Threshold convergence between the federal fund rate and South African equity returns around the colocation period

Phiri, Andrew

Department of Economics, Finance and Business Studies, CTI Potchefstroom Campus, North West, South Africa

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THRESHOLD CONVERGENCE BETWEEN THE FEDERAL FUND RATE AND SOUTH AFRICAN EQUITY RETURNS AROUND THE COLOCATION PERIOD

A. Phiri

Department of Economics, Finance and Business Studies, CTI Potchefstroom Campus, North West, South Africa.

ABSTRACT: Using weekly data collected from 20.09.2008 to 09.12.2016, this paper uses dynamic threshold adjustment models to demonstrate how the introduction of high-frequency and algorithmic trading on the Johannesburg Stock Exchange (JSE) has altered convergence relations between the federal fund rate and equity returns for aggregate and disaggregate South African market indices. We particularly find that for the post-crisis period, the JSE appears to operate more efficiently, in the weak-form sense, under high frequency trading platforms.

Keywords: Colocation; High frequency trading; Global financial crisis; Federal fund rates; Equity returns; Threshold cointegration; Johannesburg Stock Exchange (JSE).

JEL Classification Code: C32; C51; C52; E44; E52.

1 INTRODUCTION

Subsequent to the filing for Chapter 11 bankruptcy by the Lehman brothers in September 2008 and the global financial crisis which ensued afterwards, many stock exchanges worldwide have closely monitored developments of the US Federal Reserve particularly with regards to the setting of it's federal funds rates. In December 2012, the US monetary authorities implemented their final phase of the three-phase quantitative easing programme and such unconventional monetary policy was completely phased out in mid-2014. In December 2015, the Federal Reserve announced it's first increase in interest rates since 2006 and a second hike in the federal funds rate has been more recently announced in December 2016. Given the increasing degree of financial integration experienced globally over the last two decades such contractionary polices implemented by the Federal Reserve are expected to have contagion spillover effects to other developed and mature capital markets worldwide.

Of recent, many stock exchange worldwide have upgraded their trading platforms in the post-crisis period to algorithmic and high-frequency trading technologies through the instalment of colocation facilities. A handful of empirical works exist which examine the effects of colocation facilities on different stock exchanges and the empirical evidence presented thus far can be best described as inconclusive. On one end, Carrion (2013), Hansbrouck and Saar (2013), Manahov and Hudson (2014), Manahov et. al. (2014) and Virgilio (2016) observe that colocation improves stock market performance whereas on the other end, the studies of Zhang (2010), Benos and Sagade (2012) and Riordan et. al. (2012) find that colocation facilities has produced detrimental effects on stock market performance. Moreover, others such as Lee (2015) contend that high-frequency trading technologies have an insignificant effect on stock market performance whilst a smaller cluster of studies (e.g. Viljoen et. al., 2014) have advocated for a U-shaped relationship between algorithmic trading and stock performance.

So far all available empirical studies have been conducted for industrialized and other emerging economies with no empirical attempts existing for Sub-Saharan African (SSA) countries. This is indeed surprising since the Johannesburg stock Exchange (JSE) qualifies as the only SSA stock exchange to adopt such colocation facilities which initially accounted for 4 percent of equity trading activity in 2014 and this has quickly risen to 30 percent of equity trading as of 2016 (Phiri, 2016). In further considering the date of adoption of the colocation facilities (i.e. May 2013) as well as the availability of time series on a weekly basis, we argue that there exists an adequate quantity of data to conduct cointegration analysis on the effects of the fed fund rates on equity returns in South Africa for post-colocation periods. Conventional cointegration techniques strictly depend on the assumption of a symmetric steady-state adjustment process. However, it is highly likely that the federal rates and equity returns converge towards their steady state differently depending on whether there has been a positive or negative shock to the equilibrium.

Therefore, in this present study, we examine asymmetric convergence between the Federal Fund rates and the returns on four equity indices on the JSE in the post-crisis period and further disseminate the post-crisis data into periods corresponding to the pre-colocation and post-colocation. To capture the relevant type of asymmetric cointegration effects between the variables we use the momentum threshold autoregressive (MTAR) cointegration framework of Enders and Siklos (2001). We further supplement the framework with corresponding threshold error correction (TEC) components in order to conduct causality tests between the time series. To the best of our knowledge, this current study becomes the first of a kind to undertake such an empirical endeavour for any African country.

The rest of the manuscript is outlined as follows. The next section presents the methodology of the study. The third section presents the data and empirical analysis. Conclusions are drawn in the fourth section of the paper.

2 METHODOLOGY

Since we are investigating the effect of federal rate (fed_t) on equity returns (er_t), our baseline long-run cointegration model is specified by placing the er_t variable as being endogenous to the fed_t variable i.e.

$$er_t = \psi_0 + \psi_1 fed_t + \mu_t \qquad (1)$$

According to the Engle and Granger's (1987) representation theorem, if the two time series fed_t and er_t are I(1) process, then at least one stationary cointegraiton vector can be formed for the variables. Therefore the first step in our empirical analysis involves determining the level of integration of the time series variables of which we make use conventional ADF unit root tests for this purpose. Once the time series fed_t and er_t are found to integrated of order I(1), then we proceed to examine whether the cointegration residual, μ_t, are stationary variables. In following Enders and Siklos (2001) we allow for asymmetries by specifying the residual terms as a threshold process which can be decomposed into two functional forms. The first is the TAR model:

$$\Delta\mu_t = \rho_1\mu_t(\mu_t < \tau) + \rho_2\mu_t(\mu_t \geq \tau) + \nu_t \qquad (2)$$

Whereas the second is the MTAR:

$$\Delta\mu_t = \rho_1\mu_t(\Delta\mu_t < \tau) + \rho_2\mu_t(\Delta\mu_t \geq \tau) + \nu_t \qquad (3)$$

The term τ represents the unknown threshold term which is estimated using the minimization criterion described in Hansen's (2000). From equations (2) and (3), stationarity of the threshold residual terms is achieved once the following convergence condition is satisfied, $\rho_1 < 0$, $\rho_2 < 0$ and $(1 + \rho_1)(1 + \rho_2) < 0$. Moreover, Enders and Siklos (2001) suggest the tests of two hypothesis to validate threshold cointegration effects. The first involves testing the null of no convergence effects as H_{10}: $\rho_1 = \rho_2 = 0$ and this is tested using the F-Max* statistic for the TAR model and the F-Max*(M) statistic for the MTAR model. The second hypothesis tests the null of no asymmetric convergence effects (i.e. H_{20}: $\rho_1 = \rho_2$) using the Φ* and Φ*(M) statistics for the TAR and MTAR model, respectively. Once both null hypotheses are rejected then one can proceed to model the following threshold error correction (TEC) models for the TAR regression:

$$\Delta fed_t = \alpha_{01} + \sum_{i=1}^{n} \Psi_{01}\, \Delta fed_{t-i} + \sum_{i=1}^{n} \sigma_{01}\, \Delta er_{t-i} + \gamma_{01} e_{t-1}^{-} \xi_{t-1}(e_{t-1} < \tau) + \gamma_{02} e_{t-1}^{+}(e_{t-1} \geq \tau) + \mu_{t1} \quad (4)$$

$$\Delta er_t = \alpha_{11} + \sum_{i=1}^{n} \Psi_{12}\, \Delta fed_{t-i} + \sum_{i=1}^{n} \sigma_{12}\, \Delta er_{t-i} + \gamma_{11} e_{t-1}^{-}(e_{t-1} < \tau) + \gamma_{12} e_{t-1}^{+}(e_{t-1} \geq \tau) + \mu_{t2} \quad (5)$$

Whereas the TEC specification for the MTAR model is given as:

$$\Delta fed_t = \alpha_{01} + \sum_{i=1}^{n} \Psi_{01}\, \Delta fed_{t-i} + \sum_{i=1}^{n} \sigma_{01}\, \Delta er_{t-i} + \gamma_{01} e_{t-1}^{-}(\Delta e_{t-1} < \tau) + \gamma_{02} e_{t-1}^{+}(\Delta e_{t-1} \geq \tau) + \mu_{t1} \quad (6)$$

$$\Delta er_t = \alpha_{11} + \sum_{i=1}^{n} \Psi_{12}\, \Delta fed_{t-i} + \sum_{i=1}^{n} \sigma_{12}\, \Delta er_{t-i} + \gamma_{11} e_{t-1}^{-}(\Delta e_{t-1} < \tau) + \gamma_{12} e_{t-1}^{+}(\Delta e_{t-1} \geq \tau) + \mu_{t2} \quad (7)$$

Where Δ is a first difference operator and μ_{ti} is a well behaved error process. From equations (4) through (7), three hypotheses are tested. Firstly, we test the null of no threshold error correction as H_{30}: $\gamma_{1i} e_{t-1}^{-} = \gamma_{2i} e_{t-1}^{+}$. Secondly, we granger test the null of the federal rate not causing equity returns (i.e. H_{40}: $\Psi_i = 0$). Lastly, we granger test the null hypothesis of equity returns not leading the federal rate (i.e. H_{50}: $\sigma_i = 0$). All aforementioned hypotheses are tested using F-statistics denoted as F[H_{30}], F[H_{40}] and F[H_{50}], respectively.

3 DATA AND EMPIRICAL ANALYSIS

We employ week time series of the closing prices of the all share index (ALSI), the Top 40 index (Top.40), the Industrial 25 Index (Ind.25) and the Resource 10 index (Res.10) and the effective federal fund rate. The stock data is collected from the McGregor BFA database whereas the federal fund rate (fed_t) is collected from the Federal Reserve Economic Data (FRED) database. All data has been collected for the post financial crisis period (i.e 20.09.2008 - 09.12.2016) and is further disseminated to periods representing the pre-colocation (i.e. 20.09.2008 - 2013.05.03) and the post colocation era (i.e. 11. 05.11 - 09/12/2016). Since our stock data is provided as an index we compute our equity returns (er_t) variable for all stock market indices (smi_t) as:

$$er_t = \frac{smi_t - smi_{t-1}}{smi_{t-1}} \times 100 \tag{6}$$

Where t is a time subscript. We also perform ADF unit root tests with a drift as well as with a trend on all the time series to ensure that the variables are suitable for cointegration modelling with the results of this empirical exercise are found in Table 1.

Table 1: Unit root tests

	Pre-colocation		Post-colocation		Decision
	drift	Trend	drift	Trend	
fed_t	7.74	7. 90	-1.81	-1.43	I(1)
	[-10.06]***	[-10.68]***	[-13.29]	[-13.44]	
$ALSI_t$	-1.90	-1.74	-2.16	-2.99	I(1)
	[-5.34]***	[-5.40]***	[-3.50]***	[-3.49]**	
$Top.40_t$	-2.48	-2.42	-2.19	-2.85	I(1)
	[-3.03]**	[-3.25]*	[-4.88]***	[-4.86]***	
$Ind.25_t$	-1.75	-1.93	-2.86	-3.08	I(1)
	[-5.69]***	[-5.77]***	[-4.58]***	[-4.59]***	
$Res.10_t$	-2.71	-2.61	-2.53	-2.63	I(1)
	[-4.74]***	[-4.75]***	[-4.57]***	[-4.58]***	

Note: "***', '**' and '*' represent 1, 5 and 10 percent critical levels, respectively. First difference statistics reported in [].

The results reported in Table 1, show that all the time series are first difference stationary variables. This finding permits us to model convergence effects between the federal fund rate and each of the four JSE index returns. As a preliminary step to the estimation process we test for significant threshold convergence effects. To recall the testing procedure, we firstly test the null hypothesis no convergence effects using the t-Max* and the t-Max*(M) statistics for the TAR and MTAR models, respectively. Secondly we test for asymmetric effects by testing the null of linear cointegration using the Φ* and Φ*(M) statistics for the TAR and

MTAR models, respectively. The rule of thumb is that we only proceed to estimate the long-run cointegration and error correction models if computed test statistics manage to reject both null hypothesis.

Table 2: Threshold cointegration tests

	TAR		MTAR	
	t-Max*	Φ*	t-Max*(M)	Φ*(M)
Pre-colocation				
$ALSI_t$	76.64***	4.75*	77.69***	6.05*
$Top.40_t$	78.63***	4.22*	76.01***	4.67*
$Ind.25_t$	73.27***	5.20*	77.90***	11.04**
$Res.10_t$	73.13***	1.35	73.26***	1.50
Post-colocation				
$ALSI_t$	107.19***	7.85**	100.88***	4.98*
$Top.40_t$	55.59***	5.01*	55.28***	2.08
$Ind.25_t$	55.03***	1.36	53.39***	3.12*
$Res.10_t$	45.95***	0.98	47.23***	2.70

Note: "***', '**' and '*' represent 1, 5 and 10 percent critical levels, respectively. First difference statistics reported in [].

Judging from the results reported in Table 2, we find that for periods before colocation, the TAR specification is suitable for the ALSI, Top.40 and Ind.25 variable whereas the MTAR specification best models the relationship for ALSI and Ind.25 variables. This gives a total of 6 estimation regressions for periods before colocation and the estimates of these regressions are reported in Table 3. Concerning periods subsequent to colocation, we find TAR specifications for the ALSI and Top.40 variables and MTAR specifications for the ALSI and Ind.25 variables. This gives a total of 4 estimation regressions for periods subsequent to colocation and the estimates of these regressions are provided in Table 4.

Table 3: Threshold cointegration and error correction estimates: Pre-colocation period

dependent variable	independent variable	model type					
		TAR ALSI	MTAR ALSI	TAR Top.40	MTAR Top.40	TAR Ind.25	MTAR Ind.25
	ψ_0	-0.71**	-0.71**	0.72**	0.72**	0.82***	0.82***
	ψ_1	-2.74***	-2.74***	-2.83**	-2.83**	-2.53**	-2.53**
	τ	1.41	-1.63	1.63	-1.71	-1.17	-2.99
	$\rho_1\ \mu_{t-1}$	-1.35***	-1.26***	-1.36***	-1.28***	-1.30***	-1.24***
	$\rho_2\ \mu_{t-1}$	-1.07***	-0.90***	-1.10***	-0.96***	1.01***	-0.76***
error correction estimates and causality tests							
	$\gamma_{i1} e_{t-1}^{+}$	-0.001**	-0.001***	-0.001	-0.001**	-0.001	-0.001**
	$\gamma_{i2} e_{t-1}^{-}$	-0.001	-0.001	0.001	0.001*	-0.001	0.001
	$F[H_{30}]$	2.14*	5.27**	-0.24	7.54***	0.24	7.55***
Δfed_t	$F[H_{40}]$	4.40**	2.63*	4.54**	3.58**	4.54**	3.58**
	$F[H_{50}]$	2.09*	1.21	0.36	0.67	0.36	0.67
	D-W	1.92	1.92	1.90	1.95	1.90	1.95
	LB(4)	0.00	0.00	0.00	0.00	0.00	0.00
	$\gamma_{i1} e_{t-1}^{+}$	-1.34***	-1.29***	-1.31	-1.27***	-1.31***	-1.70***
	$\gamma_{i2} e_{t-1}^{-}$	-1.03***	-0.74***	-0.96	-0.67***	-0.96***	-0.67***
	$F[H_{30}]$	1.84	4.70**	2.45*	7.05***	2.45*	7.05***
Δer_t	$F[H_{40}]$	0.94	2.01*	1.30	1.81	1.30	1.81
	$F[H_{50}]$	0.07	1.13	0.40	1.56	0.40	1.56
	D-W	1.99	2.03	2.02	2.04	2.02	2.04
	LB(4)	0.00	0.00	0.05	0.00	0.05	0.04

Note: "***', '**' and '*' represent 1, 5 and 10 percent critical levels, respectively. First difference statistics reported in []. D-W and LB are the Durbin-Watson and Ljung-Box statistics for, and both statistics show that all estimated regressions are free of autocorrelation.

For pre-colocation periods as reported in Table 3, we find significant negative long run elasticities, ψ_1, ranging between -2.53 to -2.83 for all six estimated regressions and since this elasticities are greater-than-unity then this implies that an increase in the federal rate, *ceteris paribus*, decreases JSE equity returns more than proportionately, and vice versa. This points to an inverse relationship between the federal rates and equity returns. For the threshold error terms, we note that $\rho_1 > \rho_2$, a result which signifies that for the pre-colocation period positive deviations from the equilibrium are eradicated quicker than negative deviations. This generally means that responsiveness of equity returns to decreases in the federal rate (expansionary policy) is quicker when compared with the responsiveness of equity returns to increases in the federal rate (contractionary policy). Moreover, all estimated regressions satisfy the convergence condition of $\rho_1 < 0$, $\rho_2 < 0$ and $(1 + \rho_1)(1 + \rho_2) < 0$.

For the error correction estimates, when the federal fund rates is the driving variable in the system, we note that the null hypothesis of no TEC effects is rejected for four of the six regressions (i.e. ALSI(TAR), ALSI(MTAR), Top.40(MTAR), Ind.25(MTAR)). Moreover, the error correction estimates obtained from all four regressions produce the correct negative and significant estimates of -0.001 in the upper regime of the model (i.e. $e_{t-1}^{-} < \tau$). And yet we note that these error correction estimates are too low in value to represent any significant equilibrium correcting behaviour during a positive shock to the federal funds. Causality tests imply that during a shock to the federal rates, there is bi-directional causality for the ALSI(MTAR) model whereas uni-directional causality is found from federal rate to equity returns for the ALSI(MTAR), Top.40(MTAR) and Ind.25(MTAR) models.

On the other hand, when equity returns is the driving variable in the system, the null of no TEC effects is rejected for five of the six regressions (i.e. ALSI(MTAR), Top.40(TAR), Top.40(MTAR), Ind.25(TAR) and Ind.25(MTAR)). For each of these five regressions we note correct negative and highly significant error correction estimates both for positive (e_{t-1}^{+}) and negative (e_{t-1}^{-}) error correction components. Note that the error correction estimates for the positive error correction components have an absolute value exceed unity and yet are below their absolute cut-off value of -2. As explained by Burke and Hunter (2005), when the error correction term lies between -1 and -2, then this simply implies that equilibrium restoring behaviour completely occurs within the data frequency of the estimated time series, which in our case is one week. Further note that the absolute coefficient estimates of e_{t-1}^{+} exceed those of e_{t-1}^{-}, a result which implies quicker equilibrium correction behaviour during contractionary

US monetary policy than during expansionary policy. The causal tests reveal no causality effects within the estimated regressions with the exception of the ALSI(MTAR) regression in which find uni-directional causality running from the federal rate to equity returns.

Table 4: Threshold cointegration and error correction estimates: Post-colocation period

dependent variable	independent variable	model type			
		TAR ALSI	MTAR ALSI	TAR Top.40	MTAR Ind.25
	ψ_0	0.18	0.18	0.22	0.51*
	ψ_1	-0.13	-0.13	-0.42	-1.61
	τ	-2.03	-3.21	-1.75	-1.14
	$\rho_1 \mu_{t-1}$	-1.18***	-1.10***	-1.23***	-1.15***
	$\rho_2 \mu_{t-1}$	-0.77***	-0.71***	-0.90***	-0.88***
	error correction estimates and causality tests				
	$\gamma_{i1} e_{t-1}^{+}$	-0.001***	-0.001***	-0.001**	-0.001*
	$\gamma_{i2} e_{t-1}^{-}$	-0.001	-0.001	-0.001	-0.001
	F[H_{30}]	2.28*	0.21	1.45	0.11
Δfed_t	F[H_{40}]	5.26***	4.32**	4.96***	4.57**
	F[H_{50}]	1.99	1.58	2.13*	0.94
	D-W	1.96	1.97	1.95	1.97
	LB(4)	0.03	0.04	0.04	0.04
	$\gamma_{i1} e_{t-1}^{+}$	-1.24***	-1.11***	-1.25***	-1.18***
	$\gamma_{i2} e_{t-1}^{-}$	-0.79***	-0.61**	-0.82***	-0.79***
	F[H_{30}]	6.18**	4.29**	5.09**	3.32*
Δer_t	F[H_{40}]	3.80**	5.00***	3.37**	1.95*
	F[H_{50}]	0.14	0.10	0.10	0.05
	D-W	2.06	2.08	2.06	2.03
	LB(4)	0.02	0.01	0.03	0.04

Note: "***', '**' and '*' represent 1, 5 and 10 percent critical levels, respectively. First difference statistics reported in []. D-W and LB are the Durbin-Watson and Ljung-Box statistics for, and both statistics show that all estimated regressions are free of autocorrelation.

In referring to the post-colocation estimates reported in Table 4, we firstly note insignificant long run regression estimates, ψ_1, a result implying no long-run relationship between federal rates and equity returns. In differing from the results for the pre-colocation period, we find that for the threshold error terms, $\rho_2 > \rho_1$, hence implying that responsiveness of South African stock markets to increases in the federal rate is quicker compared to the responsiveness of stock markets to decreases in the federal rate. Also note that all estimated regressions satisfy the convergence condition of $\rho_1 < 0$, $\rho_2 < 0$ and $(1 + \rho_1)(1 + \rho_2) < 0$.

In testing or TEC effects we find that, when federal rates is the driving variable, the null hypothesis of no TEC effects is rejected only for the ALSI(TAR) model and similar to the results for the pre-colocation period, the only error correction term which produces a correct negative and significant estimate is positive component of the error correction term at a value of -0.001. As previously mentioned, this estimate is too low to draw any significant conclusions. For this regression we also observe uni-directional causality from the federal rate to equity returns.

On the other hand, when equity returns is the driving variable, all four estimated regressions manage to reject the null of no TEC effects. Furthermore, we find significant and correctly negative error correction estimates for both negative and positive error correction components with the absolute values of e_{t-1}^{+} being greater than those of e_{t-1}^{-}, hence implying faster equilibrium correction for positive deviations from the steady state. Moreover, all regressions verify causality running uni-directional from federal rates to equity returns.

4 CONCLUSSION

This current study is concerned with examining the threshold convergence effects between the federal rates and aggregate as well as disaggregate equity returns on the JSE using two sample periods of weekly data corresponding to pre-colocation (i.e. 20.09.2008 - 2013.05.03) and post-colocation periods (i.e. 11. 05.11 - 09/12/2016). The findings from our study can be summarized into the following three observations. Firstly, we obtain overwhelming evidence of causality running from the federal rate to equity returns in both subsample periods more especially during a shock to equity returns. Secondly, we note that during the pre-colocation period equity returns were more responsive to US expansionary monetary policy whereas in the post-colocation period equity returns respond quicker to contractionary policy. Lastly, and more importantly, we find that during the pre-colocation period there existed a significant long-run relationship between the time series which turns insignificant during the post-colocation period. Collectively, these results demonstrate on how the JSE has behaved more efficiently subsequent to the adoption of high-frequency trading technologies since, during this period, changes in the federal rate do not seem to significantly influence equity returns and thus cannot be used to predict movements in equity returns.

REFERENCES

Benos E. and Sagade S. (2012), "High-frequency trading behaviour and its impact on market quality: Evidence from the UK equity market", Bank of England Working Paper No. 469, December.

Burke S. and Hunter J. (2005), "*Modelling non-stationary economic time series: A multivariate approach*", Palgrave, Basingstoke.

Carrion A. (2013), "Very fast money: High frequency trading on NASDAQ", *Journal of Financial Markets*, 16, 637-645.

Enders W. and Silkos P. (2001), "Cointegration and threshold adjustment", *Journal of Business and Economic Statistics*, 19(2), 166-176.

Engle R. and Granger C. (1987), "Co-integration and error correction: Representation, estimation, and testing", *Economertrica*, 55, 369-384.

Hansbrouck J. and Saar G. (2013), "Low-latency trading", *Journal of Financial Markets*, 16, 741-770.

Hansen B. (2000), "Sample splitting and threshold estimation", *Econometrica*, 68, 575-603.

Lee E. (2015), "High frequency trading in the Korean index futures market", *Journal of Futures Market*, 35, 31-51.

Manahov V. and Hudson R. (2014), "The implications of high-frequency trading on market efficiency and price discovery", *Applied Economics Letters*, 21(16), 1148-1151.

Manahov V., Hudson R. and Gebka B. (2014), "Does high frequency trading affect technical analysis and market efficiency? And if so, how?", *Journal of International Financial Markets, Institutions and Money*, 28, 131-157.

Phiri A. (2016), "Long-run equilibrium adjustment between inflation and stock market returns in South Africa: A nonlinear perspective", *International Journal of Sustainable Economy*, 9(1), 19-33.

Riordan R. and Storkenmaier A. (2012), "Latency, liquidity, and price discovery", *Journal of Financial Studies*, 43, 767-797.

Viljoen T., Westerholm J. and Zheng H. (2014), "Algorithmic trading, liquidity and price discovery: An intraday analysis of the SPI 200 futures", *The Financial Review*, 49(2), 245-270.

Virgilio G. (2016), "The impact of high-frequency trading on marketing volatility", *The Journal of Trading*, 11(2), 55-63.

Zhang F. (2010), "The effect of high-frequency trading on stock volatility and price discovery", Available from SSRN: http://ssrn.com/abstract=1691679.