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Too Much Waste: A Failure of Stochastic, Competitive Markets*

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Abstract

The equilibrium of a competitive market in which firms must choose prices ex ante and demand is stochastic is shown to be second-best inefficient. Even under risk neutrality, equilibrium price exceeds the welfare-maximising predetermined price. Competition tends to eliminate rationing, but at the greater welfare cost of creating excess capacity. Entry incentives are also distorted. In low states, entrants obtain a share of revenue without increasing consumption, giving rise to a version of the common pool problem. In high states, firms do not appropriate the consumer surplus gained from marginal reductions in rationing. As a result of these offsetting externalities, the number of firms may be excessive or insufficient. Inefficiency arises whether or not the rationing rule is efficient.

Keywords: stochastic demand, rationing, waste, efficiency.

JEL classifications: D61, D81, H23.

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Introduction

Restaurants typically set prices before knowing how many customers turn up on a particular night. Sometimes they run out of specific dishes, other times they throw food away. Granted that for good reasons prices are set *ex ante*, does competition lead to an efficient outcome?[1] This paper shows that market failure is endemic, even under universal risk neutrality. The *ex ante* price that maximizes expected welfare does not normally coincide with the price in an atomistic equilibrium.[2] Waste is socially excessive and rationing insufficient. Moreover, the number of active firms in a competitive equilibrium may not be welfare maximizing. As demand is never entirely certain and it necessarily takes time to adjust price, the systematic market failure identified here is potentially widespread.

The setup involves firms with fixed capacity selling homogeneous goods to identical consumers. Firms are sufficiently small they have negligible impact on the rest of the market. Consumers know prices and can only visit one firm per period. If demand is certain and the number of firms is sufficiently large, as shown in Section 4 of Peck (2016), these assumptions yield a socially efficient, market-clearing Nash equilibrium in prices, identical to the Walrasian equilibrium. We show that the consequence of introducing aggregate demand shocks is that the equilibrium is not even second-best efficient. Firms are competitive in that they can sell to as many buyers as they want, as long as they deliver to each of them the equilibrium expected utility.[3] Utility taking does not, however, imply price taking. As Carlton (1978) noted, a firm charging more than rivals loses customers but this is compensated by a reduction in the probability of being rationed. So not all sales are lost. Conversely, deep price cuts may be necessary to utilize capacity in low demand states. This is privately expensive in terms of lost high-state revenue. The potential gains to consumers from better capacity utilization in low demand states are not captured by firms, so price tends to be above the welfare maximizing level, which in turn distorts entry incentives. In low-demand states, an entrant captures a share of revenue,

[1] Fixing prices *ex ante* may be due to menu costs, rational inattention due to difficulty of judging state, credibly preventing consumer hold-up, avoiding adverse behavioral responses from consumers, as in the snow shovel case of Kahneman, Knetsch and Thaler (1986).

[2] Under risk neutrality, the appropriate welfare criterion is expected surplus. Risk aversion requires weighting of gains and losses. Typically, there is a missing risk market and therefore market failure is immediate.

[3] The market cannot clear in all states if price must be set *ex ante*, so the concept of Walrasian equilibrium is not directly applicable. Instead, competition is again taken as Nash equilibrium with large numbers of firms. For example Rothschild and Stiglitz (1970) title their seminal paper "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information", although the equilibrium is not Walrasian.

but as there is already excess capacity, does not contribute any net benefit. This incentive to excess entry is analogous to the common pool problem. Counteracting this distortion, entry relieves rationing in high demand states. Not all of the social benefit of the increased industry capacity is obtained by an entrant as the surplus of the newly unrationed consumers is not part of the firm's gain. The balance between these externalities determines whether equilibrium involves too much waste or too much rationing. Unlike Carlton, we conclude that equilibrium is not efficient.

The paper is in four sections. Section 1 presents two simple models of rationing and waste with unit demand. Section 2 extends the models to smooth demand functions. The main result is that the equilibrium price will almost never be efficient, even with atomistic firms. Entry distortions are then analysed. To this point, the rationing rule has been first-come-fully-served. It is shown that even with efficient rationing (everyone gets an equal share of the available stock), the competitive equilibrium is inefficient.[4] In another modification, if consumers select the seller subsequent to inferring the demand state, inefficiency still arises. Section 3 places results in the context of the literature. Section 4 draws some brief conclusions.

1 Unit-demand

1.1 Shocks affect all consumers equally

A continuum of identical consumers of measure M demand at most one unit of the good. Willingness to pay, v, depends on the macro state, with support $[\underline{v}, \overline{v}]$ and probability density function $f(v)$. There are N firms, each with capacity K, initially assumed costless, so all firms are active. As $M > NK$, not all consumers can be served.

The timeline is: 1) sellers simultaneously post prices to maximize expected profits; 2) consumers observe the price and make simultaneous choices of which seller to visit. Only one seller can be visited per period; 3) the state is revealed (in Sub-section 2.6, the order of 2 and 3 is reversed). As in Deneckere and Peck (1995), given prices, consumers' choice of seller is a Nash equilibrium whilst sellers' choice of price is a Nash equilibrium. Everyone is risk neutral.

[4] Given the assumptions here, a seller could deduce the state from the demand of the first customer making it relatively easy to administer efficient rationing. Adding idiosyncratic risk to sequential arrivals would make this difficult. Burguet and Sakovics (2016) show that if sellers can make personalised offers, the unique equilibrium is the market clearing price, thereby avoiding the need to specify a rationing rule. Personalised offers may not be easily implemented though, especially where there are *ex post* macro demand shocks.

Determining the socially optimal price, p^w, is straightforward. If price exceeds $\underline{v}$, a reduction has no effect on aggregate surplus (profit plus consumer surplus) in states in which sales were already made, but creates value in the states in which sales are newly made. So, to maximise total surplus, $p^w \leq \underline{v}$.

Turning to equilibrium, suppose all firms charge p. In states with $v > p$, each seller has M/N customers, who all attempt to buy, but only K of them can be served. Revenue per firm is

$$pK \int_{p}^{\overline{v}} f(v)dv. \tag{1}$$

Consider a firm deviating to price p^* in the vicinity of p. If $p^* > p$, it attracts more consumers and it sells out in all states with $p^* < v$. When $p^* < p$, fewer consumers find the firm attractive, though this is offset by the lower probability of rationing, so the reduction in demand is finite.[5] If the upward deviation is small, fewer customers choose that seller but it still faces excess demand, selling out in all states with $p^* < v$.[6] The deviant's revenue is therefore

$$p^* K \int_{p^*}^{\overline{v}} f(v)dv. \tag{2}$$

Taking the derivative with respect to p^*, if there is an interior equilibrium, price, p^c, must satisfy

$$\int_{p^c}^{\overline{v}} f(v)dv - p^c f(p^c) = 0. \tag{3}$$

It will normally be the case that the left-hand side of (3) is positive at p^c, certainly if the tails of the distribution thin out. Even in the case of a uniform distribution, $p^c = 0.5\overline{v}$, so if $2\underline{v} < \overline{v}$, the equilibrium price exceeds the socially optimal price. Note that the equilibrium price is independent of the number of firms, as long as $N < M/K$.[7] Rationing implies that every firm has a local monopoly. Even if many

[5] Given that N is large, dispersal of the deviant's customers across the other firms has negligible effect on the expected utility they provide. The number of consumers selecting the deviant is therefore m, $dm/dp^* = -M/(v-p)N$.

[6] There is a threshold, p^*, at which rationing is eliminated and above which demand is zero. Such a large upward deviation cannot be profitable.

[7] This property is not true of other distributions, but allows for easy illustration of possibilities.

firms sell an identical product, the equilibrium coincides with the price set if there were a single owner of all the firms.[8] Increasing the number of firms, whilst decreasing the capacity of each to keep NK constant, does not affect equilibrium price, contrary to the usual expectation that making firms smaller and more numerous intensifies competition and drives price down.

Turning to entry, suppose initially that the cost of capacity is the same for all firms. First firms decide whether to enter, then they make simultaneous price choices, then consumers choose where buy and finally the state is revealed.[9] It follows from price being invariant to N in the rationing zone, that either no firms are active or else there is no rationing. An equilibrium with no sales is not necessarily socially optimal. Selling out in all states with $p^c > v$ creates consumption value at no cost. In the uniform case, an extra firm charging $\underline{v}$ creates value $0.5\left(\underline{v}+\overline{v}\right)K$, whereas the revenue earned from charging the equilibrium price is $0.5\overline{v}K$. Hence, if capacity cost, C, is such that $0.5\overline{v}K < C < 0.5\left(\underline{v}+\overline{v}\right)K$, there is no industry though it would be socially efficient to create one. Normally, it is economies of scale that create such situations but here, however small the capacity of individual firms, the problem remains.

A free entry equilibrium with positive rationing may arise if firms differ in their capacity costs. The Nth most efficient firm has capacity cost $C(N)$, with $C'(N) > 0$. So, if $0.5\overline{v}K = C(N^*)$ and $N^*K < M$, there will be a rationing equilibrium. An extra firm reduces rationing in all states with $p^c < v$, creating finite consumer surplus, so subsidising entry is expected welfare improving.

Proposition 1 *The equilibrium price exceeds the socially optimal price, given $NK < M$, and is invariant to N. There will be insufficent entry from the social perspective and possibly no firms active, even though a large industry may be socially optimal.*

1.2 Shocks affect consumers differentially

Aggregate shocks may have random effects on *ex ante* identical consumers. The natural assumption is that the distribution of valuations associated with a less negative shock first-order dominates that with a more positive shock. Carlton (1978) and

[8] The result is reminiscent of Diamond's (1971) result that, in a search model with arbitrarily large numbers of sellers, the equilibrium price equals the monopoly price. In both cases, firms do not lose customers from an incremental price increase, but the mechanism is different. In our case consumers know price before choosing a supplier so a marginal price change does affect the number of consumers. The monopoly result does not generalise to other rationing models as will be shown.

[9] Instead of entry, the results are the same if the number of firms is fixed but at the first stage they choose capacity and this is observed by buyers,

Denekere and Peck (1998) consider the case that consumers are either unaffected by a shock or else so adversely they do not value the good at all. The worse the shock, the higher the proportion of affected consumers. This is the first case considered with modifications then analysed.
There are two states. In the high state, which occurs with probability x, all N consumers value the good at $\overline{v}$. The low state involves a fraction f of the consumers valuing the good at $\overline{v}$ and the rest at $\underline{v}$. Once again, $M > NK$. Also, $fM < NK$, so there is enough capacity to satisfy all active consumers in the low-state.

Assume first that $\underline{v} = 0$. To find a pure-strategy equilibrium, suppose one seller charges p^* when all others charge p. The surplus enjoyed by a customer of the p seller is[10]

$$S = x(\overline{v} - p)\frac{NK}{M} + (1-x)f(\overline{v} - p), \tag{4}$$

where NK/M is the probability of being served in the high state. The number of customers, $m(p^*, p)$, chosing the p^* seller must therefore satisfy

$$x(\overline{v} - p^*)\frac{K}{m(p^*, p)} + (1-x)f(\overline{v} - p^*) = x(\overline{v} - p)\frac{NK}{M} + (1-x)f(\overline{v} - p). \tag{5}$$

implying that

$$m(p^*, p) = \frac{x(\overline{v} - p^*)MK}{(1-x)f(p^* - p)M + x(\overline{v} - p)NK}. \tag{6}$$

which is decreasing in p^*. The (gross) profit of the p^* seller is

$$\begin{aligned} \pi(p^*, p) &= xp^*K + (1-x)fp^*m(p^*, p) = \\ &= .xp^*K + (1-x)fp^*\frac{x(\overline{v} - p^*)MK}{(1-x)f(p^* - p)M + x(\overline{v} - p)NK}. \end{aligned} \tag{7}$$

Differentiating (7) writh respect to p^*, and then setting $p = p^* = p^c$, at an interior equilibrium,

$$\frac{[(1-x)fM + xNK][x(\overline{v} - p^c)NK - (1-x)fMp^c]}{x(\overline{v} - p^c)N^2K} = 0, \tag{8}$$

yielding equilibrium price

$$p^c = \frac{xNK\overline{v}}{(1-x)fM + xNK} < \overline{v}. \tag{9}$$

[10] Appendix A shows that we obtain the same analytical results using the Bayes' rule for calculating the customer's expected surplus.

Substituting p^c for p in (7),

$$\pi(p^*, p^c) = xK\overline{v}. \tag{10}$$

Given that the other firms charge p^c, the profit of a seller is independent of its price as long as it is in the interval that leads to rationing and waste. Outside this interval profit is lower. A deviation price below that at which a seller is at full capacity in the low state is clearly less profitable than the price at which capacity is just used and eliminating rationing by charging above $\overline{v}$ results in zero revenue. It follows that (8) does identify the unique pure-strategy price equilibrium.

Notice that from (9), price rises as N increases. With more entry, each seller has fewer low-state customers and therefore gives more weight to the price that is appropriate to the high state. This means price is increased, which just offsets the effect on profit of the decline in each firm's low-state demand .

As price is below $\overline{v}$, it is socially efficient. Adding another seller relaxes high-state rationing, augmenting expected social benefit by $xK\overline{v}$, equal to gross revenue. Hence, the equilibrium is fully efficient. This is a special case, however.

When $\underline{v} > 0$, the equilibrium price and entry may be socially inefficient. The first task is to determine the welfare maximising price. At first sight this should be $\underline{v}$ or below to make full use of capacity. This though implies rationing in the low state. Assuming it is random who gets served of those wanting to buy, some low valuation types displace those with a higher valuation. It might be preferable to set price above $\underline{v}$ to improve selection, even if total sales are thereby lower. At a price below $\underline{v}$, average value in the low state is $f\overline{v} + (1-f)\underline{v}$. The lowest $\underline{v}$ for which it is welfare maximising to set price lower is

$$v^l = \frac{\overline{v}f\,(M - NK)}{(1-f)NK}. \tag{11}$$

It will now be shown that it is possible that $\underline{v} > v^l$ so it is efficient to set a low price yet the equilibrium remains at $p^c > \underline{v}$. As a deviant's price falls from the candidate equilibrium p^c, there comes a level, p^l, at which the deviant sells out in the low state. This occurs when $m(p^l, p^c) = K$, implying

$$p^l = x\overline{v}. \tag{12}$$

If $p^l > \underline{v}$, deviation from p^c to bring in the low value types is certainly unprofitable and therefore equilibrium remains at p^c. So, if $p^l > v^l$, there is an interval into which $\underline{v}$ can fall such that equilibrium price is inefficiently high. From (11) and (12), $p^l > v^l$ iff

$$f < \frac{xNK}{M - (1-x)NK}. \tag{13}$$

When the welfare maximising price is below $\underline{v}$, it is socially efficient to have rationing in both states. The social value of an extra seller includes the value of low state sales and is $(1-x)[f\overline{v}+(1-f)\underline{v}]K+x\overline{v}K$. In a p^c equilibrium, sales are only made in the high state and profit is $x\overline{v}K$. Hence, if heterogeneous set up costs lead to an equilibrium with rationing and waste, entry is below the socially efficient level (although without price regulation, an entry subsidy would lower welfare).

Proposition 2 *If the competitive price involves rationing and waste, it rises with entry although not to the monopoly level, $\overline{v}$. If (13) holds, so $p^l > \underline{v} > v^l$, and a competitive industry is in equilibrium in the rationing/waste regime, the competitive price exceeds the socially efficient price (which involves rationing but not waste). For social efficiency, entry should exceed the competitive level.*

2 Smooth demand

When goods are perfectly divisible, it becomes apparent that inefficiency is generic. It can also be shown that entry may be excessive. Profit functions are typically not well behaved, making it necessary to use an explicit functional form, here linear demand.

The basic set up is as before. There are two macro states: in the high-state, which occurs with probability x, individual demand is $q = a - bp$, where p is price; in the low-state, the demand is $q = ad - bcp$. Demand curves cross unless $c \geq d$. If $c = d < 1$, elasticity at every price is the same in both states. More plausibly, if $c = 1$, $d < 1$, willingness to pay in the low state is reduced by the same percentage at all quantities (and therefore at any given price, elasticity is higher).[11] If there is excess demand at a shop, the rationing rule is first-come, fully-served (an alternative, "efficient rationing" is considered in Sub-section 2.5). With multiple units demanded, eliminating rationing is efficiency enhancing as the consumption value of the stock is increased.

2.1 Socially optimal price

The price that maximises aggregate welfare given the number of firms cannot involve a price so high that there is waste in both states as the extra consumption from a lower price is costless. Nor can it be optimal to set price so low there is rationing

[11] The unchanged elasticity case can also be interpreted as some consumers drop out of the market in the low state with demand by the remainder unaffected.

in both states.[12] Since marginal consumption is the least valuable, raising price increases the aggregate value of consumption even though the total number of units supplied is fixed. Hence, there are three possible optimal regimes: (*i*) rationing in the high state and waste in the low state; (*ii*) market clearing in the high state and waste in the low state; (*iii*) market clearing in the low state and rationing in the high state. The rationing/waste regime is central, with its range defining where the other regimes take over. Attention is therefore focussed on (*i*), in which a higher price enables more customers to be served in the high state which, despite each unrationed consumer buying less, increases the average utility value of consumption. The welfare cost of a higher price is the lost consumption in the low state. For social optimallity, price should be increased to the point that the value of the marginal loss of consumption in the low state equals the value of the more efficient consumption in the high state.

To maximise expected surplus, it is optimal that all firms charge the same price.[13] Every shop has an equal number of potential customers and, in a rationing/waste regime, the total value (area under the individuals' demand curve) of all goods consumed is

$$W = x\frac{(a-bp)(a+bp)}{2b}\frac{NK}{a-bp} + (1-x)\frac{(ad-bcp)(ad+bcp)}{2bc}M, \tag{14}$$

where $NK/(a-bp)$ is the number of customers served in the high state. At a maximum,

$$\frac{dW}{dp} = \frac{xNK}{2} - (1-x)bcpM = 0, \tag{15}$$

yielding welfare maximising price

$$p^w = \frac{xNK}{2(1-x)bcM}. \tag{16}$$

It is easily checked that the second-order condition necessarily holds at p^w.[14]
The parameter d, which only affects demand in the low state and not its slope, does not appear in (16). As the welfare effect of an infinitesimal price change is

[12] Rationing involves inefficiency since the value of the marginal consumption of served consumers is less than the average consumption value of excluded consumers. Under unit demand, this inefficiency would not arise.

[13] Introducing a second price in addition to p^w and maximising welfare with respect to the proportion of sales at the new price yields an optimum at zero.

[14] The second-order condition is $d^2W/dp^2 = -(1-x)bcM < 0$.

proportional to the change in low-state sales ($pdq/dp = -pbc$), which does not depend on d, neither does p^w.

For p^w to be within the rationing/waste regime,

$$M(a - bp^w) > NK \text{ and } M(ad - bcp^w) < NK, \tag{17}$$

or

$$\underline{a} = \frac{2(1-x)cNK + xNK}{2(1-x)cM} < a < \frac{(2-x)NK}{2(1-x)dM} = \overline{a}. \tag{18}$$

The length of the interval within which a must fall if the rationing/waste regime applies is therefore

$$\overline{a} - \underline{a} = \frac{[c(2 - x - 2(1-x)d) - xd]NK}{2(1-x)cdM}. \tag{19}$$

Should $c = d$, then $\overline{a} - \underline{a} = (1-c)NK/cM > 0$, implying that for a rationing/waste regime to be welfare maximising, it is required that $c < 1$. When $a > \overline{a}$, it is welfare maximising to set price to clear the market in the high state and when $a < \underline{a}$, it is optimal to set price to clear the market in the low state.
An immediate feature of (16) is that the welfare maximising price is increasing in the number of firms. Given the price, as the number of firms rises, the number of customers served in the high-state increases. This means that the benefit of raising price through more efficient rationing is greater the more firms there are. As low-state sales are independent of the number of firms, the welfare cost of increasing price is independent of the number of firms.

2.2 Competitive price

A SPNE in prices is sought. At the first stage, firms make simultaneous price choices. Then, at the second stage, consumers distribute themselves across firms to equalize their expected utility, as in Deneckere and Peck (1995). The consequences of a firm offering a different price to its rivals depend on the regime. If a proposed pure-strategy symmetric equilibrium has rationing in the high state and waste in the low state, a lower price will attract extra consumers, but only a limited number of them, as the attraction of the lower price will be offset by an increased chance of rationing.

If $m(p^*, p)$ consumers patronise a particular firm charging p^*, each of them has expected surplus

$$\begin{aligned} S &= x\left[\frac{(a-bp^*)(a+bp^*)}{2b} - p^*(a-bp^*)\right]\frac{K}{(a-bp^*)m(p^*,p)} \\ &\quad +(1-x)\left[\frac{(ad-bcp^*)(ad+bcp^*)}{2bc} - p^*(ad-bcp^*)\right], \end{aligned} \tag{20}$$

where the term in the first brackets is the surplus obtained in the high state if a consumer is served, $K/(a - bp^*)m(p^*, p)$ is the probability of being served, and the term in the second brackets is the surplus in the low state. Consumers distribute themselves between firms to maximise their utility given the choices of others. In a competitive equilibrium, firms are utility takers, so a price change by an individual store has a negligible effect the surplus obtained from the other shops. In response to a change in the price of an individual store, the number of customers per store, $m(p^*, p)$, must therefore adjust to maintain the expected surplus from visiting it. From (20),

$$\frac{dm(p^*, p)}{dp^*} = -\frac{b[2(1-x)(ad - bcp^*)m(p^*, p) + xK]m(p^*, p)}{x(a - bp^*)K}, \tag{21}$$

given the price charged by its competitors. In a rationing/waste equilibrium, every shop sells out in the high-state, so the firm's expected profit (revenue) is

$$\pi(p^*, p) = xp^*K + (1-x)(ad - bcp^*)p^*m(p^*, p). \tag{22}$$

The necessary condition for profit-maximisation by an individual firm, given the price of its competitors, is therefore

$$\frac{d\pi(p^*, p)}{dp^*} = xK + (1-x)\left[(ad - 2bcp^*)m(p^*, p) + p^*(ad - bcp^*)\frac{dm(p^*, p)}{dp^*}\right] = 0. \tag{23}$$

In a symmetric Nash, rationing/waste equilibrium, $m(p^*, p) = M/N$, and from (23) it follows that if there is an equilibrium price in this regime, p^c, it satisfies

$$xK + (1-x)\frac{M}{N}\left[(ad - 2bcp^c) - p^c(ad - bcp^c)\frac{b[2(1-x)\frac{M}{N}(ad - bcp^c) + xK]}{x(a - bp^c)K}\right] = 0. \tag{24}$$

For p^c to be an equilibrium, the second-order condition for the individual firms must also be satisfied, an issue addressed below.

2.3 The efficiency of competitive equilibrium

A tractable closed form solution for the competitive price, p^c, is not available, but evaluating (23) at $p = p^w$, $m(p^*, p^w) = M/N$, and simplifying,

$$\frac{d\pi(p^*, p^w)}{dp^*} = \frac{2(1-x)^2a^2d(c-d)M^2}{[2(1-x)acM - xKN]N}. \tag{25}$$

From (25), it is immediate that $p^w \neq p^c$ if $c \neq d$. The numerator of (25) is positive and the denominator is positive if $a > xNK/2(1-x)cM = \widetilde{a}$. We have that $\underline{a} - \widetilde{a} = NK/M > 0$, hence, if welfare maximisation involves rationing/waste, p^w is not an equilibrium. When $c = d$, at first sight the equilibrium is efficient, but although the FOC for p^w and p^c coincide, the SOC for the firms' individual maximisation problem cannot be satisfied. Specifically, the SOC evaluated at $p = p^w$, $m(p^*, p^w) = M/N$, and $c = d$ is

$$\frac{d^2\pi(p^*, p^w)}{dp^{*2}} = \frac{(1-x)[2(1-x)acM + xNK]bcM}{[2(1-x)acM - xNK]N}. \tag{26}$$

The numerator of (26) is positive, and again the denominator is positive if $a > \widetilde{a}$. Hence, p^w is not an equilibrium even if $c = d$. In addition, it is impossible that there can be an equilibrium in which the equilibrium price is below the welfare maximising price. From (25), at p^w, $\pi_1(p^w, p^w) > 0$. In Appendix B we show that $\pi_1(p^*, p^w) > 0$ for all $p^* < p^w$. Hence, the result follows and an equilibrium must involve excessive waste.

Proposition 3 *If the number of firms is fixed and welfare maximisation involves rationing/waste, equilibrium involves an inefficiently high price.*

Profit functions are not well behaved, making it hard to find equilibria. One possibility, associated with low d, is that the SOC of the invidual firms hold at the solution to (25). Small deviations are then unprofitable, but large upward price deviations into the zone where there are no low-state sales are advantageous. An equilibrium at the market-clearing price in the high state, $p^h = (aM - NK)/bM$, may then exist. The Figure reports parameter values that generate this outcome and plots the relevant functions. The first panel shows shows welfare (expected profit plus expected surplus) as a function of price. Below the market-clearing price in the low state, $p^l = (adM - NK)/bcM$, capacity is fully used even in the low-state. Above p^h, rationing is eliminated in the high-state. When price exceeds $p^0 = ad/bc$, demand is zero in the low state. Beyond p^0, price increases raise allocative efficiency by distributing the stock of the good to more consumers, so raising average consumption value. Welfare would definitely drop above p^h, as there is no gain to leaving some of the good unsold in the high-state. In this case, welfare is maximised at p^w, in the rationing/waste regime.
The second panel shows the profit function of an individual firm for $p^* \leq p^h$ when all other firms charge p^h. Initial price cuts are insufficient to attract low-state sales so lower profit. At prices below p^0, such sales are attracted despite $m(p^*, p^h) > M/N$, but the price effect dominates and decline to $\overline{p} = [adm(\overline{p}, p^h) - K]/bcm(\overline{p}, p^h)$, where

the deviant sells out in the low state.[15] Below $\overline{p}$, profit falls proportionately with price. Above p^h, consumers cannot be compensated with lower rationing for a higher price, so demand collapses to zero.[16] It follows that p^h is an equilibrium.
The final panel shows the deviant's profit when all other firms charge p^c. Small deviations are unprofitable, but a deviation to the price where the deviant sells out in the high state, $\overline{\overline{p}} = [am(\overline{\overline{p}}, p^c) - K]/bm(\overline{\overline{p}}, p^c)$, breaks the equilibrium[17] (the price at which the deviant sells out in the low state is not the same as in the second panel as the deviation is from a different price[18]).

Proposition 4 *If the number of firms is fixed and the welfare maximising price involves rationing/waste, it is possible that the market equilibrium is at $p^h > p^w$, with rationing eliminated.*

When d is relatively high, the SOC may not hold at the solution of (25). There may not be an equilibrium at p^h either as not too much of aprice cut is needed to capture substantial low-state sales. In addition, p^l is not an equilibrium as high-state rationing is high and the discouragement effect of a price increase is easily offset by a reduction in high-state rationing. It is therefore possible that no single price equilibrium exists. The parameter set of the figure with d increased to 0.49 has this property. In these cases we conjecture that equilibrium involves a continuum of prices at the highest of which there is no rationing and at the lowest, no waste. As a step to showing this, we looked at examples where the SOC does not hold at p^c but resticted firms to choosing either p^c or a finitely different price. If the two prices are not too different, in equilibrium some firms offer the higher price with low rationing and others the low price, offset for consumers by higher rationing.

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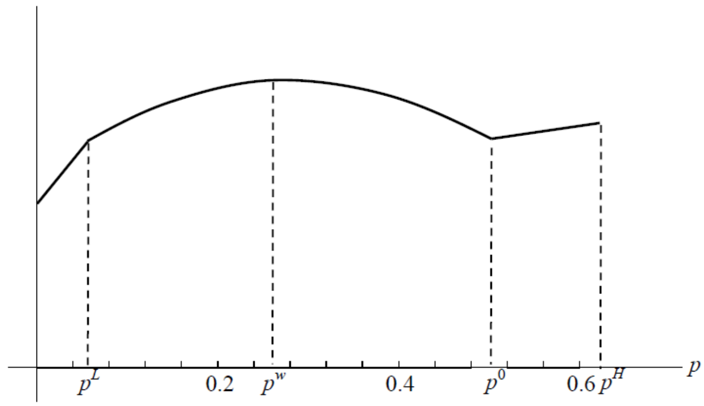
[15] The deviant's price $\overline{p}$ derives from $(ad - bc\overline{p})m(\overline{p}, p^h) = K$. At the deviant's $m(\overline{p}, p^h)$, when all other firms charge p^h, we have
$\overline{p} = \frac{ad}{bc} - K\frac{2xcN^2K}{xa(c-d)M^2+\sqrt{4xcM^2N^2K^2+(xa(c-d)M^2)^2}}$.

[16] Consumers of the deviant distribute themselves over the large number of remaining firms creating negligible rationing at each and therefore no change in the surplus available there. See Peck (2016, Section 4) for the similar deterministic case.

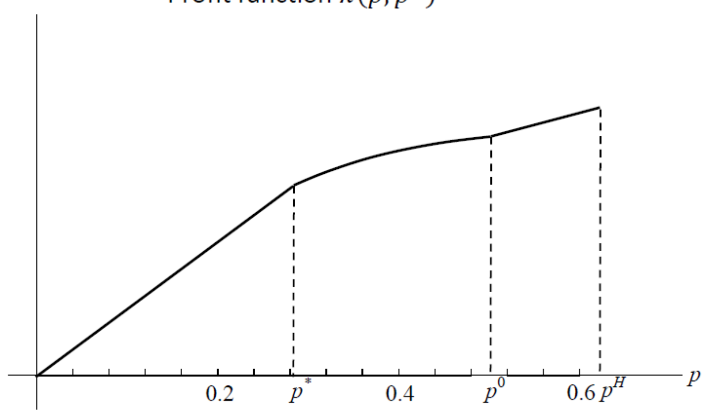
[17] The deviant's price $\overline{\overline{p}}$ derives from $(a - b\overline{\overline{p}})m(\overline{\overline{p}}, p^c) = K$. At the deviant's $m(\overline{\overline{p}}, p^c)$, when all other firms charge p^c in the rationing/waste zone, we have
$\overline{\overline{p}} = \frac{a - \frac{\sqrt{(1-x)a^2d^2M + bcp^c((1-x)bcMp - xNK) - ac(2(1-x)bdMp^c - xNK)}}{\sqrt{x}\sqrt{c}\sqrt{M}}}{b}$,
where p is the price of rivals.

[18] At the deviant's $m(\overline{p}, p^c)$, when all other firms charge a price in the rationing/waste zone, we have
$\overline{p} = \frac{aMK(xc+(2-x)d) - \sqrt{MK^2(4bcp^c((1-x)bcp^cM - xNK)) - 4ac(2(1-x)bdp^cM - xNK) + a^2M(4(1-x)d^2 + x^2(c-d)^2))}}{2bcMK}$,
where p^c is the price of rivals.

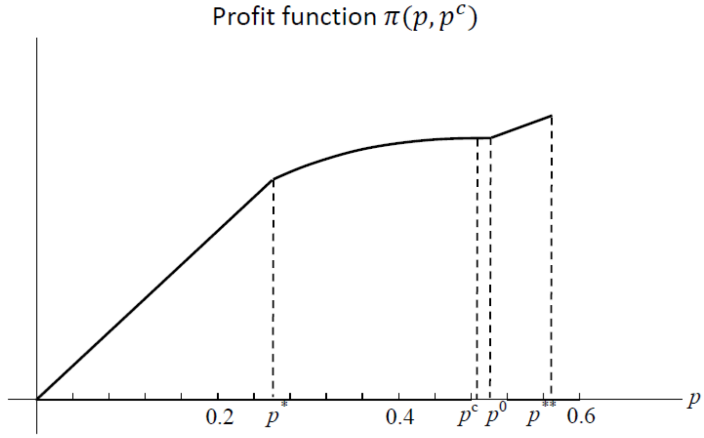
Welfare



Profit function $\pi(p, p^h)$



Profit function $\pi(p, p^c)$



$x = 0.5,\ n = 230,\ k = 4,\ d = 0.45,\ M = 2000,\ a = 1.1,\ b = 1,\ c = 1.$

2.4 Entry

So far, the number of firms has been fixed. Once again, let firms differ in their capacity costs. The Nth most efficient firm has capacity cost $C(N)$, with $C'(N) > 0$.[19] As a first step in establishing whether equilibrium is efficient, at the welfare optimum (with the optimal *ex ante price)*,

$$\frac{dW(p^w(N), N)}{dN} - C'(N) = \frac{x[2(1-x)acM + xKN]K}{4(1-x)bcM} - C'(N) = 0. \tag{27}$$

Market entry is driven by profit which, evaluated at a p^h equilibrium and common N, yields

$$\frac{dW(p^w)}{dN} - \pi(p^h, p^h) = \frac{x[NK(4(1-x)c + x) - 2(1-x)acM]K}{4(1-x)bcM}.$$

The sign of this expression is ambiguous even when evaluated at parameter sets at which p^w involves rationing/waste and a p^h equilibrium, as numerical examples confirm. In particular when the number of firms is low, the profitability of the no rationing equilibrium is high, leading to excess entry.

Proposition 5 *If welfare maximisation involves rationing/waste, equilibrium entry may be above or below the socially efficient level.*

In the numerical case above, given N, a monopolist would also set price at p^h, which is market clearing in the high state. Unlike the unit demand case, entry lowers price so as to maintain market clearing in the high state. Under competition, entry proceeds till the marginal firm just covers its cost. A monopolist recognises that this entrant depresses the profit of existing firms. So, as usual, a monopolist chooses lower industry capacity and higher price than a competitive industry.[20]

2.5 Efficient rationing

It might be thought that the inefficiency of equilibrium is due to the nature of the rationing. It has so far been assumed that consumers buy all they want at the

[19] It follows from (14) and (16) that W is increasing in N so, for an internal solution, $C'(N)$ must be sufficiently high.

[20] Under competition, if $N = 280$, rather than 230, the equilibrium is still at p^h, but a monopolist would set price above the new p^h. That is, under monopoly, there would be waste in both states, but in the competitive solution only in the low demand state. Of course, the monopolist would then cut N below 280, so again its capacity would be below the competitive level and price higher.

ruling price or else obtain nothing at all. An alternative, if feasible, is that if there is excess demand, everyone obtains an equal share of the available supply. This procedure equalises the value of marginal consumption which, granted downward sloping demand, maximises total consumption value. Under efficient rationing, price should be set to clear the market in the low state, zero if there are still unsold stocks when the good is free. This latter case is easy to analyse. Though the efficient price is zero, this cannot be an equilibrium. A firm deviating by setting a positive price still makes some sales as its remaining consumers obtain more of the good in the high state. Its profits are therefore increased by deviation. More formally, assuming that even at zero price there is excess demand in the low state, but excess demand in the high price, with everyone having an equal share,

$$W = x\frac{2a\frac{M}{N}K - K^2}{2b\left(\frac{M}{N}\right)^2}M + (1-x)\frac{(ad-bcp)(ad+bcp)}{2bc}M, \tag{28}$$

where the surplus measure in the high state is the area below the demand curve between 0 and NK/M. From (28),

$$\frac{dW}{dp} = -(1-x)bcpM < 0. \tag{29}$$

The efficient price is therefore zero. At any price above zero consumption is lost in the low state without gain, whilst in the high state consumption is unchanged (unless price is so high that there is excess supply in which case there is a welfare loss in both states). Turning to equilibrium,

$$\begin{aligned} S &= x\left[\frac{2am(p^*,p)K - K^2}{2bm^2(p^*,p)} - p^*\frac{K}{m(p^*,p)}\right] \\ &\quad +(1-x)\left[\frac{(ad-bcp^*)(ad+bcp^*)}{2bc} - p^*(ad-bcp^*)\right], \end{aligned} \tag{30}$$

where the surplus area in the high state is calculated from 0 to $K/m(p^*,p)$. From (20),

$$\frac{dm(p^*,p)}{dp^*} = -\frac{[(1-x)(ad-bcp^*)m(p^*,p) + xK]bm^2(p^*,p)}{x[(a-bp^*)m(p^*,p) - K]K}. \tag{31}$$

The equilibrium condition is,

$$\frac{d\pi(p^*,p)}{dp^*} = xK - (1-x)bcp^*m(p^*,p) + (1-x)(ad-bcp^*)\left[m(p^*,p) + p^*\frac{dm(p^*,p)}{dp^*}\right] = 0,$$

which evaluated at the welfare maximising price of zero, yields

$$\frac{d\pi(p^*,0)}{dp^*} = xK + (1-x)ad\frac{M}{N} > 0. \tag{32}$$

Proposition 6 *Under efficient rationing, the equilibrium may not be efficient.*

When market clearing in the low state involves a positive price, it is possible but not necessary that $p^c = p^w$.[21]

2.6 Consumers choose supplier post realisation

If consumers know their demand before choosing where to buy but the other assumptions are maintained they can infer the macro state. Assuming firms must still set price prior to the realisation, the welfare maximising price remains unchanged, but the equilibrium is different. Price cannot exceed market clearing in the low state. If it did, an individual firm cutting price by an infinitesimal amount would sell out in the low state so increase its profit. If price is raised above the market clearing level, no buyer would select the firm in the low state as rationing at these firms would be infinitesimal.[22] Thus, the equilibrium price is low-state market clearing.[23] From (18), the welfare maximising price is above the market clearing price in the low state if

$$a > \underline{a} = \frac{2(1-x)cNK + xNK}{2(1-x)cM}. \tag{33}$$

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[21] In this case, $adM > NK$. Substituting the market clearing price in the shop's equilibrium condition,
$\frac{d\pi(p^*,p)}{dp^*} = \frac{-(x(1-x)a^2(c-d)dM^2 - N^2K^2(1+x-2cx-x^2(1-c)) + aNKM(d+x(2-c)d-xc(2-x)-x^2(2-c)d)}{x(a(c-d)M+(1-c)NK))N}$.

Numerical examples show this expression can be positive or negative. The latter case implies $p^c = p^w$, with low state market clearing, since it cannot be profit maximising to have rationing in both states.

[22] If one firm charges a higher price p, when all others charge $\widehat{p}$, the number of consumers, n, at each low price firm satisfies $S(\widehat{p})\left(K/q(\widehat{p})n(N-1)\right) = S(p)$. So, $(N-1)(dn/dp) = q(\widehat{p})(N-1)^2 n^2/q(p)KS\left(\widehat{p}\right)$. Taking a first-order Taylor approximation, the price change required to eliminate all the deviant's customers is therefore $q(p)KS\left(\widehat{p}\right)/q(\widehat{p})M(N-1)^2$, which tends to zero as N tends to infinity.

[23] Bertrand competition with fixed capacity is often modelled as involving mixed strategy equilibria as in Kreps and Scheinkman (1983). The reason is that it is implicitly assumed that those not served at a low price firm can switch to the high price firm. This creates an incentive to raise price above market clearing. The solution here resembles the market clearing equilibrium in Peck (2016) for the case of deterministic fixed-price per unit.

Proposition 7 *Under ex post consumer choice with the number of firms given, equilibrium price is below the efficient price if $a > \underline{a}$, and is otherwise equal to it.*

3 Related work

Carlton (1978) pioneered the analysis of competitive equilibrium in the presence of stochastic demand and endogenous but predetermined price.[24] He notes that the competitive equilibrium is not welfare maximizing when there is risk aversion or consumers are heterogenious but when these features are absent efficiency is achieved. Carlton's model differs from ours in various respects, such as variable firm capacity and allowing idiosyncratic shocks as well as aggregate shocks. These are not the reasons for the different conclusions under risk neutrality and homogeneous consumers. Carlton restricts deviations to capacity and price pairs that preserve the number of buyers selecting a seller, presenting an informal dynamic justification (see also Carlton, 1991).[25] Our equilibrium allows the number of consumers selecting a seller to vary, subject to the offer remaining competitive in terms of utility.
In a further analysis, Carlton (1979) studies a vertical production chain in a framework with *ex ante* pricing. There is a private incentive for downstream firms to vertical integrate in whole or part, which affects the risk borne by the supplying industry, an externality leading to market failure. Horizontal externalities, the source of market failure in our analysis, are not examined.

Deneckere and Peck (1995) analyse a set up similar to ours but for unit demand. *Ex post,* consumers either value the good at v or not at all with the proportion in the latter state random. The efficiency of equilibrium is not analysed. Peters (1984) has smooth demand, but the randomness is due to the law of large numbers not holding in the mixed stategy buyer equilibrium. Again, welfare is not analyzed. Myatt and Wallace (2016) study a price setting industry selling differentiated goods under demand uncertainty. Their primary interest is in the causes and consequences of information acquisition. It is assumed that firms have quadratic costs, but rationing is not considered.

Our results have some resemblance to those of Mankiw and Whinston (1986) for non stochastic oligopoly. As price exceeds marginal cost under oligopoly, it is too high from the viewpoint of social efficiency. Assuming price cannot be regulated, entry

[24] Optimal stockholding with exogenous prices, sometimes known as the news vendor problem, can be traced at least to Edgeworth (1888).

[25] "When firms remain competitive by offering the given level of utility, they randomly receive their equal share of the L customer" (p. 575).
See also his eq (4), and the discussion in Deneckere and Peck (1995).

may be excessive or insufficient. The business stealing effect involves the entrant capturing some of the profit of incumbents, a negative externality. Opposing this, a new product creates consumer surplus not captured by the entrant, a positive externality. The divergence between the private and social benefit can thus be of either sign. As the number of firms becomes large (say as fixed costs are decreased), market power shrinks and efficiency prevails. In our model, as the number of firms increases they become utility takers but market failure generally persists.

4 Conclusions

The market failure analysed here is intrinsic to competitive equilibrium under stochastic demand and ex-ante price setting. Welfare naturally falls short of what is achievable if price could be adjusted always to clear the market. Our point is different. Accepting price must be chosen before demand is known, the market normally fails to provide efficient incentives. Price tends to be excessive given the number of firms. Though a high price eliminates rationing, which is in itself efficient, it does so by creating an even worse problem of wasted capacity. Entry signals are also distorted. Adding a seller relieves rationing, so yields consumer benefits not fully captured by the entrant. Conversely, when demand is low, there is excess capacity. An entrant obtains their share of revenue in this state though the they create no social benefit. The externalities arising in the two states are independent but offsetting though only in special cases exactly so. Inefficiency is not due to the nature of the rationing. Even with efficient rationing, the market equilibrium can be improved by intervention, as is also true if the order of consumer choice is reversed. The general message is that stochastic demand implies elements of price setting power, even though all firms are utility takers. A price change necessarily differentiates a firm's product by affecting rationing probabilities, even if there is no rationing in equilibrium. Price tends to be too high, with ambiguous effects on the efficiency of entry decisions. The results can be directly translated into a labor market setting. If employers post wages before supply conditions are resolved, however competitive the labor market, wages will tend to be inefficiently low with excessive vacancies.

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Appendix A

Using Bayes' rule, the number of customers, $m(p^*, p)$, choosing the deviant must satisfy

$$\begin{aligned} S &= \frac{x}{x+(1-x)f}(\overline{v}-p^*)\frac{K}{m(p^*,p)} + \left(1 - \frac{x}{x+(1-x)f}\right)(\overline{v}-p^*) = \\ &= \frac{x}{x+(1-x)f}(\overline{v}-p)\frac{NK}{M} + \left(1 - \frac{x}{x+(1-x)f}\right)(\overline{v}-p). \end{aligned}$$

We have

$$m(p^*,p) = \frac{x(\overline{v}-p^*)MK}{(1-x)f(p^*-p)M + x(\overline{v}-p)NK},$$

which is equivalent to the expression in (6).

Appendix B

Proof that $\pi_1(p^*, p^w) > 0$ for all $p^* < p^w$:
writing $p^* = p^w - \varepsilon$,
$\frac{d\pi}{d\varepsilon} = \frac{1}{xN^2K(-2(1-x)acM-2(1-x)bcM\varepsilon+xNK)^2}[(1-x)bcM8(1-x)^3a^3cd^2M^3+$
$+(4(1-x)bcM\varepsilon + xNK)(-2(1-x)bcM\varepsilon + xNK)^2+$
$+2(1-x)aM(-2(1-x)bcM\varepsilon + xNK)(2(1-x)bc(3c+2d)M\varepsilon + xNK(3c-2d))+$
$+4(1-x)^2a^2cM^2(8(1-x)bcdM\varepsilon + xNK(3c-2d))]$.
This is positive if $-2(1-x)bcM\varepsilon + xNK > 0$, which is satisfied when $\varepsilon < \frac{xNK}{2(1-x)bcM} = p^w$, that is, for each $p^* > 0$, $p^w - p^l = [(2-x)NK - 2(1-x)adM]/[2(1-x)bcM] < p^w$.

A few notes on derivations (not for publication)

Derivation of total value of consumption in (14):
the total value of consumption at price p (surplus plus revenue) is:
$p^*(a-bp^*) + \frac{1}{2}(a-bp^*)\left(\frac{a}{b} - p^*\right) = \frac{1}{2b}\left(a^2 - b^2p^{*2}\right) = \frac{(a-bp^*)(a+bp^*)}{2b}$.
Derivation of (15):
$\frac{dW}{dp^*} = \frac{1}{2}(1-x)(ad-bcp^*)M - \frac{1}{2}(1-x)(ad+bcp^*)M - \frac{1}{2}xNK = \frac{xNK}{2} - (1-x)bcp^*M$.
Derivation of (21):
$\frac{dS}{dm} = \frac{xK\left[p^*(a-bp^*) - \frac{(a-bp^*)(a+bp^*)}{2b}\right]}{(a-bp^*)m^2}$.
$\frac{dS}{dp^*} = \frac{xK\left[\frac{1}{2}(a-bp^*) - \frac{1}{2}(a+bp^*) - a + 2bp^*\right]}{(a-bp^*)m} + \frac{xbK\left[\frac{(a-bp^*)(a+bp^*)}{2b} - p^*(a-bp^*)\right]}{(a-bp)^2m} - (1-x)(ad-bcp^*)$.

Simplifying: $\frac{dm}{dp^*} = -\frac{b[2(1-x)m(ad-bcp^*)+xK]m}{x(a-bp^*)K}$.

Derivation of (27):

$W(p^w(N), N) = \frac{4(1-x)^2a^2d^2M^2-4x(1-x)acMNK-x^2N^2K^2}{8(1-x)bcM}$.

$\frac{dW(p^w(N),N)}{dN} - C'(N) = \frac{4x(1-x)acMK+2x^2NK^2}{8(1-x)bcM} - C'(N)$.

Simplifying: $\frac{dW(p^w(N),N)}{dN} - C'(N) = \frac{xK[2(1-x)acM+xKN]}{4(1-x)bcM} - C'(N)$.

Derivation of (31):

$\frac{dS}{dm} = x\left[\frac{aK}{bm^2} + \frac{(2am-K)K}{m^2} - \frac{p^*K}{bm^3}\right]$.

$\frac{dS}{dp^*} = (1-x)\left[\frac{1}{2}(ad-bcp^*) - \frac{1}{2}(ad+bcp^*) + 2bcp^* - ad\right] - \frac{xK}{m}$.

Simplifying: $\frac{dm}{dp^*} = -\frac{b[(1-x)m(ad-bcp^*)+xK]m^2}{x[m(a-bp^*)-K]K}$.