# Intermediaries and Consumer Search

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**Abstract**. This paper discusses how intermediaries, such as a search engine and an online marketplace, may affect consumer search. We propose an analytical framework that encompasses several models of search for differentiated products, with a high-quality firm being more likely to offer a product that meets each consumer's need. An intermediary improves consumer search efficiency by providing a search platform on which positions are sold to high-quality firms through competitive bidding. While the intermediary may admit too many or too few firms to its platform, compared to what would maximize consumer surplus or total welfare, its presence can nevertheless benefit consumers and improve welfare. However, the intermediary may reduce search efficiency when firms are differentiated only horizontally, when they sell experience or credence goods, or when the intermediary is biased (possibly due to vertical integration).

**Keywords**: consumer search, intermediary, search engine, search platform, online marketplace, vertical differentiation

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## 1. INTRODUCTION

Intermediaries play important roles in consumer search. Shopping malls and (multi-product) retailers, for example, have traditionally served as intermediaries for consumers who search for products from various manufacturers. As economic activities are increasingly connected through the Internet, consumers can have access to more products at lower search costs, but they also face a much larger set of sellers to choose from. Consumers are thus increasingly dependent on intermediaries to guide their search (in some deliberate order) for sellers and products. This has led to enormous commercial successes of Internet companies such as Google, Amazon, and Expedia. How can we understand the strategies and successes of these search intermediaries? How do they affect consumer search and welfare? What public policies, if any, can potentially improve performance in these markets? This paper presents some simple economics that addresses these issues.[1]

There are different ways in which search intermediaries operate. For example, an search engine provides sponsored links to sellers who win keyword auctions. The seller makes a payment to the search engine when a consumer visits the seller (i.e., clicks the seller's link), regardless of whether and how much the consumer purchases from the seller. A search platform (or an online marketplace), on the other hand, may host various sellers. Each seller could be charged a hosting fee as, for example, a flat monthly fee by Yelp SeatMe to each restaurant for reservations, or a commission as a percentage of the transaction amount, as, for example, by Expedia. It is also possible that an intermediary has multiple functions. For example, an online store like Amazon is both a multi-product retailer and a marketplace for independent sellers: as a retailer it sets prices for its various products, and as a marketplace it may charge an entry fee or collects fees from each seller depending on its transaction amount.

We propose an analytical framework with the following key features: A market contains

[1] While this paper will focus on how intermediaries may affect consumer search, there are other reasons for consumers to conduct search in a deliberate—instead of a random—order, as discussed in details in Armstrong (2106).

a unit mass of consumers and $N \geq 2$ sellers. Each consumer, who demands one unit of a product, is uncertain whether a particular seller offers the product that she desires and how much she is willing to pay for it. Specifically, seller $i$'s product $(i = 1, 2, ...N)$ will match a consumer's need with probability $\beta_i \in (0, 1]$, whereas $1 - \beta_i$ is the probability that it is not a match. A consumer's valuation of a product from $i \in \Omega$ is $u_i$, where $\Omega$ is the set of matched sellers for the consumer, and her valuation of the product from any non-matched seller is normalized to zero. After $\beta_i$ is realized and is known privately by $i$, an intermediary provides a mechanism that selects $n \leq N$ sellers into a platform, together with a search technology for consumers to search sellers on the platform. Sellers then simultaneously and independently set their prices, after which each consumer chooses whether and how to conduct sequential search, on the platform and possibly also off the platform. Each search, with a search cost, will enable the consumer to discover her valuation for and the price of the seller's product. All players are risk neutral.

This framework contains as special cases a number of models of search for differentiated products, with or without an intermediary. One way to classify these models is according to the following possible relationships between the values of different matched sellers for each consumer:

- Perfect Dependence (PD): for each consumer, the values of all her matched sellers are perfectly dependent—they are the same: $u_i \equiv u$ for all $i \in \Omega$, where $u$ is a random draw from distribution $F(\cdot)$ with density $f$ on support $[\underline{u}, \bar{u}]$, and $\bar{u} > \underline{u} \geq 0$.[2] Athey and Ellison (2011), and Chen and He (2011), among others, consider the PD case with $\beta_i < 1$.[3]

- Independence (ID): for each consumer, each of her matched sellers has an independent value: $u_i$ is an independent draw from distribution $F(\cdot)$ for every $i \in \Omega$. Anderson and

[2] Note that $\beta_i$ is firm-specific, but its realization is consumer-specific (in the sense that a match for one consumer does not necessarily imply a match for another). Also, both $\Omega$ and $u$ are consumer-specific, but for each consumer, $u$ is equal across her matched sellers.

[3] Eliaz and Spiegler (2016) also assume PD, and in their model consumers who have limited abilities to describe their needs can send inquires to an intermediary.

Renault (1999), Armstrong et al (2009), and Wolinsky (1986), among others, consider the ID case with $\beta_i = 1$.

Moreover, Eliaz and Spiegler (2011) considers the ID case with $\beta_i < 1$, while Chen and Zhang (2016) considers both the PD and the ID cases with $\beta_i < 1$.[4] The true relationship is likely neither PD nor ID, but studies in the literature have made these more extreme assumptions for analytical tractability.[5] As we shall discuss below, the PD approach is analytically more convenient because it makes the derivation of the equilibrium market price straightforward.[6]

We will proceed by considering various cases of this framework. Section 2 reviews an especially simple base model under the assumption of PD, where a search engine auctions a given number of positions to sellers. In equilibrium, a seller with a higher match quality (i.e., a higher $\beta_i$) bids more to be placed at a higher position on the search engine's list, because it has a higher expected profit from being searched by a consumer, whereas consumers will search the sellers in the order of their placement on the list. The search engine thus serves as an information intermediary, guiding consumers to search sellers more efficiently. This results in higher output, consumer surplus, and total welfare.

Section 3 discusses two more sophisticated models, in which there is (additional) incomplete information about sellers' match probabilities, respectively under the assumptions of PD and ID. These models further illustrate the beneficial effects of the search engine on

[4] Starting from the seminal work of Stigler (1961), the economics of consumer search has advanced in the directions of search for the best price among homogeneous sellers (e.g., Stahl, 1989) and of search for the best value among horizontally differentiated sellers (e.g., Wolinsky, 1986 and the other ID models above). In the more recent PD models mentioned above, firms are vertically differentiated but ex post each consumer's matched sellers are homogeneous.

[5] Analytical tractability would be a major issue for a search model in which consumers' product valuations are dependent in a general form, because in this case a consumer's search strategy is non-stationary, as she will keep updating her belief about product valuations during search. Consumer search strategies are stationary under both PD and ID, which greatly simplifies the analysis.

[6] Armstrong and Zhou (2011) consider a Hotelling model where a consumer's valuations for two products have perfect negative correlation, and hence the consumer will learn both products' valuations after searching only one firm.

consumers and total welfare, and both models also offer additional insights on how the search engine can optimally design position auctions or set a uniform fee for each position on its platform.

Section 4 analyzes a model where an intermediary endogenously chooses the size of its search platform. We find that when search cost is sufficiently low, the intermediary will charge an entry fee that is too high, or will admit too few sellers to its search platform, from the perspectives of consumers and total welfare; whereas if search cost is sufficiently high, the intermediary's entry fee is too high for consumer welfare and possibly also for total welfare. We also find that the presence of the intermediary will benefit consumers when search cost is above some threshold, but it will now harm consumers when search cost is sufficiently low.

Section 5 discusses several situations where an intermediary may reduce search efficiency and welfare, including when sellers are differentiated only horizontally, when the intermediary may have a bias due to its possible (partial) vertical integration, and when it auctions sponsored positions for sellers of experience goods or credence goods. Section 6 concludes.

## 2. A BASE MODEL

We start with an illustrative base model in which the intermediary is a search engine that has $n$ positions on its platform. Sellers can bid payments to the search engine to be placed at these positions, in the order of their bids. In equilibrium, sellers with higher match probabilities ($\beta_i$) will bid higher and be placed on the platform at higher positions, whereas consumers will first sequentially search the sellers on the platform, in the order of positions, before searching sellers off the platform. The intermediary thus guides consumers to search the more "relevant" sellers who have higher match probabilities, which improves consumer search efficiency and boosts both consumer and total welfare. The main idea for the model was developed in Athey and Ellison (2011) and Chen and He (2011) in their analysis of keyword auctions from companies such as Google in settings with consumer search.[7] The

[7] The auction of advertisement positions by a search engine has been studied by Edelman et al. (2007) and Varian (2007), among others. Athey-Ellison and Chen-He first embedded such aunctions in models of

discussion below follows closely Chen and He (2011).

Specifically, a search engine ($E$) has $n < N$ positions $E_1$, $E_2, ...,$ $E_n$, each of which can list a seller. For simplicity, suppose that there are $N$ known values of match probabilities, but consumers do not know how these values are assigned to the $N$ sellers. In other words, each seller's match probability is its private information. Let $S_i$, $i = 1, 2, ...,$ denote the sellers in the descending order of their match probability. For convenience, assume $n = 3$, $N \geq 4$, and the match probability for seller $S_i$ is

$$\beta_i = \begin{cases} \gamma^{i-1}\beta & \text{for} \quad i = 1, 2, 3 \\ \gamma^3\beta & \text{for} \quad i = 4, \ldots, N \end{cases},$$

where $\beta, \gamma \in (0, 1)$. Thus the match probability decreases among the sellers at a constant rate $\gamma$ for 3 sellers and then becomes constant for the rest of the sellers.[8] Each consumer's matched sellers have identical value $u$, which is a random draw from $F(u)$.

We assume that there exists a unique $p^m$ such that

$$p^m = \arg\max_p \{p[1 - F(p)]\}; \qquad \pi^m = p^m [1 - F(p^m)]. \tag{1}$$

Then $p^m$ is the price of a monopolist selling a product to a population of consumers whose valuations for the product follow distribution $F$.

Consumers may have some constant search cost per search initially, but the search cost becomes higher after some searches, possibly due to time constraints or search fatigue. To capture such consideration and for convenience, we assume that the cost for each consumer to conduct her $j^{th}$ search is, for $j = 1, \ldots, N$:

$$s_j = \begin{cases} s & \text{for} \quad j = 1, 2, 3, 4 \\ s^h & \text{for} \quad j > 4 \end{cases}, \text{ where } s < \gamma^3\beta \int_{p^m}^{\bar{u}} (u - p^m) f(u)\, du < s^h. \tag{2}$$

consumer search.

[8] We consider this rather special setting in order to illustrate the main idea most conveniently and transparently. In more general models, $\beta_i$ can be random draws from some distriubution, as, for example, in Athey and Ellison (2011), Elizr and Spiegler (2011), and Chen and Zhang (2016). See discussions in later sections.

The search engine auctions the positions to the sellers in a second price auction, where the seller who bids the most is listed at the highest position (at $E_1$) and pays the second highest bid, and so on. We proceed to construct the equilibrium where the search engine guides consumer search through the position auction.

First, suppose that the sellers placed on $E$ are in the order of their relevance, namely that $S_i$ takes the positions of $E_i$ for $i = 1, 2, 3$. Suppose further that all sellers set their prices equal to $p^m$. Then, a consumer's expected surplus (excluding the search cost) is

$$\gamma^{i-1}\beta \int_{p^m}^{\bar{u}} (u - p^m) f(u)\, du, \text{ for } i = 1, 2, 3,$$

from searching $E_i$ and is

$$\gamma^3 \beta \int_{p^m}^{\bar{u}} (u - p^m) f(u)\, du$$

from searching any randomly selected seller not listed on $E$.

Given the presumed placement of sellers on $E$ and the sellers' pricing, it is clearly optimal for each consumer to search sequentially, in the order of $E_1$, $E_2$, $E_3$ and then one randomly selected seller not listed on $E$.[9] The consumer stops searching when she either finds a match or has conducted these four searches without finding a match. She will purchase from a matched seller if $u \geq p^m$ but does not purchase if $u < p^m$.

Next, given consumers' search and purchase behavior described above, if a seller's product matches a consumer's need, then the seller's price that maximizes his expected profit from this consumer, without knowing the consumer's realized $u$, is $p^m$, and following Diamond (1971), $p^m$ is the unique equilibrium price with consumer search.

Therefore, given consumers' search and purchase behavior, if $S_1$, $S_2$, and $S_3$ are placed at $E_1$, $E_2$, and $E_3$, the expected profits of $S_i$, excluding their payments to the search engine,

[9] Notice that consumers can optimally choose whether to search on or off the search engine's platform, even though this is modeled in a very coarse way.

are

$$\pi_1 = \beta\pi^m; \quad \pi_2 = (1-\beta)\,\gamma\pi_1; \quad \pi_3 = (1-\gamma\beta)\,\gamma\pi_2; \tag{3}$$

$$\pi_k = \frac{1-\gamma^2\beta}{N-3}\gamma\pi_3, \text{ for } k = 4,\ldots,N. \tag{4}$$

The analysis of bidding strategies here differs from the usual second price auction, since there are multiple positions to be auctioned, and the values of $E_1$, $E_2$, $E_3$ and not winning the bid are endogenous for the bidders, depending on who will be placed at alternative positions. Because a firm with a higher match probability is more likely to yield a sale when being searched by a consumer, it has a higher expected profit being placed on $E$ and being searched earlier. Furthermore, when firms charge the same price in equilibrium, consumers will want to search the more relevant firms first, in order to reduce expected search cost. The competitive bidding for positions thus leads to an equilibrium where more relevant firms bid more, are placed higher, and are more likely searched by consumers, as Chen and He (2011) establish in the following:

**Proposition 1** *Assume* $\beta \geq \max\left\{2-\frac{1}{\gamma},\ \frac{1-\gamma}{2-\gamma}\right\} \equiv \underline{\beta}(\gamma)$. *Then, there is an equilibrium in which seller* $S_i$ *bids to pay the search engine*

$$b_1 = \gamma\beta\pi_1 + \left(1-\frac{1-\gamma^3\beta}{N-3}\right)\pi_3; \quad b_2 = \gamma^2\beta\pi_1 + \left(1-\frac{1-\gamma^2\beta}{N-3}\right)\gamma\pi_3; \tag{5}$$

$$b_3 = \left(1-\frac{1-\gamma^3\beta}{N-3}\right)\pi_3; \qquad b_k = \left(1-\frac{1-\gamma^2\beta}{N-3}\right)\gamma\pi_3,\ k = 4,\ldots,N. \tag{6}$$

$S_1$*,* $S_2$*,* $S_3$ *are placed at* $E_1$*,* $E_2$*,* $E_3$ *and pay* $b_2$*,* $b_3$*, and* $b_4$*, respectively. Each seller's price is* $p^m$*, and each consumer searches sequentially in the order of* $E_1$*,* $E_2$*,* $E_3$ *and then one randomly selected seller not listed on* $E$*. The consumer stops searching either when she finds a match, in which case she purchases if* $u \geq p^m$*, or when she has conducted these four searches without finding a match.*

Notice that at the (perfect Bayesian) equilibrium, consumers search optimally under the belief that all sellers charge price $p^m$ and that the sellers placed at $E_1, E_2$, $E_3$ and not on $E$ respectively have match probabilities $\beta_1, \beta_2, \beta_3$, and $\gamma\beta_3$; and given the consumers' search strategy, firms bid and price optimally; and consumers' beliefs are consistent with the firms' strategies. We shall focus on this equilibrium, which is quite natural in this context, even though there are other equilibria in this model.[10]

The parameter restriction on $\beta$ in Proposition 1, which is satisfied if $\beta \geq \max\left\{\frac{1}{2}, \gamma\right\}$, provides a sufficient but not necessary condition (when $N > 4$) for the equilibrium. Intuitively, if $\beta$ is too small relative to $\gamma$, the sellers will become too similar in their relative relevance, which makes the condition that no seller will mimic the other seller's bidding strategy difficult to satisfy.

The search engine's profit, which is also its revenue since it has no production cost, is

$$\Gamma = b_2 + b_3 + b_4, \tag{7}$$

which can be shown to increase in $N$. This is because as more sellers are present in the market, a seller is less likely to be selected randomly by a buyer off the platform, and thus placement on the search engine's platform is more valuable. This motivates the sellers to bid more for placement, increasing $\Gamma$.

Also, solving $\beta$ from $\lim_{N\to\infty} \frac{\partial \Gamma}{\partial \beta} = 0$, we find that, as $N \to \infty$, $\Gamma$ is maximized when $\beta$ is

$$\hat{\beta}(\gamma) \equiv \frac{1}{6\gamma^2}\left(1 + 2\gamma + 2\gamma^2 - \sqrt{4\gamma + 2\gamma^2 - 4\gamma^3 + 4\gamma^4 + 1}\right), \tag{8}$$

which decreases in $\gamma$, with $\lim_{\gamma\to 0} \hat{\beta}(\gamma) = \frac{1}{2}$ and $\lim_{\gamma\to 1} \hat{\beta}(\gamma) = \frac{5-\sqrt{7}}{6}$. Thus, when $N$ is large, $\Gamma$ has an inverted-U shape with respect to $\beta$: it increases in $\beta$ for $\beta < \hat{\beta}(\gamma)$ but decreases in $\beta$ for $\beta > \hat{\beta}(\gamma)$. Intuitively, an increase in $\beta$ has a positive effect on the value of being placed at $E_1$, since consumers are more likely to purchase at $E_1$, but it also has negative effects on the values of being placed at $E_2$ and on $E_3$, since consumers will be less likely to visit $E_2$ and $E_3$. The balance of these effects results in the search engine's revenue being first increasing and then decreasing in $\beta$.

[10] One other possible equilibrium is that $S_1, S_2$, and $S_3$ will bid some identical amount, higher than the rest of sellers' bids, and be placed on $E$ in random order, while consumers will search the sellers on $E$ in random order before they possibly search a seller off $E$, and all sellers again charge $p^m$. The search engine again guides consumer search, although its improvement on consumer search efficiency is not as high as the equilibrium in Proposition 1.

We can also investigate consumer welfare and efficiency properties of the equilibrium. One way is to see how the paid placement of sellers on the search engine impacts the probability of finding a match for a consumer for the same search cost. Without paid placement and for large $N$, the probability of a match from each search is

$$\frac{1}{N}\left[1+\gamma+\gamma^2+(N-3)\,\gamma^3\right]\beta \approx \gamma^3\beta,$$

whereas with paid placement the probability of a match from each search is respectively $\beta$, $\gamma\beta$, $\gamma^2\beta$ for the first three searchers and $\gamma^3\beta$ thereafter. Therefore, for the same search cost, the search engine increases the probability of finding a match.

Another way to evaluate the efficiency property of the equilibrium with the search engine acting as an information intermediary is to see how it impacts expected output. The expected output under paid placement is

$$q_h = \left[1-(1-\beta)(1-\gamma\beta)\left(1-\gamma^2\beta\right)\left(1-\gamma^3\beta\right)\right]\left[1-F\left(p^m\right)\right], \tag{9}$$

whereas the expected output without paid placement is approximately

$$q_l = \left[1-\left(1-\gamma^3\beta\right)^4\right]\left[1-F\left(p^m\right)\right] < q_h. \tag{10}$$

We therefore conclude:

**Remark 1** *Paid placement by the search engine leads to more efficient consumer search and to higher total output.*

The stylized model above abstracts away from other important considerations in the placement of sellers by a search intermediary. For example, Google determines ad placement using a quality measure of the sellers (click-through rate) in addition to bids.[11] The fact that the search engine may also consider factors such as the sellers' quality of service and/or prices in determining the sellers' placement is likely to reinforce the main conclusion that position auction provides useful information to consumers that facilitates consumer search and improves efficiency. Consideration of these other factors may also support the finding that the search engine's revenue may not be monotonically increasing in sellers' relevance.

[11] See, for example, Gomes (2014) for an analysis of auction design based on the mechanism-design approach.

## 3. INCOMPLETE INFORMATION AND DEPENDENCE RELATIONS

In this section, we discuss a more sophisticated way to model different sellers' match probabilities and an alternative assumption on the dependence relations between a buyer's values of the products from her matched sellers.

### 3.1 Incomplete Information about Match Probabilities

Recall that in the base model, there are $N$ known values of match probabilities among the $N$ sellers, even though the specific match probability of each seller is its private information. An alternative, possibly more realistic assumption is that there is also incomplete information about the values of the match probabilities. One way to model this is to assume that each $\beta_i$ is a random draw from some distribution and is only known to seller $i$. The problem is then much more complicated, since consumers would need to form expectations about $\beta_1, \beta_2, ...\beta_N$, even when they know from the position auction that the sellers with the highest match probabilities, again denoted as $S_1, S_2, ..., S_n$, are placed respectively at $E_1, E_2, ..., E_n$ on $E$. Furthermore, when a consumer does not find a match after inspecting a seller, say $S_1$, she may update her beliefs about the match probabilities of the remaining sellers.

Athey and Ellison (2011) analyze a model with such incomplete information, and show also that position auction by the search engine improves consumer search efficiency and welfare. In their model, when a consumer finds a seller who meets her need, the consumer and the seller each receives a payoff of 1. Consumers have different search cost, which follows some distribution on $[0, 1]$. In the equilibrium they focus on, sellers with higher match qualities bid more and are placed at higher positions on the search engine's list, and consumers whose search cost is below some critical value will search the sponsored links according to their orders on the list. The intuition behind this equilibrium is similar to that in the base model of section 2: A seller with a higher match quality is willing to bid more to be searched earlier by consumers, because it has a higher expected payoff to be searched by each consumer; and consumers will optimally search the sellers on the sponsored list in the order of their positions, because this search strategy has the highest expected net payoff.

As Athey and Ellison (2011) explain, the list of sponsored links provide consumers with two types of information. They identify a set of sellers that may meet the consumer's need, and they provide information on relative match quality that helps consumers search through this set more efficiently. Compared to the situation where there is no list of the sponsored links, the search engine benefits consumers in two ways: some consumers who would not search without the list will now search, with their expected surplus increasing from zero to positive; and the consumers who would search without the list can now potentially find a match through fewer searches. The higher search efficiency and higher output also increase total welfare.

In addition to establishing the existence of a symmetric pure strategy perfect Bayesian equilibrium for the model and characterizing the strictly monotone equilibrium bidding strategy of firms, Athey and Ellison (2011) also provide interesting insights on auction design, especially on how to set the reserve price.

### 3.2 Independent Values of Matched Sellers

Both the base model and the model in subsection 3.1 assume that a consumer's values from all her matched sellers are the same—they are perfectly dependent—and are randomly drawn from $F$. One advantage of this assumption is that the determination of the equilibrium price becomes straightforward—it is simply equal to the monopoly price $p^m$, invariant with search cost—following Diamond (1971). This allows us to focus on the search engine's role to guide consumers to find their desired products, which could be of first-order importance for consumers to conduct online keyword search.

An alternative approach is to assume that a consumer's value from each of her matched sellers is independently drawn from $F$. Then, the equilibrium price will generally depend on consumer search cost $s$, which could be more plausible.[12] To illustrate, we assume that

[12] As we shall see shortly, how the equilibrium price may vary with $s$ in this model depends crucially on the hazard rate of $F$. In other models of consumer search, prices can be either higher or lower when search cost is lower (e.g., Chen and Zhang, 2011; and Moraga-Gonzalez, et al., 2016).

there is a continuum of sellers of measure 1, whose match probability $\beta$ follows cdf $G(\beta)$, and the search engine has a range of spaces $\sigma \in (0,1)$, which can be occupied by firms who will pay some per-click price $r$.

As in Eliaz and Spiegler (2011), suppose all firms whose $\beta$ is above some threshold $t$ ($\beta \geq t$) will pay $r$ to be listed on the search platform, $E$, and charge price $p^*$.[13] Consider a consumer's search strategy on $E$. Because all firms charge $p^*$, consumers face a stationary problem, and searches optimally with reservation value $u^*$. Denote the expected match quality of sellers on $E$ by

$$\gamma = \frac{\int_t^1 \beta dG(\beta)}{1 - G(t)}. \tag{11}$$

Then, when $s$ is small enough, there is a unique $u^*$ that solves

$$\gamma \int_{u^*}^{\bar{u}} (u - u^*)\, dF(u) = s, \tag{12}$$

where the LHS is the incremental expected benefit from one more search on $E$, while the RHS is the cost per additional search. Notice that $u^*$ is higher when $s$ is lower: the consumer will have a higher reservation value when search is less costly.

Next consider the pricing decision of firms on $E$. If a firm deviates from the equilibrium price $p^*$ to another price $p$, a consumer who visits the firm and learns that the product is a match with value $u > 0$ will buy the firm's product if

$$u - p > u^* - p^*,$$

because the RHS of this inequality represents the consumer's reservation surplus conditional on a match. Thus, the probability that the consumer will buy at $p$ is $1 - F(u^* + p - p^*)$. Therefore, the firm will choose $p$ to maximize

$$p\left[1 - F(u^* + p - p^*)\right].$$

Setting $p = p^*$ in the first-order condition, we have

$$p^* = \frac{1 - F(u^*)}{f(u^*)}. \tag{13}$$

[13] We thus consider a uniform price equilibrium. Hence, the model does not capture a salint feature of many consumer markets: the prevalence of price dispersion, which has been studied in a different type of models (e.g., Varian, 1980; Stahl, 1989; Baye and Morgan, 2001).

The expected surplus for each consumer is $u^* - p^*$, which is positive when $s$ is small. Also, a lower $s$ leads to a higher $u^*$ and thus also to a lower $p^*$, provided that $F$ has an increasing hazard rate, which we shall assume in this subsection. Notice that while it is intuitive that equilibrium price will be lower under a lower search cost, (13) also indicates that price is constant (or higher) when search cost is lower if the hazard rate of $F$ is constant (or decreasing).[14] The expected profit per click for the marginal firm type, $t$, is

$$\pi_t = tp^* \left[1 - F(u^*)\right] = t\frac{\left[1 - F(u^*)\right]^2}{f(u^*)}.$$

By the same logic, the equilibrium price for the firms not on the search platform will be

$$p^{**} = \frac{1 - F(u^{**})}{f(u^{**})},$$

where $u^{**} < u^*$ solves

$$\gamma_L \int_{u^{**}}^{\bar{u}} (u - u^{**})\, dF(u) = s, \text{ with } \gamma_L = \frac{\int_0^t \beta dG(\beta)}{G(t)} < t < \gamma.$$

Therefore, under the assumption that the hazard rate of $F$ is increasing, $p^{**} > p^*$ and all consumers will only search on the platform provided by the search engine, which means that the firms not on the platform will earn zero profit. To fill exactly the $\sigma$ listing positions, we need $G(t) = 1 - \sigma$, so that every firm with $\beta \geq t$ will pay for a position and the mass of firms on the list will be $1 - G(t) = \sigma$. Firms will then bid $r$ up to

$$r_\sigma = \pi_{G^{-1}(1-\sigma)} = G^{-1}(1 - \sigma)\frac{\left[1 - F(u^*)\right]^2}{f(u^*)},$$

the marginal seller's profit per consumer visit, so that the marginal type will have zero net profit (after paying the fee to the search engine), the same as any firm off the list, while all other firms on the platform will earn positive expected profit.

Notice that if no search engine is present, the expected match quality in the market is lower than $\gamma$, which means that in equilibrium consumers will have a lower reservation value

[14] Interestingly, the price effect of market structure under horizontal differentiation depends similarly on the hazard rate of the corrosponding (marginal) distribution $F$: competition leads to a lower, the same, or a higher price compared to monopoly when the hazard rate of $F$ is respectively increasing, constant, or decresing (Chen and Riordan, 2008; and, for general preference dependence, Chen and Riordan, 2015).

than $u^*$ and firms will charge a higher price than $p^*$. Therefore, consumer surplus, which is equal to reservation value minus price, is higher due to the search engine. Moreover, total welfare is also higher in the presence of the search engine, because with a continuum of firms all consumers eventually purchase, but with the guide of the search engine consumers will be able to search firms that have higher match probabilities and thus search more efficiently. Same as in our base model, here the search engine is beneficial to consumers and total welfare because it serves as a useful information intermediary. Sellers with higher match qualities are willing to pay more to be listed on the search platform, because their expected profit from the visit by a consumer is higher. The competitive bidding for the positions on the platform thus selects firms with higher match qualities. As in the base model, consumer search on the platform is more efficient due to the higher expected match quality; but now consumers also benefit from the intermediary through another channel: the higher match quality on the search platform intensifies price competition, lowering the equilibrium market price.[15]

## 4. PLATFORM SIZE: PRIVATE VS. SOCIAL INCENTIVES

In our discussions so far, the intermediary is assumed to have a given number of positions on its platform, and its problem is how to assign these positions to sellers. In this section, we assume that there is no constraint on the size of the platform, but the intermediary can control the number of sellers who enter the platform by charging an entry fee. We are interested in two questions in this context. First, how does the equilibrium number of sellers who enter the platform under the profit-maximizing entry fee compare to the number of sellers that maximizes consumer welfare or total welfare? Second, will the intermediary be beneficial for consumer search, even if its profit maximizing entry fee possibly induces too many or too few entrants to the platform?

Suppose that the setting is similar to the base model, but now $\beta_i$ is a random draw

[15] Recall that when the values of the matched sellers are perfectly dependent, the equilibrium price will be $p^m$, the monopoly price, independent of the expected match quality.

from cumulative distribution function $G$ and that potential sellers, $i = 1, 2, ..., N$, can reach consumers only through the platform set up by the intermediary, who charges an entry fee (or listing fee) $k$ for each entrant.

The timing is as follows: First, The intermediary commits to a fee $k$, and $\beta_i$ is realized and privately learned by potential entrants $i = 1, 2, ..., N$. Next, potential entrants, based on their private $\beta_i$, simultaneously and independently choose whether to pay the fee to be listed on the platform. The number of sellers on the platform is then known publicly and is denoted as $n \geq 0$. The sellers then simultaneously and independently set prices—which will all be equal to $p^m$— because, under the assumption of the base model, for each consumer the product values of her matched sellers are identical. Finally, each consumer can choose sequential search to discover whether any particular seller is a match, her $u$ from the match, and the seller's price. Each search costs $s$, and at least one search is needed for purchase.

We consider symmetric perfect Bayesian equilibrium of this dynamic game of incomplete information. From Proposition 1 in Chen and Zhang (2016), for any given fee $k \in [0, \pi^m]$, there is a unique symmetric perfect Bayesian equilibrium, with a unique threshold $t$ of match probability, such that $i$ will pay $k$ to enter iff her $\beta_i \geq t$. Denote the equilibrium expected number of entrants by $E[n|k]$. It can be shown that

$$E[n|k(t)] = N[1 - G(t)].$$

The intermediary chooses $k$ to maximize its profit

$$\Gamma = kE[n|k].$$

In equilibrium, $t$ is an increasing function of $k$ (i.e., the marginal entrant has a higher match probability when the entry cost is higher). Hence, we can write $k \equiv k(t)$ in equilibrium and view the intermediary's problem as equivalently choosing $t$ to maximize $\Gamma$.

The marginal entrant, whose $\beta_i = t$, will earn zero net expected profit, so that

$$E(\pi|t) = k.$$

Following Chen and Zhang (2016), we can show

$$E(\pi|t) = \frac{t}{\gamma}\pi^m \left\{ \frac{1 - M(t)^N}{N[1 - G(t)]} \right\},$$

where $\pi^m$ is defined in (1), and

$$M(t) = 1 - \gamma[1 - G(t)] \qquad \gamma \equiv \gamma(t) = \frac{\int_t^1 xg(x)\,dx}{1 - G(t)}.$$

Notice that $M(t)$ indicates the probability that a potential entrant will not be a match when the entry threshold is $t$, while $\gamma$ is the expected quality (match probability) of sellers who enter. It follows that in equilibrium:

$$k(t) = \frac{t}{\gamma}\pi^m \left\{ \frac{1 - M(t)^N}{N[1 - G(t)]} \right\},$$

and hence

$$\Gamma = \left[1 - M(t)^N\right] \frac{t}{\gamma}\pi^m. \tag{14}$$

**Lemma 1** *In the equilibrium associated with a given $k \in [0, \pi^m]$ and a given $s$, there is a match probability threshold $t$ such that $i$ will enter iff $\beta_i \geq t$. The intermediary's profit and the total profit of all sellers are respectively*

$$\Gamma = \left[1 - M(t)^N\right] \frac{t}{\gamma}\pi^m, \qquad \Pi = \left[1 - M(t)^N\right] \left(1 - \frac{t}{\gamma}\right)\pi^m; \tag{15}$$

*consumer welfare, measured by aggregate consumer surplus, and total welfare are respectively*

$$V = \left[1 - M(t)^N\right] \left(\Phi - \frac{s}{\gamma}\right), \qquad W = \left[1 - M(t)^N\right] \left(\Phi - \frac{s}{\gamma} + \pi^m\right), \tag{16}$$

*where $\Phi = \int_{p^m}^{\bar{u}} (u - p^m) f(u)\,du$. Moreover, both $V$ and $W$ are single-peaked functions of $t$, and, hence, also of $k$.*

**Proof.** See the appendix. ■

Define the entry fees that maximize intermediary's profit, consumer welfare, and total welfare respectively by

$$k_f = \arg\max_k \Gamma, \qquad k_V = \arg\max_k V \qquad \text{and} \qquad k_W = \arg\max_k W, \tag{17}$$

with $k_f = k(t_f)$, $k_V = k(t_V)$, and $k_W = k(t_W)$. Notice that $k_V$ and $k_W$ are unique from Lemma 1, and we assume $k_f$ is unique as well.

**Lemma 2** *There exists $\hat{s} > 0$ such that $k_V \lesseqgtr k_f$ if $s \lesseqgtr \hat{s}$.*

**Proof.** See the appendix. ■

Therefore, when $s$ is below $\hat{s}$, the intermediary will charge an entry fee that is too high, or admits too few sellers to the platform, from the perspective of consumer welfare; while when $s$ is above $\hat{s}$, the intermediary will admit too many sellers than consumers would like.

Next, we consider total welfare. Notice that

$$\frac{dW}{dt} = \frac{dV}{dt} + \frac{d\left[1 - M\left(t\right)^N\right]}{dt}\pi^m < \frac{dV}{dt}. \tag{18}$$

Defining the elasticity of $\gamma$ with respect to $t$ as

$$\eta\left(t\right) \equiv \frac{t}{\gamma}\frac{d\gamma}{dt},$$

we have:

**Lemma 3** *$k_W < k_V$, and there exists $\tilde{s} > \hat{s}$ such that $k_W < k_f$ if $s < \tilde{s}$ or $\eta\left(t_f\right) < \frac{\pi^m}{\Phi+\pi^m}$, but $k_f < k_W$ if $s > \tilde{s}$ and $\eta\left(t_f\right) > \frac{\pi^m}{\Phi+\pi^m}$.*

**Proof.** See the appendix. ■

Summarizing the analysis above, we have:

**Proposition 2** *There exist $\tilde{s}$ and $\hat{s}$, with $\tilde{s} > \hat{s} > 0$, such that: (i) if $s < \hat{s}$, then the intermediary will set an entry fee that is too high from the perspectives of consumer welfare and total welfare ($k_W < k_V < k_f$); (ii) if $\hat{s} < s < \tilde{s}$, then the fee is too high for total welfare but too low for consumer welfare ($k_W < k_f < k_V$); and (iii) if $s > \tilde{s}$, then the fee is too low for both consumer and total welfare ($k_f < k_W < k_V$) when $\eta\left(t_f\right) > \frac{\pi^0}{\Phi+\pi^0}$ but too low only for consumer welfare ($k_W < k_f < k_V$) when $\eta\left(t_f\right) < \frac{\pi^o}{\Phi+\pi^o}$.*

Notice that $\frac{\pi^o}{\Phi+\pi^o}$ depends only on $F\left(u\right)$ and $\eta\left(t\right) = \frac{t}{\gamma}\frac{d\gamma}{dt}$ depends only on $G\left(\beta\right)$. Interestingly, the monopoly intermediary's entry fee can be either too high or too low for consumer welfare and/or for total welfare, depending on the search cost. An entry fee to the search platform has both a variety effect and a quality effect on consumer search. It raises $t$, so that fewer potential sellers will participate in the platform, which reduces the variety of goods that consumers can search. On the other hand, a higher $t$ increases the average quality (match probability) of sellers in the market, which means that consumer search will be more efficient.[16] When search cost is sufficiently low ($s < \hat{s}$), the variety benefit to consumers from more entry dominates, but the intermediary does not internalize such consumer benefits. Consequently, its entry fee is too high for consumers and for total welfare. When search cost is in the intermediate range ($\hat{s} < s < \tilde{s}$), the quality effect becomes more important, and the intermediary sets an entry that introduces too many sellers from the perspective of consumer welfare, but still too high for total welfare because the total profit of sellers is not maximized. When search cost is high enough ($s > \tilde{s}$), the entry fee may be too low for both consumer and total welfare due to the excessive number of sellers in the market, and this result depends on how the average seller quality in the market changes as additional sellers enter.

Although the intermediary's choice of the entry fee generally differs from what would maximize consumer welfare, the presence of the intermediary can nevertheless benefit consumers. Suppose that without the intermediary, consumers can sequentially search all the $N$ potential sellers, with each search still costing $s$. Consumer welfare in this case is the same as $V$ with the intermediary who sets $k = 0$. Since $V$ is single-peaked from Lemma 1, proposition 2 then implies that the intermediary increases $V$ if $s \geq \hat{s}$ and can also increase $V$ when $s$ is not too much lower than $\hat{s}$. However, as $s \to 0$, $V$ is maximized when $k \to 0$, but $k_f > 0$ is bounded away from 0, and hence the intermediary reduces consumer welfare. We thus have:

**Corollary 1** *Under the assumption of this section, the intermediary benefits consumers when search cost is above some threshold, but harms consumers when search cost is sufficiently low.*

A parallel, but more complicated analysis can be carried out under the assumption that each consumer's valuation for any of her matched sellers is an independent draw from cdf

[16] See Chen and Zhang (2016) for a related discussion.

$F$ for $i = 1, ..., N$. The model is then one of both vertical and horizontal differentiation, where each seller differs in their match quality while each consumer's matched sellers also differ horizontally. The equilibrium price when the market has $n > 1$ sellers (entrants) then has a complex expression—is no longer $p^m$—and it is no longer clear that $V$ and $W$ will be single-peaked functions of $k$. Part of the complication is that with a higher $k$, there will be fewer sellers in the market (a lower $n$), which tends to raise equilibrium price; but a higher $k$ also raises the match quality of the marginal entrant and the average seller in the market, which exerts downward price pressures. The price and welfare effects of a marginal increase in $k$ are thus more subtle in this setting, but the qualitative result of this section still holds, with the intermediary still being able to benefit consumers when $s$ is relatively large but to reduce consumer welfare when $s$ is small.[17]

So far in this section, we have focused on the intermediary's role to change the number of sellers in the marketplace, which affects search efficiency through the "extensive" margin. There are other ways an intermediary can affect consumer search by organizing a marketplace. For example, by gathering sellers at a common physical location (such as in a shopping mall) or hosting them on a single website (such as an online shopping center), the intermediary may reduce the consumer search cost on the marketplace, say from $s$ to $s'$, which would enhance the beneficial impact of the intermediary. Also, the intermediary can list the sellers who enter the marketplace in the order of their match quality (instead of randomly), perhaps through paid advertising or from consumer search data, which could further provide useful information for consumer search. However, different from such practices that can enhance search efficiency, the intermediary may also charge a commission for each transaction, which can potentially raise sellers' marginal costs and hence their prices.

[17] See Chen and Zhang (2016) for a detailed analysis of the case where matched sellers are ex post horizontally differentiated. Then, as explained there, when the match qualities of sellers are higher, consumers have higher incentives to search, which leads to lower prices in equilibrium.

## 5. WHEN INTERMEDIARIES MAY NEGATIVELY IMPACT SEARCH

In earlier discussions, we have identified several ways in which an intermediary can facilitate consumer search, improving both consumer and total welfare. This section will consider situations where an intermediary may negatively impact consumer search.

### 5.1 Firms Are Differentiated only Horizontally

A search intermediary can reduce search efficiency and consumer welfare. This may happen, for instance, when firms are only horizontally differentiated and the intermediary directs the order in which consumers search. To illustrate, consider another case of our general framework, where $\beta_i = \beta = 1$ for all $i$, so that every seller's product will meet each consumer's need, but each consumer's value for any seller's product is independently drawn from $F$. Then, this is the setting of the Wolinsky model (Wolinsky, 1986).[18]

Now suppose that the intermediary can auction a position to make a seller "prominent", which all consumers will search first before they randomly and sequentially search other sellers. As shown in Armstrong et al (2009), the demand for the prominent seller will become more elastic—because all the consumers inspecting its product have not yet visited other sellers— than demand for any other seller and than demand if no seller is made prominent so that they are all searched in random order. Consequently, in equilibrium the prominent seller will set a price lower than that charged by the other sellers (and also lower than the price if every seller is searched randomly), and earn a higher profit than any other seller because its higher demand; while each consumer is optimal to search the prominent firm first given that all other consumers will search the firm first. The intermediary is thus able to extract a fee from the firm being placed at the prominent position, as high as the difference between the prominent firm's profit and the profit of any other firm.[19] However,

[18] Search models with horizontally dfferentiated sellers following Wolinsky (1986) include, for example, Anderson and Renault (1999), Bar-Isaac et al. (2012), Haan and Moraga-González (2011), and Zhou (2011).

[19] Rhodes (2011) shows that a prominent firm earns significantly more profit than other sellers even when consumers' cost of searching and comparing products is essentially zero.

the prices of the “non-prominent” firms will be higher than those when no firm is made prominent, because their demands become less elastic.

On balance, the prominence created by the intermediary guides consumers to search the low-priced firm first, but it nevertheless can harm consumers and reduce total welfare, because the product each consumer purchases on average has lower value than when search is random. It may not be entirely surprising that ordered search in this case leads to lower consumer surplus and total welfare than random search, because firms are ex ante symmetric with purely horizontal differentiation. The intermediary is able to coordinate consumers’ search order, but does not provide useful product information as in the earlier models, or leads to economies in production as, for example, in Bagwell and Ramey (1994).

### 5.2 Search Engine Bias

In the simple models we have discussed, a search engine generally has no incentive to direct consumers to products with lower match quality or with higher prices: as an information intermediary it is unbiased. However, there are situations where the search engine may have a conflicting interest that causes it to be biased. One possibility is that, in addition to the product information from sellers who display paid ads, consumers also rely on the search engine to display “organic results”, information about products from firms who do not pay to advertise. The relevance of the organic results is important for a search engine’s reputation and helps it to attract consumers to its platform. But when the reliability of the organic results becomes higher, sellers with highly relevant products may choose to depend more on displays of organic results, lowering their incentives to bid for placement at paid positions. The desire to increase its revenue from paid placement could then distort the search engine’s incentive to improve the search efficiency for the organic results (see, e.g., White, 2013).

Bias of the search engine may also arise when it is (partially) vertically integrated, for instance, by having its own (or affiliated) shopping services or content providers (see, e.g., Burguet, et al., 2015; de Cornière and Taylor, 2014). Google, for example, has its own shopping site Google Shopping and content providers such as Google Finance. Vertical integration can encourage the intermediary to improve search reliability by internalizing its benefits, but may also motivate the intermediary to divert consumers' organic search towards its affiliates, biased against unaffiliated sites. This may not only lead to inefficiency in consumer search, but also potentially harm competition in the providers' market. However, as Burguet, et al. (2015) points out, if the intermediary already has a bias towards sponsored searches, then some integration could alleviate the distortion between sponsored and organic results.

While the search engine may have a bias in directing consumer search, consumer search efficiency can still be higher with the search engine than without it. Moreover, reputation concerns, (potential) competition from alternative search intermediaries, and antitrust enforcement can provide countervailing incentives and forces to alleviate the search engine's bias in displaying search results.

### 5.3 Experience or Credence Goods

A key assumption in the literature on search and search intermediaries is that the goods concerned are "inspection goods": a consumer can determine whether a product meets her need after an inspection, and there can be no quality variations for any product that appears the same from the inspection. However, in many "real-world" situations, it is possible that the quality of a good is learned only after consumption (i.e., the case of an experience good) or is not known even after the consumption (i.e., the case of a credence good). It is also possible that a seller may provide false product information.

Suppose, for instance, consumers searching for an experience good will purchase if the product matches their needs from its description. Suppose further that the matched products now differ vertically, with a high-quality product having a high (marginal) cost. A low-quality producer would then be willing to bid more to be placed at top positions—because it has higher profit margins—and describes its product as being of high quality. If consumers are sophisticated, they may not want to purchase from the advertising sellers based on their claims, and it would then not be profitable for the low-quality sellers to bid more for the high positions. However, some consumers might be naive and would first search the advertised positions and make purchase decisions based on product descriptions, in which case the low-quality sellers would advertise to exploit the naive consumers.

The intermediary can alleviate this problem by directing consumers to search "relevant" sellers, not necessarily those who are willing to pay more, where "relevance" may include broad information about a seller's quality and reputation. It may also choose to remove certain ads or search results.[20] However, an intermediary may be tempted to accept the high payments from advertisers to boost short-term profit, without exerting enough efforts to screen and keep out the low-quality sellers. It would be important to understand the role of intermediaries and the functioning of search markets in such environments, a goal that is being pursued in our on-going research.

## 6. CONCLUSION

This paper has proposed an analytical framework, where a consumer may incur a search cost to discover whether a firm's product matches her need and her valuation for a matched product, to discuss how an intermediary may affect consumer search. When firms are vertically differentiated in their "quality", in the sense that they differ in the probability of matching each consumer's need, an intermediary can direct consumers to search higher-quality firms by selling the ad positions on its platform through competitive bidding, thereby improving search efficiency. The intermediary can also be beneficial to consumers and total welfare when the number of ad positions on its platform is endogenous, even though in this case it will generally place either too many or too few sellers on its platform, compared to what would maximize consumer or total welfare. While intermediaries such as search engines and online marketplaces have facilitated consumer search and enjoyed enormous

[20] According to MSE news, in May 2016 Google announced that it would ban payday loan ads to protect consumers from "harmful financial products". It was further reported that Google disabled more than 780 million ads in 2015 for reasons ranging from counterfeiting to phishing (trying to take sensitive information from people by pretending to be a trustworthy source).

commercial successes, we also identify several situations where they can reduce the efficiency of consumer search, namely when firms are differentiated only horizontally,[21] when a search intermediary is (partially) vertically integrated, or when firms sell experience or credence goods. Markets and policy design in such situations are important topics for future research. There are other interesting topics for future research, such as how a search intermediary may contract with sellers, and competition among search intermediaries.

## APPENDIX

The appendix contains proofs for Lemmas 1-3.

**Proof of Lemma 1.** Notice that $\left[1 - M(t)^N\right]$ is the probability that at least one potential entrant will be a match for a consumer under entry threshold $t$, and $\pi^m$ is the expected industry profit when that happens. Given that there is a unit mass of consumers, it follows that the expected industry profit is $\left[1 - M(t)^N\right]\pi^m$. The rest of the lemma then follows immediately from (14) and from Lemma 3 and Theorem 1 of Chen and Zhang (2016): ■

**Proof of Lemma 2.** It suffices to show that $t_V \lesseqqgtr t_f$ if $s \lesseqqgtr \hat{s}$.

At $t = t_f$, we have

$$\frac{d\Gamma}{dt}|_{t=t_f} = \frac{d\left[1 - M(t)^N\right]}{dt}\frac{t}{\gamma}\pi^m + \left[1 - M(t)^N\right]\frac{d\left(\frac{t}{\gamma}\right)}{dt}\pi^m = 0,$$

or

$$\frac{d\left[1 - M(t)^N\right]}{dt} = -\left[1 - M(t)^N\right]\frac{d\left(\frac{t}{\gamma}\right)/dt}{t/\gamma}.$$

[21] For example, the match probability of each firm for every consumer is equal to 1, and each consumer's valuation for any matched product is an independent random draw from some known distribution.

Then:

$$\begin{aligned}
\frac{dV}{dt}|_{t=t_f} &= \frac{d\left[1-M(t)^N\right]}{dt}\left(\Phi-\frac{s}{\gamma}\right)+\left[1-M(t)^N\right]\frac{s}{\gamma^2}\frac{d\gamma}{dt} \\
&= -\left[1-M(t)^N\right]\frac{d\left(\frac{t}{\gamma}\right)/dt}{t/\gamma}\left(\Phi-\frac{s}{\gamma}\right)+\left[1-M(t)^N\right]\frac{s}{\gamma^2}\frac{d\gamma}{dt} \\
&= \left[1-M(t)^N\right]\left[-\frac{d\left(\frac{t}{\gamma}\right)/dt}{t/\gamma}\Phi+\left(\frac{\gamma^2 d\left(\frac{t}{\gamma}\right)/dt}{t}+\frac{d\gamma}{dt}\right)\frac{s}{\gamma^2}\right] \\
&= \left[1-M(t)^N\right]\left[-\frac{d\left(\frac{t}{\gamma}\right)/dt}{t/\gamma}\Phi+\frac{s}{t\gamma}\right],
\end{aligned}$$

where the last equality holds because, from Lemma 1 of Chen and Zhang (2016),

$$(i)\ \frac{d\gamma}{dt}=\frac{g(t)}{1-G(t)}(\gamma-t)>0; \quad (ii)\ \frac{d\left(\frac{t}{\gamma}\right)}{dt}=\frac{\gamma-\frac{g(t)t(\gamma-t)}{1-G(t)}}{\gamma^2}>0,$$

and thus

$$\frac{\gamma^2}{t}\frac{d\left(\frac{t}{\gamma}\right)}{dt}=\frac{1}{t}\left[\gamma-\frac{g(t)\,t\,(\gamma-t)}{1-G(t)}\right]=\frac{\gamma}{t}-\frac{d\gamma}{dt}.$$

Notice that if $s\to 0$, we have $\frac{dV}{dt}|_{t=t_f}<0$. Furthermore, for a given $t$, search cost is smaller than the expected search benefit: $s<\gamma\Phi$. When $s\to\gamma\Phi$,

$$-\frac{d\left(\frac{t}{\gamma}\right)/dt}{t/\gamma}\Phi+\frac{s}{t\gamma}=-\frac{1}{\gamma}\left(\frac{\gamma}{t}-\frac{d\gamma}{dt}\right)\Phi+\frac{1}{t}\Phi=\frac{1}{\gamma}\frac{d\gamma}{dt}\Phi>0$$

and thus $\frac{dV}{dt}|_{t=t_f}>0$. Moreover, it can be verified that $\frac{dV}{dt}$ increases in $s$. Therefore, there exists a unique $\hat{s}$ such that $t_V \lesseqqgtr t_f$ if $s \lesseqqgtr \hat{s}$ ■

**Proof of Lemma 3.** From (18), $t_W<t_V$, and hence $k_W<k_V$.

Moreover, if $s<\hat{s}$, $k_V<k_f$ and thus $k_W<k_f$.

Next consider the case $s>\hat{s}$. Similar to the analysis of deriving $\frac{dV}{dt}$, we can show (replacing $\Phi$ with $\Phi+\pi^m$)

$$\frac{dW}{dt}|_{t=t_f}=\left[1-M(t)^N\right]\left[-\frac{d\left(\frac{t}{\gamma}\right)/dt}{t/\gamma}(\Phi+\pi^m)+\frac{s}{t\gamma}\right].$$

If $s \rightarrow \hat{s}$, $\frac{dW}{dt}|_{t=t_f} < \frac{dV}{dt}|_{t=t_f} = 0$. If $s \rightarrow \gamma\Phi$,

$$\begin{aligned} -\frac{d\left(\frac{t}{\gamma}\right)/dt}{t/\gamma}\left(\Phi+\pi^0\right)+\frac{s}{t\gamma} &= -\frac{1}{\gamma}\left(\frac{\gamma}{t}-\frac{d\gamma}{dt}\right)\pi^m+\frac{1}{\gamma}\frac{d\gamma}{dt}\Phi \\ &= -\frac{1}{t}\pi^m+\frac{1}{\gamma}\frac{d\gamma}{dt}\left(\Phi+\pi^m\right), \end{aligned}$$

which is positive (negative) if

$$\eta\left(t\right)=\frac{t}{\gamma}\frac{d\gamma}{dt}>(<)\frac{\pi^m}{\Phi+\pi^m}.$$

Therefore, if $\eta\left(t_f\right) < \frac{\pi^m}{\Phi+\pi^m}$,

$$\frac{dW}{dt}|_{t=t_f} < 0 \quad \text{(i.e., } t_W < t_f, \text{ or } k_W < k_f \text{ )},$$

and if $\eta\left(t_f\right) > \frac{\pi^m}{\Phi+\pi^m}$, there exists $\tilde{s} > \hat{s}$ such that

$$\frac{dW}{dt}|_{t=t_f} \gtrless 0 \quad \text{(i.e., } t_W \gtrless t_f, \text{ or } k_W \gtrless k_f\text{)} \quad \text{if} \qquad s \gtrless \tilde{s}.$$

■

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