability measure on $\mathcal{B}_D$ for every $c \in C_\Psi$, and $\bar{U} : D \mapsto \mathbb{R}_+$. The allocation of a competitive search equilibrium $\{\Psi, \vartheta, \Phi, \bar{U}\}$ is $\{\Psi, \tilde{\vartheta}, \tilde{\Phi}, \bar{U}\}$, where $\tilde{\vartheta}$ and $\tilde{\Phi}$ are the restricted domain functions of ϑ and Φ respectively.

An allocation $\{\Psi, \tilde{\vartheta}, \tilde{\Phi}, \bar{U}\}$ specifies aggregate decisions, an outcome, and implied payoffs. The *allocation of vacancies* Ψ depicts banks' aggregate decisions. Let $\mathcal{H}(\delta,\cdot)$ be the measure that specifies how many type δ borrowers apply to each posted contract: for every $C_0 \in \mathcal{B}_{C_\Psi}$, $\mathcal{H}(\delta, C_0)$ indicates the mass of type δ borrowers apply to some contract in C_0. The *allocation of borrowers* $\mathcal{H} : D \times \mathcal{B}_C \mapsto \mathbb{R}_+$ describes borrowers' aggregate decisions and is given by

$$\mathcal{H}(\delta, C_0) = \int_{C_0} [\tilde{\phi}(c,\delta)/\tilde{\vartheta}(c)]\Psi(dc)\,,\ \forall \delta \in D,\ C_0 \in \mathcal{B}_{C_\Psi}.$$

The outcome $\mathcal{G}$ is determined by $\mathcal{H}$ as well as the *trading function* $\alpha \circ \tilde{\vartheta} : C_\Psi \mapsto [0,1]$, which specifies the share of applicants to each posted contract who success to obtain it, and the *payoff function* $\bar{U} : D \mapsto \mathbb{R}_+$ indicates the expected surplus of each type from the outcome.

Definition 3. An allocation $\{\Psi, \tilde{\vartheta}, \tilde{\Phi}, \bar{U}\}$ is attainable if

(i) For every $(c,\delta) \in C_\Psi \times D$ such that $\tilde{\phi}(c,\delta) > 0$, $U(c,\delta;\tilde{\vartheta}) = \bar{U}(\delta)$;

(ii) For every $c \in C_\Psi$, $\pi(c;\tilde{\vartheta},\tilde{\Phi}) = 0$;

(iii) For every $\delta \in D$, $\int_{C_\Psi} [\tilde{\phi}(c,\delta)/\tilde{\vartheta}(c)]\Psi(dc) \leq g(\delta)$, with equality if $\bar{U}(\delta) > 0$.

Clearly a competitive search equilibrium has an attainable allocation. The zero profit condition (ii) in Definition 3 implies that banks' profit maximization and their competition under free entry of vacancies lead to zero expected profits from any posted contract. Since $\vartheta(c) = \infty$ makes negative profit $-\gamma$, it also implies that every posted contract has a positive mass of applicants: $\tilde{\vartheta}(c) < \infty$ for all $c \in C_\Psi$. The borrowers' optimality condition (i) then implies that, if a borrower applies to a posted contract, this choice over all the contracts posted is optimal for his type. The market clearing condition (iii) implies that every borrower applies to a posted contract unless his type is indifferent about participating in the market.

Notice that not every attainable allocation is of equilibrium, the definition of which imposes restrictions on contracts not posted in equilibrium, i.e., $c \notin C_\Psi$. To show the role for these restrictions, introduce some additional notations. Given equilibrium payoff function $\bar{U}$, for every contract c and type δ, let $P(c,\delta) \equiv \left\{p \in \mathbb{R}_{++} \mid pu(c,\delta) \geq \bar{U}(\delta)\right\}$, and define $p(c,\delta) \equiv \inf P(c,\delta)$.[11] In addition, let $p^*(c) \equiv \inf_{\delta \in D} p(c,\delta)$. Among all the types, we could say, a type δ is most likely to apply for a contract c if $p(c,\delta) = p^*(c) \leq 1$, in the sense that this type borrowers are willing to do so at the highest rationing probability $1 - \alpha(\vartheta(c))$.

Lemma 1. *In any competitive search equilibrium* $\{\Psi, \vartheta, \Phi, \bar{U}\}$, $\alpha(\vartheta(c)) = p^*(c)$ *if* $p^*(c) \leq 1$, *and* $\vartheta(c) = \infty$ *otherwise. Moreover, if* $p^*(c) \leq 1$ *and* $p(c,\delta) > p^*(c)$, $\phi(c,\delta) = 0$.

Arguing by analogy with the forward induction, the equilibrium conditions in Definition 1 require that an equilibrium must not be supported by implausible beliefs about ϑ and Φ on contracts not posted.[12] Following the equilibrium refinement proposed by Gale (1996), equilibrium condition (i) restricts banks' beliefs about the composition of borrowers attracted to a deviating contract, imposing that their probability assessment of its implementation should be concentrated on the set of types most likely to apply for it.[13] In addition, banks should anticipate that the market tightness will make such types indifferent about applying for it. Imposing these restrictions on all the possible deviations pins down ϑ and Φ. Equi-

[11] Notice that the infimum is taken over $p > 0$. This is to preclude that a type δ with $\bar{U}(\delta) = 0$ is defined to be most likely to apply for a contract c such that $u(c,\delta) < 0$.

[12] For concreteness, consider an equilibrium candidate, and suppose that it is profitable for banks to post a deviating contract $c \notin C_\Psi$ as long as this contract attracts the types most likely to apply for it, inducing the highest probability of success to implement it. Nevertheless it is possible that the candidate has an attainable allocation, the definition of which allows arbitrary beliefs about $\vartheta(c)$ and $\Phi(c,\cdot)$, if banks anticipate that the contract will attract borrowers whose application requires a low level of the market tightness, making the deviation unprofitable. However such beliefs would be refuted once a bank did post the contract and borrowers take the reason for this deviation into account in forming their beliefs about the market tightness. Such beliefs are implausible since they could survive only if banks did not perceive this.

[13] As shown in Lemma 1, for any contract $c \in C$ such that $\vartheta(c) < \infty$, banks' belief about the distribution $\Phi(c,\cdot)$ should put zero weights on all the types that are not most likely to apply for it. As pointed out by Gale (1996), this is analogous to the "universal divinity" refinement proposed by Banks and Sobel (1987).

librium condition (ii) then imposes that, given these ϑ and Φ, no deviating contract could yield positive profit in an equilibrium.

Notice that an attainable allocation might not be of equilibrium if some posted contract attracts more than two types, with some of which entering into it makes negative expected profits. Suppose that banks post such non-distorting pooling contract in a proposed equilibrium, same in spirit to Rothschild and Stiglitz (1976). Since this contract cross-subsidizes low types at the expense of high types, low types have more to gain from the deviation and are, therefore, the ones who actually search for the deviating contract. Thus the proposed equilibrium is not destroyed by the possibility of such deviation, which is supported by the belief that only bad types search for deviating contracts. To destroy a proposed equilibrium, together with borrowers who can freely redirect their search, it must be possible for a bank to post a deviating contract that will be strictly profitable and that will not become strictly unprofitable even if other banks are allowed to deviate by posting still more contracts. This implies that competitive search equilibrium might be regarded as a version of Riley's reactive equilibrium (Riley, 1979) with the Pareto-dominating strongly informationally consistent (SINC) outcome.[14] Formalize this idea to find a way to characterize equilibrium allocations.

Definition 4. An allocation $\{\Psi, \tilde{\vartheta}, \tilde{\Phi}, \bar{U}\}$ is SINC if it is attainable, and for every $(c, \delta) \in C_{\Psi} \times D$ such that $\tilde{\phi}(c, \delta) > 0$, $\beta(\tilde{\vartheta}(c)) v(c, \delta) \geq \gamma$.

Consider an allocation $\{\Psi, \tilde{\vartheta}, \tilde{\Phi}, \bar{U}\}$. Let $C^*(\delta) \equiv \{c \in C_{\Psi} \mid \tilde{\phi}(c, \delta) > 0\}$, and define $C(\delta)$ by $C(\delta) \equiv C^*(\delta)$ or $C(\delta) \equiv \{\varnothing\}$ if $C^*(\delta) = \emptyset$. The allocation is SINC if and only if, for all $\delta, \delta' \in D$ and $c(\delta') \in C(\delta')$, every $c(\delta) \in C(\delta)$ satisfies

[14] It might be noticeable that the universal divinity refinement, to which the requirement of equilibrium condition (i) is analogous, selects the "Riley outcome" in a standard signalling game. To be concrete, we might imagine the following hypothetical adjustment process. Given contracts posted by banks, borrowers choose not only to which contract to apply but also which vacancy they approach for the contract. Applicants who approach each vacancy form a queue in front of it. After all borrowers make the decision, they are given the opportunity to change the contract to which they apply as well as the vacancy at which they apply. If any borrower moves, borrowers are given a further opportunity to move, and so on, until no borrower moves. And then banks are given the opportunity to post new contracts. If any bank posts a new contract, banks are given a further opportunity to post, and so on, until no bank posts any new contracts. Then borrower are given the opportunity to move one more time, and this process iterates until neither any borrower nor any bank moves. After the process ends, borrowers approach the vacancies finally chosen to apply for a loan.

(C1) $\bar{U}(\delta) = \alpha(\theta(\delta))\, u(c(\delta), \delta);$ (C2) $\bar{U}(\delta) \geq \alpha(\theta(\delta'))\, u(c(\delta'), \delta);$

(C3) $\beta(\theta(\delta))\, v(c(\delta), \delta) = \gamma;$ (C4) $c(\delta) \in C \cup \{\varnothing\}$, $\theta(\delta) \in \bar{\mathbb{R}}_+$, $\bar{U}(\delta) \in \mathbb{R}_+$

where $\theta(\cdot) \equiv \tilde{\vartheta}(c(\cdot))$. Thus one can find a Pareto dominating SINC allocation by solving

$$\text{(P)} \qquad \max_{\bar{U}(\cdot), c(\cdot), \theta(\cdot)} \int_{\delta_0}^{\delta_I} \bar{U}(\delta)\, \hat{g}(\delta)\, d\delta$$

subject to (C1)-(C4) for all $\delta, \delta' \in D$, where $\hat{g} : D \mapsto \mathbb{R}_{++}$ is a continuos density function that assigns welfare weights. Notice that (P) is an optimal control problem with the state $\bar{U}(\delta)$ and the control $\Gamma(\delta) \equiv (c(\delta), \theta(\delta))$. As will be shown in the next section, this problem can be restated as a standard program, to which a solution has the form of state-control trajectory $(\bar{U}, \Gamma)$, a single-valued vector function with $\bar{U} : D \mapsto \mathbb{R}_+$ and $\Gamma : D \mapsto (C \cup \{\varnothing\}) \times \bar{\mathbb{R}}_+$.

Lemma 2. *A solution to* (P) *exists. Every solution has the same state trajectory* $\bar{U}$ *such that* $\bar{U}(\delta) > 0$ *for all* δ*.*

The above lemma states that there exists a Pareto dominating SINC allocation. Moreover it is unique in terms of payoffs. Let $\{\Psi, \tilde{\vartheta}, \tilde{\Phi}, \bar{U}\}$ be a Pareto dominating SINC allocation associate with a single solution $(\bar{U}, \Gamma)$. The support of Ψ, or the set of posted contracts is $C_\Psi = c(D)$. Take an arbitrary posted contract $c \in C_\Psi$, and fix it. In the next section, it will be shown that only one type applies for each posted contract. Thus there exists unique δ such that $c = c(\delta)$. The vacancy-applicant ratio at the contract is $\tilde{\vartheta}(c) = \theta(\delta)$, and $\tilde{\Phi}(c, \cdot)$ is the Dirac measure at point δ. The market clearing condition then implies that $\psi(c) = \theta(\delta)\, g(\delta)$, where ψ is the Radon-Nikodym derivative of Ψ with respect to the Lebesgue measure. The payoff function $\bar{U}$ is the same to the optimal state trajectory. As will be shown in the next section, however an optimal control trajectory is not necessarily unique. Consider a Pareto dominating SINC allocation associate with a pair of solutions $(\bar{U}, \Gamma_0), (\bar{U}, \Gamma_1)$. In this case, for each $c \in C_\Psi$, there exists unique δ such that $c \in \{c_0(\delta), c_1(\delta)\}$, and $\tilde{\Phi}(c, \cdot)$ is the Dirac measure at point δ. The market clearing condition implies that $\sum_i \psi(c_i(\delta)) / \theta_i(\delta) = g(\delta)$. Notice that the allocation of vacancies Ψ is not pinned down because banks' beliefs about the distribution of type δ applicants over the indifferent contracts $c_0(\delta), c_1(\delta)$ is arbitrary.

The next proposition establishes that the model has outcomes characterized by solutions to (P), and that hence they are equivalent in terms of payoffs. This follows directly from results in GSW. See the proof of Lemma 2. In an equilibrium, the type distribution g only affects the allocation of vacancies Ψ. As pointed out by GSW, this is consistent with known results in competitive search models with heterogenous agents (e.g. Moen, 1997).[15]

Proposition 1. *A competitive search equilibrium exists, and any competitive search equilibrium has a Pareto dominating SINC allocation.*

The next proposition implies that the restriction to contract posting is without loss of generality in terms of outcomes. The definition of competitive search equilibrium with revelation policies is provided in its proof.

Proposition 2. *Any competitive search equilibrium with contract posting is a competitive search equilibrium with revelation policies. Any competitive search equilibrium with revelation policies has the outcome same to a competitive search equilibrium with contract posting.*

4 Characterization

To solve (P), first restate it in the form of a standard program. The definitional constraint on payoffs (C1) can be written as

$$\bar{U}(\delta) = \alpha(\theta(\delta))\left[\delta f(q(\delta), \delta) - x(\delta) + (1-\delta)\, y(\delta)\right], \tag{2}$$

where $y(\cdot) \equiv \max\{x(\cdot) - k(\cdot), 0\}$. As is standard in optimal mechanism design, write the incentive compatibility constraint (C2) as

$$\bar{U}(\delta) = \max_{\hat{\delta} \in D} \mathcal{U}(\hat{\delta}, \delta) \equiv \alpha(\theta(\hat{\delta}))[\delta f(q(\hat{\delta}), \delta) - x(\hat{\delta}) + (1-\delta)\, y(\hat{\delta})].$$

By the envelope theorem, the first order condition $\mathcal{U}_{\hat{\delta}}(\delta, \delta) = 0$ is equivalent to

$$\bar{U}'(\delta) = \alpha(\theta(\delta))\left[f(q(\delta), \delta) + \delta f_q(q(\delta), \delta) - y(\delta)\right]. \tag{3}$$

[15] A solution to (P) is not affected by $\hat{g}$, implying that the Pareto dominating SINC allocation is independent of the welfare weights as in BT. This is because (P) has an equivalent representation in a recursive structure. See the proof of Lemma 2.

Since the first order condition is only local and not sufficient even locally, so is (3). The next lemma provides a sufficient condition under which it replaces the global constraint (C2). In addition, if the condition holds as the inequality almost everywhere, then that only one type applies for each posted contract.

Lemma 3. *Let* (3) *be satisfied for almost every* δ *and* (C1) *for all* δ*. Then* (C2) *hold for all* δ, δ' *if* $\bar{U}'(\delta) \geq 0$ *for almost every* δ*.*

Notice that the zero profit condition (C3) can be written as

$$y(\delta) = (1-\delta)^{-1}\left[x(\delta) - \rho q(\delta) - \gamma\zeta(\theta(\delta))\right], \tag{4}$$

where $\zeta(\theta) \equiv \theta/\alpha(\theta) = 1/\beta(\theta)$, and that this is equivalent to

$$\begin{aligned} x(\delta) &\geq \rho q(\delta) + \gamma\zeta(\theta(\delta)), && \text{(5a)} \\ \delta x(\delta) + (1-\delta)k(\delta) &\geq \rho q(\delta) + \gamma\zeta(\theta(\delta)), && \text{(5b)} \end{aligned}$$

with either (5a) or (5b) holding as an equality. Substituting (4) into (2) and (3) results in

$$\begin{aligned} \bar{U}(\delta) &\leq \alpha(\theta(\delta))S(q(\delta),\delta) - \gamma\theta(\delta), && (6) \\ \bar{U}'(\delta) &= \alpha(\theta(\delta))\left[S_\delta(q(\delta),\delta) - (1-\delta)^{-1}\{x(\delta) - \rho q(\delta)\}\right] + (1-\delta)^{-1}\gamma\theta(\delta), && (7) \end{aligned}$$

with (6) holding as the equality, respectively.

Now (P) can be restated as follows. Maximize the objective function subject to, for all δ, (5a), (5b), with either holding as an equality, (6), $0 \leq k(\delta) \leq \kappa$, and for almost every δ, (7). Lemma 2 allows us not to deal with the state-space constraint $\bar{U}(\delta) \geq 0$, establishing that it never binds for any δ in a solution. In addition, as shown in its proof, if the nonnegative profit condition were introduced, it would be never slack. Hence the solution remains unchanged by replacing the equality constraint with (6). As is usual, solve the relaxed problem obtained by ignoring that (7) is not sufficient. It will be shown that a resulting solution satisfies the condition in Lemma 3.

4.1 Full Information Benchmark

First describe the allocation that would arise under full information, the case in which banks observe each applicant's type, as a benchmark.

A full information allocation is a Pareto optimum and is characterized by $(\bar{U}^\star, \Gamma^\star)$ that solves (P) without the incentive compatibility constraint (7). In a solution, the loan size and the market tightness for each type is given by

$$\left(q^\star(\delta), \theta^\star(\delta)\right) = \arg\max_{(q,\theta)\in\bar{\mathbb{R}}_+} \left\{\alpha(\theta) S(q(\delta), \delta) - \theta\gamma\right\}, \ \forall\delta. \tag{8}$$

Here an optimal choice of q does not depend on the level of θ. Thus $q^\star : D \mapsto \mathbb{R}_{++}$ defined before constitutes a solution. Given $q^\star$, $\theta^\star : D \mapsto \mathbb{R}_{++}$ is characterized by

$$\alpha'(\theta^\star(\delta)) S^\star(\delta) = \gamma, \ \forall\delta. \tag{9}$$

Each borrower receives a loan that maximizes the expected social surplus of his project with the socially optimal probability given matching technology and capacity expansion costs. Plugging (C3) into (9), we see that

$$v(c^\star(\delta), \delta) / S^\star(\delta) = \alpha'(\theta^\star(\delta)) \theta^\star(\delta) / \alpha(\theta^\star(\delta)), \ \forall\delta,$$

where the elasticity in the right hand side measures banks' contribution to borrowers' probability of transaction. This demonstrates that, for all δ, $c^\star(\delta)$ *endogenously* satisfies the famous Hosios condition (Hosios, 1990): entry is efficient if and only if agents' share of the surplus from trade equals the elasticity. As pointed out by Rocheteau and Wright (2005), under full information, the competitive search market structure internalizes the optimality conditions in both intensive and extensive margins.

Lemma 4. *Define $\check{\delta}$ such that $S^\star(\check{\delta}) = \gamma$. Let $\bar{\delta} \equiv \check{\delta}$ if $\rho q^\star(\check{\delta}) \geq \kappa$ and $\bar{\delta} \equiv 1$ if $\rho q^\star(1) \leq \kappa$. Then there exists unique $\bar{\delta} \in [\check{\delta}, 1]$ such that $\rho q^\star(\delta) + \gamma\zeta(\theta^\star(\delta)) > \kappa$ if and only if $\delta > \bar{\delta}$.*

For each type, the collateral requirement maximizes expected profit given interest rate, while the interest rate yields zero expected profits given collateral requirement. Notice that we have restricted attention on the case all $\delta > \check{\delta}$. By the zero profit condition, for $\delta \leq \bar{\delta}$,

$$x^\star(\delta) = \rho q^\star(\delta) + \gamma\zeta(\theta^\star(\delta)), \;\; k^\star(\delta) \geq \rho q^\star(\delta) + \gamma\zeta(\theta^\star(\delta)).$$

The size of collateral requirement is arbitrary as long as $k^\star(\delta) \geq x^\star(\delta)$, but a dispersion due to this arbitrariness is immaterial. Assume that these payoff equivalent contracts are traded in the same submarket. For $\delta > \bar{\delta}$, the zero profit condition yields

$$x^\star(\delta) = \delta^{-1}\left[\rho q^\star(\delta) + \gamma\zeta(\theta^\star(\delta)) - (1-\delta)\kappa\right] > \rho q^\star(\delta) + \gamma\zeta(\theta^\star(\delta)), \;\; k^\star(\delta) = \kappa.$$

Here $\bar{\delta}$ is a cutoff success probability. Assume that $\delta_0 < \bar{\delta} < \delta_I$ in what follows. A borrower with a success probability lower than the cutoff makes small investments and pays the riskless rate and the spread due to bank's administrative costs only. Limited commitment does not matter for this kind of high-risk borrowers because they offer enough collateral to ensure full loan repayment. However a borrower with a higher success probability makes larger investments and puts up all of his collateralable wealth as collateral. Given collateralable wealth, he has a positive probability of default. The interest rate spread includes a *default premium* that compensates the bank for this risk.

A positive probability that a borrower fails to obtain a mutually beneficial contract under full information is not 'pure credit rationing' but close to 'redlining' (see Stiglitz and Weiss, 1987). Moreover this probability is socially optimal. However it might be noticeable that interest rate spreads are affected by such probabilities because interest rates compensate banks for costs of creating all vacancies, though some of them fail to sell a contract.

4.2 Allocation under Asymmetric Information

To describe the equilibrium allocation, investigate the optimal control solution to (P) separately over $[\delta_0, \bar{\delta}\,]$ and $(\bar{\delta}, \delta_I]$ and then join the two solutions.

First consider the problem same to (P) but only with $\delta \in [\delta_0, \bar{\delta}\,]$. Its solution is the same to the part of solution to (P) over $[\delta_0, \bar{\delta}\,]$ because (P) has the recursive structure previously mentioned. The following proposition shows that, in a competitive search equilibrium, every type $\delta \le \bar{\delta}$ obtains the first-best contract with the socially optimal probability as it would under full information. This is the same to the result in BT, though the full information allocation as well as the cutoff probability is different.

Proposition 3. *If* $(\bar{U}, \Gamma)$ *solves* (P)*,* $(\bar{U}(\delta), \Gamma(\delta)) = (\bar{U}^\star(\delta), \Gamma^\star(\delta))$ *for all* $\delta \in [\delta_0, \bar{\delta}]$*.*

Now consider the problem same to (P) but only with $\delta \in [\bar{\delta}, \delta_I]$, denoted by $(\mathbb{P})$, and let $(\bar{U}^*, \Gamma^*)$ be its solution. Since type $\bar{\delta}$ is the lowest one in $(\mathbb{P})$ with the recursive structure, it is awarded the first-best outcome as under full information: $\bar{U}^*(\bar{\delta}) = \bar{U}^\star(\bar{\delta})$ and $\Gamma^*(\bar{\delta}) = \Gamma^\star(\bar{\delta})$. This implies that the initial state of $(\mathbb{P})$ is equal to the optimal terminal state of the problem with $\delta \in [\delta_0, \bar{\delta}]$. As long as every type $\delta \le \bar{\delta}$ has no incentive to deviate, by the recursive structure of (P), the part of its solution over $[\bar{\delta}, \delta_I]$ is the same to $(\bar{U}^*, \Gamma^*)$.

Lemma 5. *If* $(\bar{U}, \Gamma)$ *solves* (P)*,* $(\bar{U}(\delta), \Gamma(\delta)) = (\bar{U}^*(\delta), \Gamma^*(\delta))$ *for all* $\delta \in (\bar{\delta}, \delta_I]$*.*

Notice that the full information Pareto optimum cannot be achieved under asymmetric information. In the allocation, every type $\delta > \bar{\delta}$ deviated to the contract designed for some higher type because for $\delta > \bar{\delta}$

$$\mathcal{U}_{\hat{\delta}}(\delta, \delta) = \alpha(\theta^\star(\delta))[\delta f_q(q^\star(\delta), \delta) - \rho] q^{\star\prime}(\delta) + [\alpha'(\theta^\star(\delta)) S^\star(\delta) - \gamma] \theta^{\star\prime}(\delta)$$
$$+ \alpha(\theta^\star(\delta)) \delta^{-1} [\rho q^\star(\delta) + \gamma\zeta(\theta^\star(\delta)) - \kappa] > 0.$$

As long as the limit of collateralization does not matter, banks can screen borrowers without welfare loss by lending $q^\star(\delta)$ at the interest rate with no default premium to a type δ borrower and asking for title to $x^\star(\delta)$ of the borrower's collateralable wealth in the event the project fails. The limit does not matter, and such screening is possible, if and only if the borrower's collateralable wealth exceeds the collateral required for this, i.e., $\kappa \ge \rho q^\star(\delta) + \gamma\zeta(\theta^\star(\delta))$.

The remaining thing is to characterize $(\bar{U}^*, \Gamma^*)$, solving the optimal control problem $(\mathbb{P})$. As a first step, the following lemma establishes that the repayment of every type $\delta > \bar{\delta}$ in the unsuccessful state equals the collateral in an equilibrium.

Lemma 6. *In an optimal solution to* (P)*,* (5b) *is binding over* $(\bar{\delta}, \delta_I]$*.*

Proposition 4. *If* $(\bar{U}, \Gamma)$ *solves* (P), *then for every* $\delta \in (\bar{\delta}, \delta_I]$,

$$q(\delta) > q^{\star}(\delta), \; x(\delta) = \delta^{-1}\left[\rho q^{*}(\delta) + \gamma \zeta(\theta^{*}(\delta)) - (1-\delta)\kappa\right] > x^{\star}(\delta), \; k(\delta) = \kappa, \tag{10}$$

and generically $\theta(\delta) \gtreqless \theta(\delta)$, *where* $\theta(\delta)$ *satisfies*

$$\alpha'(\theta(\delta))\left[S(q(\delta), \delta) + \Lambda(q(\delta), \delta)\left\{\delta f_{\delta}(q(\delta), \delta) + \delta^{-1}\kappa\right\}\right] = \gamma. \tag{11}$$

where $\Lambda(q, \delta) \equiv [\delta f_{q\delta}(q, \delta)]^{-1}[\rho - \delta f_q(q, \delta)]$. *In addition,* $\bar{U}'(\delta) > 0$ *for all* $\delta \in (\bar{\delta}, \delta_I]$.

Regarding the intensive margin, the equilibrium contracts are not so much different from those in BT without search frictions. The loan sizes and the collateral requirements are exactly same to those in it, and the interest rates are changed only for zero profits under the matching technology and capacity expansion costs. For the reason same as under full information, the low-risk borrowers, those with $\delta > \bar{\delta}$, puts up the maximum available collateral. Then collateral cannot be used as a screening device under asymmetric information. However a contract specifying a suboptimally large loan in conjunction with a high interest rate is relatively more attractive for a borrower with a higher type.[16] Thus, banks sort borrowers by offering a set of such contracts, and a high type borrower receives a larger loan and pays more interest than under full information.

A key finding distinguished from BT, in which a randomized loan granting strategy does not emerge, is the possibility of credit rationing in the extensive margin. Define $\theta^{\circ}(q, \delta)$ such that $\alpha'(\theta^{\circ}(q, \delta)) S(q, \delta) = \gamma$, which indicates the efficient level of market tightness given loan size q. Because $q^{*}(\delta) > q^{\star}(\delta)$ implies that $\delta f_q(q^{*}(\delta), \delta) < \rho$, the condition (11) yields $\theta^{*}(\delta) > \theta^{\circ}(q^{*}(\delta), \delta)$ for all $\delta > \bar{\delta}$. Together with competitive search, screening under asymmetric information requires that not only borrowers should overinvest, but that banks should create vacancies more than the efficient number under the overinvestment. However such overinvestment lowers down the efficient level of vacancy creation: $\theta^{\circ}(q^{*}(\delta), \delta) < \theta^{\star}(\delta)$

[16]This is because, when $f_{q\delta} > 0$ in the quasi-linear preference, the marginal rate of substitution between investment and interest rate is increasing in the success probability δ. That is, a borrower with a higher type is willing to pay more for an incremental amount of investment. The sorting property of u stated in the Appendix holds for this reason.

for all δ since $S(q^*(\delta), \delta) < S(q^\star(\delta), \delta)$ in an equilibrium. Because the screening reduces socially gains from contracts, it is efficient that banks do not create vacancies as much as under full information. Clearly the vacancy creation in an equilibrium is less than under full-information, i.e., $\theta^*(\delta) < \theta^\star(\delta)$, if and only if the latter channels a stronger effect than the former, and this situation can be regarded as pure credit rationing. In this case, if banks cut down interests reducing loan sizes per borrower, the supply of credit would be enhanced in the extensive margin, and it would be more efficient. Nevertheless this does not occur in an equilibrium because banks screen borrowers to maximize profits under hidden information.[17]

Precise analytical analysis for the effects of changes in the model parameters is difficult without a closed form solution. However, assuming the environment in which the effects on the loan sizes are relatively small, focus on changes in the market tightness. Then, the condition (11) shows that a rise in banks' costs γ as well as a fall in the value of collateralalable wealth κ make the market more tight, lowering down the vacancy-applicant ratio $\theta^*(\delta)$ for all δ. This amplifies the increase in interest rate spread $x^*(\delta)/q^*(\delta) - \rho$ for every δ due to the direct effect. The condition also shows that a fall in the productivity of matching technology, i.e., decreases in both $\alpha(\theta)$ and $\alpha'(\theta)$ for every θ, has the same effects. These results might be important for understanding the occurrence of a credit crunch. For example, when banks have more difficulties in finding qualified borrowers, the tightness of credit market may be endogenously amplified. As discussed in the introduction, the results also help to explain small enterprises' limited access to bank finance.

Another key finding of this paper is that there can exist an equilibrium in which some borrowers of the same type obtain different contracts. The following proposition establishes the possibility of such contract dispersion.

Proposition 5. *An optimal control trajectory is unique if $f_{qq\delta} \leq 0$ but not in general. If both Γ_0 and Γ_1 are optimal control trajectories, $q_0(\delta) < q_1(\delta)$ implies that $\theta_0(\delta) > \theta_1(\delta)$.*

[17] The cost of screening is independent of the distribution of borrower types, whereas the collective benefit of screening depends on the distribution. Thus high type borrowers may collectively prefer to cross-subsidize low type borrowers rather than take costly screening. However, any individual borrower would prefer a contract that screens out all the lower types, and banks know this.

Suppose that, for each type δ, the arrival rate of loan offers $\alpha(\theta(\delta))$ were fixed at one, or any exogenously given level, as in BT. Then the equilibrium payoff $\bar{U}(\delta)$ only depends on the surplus $S(q(\delta),\delta)$, which is entirely determined by the loan size $q(\delta)$ given δ. Thus, in the absence of extensive margin, there could not exist more than one optimal contract since S is strictly decreasing on $q > q^{\star}$. However the competitive search market structure allows that different combinations of loan size $q(\delta)$ and market tightness $\theta(\delta)$ in each margin yields the same equilibrium payoff. If both Γ_0 and Γ_1 are optimal control trajectories and $q_0(\delta) < q_1(\delta)$, contract $(q_0(\delta), x_0(\delta), \kappa)$ yields less gains from implementation for type δ.[18] Nevertheless it can survive in the market with a higher probability of implementation: $\theta_0(\delta) > \theta_1(\delta)$. This is similar in spirit to wage dispersion in wage posting models with more than one offer (e.g. Burdett and Mortensen, 1998) as well as price dispersion in the price posting model of Curtis and Wright (2004). This result is noticeable because it provides an account for a source of dispersion in the terms of loan other than heterogeneity of borrowers. It also demonstrates the possibility that sunspots affect the distribution of credit supply. As discussed in the last section, multiple solutions to (P) does not only allow contract dispersion in an equilibrium but also yields multiple equilibria. If borrowers obtain different contracts in an equilibrium, there is an equilibrium in which borrowers obtain only one of them.

5 Concluding Remarks

This paper develops a model of a competitive search credit market under hidden information. The novelty of the model is that it explicitly captures both intensive and extensive margins of credit supply, and hence that it is appropriate to show how they jointly operate under hidden information. Using the model, this paper sheds light on and explains the possibility of pure credit rationing and contract dispersion among homogeneous borrowers. These are key findings distinguished from the results of existing works on a competitive credit market under

[18]This highlights the role of capacity expansion costs. It states that $\alpha(\theta_0(\delta)) S(q_0(\delta),\delta) - \gamma\theta_0(\delta)$ is less than the other though $S(q_0(\delta),\delta) > S(q_1(\delta),\delta)$. This is possible because banks ask more interests $x_0(\delta)$ that compensate $\gamma\zeta(\theta_0(\delta)) > \gamma\zeta(\theta_1(\delta))$ due to lower success probability. See the proof of Proposition 5.

hidden information. This paper also provides a theoretical account that links unproductive banking as well as lack of collateral to the credit market tightness and interest rate spreads.

The analysis of this paper relies on simplifying assumptions, which preclude us from analyzing dynamic general equilibrium effects. However the model developed here could be used as a module of a macroeconomic model, which properly take into account both intensive and extensive margins of credit supply under asymmetric information. A straightforward way for this is to use the two-sector framework in new monetarist economics (see Williamson and Wright, 2011). It is obvious that not only total amount of credit supply but its distribution matters for macroeconomic performance. Therefore such integration is important in studying many central issues like monetary policy, credit cycles, and credit market regulation. Clearly extending the work in this paper by introducing moral hazard as well as potential borrower's signaling is worthwhile for a more plausible model. Anyhow this paper can be considered as a reasonable starting point.

Appendix

1 Properties of Preferences

First of all, it is obvious that both $u : C \times D \mapsto \mathbb{R}$ and $v : C \times D \mapsto \mathbb{R}$ are continuous. In addition, C is nonempty and compact in $(\mathbb{R}^3, \|\cdot\|)$, where $\|\cdot\|$ is the Euclidean norm. Let

$$\bar{C}(\delta) \equiv \{c \in C \mid u(c, \delta) \geq 0,\ v(c, \delta) \geq \gamma\}$$

denote the set of contracts of which implementation with type δ yields nonnegative gains for both sides, and let $N_\epsilon(c) \equiv \{c' \in C \mid \|c' - c\| < \epsilon\}$ be the neighborhood of c with radius ϵ.

Monotonicity: For every $c \in C$, $v(c, \cdot)$ is increasing.

Proof. Notice that, for $c = (q, x, k)$,

$$v(c, \delta) = x - \rho q - (1 - \delta)\max\{x - k, 0\}.$$

For each $c \in C$, $v(c, \cdot)$ is constant if $x \leq k$, and it is strictly increasing otherwise. □

Local Nonsatiation: For any $c \in \cup_{\delta \in D} \bar{C}(\delta)$ and $\epsilon > 0$, there exists $c' \in N_\epsilon(c)$ such that $v(c', \delta) > v(c, \delta)$ and $u(c', \delta) < u(c, \delta)$ for all δ.

Proof. Take arbitrary $\delta \in D$ and $c = (q, x, k) \in \bar{C}(\delta)$. Then $0 < x < S^\star(\delta)$ since $c \in \bar{C}(\delta)$. Thus, for any $\epsilon > 0$, there is $\Delta x > 0$ that allows $c' = (q, x - \Delta x, k) \in N_\epsilon(c)$. This c' satisfies the inequalities since u is strictly decreasing in x while v is strictly increasing in x. $\square$

Lemma 7. *For any δ, $c \in \bar{C}(\delta)$, and $\epsilon > 0$, there is $c' \in N_\epsilon(c)$ such that $u(c', \delta') > u(c, \delta')$ for all $\delta' > \delta$ and $u(c', \delta') < u(c, \delta')$ for all $\delta' < \delta$.*

Proof. Take arbitrary $\delta \in D$ and $c = (q, x, k) \in \bar{C}(\delta)$. Consider $c' = (q + \Delta q, x - \Delta x, k)$ with $\Delta q > 0$. Notice that $\delta f(q + \Delta q, \delta) - \delta f(q, \delta)$ is positive and is strictly increasing on δ. Since u is continuous and strictly decreasing in x, given Δq, there exists $\Delta x > 0$ such that $u(c', \delta) = u(c, \delta)$. It is obvious that this c' satisfies the inequalities, and that such Δx approaches 0 as Δq approaches 0. Since $c \in \bar{C}(\delta)$ implies that $q < \bar{q}(\delta)$ and $0 < x < S^\star(\delta)$, there exists Δq able to ensure that $c' \in N_\epsilon(c)$ for any $\epsilon > 0$. $\square$

Sorting: For any δ, $c \in \bar{C}(\delta)$, and $\epsilon > 0$, there exists $c' \in N_\epsilon(c)$ such that $u(c', \delta) > u(c, \delta)$ and $u(c', \delta') < u(c, \delta')$ for all $\delta' < \delta$ such that $c \in \bar{C}(\delta')$.

Proof. Take arbitrary pair $(\delta, \delta') \in D^2$ such that $\delta' < \delta$ and $c = (q, x, k) \in \bar{C}(\delta') \subset \bar{C}(\delta)$. Once it is shown that there exists $\delta'' \in (\delta', \delta)$ such that $u(c, \delta'') \geq 0$, Lemma 7 completes the proof since $v(c, \delta') \leq v(c, \delta'') \leq v(c, \delta)$ by the monotonicity. Since u is continuos, it is obvious that such δ'' exists in case that $u(c, \delta') > 0$. If $u(c, \delta') = 0$, its existence is ensured by $u_\delta(c, \delta') > 0$. Notice that $u_\delta(c, \delta) = f(q, \delta) + \delta f_\delta(q, \delta)$ if $x \leq k$, and that $u(c, \delta) = 0$ yields $u_\delta(c, \delta) = \delta f_\delta(q, \delta) + k/\delta$ otherwise. $\square$

2 Proof of Lemma 1

First notice that we can rewrite equilibrium condition (i) as: for every $c \in C$, $\alpha(\vartheta(c)) \leq p^*(c)$, and $\alpha(\vartheta(c)) = p(c, \delta)$ if $\vartheta(c) < \infty$ and $\phi(c, \delta) > 0$ for some $\delta \in D$.

Take an arbitrary contract $c \in C$, and fix it. Notice that there should exist some type δ such that $\phi(c,\delta) > 0$ since $\Phi(c,\cdot)$ is a probability measure. Hence the condition implies that $\vartheta(c) = \infty$ if $p^*(c) > 1$. It also implies that, in case that $p^*(c) \leq 1$, $p^*(c) = p(c,\delta)$ for some δ such that $\phi(c,\delta) > 0$, or $\vartheta(c) = \infty$, and hence $\alpha(\vartheta(c)) = p^*(c)$. If $p^*(c) > 1$, $\Phi(c,\cdot)$ is arbitrary and immaterial. Otherwise $\phi(c,\delta) > 0$ implies that $p(c,\delta) = p^*(c)$. □

3 Proof of Lemma 2

For the standard program in Section 4 equivalent to (P), there exists mathematical theory that proves the lemma directly (see, e.g., Weber, 2011). However, rather than verifying the applicability of the theory, here it is proved by deriving the continuos type results as the limit of the discrete type results in GSW.

Suppose that there exist only finite number of borrowers' types as in GSW. For any given $\Delta \in \mathbb{R}_{++}$, let $\delta_i \equiv \delta_0 + i\Delta$ for $i = 0, 1, \cdots, n(\Delta)$, where $n(\Delta) - 1 < \delta_I - \delta_0 \leq n(\Delta)$. Then define $\left\{\bar{U}_i, \hat{g}_i, c_i, \theta_i\right\}_{i=0}^{n(\Delta)}$ by $\bar{U}_i \equiv \bar{U}(\delta_i)$, $\hat{g}_i \equiv \hat{g}(\delta_i)$, $c_i \equiv c(\delta_i)$, and $\theta_i \equiv \theta(\delta_i)$ for every i. In this case, a Pareto dominating SINC allocation can be found by solving

$$(\bar{\mathrm{P}}) \qquad \max_{\{c_i,\theta_i\}} \sum_{i=0}^{n(\Delta)-1} \bar{U}_i \hat{g}_i \Delta$$

subject to (C1)-(C4) for all δ_i, δ_j. The continuous type problem (P) is the limit of ($\bar{\mathrm{P}}$) as Δ approaches to 0 because

$$\int_{\delta_0}^{\delta_I} \bar{U}(\delta)\,\hat{g}(\delta)\,d\delta = \lim_{\Delta\to 0} \Delta \sum_{i=0}^{n(\Delta)-1} \bar{U}(\delta_i)\,\hat{g}(\delta_i)\,.$$

Notice that ($\bar{\mathrm{P}}$) has an equivalent representation in the form of a nested sequence of smaller optimization problems (see Spence, 1978). For any type i, consider a problem

$$(\mathrm{P}i) \qquad \bar{U}_i = \max_{c_i \in C\cup\{\varnothing\},\theta_i\in\bar{\mathbb{R}}_+} \alpha(\theta_i)\,u(c_i,\delta_i)$$

subject to

$$\beta(\theta_i)\,v(c_i,\delta_i) = \gamma,\ \ \alpha(\theta_i)\,u(c_i,\delta_j) \leq \bar{U}_j,\ \ \forall j < i. \tag{12}$$

The larger problem ($\tilde{\mathrm{P}}$) of solving (Pi) for all i is equivalent to the problem ($\bar{\mathrm{P}}$). Notice that the solution will be unchanged as the first equality constraint in (12) is replaced by

$$\beta(\theta_i)\, v(c_i, \delta_i) \geq \gamma \tag{13}$$

since (13) will never be slack. In addition, for all i, the optimal decision about θ_i in $\bar{\mathbb{R}}_+$ is the same in $\mathbb{R}_+$ since $\theta_i = \infty$ violates the constraint (13). Therefore the problem ($\bar{\mathrm{P}}$) as well as ($\tilde{\mathrm{P}}$) is equivalent to (P) in GSW. Now one can use all the results in GSW as long as their assumptions on preferences u and v hold for arbitrarily small Δ. The preferences in our model satisfy this condition as shown above in the Appendix.

The existence of solution and the uniqueness of optimal state trajectory follow directly from Lemma 1 and Proposition 3 in GSW. The assumption (1) guarantees that, for every δ, there exists $c \in C$ with $u(c, \delta) > 0$ and $v(c, \delta) > \gamma$. By Proposition 4 in GSW, this implies that $\bar{U}(\delta) > 0$ for all δ on the optimal state trajectory. □

4 Proofs of Proposition 2

The following definition of competitive search equilibrium in the unrestricted model with policy posting is a natural generalization of its definition in the restricted model with contract posting. Let $\phi^p(c, \cdot)$ denote the probability density function associated with $\Phi^p(c, \cdot)$.

Definition 5. A competitive search equilibrium with revelation policies is a list of functions $\{\Psi^p, \vartheta^p, \Phi^p, \bar{U}\}$, where Ψ^p is a measure on a σ-algebra of $\mathcal{C}$ with support $\mathcal{C}_{\Psi^p}$, $\vartheta^p : \mathcal{C} \mapsto \bar{\mathbb{R}}_+$, $\Phi^p : \mathcal{C} \times \mathcal{B}_D \mapsto [0, 1]$, and $\bar{U} : D \mapsto \mathbb{R}_+$, that satisfies the following conditions:

(i) For every $(\mathring{c}, \delta) \in \mathcal{C} \times D$, $U^p(\mathring{c}, \delta; \vartheta^p) \leq \bar{U}(\delta) \equiv \max_{\hat{c} \in \mathcal{C}_{\Psi^p} \cup \{\varnothing\}} U^p(\hat{c}, \delta; \vartheta^p)$, with equality and $u(\mathring{c}(\delta), \delta) \geq 0$ if $\vartheta^p(\mathring{c}) < \infty$ and $\phi^p(\mathring{c}, \delta) > 0$;

(ii) For every $\mathring{c} \in \mathcal{C}$, $\pi(\mathring{c}; \vartheta^p, \Phi^p) \leq 0$, with equality if $\mathring{c} \in \mathcal{C}_{\Psi^p}$;

(iii) $\int_{\mathcal{C}_{\Psi^p}} [\phi^p(\mathring{c}, \delta) / \vartheta^p(\mathring{c})]\, \Psi^p(d\mathring{c}) \leq g(\delta)$ for every $\delta \in D$, with equality if $\bar{U}(\delta) > 0$.

The proposition follows directly from the proof of Proposition 5 in GSW. □

5 Proof of Lemma 3

Assume that $\bar{U}'(\delta) \geq 0$ for almost every δ. Since (P) has the recursive structure, it is enough to show that there is no incentive to deviate to the contract designed for a higher type. See the proof of Lemma 2. Suppose that there exists $\delta' > \delta$ such that $\mathcal{U}(\delta', \delta) > \mathcal{U}(\delta, \delta)$ for some δ. Then $\int_{\delta}^{\delta'} \mathcal{U}_{\hat{\delta}}(\tilde{\delta}, \delta) d\tilde{\delta} > 0$. The assumption implies that $\mathcal{U}_{\hat{\delta}}(\tilde{\delta}, \tilde{\delta}) \geq \mathcal{U}_{\hat{\delta}}(\tilde{\delta}, \delta) \geq \mathcal{U}_{\hat{\delta}}(\delta, \delta)$ for almost every $\tilde{\delta} \in [\delta, \delta']$, and hence that $\int_{\delta}^{\delta'} \mathcal{U}_{\hat{\delta}}(\tilde{\delta}, \tilde{\delta}) d\tilde{\delta} > 0$. This contradicts the first order condition that $\mathcal{U}_{\hat{\delta}}(\tilde{\delta}, \tilde{\delta}) = 0$ for almost every $\tilde{\delta}$. □

6 Proof of Lemma 4

Since $\theta^\star$ as well as $q^\star$ is strictly increasing, it is obvious that

$$\mathcal{J}(\delta) \equiv \alpha(\theta^\star(\delta))(\rho q^\star(\delta) - \kappa) + \theta^\star(\delta)\gamma > 0$$

for all $\delta > \check{\delta}$ if $\rho q^\star(\check{\delta}) \geq \kappa$, and that $\mathcal{J}(\delta) \leq 0$ for all δ if $\rho q^\star(1) \leq \kappa$. In case that $\rho q^\star(\check{\delta}) < \kappa$ and $\rho q^\star(1) > \kappa$, $\lim_{\delta \to \check{\delta}} \mathcal{J}(\delta) < 0$ since $\theta^\star(\delta) \to 0$ as $\delta \to \check{\delta}$, and $\mathcal{J}(1) > 0$. Since $\mathcal{J}$ is strictly increasing, this implies that there exists unique $\bar{\delta} \in (\check{\delta}, 1)$ such that $\mathcal{J}(\bar{\delta}) = 0$. □

7 Proof of Proposition 3

Taking $\Gamma = \Gamma^\star$, we have

$$\mathcal{U}(\delta', \delta) = \alpha\left(\theta^\star(\delta')\right) S\left(q^\star(\delta'), \delta\right) - \gamma\theta^\star(\delta'), \ \forall \delta, \delta' \in [\delta_0, \bar{\delta}\,].$$

For every δ, $\left(q^\star(\delta), \theta^\star(\delta)\right)$ is a solution to the maximization problem in (8). Thus, for any $\delta \in [\delta_0, \bar{\delta}\,]$, there cannot exist $\delta' \in [\delta_0, \bar{\delta}\,]$ such that $\mathcal{U}(\delta', \delta) > \mathcal{U}(\delta, \delta)$, and $(\bar{U}^\star, \Gamma^\star)$ satisfies the global incentive compatibility constraints. Since the solution to the unconstrained problem satisfies the constraints, it is the solution to the constrained problem. □

8 Proof of Lemma 5

The only thing we need to prove is that any type $\delta \in [\delta_0, \bar{\delta}\,]$ has no incentive to deviate from $\Gamma^\star$. Deviating to the contract designed for a type $\delta' > \bar{\delta}$, the type δ obtains

$$\mathcal{U}(\delta', \delta) = \alpha\left(\theta^\star(\delta')\right) S\left(q^\star(\delta'), \delta\right) - \gamma\theta^\star(\delta')$$

if $k^\star(\delta') \geq x^\star(\delta')$. Clearly, in this case, it has no incentive to deviate since $(q^\star(\delta), \theta^\star(\delta))$ is a solution to the maximization problem in (8). If $k^\star(\delta') < x^\star(\delta')$, the type δ obtains

$$\mathcal{U}(\delta', \delta) = \alpha(\theta^\star(\delta'))[S(q^\star(\delta'), \delta) - (\delta' - \delta) k^\star(\delta')] - \gamma\theta^\star(\delta').$$

Thus it has no incentive to deviate to the contract designed for a type $\delta' > \delta$. □

9 Proof of Lemma 6

Suppose not, and let $(\bar{U}, \Gamma)$ be a solution such that, for some $\delta > \bar{\delta}$,

$$x(\delta) + (1 - \delta) k(\delta) > \rho q(\delta) + \gamma\zeta(\theta(\delta)). \tag{14}$$

Then (5a) holds as an equality for δ, and this implies that

$$\mathcal{U}(\delta', \delta) = \alpha(\theta(\delta')) S(q(\delta'), \delta) - \gamma\theta(\delta').$$

For the incentive compatibility, i.e. $\delta \in \arg\max_{\hat{\delta}\in D} \mathcal{U}(\hat{\delta}, \delta)$, it must hold that $q(\delta) = q^\star(\delta)$ and $\theta(\delta) = \theta^\star(\delta)$. But this contradicts the assumption because

$$x^\star(\delta) = \rho q^\star(\delta) + \gamma\zeta(\theta^\star(\delta)) > \kappa \geq k^\star(\delta)$$

for $\delta > \bar{\delta}$, whereas (14) implies that $k(\delta) < x(\delta)$ if (5a) holds as an equality. □

10 Proof of Proposition 4

First notice that, by Lemma 6, (7) becomes

$$\begin{aligned} \bar{U}'(\delta) &= \alpha(\theta(\delta))\left[f(q(\delta), \delta) + \delta f_\delta(q(\delta), \delta) - \delta^{-1}\rho q(\delta) + \delta^{-1} k(\delta)\right] - \delta^{-1}\gamma\theta(\delta) \quad (15) \\ &\equiv \varphi(q(\delta), \theta(\delta), k(\delta), \delta). \end{aligned}$$

Consider an optimal control problem

$$(\bar{\mathbb{P}}) \qquad \max_{\bar{U}(\cdot), q(\cdot), \theta(\cdot), k(\cdot)} \int_{\bar{\delta}}^{\delta_I} \bar{U}(\delta) \hat{g}(\delta) \, d\delta$$

subject to, for all $\delta \in [\bar{\delta}, \delta_I]$, (6), (15), $0 \leq k(\delta) \leq \kappa$, and the initial condition $\bar{U}(\bar{\delta}) = \bar{U}^\star(\bar{\delta})$. The relaxed version of $(\mathbb{P})$ consists of $(\bar{\mathbb{P}})$ and (5b) holding as an equality.

Letting $\bar{U}$ be the state variable, $\Upsilon \equiv (q, \theta, k)$ the vector of control variables, and ς the costate variable, Lagrangian for $(\bar{\mathbb{P}})$ is

$$\mathcal{L}\left(\bar{U}, \Upsilon, \varsigma, \nu, \delta\right) = \bar{U}\hat{g}\left(\delta\right) + \varsigma\varphi\left(q, \theta, k, \delta\right) + \lambda[\alpha\left(\theta\right)\left[\delta f\left(q, \delta\right) - \rho q\right] - \gamma\theta - \bar{U}] + \mu\left(\kappa - k\right), \quad (16)$$

where $\nu \equiv (\lambda, \mu)$ is the vector of multipliers. The Pontryagin maximum principle calls for (i) maximality: for all δ,

$$\begin{aligned}
\mathcal{L}_q &= \left(\lambda + \delta^{-1}\varsigma\right)\left[\delta f_q\left(q, \delta\right) - \rho\right] + \varsigma\delta f_{q\delta}\left(q, \delta\right) = 0, && (17)\\
\mathcal{L}_\theta &= \left(\lambda + \delta^{-1}\varsigma\right)\left[\alpha'\left(\theta\right)\left\{\delta f\left(q, \delta\right) - \rho q\right\} - \gamma\right] + \varsigma\alpha'\left(\theta\right)\left[\delta f_\delta\left(q, \delta\right) + \delta^{-1}k\right] = 0, && (18)\\
\mathcal{L}_k &= \delta^{-1}\varsigma\alpha\left(\theta\right) - \mu \leq 0, \quad = 0 \text{ if } k > 0, && (19)
\end{aligned}$$

together with $\lambda, \mu \geq 0$ and the complementary slackness conditions

$$\lambda[\alpha\left(\theta\right)\left\{\delta f\left(q, \delta\right) - \rho q\right\} - \gamma\theta - \bar{U}] = 0, \;\; \mu\left(\kappa - k\right) = 0;$$

(ii) adjoint equation:

$$-\varsigma'\left(\delta\right) = \mathcal{L}_{\bar{U}} = \hat{g}\left(\delta\right) - \lambda\left(\delta\right) \;\; \forall\delta; \quad (20)$$

(iii) transversality: $\varsigma\left(\delta_I\right) = 0$.

Let $(\bar{U}^*, \Gamma^*)$ be a solution to $(\mathbb{P})$. Notice that $(\bar{\mathbb{P}})$ has an equivalent representation

$$\max_{\bar{U}(\cdot),\Upsilon(\cdot)} \int_{\bar{\delta}}^{\delta_I} \left[\alpha\left(\theta\left(\delta\right)\right)\left\{\delta f\left(q\left(\delta\right), \delta\right) - \rho q\left(\delta\right)\right\} - \gamma\theta\left(\delta\right)\right]\hat{g}\left(\delta\right) d\delta$$

subject to the same constraints but

$$\alpha\left(\theta\left(\delta\right)\right)\left\{\delta f\left(q\left(\delta\right), \delta\right) - \rho q\left(\delta\right)\right\} - \gamma\theta\left(\delta\right) \leq \bar{U}\left(\delta\right) \quad (21)$$

instead of (6). By rearranging (6), we see that $\mathcal{L}_{\bar{U}} = \hat{g}\left(\delta\right) - \lambda\left(\delta\right)$ is the Lagrange multiplier for (21). Since $\mathcal{L}_{\bar{U}} > 0$ if and only if $\bar{U}^*\left(\delta\right) < \bar{U}^\star\left(\delta\right)$, by the adjoint equation (20), $\varsigma'\left(\delta\right) < 0$ for all $\delta \in (\bar{\delta}, \delta_I]$. Together with the transversality condition, this implies that $\varsigma\left(\delta\right) > 0$ for all $\delta \in [\bar{\delta}, \delta_I)$. In addition, $\mathcal{L}_{\bar{U}} = \hat{g}\left(\delta\right)$ if and only if $\varsigma\left(\delta\right)\bar{U}^{*\prime}\left(\delta\right) = 0$ as well as $\bar{U}^*\left(\delta\right) < \bar{U}^\star\left(\delta\right)$. If $\bar{U}^*\left(\delta\right) < \bar{U}^\star\left(\delta\right)$, $\varsigma\left(\delta\right) > 0$ implies that $\bar{U}^{*\prime}\left(\delta\right) > 0$ because (18) yields

$$\alpha'\left(\theta\right)\left[f\left(q, \delta\right) + \delta f_\delta\left(q, \delta\right) - \delta^{-1}\rho q + \delta^{-1}k\right] \geq \gamma, \;\; = 0 \text{ if } \lambda = 0$$

and $\alpha(\theta) > \theta\alpha'(\theta)$. Therefore $\lambda(\delta) > 0$ for all $\delta \in [\bar{\delta}, \delta_I)$ and $\lambda(\delta_I) = 0$.

Since $\lambda(\delta), \varsigma(\delta), \bar{U}^{*\prime}(\delta) > 0$ for all $\delta \in (\bar{\delta}, \delta_I)$, for every $\delta \in (\bar{\delta}, \delta_I]$, $\delta f_q(q^*(\delta), \delta) < \rho$ by (17) implies that $q^*(\delta) > q^\star(\delta)$, and $\mu(\delta) > 0$ by (19) implies that $k^*(\delta) = \kappa$. Together with $k^*(\delta) = \kappa$, Lemma 6 yields $x^*(\delta)$ in (10), and combining (17) and (18) leads to the condition (11). Though this characterizes a solution to the relaxed problem with a local representation of the incentive compatibility constraint, Lemma 3 ensures that the solution is globally incentive compatible. □

11 Proof of Proposition 5

Take an arbitrary $\delta \in (\bar{\delta}, \delta_I]$, and fix it. The condition for the uniqueness comes from the condition (17). Since $f_{qq} < 0$, $\rho - \delta f_q(q, \delta)$ is strictly increasing on q. Thus, given $\varsigma, \lambda > 0$, a loan size q that satisfies the condition is unique in case that $f_{q\delta}$ is decreasing on q.

Let q_0 and $q_1 > q_0$ satisfy (17), and let θ_0 and θ_1 satisfy (11) given q_0 and q_1 respectively. In principle, the relative size between the two levels of market tightness is arbitrary, depending on the shapes of f and α. However, since the optimal payoffs must be the same,

$$\Delta\bar{U} \equiv J(S(q_0, \delta), \theta_0) - J(S(q_1, \delta), \theta_1) = 0,$$

where $J(S, \theta) \equiv \alpha(\theta) S - \gamma\theta$. Notice that the condition (11) with $\delta f_q(q, \delta) < \rho$ ensures that $J_\theta = \alpha'(\theta) S - \gamma < 0$. In addition, $J_S \cdot S_q < 0$ since $J_S = \alpha(\theta) > 0$ and $S_q < 0$ for $q > q^\star$. Thus $\Delta\bar{U} = 0$ implies that $\theta_0 > \theta_1$. □

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