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Liu, Liyuan and Peng, Fei

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Liyuan Liu
School of Finance
Shanghai Lixin University of Accounting and Finance
No.2800 Wenxiang Road, Song Jiang District, Shanghai, 201620, China
Email: liuliyuan200@sina.com

Fei Peng
Department of Economics
Shanghai Lixin University of Accounting and Finance
No.2800 Wenxiang Road, Song Jiang District, Shanghai, 201620, China
Email: pengfei@lixin.edu.cn

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Abstract

Environmental protection against pollution has become a common issue faced by the whole world. In the case of the international cooperation on controlling the environmental pollution, the developing and developed countries have different understanding on the principle of "common but differentiated responsibilities". This paper has set up an optimal pollution controlling model for the developing and developed countries to incorporate environmental protection and economic growth. Based on a dynamic differential game, we find that the increasing environmental expenditure of developed countries in the initial stage of the economic growth path of the developing country can stimulate more international cooperation on pollution controlling. The developing and developed countries can control the environment pollution without significant loss of social welfare.

JEL Classifications: C71, O44, Q52, Q56

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1. Introduction

Increasingly worsening environment has become a common international issue faced by the whole human society. Global environmental problems such as greenhouse gas have drawn wide attention of developing and developed countries. Environmental pollution problems have suffered serious problems such as negative externality, free-rider problems and inadequate public good supply which call for efficient governance and cooperation of international community. List and Mason (1999) utilize an asymmetric information differential game to explore whether environmental regulations should be carried out locally or centrally. They argue that local control Pareto dominates central control when enough synergism occurs between pollutants. Conconi (2003) examines the determination of trade and environmental policies in two large countries that are linked by trade flows and transboundary pollution. They find that the outcomes of environmental policy depend on the prevailing cooperative or non-cooperative trade regime and on the size of the emission leakages and transboundary spillovers. Thus, a cooperative mechanism should be set up to resolve the transboundary pollution problem.

Hoel (2005) shows that a domestic inefficiency may arise in addition to the well-known inefficiencies at the international level as pollution is transboundary and there is international trade. Eyckmans and Finus (2007) analyze the important forces that hamper the formation of successful self-enforcing agreements to mitigate global warming. The success of international environmental treaty-making is enhanced by two types of measures: transfers to balance asymmetric gains from cooperation as well as institutional changes to keep the stability of a treaty. Institutional changes may be as important as transfers and should therefore receive more attention in future international negotiations on the transboundary pollution problem.

Bhagwati (2006) suggests a global warming fund which can legitimize the common responsibility of the developing and developed countries for emission reduction. Funfgelt and Schulze (2011) analyze the formation of environmental policy to regulate transboundary pollution. Governments systematically deviate from socially optimal environmental policies and may actually subsidize the production of a polluting good. Thus, politically motivated environmental policy thus may be more harmful to the environment and may enhance environmental quality and welfare beyond what a benevolent government would achieve.

This paper aims to explore an optimal controlling model for developing and developed countries incorporating both environment protection and economic growth. We improve upon the existing literature in various dimensions. First, a theoretic model based on dynamic differential game is set up to consider the optimal pollution controlling approach of developing and developed countries. Second, we differentiate four regions of environmental expenditure of both developing and developed countries in a capital accumulation and capital return panel. The optimal economic growth path of the developing country is deduced to explore the international cooperation for the pollution controlling.

Last but not least, we find that the increasing environmental expenditure of developed countries can initiate the optimal economic path for the developing country and control the environment pollution without significant loss of social welfare in both developing and developed countries. The rest of this paper is structured as follows. Section 2 introduces the theoretic model involving environmental protection cost. Section 3 differentiates the four regions of environmental expenditure of the developing and developed countries and the common border lines between four regions. Section 4 discusses the equilibrium of the optimal pollution controlling model and the economic growth path of the developing country. Section 5 contains some concluding remarks.

2. Model Setup

The world economy is composed of a developing and a developed country, respectively, indicated by superscript of 1 and 2. The population is constant in both countries. The GDP of the developed country 2 is divided into two parts:

1) Fixed expenditure: GDP-Y^2 =fixed consumption + capital accumulation;

2) Disposable income: Y^2= variable consumption C^2 + environment expenditure E^2.

The utility from fixed consumption of GDP-Y^2 in the developed country (i.e. U_B^2) will be dropped in the later discussion since consumption is very smooth in long run (Deaton and Muellbauer, 1980; Campbell and Deaton, 1989). Environment expenditure E^2 includes the costs of environmental protection in the developed country and its technological aids to the developing countries. Social welfare depends on the consumption and the quality of environment. According to the assumption, developing and developed countries try to maximize their social welfare in a dynamic model:

$$\max \quad \int_0^\infty [U^1(C^1, Z) + U_B^2 + U^2(C^2, Z)]e^{-\theta t}dt$$
$$= \int_0^\infty [U^1(C^1, Z) + U^2(C^2, Z)]e^{-\theta t}dt$$

subject to

$$\dot{K}^1 = F^1(K^1) - C^1 - E^1 - \delta^1 K^1$$

$$Z = G^1(K^1) - j^1 E^1 - j^2 E^2$$

$$Y^2 = C^2 + E^2, \qquad K^1(0) = K_0^1 \tag{1}$$

The objective function is the sum of welfare utility discounted value for the two countries, while the marginal utility of consumption is positive, i.e. $U_C^i > 0$ and the second order condition is negative, i.e. $U_{CC}^i < 0$; the marginal utility of pollution material stock Z is non-positive, i.e. $U_Z^i \leq 0$; and the second order condition and the initial condition are as follows: $U_{ZZ}^i \leq 0$, $U_{CZ}^i = U_{ZC}^i = 0$, $U_Z^i(C^i, 0) = 0$. It suggests the negative utility from pollution will become much worse as pollution stock is accumulated. We assume the pollution is only from the production in the developing country but will affect both countries. A proper example is the emission of greenhouse gas. Hence, the welfare utility of each country is dependent on consumption C^i and pollution stock Z.

In this equation, θ indicates time discount rate; K^1 indicates capital stock in the developing country so that $F^1(K^1)$ indicates its output; $\mathrm{G}^1(K^1)$ indicates the emission rate of pollution in the developing country and follows the first and second order conditions and initial condition as follow: $G_K^1 > 0$, $G_{KK}^1 \geq 0$, $G^1(0) = 0$; $\delta^1 K^1$ indicates depreciation of capital stock as δ^1 is the depreciation rate.

Being in the upstream of the global value chain, the emission rate of pollution material from the output of the developed country is assumed to be stable which has the advanced environment technology and low rate of economy growth. Hence, the environment pollution intensity of the whole economy system is only dependent on the capital stock in the developing country (K^1), and environmental expenditure ($E^i \geq 0, i = 1,2$) and environment technology coefficient ($j^i > 0, i = 1,2$) in each country. It is reasonable to assume these relationships as $Z_{K^1} > 0, Z_{E^1} < 0, Z_{E^2} < 0, Z_{j^1} < 0, Z_{j^2} < 0$.

Next, we set the Hamilton function to get the solution to the dynamic optimization in equation (2):

$$H = U^1(C^1, Z) + U^2(C^2, Z) + \lambda[F^1(K^1) - C^1 - E^1 - \delta^1 K^1] \tag{2}$$

λ indicates the marginal efficiency of capital stock. The necessary conditions of the maximum of welfare are:

$$\frac{\partial H}{\partial C^1} = 0 \quad \rightarrow \quad U_C^1 = \lambda \tag{3}$$

$$E^1 \frac{\partial H}{\partial E^1} = 0 \quad \rightarrow \quad E^1[(U_Z^1 + U_Z^2)(-j^1) - \lambda] = 0 \tag{4}$$

$$E^2 \frac{\partial H}{\partial E^2} = 0 \quad \rightarrow \quad E^2[-(U_Z^1 + U_Z^2) j^2 - U_C^2] = 0 \tag{5}$$

Euler equations: $$\frac{d(e^{-\theta t}\lambda)}{dt} = -\frac{\partial H}{\partial K^1} \quad \rightarrow \quad \dot{\lambda} = \lambda(\theta + \delta^1 - F_K^1) - (U_Z^1 + U_Z^2) G_K^1 \tag{6}$$

Transversality condition: $$\lim_{t \to \infty} K^1(t) \lambda(t) e^{-\theta t} = 0 \tag{7}$$

3. Four Regions of Environmental Expenditure

According to above conditions, the optimal solution cannot be deduced directly. Hence, the further analysis is dependent on different environment which is based on the four regions in the Figure 1.

(Figure 1 is around here)

3.1 Region a

In region a, both developing and developed countries invest the environment protection: $E^1 > 0, \quad E^2 > 0$. From equation (4), we can get $(U_Z^1 + U_Z^2)j^1 = -\lambda$. Since the second order condition of utility function is $U_{ZC} = 0$ and $U_{CZ} = 0$, the partial differentiation of utility on pollution material stock Z, i.e. U_Z is decided only by C and Z. We will have the equation: $[U_Z^1(C, Z) + U_Z^2(C, Z)]j^1 = -\lambda$. The increase of the marginal efficiency of capital can increase the total output which will next increase the pollution stock Z:

$$Z_\lambda = \frac{\partial Z}{\partial \lambda} = -\frac{1}{j^1(U_{ZZ}^1 + U_{ZZ}^2)} > 0 \tag{8}$$

Hence, the pollution stock function has the form as $Z = Z(\lambda)$, which is a monotonically increasing function of the marginal efficiency of capital λ. Since $U_C^1 = \lambda$ in equation (3), we can derive the monotonically decreasing function of λ for the consumption of the developing country, $C^1 = C^1(\lambda)$:

$$C_\lambda^1 = \frac{\partial C^1}{\partial \lambda} = \frac{1}{U_{CC}^1} < 0 \tag{9}$$

From equation (4) and (5), we can get: $\lambda = \frac{j^1}{j^2} U_C^2$. Do the partial differential on it and we have the monotonically decreasing function of λ for the consumption of developed country, $C^2 = C^2(\lambda)$:

$$C_\lambda^2 = \frac{\partial C^2}{\partial \lambda} = \frac{j^2}{j^1 U_{CC}^2} < 0 \tag{10}$$

In the developed economy, the total expenditure of variable consumption and environmental expenditure is $Y^2 = C^2(\lambda) + E^2$. Do the partial differentiation for λ and Y^2, we can find the environmental expenditure E^2 is a monotonically increasing function of λ and Y^2, $E^2 = E^2(\lambda \quad, Y^2)$. Therefore, the environment expenditure of the developed country is decided by marginal efficiency of capital stock λ and level of disposable income Y^2:

$$E_\lambda^2 = \frac{\partial E^2}{\partial \lambda} = -C_\lambda^2 > 0, \qquad \frac{\partial E^2}{\partial Y^2} = 1 > 0 \tag{11}$$

The intensity of environment pollution is: $Z(\lambda) = G^1(K^1) - j^1 E^1 - j^2 E^2(\lambda, Y^2)$, which can be transformed to $E^1 = \frac{1}{j^1}[G^1(K^1) - Z(\lambda) - j^2 E^2(\lambda, Y^2)]$. Do the partial differentiation for λ, K^1 and Y^2, we have the environment expenditure of the developing country decided by marginal efficiency of capital stock, capital stock in developing country and the disposable income in the developed country, $E^1 = E^1(\lambda, K^1, Y^2)$. Using equation (8) and (11), we derive the environment expenditure function in developing country is a monotonically decreasing function of λ and Y^2, but a monotonically increasing function of K^1. Thus, as the marginal efficiency of capital and the disposable income in the developed country increase, the environmental expenditure of the developing country will decrease. If the domestic capital stock increased, the environmental expenditure of the developing country would also increases:

$$\frac{\partial E^1}{\partial \lambda} = \frac{1}{j^1}(-Z_\lambda - j^2 E^2_\lambda) < 0, \ \frac{\partial E^1}{\partial K^1} = \frac{1}{j^1} G^1_K > 0, \ \frac{\partial E^1}{\partial Y^2} = -\frac{j^2}{j^1}\frac{\partial E^2}{\partial Y^2} < 0 \tag{12}$$

According to equation (4), (6) and (8), we can derive the locus of $\dot{\lambda}$ is:

$$\dot{\lambda} = -(U^1_{ZZ} + U^2_{ZZ}) j^1 \dot{Z} \tag{13}$$

when $E^1 > 0, E^2 > 0$, the fluctuation of environment pollution intensity is moving in tandem with the shadow price of capital stock in the developing country.

3.2 Region b

In region b, $E^1 > 0, E^2 = 0$, the environment expenditure of the developed country is zero when the developing country has made efforts on governing environment pollution. From equation (5), we can assume the marginal utility of consumption in the developed country is greater than the marginal utility of decreasing pollution in both developing and developed countries from the environmental protection expenditure in the developing country, $-(U^1_Z + U^2_Z\big|_{E^2=0}) j^2 < U^2_C$. Equation (8) still holds as $E^1 > 0$. Additionally, pollution stock is $Z(\lambda) = G^1(K^1) - j^1 E^1$ which can be transformed to $E^1 = \frac{1}{j}[G^1(K^1) - Z(\lambda)]$. Do the partial differentiation for λ and K^1, we have the environment expenditure of the developing country decided by marginal efficiency of capital stock and capital stock in developing country. Using equation (8) and (11), we derive the environment expenditure function in developing country is a monotonically decreasing function of λ, but a monotonically increasing function of K^1 : $E^1 = E^1(\lambda, K^1)$. Thus, as the marginal efficiency of capital increase, the environmental expenditure of the developing country will decrease. If the domestic capital stock increased, the environmental expenditure of the developing country would also increases:

$$\frac{\partial E^1}{\partial \lambda} = -\frac{1}{j^1} Z_\lambda < 0, \quad \frac{\partial E^1}{\partial K^1} = \frac{1}{j^1} G_K^1 > 0 \tag{14}$$

Without the environmental aids from the developed country, the developing country should increase the environment expenditure along with the capital accumulation and economic development. Equation (13) still holds, $\dot{\lambda} = -(U_{ZZ}^1 + U_{ZZ}^2) j^1 \dot{Z}$, the fluctuation locus of economy system's environment pollution intensity is as same as the track of capital stock shadow price of developing countries.

Proposition 1 Let $E^1 > 0, E^2 \geq 0$, environment pollution stock is moving in tandem with the shadow price of capital stock in the developing country, $\dot{\lambda} = -(U_{ZZ}^1 + U_{ZZ}^2) j^1 \dot{Z}$.

3.3 Region c

In region c, $E^1 = 0, \quad E^2 > 0$, the environment expenditure of the developing country is zero when the developed country has made efforts on governing environment pollution. From equation (4), we can assume the marginal utility of consumption in the developing country is greater than the marginal utility of decreasing pollution in both developing and developed countries from the environmental expenditure in the developed country, $-(U_Z^1 + U_Z^2\big|_{E^1=0}) j^1 < U_C^1$. From equation (3), we get $-(U_Z^1 + U_Z^2) j^1 < \lambda$. According to $U_{CZ}^1 = 0$ and $\lambda = U_C^1(C^1)$ in equation (3), we can have equation (9) as before.

And, from equation (5), we can get $-(U_Z^1 + U_Z^2) j^2 = U_C^2$ .The marginal utility of consumption in the developed country is equal to the marginal utility of decreasing pollution in both developing and developed countries from the environmental protection expenditure in the developing country. Because the U_Z and U_C are decided by C and Z: $-[U_Z^1(C^1,Z)+U_Z^2(C^2,Z)]j^2=U_C^2(C^2,Z)$, we can do the partial differential for C^2 and Z. The consumption in the developed country will decrease as the pollution stock increases, as the developed country need spend more on environmental protection.

$$\frac{\partial C^2}{\partial Z} = -\frac{j^2(U_{ZZ}^1 + U_{ZZ}^2)}{U_{CC}^2} < 0 \tag{15}$$

When the environment expenditure of the developing country is zero, the pollution stock is $Z=G^1(K^1)-j^2E^2$. Using $Y^2=C^2+E^2$, the pollution function is transformed into the form: $Z=G^1(K^1)-j^2(Y^2-C^2)$. The following derivation proves that Z is the increasing function of K^1:

$$\frac{dZ}{dK^1} = \frac{G_K^1}{1 - j^2C_Z^2} > 0 \tag{16}$$

Consumption in the developed country can be presented as a function of Z and K^1: $C^2 = C^2(Z) = C^2(Z(K^1))$. Using equation (15) and (16), we can conclude that the consumption in the developed country would decrease if the capital has been accumulated faster in the developing country:

$$\frac{dC^2}{dK^1}=\frac{dC^2}{dZ}\times\frac{dZ}{dK^1}=-\frac{j^2(U_{ZZ}^1+U_{ZZ}^2)}{U_{CC}^2}\times\frac{G_K^1}{1-j^2C_Z^2}<0 \tag{17}$$

Consequently, environmental expenditure in the developed country is in the form: $E^2 = E^2(K^1, Y^2) = Y^2 - C^2(Z(K^1))$. Do the partial differential for K^1 and Y^2 in the environmental expenditure function:

$$\frac{\partial E^2}{\partial K^1} = -\frac{\partial C^2}{\partial K^1} > 0\,,\ \frac{\partial E^2}{\partial Y^2} > 0 \tag{18}$$

Proposition 2 Let $E^1 = 0, \quad E^2 > 0$, environment expenditure in the developed country increases as the capital stock and disposable income increase in the developing country.

3.4 Region d

In region d, the marginal utility of consumption in both developing and developed countries is greater than the marginal utility of decreasing pollution in both developing and developed countries. Hence, environment expenditure is zero in both countries, $E^1 = E^2 = 0$. From equation (4) and (5), we can have $-(U_Z^1 + U_Z^2\big|_{E^1=0})j^1 < \lambda = U_C^1(C^1, Z)$ and $-(U_Z^1 + U_Z^2\big|_{E^2=0}) < U_C^2$. Equation (3) and (9) still hold.

The environment expenditure is zero, $E^1 = E^2 = 0$. Hence, all disposable income in the developed country is used for consumption, $Y^2 = C^2$. The derivation of $Z = G^1(K^1)$ can get $Z = Z(K^1)$:

$$\dot{Z} = G_K^1 \dot{K}^1 \tag{19}$$

Proposition 3 Let $E^1 = 0, \quad E^2 = 0$, there is no environment expenditure in the developing and developed countries. The pollution stock changes in the same way of the shadow price of the capital stock in the developing country.

3.5 The common border lines of 4 regions

In above four statuses, the pollution stock and the shadow price of capital stock in the developing country change in the same direction. It suggests that the economic growth is an important factor of the pollution emission. The rapid economy growth as well as the environment deterioration in the developing country might be two sides of the same coin. With the increasing environmental expenditure, a optimal environmental protection path could be achieved.

We first derive the common border lines between the four states (see the derivation process in Appendix A):

1) $E^2 > 0$ is corresponding to region a and c, with the common border line: $-(U_Z^1 + U_Z^2\big|_{E^1=0})j^1 = \lambda$;

2) $E^2 = 0$ is corresponding to region b and region d, with the common border line: $-(U_Z^1 + U_Z^2\big|_{E^1=E^2=0})j^1 = \lambda$;

3) $E^1 > 0$ is corresponding to region a and region b, with the common border line: $\lambda = \left.\frac{j^1 U_C^2}{j^2}\right|_{C^2=Y^2}$;

4) $E^1 = 0$ is corresponding to region c and region d, with the common border line: $-(U_Z^1 + U_z^2 \big| Z_{E^1=E^2=0}) = \left.\frac{U_C^2}{j^2}\right|_{C^2=Y^2}$.

All of the four the common border lines intersect at a point ($\overline{K}$, $\frac{j^1}{j^2}U_C^2$, see the derivation process in Appendix B). The classification of two countries by environment expenditure is presented in Figure 1.

4. An Optimal Pollution Controlling Model

In this section, we explore the optimal pollution protection and economic growth path. The general forms of utility, pollution and production functions cannot give the equilibrium of the pollution controlling model. In order to clear the market and decide the optimal economic growth path, function forms of utility, pollution and production are set as follows:

$$U = \frac{C^{1-\sigma}-1}{1-\sigma} - \frac{Z^{1+\rho}}{1+\rho}, \quad 0 < \sigma \le 1, \quad \rho > 0 \tag{20}$$

$$Z = K^{\beta} - j^1 E^1 - j^2 E^2, \qquad j > 0, \quad \beta \ge 1 \tag{21}$$

$$F = bK^{\alpha}, \qquad b > 0, \quad 0 < \alpha < 1 \tag{22}$$

Therefore, $U_C = C^{-\sigma} > 0$, $U_{CC} = -\sigma C^{-\sigma-1} < 0$; $U_Z = -Z^{\rho} < 0$, $U_{ZZ} = -\rho Z^{\rho-1} < 0$; $U_{ZC} = 0$; $Z_K = \beta K^{\beta-1}$, $Z_{KK} = \beta(\beta-1)K^{\beta-2} > 0$; $Z_E = -j < 0$; $F_K = \alpha b K^{\alpha-1} > 0$, $F_{KK} = \alpha b(\alpha-1)K^{\alpha-2} < 0$; σ is the elasticity coefficient of marginal utility from consumption: $U_{CC}\frac{C}{U_C} = -\sigma < 0$; ρ is the elasticity coefficient of marginal negative utility from pollution: $U_{ZZ}\frac{Z}{U_Z} = \rho > 0$; $\alpha - 1$ is the elasticity coefficient of marginal production: $F_{KK}\frac{K}{F_K} = \alpha - 1 < 0$.

The optimal equilibrium of the pollution controlling model should distribute the cost of environmental protection between the developing and developed countries. Hence, the optimal economic growth path of the developing country need convergence to a point within the region a. After the derivation process presented in Appendix C, we can find that if the disposable income of the developed country were more than the value in equation (23), equilibrium point (K^{1*}, λ^*) should be within the region a (see Figure 2).

$$Y^2 = \frac{2(\theta + \delta^1 + \frac{1}{j^1})^2}{b^2 j^2} \tag{23}$$

It suggests that the developed country should have enough disposal income before they decide to spend money on controlling the pollution emitted by the production and capital accumulation in the developing country. As the disposable income of the developed country is more than the value in equation (23), the capital stock of the developing country can be accumulated beyond the region d and afford the cost of environmental protection. Thus, the economic growth and technological improvement could increase the capital return λ as well as the capital formation K^1 into region a, which distribute the cost of environmental protection between the developing and developed countries and finally reduce the pollution. It is consistent with the environmental Kuznets curve (EKC) hypothesis (see a survey in Dinda, 2004).

(Figure 2 is around here)

Moreover, according to the accumulation of capital stock K^1, the optimal economic growth approach of the developing country can be divided into 3 stages as in Figure 2. First, in region d, the capital stock level of the developing country is too low to afford any cost of environmental protection, $E^1 = 0$. The developed country has no enough disposal income to invest in governing environment pollution from the developing country ($E^2 = 0$). The consumption in the developing country i.e. C^1 increases continuously so that its marginal utility i.e. U_C^1 has been declining with $U_{CC}^1 < 0$. According to equation (3), the capital return is equal to the marginal utility of consumption in the developing country i.e. $U_C^1 = \lambda$. Hence, as the capital stock has been increasing in the developing country, the capital return λ has been declining along the optimal economic growth curve $(\dot{K}^1 = 0)$ in region d (see Figure 2). The consumption of the developed country is at steady state, but the sum of pollution stock $Z = G^1(K^1)$ increases continuously which deteriorates the environment of both countries.

Second, after the capital stock goes beyond the region d, $\tilde{K} > \overline{K}$, the optimal economic path is in region c. However, the capital stock of developing countries is still too low to afford any cost of pollution controlling. The environment expenditure in the developing is

$E^1 = 0$, while the developed country now has enough disposable income to invest in the pollution controlling ($E^2 > 0$).

As the capital stock accumulates in the developing country, the economy grows and the consumption (C^1) increases continuously. The marginal utility of consumption (U_C^1) and the capital return (λ) have been declining along the optimal economic growth curve ($\dot{K}^1 = 0$) in region c (see Figure 2). On the one hand, according to equation (18), environmental expenditure in the developed country (E^2) increases with its more and more disposable income (Y^2) and the capital stock accumulation in the developing country (K^1). On the other hand, along with the increasing consumption (C^1) in the developing country, the sum of pollution stock Z also increases continuously. The developed country need decrease its domestic consumption (C^2) to increase environment expenditure (E^2) as follows:

$$C^1 \uparrow \rightarrow \lambda \downarrow \rightarrow K^1 \uparrow \begin{pmatrix} \dfrac{\partial E^2}{\partial K^1} > 0 \\ \dfrac{\partial Z}{\partial K^1} > 0 \end{pmatrix} \rightarrow \begin{cases} E^2 \uparrow \rightarrow C^2 \downarrow \\ Z \uparrow \end{cases} \tag{24}$$

Third, in region a ($E^1 > 0, E^2 > 0$), when the capital stock level of developing countries is high enough to invest the environmental protection, the environmental expenditure from both developing and developed countries increase continuously to reduce the pollution. As the capital stock has been increasing, the capital return (λ) decreases in the developing country. According to equation (9), consumption in the developing country (C^1) has been increasing continuously. At the same time, equation (11) and (12) suggest increasing environment expenditure in the developing country (E^1), but decreasing environment expenditure in the developed country (E^2). According to equation (10), consumption in the developed country (C^2) can resume step by step as they can save environment expenditure. From equation (8), the pollution stock (Z) begins to decrease in this phase.

$$C^1 \uparrow \to \lambda \downarrow \;\; \to \left.\begin{matrix} \dfrac{\partial C^2}{\partial \lambda} < 0, \dfrac{\partial E^1}{\partial \lambda} < 0 \\ \dfrac{\partial E^2}{\partial \lambda} > 0, \dfrac{\partial Z}{\partial \lambda} > 0 \end{matrix}\right\} \Rightarrow \;\; C^2 \uparrow, \;\; E^1 \uparrow, \;\; E^2 \downarrow, \;\; Z \downarrow \qquad (25)$$

It is assumed that the developing country begins to develop economy from a very low productivity level. Hence, region b ($E^1 > 0, E^2 = 0$) is irrelevant to the optimal economic growth path of the developing country.

Based on above analysis, we can explore the effect of increasing environment expenditure of the developed country. In region c and d, the developing country has no environmental expenditure ($E^1 = 0$). The consumption in the developed country (C^2) increases along with its disposable income (Y^2). The marginal utility of consumption in the developed country (U_c^2) has been decreasing. Hence, the common border line $-(U_z^1 + U_z^2\big|_{E^1=E^2=0}) = \dfrac{U_c^2}{j^2}\bigg|_{c^2=Y^2}$ of region c and d would move leftwards.[1]

In region a and b, the developing country invests in environmental protection $E^1 > 0$. The consumption in the developed country (C^2) increases along with its disposable income (Y^2). The marginal utility of consumption in the developed country (U_c^2) and capital return (λ) decrease, the common border line $\lambda = \dfrac{j^1 U_c^2}{j^2}\bigg|_{c^2=Y^2}$ of region a and b moves downwards.

In region a and c, the developed country invests in environmental protection ($E^2 > 0$). The common border line between region a and c is $-(U_z^1 + U_z^2\big|_{E^1=0}) j^1 = \lambda$. In region c, $E^1 = 0$, $Z = G^1(K^1) - E^2 j^2$, using equation (18), we find that as Y^2 increases, the pollution stock Z decreases. :

[1] Also see equation (C9) in Appendix C. As the disposable income (Y^2) increases in the developed country, the common border line between region c and d, i.e. $K^1 = \bar{K} = \left(\dfrac{(Y^2)^{-\sigma}}{2j^2}\right)^{\frac{1}{\rho}}$ would also move leftwards.

$$\frac{\partial Z}{\partial Y^2} = \frac{\partial Z}{\partial E^2}\frac{\partial E^2}{\partial Y^2} = -E_Y^2 j^2 = -j^2 < 0 \qquad (26)$$

At the same time, the decreasing Z makes $-U_z^1$ and $-U_z^2$ decreases, so that the capital return (λ) decreases with the increasing capital stock in the developing country (K). Therefore, the common border line between region a and c moves downwards-and-rightwards.

In region b and d, the developed country does not invest in environmental protection ($E^2 = 0$). The common border line between region b and region d is $-(U_z^1 + U_z^2\big|_{E^1=E^2=0})j^1 = \lambda$. In region d, E^1=0, so the pollution stock is only dependent the pollution emission of the production in the developing country: $Z = G^1(K^1)$. Even if the disposable income of developed country (Y^2) increases, the pollution stock Z does not change with Y^2. The common border line between region b and region d does not move. These results are depicted in Figure 3.

(Figure 3 is around here)

It's more difficult for the developing country to reduce consumption, capital accumulation and economic growth at the initial stage to protect environment. With very low levels of consumption and capital at beginning, the marginal utility of consumption and capital return are too high in the developing country (see equation (3)). The pollution stock will increase with the higher consumption and capital accumulation in the developing country, which will decrease the welfare of two countries.

The developed country increases the environment expenditure and lowers the marginal output of K^1 to give more space for region a. In order to stimulate the developing country to reduce environment pollution, the developed country need contain its domestic consumption in advance and find a balance between domestic consumption and pollution controlling. It also needs to smooth out the current and future consumption in the developed country. At the later stage, as the developing country join the pollution controlling and share the cost of environmental expenditure, the developed country can resume the consumption with a better environment. Therefore, there is an optimal economic growth path in the pollution controlling model for both developing and developed countries which are facing the same dual targets of environmental protection and economic growth.

5. Concluding Remarks

This paper sets up a pollution controlling model for a representative pollution substance like greenhouse gas. We find an optimal economic growth path of a developing country with pollution emission from its production process. There is a solid theory basis for the developing country to share the "common but differentiated responsibilities" with the developed countries on climate negotiation. The developed country need take more responsibility at initial stage to stimulate the developing country to follow the obligation of emission.

Moreover, the deterioration of environment pollution can be avoided along with the economic growth of the developing countries. The existing greenhouse gas accumulation was discharged by developed countries during early economic growth period. Thus, it is unfair and impossible for only the developing country to take all responsibility (region b in Figures). This paper indicates that the capital accumulation and economic growth in the developing countries is the continuous impetus to solve the environment problems. For developed countries, the optimal pollution controlling model provides a compensating mechanism to the decreasing consumption at the initial stage. The developed country need increase environment expenditure to assist the developing countries in economic growth and capital accumulation. On the other hand, the R&D investment in the pollution controlling technology can help the developing countries join the international cooperation project without too much environmental expenditure at the early stage of economic growth.

The future research need apply this model to a more realistic background of the international cooperative program of pollution controlling. More factors such as international trade, technological spillovers and global value chain embedment should be considered. Moreover, the model can be also applied into regional studies within a developing country which are in different development stages with segmentation of pollution emission.

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Figure 1 environment expenditure classifications of two countries

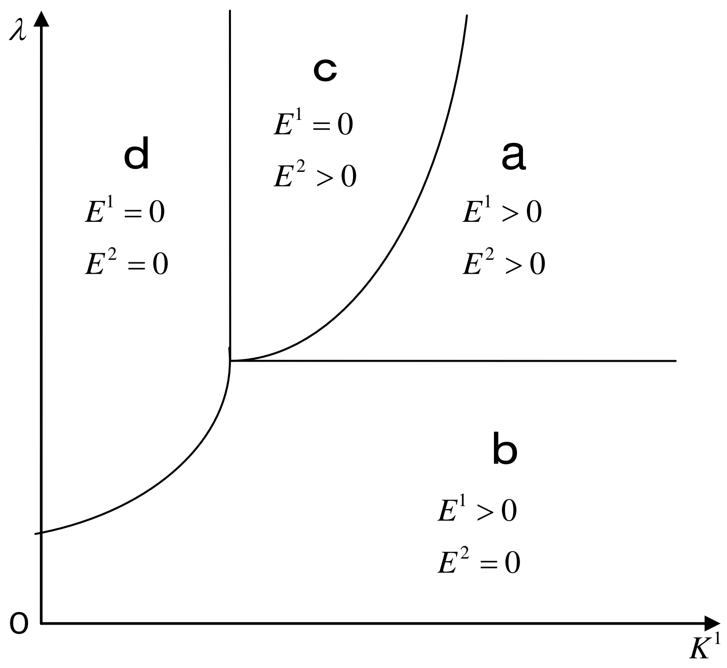


Figure 2 Optimal economic growth path of the developing country

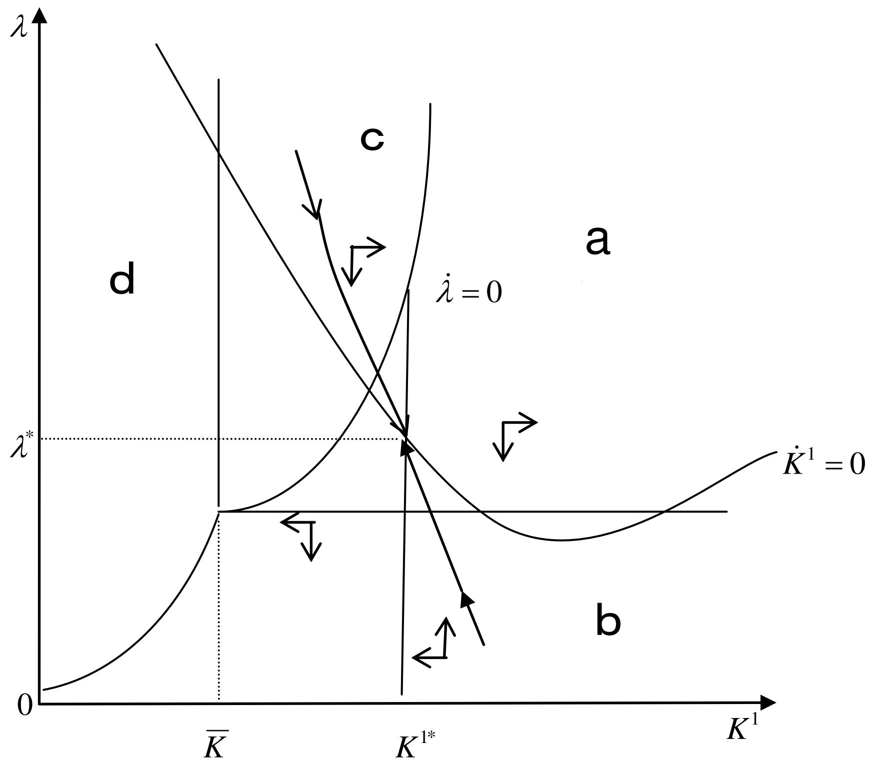
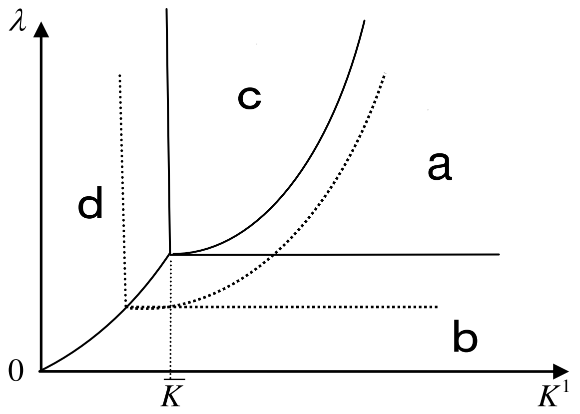


Figure 3 Effect of environment expenditure of the developed country



Appendix A

Common border lines between regions

If $E^1 > 0$, in region a, equation (4) and (5) can hold, and the pollution stock function has the form: $Z = G^1(K^1) - j^1E^1(\lambda, K^1, Y^2) - j^2E^2(\lambda, Y^2)$, equation (12) is derived, and we can have the horizontal common border line between region a and region b in the (K^1, λ) panel (see Figure 1) as follows:

$$\frac{\partial E^1}{\partial \lambda} = \frac{1}{j^1}(-Z_\lambda - j^2E_\lambda^2) < 0 \qquad \text{(A1)}$$

$$\frac{\partial E^1}{\partial K^1} = \frac{1}{j^1}G_K^1 > 0 \qquad \text{(A2)}$$

$$\frac{\partial \lambda}{\partial K^1} = \frac{G_K^1 - j^1E_K^1}{j^1E_\lambda^1 + j^2E_\lambda^2} = 0 \qquad \text{(A3)}$$

Replace E_K^1 in (A3) with (A2), we can see capital return λ does not change with capital stock in the developing country K^1, suggesting the common border line is horizontal. In region a and b, $E^1 > 0$. The developed country is indifferent on choosing zero or a positive environmental expenditure at the common border line between region a and b. From equation (4) and (5), the horizontal common border line between region a and b is:

$$\lambda = \left.\frac{j^1U_C^2}{j^2}\right|_{C^2=Y^2} \qquad \text{(A4)}$$

In region a and c, $E^2 > 0$. If $E^1 = 0$, in region c, from equation (3) and (5) we derive equation (15)-(18). Environment expenditure in the developed country (E^2) is a monotonically increasing function of the capital stock in the developing country (K^1): $E^2 = E^2(K^1)$,

$$E_K^2 = -\frac{C_Z^2G_K^1}{1 - j^2C_Z^2} > 0 \qquad \text{(A5)}$$

In the common border line between region a and c, the developing country is indifferent on choosing zero or a positive environmental expenditure: $E^1 \geq 0$. Equation (4) suggests $-(U_Z^1 + U_Z^2\big|_{E^1=0}) j^1 = \lambda$, and the pollution stock function $Z = G^1(K^1) - j^2 E^2(K^1)$, we can derive the partial differentiation of λ on K^1 in the common border line:

$$\frac{\partial \lambda}{\partial K^1} = -j^1(U_{ZZ}^1 + U_{ZZ}^2)\,(G_K^1 - j^2 E_K^2) > 0 \qquad \text{(A6)}$$

We can replace E_K^2 in equation (A6) with the equation (A5). Using equation (15) C_Z^2 <0 and G_K^1 >0, we can find: $G_K^1 - j^2 E_K^2 = G_K^1 + j^2 \dfrac{C_Z^2 G_K^1}{1 - j^2 C_Z^2} = \dfrac{G_K^1}{1 - j^2 C_Z^2} > 0$. Hence, the common border line between region a and c is a curve with positive slope which can be derived from $-(U_Z^1 + U_Z^2\big|_{E^1=0}) j^1 = \lambda$.

In the same vein, we derive the common border line between region b and d is $-(U_Z^1 + U_Z^2\big|_{E^1=E^2=0}) j^1 = \lambda$. And, the common border line between region c and d is a vertical line derived from $-(U_Z^1 + U_z^2 \big| Z_{E^1=E^2=0}) = \left.\dfrac{U_C^2}{j^2}\right|_{C^2=Y^2}$.

Appendix B

The intersection of four common border lines

The intersection point of the common border line between region b and d and the common border line between region c and d can be derived from the simultaneous equations as follow:

$$\begin{cases} -(U_Z^1 + U_Z^2\big|_{E^1=E^2=0})j^1 = \lambda & \text{(B1)} \\ -(U_Z^1 + U_z^2\big|Z_{E^1=E^2=0}) = \dfrac{U_C^2}{j^2}\bigg|_{C^2=Y^2} & \text{(B2)} \\ Z = G^1(K^1) & \text{(B3)} \end{cases}$$

Equation (B1) can be transformed to $-(U_Z^1 + U_Z^2\big|_{E^1=E^2=0}) = \dfrac{\lambda}{j^1}$, replacing the left hand side of equation (B2), we get $\lambda = \dfrac{j^1}{j^2}U_C^2$. Let the capital stock equalize the equation (B2) and (B3) be a constant $\overline{K}$, we have the intersection point ($\overline{K}$, $\dfrac{j^1}{j^2}U_C^2$).

The intersection point of the common border line between region a and b and the common border line between region b and d can be derived from the simultaneous equations as follow:

$$\begin{cases} \lambda = \dfrac{j^1 U_C^2}{j^2}\bigg|_{C^2=Y^2} & \text{(B4)} \\ -(U_Z^1 + U_Z^2\big|_{E^1=E^2=0})j^1 = \lambda & \text{(B5)} \end{cases}$$

The point of intersection is still ($\overline{K}$, $\dfrac{j^1}{j^2}U_C^2$). In the same vein, the interaction point of the common border line between region a and c and the common border line between region a and b can be solved by same way.

Appendix C

In region a, $E^1 > 0, \quad E^2 > 0$, from equation (4) and (6), $\dot{\lambda} = -(U_{ZZ}^1 + U_{ZZ}^2)[j^1(\theta + \delta^1 - F_K^1) - G_K^1)]$. At the steady state of the economic growth path of the developing country, the capital return should be stable. $\dot{\lambda} = 0$ and $U_{ZZ} < 0$ will give $j^1(\theta + \delta^1 - F_K^1) - G_K^1 = 0$. Hence, the economic growth path of the developing country should satisfy the following condition:

$$\theta + \delta^1 + \frac{G_K^1}{j^1} - F_K^1 = 0 \qquad \text{(C1)}$$

Using equation (21) and (22), we can calculate $G_K^1 = \beta K^{\beta-1}$ and $F_K^1 = b\alpha K^{\alpha-1}$. And, equation (C1) can be transformed into

$$\theta + \delta^1 + \beta \frac{K^{\beta-1}}{j^1} - b\alpha K^{\alpha-1} = 0 \qquad \text{(C2)}$$

We further calculate the capital stock in the equation by assuming $\beta = 1$. And then, we can get the optimal capital stock of the developing country at the steady state as:

$$K^1 = \tilde{K} = \left(\frac{\theta + \delta^1 + \frac{1}{j^1}}{b\alpha} \right)^{\frac{1}{\alpha-1}} \qquad \text{(C3)}$$

Since $U_C = C^{-\sigma}$, $U_Z = -Z^{\rho}$ and $U_C^1 = \lambda$ in equation (3), we get the optimal consumption in the developing country:

$$C^1 = \lambda^{-\frac{1}{\sigma}} \qquad \text{(C4)}$$

From equation (4) and (5), $U_C^2 = (C^2)^{-\sigma} = \frac{j^2\lambda}{j^1}$, we have the optimal consumption in the developed country:

$$C^2 = \left(\frac{j^2\lambda}{j^1}\right)^{-\frac{1}{\sigma}} \tag{C5}$$

From equation (4) and $U_Z = -Z^{\rho} - j^1(U_Z^1 + U_Z^2) = \lambda$, we derive the optimal pollution stock as follows:

$$Z = \left(\frac{\lambda}{2j^1}\right)^{\frac{1}{\rho}} \tag{C6}$$

Using $G^1(K^1) = K^1$, $Y^2 = C^2 + E^2$ and equation (C5)-(C6), equation (12) can be transformed into a new form:

$$E^1 = \frac{1}{j^1}\{K^1 - \left(\frac{\lambda}{2j^1}\right)^{\frac{1}{\rho}} - j^2Y^2 + j^2\left(\frac{j^2\lambda}{j^1}\right)^{-\frac{1}{\sigma}}\} \tag{C7}$$

The steady state of the capital accumulation in the developing country suggests $\dot{K}^1 = 0$. Hence, the capital accumulation condition in equation (1) is $\dot{K}^1 = F^1(K^1) - C^1(\lambda) - E^1(K^1, \lambda) - \delta^1K^1 = 0$. Using equation (22), (C4) and (C7), the optimal condition of the capital accumulation in the developing country, also the optimal economic growth path is given in the K^1 and λ panel as in Figure 2:

$$b(K^1)^{\alpha} - (\lambda)^{-\frac{1}{\sigma}} - \frac{1}{j^1}\{K^1 - \left(\frac{\lambda}{2j^1}\right)^{\frac{1}{\rho}} - j^2Y^2 + j^2\left(\frac{j^2\lambda}{j^1}\right)^{-\frac{1}{\sigma}}\} - \delta^1K^1 = 0 \tag{C8}$$

(C3) and (C8) jointly decide the equilibrium point ($\tilde{K}$, λ^*). Next, we take further analysis on the common border line between region c and d. When the developing country provide no expenditure on the environmental protection, i.e. $E^1 = 0$, the common border line between region c and d gives the contingent condition that the developed country is indifferent on whether provide expenditure on the environmental protection, $E^2 > 0$, or not, $E^2 = 0$. Hence, given $E^1 = 0$ and $E^2 = 0$, $Z = G^1(K^1) = K^1$ and $Y^2 = C^2$. The common border line can be rewritten as: $-j^2[U_Z^1(G^1(K^1)) + U_Z^2(G^1(K^1))] = U_C^2(Y^2)$. Using $U_C = C^{-\sigma}$ and $U_Z = -Z^{\rho}$, the common border line is $2j^2(K^1)^{\rho} = (Y^2)^{-\sigma}$. We can calculate the common border line is:

$$K^1 = \overline{K} = \left(\frac{(Y^2)^{-\sigma}}{2j^2}\right)^{\frac{1}{\rho}} \tag{C9}$$

Comparing (C3) and (C9), we can find that the necessary condition of the equilibrium in region a is $\tilde{K} > \overline{K}$, that is:

$$\left(\frac{\theta + \delta^1 + \frac{1}{j^1}}{b\alpha}\right)^{\frac{1}{\alpha-1}} > \left(\frac{(Y^2)^{-\sigma}}{2j^2}\right)^{\frac{1}{\rho}} \tag{C10}$$

We further simply the inequality by assuming $\sigma = 1, \rho = 1, \alpha = \frac{1}{2}$, and get the necessary condition of equilibrium being within region a:

$$Y^2 > \frac{2(\theta + \delta^1 + \frac{1}{j^1})^2}{b^2 j^2} \tag{C11}$$

Therefore, as the disposable income of the developed country is more than the value in equation (C11), the capital stock of the developing country can be accumulated beyond the region d and afford the cost of environmental protection.