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CYCLICAL MACKEY-GLASS MODEL FOR OIL BULL SEASONALS

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Forecasting crude oil prices remains one of the greatest challenges encountered by economists and econometricians. Oil prices are clearly characterized by unpredictable and volatile price movements. Supply is inelastic in the short run, and the future position of prices depends on how future demand evolves. Thus, an additional demand for oil triggers speculation and prices escalate more quickly than they would have done otherwise.[1] Demand for crude oil is influenced by two major seasonal variations in the year with the summer driving/hurricane season and the winter heating season. In order to better understand underlying trends in

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the market, there is clearly a need to devote greater attention to these seasonal effects impacting oil prices and to develop the appropriate models.

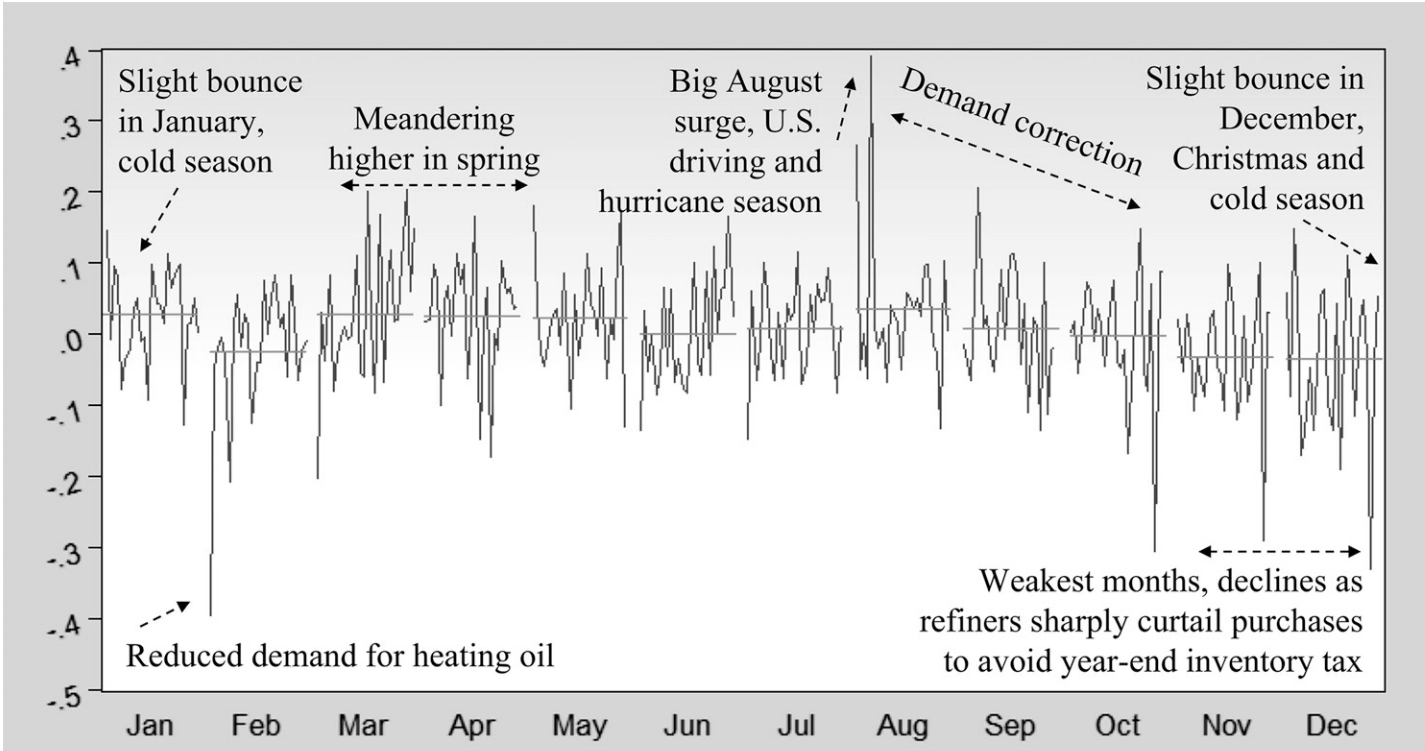
During the study period 1973 to 2008, crude oil prices showed some interesting seasonal features (figure 1). From year to year, the price showed greatest strength on average in August. This surge can be attributed to a number of factors. One is anticipation of the hurricane season in the Gulf of Mexico and hence possible disruptions in supply spawned by meteorological phenomena. Another is the fact that August is often the heaviest vacation month, driving very strong demand for gasoline and thus exerting upward pressure on oil prices. Once a rational bull begins, oil prices rise. As the summer driving season ends and the weather gets colder, the demand for all petroleum products wanes. This slowdown in demand is coupled with an increase in supply caused by the sharp curtailing of purchases by refiners to avoid year-end inventory taxes, which explains the pullback in prices during the October–November period.[2]

Then, starting from December through January, the crude oil seasonal uptrend develops. For a variety of reasons, including high heating fuel demand and the Christmas travel season, oil prices and oil stocks tend to do well in the winter months. They are a great winter speculation as demand is tied to temperature, rising as the temperatures drops. Given that seasonal weather variations remain largely unpredictable, forecasting demand remains well-nigh impossible. Consequently, additional demand caused by a cold spell triggers speculation and drives up prices. Thus, a seasonal rational bubble can form.

Once the underlying assumption that there are different types of agents with heterogeneous expectations active in the market has been acknowledged, such heterogeneous speculators' interactions on both the supply and demand sides can be seen to be responsible for major swings in oil prices over both periods. The resultant uncertainty and anxieties as to future supply exert pressure to make crude oil prices increasingly volatile, encouraging even more speculative behavior. Then all it takes is additional demand caused by the emergence of a fear factor or a climate hazard to turn the seasonal speculative situation into a speculative bull.[3]

Econometrically, modeling oil seasonal speculation by nonlinear dynamics has aroused considerable interest. The classical seasonal linear models, based on Seasonal Autoregressive Integrated Moving Average (SARIMA) models, are not strongly fitted. Thus, analyzing seasonal behavior in the presence of terms of nonlinear structures has become a necessity. Many studies have used nonlinear processes to detect the presence of seasonal behaviors in time series structures. This class of models was introduced by P. Franses and M. Ooms, who suggested a periodic autoregressive procedure.[4] Their model highlights the importance of considering seasonal behavior in the presence of a nonlinear stochastic process. Meanwhile, W. Guiming and L. Getz used the stochastic approach of a Basic Structural Model (BSM)—a state-space time-series model—exhibiting seasonal

Figure 1
OIL BULL SEASONALS, INDEXED MONTHLY, 1973 TO 2008



and multi-annual variations in abundance.[5] Then, in order to take into account complex structures, C. Kyrtsou and M. Terraza introduced seasonal chaos-stochastic processes that allowed seasonal fluctuations in stock prices to be better understood.[6] L. Ferrara and D. Guégan proposed a Seasonal Cyclical Long-Memory model, which includes generalized long-memory processes and seasonal long-memory processes.[7]

Despite the fact that oil-stock bull seasonals form a significant aspect of oil price time series, the models cited above do not begin to address this kind of anomaly. The present paper seeks to investigate the role of market speculation in oil price hikes during the two identified periods (winter and summer). Special attention is devoted to the hypothesis whereby seasonal speculative activities by heterogeneous speculators are responsible for prices swings. To this purpose, a modification of the Mackey-Glass equation is proposed that takes the rhythm of seasonal frequency into account (referred to hereafter as the Seasonal Cyclical Mackey-Glass model). Using this kind of modeling to forecast oil prices appears to be an attractive alternative due to its unique ability to model seasonal effects in the presence of deterministic behavior.

The structure of the paper is as follows. We provide a description of our stylized model of the oil market with heterogeneous interacting traders and an introduction to Seasonal Cyclical Mackey-Glass models. This is followed by a description of the data used and offers the empirical and estimated results. The paper ends with concluding remarks as to the possible practical applications of the method.

A Stylized Model

The model proposed in this paper is inspired by the chartist-fundamentalist approach, which has proven to be fairly successful in replicating some significant stylized facts relating to the oil market.[8] This model's underlying assumption is that there are different types of agents with seasonal heterogeneous expectations active in both winter-summer seasons in the market.

The group of speculators is divided into fundamentalist and chartist groups. The fundamentalists' seasonal demand for oil is based on the difference between the price at time t and the expected price at time $t + 1$ for season s, where s represents the winter and summer seasons.

$$D^F_{t,s} = a^F \left[E^F_{t,s}\left(P_{t+1,s}\right) - P_{t,s} \right] \tag{1}$$

where $P_{t,s}$ is the price over period t and season s. a^F represents a positive reaction parameter and E the expectation operator. The seasonal demand under the fundamentalists' approach will increase as they expect the future price to be higher than the current price during the season (s) and lower outside it. The fundamentalist expected seasonal price is given by:

$$E^F_{t,s}\left(P_{t+1,s}\right) = P_{t,s} + b^F_1\left(P_{t,s} - F_{t,s}\right)^+ + b^F_2\left(P_{t,s} - F_{t,s}\right)^- \tag{2}$$

where $F_{t,s}$ is the fundamental price in period t at season s. The equation shows that the price movement expected by fundamentalists is caused by deviation of the price from the fundamental value during the season. Fundamentalists' seasonal reactions to overvaluation (undervaluation) is expressed by $b_1^F \in [-1,0]$($b_2^F \in [-1,0]$) and expected to be negative since, over the season s, fundamentalists will expect the oil price to decrease (increase) if the current price is above (below) the fundamental value. Whenever b_1^F equals b_2^F, there is a symmetric reaction to overvaluation and undervaluation.

The second group of speculators goes under the name of chartists. These speculators apply a very simple type of technical analysis to form their expectations about future prices. The seasonal demand of chartists is linearly conditional on the expected price changes.

$$D^C_{t,s} = a^C\left[E^C_{t,s}\left(P_{t+1,s}\right) - P_{t,s}\right] \tag{3}$$

where a^C denotes a positive reaction parameter. This implies that demand will rise as chartists expect the future price to be higher than the current price in the same season s. Chartists' seasonal expectations are given by:

$$E^C_{t,s}\left(P_{t+1,s}\right) = P_{t,s} + b^C_1\left(P_{t,s} - P_{t-1,s}\right)^+ + b^C_2\left(P_{t,s} - P_{t-1,s}\right)^-. \tag{4}$$

A distinction is made between an upward or downward trend, or past price decrease and increase. Since technical traders expect trend movements to continue in the same direction, we expect both b_1^C and b_2^C to be positive. Negative parameters would imply contrarian behavior. If $b_1^C > b_2^C$, chartists react more to an increase in price. On the other hand, if $b_1^C < b_2^C$, chartists will be more eager to sell in a downtrend than to buy in an uptrend.

Total market seasonal demand for oil consists of the real demand plus the weighted average for seasonal demand from technical traders and fundamentalists:

$$D^M_{t,s} = D^R_t + W_{t,s}D^F_{t,s} + \left(1 - W_{t,s}\right)D^C_{t,s} \text{ and}$$

$$W_{t,s} = \left[1 + exp\left(-\gamma\left[\frac{A^F_{t,s} - A^C_{t,s}}{A^F_{t,s} + A^C_{t,s}}\right]\right)\right]^{-1} \tag{5}$$

where $W_{t,s} \in$ <0,1> is the share of the fundamentalists in the market, such that $1 - W_{t,s}$ is the chartist fraction in period t over the same season. Parameter γ is the intensity of choice and represents the extent to which performance of a given strategy determines whether it is adopted or not. With $\gamma > 0$, a strategy that performs better in period t is more broadly applied over time $t + 1$ during season s, and therefore the seasonal demand of that group will weigh more heavily on period $t + 1$. Conversely, if $\gamma = 0$, the reverse situation will be obtained.[9] Price changes, finally, are a function of excess seasonal demand plus a noise term.

$$P_{t+1,s} = P_t + \varphi\left[D^M_{t,s} - S_{t,s}\right] + \varepsilon_t \tag{6}$$

where S is the supply of oil and assumed to be a linear function of price; φ is a positive price adjustment parameter governing market frictions; and ε_t is taken to be a random noise term. Therefore, the solution for the oil price can be derived as:

$$\begin{aligned}\Delta P_{t+1,s} = a + bP_t + W_{t,s}\left(\alpha_1\left(P_{t,s} - F_{t,s}\right)^+ + \alpha_2\left(P_{t,s} - F_{t,s}\right)^-\right) + \\ \left(1 - W_{t,s}\right)\left(\beta_1\left(P_{t,s} - P_{t-1,s}\right)^+ + \beta_2\left(P_{t,s} - P_{t-1,s}\right)^-\right) + \varepsilon_{t,s}.\end{aligned} \tag{7}$$

From equation (7) it can be seen that, for a given value of α and β, fundamentalist and chartist traders' stabilizing seasonal impact on the oil price increases nonlinearly with their confidence in fundamental and technical analysis. We now turn to the empirical implementation of the model.

The Seasonal Cyclical Mackey-Glass Model

The specific system chosen for the present study was the well-known Mackey-Glass (MG) nonlinear time delay differential equation. The model is given by the following:[10]

$$\frac{d_P}{d_t} = \alpha\frac{P_{t-\tau}}{1+P^C_{t-\tau}} - \delta P_{t-1} \quad \text{where } c > 0. \tag{8}$$

It should be noted that the choice of lags τ and c is crucial since they determine the system dimensionally. α and δ are parameter estimates. The MG equation can now be modified to take into account the rhythm of seasonal frequency. This rhythm defines the frequency of separate seasons. During each cycle period, prices increase to some maximal value and then revert to the normal situation. Seasonal frequencies are defined as $\omega = 2\pi/s = 2\pi f$ where $f = 1/s$ and s is the number of observations per year (for example, $s = 1$ for annual data, $s = 12$ for monthly data, etc.). Hence, for oil prices the instantaneous ventilation V is a non-negative periodic function. The assumption is made that it can be modeled as $V = [1 + \sin\omega(t - \tau)]$.

$$P_t = \alpha\frac{P_{t-\tau}}{1+P^C_{t-\tau}}\left(1 + \sin\omega(t-\tau)\right) - \delta P_{t-1} + \varepsilon_t \tag{9}$$

where ε_t is *i.i.d.* Many chaos properties remain valid when noise is added to the system, provided the noise level is not too high. Furthermore, the stochastic part added in the Mackey-Glass equation can assume two different forms. In the first instance, white noise is added to the Modified Mackey-Glass equation (homoskedastic errors) where $\varepsilon_t \sim N(0,1)$. In the second, when anomalies are heteroskedastic, the stochastic part added onto the Modified Mackey-Glass equation follows an ARCH(1) process, where $\varepsilon_t/I_t \sim N(0,h_t)$. h_t is the conditional variance.[11]

The local asymptotic stability for the equilibrium of equation (9) implies global asymptotic stability, meaning that all solutions converge to zero while t tends towards infinity. To formulate a criterion of asymptotic stability for equation (9), the stability of the seasonal point can be studied as suggested by P. Landa and M. Rosenblum.[12] As a result, $(P_{t-\tau} - P^* \equiv 0)$ is subject to oscillatory instability if $V^* \leq SP^*$ and

$$\tau > \tau_{cr} = \frac{\arcsin[-V^*/SP^*]}{\alpha\sqrt{S^2P^{*2} - V^{*2}}} \tag{10}$$

where P^* is the singular point of equilibrium, $V^* = V|_{Pt-\tau=P^*}$, $S = (dV/dX_{t-\tau})|_{Pt-\tau=P^*}$. The period of oscillations close to the stability boundary is approximately equal to 6τ (approximately one peak every 6 months). It is assumed that the frequency of the seasonal rhythm f weakly depends on the level of prices at some previous moment in time, and that the purpose of this control is to maintain linearity of price levels. It also is assumed that the estimated price level may not necessarily be in an equilibrium state but may vary in its immediate neighborhood (P^*). This suggests that minor modulations in the frequency of the seasonal rhythm can be considered:

$$f = f_0 + \beta(P_{t-\tau} - P^*), \quad \text{where } 0 \leq |\beta| \leq 1 \tag{11}$$

where β is the parameter of modulation and the frequency of normal seasonality rhythm f_0 is equal to $\pi/3$ (referring to a period of 6 months). As shown previously, this small modulation leads, nevertheless, to a nontrivial effect. For $\tau < \tau_{cr}$ and β varying closely around zero, periodic effects in time and level are obtained. For $\tau > \tau_{cr}$ and $\beta = 0$ a quasi-periodic regime with a basic frequency f_0 can be observed. For $\tau > \tau_{cr}$ and $\beta > 0$, irregular seasonal effects in level can be observed.

Empirical Results

The data consists of real monthly spot prices on the New York Mercantile Exchange (NYMEX) for West Texas intermediate (WTI) light crude oil from January 1973 to December 2008. However, here the focus is on market returns on these spot prices. The data were obtained from the U.S. Energy Information Administration (EIA). In order to proceed with an unbiased and unambiguous interpretation of long-memory and nonlinearity phenomena, oil prices first have to be rendered stationary. The augmented Dickey-Fuller (ADF) test applied to oil raw series showed the presence of unit root properties in oil spot prices (table 1). Therefore, first order differencing of raw series is considered as denoted (DLOIL) and defined by $DLOIL_t = \ln(OIL)_t - \ln(OIL)_{t-1}$.

Table 1 presents the descriptive statistics for oil returns. Using ADF to test the unit root, the stationary nature of oil series at the 5-percent significance level is firmly accepted. It can be observed that there is excess kurtosis relative to the standard distribution. The distribution is positively skewed. The combination of a significant asymmetry and leptokurtosis indicates that oil price series are not normally distributed (as suggested by the Jarque-Bera statistic). The Engle test result confirms the presence of heteroskedasticity and residuals are autocorrelated.

The fractional integration parameter d is provided by the GPH estimator. The null hypothesis of interest relates to whether the return series comes under integration of order zero (H_0: $d = 0$) versus the alternative of fractional integration (H_1: $d \neq 0$). Estimates for the fractional integration parameter d are provided in table 1, along with t-statistics for the null hypothesis $d = 0$. The point estimates provided by the GPH estimator are considered with an estimation window of $T^{0.8}$. These estimates indicate evidence of long memory in oil spot prices, but with $d_{GPH} > 0$. Positive values for the fractional differencing parameters indicate predictability in variance. The point estimates are characterized by persistent processes, suggesting that the variance in the series is dominated by low frequency (slow cycle) and spectral density tends towards infinity when the frequency moves towards zero. The statistical test shows that the movement in oil prices appears as the result of an exogenous shock affecting the oil market.[13]

In testing for the presence of seasonal effects in oil prices, autoregressive models for oil series with control for possible seasonal effects are first estimated, as in:

$$DLOIL_t = \sum_{i=1}^{p} \alpha_i DLOIL_{t-1} + \sum_{j=1}^{12} \beta_j D_{jt} + \varepsilon_t \tag{12}$$

where D_{jt} represents 12th month-of-the-year dummies. The lag length is selected based upon the Akaike criterion. Table 2 reports the results from the ordinary least squares (OLS) regressions. There is evidence of seasonal effects in oil return

Table 1

STATISTICS SUMMARY FOR DLOIL

			Augmented Dickey-Fuller				
Skew	**Kurtosis**	**Jarque-Bera**	**Raw**	**Δ**	**ARCH(12)**	**Q(12)**	**GPH**
1.97	26.8	10513[a]	0.48	–49.25[a]	24.87[a]	27.92[a]	0.274[a]
		(0.00)	(0.81)	(0.00)	(0.01)	(0.00)	(0.02)

[a]The null hypothesis rejected at the 5-percent significance level. The Q(12) statistic represents the Ljung-Box (Q) statistics for autocorrelations in the residuals.

Table 2
OIL SEASONALITY TEST[a]

Oil	Jan	Feb	Mar	April	May	June
Est.	–0.035[W] (0.01)[b]	–0.021 (0.10)[c]	0.006 (0.64)	0.020 (0.08)[c]	–0.013 (0.35)	0.047 (0.01)[b]
Oil	**July**	**Aug**	**Sept**	**Oct**	**Nov**	**Dec**
Est.	0.022 (0.87)	0.026[S] (0.04)[b]	–0.003 (0.80)	–0.014 (0.34)	–0.02 (0.03)[b]	–0.033[W] (0.02)[b]

[a]W is the winter effect; S is the summer effect.
[b]Represents the significance at the 5-percent significance level.
[c]Represents the significance at the 10-percent significance level.

series. A significantly positive coefficient was found in August while a negative one was found in January. These results concur with those of A. Hamilton, who showed that the demand for crude oil in both August and January are the highest in the year, with prices thus logically being at their highest in both those months.[14] Moreover, in the context of descriptive tests, it has to be ascertained whether the structure of oil prices contains nonlinear and chaos processes. However, the presence of a linear structure may be responsible for the rejection of chaos. Thus, there is a need to eliminate low frequency signals from the oil prices structure.

However, the series adopted is filtered using the ARFIMA model. Then, after controlling for long memory, the nonlinear statistics test is applied to ARFIMA filtered residuals (RFDLOIL) to investigate the hypothesis of a nonlinear seasonal process. The statistical results of ARFIMA (p,d,q) processes are summarized in table 3. The value of the fractional integration parameter is $\hat{d} = 0.39$ and is accepted at a 1-percent significance level (between ±0.5). Applying the ARCH-LM test to the ARFIMA filtered residuals (RFDLOIL) confirms that the errors are heteroskedastic but are not autocorrelated and also that the RFDLOIL series is not normally distributed as suggested by the Jarque-Bera test statistics.

Due to the fact that nonlinearity is a necessary (but not sufficient) condition for chaos, the BDS test is used to assess the null of whiteness against the alternative of

Table 3
ARFIMA TESTS ON DLOIL

$M\hat{A}$	$\hat{d}$	ARCH-LM(12)	Q(12)	Jarque-Bera
0.10 (0.00)[a]	0.39 (0.00)[a]	11.83 (0.00)[a]	17.68 (0.12)	3687 (0.00)[a]

[a]Results accepted at a 1-percent significance level.

non-white linear and non-white nonlinear dependence.[15] This is based on an estimation of the correlation integral as introduced in the context of dynamic systems by P. Grassberger and I. Procaccia.[16]

Practitioners of the BDS test usually consider different embedding dimensions. Our study used six embedding dimensions for this test. Error was set at $\varepsilon = 0.5\sigma$. It is obvious from table 4 that the null of whiteness is rejected according to all computed statistics, and hence the remaining dependence is consistent with a nonlinear dynamic explanation. It can be concluded that there is evidence of nonlinearity of the general form.

To test chaos, the A. Wolf et al. test was applied to compute the Lyapunov exponents.[17] To this purpose, the notion of the Lyapunov exponent was introduced since it is usually taken as an indication of the underlying dynamic system characteristic. In the presence of noise, as is often the case with real world data sets, the meaning of "detecting deterministic chaotic dynamics" is ambiguous. Thus, the algorithm developed by A. Wolf et al., which is used to estimate the growth rate of the propagation of small perturbations in the initial conditions, appears to be non-robust.[18] Therefore, given that there is considerable exogenous influence perturbing the endogenous dynamics, it is necessary to define the Lyapunov exponent in a stochastic context. D. Nychka et al. defined this as $X_{t+1} = F(X_t) + \varepsilon_{t+1}$.[19]

Since the largest Lyapunov exponent λ_1 has often been of primary interest in the literature, our study mainly focused on analysis of the largest Lyapunov exponent, simply denoting it as λ. However, it should be noted that other exponents λ_1 for $2 \le i \le d$ also contain some important information relating to the stability of the system, including the directions of divergence and contraction of trajectories and the types of non-chaotic attractors.[20] The presence of a positive exponent is sufficient for diagnosing particular classes of chaos and presents local instability in a given direction. The results obtained from the log-differenced price series are reported in table 5. The best Lyapunov exponent is that which minimizes the SIC criterion. Results show that the minimum SIC value occurs when we use six hidden units. In this case, the corresponding Lyapunov exponents are $\lambda_1 = 0.1239$ e-05 and $\lambda_2 = -0.1972$. For both cases, λ_1 is positive and λ_2 is negative.

Table 4
BDS TEST RESULTS (RFDLOIL)

ε/σ	BDS Statistic (0.5)	P-Value
m=2	0.05	0.00[a]
m=3	0.11	0.00[a]
m=4	0.15	0.00[a]
m=5	0.17	0.00[a]
m=6	0.18	0.00[a]

[a]The critical value is 1.96 for the 5-percent significance level.

Table 5
LYAPUNOV EXPONENT ESTIMATES ON ARFIMA FILTERED RESIDUALS

Hidden	λ_1	λ_2	SIC
1	–0.2934 e-05	–1.9076	–12.98
2	0.2798 e-05	–0.9528	–12.94
3	0.1955 e-05	–0.5671	–13.24
4	0.1788 e-05	–0.4190	–13.38
5	0.1388 e-05	–0.2918	–13.47
6	0.1239 e-05	–0.1972	–13.55
7	–0.1299 e-05	–0.1311	–13.52
8	0.1179 e-05	–0.0576	–13.47
9	0.2100 e-05	–0.0202	–13.49
10	–0.2222 e-05	–0.0418	–13.57

Consequently, it can be concluded that there is clear evidence for a mixture of processes. The fact that λ_1 is slightly positive could be due to the existence of high-dimensional chaos, which could be confused with stochastic processes.[21]

On the other hand, since λ is negative, it cannot be concluded that there is stochastic behavior; behaviors may be periodic.[22] In the case where high periodic effects are obtained, seasonality can produce high variance, similar to that of stochastic behavior. In other words, the presence of heteroskedasticity in the series may result from seasonal effects. Therefore, it can be assumed that there is a complex structural composite of a mixture of processes with slightly irregular behavior sensitive to small perturbations (chaos) and periodic behavior.

From the preceding applications, it can be concluded that the hypothesis of nonlinearity in oil spot price movements cannot be rejected and is not an *i.i.d.* process. Moreover, it is not clearly determined, according to Lyapunov exponent statistics, what is exactly the source of nonlinear behavior. The plausible explanation is that there are both chaotic and periodic behaviors in log-differenced oil price returns. To this hypothesis, Seasonal Cyclical Mackey-Glass models are applied. Using $\tau = 5$ and $c = 2$ (selection with SIC criterion) with frequency $\pi/3$, the SCMG parameters estimated are significant at a 5-percent level (table 6). Thus, the model detects significant evidence of nonlinearity in the seasonal bull in oil returns. Moreover, the seasonal solution found for $\tau = 5$ ($\tau < \tau_{cr}$) and $\beta = 0.0029$ (around zero) seems to be a chaotic solution. Furthermore, it can be said to be quasi-periodic. The difficulty in distinguishing clearly between both processes may be due to the high noise level. As a result, the seasonal bull detected in oil prices at frequency $\pi/3$ appears to be persistent over time. Finally, tests on SCMG filtered residuals showed that residuals are empty of heteroskedasticity and autocorrelation, despite the presence of non-normality (Jarque-Bera).[23] This suggests that the heteroskedasticity in residuals may be due to periodic behaviors in oil prices.

Table 6
PARAMETER RESULTS OF THE SCMG MODEL AT FREQUENCY $\omega = \pi/3$

Coefficient	Coefficients	P-Value	ARCH-LM	Q(12)	Jarque-Bera
$\hat{\alpha}$	0.4016	0.05[a]	2.56	12.66	124[a]
$\hat{\delta}$	0.1493	0.00[a]	(0.24)	(0.39)	(0.01)
$\hat{\beta}$	0.0029	0.06[b]			

[a] $\hat{\alpha}$ and $\hat{\delta}$ are accepted at the 5-percent significance level.

[b] Represents the coefficients that are accepted at the 10-percent significance level.

Equation (5) is applied to SCMG residuals to verify whether the residuals' structures contain cyclical effects. Statistical tests showed that residuals are empty of cyclical effects.

We will now present the forecasting experience. Four models are applied to the RFDLOIL series to compute the Root-Mean-Squared Error (RMSE hereafter) of each model. The models are SARIMA, SCLM, SMG-GARCH, and SCMG. To obtain a quick summary of the results, the ratios of the RMSE are calculated, dividing the RMSE from the SCMG model by the one from each other model. Thus, a ratio of lower than one indicates better forecasting performance of the SCMG model (table 7).

The table reports forecast evaluation statistics for a full sample horizon. The sample covers a total of 433 forecasts for the horizon considered. The forecasting models are: $SARIMA(3,0,0)(2,1,0)_6$, where the estimates models are as follows $(I–B^6)(I–0.224B–0.279B^2–0.132B^3)(I–0.542B^6–0.188B^{12})X_t^1 = \varepsilon_t$, $R^2 = 0.147$. The second is the Seasonal Cyclical Long-Memory model. The parameter estimate of the model associated with the cycle of period of six months is $\pi/3$. The estimate model is defined as: $(I - 0.321B + 0.244279B^2)^{0.065}\,(I + 0.119B)^d X_t^1 = \varepsilon_t$, $R^2 = 0.23$. Finally, the seasonal MG-GARCH is (1,1). The dummy variable from the period of December 15 to January 30 and from the period of August 1 to September 30 equal to 1 and 0 otherwise was used. Taking $\tau = 1$ and $c = 2$, the model is accepted at a 5-percent significance level; $R^2 = 0.192$.

Conclusion

In this paper an empirical oil market model was developed to detect dynamic seasonal cyclical behaviors in oil price series. The main conclusion obtained from this application is that oil prices have greater potential to show strong seasonality in both

Table 7
RATIOS OF THE RMSE FOR THE SCMG MODEL OVER THE RMSE OF EACH MODEL

	SCMG/SARIMA	SCMG/SCLM	SCMG/SMG-GARCH
Ratio of RMSE	0.919	0.992	0.92

the December–January and August periods of the year. The movements associated with frequency $\pi/3$ appeared to be persistent over time. Moreover, results suggest that speculative activities are responsible for changes in spot prices in both peaks in the year, especially when speculative trading strategies are influenced by periodic information. Thus, the heterogeneous agents' hypothesis and their nonlinear trading impact influenced by seasonal effects may explain the pronounced swings in oil prices, as witnessed in recent years. Clearly, these results are of interest to investors in this crucial commodity as they contain valuable information to assist in fine-tuning the timing of entry and points for oil-stocks investors and speculators to maximize gains in this ongoing oil-stock bull, though it remains essential to consider all other features of the markets. Finally, SCMG models can be extremely competitive in terms of forecasting as compared with conventional linear and nonlinear models.

NOTES

[1]United States Senate, Permanent Subcommittee on Investigations of the Committee on Homeland Security and Governmental Affairs, "The Role of Market Speculation in Rising Oil and Gas Prices: A Need to Put the Cop Back on the Beat," Staff report Senate Prints 109-65, 109th U.S. Congress, 2nd session, Washington, D.C., June 27, 2006, available at http://www.gpo.gov/fdsys/pkg/CPRT-109SPRT28640/html/CPRT-109SPRT28640.htm; Paul Krugman, "Speculation and Signatures," Princeton University, Princeton, New Jersey, June 24, 2008 (mimeograph); Robert Kaufmann and Ben Ullman, "Oil Prices, Speculation, and Fundamentals: Interpreting Causal Relations among Spot and Future Prices," *Energy Economics,* vol. 31, no. 4 (2009), pp. 550–58; and Bassam Fattouh, "The Dynamics of Crude Oil Prices Differentials," *Energy Economics,* vol. 32, no. 2 (2010), pp. 334–42.

[2]John Winston "Gold and Crude Seasonal Analysis," *The Market Oracle*, December 11, 2009, available at http://www.marketoracle.co.uk/Article15730.html.

[3]Alan Greenspan, "Energy and Security: Prepared Statement by Alan Greenspan, President, Greenspan Associates LLC, Washington, D.C., before the U.S. Senate Committee on Foreign Relations," Hearing before the Committee on Foreign Relations, Staff report Senate Prints 109-64, 109th U.S. Congress, 2nd session, Washington, D. C., June 7, 2006, available at http://lugar.senate.gov/energy/press/pdf/committee_print.pdf.

[4]P. H. Franses and M. Ooms, "A Periodic Long Memory Model for Quarterly UK Inflation," *International Journal of Forecasting*, vol. 13, no. 1 (1997), pp. 117–26.

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[17]A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano, "Determining Lyapunov Exponents from a Time Series," *Physica D*, vol. 16, no. 3 (1985), pp. 285–97.

[18]Lyapunov exponents test of A. Wolf et al., op. cit., is highly sensitive to noise level. Thus, it cannot be confirmed that the test is robust when a high noise level obtains in financial series.

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[21]Noise can always be interpreted as a deterministic time evolution in infinite dimensions.

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[23]The residuals' remaining structure is not identified. This may be due to an unknown structure or to a misspecification of one of the parameters.