Inflation and the underground economy

Stephen Ahiabu

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Stephen Eli Ahiabu†
University of Toronto

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Abstract

This paper studies the optimal rate of seigniorage in an economy characterized by decentralized trade and a tax-evading underground sector. The economy has buyers, some of whom visit the formal market, while others visit the underground market. I find that the optimal rate of inflation depends on which of the two sectors, formal or underground, is more crowded/congested with buyers. If the underground sector is more crowded, the optimal inflation rate is as high as 42% per annum for Peru. That is, I offer a possible motivation for the high rates of inflation observed in that country from the mid 1970s up to the mid 1990s. If the formal sector is more crowded, optimal inflation falls to about 1.4%, which is close to the rate in 2005. Friedman rule is not optimal.

Keywords: Inflation, Market Congestion, Ramsey Equilibrium, Underground Economy

JEL classification: E26, H21, H23, H26, O17, O23

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†Department of Economics, Sidney Smith Hall, Room 4072, 100 St. George Street, University of Toronto, Toronto, Ontario, M5S 3G3, Canada. E-mail: sahiabu@chass.utoronto.ca, Tel/Fax: +1(416) 963 4457

1 Introduction

This paper studies the optimal rate of seigniorage in an economy characterized by decentralized trade and a tax-evading underground sector. Previous authors have addressed policy questions using environments in which both the formal and underground markets are perfectly competitive (centralized markets). To the contrary, one-on-one meetings between buyers and sellers (decentralized exchange) seem to be the more plausible trade arrangement that facilitates tax evasion. I show that when trade is decentralized in both the formal and underground sectors, the optimal rate of inflation is widely different from what has been suggested in the literature. That is, policymakers must revise their notions of optimal inflation policy if they agree with the premise that decentralized trade is the better way to model tax evasion. In terms of magnitudes, the underground-to-formal sector output ratio is estimated to be about 8.8% in the US, 44% in Peru and 76% in Nigeria [see Schneider and Enste (2000)].

The literature notes that only the formal sector is directly taxable. Taxes are thus distortionary. An increase in the inflation rate increases seigniorage incomes. With higher income from seigniorage, government no longer needs to tax as much and the tax rate can be reduced, along with tax distortions. Lower taxes in turn encourage formal sector production, as opposed to underground production. In short, inflation causes a decline in underground output.[1] This logic explains the conventional wisdom claiming a negative relationship between changes in the rate of inflation and changes in underground output. This negative "seigniorage relationship" is well-captured in the models with competitive markets.[2] The data on the other hand is far less conclusive. In Figure 3, I compare changes in inflation to changes in underground output for several countries. There is very little if any such negative correlation.[3] Although there may be an endogeneity problem, this

[1] Specifically, I mean a decline in the underground-to-formal sector output ratio.

[2] See Cavalcanti and Villamil (2003) and Koreshkova (2006).

[3] In fact, regressing changes in inflation on changes in the underground-to-formal sector "output ratio" generates coefficients that are statistically not different from zero.

only cements the need for comprehensive modeling of the underground economy to investigate the evidence at hand.

This paper seeks to provide a better understanding of how inflation affects underground output and how this may influence the optimal rate of inflation. The working definition of the underground economy is that buyers in this sector evade consumption taxes. Secondly, underground goods are of inferior quality.[4] The model predicts that depending on certain market conditions (which I explain next), inflation can either increase or decrease the underground sector. This reconciles well with the data. The fact that inflation can move underground output in both directions has pivotal implications for optimal inflation. That is, when inflation reduces the underground sector (reduces tax evasion), seigniorage financing becomes very attractive. The optimal rate of inflation is therefore high. The reverse is also true.

In the environment examined, households have buyers. Some buyers are sent to the formal market, while others are sent to the underground market. If underground goods are extremely poor in quality, a household sends relatively more buyers to the formal market. Every household acts in the same way and private interest overwhelms the social optimum. Therefore, there is the tendency for overcrowding of buyers in the formal sector and trade opportunities become few for each buyer in this sector.[5] If the inflation rate increases, households try to spend money faster.[6] They divert buyers to the underground market, where the overcrowding of buyers is less. The turnover of goods in the underground market increases and underground output increases.

Notice that in the above analysis, inflation delivers the negative seigniorage effect; similar to the competitive models (thus, seigniorage income reduces taxes,

[4] Admittedly, there are a wide range of definitions of the underground economy. For the purpose of this paper, I focus on this narrow definition.

[5] This of course depends on the allocation of sellers as well. For a full description of how I treat sellers, see section 2. Also, one can think of "fewer trade opportunities" as equivalent to a "lower probability of finding a match with a seller".

[6] With higher inflation, households' money stocks lose value quicker and they try to spend money faster at currently prices rather than at future higher prices.

which in turn reduce underground output). Here however, the overcrowding effect reverses the decline in underground output. Since inflation increases tax evasion, seigniorage financing becomes less attractive and the optimal rate of inflation is low compared to the literature.

On the other hand, if underground goods are of considerably good quality, the underground market tends to be more crowded for underground buyers. Each underground buyer has fewer trade opportunities. In response to higher inflation, buyers move to the "less-crowded" formal market to spend money faster. Thus, inflation reduces tax evasion on both fronts (seigniorage effect and crowding effect) and seigniorage financing becomes more attractive. The optimal inflation rate is high, as observed in some poor countries.

In relation to Figure 3, the results can be interpreted as follows. At a given point in time, two countries can take opposite positions on the relative overcrowding of their formal and underground markets for buyers. Inflation hence impacts their underground sectors in opposite directions. Secondly, over time, a single country can switch states in the relative overcrowding of the two sector markets for buyers. Thus, inflation moves underground output in reverse directions over time. Putting these together, one can get data points that wrongly suggest no relationship between changes in inflation and changes in the output ratio, similar to Figure 3.

An environment with decentralized trade and market crowding is essential for generating the results outlined. In the equivalent economies with perfectly competitive markets, the distribution of goods from sellers to buyers is fully and equally efficient in both sectors. Money can thus be spent equally fast in both sectors and households do not need to adjust buyer allocations in order to accomplish this goal. Since the overcrowding effect does not exist, this dimension has no effect on tax evasion. The optimal rate of inflation is thus unaffected. The overcrowding effect is peculiar to decentralized markets and is sometimes termed the "extensive margin" or the "market congestion effect".

The contributions of this paper are as follows. First, I build strong micro foundations for the underground economy by including anonymity, which directly motivates tax evasion. I show that the relative crowding/congestion of the two markets is important for optimal inflation. I make a significant contribution towards the integration of elaborate schemes of public finance into the monetary search literature, following recent progress by Aruoba, Waller and Wright (2006). I show that these models are indeed computable and that they can generate conclusions that are relevant for policy.

This paper adds to the existing monetary literature on the informal sector, along side Koreshkova (2006), Cavalcanti and Villamil (2003) and Nicolini (1998). Optimal policy in the presence of externalities follows fundamentals laid by Sandmo (1975).[7] The next section presents a two-sector monetary search framework, replicating properties of the underground-formal dichotomy. In section 3, I characterize the model and describe the equilibrium. Section 4 derives the price and output ratios and examines how households adjust decisions when inflation changes. In section 5, I calibrate the model to data from Peru and the US and present quantitative estimates of the impact of inflation. Section 6 considers robustness and compares the results to some forerunners. I conclude in section 7.

2 Economic Environment

I extend the tractable framework introduced by Shi (1999) to allow for two sectors, formal and underground/informal. These are denoted by the subscripts f and i respectively and are assumed to be on separate islands. Goods are perishable between periods, irrespective of the sector in which they are produced. By this, I preclude the emergence of commodity money. Self-produced goods yield no utility and hence trade is essential for worthwhile consumption. Some of these restrictions are standard in monetary search models, as they permit trade and an endogenous

[7] Also see Ng (1980), Goulder (1995) and Bovenberg and van der Ploeg (1998).

role for fiat money.

Time is discrete, denoted t. Money is the sole state variable. The economy is inhabited by a large number of anonymous and infinitely-lived agents who are either buyers or sellers/producers. For tractability, I collect agents into decision-making families or households.[8] A household is constituted by the measure s of sellers and b of buyers; $s \in (0, \infty)$, $b \in (0, s]$. There are a large number of households, and each household is infinitesimal compared to the aggregate. The focus is on the representative household, who's state and choice variables are in lower-case letters. Capital-case variables represent those of other households and the aggregate economy, which the representative household takes as given. Economy-wide money supply is M_t, of which the representative household has m_t. There is no population growth; the number of households, sellers and buyers being exogenous constants.

2.1 Market Congestion (Market Overcrowding)

The key mechanism driving the results in this paper is the potential for differences in market congestion in the two sectors. Hence, I present this mechanism first.

Each household sends a fraction of its buyers and sellers to each sector market. Let B_{jt} and S_{jt} be the aggregate number of buyers and sellers entering market j, $j = f, i$. These agents match one-on-one and may trade if the match is successful. A successful match occurs when any buyer meets a seller from a household other than his own. The total number of successful matches, $\mathcal{X}_{jt}$, is derived from the matching function:

$$\mathcal{X}_{jt} = B_{jt}^{\alpha} S_{jt}^{1-\alpha} \ , \ \alpha \in (0,1) \ , \ j = f, i \ .$$

[8] A related tractable environment proceeds with agents rather than households [see Lagos and Wright (2005)].

Also, define $\mathcal{B}_{jt}$ and $\mathcal{S}_{jt}$ as:

$$\begin{aligned} \mathcal{B}_{jt} &= \frac{\mathcal{X}_{jt}}{B_{jt}} = \left(\frac{S_{jt}}{B_{jt}}\right)^{1-\alpha} \quad \text{and} \\ \mathcal{S}_{jt} &= \frac{\mathcal{X}_{jt}}{S_{jt}} = \left(\frac{B_{jt}}{S_{jt}}\right)^{\alpha}, \quad j = f, i. \end{aligned}$$

Then $\mathcal{B}_{jt}$ and $\mathcal{S}_{jt}$ are the market congestion/crowding rate for buyers and sellers respectively.[9] These can also be interpreted as the average matching rates per buyer and per seller respectively. Since each household is infinitesimal, they take market congestion rates as given. Holding sellers constant, the larger the number of buyers entering market j, the higher is the market congestion/crowding for buyers in that sector and the fewer the trade opportunities for each buyer in that sector.

Suppose there are more trade opportunities for each underground buyer than for each formal buyer: $\mathcal{B}_{it} > \mathcal{B}_{ft}$. In other words, the formal market is more crowded for buyers than the underground sector. Then, an increase moves buyers to the "less-crowded" underground market, given higher urgency to spend money stocks. Buyers are moved underground to take advantage of better trade opportunities there. That is, if output per trade is unaffected, then aggregate underground output/turnover increases relative to formal sector output. Since inflation can increase tax evasion, this makes seigniorage financing unattractive. The opposite is the case when $\mathcal{B}_{it} < \mathcal{B}_{ft}$. I focus on the market congestion rate for buyers only, the reason for which will become apparent.

2.2 Household's Problem

Household agents are altruistic towards fellow members. Let U_t be instantaneous utility from consumption, net of the disutility of production. $\Phi(Q_{jt}) = Q_{jt}^{\phi}$, $\phi > 1$ is the disutility of producing Q_{jt} units inside a match. Also, let the pair $\{q_{jt}, x_{jt}\}$ be the terms of trade whenever the representative household's buyers engage in

[9] Note that $B_{jt}\mathcal{B}_{jt} = S_{jt}\mathcal{S}_{jt}$, $j = f, i$. Since it takes two to trade, one successfully matched seller implies a successfully matched buyer. See Petrongolo and Pissarides (2001) for a survey of related matching functions.

purchases and $\{Q_{jt}, X_{jt}\}$ when the sellers engage in sales. Here, q_{jt} (or Q_{jt}) is the quantity to be traded and x_{jt} (or X_{jt}) is the monetary payment in currency. The terms of trade will be discussed later but for now, it suffice to take these values as given. The household's problem is:

$$v(m_t) = \max_{s_{jt}, b_{jt}, m_{jt}, m_{t+1}, j=f,i} U_t + \beta E v(m_{t+1}) \quad , \quad \beta \in (0,1) \ ,$$

subject to the terms of trade as well as:

$$U_t = c_{ft} + \eta c_{it} - s_{ft}\mathcal{S}_{ft}\Phi(Q_{ft}) - s_{it}\mathcal{S}_{it}\Phi(Q_{it}) \ , \tag{1}$$

$$c_{ft} = (1-\tau) b_{ft}\mathcal{B}_{ft}q_{ft} - Q_t^g \ , \tag{2}$$

$$c_{it} = b_{it}\mathcal{B}_{it}q_{it} \ , \tag{3}$$

$$b_{ft} + b_{it} \leq b \ , \tag{4}$$

$$s_{ft} + s_{it} \leq s \ , \tag{5}$$

$$m_{ft} + m_{it} \leq m_t \ , \tag{6}$$

$$m_{t+1} - m_t \leq s_{ft}\mathcal{S}_{ft}X_{ft} + s_{it}\mathcal{S}_{it}X_{it} + P_{ft}Q_t^g - b_{ft}\mathcal{B}_{ft}x_{ft} - b_{it}\mathcal{B}_{it}x_{it} \ , \tag{7}$$

$$m_{jt}, x_{jt}, c_{jt}, b_{jt}, s_{jt} \geq 0 \quad , \quad j = f, i \ . \text{ and } m_t \geq 0 \ \forall t.$$

In the above problem, b_{jt} and s_{jt} are respectively buyer and seller allocations to sector j , $j = f, i$. Given the market crowding/congestion rates, total successful matches for household agents sent to market j are $b_{jt}\mathcal{B}_{jt}$ for buyers and $s_{jt}\mathcal{S}_{jt}$ for sellers. Total purchases are thus $b_{jt}\mathcal{B}_{jt}q_{jt}$, while total disutility is $s_{jt}\mathcal{S}_{jt}\Phi(Q_{jt})$, $j = f, i$. In (1), formal and underground goods are perfect substitutes in consumption but underground goods may be of inferior quality: $\eta \leq 1$.[10] I define composite

[10] An alternative interpretation of η is that underground sellers use a less efficient production technology.

consumption as $c_t = c_{ft} + \eta c_{it}$, where c_{jt} is consumption of sector j goods. A fraction, τ, of formal sector purchases is paid as tax. Also, the government buys off the quantity Q_t^g from formal buyers and pays for these units with money. Due to perishability, the household consumes all goods brought home instantly. In (7), incoming funds from sales, X_{jt}, arrive simultaneously as outgoing funds x_{jt} during purchases, $j = f, i$. Hence the former cannot be used to finance the latter within the same period. Nominal income from sales to the government is $P_{ft}Q_t^g$, where P_{ft} is the price (more on this later).

I specify the timing of events next. Starting a period with money holdings m_t, the representative household makes decisions on the allocation of buyers, sellers and money. The household also instructs its buyers and sellers on the terms of trade, which include the offers to make and/or accept in all successful matches.

t			$t+1$
Decisions	Markets Open	Markets Close	Pooling
$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$
b_{jt}, s_{jt}	Buyers $\rightarrow \frac{m_{jt}}{b_{jt}}$	Taxes Paid	Consumption
m_{jt}, m_{t+1}	Match, Bargain	Govt. Purchases: Q_t^g	
Terms of trade	Produce, Trade		

Next, the markets open. Formal agents visit only the formal market while informal agents go to the underground market. Once in the market, agents match randomly and one-on-one. Anonymity forbids credit transactions and trade is *quid pro quo.* After a bargain is reached, a successfully matched seller produces the desired output and trade is then finalized. As markets close, goods exiting the formal market gates are all taxed. Each formal buyer compulsorily sells some quantity Q_t^g to the government and receives money. Agents return to their respective households where purchased goods and sales receipts are gathered. There is consumption and the period ends.

2.3 Terms of Trade

Notice that the terms of trade, $\{q_{jt}, x_{jt}\}$, essentially establishes the per-unit price, p_{jt}, which is implied by $p_{jt} = \frac{x_{jt}}{q_{jt}}$, $j = f, i$. After the money and buyer allocations, a representative buyer enters his assigned market j with $\frac{m_{jt}}{b_{jt}}$ units of money, $j = f, i$. In each successful match, trade can occur if the offer is acceptable to both sides. For each implementable offer, monetary payments cannot exceed the buyer's money holding upon entering the match: $x_{jt} \leq \frac{m_{jt}}{b_{jt}}$, $j = f, i$. This feasibility constraint is intrinsic to the environment, given that trade is *quid pro quo*.[11]

Let ω_t be the value of money. Then, for an offer to be accepted, it must satisfy the seller's individual rationality constraint. This is simply $x_{jt}\omega_t \geq \Phi(q_{jt})$, $j = f, i$. In both sectors, I allow buyers to hold all the bargaining power and to make take-it-or-leave-it offers. Optimal offers ensure that the individual rationality constraint holds with equality. Combined with the feasibility constraint, we have:

$$\frac{m_{jt}}{b_{jt}} \geq \frac{\Phi(q_{jt})}{\omega_t} \ , \ j = f, i \ . \tag{8}$$

Inequality (8) is named the cash-and-carry constraint and is the final constraint on the household's problem.

Sellers act as "offer takers", and take the quantity requested as given. Temporarily assume that money is valued, allowing the cash-and-carry constraint to bind in both sectors. Then one can rewrite the level of output-per-trade in each sector as:

$$q_{jt} = \left[\frac{m_{jt}}{b_{jt}}\omega_t\right]^{\frac{1}{\phi}} \ , \ j = f, i \ . \tag{9}$$

With quantities determined, the *quantity-per-trade ratio*, $\frac{q_{it}}{q_{ft}}$, can be readily derived. I return to this later.

[11] Walrasian models of the underground economy are useful due to the ease of incorporating credit. For ways to include credit in models with anonymous agents, see Berentsen, Camera and Waller (2005).

In summary, the terms of trade in matching between agents is simply $x_{jt} = \frac{m_{jt}}{b_{jt}}$ and q_{jt} given by (9). Having established this, I next address the price P_{ft} which the government pays for goods. I allow agents to charge a premium on all sales to the government to take account of matching costs. Specifically, I assume $P_{ft} = \frac{p_{ft}}{\mathcal{X}_{ft}}$.

2.4 Government

The definition of a sector as "underground" suggests the existence of an authority that makes this distinction. There is a centralized government which implements both monetary and fiscal policies. Money supply, M_t per capita household, grows at the rate γ per period. Newly printed money, $T_t = (\gamma - 1) M_t$, is used by the government in the market for payments for Q_t^g. Government's real budget constraint is:

$$G = \tau b_{ft} \mathcal{B}_{ft} q_{ft} + Q_t^g \ , \tag{10}$$

where G is an exogenous expenditure each period. Since part of government revenues are nominal while expenditure is real, the government faces a liquidity constraint much like private households. This is:

$$(\gamma - 1) M_t = P_{ft} Q_t^g \ . \tag{11}$$

Note that the money growth rate and tax rate are endogenous. Consider a reduction in τ. The government's liquidity constraint goes into deficits as consistent with the optimal region of the Laffer curve. This requires an adjustment in transfers T_t to supply the funds necessary to alleviate the fiscal position, which in turn changes γ. Thus, (10) and (11) emphasize the inherent interaction between the fiscal and monetary policy variables τ and γ.

3 Characterizing the equilibrium

This section examines the euler conditions that characterizes the equilibrium. Since buyers make take-it-or-leave-it offers on both islands, sellers exit each match with zero net surplus. Households are thus indifferent and randomize sellers between sectors. This indifference explains why I focus on market congestion for buyers only. I implement the equilibrium with a constant sector choice: $\frac{s_i}{s} \in (0,1)$ such that (5) is satisfied with equality.[12]

Let λ_{jt}, $j = f, i$, be the Lagrange multiplier on the cash-and-carry constraint in each successful match. m_{jt} is chosen such that the cash-and-carry constraint binds to an equal extent in expectation in each sector: $\mathcal{B}_f \lambda_{ft} = \mathcal{B}_i \lambda_{it}$. The implied euler condition for money is:

$$\frac{\omega_t}{\beta} = \omega_{t+1} + \mathcal{B}_{jt+1}\lambda_{jt+1} \ , \ j = f, i \ . \tag{12}$$

Money kept between periods delivers its discounted value in the next period as well as helps alleviate the cash-and-carry constraint in future trade matches. From (12), it can be shown that both cash-and-carry constraints bind in all successful matches in equilibrium if the return on money is sufficiently low: $\gamma > \beta$. From this point on, I assume this to be the case.

Next, I turn to the optimal quantity that is demanded in each trade match. The associated first order conditions are derived as:

$$1 - \tau = \lambda_{ft} \frac{\Phi\left(q_{ft}\right)}{\Omega_t} \frac{\phi}{q_{ft}} + \omega_t \frac{dx_{ft}}{dq_{ft}} \quad \text{and} \tag{13}$$

$$\eta = \lambda_{it} \frac{\Phi\left(q_{it}\right)}{\Omega_t} \frac{\phi}{q_{it}} + \omega_t \frac{dx_{it}}{dq_{it}} \quad , \tag{14}$$

which are similar to Shi (1999). Demanding a higher quantity yields marginal utility from the additional units. The marginal cost is incurred at two levels. At

[12] In the calibration, I choose s_f (and s_i) so that the model matches the underground-to-formal sector output ratio as observed in data.

the buyer level, demanding a larger quantity requires of the buyer to pay more money, thus making the corresponding cash-and-carry constraint more binding. The rate at which this constraint becomes more binding depends on how much is required to motivate the seller to deliver the additional quantity, which in turn depends on the seller's production disutility costs on the margin. Secondly, as buyers purchase higher quantities from the market and need more money to do so, the household is pressured to deliver more money to its buyers. This causes the liquidity constraint (7) to become more binding. Using (13) and (14), it is easy to show that the first order condition for s_{ft} holds true for all values of $s_{ft} \in [0, s]$. Households are thus indifferent on the allocation of sellers between sectors.

The first order condition for b_{ft} is given as:

$$\mathcal{B}_{ft}\left[(1-\tau)\,q_{ft} - \lambda_{ft}\frac{\Phi\left(q_{ft}\right)}{\Omega_t} - \omega_t x_{ft}\right] = \mathcal{B}_{it}\left[\eta q_{it} - \lambda_{it}\frac{\Phi\left(q_{it}\right)}{\Omega_t} - \omega_t x_{it}\right]. \quad (15)$$

Allocating more buyers to the formal sector generates more formal sector purchases and yields the associated marginal benefits in consumption utility. All things being equal, as more buyers visit the formal sector, $\frac{m_{ft}}{b_{ft}}$ declines and the cash-and-carry constraint binds further in this sector. The household is pressured to deliver more money to formal sector buyers, causing the liquidity constraint to become more binding as well. A similar effect pertains to the underground sector. For the marginal buyer, the net benefits must be equal between sectors in expectation.

All households are alike and so I apply symmetry as usual. The only state variable is money. To proceed to describe an equilibrium therefore, it is essential to ensure that this variable evolves at a constant rate. Assuming fixed inflation rate γ, the euler condition for money holding in steady state reduces to:

$$\lambda_{jt} = \frac{\gamma - \beta}{\beta \mathcal{B}_{jt}}\Omega_t \ , \ j = f, i \ .$$

Substituting this into (13) to (15) gives (16) to (18) below.

3.1 The Equilibrium

Definition 1 *A symmetric monetary search equilibrium is defined as the tax rate* τ*, the set of household choices* $(s_f, b_f, m_{ft})_{t=0}^{\infty}$ *and the implied value of money* $(\omega_t)_{t=0}^{\infty}$ *such that given* γ*, the following requirements are met: (i) each household solves its optimization problem; (ii) the representative household's variables replicate the aggregate equivalents; (iii) prices are positive, though bounded (the value of money is positive and bounded); and (iv) the government budget balances.*

In particular, an equilibrium involves a solution to a system of four equations for b_f, m_{ft}, ω_t and τ:

$$1 - \tau = \left[1 + \frac{\gamma - \beta}{\beta \mathcal{B}_f}\right] \Omega_t \frac{m_{ft}}{b_f} \frac{\phi}{q_f} , \tag{16}$$

$$\eta = \left[1 + \frac{\gamma - \beta}{\beta \mathcal{B}_i}\right] \Omega_t \frac{m_{it}}{b_i} \frac{\phi}{q_i} , \tag{17}$$

$$\frac{\frac{m_{it}}{b_i}}{\frac{m_{ft}}{b_f}} = \frac{\gamma - \beta (1 - \mathcal{B}_f)}{\gamma - \beta (1 - \mathcal{B}_i)} , \tag{18}$$

$$G = \tau b_f \mathcal{B}_f q_f + (\gamma - 1) \frac{M_t}{P_{ft}} \tag{19}$$

and for completeness,

$$s_{ft} = s_f, \text{ and } s_{it} = s_i \ .$$

Variables without the time subscript represent equilibrium real values. Those with time subscripts are nominal values, which depend on the money stock at date t.

Given γ, there exists an equilibrium. The equations (16), (17) and (18) deliver values for the household variables b_f, m_{ft} and ω_t, all in terms of τ. The required tax rate that balances the budget, given γ, is then derived from (19). All other variables - such as q_j, λ_{jt}, c_j, x_{jt}, p_{jt}, $\mathcal{B}_{jt}$ and P_{ft} - can be derived as functions of the four in the definition [see the appendix for further details].

Equation (18) plays a central role in understanding the implications of the model. First, the sector with the fewer trade opportunities per buyer always has the higher money holding per buyer. If market overcrowding/congestion is worse for formal buyers, each is compensated with higher sums of money. In other words, if $\mathcal{B}_f < \mathcal{B}_i$, households take advantage of the intensive margin when buying from the formal sector and the extensive margin when buying underground goods. Secondly, suppose there is an increase in inflation, γ, with $\mathcal{B}_f < \mathcal{B}_i$. All things being equal, more money is diverted to underground buyers per capita and q_{it} increases relative q_{ft}. That is, the erosive effect of inflation on household money stock increases tax evasion and seigniorage financing becomes less attractive. The reverse is the case when market congestion is worse for buyers in the underground market. A discussion of the effect of inflation follows in the next section.

4 Size, Prices and Inflation

The quantity-per-trade ratio describes trade within an underground match relative to a formal sector match and is denoted $R_I = \frac{q_i}{q_f}$. Summing over all such trade encounters in each sector gives the aggregate output ratio in trades involving all household buyers. This is denoted $R = \frac{b_i^\alpha s_i^{1-\alpha}}{b_f^\alpha s_f^{1-\alpha}} \frac{q_i}{q_f}$. The subscript I is used to denote the intensive margin.

4.1 Relative Quantities and Relative Price

Since the cash-and-carry constraint binds in both sectors, (9) gives the quantity-per-trade in each sector. Using this outcome together with (18), the equilibrium *quantity-per-trade ratio* becomes:

$$R_I = \left[\frac{\gamma - \beta \left(1 - \mathcal{B}_f\right)}{\gamma - \beta \left(1 - \mathcal{B}_i\right)} \right]^{\frac{1}{\phi}} ,$$

which completely describes the *intensive margin.* The intensive margin concerns the quantity traded within each successful match, which depends on the amount of money each buyer takes into a match. If trade opportunities are few for formal buyers, households take advantage of each successful formal match to acquire large quantities, which implies the expense of higher sums of money in formal matches compared to underground matches. In other words, high market congestion for formal buyers reduces the intensive ratio.[13]

Next, the aggregate trades equivalent is:

$$R = \frac{b_i \mathcal{B}_i}{b_f \mathcal{B}_f} R_I \ , \tag{20}$$

which is the underground-to-formal sector *output ratio.* Comparing with the intensive ratio, R stresses the effect of the matching rate on aggregate market outcomes. Suppose R_I is given. Then for the representative buyer sent to each island, the congestion of the underground market relative to the formal market, $\frac{\mathcal{B}_i}{\mathcal{B}_f}$, determines the quantity of expected purchases by an underground buyer relative to a formal buyer: $\frac{\mathcal{B}_i}{\mathcal{B}_f} R_I$. Preference and policy parameters η and τ are indirectly reflected in R because households are mindful of the effect of their buyer allocation decisions on the eventual mix of goods that they consume. Given the bargaining outcome and market crowding conditions, households employ their buyer allocation decision to edge closer to their preferred mix of goods. The allocation of buyers and its effect on market crowding and aggregate trade outcomes is termed the *extensive margin.* This margin is conclusively captured by R and decentralized trade is essential for separating R from R_I.

Again, price in each transaction as determined from the terms of trade is $p_{jt} = \frac{m_{jt}}{b_j} \frac{1}{q_j}$, $j = f, i$ in equilibrium. Using (18), the relative price ratio in private trades

[13] One can consider the effect of technology as another dimension of the intensive margin. Superior technology in the formal sector means that even with equal financial compensation, formal sector sellers can deliver higher quantities within each trade meeting.

reduces to $\frac{p_{it}}{p_{ft}} = \frac{\frac{m_{it}}{b_i}}{\frac{m_{ft}}{b_f}} \frac{1}{R_I}$, or:

$$\frac{p_{it}}{p_{ft}} = \left[\frac{\gamma - \beta\left(1 - \mathcal{B}_f\right)}{\gamma - \beta\left(1 - \mathcal{B}_i\right)}\right]^{1-\frac{1}{\phi}} . \tag{21}$$

The relative price is not only a function of preferences and taxes but also an endogenous outcome of monetary policy, unlike in the earlier papers.[14] With relatively high market crowding for formal buyers, each brings more money into a match and this increases the formal sector price relative to that underground. If $\mathcal{B}_i < \mathcal{B}_f$, it is possible to generate higher prices in the underground sector. It is worth noting however that p_{ft} is price before taxes. The effective price ratio after tax is $\frac{p_{it}}{p_{ft}}\left(1 - \tau\right)$, which I report in section 5.

The ratio R has been the subject of virtually all of what is known in the literature about the underground economy. The environment presented above enables us to use published empirical estimates of R and back out the micro level ratio R_I as well as the price ratio $\frac{p_{it}}{p_{ft}}$ as demonstrated. Some of these results may be particularly useful since empirically, micro level data is unattainable in studies on the underground economy.

4.2 Erosive Effect of Inflation

Temporarily assume that monetary policy is via lump sum transfers to households and also that $\frac{d\tau}{d\gamma} = 0$.[15] Even in the face of changes in the inflation rate, households are fully protected since they are recipients of transfers equal to the going rate of inflation. In the equivalent case in Cavalcanti and Villamil (2003) as well as Nicolini (1998), firms and households will not adjust their portfolios since the erosive effect

[14] The price ratio (21) indirectly includes the parameters η and τ, since it affects the buyer allocation decision, which in turn affects market congestion rates $\mathcal{B}_i$ and $\mathcal{B}_f$.

[15] Specifically, government simply hands money to each buyer, instead of requesting Q_t^g units of output. Temporarily ignore the effect on the government budget.

of inflation has no effect on sectoral allocations. In the current paper however:

$$\left.\frac{dR_I}{d\gamma}\right|_\tau = \left[\mathcal{B}_i - \mathcal{B}_f + \varphi\left(\frac{d\mathcal{B}_f}{d\gamma}, \frac{d\mathcal{B}_i}{d\gamma}\right)\right]\frac{R_I}{A}$$

where $\varphi(\cdot)$ is a function that is of the same sign as $\mathcal{B}_i - \mathcal{B}_f$ and A is a positive value.[16]

Assume that underground buyers have better matching success: $\mathcal{B}_i - \mathcal{B}_f > 0$. When γ increases, households seek to spend nominal balances faster and they commit more money to underground buyers compared to previously and this increases R_I. Further, households divert some buyers from the formal market to the less congested underground market, as consistent with (18). Since b_i increases, $\frac{d\mathcal{B}_i}{d\gamma} < 0$ and the household compensates underground buyers with even more money [this part is captured by $\varphi(\cdot) > 0$], which further increases R_I. The effect on the extensive ratio R is in the same direction. Given that b_i increases, aggregate matches increase underground relative to the formal sector, holding sellers constant.

To summarize, when market congestion is worse for buyers in the formal market, $\left.\frac{dR}{d\gamma}\right|_\tau > 0$. Seigniorage spending may help alleviate the tax rate and encourage formalization. However, the erosive effect of inflation on household money stock reverses the seigniorage effect. If this reversal is strong enough, it can generate a total effect $\frac{dR}{d\gamma} = 0$ as suggested by Figure 3. On the other hand, if the underground market holds fewer trade opportunities for each buyer ($\mathcal{B}_i - \mathcal{B}_f < 0$), then the erosive effect of inflation reinforces the seigniorage effect in reducing the *output ratio*. This makes seigniorage spending even more attractive.

Economic policy does not leave the relative price unchanged, unlike in several earlier papers. If $\mathcal{B}_i - \mathcal{B}_f > 0$, underground prices rise and approaches formal sector levels as the rate of inflation increases:

$$\left.\frac{d\frac{p_{it}}{p_{ft}}}{d\gamma}\right|_\tau = \left[\mathcal{B}_i - \mathcal{B}_f + \varphi\left(\frac{d\mathcal{B}_f}{d\gamma}, \frac{d\mathcal{B}_i}{d\gamma}\right)\right](\phi - 1)\frac{\frac{p_{it}}{p_{ft}}}{A}.$$

[16] See the appendix for A and φ.

Intuitively, increased inflation implies that each underground buyer starts to hold more money compared to previously (if $\mathcal{B}_i - \mathcal{B}_f > 0$). Thus, underground buyers begin to demand higher quantities in each trade and they need to pay higher prices to motivate the additional units. This change in the relative price implies a marginal decline in R_I, however this effect is of second order and does not reverse the initial rise in R_I and R. While changes in fiscal policy can affect the output ratio in some of the environments with centralized markets, the isolated effect of inflation on household portfolio allocations and the relative price is a unique outcome of decentralized exchange [see the appendix for a comparison].

4.3 The Ramsey Problem (Optimal Inflation)

Bailey (1956) and Phelps (1973) brought the subject of optimal inflation into the fold of public finance. In this seminal contribution, Phelps advocates for a positive tax on the liquidity services that money provides if taxes on other goods and services are distortionary. This argument favours a positive nominal interest rate, or simply, positive inflation. Intuitively, tax distortions are socially costly while inflation presents the usual welfare consequences. The task facing a benevolent government is to find the best trade-off between the deadweight loss from tax financing and that from seigniorage financing.

The focus of the Ramsey problem is to find the optimal mix of consumption and inflation taxes. Without money, the cash-and-carry constraint (8) cannot be satisfied and the economy described in this paper degenerates into autarky. In the spirit of Kiyotaki and Wright (1989), money acts as an intermediate commodity that facilitates trade. Diamond and Mirrlees (1971) established a general result emphasizing the undesirability of taxing the intermediate goods sector when *all* final goods and services fall under the tax radar. In monetary economics, their conclusion implies that inflationary tax should not be used despite the distortions caused by taxes on the final goods sector.[17] However, where there is a third sector

[17] Also, see Kimbrough (1986), Faig (1988), Guidotti and Vegh (1992), Chari, Christiano and

that evades regular taxes, the optimal policy set may include positive seigniorage.

The Trade-off

The formalized Ramsey problem is to solve the household's problem subject the first order conditions and the government's budget constraints. This problem in the typical environment with competitive trade and evadable taxes involves a trade-off between (i) *output ratio* distortions created by formal sector taxes and (ii) the welfare cost of inflation. Assume that the optimal policy set from this trade-off is the pair $\{\tau_a, \gamma_a\}$.

The additional dimension provided in the framework with decentralized exchange is the coordination problem that arises from market congestion.[18] To illustrate, suppose sellers are distributed evenly between the two sectors. Then, to minimize search frictions and maximize aggregate matches $(\mathcal{X}_f + \mathcal{X}_i)$, buyers must also be allocated equally between sectors. Suppose instead that the allocation of buyers is skewed towards the underground sector, causing high market congestion for buyers in that sector. The optimal policy set includes low taxes: $\tau_1 < \tau_a$ and high seigniorage: $\gamma_1 > \gamma_a$. Low taxes edge buyers back into the formal market and improves the coordination problem. Here, lowering the tax rate implies government needs more seigniorage income, hence high γ. Two factors account for this negative relationship between τ and γ in this case. The first is the traditional argument that as γ increases, seigniorage income rises, which finances the government and helps reduce τ. The second is that as γ increases, buyers move to the formal sector via the extensive margin in order to spend money faster. Thus, more goods become taxable, which also means the tax rate can adjust downwards even further. For both of these factors, $\frac{d\tau}{d\gamma} < 0$ when the underground market has the higher market crowding for buyers.[19] Apart from the trade-off between distorting taxes

Kehoe (1996).

[18] For more on second best taxation in environments with externalities, see Sandmo (1975), Ng (1980), Goulder (1995) and Bovenberg and van der Ploeg (1998).

[19] See the upper right panel of Figure 1.

and the welfare cost of inflation, the Ramsey problem seeks to even out market crowding rates in the two sectors and improve the coordination problem.

When the market is more congested for formal buyers, the opposite is generally the case and optimal policy involves $\tau_2 > \tau_1$ and $\gamma_2 < \gamma_1$. In comparison to $\{\tau_a, \gamma_a\}$ however, the effect of inflation is less clear-cut. There are three factors of importance, being tax distortions, the welfare cost of inflation and market congestion. Temporarily consider the first two only. Taxes move buyers out of the formal market. When $\mathcal{B}_f < \mathcal{B}_i$, inflation also moves buyers out of the formal market, since buyers go to the informal market where trade opportunities are better. Thus, inflation and taxes both increase tax evasion in this case. Holding taxes constant, marginally reduce inflation. Seigniorage incomes decline, but tax revenues increase even with no change to the tax rate. This is because buyers return to the congested formal market, given less urgency to spend money. Depending on the influx of buyers into the formal market, the rise in tax revenues can outweigh losses in seigniorage income. Thus, a lower tax rate becomes possible: $\frac{d\tau}{d\gamma} > 0$.[20] This underscores the solution that lowering seigniorage financing and lowering taxes can work together to improve welfare and yet balance the budget.

In this second case (the case where $\mathcal{B}_f < \mathcal{B}_i$), does a benevolent policymaker choose some low pair $\{\tau; \gamma = \beta\}$, so long as it balances the budget? This is where the final dimension, market crowding, plays a role. Recall that since $\mathcal{B}_f < \mathcal{B}_i$, lower taxes and lower inflation both have the same effect in moving buyers to the crowded formal market. Thus, for low-enough levels of γ and τ, too many buyers enter the already-crowded formal market and the coordination problem worsen. This hinders trade and reduces welfare. In summary, the optimal policy may include $\tau_2 < \tau_a$ and $\gamma_2 < \gamma_a$ when $\mathcal{B}_f < \mathcal{B}_i$, but this does not guarantee that Friedman rule becomes optimal.[21] Here again, the Ramsey problem finds optimal policy after considering not only tax distortions and the welfare cost of inflation,

[20] The analysis here is aimed at explaining our simulation results as in section 5, for the case where $\mathcal{B}_f < \mathcal{B}_i$. See the upper right panel of Figure 2.

[21] See upper left panel of Figure 2.

but also market congestion.

5 Calibration and Results

This section calibrates the model to match data from Peru and conducts three policy experiments. The first experiment is to vary γ, with $\frac{d\tau}{d\gamma} = 0$. This isolates the change in households decisions with higher inflation and shows the effect on R_I and R. The second is to vary τ alone and evaluate the effect on these ratios. In the third experiment, I endogenize the interaction between γ and τ and evaluate the optimal policy set, or the Ramsey solution. In each of these experiments, I consider the case where the formal sector has the higher market congestion for buyers as well as that in which the opposite is the case.

I normalize the number of sellers, s, to unity. Using time diary data, Juster and Stafford (1991) estimate that US residents spend on average 23.9 hours on paid work and 6.8 hours shopping per week. b is set to $\frac{6.8}{23.9}$. This figure is adopted for Peru, but considered a lower bound for time spent shopping in that country.[22]

TABLE 1

PERU

Parameters							Economic Indicators		Target
Period	β	ϕ	α	s	b	M_t	τ	$\gamma^{12}-1$	R
1 Month	.997	1.2	.5	1	$\frac{6.8}{23.9}$	1	12.71%	2.24%	44%

Data on tax revenue as a percentage of GDP is retrieved from the World Development Indicators (WDI) database of the World Bank. This is used to approximate τ. Also collected from the same database is average annual CPI inflation for 2000 to 2005, which is used to represent $\gamma^{12} - 1$. Finally, an estimate of the underground-to-formal output ratio is taken from Schneider and Enste (2000) and used to represent R.

[22] The appendix includes sensitivity analysis on b, ϕ and α.

Specifically, the equations I calibrate are (16) to (20). Temporarily assume that we know s_f (and hence s_i). Then, given the above values for τ, G, γ, b, β and α, equations (16), to (19) are used to get b_f, m_{ft}, ω_t and Q^g. The remaining requirement is to verify s_f. The model is simulated for the value s_f such that the relative size of the underground economy, R, equals 0.44, as consistent with (20). This completes the calibration. The value of s_f derived from each scenario is retained for all other simulations. (20) is thus used to evaluate the new level of R, given each policy set that is fed into the model.[23]

Suppose underground goods are just as good as formal sector goods: $\eta = 1$. Taxes cause households to send relatively fewer buyers to the formal sector and there is a tendency for high market congestion for underground buyers ($\mathcal{B}_i < \mathcal{B}_f$).

TABLE 2

CALIBRATION OUTCOMES

η	1		.85	
	Peru	US	Peru	US
$\frac{b_i}{b}$	.3351	.0897	.2999	.0773
$\frac{s_i}{s}$	.0920	.0253	.3699	.1303
$\frac{m_{it}}{m_t}$	.5285	.1609	.2680	.0590
$\mathcal{B}_i$	.9824	.9947	2.0820	2.4335
$\mathcal{B}_f$	2.1908	1.9400	1.7786	1.8202
q_i	.3921	.3913	.3972	.3975
q_f	.2014	.2247	.4528	.5060
R_I	1.9466	1.7411	.8773	.7855
R	.4400	.0880	.4400	.0880
$\frac{p_{it}}{p_{ft}}(1-\tau)$	.9973	.9974	.8503	.8506
Q^g	.0001	.0001	.0001	.0002
G	.0107	.0122	.0205	.0261

[23] For the sake of comparison, I also calibrate the US economy for which data is collected similarly and from the same sources, with $R = .088$, $\tau = .1073$ and $\gamma^{12} = 1.028262$.

Each underground buyer is handed a relatively high sum of money: $\frac{m_{it}}{m_t} > \frac{b_i}{b}$, or alternatively put, $\frac{m_{ft}}{m_t} < \frac{b_f}{b}$. Since each underground buyer holds more money per capita, they can buy more units and the intensive margin thus ensures that $R_I > 1$. In order to match the *output ratio* with $R < 1$, few sellers are allocated to the underground market so that the aggregate number of matches underground is low.

For sufficiently low η however, market congestion is reversed, with the formal sector being more congested for buyers. Market congestion improves for each remaining underground buyer $\left(\frac{b_i}{b} < \frac{s_i}{s}\right)$, requiring lower money allocation to these buyers: $\frac{m_{it}}{m_t} < \frac{b_i}{b}$. Since each underground buyer bears lower money stocks, they buy fewer units per capita compared to formal buyers and $R_I < 1$. Since there are more formal matches than underground, the extensive margin reinforces the intensive margin with $R < R_I$. The quantity Q^g is real government revenue from seigniorage spending. The values are however small for both countries compared to the total government budget G.

Next, I turn to the policy experiments. The upper panel of Table 3 reports simulations of the model for different levels of money growth for a given tax rate for the case where trade opportunities are better for formal buyers than for underground buyers (ignoring the consequence on the government budget). As the inflation rate increases, there is higher urgency to spend nominal balances and households move buyers to the formal market. Thus, $\frac{b_i}{b}$ falls steadily. Also, each formal buyer is handed more money compared to previously and $\frac{m_{it}}{b_i}$ falls. On both the intensive and extensive margins, the output ratio declines. The negative effect of inflation on the output ratio - holding taxes constant - is consistent with Koreshkova (2006) in which credit services are employed to attain a similar effect with centralized markets. Due to the erosive effect of inflation, consumption declines in both sectors. As inflation increases from 2.24% to 10% per annum, the required compensation in composite consumption units is only 0.13% of the previous consumption level. In the lower panel, I vary the tax rate while keeping

TABLE 3

PERU: The Case of $\eta = 1$

	EFFECT OF INFLATION FOR A GIVEN TAX RATE													
γ^{12}	τ	$\frac{b_i}{b}$	$\frac{m_{it}}{b_i}$	$\frac{m_{it}}{m_t}$	$\mathcal{B}_i$	$\mathcal{B}_f$	R_I	R	U_t	c_f	c_i	$DisU$	Δc	$\%\Delta c$
β^{12}	.1271	.3412	5.5556	.5394	.9736	2.2010	1.9732	.4521	.0185	.0733	.0380	.0928	-.0000	-.00
1.0224	.1271	.3351	5.5429	.5285	.9824	2.1908	1.9466	.4400	.0185	.0728	.0367	.0910	0	0
1.1	.1271	.3279	5.5255	.5155	.9932	2.1790	1.9151	.4259	.0185	.0722	.0352	.0889	.0000	.13
	EFFECT OF TAX ADJUSTMENTS FOR A GIVEN MONEY GROWTH RATE													
τ	γ^{12}	$\frac{b_i}{b}$	$\frac{m_{it}}{b_i}$	$\frac{m_{it}}{m_t}$	$\mathcal{B}_i$	$\mathcal{B}_f$	R_I	R	U_t	c_f	c_i	$DisU$	Δc	$\%\Delta c$
.1	1.0224	.2602	5.3403	.3954	1.1149	2.0769	1.6765	.3166	.0210	.0922	.0325	.1037	-.0025	-2.31
.1271	1.0224	.3351	5.5429	.5285	.9824	2.1908	1.9466	.4400	.0185	.0728	.0367	.0910	0	0
.15	1.0224	.4080	5.5281	.6417	.8904	2.3217	2.2162	.5857	.0167	.0586	.0404	.0823	.0018	1.60

TABLE 4

PERU: The Case of $\eta = .85$

	EFFECT OF INFLATION FOR A GIVEN TAX RATE													
γ^{12}	τ	$\frac{b_i}{b}$	$\frac{m_{it}}{b_i}$	$\frac{m_{it}}{m_t}$	$\mathcal{B}_i$	$\mathcal{B}_f$	R_I	R	U_t	c_f	c_i	$DisU$	Δc	$\%\Delta c$
β^{12}	.1271	.2991	3.1348	.2668	2.0849	1.7775	.8755	.4383	.0039	.1419	.0713	.1987	.0005	.23
1.0224	.1271	.2999	3.1406	.2680	2.0820	1.7786	.8773	.4400	.0044	.1399	.0706	.1956	0	0
1.1	.1271	.3010	3.1478	.2695	2.0785	1.7799	.8794	.4421	.0049	.1374	.0697	.1918	-.0006	-.28
	EFFECT OF TAX ADJUSTMENTS FOR A GIVEN MONEY GROWTH RATE													
τ	γ^{12}	$\frac{b_i}{b}$	$\frac{m_{it}}{b_i}$	$\frac{m_{it}}{m_t}$	$\mathcal{B}_i$	$\mathcal{B}_f$	R_I	R	U_t	c_f	c_i	$DisU$	Δc	$\%\Delta c$
.1	1.0224	.2297	2.6834	.1753	2.3793	1.6955	.7545	.3157	.0050	.1762	.0618	.2238	-.0006	-.32
.1271	1.0224	.2999	3.1406	.2680	2.0820	1.7786	.8773	.4400	.0044	.1399	.0706	.1956	0	0
.15	1.0224	.3699	3.5147	.3699	1.8748	1.8748	1.0000	.5871	.0039	.1133	.0783	.1759	.0004	.22

the inflation rate fixed (again, ignore the government budget). As expected, higher taxes cause measurable declines in welfare.

In Table 4, I present the same simulations for an economy in which $\eta = 0.85$. Notice that I choose s_f (and hence s_i) appropriately such that for the benchmark policy values $\{\tau, \gamma\} = \{.1271, 1.0244\}$, the economy is characterized by the observed output ratio, being $R = 0.44$. Thus, the initial degree of tax evasion is the same as in Table 3. When we increase inflation with taxes constant, again buyers move to the market with lower market congestion, which in this case is the underground market. This sector increases in both the intensive and extensive margins. This confirms the theoretical result that when market congestion is higher for formal buyers, inflation increases R. In absolute terms, consumption of both formal and underground goods decline since inflation is a tax on money. However, larger declines in production disutility $\left(DisU = s_{ft}\mathcal{S}_{ft}q_{ft}^{\phi} + s_{it}\mathcal{S}_{it}q_{it}^{\phi}\right)$ contributes to marginal improvements in welfare. An increase in taxation at constant money growth also moves buyers underground and increases R. As expected, this is welfare-costly, although to a much smaller than the lower panel of Table 3. The difference is that in Table 4, taxation is not so evil since it forces buyers to exit the formal market to the underground sector, which improves the coordination problem.

The introduction outlined how the data suggests that changes in inflation leave the relative size unchanged. Subsection 4.2 argued that although the seigniorage effect reduces R, the erosive effect of inflation can cause a reversal. We do achieve the reversal in the upper panel of Table 4, unlike in earlier papers. However, the margin is rather modest [but see the next subsection].

5.1 Optimal Inflation Tax

The optimal policy set are in Table 5. It is important to note that the higher optimal inflation recommended for the economy with $\eta = 1$ is not because that economy has higher tax evasion. In fact, in both economies, I start off with $R = 0.44$ as shown in Table 2. Instead, the economy with $\eta = 1$ has higher optimal inflation because of higher market congestion for buyers in the underground market. Inflation does not only bring seigniorage income, it also reduces tax evasion as buyers start to take advantage of lower market crowding in the formal market. This acts as an additional incentive for seigniorage financing, which explains the optimal rate of 42.69% in Peru. We still get such high optimal rates even for $\eta = 1 - \varepsilon$; ε being an arbitrarily small positive number. In that case, inflation increases the relative consumption of higher-quality formal sector goods. This outcome is new, and opposite to that found in Peterson and Shi (2004), where inflation increases the consumption of lower quality goods and strictly reduces welfare.

Table 5

Peru

η	1	.85	Nicolini (1998)[24]	Data	
$\tau(\gamma)$	.0998	0.1246	not comparable	Mean 1976-1995	2005
$\gamma^{12} - 1$	.4269	.014	.1495 to .0354	6.25	.016

On the other hand, when the overcrowding of buyers is higher in the formal sector ($\eta = .85$), inflation increases in R, which acts as a disincentive to seigniorage financing. The optimal inflation rate here is 1.4%, despite the large tax-evading sector. Figure 2 warrants further explanation. As γ increases, seigniorage income (Q^g) rises, as consistent with models with centralized markets. Given G, seigniorage helps alleviate tax financing. However, as γ increases, buyers exit the formal market in search for better matching rates underground ($\mathcal{B}_i > \mathcal{B}_f$). As they do

[24] Nicolini (1998) studies optimal inflation when relative credit use is different between the formal and underground sectors. For different configurations of relative credit-use, he finds optimal annual interest rates between 7.34% and 19.17%. In Table 5, I convert these estimates into inflation rates using the Fisher equation as in section 6. The tax rate in that paper is calibrated differently and not compared above.

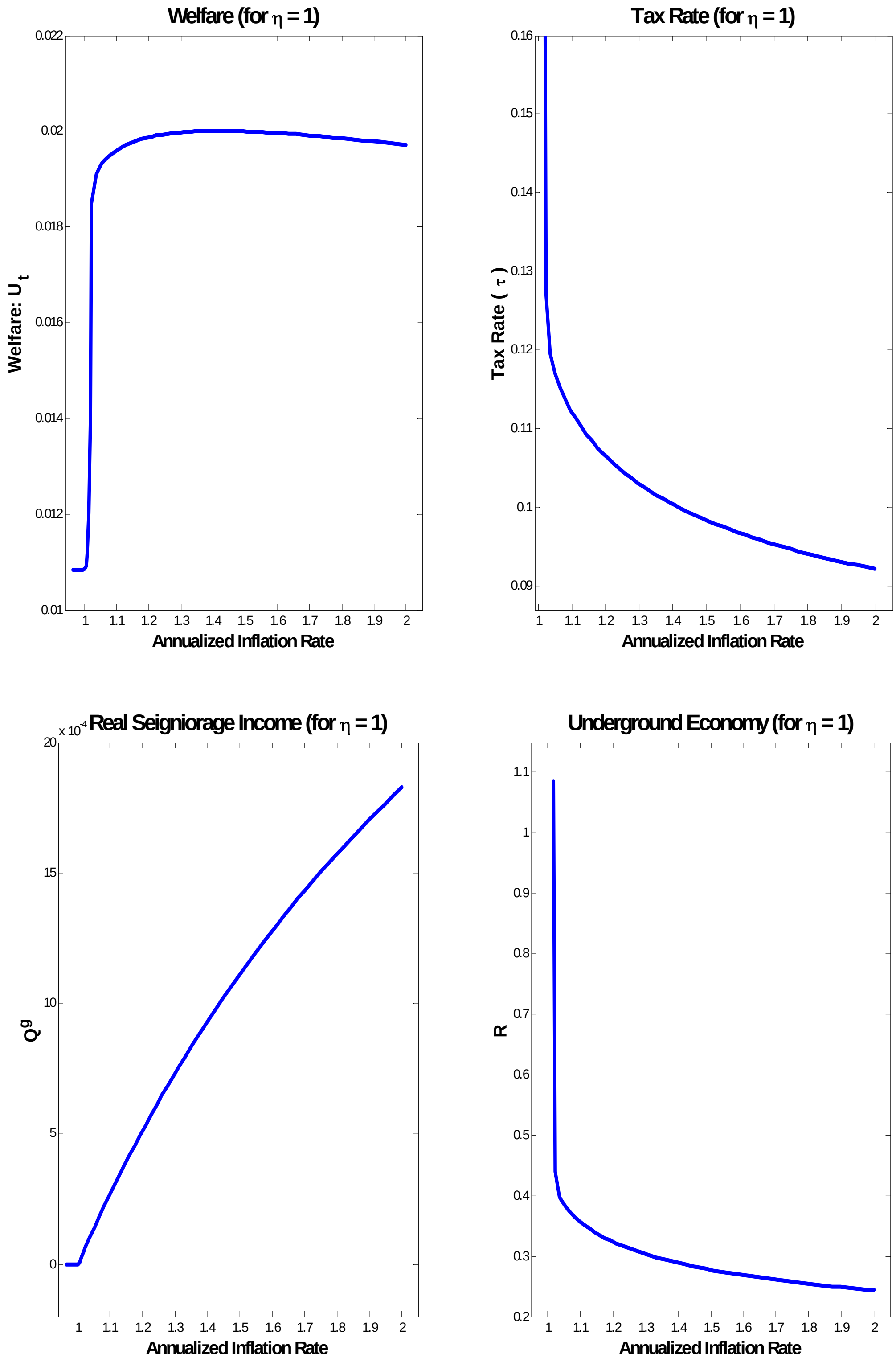

Figure 1: Peru

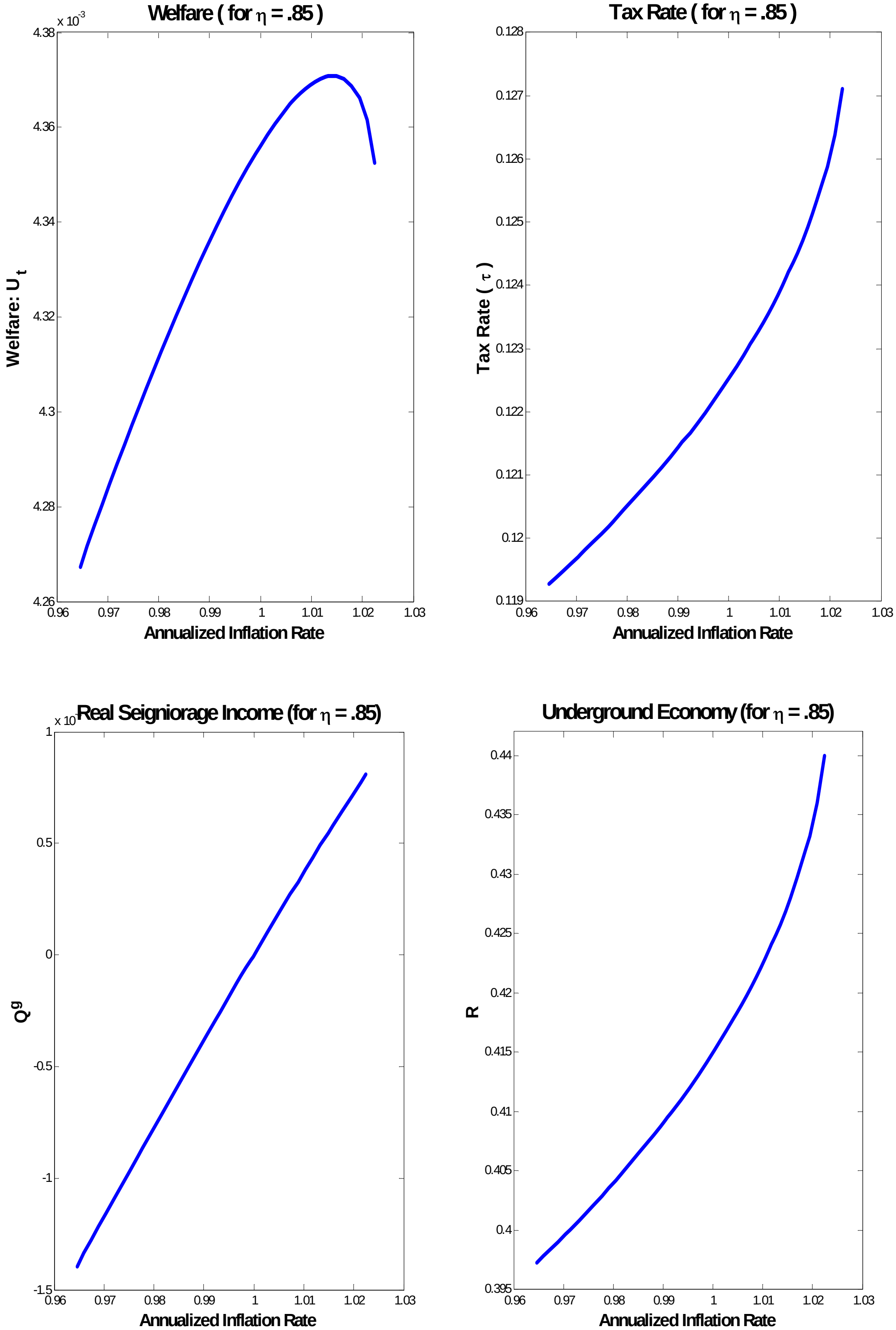

Figure 2: Peru

so, the turnover of taxable goods decline, along with tax revenues, at the going tax rate. Tax revenues decline at a rate faster than the gains from seigniorage, requiring τ to rise along with γ.

The results are worth comparing with those found in Nicolini (1998); also calibrated to Peru. Nicolini considers an economy in which the relative use of credit is higher in the formal market. Inflation thus taxes underground cash-denominated transactions more than formal credit-denominated trades. For different relative credit-use levels, he documents the optimal rate of inflation. The results show that even without any assumptions regarding credit-use, there can be large variations in the optimal rate of inflation if trade is decentralized. Integrating credit can further diverge these rates and better account for the prevalence of inflation in poor countries.

6 Discussion

The economic environment discussed above is directly equivalent to one in which households interact with a centralized market for government bonds. Augmenting the household's liquidity constraint with bonds, the euler condition for bonds is $\frac{\omega_t}{\beta} = \omega_{t+1}(1 + r_{t+1})$, where r_t is the net nominal interest rate. Comparing this euler condition with (12), the interest rate is derived as:

$$r = \mathcal{B}_f \frac{\lambda_{ft}}{\omega_t} = \mathcal{B}_i \frac{\lambda_{it}}{\omega_t} \equiv \frac{\gamma - \beta}{\beta} \ .$$

Friedman rule involves setting γ to β, or alternatively, r to zero.

In the environment presented in this paper, households randomize sellers between sectors due to take-it-or-leave-it offers by buyers. Employing Nash bargaining can allow interesting dimensions on seller allocation. Such an extension can strengthen results discussed in this paper. In response to changes in monetary and/or tax policies, sellers are likely to move in the same direction as buyers, further strengthening the results on the extensive margin.

Some contributions in the literature stress the importance of credit in formal sector trades, unlike in the informal sector. With higher inflation, agents resort to more credit trades, which increases the formal sector in relative terms [see Koreshkova (2006)]. The alternative approach used in this paper is motivated by the concern that credit services may not be exclusively produced by - nor exclusively used in - the formal sector. It is not clear how credit-use will affect relative underground output when inflation changes [see Besley and Levenson (1996)].[25]

This paper generate endogenous micro level trade ratios including the quantity-per-trade ratio and the relative price. A somewhat related paper in the literature is McLaren (1998). He considers a non-monetary economy with markets for imported goods. There are several markets, each for a specific class of imported goods. Depending on the tax rate and the concentration of tax inspectors in a given market, traders decide either to import legally and pay the associated taxes or to smuggled at a risk of detection. Quantity per importer is fixed and only the choice of sector is endogenously influenced by policy. In equilibrium, traders in the market for a particular class of good are all simultaneously legitimate importers or all smugglers. Although separate prices can be derived for the two sectors, only one is operational for each commodity class. He then studied the optimal tax and audit rates in a Ramsey-type equilibrium. The current paper on the other hand endogenizes production quantities, prices and sector choice, and these depend on fundamentals as well as economic policy, including money.

A possible extension is to introduce capital into the environment examined in this paper. First, notice that the model presented above can be interpreted as one with constant returns to scale production technology involving labour: $q_{jt} = l_{jt}$, $j = f, i$, where l_{jt} is labour input. In this case, the disutility of production reverts to disutility of labour: $\Phi(l_{jt})$. The introduction of capital simply involves employ-

[25] Besley and Levenson (1996) documents high prevalence of Rotating Saving and Credit Associations in many developing countries allowing informal sector agents access to financial intermediation. Participation rates are as high as 45% among high income groups in Taiwan, which is significant relative to participation rates in underground production.

ing a more general production function and an appropriate capital accumulation equation. In this vain, this extension compels one to take a stand on which good(s), formal or underground, can be accumulated into capital, if not both. How exactly are they combined in the constitution of a uniform capital stock?

7 Conclusion

The conclusions to draw from this paper are two-fold. First, the data fails to support conventional wisdom that inflation reduces the relative size of the underground economy. Changes in the underground-to-formal sector output ratio seem unaffected by changes in the rate of inflation. I develop a theoretical framework that explains the evidence. The solution I propose is that over time (or across countries), the relative overcrowding of the formal and underground markets for buyers can change (can be different). Where the formal market is more crowded for buyers, inflation can cause households to compromise on the quality of goods they consume and commit more money and more buyers to the underground sector. In this case, relative underground output increases both on the intensive and extensive margins. When the underground sector is more overcrowded for buyers however, inflation achieves the opposite result. The relative size of the underground economy declines and consumption of higher-quality formal sector goods increases. Inflation can thus move underground output in both directions, as consistent with data.

The second conclusion is as follows. In the presence of an underground sector, tax distortions are socially costly while inflation presents the usual welfare consequences. If both sector markets are perfectly competitive, the task facing a benevolent government is to find the best trade-off between the deadweight loss from tax financing and that from seigniorage financing. With decentralized trade however, optimal policy also seeks to correct the coordination problem that arises when market overcrowding is unbalanced between sectors. When underground goods are of considerably good quality, there is overcrowding of buyers underground. The benevolent government reduces the formal sector tax rate to encourage buyers back into this sector. Optimal policy thus involves high seigniorage financing and low taxes. I find optimal inflation rate as high as 42% per annum for Peru. Although this rate is lower than the rates observed in that country from the mid 1970s to the mid 1990s, it does offer a general explanation for the high rates of inflation in some developing countries within the context of optimal public finance policy.

When underground goods are very inferior, the formal sector tends to be more crowded for buyers. Optimal policy seeks to reduce the overcrowding of buyers in the formal sector. This requires high taxes combined with low seigniorage spending. For the relevant configuration of the model, I generate an optimal annual inflation rate of 1.4% for Peru, which is close to the rate observed in that country in 2005. In Peru, the size of underground output relative to the formal sector is estimated at 44%.[26] With such high rates of tax evasion, a familiar assertion in the literature calls for high reliance on seigniorage financing. The results in this paper show that the optimal inflation rate can be far lower than suggested in the literature.

On the theory front, this paper makes significant inroads towards the integration of fiscal policy instruments into the monetary search literature. I showed that the model is also adaptable for the inclusion of capital, thus allowing the familiar neoclassical growth theory analysis. Finally, the environment proposed is flexible and permits applications to other sectoral divisions of the economy.

[26] See Schneider and Enste (1998).

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Appendix

The household solves:

$$\begin{aligned} v\left(m_t\right) &= \max_{b_{jt},s_{jt},m_{jt},q_{jt},m_{t+1},j=f,i} c_{ft}+\eta c_{it}-s_{ft}\mathcal{S}_{ft}\Phi\left(Q_{ft}\right)-s_{it}\mathcal{S}_{it}\Phi\left(Q_{it}\right) \\ &\quad +\beta E v\left(m_{t+1}\right)+b_{ft}\mathcal{B}_{ft}\lambda_{ft}\left[\frac{m_{ft}}{b_{ft}}-\frac{\Phi\left(q_{ft}\right)}{\Omega_t}\right]+b_{it}\mathcal{B}_{it}\lambda_{it}\left[\frac{m_{it}}{b_{it}}-\frac{\Phi\left(q_{it}\right)}{\Omega_t}\right] \\ &\quad +\omega_t\left[m_t+s_{ft}\mathcal{S}_{ft}X_{ft}+P_{ft}Q_t^g+s_{it}\mathcal{S}_{it}X_{it}-b_{ft}\mathcal{B}_{ft}x_{ft}-b_{it}\mathcal{B}_{it}x_{it}-m_{t+1}\right] \end{aligned}$$

The Euler conditions (12) to (15) follow direct from the above set up. Now, I proceed to show how I arrived at (16), (17) and (18). First, note that if money is valued, $\lambda_{jt}\geq 0$, $j=f,i$ and hence $x_{jt}=\frac{\Phi(q_{jt})}{\Omega_t}$ and $\frac{dx_{jt}}{dq_{jt}}=\frac{\Phi(q_{jt})}{\Omega_t}\frac{\phi}{q_{jt}}$, $j=f,i$. This substituted into (13), (14) and (15) yield:

$$1-\tau=\left[\lambda_{ft}+\omega_t\right]\frac{\Phi\left(q_{ft}\right)}{\Omega_t}\frac{\phi}{q_{ft}}\ ,$$

$$\eta=\left[\lambda_{it}+\omega_t\right]\frac{\Phi\left(q_{it}\right)}{\Omega_t}\frac{\phi}{q_{it}}\quad \text{and}$$

$$\mathcal{B}_f\left[\left(1-\tau\right)q_{ft}-\left\{\lambda_{ft}+\frac{\omega_t}{1-\tau}\right\}\frac{\Phi\left(q_{ft}\right)}{\Omega_t}\right]=\mathcal{B}_i\left[\eta q_{it}-\left\{\lambda_{it}+\frac{\omega_t}{1-\delta}\right\}\frac{\Phi\left(q_{it}\right)}{\Omega_t}\right].$$

To simplify these three FOCs further, consider the following two properties of the equilibrium. First, with a constant money growth rate $m_{t+1}=\gamma m_t$, the value of money declines at the growth rate of money: $m_{t+1}\omega_{t+1}=m_t\omega_t$. Thus the euler for money gives $\gamma\omega_t m_t=\beta\omega_{t+1}m_{t+1}+\beta\mathcal{B}_f\lambda_{ft+1}m_{t+1}$. Rearranging,

$$\mathcal{B}_f\lambda_{ft}=\left(1-\delta\right)\mathcal{B}_i\lambda_{it}=\frac{\gamma-\beta}{\beta}\omega_t\ . \tag{22}$$

Second, due to the restriction $\gamma\geq\beta$, $\lambda_{jt}\geq 0$, $j=f,i$ and the cash and carry constraints bind in all transactions:

$$\frac{m_{jt}}{b_{jt}}\omega_t=q_{jt}^{\phi}\quad ,\ j=f,i \tag{23}$$

Substituting (22) and (23) in the FOCs for output and imposing symmetry ($\omega_t =$

Ω_t and $Q_{jt} = q_{jt}$, $j = f, i$ etc), we have:

$$1 - \tau = \frac{\gamma - \beta\left(1 - \mathcal{B}_f\right)}{\beta \mathcal{B}_f} \omega_t \frac{m_{ft}}{b_{ft}} \frac{\phi}{q_{ft}} \tag{16}$$

$$\eta = \frac{\gamma - \beta\left(1 - \mathcal{B}_i\right)}{\beta \mathcal{B}_i} \omega_t \frac{m_{it}}{b_{it}} \frac{\phi}{q_{it}} \tag{17}$$

Again, substituting (22) and (23) in the first order condition for buyers and imposing symmetry gives:

$$\mathcal{B}_f \left[\left(1 - \tau\right) q_{ft} - \frac{\gamma - \beta\left(1 - \mathcal{B}_f\right)}{\beta \mathcal{B}_f} \omega_t \frac{m_{ft}}{b_{ft}}\right] = \mathcal{B}_i \left[\eta q_{it} - \frac{\gamma - \beta\left(1 - \mathcal{B}_i\right)}{\beta \mathcal{B}_i} \omega_t \frac{m_{it}}{b_{it}}\right].$$

Simplifying using (16), (17) gives:

$$\frac{\frac{m_{it}}{b_i}}{\frac{m_{ft}}{b_f}} = \frac{\gamma - \beta\left(1 - \mathcal{B}_f\right)}{\gamma - \beta\left(1 - \mathcal{B}_i\right)} \quad . \tag{18}$$

At the government side (19) follows easily from (10) and (11).

The derivation of all the ratios are explained in the paper. $\frac{dR_I}{d\gamma}$ follows directly from quotient rule, where $\varphi\left(\cdot\right) = \left[\gamma - \beta + \beta \mathcal{B}_{it}\right] \frac{d\mathcal{B}_{ft}}{d\gamma} - \left[\gamma - \beta + \beta \mathcal{B}_{ft}\right] \frac{d\mathcal{B}_{it}}{d\gamma}$ and $A = \frac{\phi}{\beta}\left[\gamma - \beta\left(1 - \mathcal{B}_{ft}\right)\right]\left[\gamma - \beta\left(1 - \mathcal{B}_{it}\right)\right]$.

The cash-in-advance model:

$$\begin{aligned} v\left(m_t\right) &= \max_{c_{jt}, m_{jt}, q_{jt}, m_{t+1}, j=f,i} \left(1 - \tau\right) c_{ft} - Q_t^g + \eta c_{it} - \Phi\left(q_{ft}\right) - \Phi\left(q_{it}\right) + \beta E v\left(m_{t+1}\right) \\ &\quad + \lambda_{ft}\left[m_{ft} - p_{ft} c_{ft}\right] + \lambda_{it}\left[m_{it} - p_{it} c_{it}\right] \\ &\quad + \omega_t\left[m_t + p_{ft} q_{ft} + p_{it} q_{it} - p_{ft}\left(c_{ft} - Q_t^g\right) - p_{it} c_{it} - m_{t+1}\right] \end{aligned}$$

The Money Euler is $\frac{\omega_{t-1}}{\beta} = \lambda_{jt} + \omega_t$, $j = f, i$, which implies that $\lambda_{jt} = \frac{\gamma - \beta}{\beta}\omega_t$. The FOC q_{jt} are $\Phi'\left(q_{jt}\right) = \omega_t p_{jt}$, $j = f, i$. The FOC c_{jt} are:

$$\begin{aligned} 1 - \tau &= \left(\lambda_{ft} + \omega_t\right) p_{ft} \\ \eta &= \left(\lambda_{it} + \omega_t\right) p_{it} \end{aligned}$$

These simplify to $1-\tau = \frac{\gamma}{\beta}\omega_t p_{ft}$ and $\eta = \frac{\gamma}{\beta}\omega_t p_{it}$. Thus, the ratios become:

$$\frac{p_{it}}{p_{ft}} = \frac{\eta}{1-\tau}$$

$$\frac{q_{it}}{q_{ft}} = \left[\frac{\eta}{1-\tau}\right]^{\frac{1}{\phi-1}}$$

A summary comparison of the models is in Table 6:

TABLE 6

COMPARISON OF MODELS

	Walrasian	**Search**
Price Ratio		
$\frac{p_{it}}{p_{ft}}$	$\frac{\eta}{1-\tau}$	$\left[\frac{\gamma-\beta\left(1-\mathcal{B}_{ft}\right)}{\gamma-\beta(1-\mathcal{B}_{it})}\right]^{1-\frac{1}{\phi}}$
Quantity Ratios		
R_I	n.a	$\left[\frac{\gamma-\beta\left(1-\mathcal{B}_{ft}\right)}{\gamma-\beta(1-\mathcal{B}_{it})}\right]^{\frac{1}{\phi}}$
R	$\left[\frac{\eta}{1-\tau}\right]^{\frac{1}{\phi-1}}$	$\frac{\mathcal{X}_{it}}{\mathcal{X}_{ft}}\left[\frac{\gamma-\beta\left(1-\mathcal{B}_{ft}\right)}{\gamma-\beta(1-\mathcal{B}_{it})}\right]^{\frac{1}{\phi}}$

Data

1. Data on the relative size of the underground economy (R) was culled from Schneider and Enste (2000). The other ratio, R_I, is derived using these values of R and formulas outlined in section 4.

2. Figure 3 does not show country names against the data points due to overcrowding. The data is available upon request. The regression $\Delta UE = \beta_0 + \beta_1 \Delta\gamma$ gives $\beta_0 = 1.2486$, $\beta_1 = 0.0872$, $R^2 = 0.0007$ and p-values of 0.7913 and 24.9586 respectively.

3. In Figure 7, I retrieve annual data on fines and forfeits (F_t) from the IMF's Government Finance Statistics. The ratios plotted, δ, are then calculated as the ratio of these revenues to underground output (UE_t) in each country. For example, according to Schneider and Enste (2000), for every dollar of output produced in the official sector, there is \$0.088 worth of output produced underground in the US. GDP values were retrieved from the International Financial Statistics database, from which UE_t was derived as 8.8% in the case of the US. δ^{US} is then estimated as:

$$\delta^{US} = \frac{1}{T}\sum_t \frac{F_t^{US}}{UE_t^{US}} \times 100\% ,$$

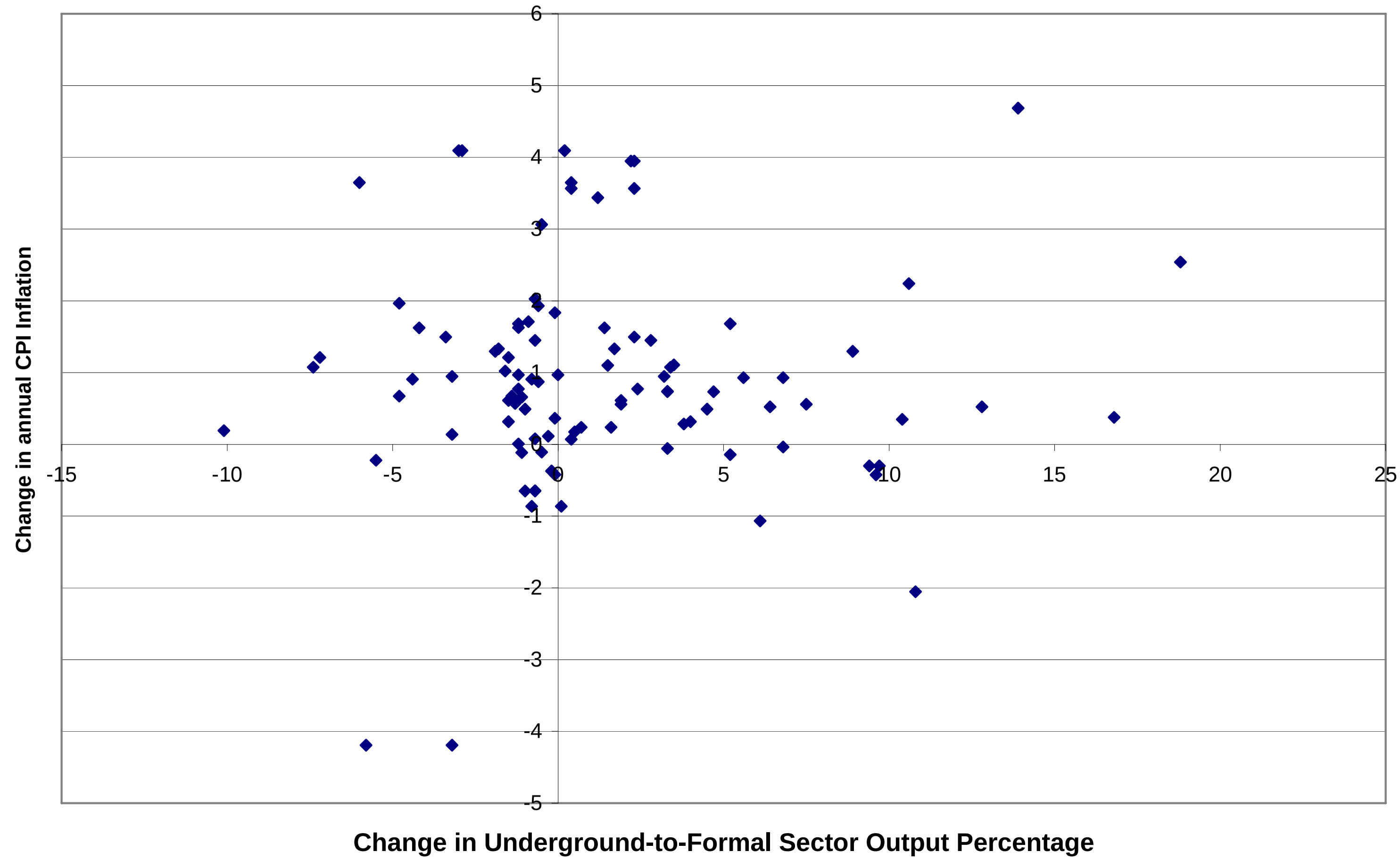
   where t is the year. F_t and UE_t are both in local currency units. The data period ranged from 1965 to 1995. Unreported years were omitted.

TABLE 7

SENSITIVITY ANALYSIS

	Benchmark ($\eta = .85$)		Parameter Variations						
ϕ	1.2		2		1.2		1.2		1.2
α	.5		.5		.75		.9		.5
b	$\frac{6.8}{23.9}$		$\frac{6.8}{23.9}$		$\frac{6.8}{23.9}$		$\frac{6.8}{23.9}$		1
	USA	Peru	USA	Peru	USA	Peru	USA	Peru	Peru
$\frac{b_i}{b}$	.0773	.2999	.0773	.2999	.0773	.2999	.0773	.2999	.2999
$\frac{s_i}{s}$	.1303	.3699	.0925	.3227	.2103	.4451	.5981	.6713	.3691
$\frac{m_{it}}{b_i}$	2.6830	3.1406	3.2113	3.3859	2.6872	3.1427	2.6911	3.1443	.8949
$\frac{m_{it}}{m_t}$	.0590	.2680	.0706	.2890	.0591	.2682	.0592	.2683	.2684
p_{it}	6.7497	7.9067	6.4393	6.7888	6.7888	7.9467	6.8259	7.9802	2.2761
p_{ft}	7.0836	8.1164	6.7610	6.9708	7.1226	8.1563	7.1597	8.1898	2.3357
$\mathcal{B}_i$	2.4335	2.0820	2.0502	1.9445	1.7583	1.5112	1.3914	1.2291	1.1094
$\mathcal{B}_f$	1.8202	1.7786	1.8593	1.8441	1.3170	1.2919	1.0435	1.0514	.9493
c_i	.0213	.0706	.0225	.0828	.0153	.0510	.0121	.0413	.1308
c_f	.2157	.1399	.2280	.1641	.1552	.1011	.1223	.0819	.2590
R_I	.7855	.8773	.9524	.9739	.7866	.8779	.7877	.8785	.8788
R	.0880	.4400	.0880	.4400	.0880	.4400	.0880	.4400	.4400

Figure 3: Change in Inflation and Change in Underground Economy



See point 2 on page 40 for comments on countries associated with these data points.

Figure 4: Inflation and the Underground Economy

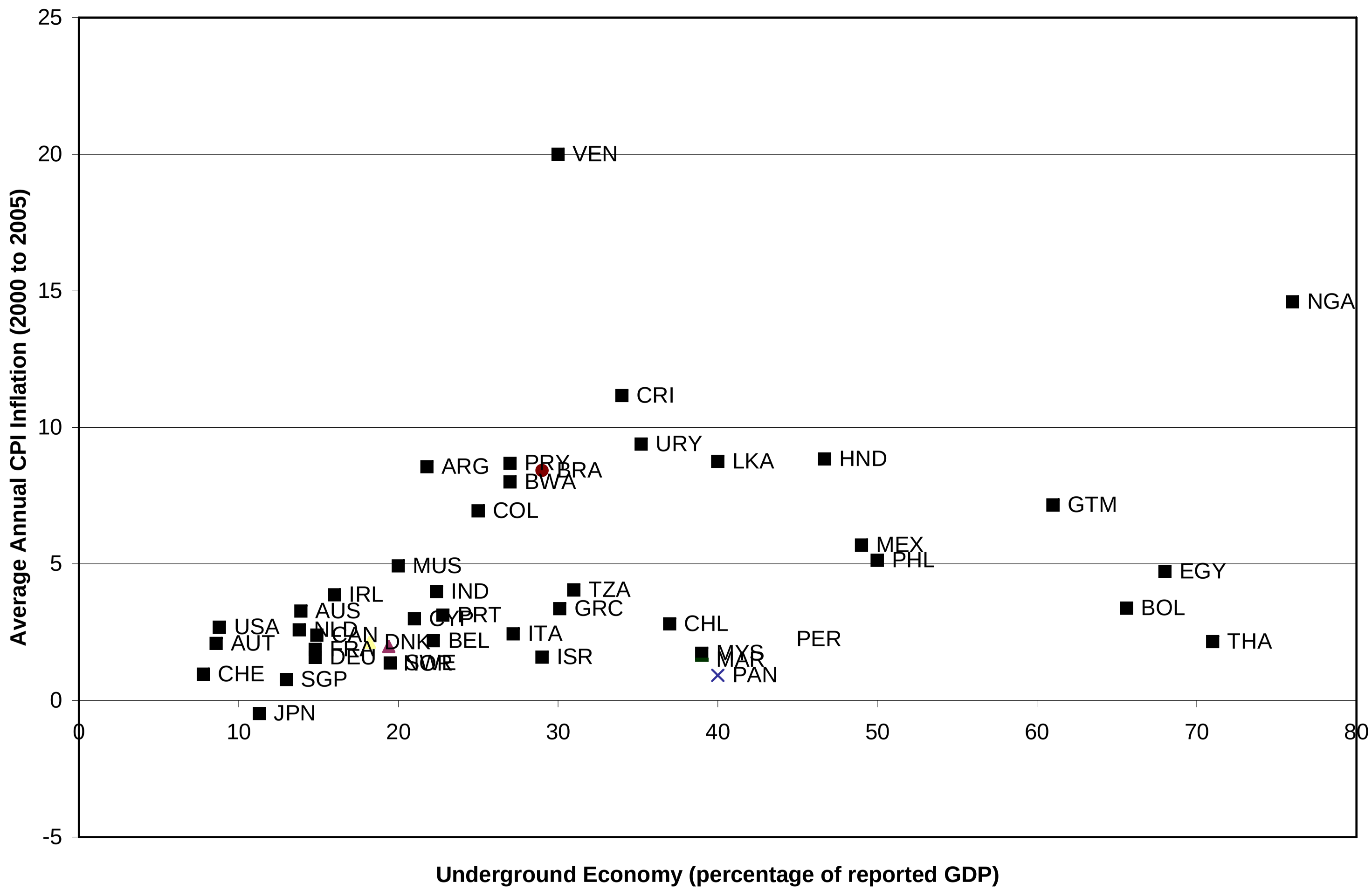


Figure 5: Government Spending and Underground Economy

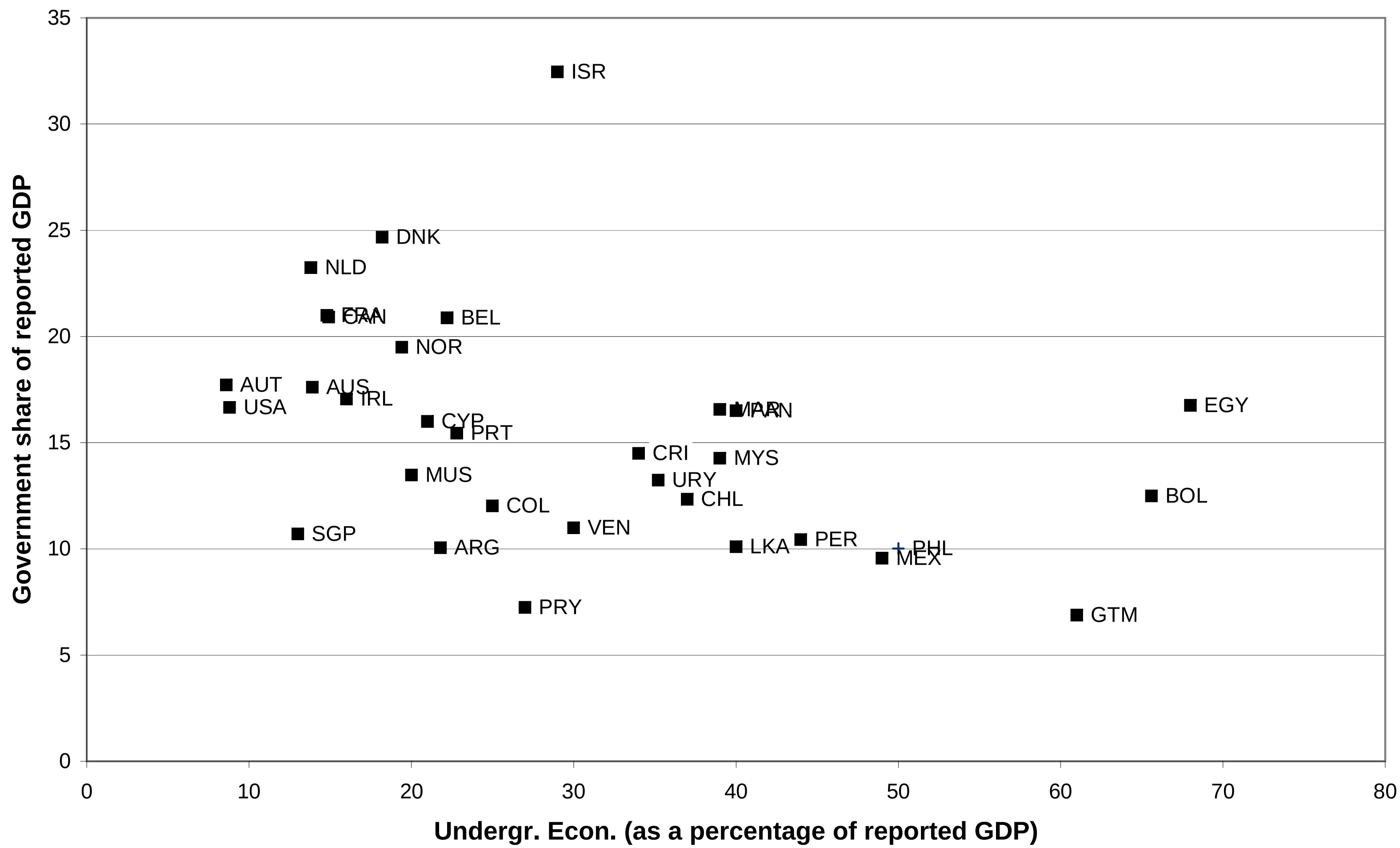


Figure 6: Taxation and the Underground Economy

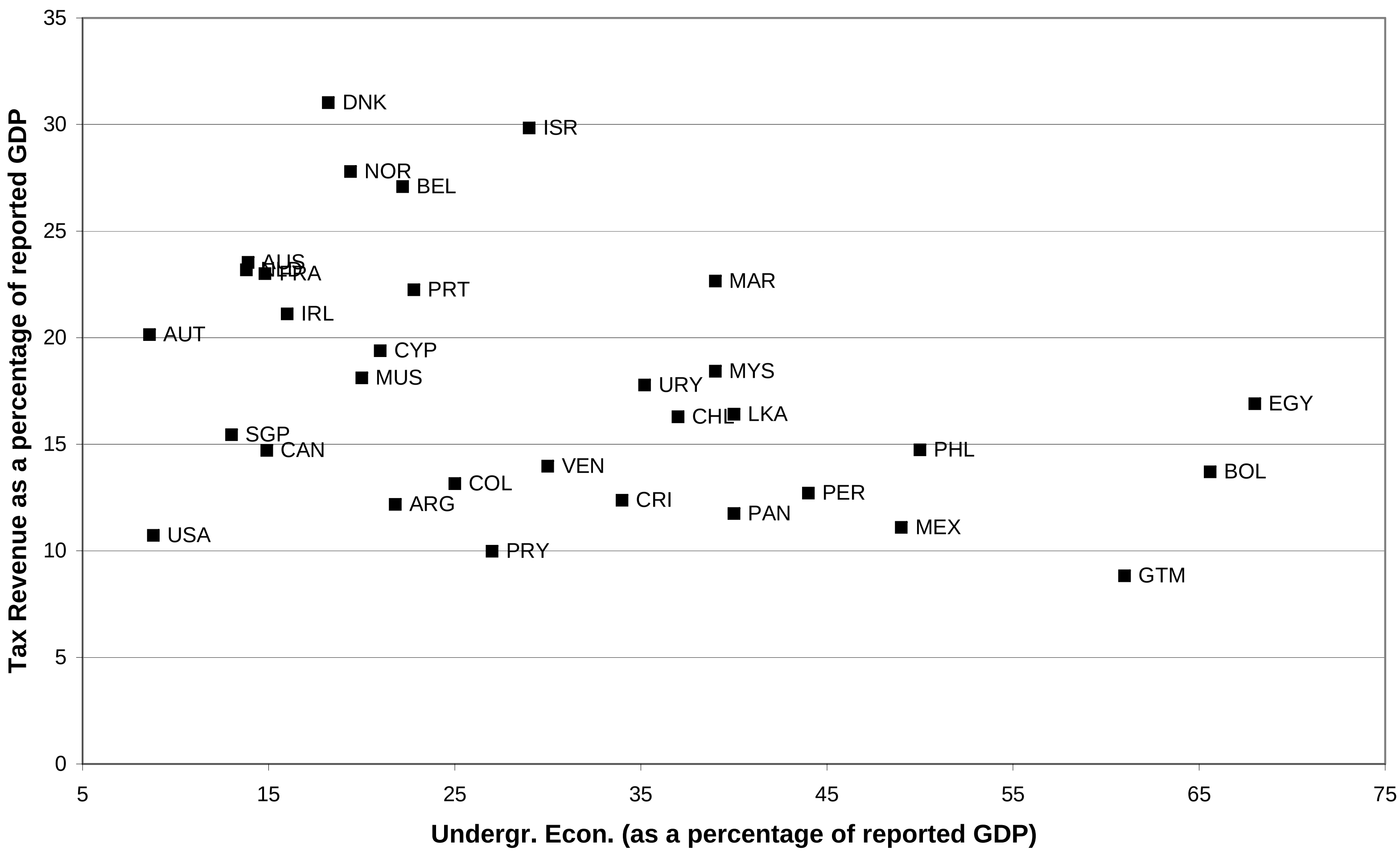


Figure 7: Fines and Forfeits as a percentage of Undergrd. Econ.