Rare Events in the American GDP Time Series, 1790-Present: Fact or Artefact

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RARE EVENTS IN THE AMERICAN GDP TIME SERIES, 1790 - PRESENT: FACTOR ARTEFACT?

Mohamed CHIKHI
Claude DIEBOLT

ABSTRACT

This paper studies the cyclical behaviour of the annual American Gross Domestic Product (GDP) series. We show that rare events have varied effects, which give useful information on the nature and the amplitude of economic shocks. Our methodology is, more precisely, an efficient testing procedure to control the robustness of historical time series constructions.

KEYWORDS

Cliometrics, GDP, Historical Time Series, Outliers, USA.

JEL-CLASSIFICATION: B22, B23, C13, C22, C82, N11, N12.

INTRODUCTION

A generally accepted definition of business cycles is one presented by Arthur F. Burns and Wesley C. Mitchell in their work Measuring Business Cycles. According to Burns and Mitchell:

“Business cycles are a type of fluctuation found in the aggregate economic activity of nations to organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general
recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles with amplitudes approximating their own. (Burns & Mitchell, 1946, p.3).

The definition of business cycles advanced by Burns and Mitchell emphasizes that cycles are recurrent but not periodic. For some, the term business cycles implies a certain rhythm of business activity. To describe the cycle as recurrent means that it possesses a repetitive pattern of development—a pattern of expansion, recession, contraction, and revival, followed by renewed expansion. The cycle, however, is uniform neither in time periods nor in amplitude. We cannot say that the expansion phase always lasts X months and measures of aggregate activity rise Y per cent above the preceding low point. There is a high degree of uniformity from one cycle to the next in the forces of cumulation. Upswings and downswings are self-reinforcing; they feed on themselves, possess similar characteristics, and show approximately concurrent movements in many different series. However, there is no evidence that they recur again and again in virtually the same form and amplitude. The completion of a cycle from trough to trough or peak to peak may take from approximately two to more then ten years, and the proportions of the upswing and downswing may vary all the way from mild to catastrophic.

Let us now turn to a consideration of the four phases of the business cycle—expansion, recession, contraction, and revival—and attempt to summarize the salient features of each phase. It is well to remember that this description is one of possible features found only in a composite view of all cycles. Features found in an individual cyclical experience may differ in some measure from the generalizations that follow.

The features typical of expansion are a large volume of production and trade; a high level of employment and job opportunities in sufficient abundance to permit a good deal of labor mobility; rising commodity prices; a rising structure of interest rates; advancing prices of equity securities; a growing volume of borrowings at both the business and consumer level; and a substantial volume of investment in machinery, alterations in plant, and additions to capacity by business.
The essence of expansion is the way in which each of these developments reinforces the others. There is a snowballing effect. As trade and production expand, the demand for labor is thereby stimulated, and hours of work are lengthened. Increased employment in its turn leads to larger wage payments that extend the market for consumers goods and so further stimulate trade and production. The expansion in demand brings increases in the prices and finished goods, of the materials used in their processing, and of services provided business and consumers. Favorable profit prospects growing out of the general expansion in trade and production give, in their own turn, a further stimulus, thus promoting a still further increase in employment and new advances in prices. Economic opportunities seem to exist in abundance, and the problem for most firms is likely to be squeezing the maximum output from existing facilities. Investment in capital facilities is encouraged, which gives renewed impetus to the demand for goods and services, to employment, to income receipts, to price level, and to production and trade. This advance may in turn improve profit prospects, maintain optimism, and so stimulate investment outlays once again.

Yet factors emerge during the expansion phase which limits its extent and duration. Stresses appear which, though they may be temporarily overcome, reappear, seemingly with renewed vigor. Business weaknesses accumulate toward the latter part of the expansion phase. These weaknesses are eventually reflected in a squeeze on business profits. Unit costs of output rise with the increasing price of raw materials and with the tendency of wage rates to outstrip the advance in labor productivity because of the breaking in of new and less efficient employees and the use of substandard equipment in order to achieve capacity output; the increased demand for credit raises the interest cost to borrowers; the rush to maximize production to meet market demand means the business expenses tend to be scrutinized less closely.

So long as higher costs can be passed along to consumers in the form of higher prices, profit margins are little affected. However, it becomes increasingly difficult to adopt this remedy universally. In some industries the optimism of the expansion phase has probably found expression in expansion of capacity somewhat in excess of current market needs and whatever accretions in demand that will take place in the immediate future. New equipment and facilities add
greatly to the supply of goods increases, and it becomes difficult to raise selling prices further. Profits are caught between advancing costs and a level of prices that is under growing competitive pressure. Less satisfactory profits are likely to force a reassessment of market prospects and to influence adversely the climate of business opinion, curtailing the spending plans of business. Reduced spending by business is reflected in reduced employment opportunities, less overtime, probable layoffs, and smaller income disbursements. Accompanying these developments may be a desire to liquidate inventories to bring their level more into line with new and less buoyant prospects for sales. Related industries may now be forced to similarly revise their spending and scheduling plans, as will those consuming units dependent for their incomes upon these areas of production. These developments, in their turn, influence retail spending.

Accompanying the new situation in production and trade are changed conditions in the money and credit markets. During the early phases of expansion, bank loans expand in unison with the growth in bank deposits. However, in the latter stages banks become loaned up, that is, the ratio of loans to deposits approaches a level beyond which it is not safe to go lest liquidity be unduly impaired. Interest rates rise, and more exacting standards for new loans and for renewals are set. A higher structure of interest rates prevails throughout the money market. Higher loan rates and greater difficulties in obtaining credit place yet other restrictions upon the ability of enterprises to continue expanding.

Even though some incomes are unaffected by the changing business prospects, total income disbursements fall. Those whose incomes are maintained may attempt to consolidate and make their financial positions more liquid. Consuming and business units which may have taken on debts during the expansion phase deem it unwise to add to their indebtedness in the face of less favorable prospects. Their emphasis is upon debt reduction; and major outlays, for which new financing might be necessary, are postponed.

The feature of contraction is the reverse of those of expansion. The volume of production and trade shrinks, employment declines, and unemployment rises; job opportunities diminish; commodity prices fall; the structure of interest rates falls, and the prices of high-grade bonds rise; the prices of equity securities decline; the demand for borrowings on the part of both business and the consumer is reduced;
there are retrenchments in capital outlays, particularly in business spending on machinery, equipment, and plant additions. Optimism gives way to pessimism. These developments accumulate and reinforce themselves. Declining production and a reduced volume of trade curtail the demand for labor. Overtime is eliminated, and employment reduced. Pay envelopes are smaller and fewer, with the consequent unfavorable effect upon retail trade, thus further depressing production. Reduced demand brings reductions in the price of raw materials and in wholesale prices. Prices of retail goods and of services are likely to be more sluggish in their decline. Wage rates and long-term interest rates also respond rather slowly. The fall in expenses lags behind the fall in revenues, and this squeezes profits further; many firms show losses. Pessimism becomes widespread and leads to further retrenchment in the spending plans of business and consuming units.

The burdens of contraction are not shared equally by all segments of the economy. Consumer demand for durable goods is more affected than that for non-durable. Durable goods are particularly sensitive to lower wage disbursements and poorer earnings prospects. In the face of falling demand businessmen increase their efforts to liquidate stocks and reduce their inventory-sales ratios. The inventory accumulation of expansion turns to liquidation in contraction. Especially affected is business spending for plant and equipment. New orders for equipment sink, for incentive to add to capacity or to replace equipment is nonexistent when operating levels are low and industry is generally burdened with over-capacity. Inventory liquidation and a reduced volume of production and trade diminish demand by business for working capital and for long-term credit. Bank loans fall, and there is increased competition among long-term lenders and banks to place funds. The structure of interest rates declines as the demand for high-grade debt instruments grows.

In contraction as in expansion all forces seem to be operating in one direction. One shrinkage forces others and through the stream of spending comes back to react upon itself. Yet, in contraction as in expansion, relations change and accumulate to bring about a reversal in the direction of the aggregate. Often, one of the first points of reversal is in the market for equities. The decline in security prices lowers the price-earnings ratio so that equities once again become attractive and arouse renewed investor interest. Within business firms
adjustments are made to improve profit margins. Expenses are brought into line, wastes are eliminated, and efficiency is empha-sized. Reorganization may serve to reduce overheads costs, and operating expenses in manufacturing firms are reduced by lower costs of raw materials and employment of only the most experienced workers.

The lower level of interest rates, "cheap money", means that both short-term credit and long-term financing can be had on very favorable terms. Further, manufacturing and trading firms may have allowed inventories to reach minimum levels; to reduce them further would be to run the risk of losing sales. Goods sold from a minimum stock level require replacement, and new orders ensue. Demand for replacement may be forthcoming from both consumers and business firms. Many family units will be called upon to replace goods no longer serviceable. Some purchases can no longer be postponed. Favorable installment credit terms are available and can provide another impetus to production and trade. Industry will possibly be faced with the need of replacing equipment or plant that has become technologically obsolescent or for which further repair is uneconomical. To the extent that such orders are forthcoming, demand is stimulated.

Business and consumer demand may receive further impetus from the knowledge that prices have fallen during contraction and that there are limits to the extent of the decline. Purchases can be made at bargain rates, and there is little advantage in delaying longer.

These encouraging signs become more and more numerous. They break a wall of pessimism, and gradually a measured optimism emerges. Tangible improvements in particular industries arising from increased spending by businesses and consumers augment the more encouraging circumstances. Revival is under way, and it gathers momentum as expansion in particular areas spreads to related segments of business activity. It may be speeded by any favorable development, such as an increase in foreign buying or the introduction of a major cutting technique affecting many areas of business. Conversely, revival may be checked or slowed by unfavorable developments from outside the business system. Be that as it may, there is a snowballing which sooner or later will lead aggregate activity once again into the expansion phase.

In view of this historical benchmark, the present article is concerned with the cyclical behaviour of the annual American Gross
Domestic Product (GDP) series for the period 1790-2007. See Johnson and Williamson (2007), for a complete description of the sources that were used and the techniques applied to those sources in constructing these series. We aim here to show that rare events can have varied effects on time series. A movement of social turbulence will thus usually have a temporary effect on the series observed. Conversely, the effects of financial slumps or changes in the choice of economic policy will be qualified respectively as temporary and permanent.

In contrast with the founding theoretical work who initiated analysis by using a deductive approach, we opted for a more inductive procedure here. In other words, we seek to detect the possible existence of atypical observations in the growth of the American GDP, or the search for outliers. Much work has been devoted to the effects of outliers in the estimation of underlying processes. All the authors agree to show that not taking such values into account leads to bias in traditional tests, the estimation of models and forecasting.

More generally, two econometric methods can be envisaged in approaches in economic history to the analysis of shocks. Either, as in the traditional approach, shocks are studied in the form of impulsive response functions. In this case, analysis is based on the estimation of a VAR model and is essentially an analytical, forecasting approach as the shocks envisaged are simulated and hence fictitious. Or, as in the most recent work in econometric history, shocks are analysed in the form of atypical points referred to as outliers. In this case, analysis of shocks is part of an analytical and historical approach as they were actual shocks.

Our paper refers to this outliers methodology. The first part defines the methodological constraints. The second presents our cliometric results.

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1 Readers interested in other cliometric applications or a full mathematical and statistical presentation of outlier methodology should see Darné and Diebolt, 2004, 2005, 2006, Diebolt, 2007.
1 - OUTLIERS METHODOLOGY

The outliers represent the infrequent large temporary and permanent shocks that have affected a time series. There are several methods for detecting the outliers [see for example Box and Tiao (1975)]. We retain here the procedure developed by Gómez and Maravall (1997) using the approach proposed by Chen and Liu (1993).

We consider a univariate time series $y_i^*$ fitted by $ARIMA(p,d,q)$ model:

$$\alpha(B)\phi(B)y_i^* = \theta(B)\epsilon_i, \quad t = 1,\ldots,T$$  \hspace{1cm} (1)

Where $B$ is backshift operator, $\alpha(B)$, $\phi(B)$ and $\theta(B)$ are the polynomials in $B$ of order $d$, with no common factors and all roots outside the unit circle $p$ and $q$ and $\epsilon_i$ is a White Noise. The outliers are classified and modelled by polynomials of regression, which can be defined as:

$$y_i = y_i^* + \sum_i \omega_i \nu_i(B)I_i(\tau) \quad i = 1,\ldots,4$$  \hspace{1cm} (2)

Where $y_i^*$ is a $ARIMA$ process, $\nu_i(B)$ the polynomial characterizing the outlier that have produced at time $t = \tau$, $\omega_i$ its impact on the series, and $I_i(\tau)$ a dummy variable such as:

$$I_i(\tau) = \begin{cases} 1, & t = \tau \\ 0, & \text{otherwise} \end{cases}$$

To treat the outliers, there are four types of variables for the regression:

- **Additive Outliers** [AO]: they affect only observation at one moment of time in the series. The polynomial of regression is generated by fitting $\nu_i(B) = 1$;

- **Level Shifts** [LS]: they have a permanent effect on the series in level. In this case, the polynomial is defined as:

  $$\nu_i(B) = 1/(1 - B);$$

- **Temporary Changes** [TC]: they affect temporarily the series, which returns exponentially to its previous level. The quickness of reversion depends on $\delta$ in the polynomial

  $$\nu_i(B) = 1/(1 - \delta B) \quad \text{With } 0 < \delta < 1;$$
• **Innovative Outliers** [IO]: Their effect is very complex\(^2\) according to the type of series, and the polynomial is defined by:

\[ V(B) = \theta(B) / \phi(B) \]

For a stationary series, they affect temporarily the series because its impact decreases in the time. On the other hand, for a non stationary series, the IOs have an initial effect at one moment of time then a permanent effect [Chen and Liu (1993)].

We consider that the AO and IO are the outliers and that the TC and LS are the structural changes. The TCs represent ephemeral on the series in level whereas the LSs reflect a permanent shock on the level and only a transitory effect on the growth rate. Moreover, the IOs have a permanent effect if the series is not stationary. We note that the LS and IO (“non stationary”) detected on the series in level correspond to the AO and IO (“stationary”) for the differenced series, in other words the growth rate (Balke and Fomby, 1991, 1994, Maddala and Kim, 2000).

From the equation (1), we obtain the residuals, which are defined as:\( \hat{\epsilon}_t = \pi(B)y_t \) (3)

where:\( \pi(B) = \alpha(B)\phi(B)/\theta(B) = 1 - \pi_1 B - \pi_2 B^2 - ... \)

For the four types of outliers, which are defined previously, the equation (3) can be written as:

\[ \text{AO} : \hat{\epsilon}_t = \epsilon_t + \omega_{1}\pi(B)l, (\tau) \]
\[ \text{IO} : \hat{\epsilon}_t = \epsilon_t + \omega_{1}l, (\tau) \]
\[ \text{LS} : \hat{\epsilon}_t = \epsilon_t + \omega_{1}\left[\pi(B)/(1-B)\right]l, (\tau) \]
\[ \text{TC} : \hat{\epsilon}_t = \epsilon_t + \omega_{1}\left[\pi(B)/(1-\delta B)\right]l, (\tau) \]

These expressions may be seeing as a regression for residuals \( \hat{\epsilon}_t \):

\[ \hat{\epsilon}_t = \omega_0 x_{i,t} + \epsilon_t \quad i = 1, 2, 3, 4 \]

with \( x_{i,t} = 0 \) for all \( i \) and \( t < \tau \), \( x_{i,t} = 1 \) for all \( i \) and \( t = \tau \), and for \( t > \tau \) and \( k \geq 1 \):

\[ \text{AO} : \quad X_{i,t+k} = -\pi_k \]

\(^2\) Indeed, the outlier effects on the observed series are independent of its modelling except to the innovative outliers.
IO: $x_{2,t+k} = 0$;
LS: $x_{3,j+k} = \sum_{j=1}^{k} \pi_j$;
and TC: $x_{4,j+k} = \delta^k - \sum_{j=1}^{k-1} \delta^{k-j} \pi_j - \pi_k$

The test statistics for the four types of outliers are given by:

**AO:**
$$\hat{\tau}_1(\tau) = \left[ \hat{\omega}_1(\tau)/\hat{\sigma}_x \right] \left( \sum_{t=\tau}^{n} x_{1,t}^2 \right)^{1/2}$$

**IO:**
$$\hat{\tau}_2(\tau) = \hat{\omega}_2(\tau)/\hat{\sigma}_x$$

**LS:**
$$\hat{\tau}_3(\tau) = \left[ \hat{\omega}_3(\tau)/\hat{\sigma}_x \right] \left( \sum_{t=\tau}^{n} x_{3,t}^2 \right)^{1/2}$$

**TC:**
$$\hat{\tau}_4(\tau) = \left[ \hat{\omega}_4(\tau)/\hat{\sigma}_x \right] \left( \sum_{t=\tau}^{n} x_{4,t}^2 \right)^{1/2}$$

With $\hat{\omega}_i(\tau) (i = 1, 2, 3, 4)$ represents the estimation of the impact of the outlier at $t = \tau$, and $\hat{\sigma}_x$ is an estimation of the residual variance.

### 2 - EMPIRICAL RESULTS

We analyse the cyclical behaviour of the annual American GDP series for the period 1790-2007 with 218 observations (see Figure 1). We are interested in applications to annual data in logarithm (see Figure 1). As shown in Figure 2 and 3, the Hodrick-Prescott trend shows both a considerable smoothness and some cyclical movements in this case. These results are confirmed with Baxter-King filter.

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3 See Chang and Alii (1988)

4 For HP trend, Hodrick and Prescott (1997) suggest that $\lambda$ values of 100 or 400 for annual data and Baxter and King (1999) argue that $\lambda = 10$ gives better results. Note that the parameter $\lambda$ is positive and penalizes variability in growth (trend) component.
Figure 1: U.S. Real GDP (in level and in logarithm)

Figure 2: U.S. Real GDP and Phase Average Trend (Hodrick-Prescott filter)

Figure 3: Cyclical movements of U.S. Real GDP in logarithm
(Using HP and Baxter-King filters)
We try especially to detect the outliers in the annual US GDP in logarithm. To identify the outliers, we use a sequential detection procedure, including an internal iteration and other external. In the external iteration, by supposing that there are any outliers, the ARIMA\((p,d,q)\) is estimated giving the residuals. The results of external iteration are used in the internal iteration to identify the outliers. The test statistics for the four types of outliers are calculated for each observation. The largest absolute value of these statistics \(\hat{r}_\text{max} = \max_i|\hat{r}_i(\tau)|\) is compared to a predetermined critical value. If the test statistic is greatest, then an outlier is identified at time \(\tau = t\).

In TRAMO\(^5\), the critical value is determined by the number of observations, and it is calculated from a simulation. Here, we have a value of 3.5. When a outlier is detected, the observation \(y_t\) at \(t = \tau\) is adjusted to obtain the corrected observation \(y^*_t\) from the equation (2) by using \(\hat{\omega}_t\). This process is repeated until any outlier is found. After, we return to the external iteration to re-estimate the ARIMA model by using the corrected data, and we start again the internal iteration. This procedure is repeated until any outlier is detected. Finally, a multiple regression on the series \(y^*_t\) is performed on the different identified outliers in order to determine the fallacious outliers.

The correction of outliers is realized on the US GDP in logarithm (see Figure 4). As shown in Table 1, the ARIMA\((0,1,1)\) model is selected because the information criterions are minimal. The coefficient is significant and the residuals don’t present any serial correlation (see Breusch-Godfrey statistic). We note that the US growth series, even derived from strongly cyclical level series, often have dominant irregular components, which obscure their underlying cyclical movements (see Figure 5).

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\(^5\) Time Series Regression with ARIMA Noise, Missing Observations, and Outliers.
Table 2 reports the identified outliers, with their date, their type of points, their amplitude and their test statistic. The last columns give an economic, political or financial explication of the occurrence of shocks. It shows that there are six outliers detected in US GDP series. The United states have a major economic shock having a permanent effect.

The US GDP may be characterized by permanent shocks, which are generated randomly in each period of observation. We note that the TRAMO technique give us a useful information on the nature and the amplitude of economic shocks but the deterministic or the
stochastic nature of its trend can be examined by using the unit root tests on the corrected US GDP series.

The results are presented in Table 3. The Unit root test of Elliott and al. (1996) accept the unit root hypothesis at significance level 5%. The stationarity test of Kwiatkowski et al. (1992) rejects also the null hypothesis of trend stationarity at significance level 5%. In order to confirm these results, we apply two efficient tests: Bierens-Guo and Breitung tests. The first test is proposed by Bierens and Guo (1993). They test a stationarity and trend stationarity against the unit root hypothesis by employing a Newey-West type variance estimator of the long run variance of innovations and the second nonparametric test is introduced by Breitung (2002). Therefore, the critical values used in this test are based on linear interpolation. These two tests show also that the presence of a unit root in corrected US GDP series is confirmed. Consequently, we conclude that the US GDP is generated by a random walk and characterized by a permanent shock because the trend is stochastic.

CONCLUSION

In most economic history cases, the earlier the period, the less accurate are the observations. This is the same for GDP data as shown by Johnston and Williamson (2007). They were not collected or even defined before the 1930s and thus any measures for years before 1932 rely on sources that were not collected for the purpose of constructing national income and product accounts.

“The construction of the GDP series before 1909 starts with an estimate of the observation for each benchmark year. The benchmarks are either the census or calendar year that comes every ten years beginning in 1800. The values for the years between benchmark years are computed by interpolation. Part of this interpolation is based on annual observations of related series, and part is based on an assumption of constant growth. The data for these benchmark years can be assumed to be more accurately measured than the nine years between each of them. Because of the method of construction, we do not recommend the data be used for sophisticated time series analysis. The data from 1790 to 1868 are for census years, and from 1869 to the present for calendar years. Each census year includes the first five months of the following calendar year. The reason is that most of the
data used are based on information from the censuses that were
collected every ten years. The early census data were collected from
June 1 of years ending in 9 through May 31 of years ending in 0. Thus
the pre 1869 benchmark years are based on these twelve months.”
This being said, the cliometric results of this paper show clearly
that over the complete statistical window only three outliers occur, in
1908, 1916 and 1932. 1916 can easily be related to WW1. The other
two points appear as level shift and have a significant impact of the
time series dynamic. Therefore and as a conclusion of this study, we
put forward the hypothesis that these points are clearly artefacts and
the results of the construction of the time series. Our research revealed
in his manner that the outliers methodology is a possible and efficient
testing procedure to control the robustness of historical time series
constructions.
The next step is probably the reinvestigate a more institutional
framework and to reconsider the time series produced. In fact,
“beyond the study of long run quantitative data sets, a branch of
cliometrics is more and more focused on the role and evolution of
institutions by aiming at combining both the desire for generality of
the economists and the concern for the precise context in which
economic players act that characterize both the historians and other
social scientists. The middle road between pure empiricism and
disincarnate theory might perhaps open the door to a better economic
time theory, enabling economists to understand more deeply the working of
economies and societies and by the way offer better policy advice.”
(Demeulemeester & Diebolt, 2007, p.16.).

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**APPENDIX**

<table>
<thead>
<tr>
<th>Selected Lags (p, q)</th>
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<tbody>
<tr>
<td></td>
<td>0.369</td>
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<tr>
<td></td>
<td>(5.87)</td>
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<tr>
<td>AIC / BIC</td>
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<td>Normality test</td>
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<td>Kurtosis</td>
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<tr>
<td>SSR</td>
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<tr>
<td>Breusch-Godfrey statistic</td>
<td>0.0667</td>
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(*) The values between the parentheses are the student statistics

**Table 1:** Estimation of ARIMA(0,1,1) by the exact maximum likelihood using the Kalman filter

<table>
<thead>
<tr>
<th>Date</th>
<th>Type</th>
<th>Amplitude</th>
<th>Event</th>
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<tr>
<td>1908</td>
<td>LS</td>
<td>-0.016</td>
<td>Artefact</td>
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<tr>
<td></td>
<td></td>
<td>(-6.48)</td>
<td></td>
</tr>
<tr>
<td>1916</td>
<td>AO</td>
<td>0.074</td>
<td>WW1</td>
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<tr>
<td></td>
<td></td>
<td>(4.99)</td>
<td></td>
</tr>
<tr>
<td>1932</td>
<td>LS</td>
<td>-0.013</td>
<td>Artefact</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-4.74)</td>
<td></td>
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(*) The values between the parentheses are the Student statistics

**Table 2:** Detection of Outliers

<table>
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<tr>
<th>Testing procedure</th>
<th>KPSS(^6)</th>
<th>DF-GLS(^7)</th>
<th>Bierens-Gao</th>
<th>Breitung</th>
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<td>m = 1</td>
<td>m = 2</td>
<td>m = 3</td>
<td>m = 4</td>
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<tr>
<td>PRSS</td>
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<td>-2.308</td>
<td>126.03</td>
<td>217.96</td>
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<td>(-2.89)</td>
<td>(12.706)</td>
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<td>1194.17</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.010)</td>
</tr>
</tbody>
</table>

(*) The values between the parentheses are the critical values and the lags are selected by minimizing the MIC criteria “Modified Information Criteria”, Ng and Perron (2001) for DF-GLS test.

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\(^7\) Elliott and Alii (1996) develop a test based on DF-GLS statistic. They suggest using the LM test for the null of unit roots against the alternative of a stationarity.