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Abstract

This paper develops a quantitative model that can rationally explain a substantial part of the dramatic rise and fall of house prices in the 2000-2009 period. The model is driven by the assumption that the government cannot resist bailing out large financial institutions, but can mitigate the consequences of that through regulation of risk. An episode of relaxed regulation is welfare-reducing, results in opportunistic behavior by lenders, and for plausible parameters inflates house prices and price/rent ratios by nearly twenty percent. This “boom” is followed by a collapse in house prices accompanied by high default rates.

The housing boom and bust cycle of the 2000s continues to attract a wide range of explanations. As is well known, real house prices rose some 30 to 40 percent relative to trend between 1998 and 2007. By 2010, in the wake of the financial crisis and recession, house prices had fallen back nearly to 1998 levels. The ratio of nominal house prices to rents experienced a similar increase and crash. (See Figures 1 and 2.) While some portion of the rise may be attributable to macroeconomic factors such as income growth and low interest rates, the magnitude of the boom appears to have gone far beyond what standard fundamentals can explain. For example, in the two prior business cycle expansions, the average real increase in house prices and price-rent ratios was only on the order of ten percent, which suggests an "excess" real appreciation of approximately 25 percent.

Much of the research on the so-called "housing bubble" has focused either on unconventional beliefs or on exogenously imposed shocks to credit availability. As examples, Fostel and Geanakoplos (2008), Burnside, Eichenbaum, and Rebelo (2011), and Boz and Mendoza (2014) incorporate heterogeneous beliefs or other departures from rational expectations, motivated by the notion (one we seek to refute) that it is otherwise not possible to explain the boom and bust. Other authors, e.g. Favilukis, Ludvigson, and Van Nieuwerburgh (2010), obtain price effects by alternately imposing or relaxing exogenous changes in credit limits for which there is no clear rationale to begin with. If a borrowing limit is just exogenously imposed (as opposed to being motivated by some other market failure), eliminating it tends to make agents better off by increasing their options. Absent any reason for credit restrictions, naturally they have adverse consequences for asset prices and welfare, and they can have the awkward implication that high price-low friction regime is preferred (see, for example, Guerrieri and Lorenzoni (2011) and Kocherlakota (2009)), which is not how most observers see the boom leading up to the 2008 crisis. Papers that focus on the adverse consequences of relaxed credit, e.g. Jeske, Krueger, and Mitman (2013), Corbae and Quintin (2015), tend to limit their analysis to credit outcomes and treat house prices as exogenous. Chatterjee and Eyigungor (2015) and Arslan, Guler, and Taskin (2015) have endogenous house prices, but only model the decline in prices resulting from exogenous increases in financial frictions and other macroeconomic factors.

By contrast, we offer a model in which binding credit constraints are a welfare-improving response to the government's presumed inability to allow large financial institutions to fail. In our baseline case, the combination of the credit limit and the pre-commitment problem results in a benign outcome, with house prices close to fundamentals and low default rates on mortgages. A relaxation of the credit standards (modeled as the eligibility requirements for mortgage to be repurchased and guaranteed by a government-sponsored enterprise such as Fannie Mae) then results in a distortion of house prices of approximately 18 percent above fundamentals, or some three-fourths of the 25 percent excess appreciation in the market suggested in figure 1.

While our paper has in common with the above-mentioned papers the exogeneity of the policy change (in our case, a relaxation of credit standards), we argue that relaxation or regulation is common, and readily explained by complacency or regulatory capture. The motivation for a welfare-reducing imposition of credit constraints in the literature is less clear. In addition, our model maintains standard assumptions that beliefs are rational and homogeneous. In our setting, house prices are bid up as a consequence of increased leverage coupled with a system of guarantees or implicit promises of bailouts.[1] That system, the intent of which is to support home ownership by subsidizing borrowers, gives rise to an ever-present incentive toward excessive leverage, as borrowers and lenders do not face the full consequences of higher default risk. Normally that incentive is blunted by strict limits on leverage as well as scrutiny of borrowers to weed out bad risks, and the result is a system that indeed supports expansive borrowing with little impact on either defaults or house prices.

With that benign outcome as a baseline, we then examine the impact of relaxing the limits on borrowing. Again, why this regulatory relaxation occurred is something we do not model explicitly, though we review some of the possible explanations in the next section. There is substantial evidence that it did occur, however. There are documented increases in loan-to-value (LTV) ratios, as well as the apparently increased disregard for other risk-related characteristics of borrowers (see Demyanyk and Hemert (2011), for example). In fact, a large number of mortgages in the period leading up to the crisis had combined LTVs (including second mortgages, home equity loans, etc.) of 100 percent.

While aspects of this story are certainly not new, this is the first effort we are

[1] Neuberg et al. (2016) find evidence in credit default swap pricing that in Europe, at least, perceived bailout probabilities have declined since 2014.

aware of to quantify in a general equilibrium model the impact of this regulatory lapse on house prices, during both the boom and the bust, and to depict the harm of the boom as leading inexorably to the debacle of a crisis. The model also realistically captures the impact on leverage and subsequent default rates. To accomplish all this we have to make certain restrictions for tractability (though we believe the model could be extended to relax these assumptions without significant impact to the main results):

- We assume a fixed stock of housing, i.e. we rule out construction
- We have a perfect foresight model with only idiosyncratic risk.
- We assume one-period debt contracts with a simple default rule.

Even so we are able to make the government's intervention contingent realistically on aggregate losses in the financial industry.[2]

Of course the United States is not the only country to have experienced a significant boom-bust cycle in housing market. While it is beyond the scope of this paper to lay out the details of the events in other countries, we would note that many other economies experienced a relaxation of lending standards in conjunction with the boom in their house prices. More important, and consistent with our story, many of these episodes were followed by crashes and bailouts of financial institutions in distress. For example, the Swedish government bailed out numerous banks in 1992 following a deregulation-fueled housing boom and bust. (See Drees and Pazarbasioglu (1995).) Spain and Ireland experienced similar episodes in which home prices were significantly inflated above the trend during the decade or so prior to 2007. In both cases financial sector supervision was lax and permitted arguably reckless lending practices (Akin et al. (2014)). In addition, bailouts of financial institutions occurred in the aftermath of the crash. (See Waldron and Redmond (2014), Beck et al. (2010).)

In any case, we do not claim to explain the entire boom and bust in U.S. housing. Rather, we isolate one set of factors which we argue played an important role,

[2]In this regard the paper differs from Jeske, Krueger, and Mitman (2013), who only compare across steady states, and who fix the price of housing (by having a linear transformation between housing and non-housing consumption) and consequently focus on default risk. We endogenize house prices by fixing the stock but, like Jeske, Krueger, and Mitman (2013), impose discipline by having realistic default rates and default costs in our baseline.

accounting for the majority of the excess movement in house prices, along with the explosion of defaults.

1 Background: Government-Sponsored Enterprises and Mortgage Lending

A complete history of government involvement in mortgage lending is beyond the scope of this paper. Suffice it to say that since the Great Depression, the government has had a major role in making mortgages more widely available and affordable to borrowers, and more liquid for lenders. The primary mechanisms have been the purchasing, insuring, and securitizing of mortgages. These efforts were successful in greatly expanding mortgage loans and, arguably, home ownership. For most of this period, through the mid-1980s, government agencies such as the Federal Housing Administration (FHA), and government-sponsored enterprises (GSEs) such as Fannie Mae, confined their involvement to loans that met relatively strict and objective standards for quality. The FHA, which insured private mortgages, was in principle self-financing, i.e. the insurance premiums were set to price default risk accurately.

Beginning with the Fair Housing Act of 1968, policy began to focus on expanding the availability of credit to those who had previously found it difficult to obtain, first by outlawing discrimination, but then by encouraging the extension of credit to riskier pools of borrowers. During this same period, Fannie Mae was privatized (though it was widely perceived to have implicit government backing), and was allowed to purchase private non-insured mortgages (as opposed to those insured by the FHA or other government agencies). By the 1990s, the GSEs were required to meet "affordable housing" goals, meaning targets for mortgages of low-income homeowners. These goals became more ambitious by the late 1990s, with private lenders also getting into the act with "subprime" and other loans that did not conform to GSE standards. Ultimately these markets grew enormously, and lenders, both government and private, took on more risk and became highly vulnerable to an economic downturn.

Nonetheless there is considerable debate over the extent to which the implicit government backing of the GSEs, as well as the "too big to fail" nature of the largest private financial institutions, contributed to this process and ultimately to the magnitude of the crisis that developed in 2008. For example, Paul Krugman writes that "Fannie and Freddie didn't do any subprime lending, because they can't: the definition of a subprime loan is precisely a loan that doesn't meet the requirement[s]" imposed on the agencies.[3]

While it is true that default rates on mortgages in GSE portfolios were substantially lower than overall rates, even within the category of loans labeled as "prime," this sanguine view of the GSEs overlooks several important facts.[4] First, as Krugman acknowledges, the GSEs were undercapitalized. Acharya et al. (2011), for example note that Fannie and Freddie had only a 0.45% capital requirement on mortgages they insured, and 2.5% for their balance sheet assets (p. 24). This of course made them more profitable during the boom, but vulnerable to a downturn in house prices or to an uptick in defaults, both of which occurred by 2007. Second, Fannie and Freddie purchased billions of dollars of subprime-backed securities for their own investment portfolios, in part as a means of meeting affordable housing toals. Finally, while the GSEs historically had been constrained to limit their purchases to mortgages with no more than 80 percent LTV, moderate debt-to-income ratios, and to borrowers with good credit scores, as Acharya et al. (2011) write (p. 39) that "After the mid-1990s . . . the GSE's mortgage underwriting standards deteriorated." They add that the lower quality of their mortgages "was masked by the continued rise in housing prices through mid-2006." Figures 3 and 4 depict the GSE's increased involvement in high-LTV mortgages and private-label securitizations that (along with other risks and high leverage) ultimately put their solvency in jeopardy. Whether this was due to pressure from HUD to reach "expanded affordability" goals, or was driven by the GSE's own quest for profit, is not important for our story.

Acharya et al. (2011) pinpoint the origin of the problem to the ironically-named Federal Housing Enterprises Financial Safety and Soundness Act (FHEF-SSA), somewhat reluctantly signed into law by President George H.W. Bush in 1992. The intent of the legislation had been to restrain the GSEs, but political

[3] *New York Times*, July 14, 2008. See also Pressman, "Fannie Mae and Freddie Mac were victims, not culprits" *Business Week*, September 26, 2008.

[4] For example, David Fiderer (http://www.fidererongses.com/params/post/695326/martin-fridson-embraces-the-big-lie-to-challenge-the-big-short) presents data that in early 2009 Fannie and Freddie's delinquency rates were below 3%, compared to a 6% overall delinquency rate.

compromises led to its containing a major Trojan horse: "mission goals" to support housing and mortgages for "underserved areas." In addition, the newly created regulator, the Office of Federal Housing Enterprise Oversight (OFHEO), was placed in the Department of Housing and Urban Development (HUD) rather than a more politically independent entity such as the Federal Reserve. The presence of these goals facilitated massive growth of low-quality mortgages, both through the increased ability of the GSEs to repurchase them as well as the participation of arguably too-big-to-fail so-called "large complex financial institutions" (LCFIs).[5]

Thus there is a strong case that the GSEs, armed with what was widely viewed as government backing, played a role in the expansion of credit that was potentially much larger than their direct role in subprime lending, while at the same time their high leverage made them vulnerable to insolvency and illiquidity. Large private financial institutions were not quite as highly levered as Fannie and Freddie, but likely saw themselves as too big to fail (i.e. subject to government bailouts) or as being able to sell low quality mortgages to investors up a food chain that was ultimately backed by the government, either implicitly or explicitly.

There are of course many other aspects to the financial crisis, notably the errors of rating agencies and private mortgage insurers, and, related, apparent underestimation of the risk of aggregate declines in house prices. While we ostensibly lump all of this together as the consequence of TBTF, we do not intend to rule out other factors as being important. In particular, there are sources of moral hazard within financial institutions that might lead to excessive risk-taking, such as the misalignment of incentives between creditors and shareholders.[6] The goal of the paper is simply to quantify what we believe to be one important contributor to the boom and bust. In particular, the model does not rely critically on the role of the GSEs; large private financial institutions, provided they have some expectation of being bailed out in a crisis, could play the same role.

[5]The 14 LCFIs were considered to be, according to Acharya et al. (2011), Citigroup, Bank of America, JP Morgan Chase, Morgan Stanley, Merrill Lynch, AIG, Goldman Sachs, Fannie Mae, Freddie Mac, Wachovia, Lehman Brothers, and Wells Fargo. Arguably 11 of the 14 were at risk of failure at some point in 2008, and all but one of those eleven were either bailed out by the governnment or folded into one of the three relatively healthy institutions (Bank of America, Wells Fargo, and JP Morgan). Lehman, of course, was the unique case of an LCFI that was allowed to fail.

[6]A challenge to such moral hazard explanations, however, is to explain the timing of the crisis, since these problems have been around for as long as there have been corporations.

In the next several sections, we formalize these ideas in a general equilibrium model of housing and mortgage markets, with a government that attempts to support the housing market while regulating risk, but that also cannot credibly commit to let large financial institutions fail.

2 A Dynamic Model with Heterogeneous Agents

Our ultimate goal is to assess the consequences of a policy change, specifically a change in conforming loan limits. For our purposes, "conforming" refers to a mortgage that qualifies for purchase or securitization by GSEs, and consequently for any favorable treatment or subsidy via government policy. We will begin with a description of stationary competitive equilibrium, and later build a dynamic analysis on this foundation.

Time $t \in \{0, 1, ...\}$ is discrete. There is a continuum of households of measure 1, and a large number $\bar{N} \gg 1$ of potential entrants/competitors in the financial sector. A competitive representative firm produces consumption and capital goods. The housing stock of the economy is in fixed supply, equal to 1, and there is no explicit rental market for housing.[7] There is a government that taxes household labor income, and uses the proceeds to finance mortgage guarantees whenever necessary. In what follows, we suppress individual subscripts, but in general, all quantities vary across agents.

2.1 Households

Households derive utility from consumption c_t and housing services h_{t+1}, discounting the future at rate $\beta \in (0, 1)$. The preferences over consumption goods and housing services are represented by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_{t+1}). \tag{1}$$

Housing services at time t are produced by a linear technology that uses the stock of housing the household owns. With some abuse of notation, we use h_{t+1} to

[7] We will, however, consider the behavior of implied rents, so that the model can address the behavior of the ratio of house prices to rents.

denote both. The price of consumption is normalized to 1, and the price of housing at date t is P_t. It will be clear that income does not affect default decisions in our framework; therefore we simplify the analysis by assuming identical labor income across households. In particular, households supply labor inelastically at a common post-tax wage rate $\bar{w}_t = (1 - \tau_t)w_t$. Nothing of any importance changes if we add idiosyncratic income risk.

Households may only borrow using housing as collateral, as unsecured borrowing is assumed to be unenforceable. We use b_{t+1} to denote the stock of mortgage debt acquired at time t, and a_{t+1} to denote the holdings of risk-free assets acquired at time t. Given the "no unsecured borrowing" assumption, we have $b_{t+1} \geq 0$ and $a_{t+1} \geq 0$ for all time periods. Specifically, households borrow through financial intermediaries, modeled as a sequence of one-period mortgage contracts similar to the treatment in Jeske, Krueger, and Mitman (2013). However, under our assumptions below, mortgages will in fact look like adjustable-rate mortgages of stochastic duration. Asset markets are incomplete, given the lack of insurance with respect to the idiosyncratic risks. We will use r_t to denote the risk-free rate on a_{t+1} and ρ_t to denote the mortgage interest rate that depends on the characteristics of the loan as well as other relevant macroeconomic variables. We assume that interest payments on the mortgage contracts are enforceable, but repayment of principal is only backed by the risky housing collateral of the individual borrower.[8]

We assume that there is no aggregate risk, and two sources of idiosyncratic uncertainty. One shock is an idiosyncratic i.i.d. "quality" shock $x_t \geq 0$ to housing. These quality shocks occur prior to the households' decisions about consumption, housing, and borrowing, and are distributed across households according to cumulative distribution function $G(x)$ and density $g(x)$ with support $[\underline{x}, \bar{x}]$, and $\mathbb{E}(x) = 1$. They can be thought of as neighborhood effects that result in unpredictable cross-sectional variation in house prices.

The second source of idiosyncratic uncertainty relates to inertial frictions. We observe that households do not freely vary their choice of housing or their financing at every opportunity. Presumably this is because of some combination of transactions costs and inattention. In lieu of modeling the micro-foundations of this inertia in detail, we impose "Calvo-style" adjustment costs: Poisson probabili-

[8]This assumption is only aesthetic, so that an LTV of one is a natural limit.

ties of being allowed to move or to refinance. These have the effect of realistically slowing the response of households to changes in their environment. Specifically, we assume that, with probability $m \in [0,1]$, a household becomes a *mover* (type m). A mover is free to choose the housing stock h_{t+1}, and can borrow b_{t+1} against the value of the new dwelling, subject to the relevant debt constraints which we will clarify shortly. With probability $(1-m)f$, where $f \in [0,1]$, the household becomes a *refinancer* (type f). This household cannot move, but is free to adjust its debt level, subject to borrowing constraints. If a household is neither a mover nor a refinancer, then it becomes type n, which occurs with probability $(1-m)(1-f)$. These households are stuck with their previous choice of h and b, but must pay a fraction of *at least* $\theta \in [0,1]$ of the existing debt. That is, if a type n household enters period t with (h_t, b_t), it must choose $h_{t+1} = h_t$ and $b_{t+1} \leq (1-\theta)b_t$.

While this approach to modeling moving and refinancing lacks micro foundations, we adopt it for tractability, and note that these frictions play very little role, if any, in our quantitative results on the magnitude of home price inflation. We will see that with or without these frictions, the magnitude of the price response to changes in lending standards in our model is virtually identical. The frictions are crucial only to the extent that they result in more realistic dynamics for both prices and quantities. They do so by preventing sudden "jumps" in variables of interest (such as high-LTV mortgages) as a response to news or to policy changes, similar to the motivation for search frictions. We should also note that there was considerable geographic variation in the incidence of subprime lending, which suggests that the availability of such loans was in part due to factors beyond individual borrowers' control. This provides additional justification for our modeling.

In what follows we distinguish between *default* and *foreclosure*. Default is a simple failure to repay the principal of the loan. It does not by itself trigger any dead-weight losses (such as legal costs) or "moving" (in the sense of the m shock described above). Foreclosure is a costly legal process that involves the owner moving in that same sense. We discuss foreclosure costs in more detail, and the implications of foreclosure versus default from the perspective of the lender, below in our discussion of financial markets (Section 2.4).

Any household can choose to default. To simplify the analysis—in particular to make the default decision simple and non-strategic in a sense to be described below, we make two assumptions about the consequences of default.

Assumption 1 *Households cannot move unless they receive the moving shock, even if they default.*

A direct consequence of this assumption together with non-enforceability of the repayment of principal, is that a non-mover borrower (type f or n) who defaults has his debt level written down to 100% of the value of the house. This arrangement serves two purposes: The first is empirically motivated, as a fraction of defaults end up in foreclosures in the U.S. Rather, the majority result in banks accepting a loss on the difference between the sale price of the house and the remaining principal on the mortgage; our assumption mimics this outcome. Second, it eliminates the incentive for a "strategic" default decision where, absent any default costs for the borrowers, an agent might choose to default just for the opportunity to move or refinance that comes with it.

Note that even with the above assumption in place, a type n household might choose to default strategically to avoid having to make the required payment θb_t. This motivates the second assumption:

Assumption 2 *Type n households are required to pay at least a share $\theta \in [0, 1]$ of their debt even if they default.*

Under these two assumptions, households default on their mortgages if and only if they have negative equity. We make these assumptions purely for tractability. The fact that owners frequently repay their loans even when they are "under water," and indeed many optimal default models suggest they should (see, for example, Krainer and LeRoy (2010) and references therein) does not undermine our findings, because—as will become clear—the crucial variables are the probability of default and losses conditional on default. As long as our model has realistic predictions for those quantities, our quantitative results are robust to the behavioral assumptions about default. If anything, our approach underestimates the impact: Were we to assume, for example, that homeowners only defaulted when substantially under water, and recalibrated the distribution of x to get the same probability of default, losses conditional on default would be greater, and consequently our model would predict even larger house price distortions than we find in our calibration.

As to foreclosure, since it almost invariably involves relocation of the former owner, for simplicity we assume that foreclosure occurs when a defaulter receives the m shock. With foreclosure, a defaulter cedes the house to the bank in lieu of repayment of the principal. So to summarize: A default occurs if and only if the value of the house falls below the value of the principal on the debt. A foreclosure occurs if a defaulter receives the m shock. This is clearly a simplification: For example, many defaults result in a voluntary or negotiated sale of the property and relocation by the former owner. Our assumptions are for the sake of parsimony and simplicity and have little impact on the main results of the paper.

The timing within a period t is as follows:

1. Households make interest payments on their existing mortgage.
2. Households observe x and their type $\{m, f, n\}$, and make default decisions.
3. Given prices, households choose c_t, a_{t+1}, b_{t+1}, h_{t+1} subject to the budget constraint and restrictions imposed by their types. Housing h_{t+1} can be used immediately for housing services at time t.

As mentioned above, our assumptions imply that the household will choose to default at time $t+1$ if and only if

$$x_{t+1}P_{t+1}h_{t+1} - b_{t+1} < 0 \tag{2}$$

This defines a threshold value of shock, $z_{t+1} \equiv \frac{b_{t+1}}{P_{t+1}h_{t+1}}$. If the household draws a value $x_{t+1} < z_{t+1}$ next period, which happens with probability $G(z_{t+1})$, default occurs.

Assuming for now that the economy is at a steady state, we drop the time subscripts from all prices, assuming $P_t = P$, $r_t = r$, $\bar{w}_t = \bar{w}$, and $\rho_t(.) = \rho(.)$ for all t. In equilibrium, due to competition among the financial intermediaries, and the fact that loan-to-value (LTV) ratio z_{t+1} alone captures the default risk of a borrower, the intermediaries will offer mortgages with interest rates that take the form $\rho(z_{t+1})$. This will be clarified further in the next few sections when we discuss the nature of competition in the financial sector. Due to the simple structure of mortgage loans, we prefer to formulate the budget constraint of a household using the LTV ratio z_{t+1} rather than b_{t+1} by using the transformation $b_{t+1} = Ph_{t+1}z_{t+1}$.

$$c_t + Ph_{t+1}(1-z_{t+1}) + a_{t+1} \leq \bar{w} + a_t(1+r) + Ph_t\big(\max\{0, x_t - z_t\} - \rho(z_t)z_t\big) \equiv I_t \quad (3)$$

$$c_t, h_{t+1}, a_{t+1} \geq 0, \text{ and } z_{t+1} \in [0,1]$$

$$h_{t+1} = h_t x_t \text{ for types } \{f, n\}$$

$$z_{t+1} \leq (1-\theta)\min\{1, \frac{z_t}{x_t}\} \text{ for type } n$$

A household enters period t with after-tax wage $\bar{w}$, assets a_t and housing h_t net of the interest payment $Ph_t\rho(z_t)z_t$. Having chosen an LTV ratio z_t in period $t-1$, upon realization of shock value x_t, the household receives a net return (or capital gain) of $Ph_t \max\{0, x_t - z_t\}$ from housing, *after* taking the optimal default decision captured by the max operator. Including the wage level and return on assets, the total resources available to the household, after the default decision, is represented above by the term I_t. These resources are spent on consumption c_t, housing h_{t+1}, and assets a_{t+1}. A non-mover household is restricted to "choose" the current housing stock $h_t x_t$. Moreover, the household can get a loan of $Ph_{t+1}z_{t+1}$ against the value of current housing Ph_{t+1} by writing a new mortgage contract. The type of contracts available depends on the household type. In particular, a type n household is restricted to choose a debt level that is lower than a share $(1-\theta)$ of the existing debt *after* taking the default decision, i.e. after the debt is written down by the lender.

As mentioned, with the frictions we impose on the model, we can interpret the time period between two moving or refinancing shocks (i.e. remaining a type n borrower) as the (stochastic) duration of a multi-period mortgage contract. Over the lifetime of a mortgage contract, the borrower needs to pay at least a constant share of the debt every period and is subject to an "adjustable rate" based on a re-evaluation of default probability by the lender.

Although mortgages with LTVs in excess of 100 percent were not unheard of during the housing boom, these were primarily to cover both the price of the house and closing costs. For now, our setting with no closing costs simply requires $z_{t+1} \leq 1$. (Otherwise, the household would default upon receipt of the loan and pocket the difference between the loan and the value of the house.) Below we will motivate government intervention in the form of an LTV limit $\zeta < 1$ on conforming loans.

We now omit time subscripts on choice variables, and use $'$ to indicate those choice variables formerly dated $t+1$. The household's decision problem at the point of choosing c, h', z', a'—that is, after the idiosyncratic shocks have been realized and any default decision has occurred—can be written recursively using its type $i \in \{m, f, n\}$, total resources I, housing h, LTV for the existing debt z (after any write-down by the lender if the household defaulted) as state variables. For what is to follow let, $\pi_m \equiv m$, $\pi_f \equiv (1-m)f$, $\pi_n \equiv (1-m)(1-f)$ represent type probabilities:

$$V_i(I, h, z) = \max_{\{c, a', z', h'\}} u(c, h') + \beta \sum_{j \in \{m,f,n\}} \pi_j \mathbb{E} V_j(I', h'x', \min\{1, \frac{z'}{x'}\}) \tag{4}$$

subject to

$$c + Ph'(1 - z') + a' \leq I$$

$$c, a', h' \geq 0, \text{ and } z' \in [0, 1]$$

$$I' \equiv I'(a', h', z') = \bar{w} + a'(1 + r) + Ph'\big(\max\{0, x' - z'\} - \rho(z')z'\big)$$

$$h' = h \text{ for types } i \in \{f, n\}$$

$$z' \leq (1 - \theta)z \text{ for type } i = n$$

2.2 Production

There is a representative firm that uses capital K and labor L, producing consumption and capital goods using a Cobb-Douglas production function. The output of the representative firm is

$$Y = AK^{\alpha} L^{1-\alpha}$$

For convenience, we also define $F(K, L) \equiv AK^{\alpha}L^{1-\alpha} - \delta K$, the output net of depreciation. The firm rents capital at rate r and labor at rate w in competitive factor markets.

2.3 Government

We now introduce key features of the government's role in the mortgage market: First, we posit that the government cannot credibly commit to let large financial institutions fail, or in the context of this model, incur huge losses-the "Too Big to Fail" (TBTF) phenomenon. Below we will define "large" in terms of market share $s \in [0,1]$. Second, as a consequence, financial regulators take measures to limit large institutions' risk-taking, which because of TBTF would tend to be excessive.[9] Indeed the key risk-reduction measures that we focus on, which in the model are distilled down to LTV limits, are those that historically applied particularly to large protected institutions such as Fannie Mae. The GSEs were long restricted to purchasing only "conforming" mortgages that were limited in size and LTV ratio.[10] Other institutions had more flexibility, though for the most part they historically could not sell non-conforming mortgages to the GSEs. Under our baseline assumptions, the equilibrium involves low default risk and house prices very close to what their values would be in the absence of TBTF. When the regulations are relaxed or circumvented, the equilibrium changes to one in which TBTF institutions take over the market. As a consequence, default risk increases, of course, but our model also implies a large increase in house prices, with no corresponding change in (implicit) rental prices.

To clarify the notation that follows, for any parameter or variable set by the government, a bar will indicate its baseline or "normal" value, while a ˆ will indicate its value during the boom. We assume that the initial policy change is unanticipated, and that once a bailout occurs, it is common knowledge that the policy will revert to the baseline forever.

We assume that in "normal" times the government, for reasons that we do not model, wishes to support the housing market, and does so by modestly subsidizing mortgage lending.[11] In our simplified setup, in which credit risk derives only from the idiosyncratic x shock, the distribution of which is assumed to be identical

[9]This is not to suggest that small banks are not also regulated in their risk-taking, but that is largely because of deposit insurance as opposed to discretionary bailouts.

[10]There have been other requirements as well: Debt to income ratio, credit score, income documentation, etc., though these appear to have varied over time.

[11]In a world with a choice between home ownership and renting, there may be positive externalities to ownership that the government wishes to subsidize. The government may also see itself as helpful in creating markets, as it has for securitized mortgages.

across all agents and houses, LTV is a sufficient statistic for default risk. Hence, to mitigate the potential moral hazard of excessive risk-taking, it suffices for government to control LTVs. We assume this takes the form of a simple LTV limit $\zeta = \bar{\zeta} \in (0, 1]$, so that $z \leq \zeta$ is required for a mortgage to be conforming. Below we will show that this policy is in fact effective. The subsidy takes the form of a government guarantee to the lender of a share $\bar{\eta} \in [0, 1]$ of any unpaid principal on a conforming mortgage.[12]

The fact that the model contains no reasons for the government's temptation to bail out large financial institutions, or for the subsidies to home ownership, does not mean no such reasons exist. There may be substantial "collateral damage" from failures of such institutions, and there may be, for example, positive externalities from home ownership that are missing from the model.[13] Our point is not that these policies are intrinsically bad—indeed in our baseline they are relatively benign. Rather it is to point out the fragility of the benign outcome, and to illustrate how modifications to the policies could have a dramatic impact. It is our contention that enriching the model to motivate the policies would not alter the main results.

The TBTF aspect of policy works as follows: Let λ denote the aggregate losses incurred by banks on conforming mortgages as a share of GDP, *when the government offers the baseline subsidy* $\bar{\eta} \in [0, 1]$. In addition to the baseline subsidy, a government "bailout" (meaning some $\hat{\eta} \gg \bar{\eta}$ on conforming loans) is triggered for one period, for any financial institution with $s > s^*$, in the event that aggregate losses as a share of GDP exceeds some threshold λ^*.[14] We call such an event a "crisis." After a bailout, it is understood that η reverts to $\bar{\eta}$ forever.

We shall see below that as long as the LTV limit on conforming loans ζ is such that aggregate default risk is low, and provided the modest baseline $\bar{\eta}$ is sufficiently high to encourage borrowing despite foreclosure costs, then mortgage lending will occur, but the (small) subsidy will have a negligible impact on the market.

[12] This can be shown to be equivalent in our model to subsidizing the interest rate on conforming mortgages, or to underpricing mortgage insurance. We use the "guarantee" language (meaning *ex post* replacement of losses) so that η can also serve as the measure of bailouts.

[13] Others, e.g. Morgenson and Rosner (2011), suggest more sinister motives involving corruption and cronyism.

[14] We assume the government can pre-commit to limit its bailout to conforming loans. This could be either because the government can prevent institutions from making so many risky loans, or politically it is feasible to let banks that plunged into non-conforming loans fail.

There are no aggregate losses, and no crisis or bailout occurs in equilibrium. This is the case because aggregate losses would be limited quantitatively even in the (off-equilibrium) contingency that banks engage in reckless lending practices by ignoring default risk completely.[15] On the other hand, should the government choose a sufficiently high ζ, the resulting equilibrium would culminate in a crisis and bailout, the dynamics of which depend on the inertial friction parameters m and f. Movers would then borrow up to the now higher LTV limit, and eventually the debt levels and default risk build up to the point that a crisis and bailout occur. In the absence of the frictions this would happen immediately when LTV limits are relaxed.

To finance the subsidy (or the bailout in the event that occurs), government taxes labor income linearly at rate τ. Since labor supply is inelastic and the households are identical in terms of their labor endowment, this is effectively a lump-sum tax equal to τw. We assume that the government runs a balanced budget each period.

The baseline government policy is thus defined by LTV limit, subsidy, and bailout threshold parameters $\{\bar{\zeta}, \bar{\eta}, s^*, \lambda^*, \hat{\eta}\}$. In our baseline quantitative exercise, it will turn out, realistically, that $\bar{\eta}$ (calibrated from the spread between loans at the boundary between conforming and non-conforming) is high enough to result in widespread mortgage finance, but low enough to ensure that a crisis never occurs in equilibrium, i.e. the aggregate default risk and therefore *potential* aggregate losses remain in check even when banks ignore the default risk. Even so, for the sake of completeness, we allow our definition of competitive equilibrium potentially to involve a steady-state with losses exceeding λ^*.[16] This necessarily involves a higher LTV limit than in our preferred baseline. In our dynamic analysis we rule this out, and in fact assume that should ζ for some reason be increased to the point that a crisis and bailout occurs, the government responds by resetting ζ back to the lower baseline level.

It should be stressed that we define government policy in terms of LTV limits simply because in our setting that is the only variable that matters for risk. In this

[15]Under the quasi-linear preferences we consider below, positive foreclosure costs would mean that virtually no risky borrowing would occur if $\bar{\eta} = 0$. This is not the case for more standard convex preferences, where borrowing also serves to share risk.

[16]In such an equilibrium, the economy would experience a bailout every period, and the stationary prices would be consistent with this outcome.

sense the LTV ratio is just a stand-in for credit risk from any source. If we had heterogeneity in other borrower or loan characteristics that affected default rates, policy could regulate those aspects of loans as well to mitigate excessive risk-taking (given the subsidy and bailout assumptions). For example, if borrowers varied in their ex ante default probabilities (as captured by FICO scores, say), which in our model could occur if the x distribution varied across individuals, then it would be possible for lenders to adhere to a fixed LTV requirement but still increase risk by extending loans to a wider range of individuals. It is just for the sake of parsimony that we limit the focus to LTV ratios, but similar results would obtain for any characteristic that affects credit risk.

2.4 Financial Markets

We assume free entry of financial institutions with positive measure (so that they can rely on the law of large numbers). These "banks" have constant returns to scale and in the baseline at least are of indeterminate size. They engage in Bertrand competition for both their rates for "depositors" and for home mortgage borrowers. They set mortgage interest rates contract by contract, depending on the mortgage's LTV. These assumptions imply that expected profits are zero for each contract; and with the law of large numbers assumption, banks make zero profits every period. In this sense, the equilibrium implications are very similar to those in the model by Chatterjee et al. (2007), where banks are price-taking Walrasian actors.

Bertrand competition among finitely many banks serves an important purpose, however, because of the role market share plays in a model of TBTF, it is indispensable. As we elaborated in the previous section, the government's bailout policy is contingent on two macroeconomic variables: aggregate losses as a share of GDP, and the bank's market share. Non-atomistic profit-maximizing competitors effectively internalize the impact they have on the aggregate default rate and losses. For instance, our setup allows a bank to pick a lower interest rate than its competitors for a risky loan, attract all consumers eligible to borrow, drive up the equilibrium default rate and potential losses significantly, and enjoy the bailout subsidy. In our baseline scenario this will be an off-equilibrium outcome: When the conforming loan limit is high and any bank can drive the potential losses above the bailout rule, all active banks would take the same action, exhausting all such gains from deviation. It is, however, precisely these potential off-equilibrium gains that lead to an inevitable bailout. Observe that if banks were atomistic price takers, in principle, a “good equilibrium” could be supported despite the high LTV limits, where mortgage interest rates remain high, risky loans are not traded, and equilibrium losses remain low, rationalizing the high mortgage interest rates (due to absence of a bailout).[17]

Since banks are bailed out only if they are TBTF, i.e. their market share exceeds s^*, the number of active banks N during the boom period would satisfy $1/N > s^*$. The size of any single institution is indeterminate. Note that the relevant institutions here are not the originators of mortgages; these could be any size. The large firms are those that in equilibrium actually hold the mortgages and/or bear the credit risk. Small institutions such as regional or local banks could originate the mortgages but then sell them to large institutions.[18]

While we do not model banks' capital structure or bank failure in any detail, we note that the typical bank is highly levered, particularly during the era in question. We have already discussed how highly levered the GSEs were. Large investment banks were as well, with leverage ratios in excess of 30. Commercial banks were not as highly levered as investment banks (though their ratios were still in excess of 10) but many also had off-balance sheet exposures that made their effective leverage higher than was evident from their balance sheet (see, for example, Kalemli-Ozcan, Sorensen, and Yesiltas (2012).)

In a standard bailout, creditors are made whole, while equity holders may have substantial losses. In the case of Bear Stearns, for example, the Federal Reserve Bank of New York facilitated a sale of the firm to J.P. Morgan by making $29 billion in loan guarantees. Bear Stearns shareholders were originally to receive just $2 per share (the stock had traded at $93 just weeks earlier), but this was subsequently revised to $10 per share. Bear Stearns had a leverage ratio in 2007 of 33.5, meaning that over 97% of the firm's liabilities were covered by the bailout,

[17]The action space and payoffs for the dynamic Bertrand game between banks is very sophisticated, and a complete specification of this game is beyond the scope of this paper. On the other hand, the Nash equilibrium outcome is trivial due to the assumption of risk-neutrality. Motivated by the latter, we choose to adopt a reduced-form approach focusing on the equilibrium implications only.

[18]Small institutions in principle could retain the highest quality, essentially risk-free mortgages.

and that only includes the debt.[19] Since the equity holders were not wiped out, the 97% figure is a lower bound.

Other examples of bailouts and interventions during the 2008 crisis, as well as in previous episodes, exhibit a similar pattern: Creditors are protected essentially 100%, and equity holders are not wiped out entirely. Thus, the high leverage of TBTF financial institutions suggests that a bailout effectively covers nearly 100% of such institutions' liabilities. We will use this to motivate our choice of $\hat{\eta}$ in our calibration.

Assuming that interest payments are enforceable, a contract between the bank and the household yields interest payments to the bank with certainty. For a contract with LTV ratio z, the household defaults with probability $G(z)$. We assume that if the property is foreclosed after default, the bank loses a fraction $\gamma \in [0,1]$ of the value of the house. We model this cost as dead-weight loss measured in terms of consumption goods. It is clear that due to foreclosure costs, there are some gains from renegotiation ex-post. The reasons lenders want to avoid foreclosures are well-documented in the literature; Ghent and Kudylak (2011) and Adelino, Gerardi, and Willen (2013) discuss them in detail. First, properties depreciate significantly (formalized by γ in the model) when the borrowers are in default, because the occupants have no incentive to maintain the property.[20] Second, there are legal and administrative costs. Lenders can eliminate most, if not all, of these costs by taking alternative actions. For instance, the parties can negotiate on a short sale agreement in which borrower sells the property at a price lower than the purchase price, remitting the proceeds to the lender, and the lender waves the right to a deficiency. Another option is a voluntary conveyance where the borrower hands over the deed to the property to the lender, and the lender forgives the debt owed.[21]

Motivated by the empirical evidence that only a fraction of defaults results in foreclosure, for the sake of simplicity we assume that among the defaulters,

[19] Kalemli-Ozcan, Sorensen, and Yesiltas (2012), Table 6.

[20] Consistent with this view, Ghent and Kudylak (2011) points out that the common view among foreclosure attorneys is that if the lenders decide to exercise the option of foreclosure, they have a strong interest in foreclosing quickly.

[21] We sidestep the question of why foreclosure ever occurs, given the alternative of a voluntary liquidation or other arrangement that avoids the dead-weight costs of foreclosure. Presumably this is related to strategic negotiation issues beyond the scope of this paper.

lenders foreclose only on those who receive the moving shock. Consequently only the share m of defaults end up in costly foreclosure. The others cause the bank to lose the amount by which the home value falls short of the remaining mortgage principal.

Before we formally define an equilibrium, we characterize mortgage interest rates in equilibrium. Under our competitive assumptions, the interest rates for contract x must satisfy the following zero-profit condition derived from the expected present value of the returns for a financial intermediary, taking the degree of government intervention, η, and the conforming loan limit ζ into account:

$$\rho(z;\eta,\zeta)z = \begin{cases} rz + (1-\eta)\int_{\underline{x}}^{z}[z-(1-m\gamma)x]dG(x) & z \le \zeta \\ rz + \int_{\underline{x}}^{z}[z-(1-m\gamma)x]dG(x) & z > \zeta \end{cases} \tag{5}$$

Note that this expression confirms our earlier claim that the mortgage interest rate depends only on the LTV ratio z, since it is a sufficient statistic to assess all risks in a contract from the perspective of a bank. Also note that $\eta = 1$ (a complete guarantee) implies a risk-free borrowing rate independent of the default probability, i.e. $\rho(z;\eta,\zeta) = r$ for all $z \in [\underline{x}, \zeta]$.

We assume that $G(.)$ is continuously differentiable everywhere in $(\underline{x}, \bar{x})$, and that $g(\underline{x}) = G(\underline{x}) = 0$. Using these assumptions, it is easy to show that

1. Function $\rho(z;\eta,\zeta)z$ is continuously differentiable in $z \in (\underline{x}, \zeta) \cup (\zeta, \bar{x})$.
2. $\lim_{z \downarrow \underline{x}} \rho(z;\eta,\zeta) = r$.
3. $\rho'(z;\eta,\zeta) > 0$ and $\rho(\bar{z};\eta,\zeta) > r$ hold for all $\eta \in (0,1)$ and $z \in (\underline{x}, \bar{x})$.

For the rest of the exposition, unless the effect of a change in parameters is analyzed explicitly, the dependence of ρ on (η,ζ) will be suppressed for notational simplicity.

2.5 Equilibrium

To investigate the long-run effects of policy on the economy, we proceed with defining a stationary recursive competitive equilibrium for this environment.

For what is to follow, let $\mathcal{S} = \mathbb{R}_+ \times \mathbb{R}_+ \times [0,1]$ represent the space for total resources I, housing h, and LTV z. We let Σ represent the Borel σ-algebra on $\mathcal{S}$, and $\mathbb{P}$ represent all probability measures over the measurable space $(\mathcal{S}, \Sigma)$.

Definition 1 *A* ***stationary recursive competitive equilibrium*** *with government policy* $\{\bar{\zeta}, \bar{\eta}, s^*, \lambda^*, \hat{\eta}\}$ *is a set of prices* $P, r, w \in \mathbb{R}_{++}$*, tax rate* $\tau \in \mathbb{R}_+$*, mortgage interest rates* $\rho : [0,1] \to \mathbb{R}_{++}$*; policy functions* $c_i, a_i', h_i', z_i' : \mathcal{S} \to \mathbb{R}_+$ *for* $i \in \{m, f, n\}$*; steady-state distribution* $\mu \in \mathbb{P}$*; number of active banks* $N \leq \bar{N}$*; aggregate losses (as a share of GDP)* $\lambda \in [0,1]$*; conforming loan limit* $\zeta \in [0,1]$*; and subsidy* $\eta \in [0,1]$*, such that*

1. *Given prices, tax rate, government policy, and* ζ*, policy functions solve the households' problem* (4).

2. *Given factor prices* (r, w)*, firms maximize profits, therefore*

$$F_K(K, L) = r$$

$$F_L(K, L) = w$$

3. *Given household policy functions, intermediaries maximize profits by choosing mortgage interest rates, i.e. they satisfy equation* (5).

4. *The equilibrium aggregate losses* λ*, number of active banks* N*, subsidy* η*, and* ζ *satisfy*

$$\lambda = \frac{P \sum_{i \in \{m,f,n\}} \int h_i'(.) z_i'(.) [\rho(z_i'(.); \bar{\eta}, \zeta) - \rho(z_i'(.); \eta, \zeta] d\mu_i}{Y} \tag{6}$$

$$N < \frac{1}{s^*} \text{ if } \lambda \geq \lambda^*$$

$$\eta = (1 - \mathbf{1}[\lambda \geq \lambda^*])\bar{\eta} + \mathbf{1}[\lambda \geq \lambda^*]\hat{\eta}$$

$$\zeta = \bar{\zeta} \text{ if } \lambda \geq \lambda^*$$

where $\mathbf{1}[.]$ *is an indicator function, taking value 1 if the condition in brackets is true and 0 otherwise.*

5. *Given policy functions, prices clear all markets:*

(a) *Labor market*

$$L = 1$$

(b) *Housing market*

$$\sum_{i \in \{m,f,n\}} \int h_i'(.) d\mu_i = 1 \tag{7}$$

(c) *Capital market*

$$K' = \sum_{i \in \{m,f,n\}} \left(\int a_i'(.) d\mu_i - P \int z_i'(.) h_i'(.) d\mu_i \right) \tag{8}$$

(d) *Goods market*

$$C + K' + DWL = Y + (1 - \delta) K$$

where aggregate dead-weight loss DWL equals

$$DWL = \gamma m P \sum_{i \in \{m,f,n\}} \int h_i'(.) \left(\int_{\underline{x}}^{z_i'(.)} x dG \right) d\mu_i$$

6. *The government runs a balanced budget and the tax rate τ satisfies*

$$\tau w L = \eta P \sum_{i \in \{m,f,n\}} \int h_i'(.) \left[\int_{\underline{x}}^{z_i'(.)} [z_i'(.) - (1 - \gamma m) x] dG(x) \right] d\mu_i$$

7. *The stationary distribution of households μ is invariant with respect to the transition function $Q_i(.)$ $i \in \{m, f, n\}$ induced by the policy functions.*

$$\mu_i(C) = \pi_i \sum_{j \in \{m,f,n\}} \int Q_j(s, C) d\mu_j(s) \text{ for each } C \in \Sigma$$

Some parts of the definition above require some clarification: First, our definition allows, for the sake of completeness, a "perpetual bailout" stationary equilibrium in which $\lambda \geq \lambda^*$ and $\eta = \hat{\eta}$. But our baseline calibration will realistically feature a stationary equilibrium in which $\lambda < \lambda^*$ and $\eta = \bar{\eta}$, which will be the case for $\bar{\zeta}$ sufficiently low. Second, our definition of aggregate losses as a percentage of GDP in section 2.3 is a calculation based on a no-bailout contingency, i.e. when $\eta = \bar{\eta}$. For an equilibrium with no bailout (the baseline case), $\eta = \bar{\eta}$ holds, and under rational expectations, all banks make zero profits by charging $\rho(z';\eta,\zeta) = \rho(z';\bar{\eta},\zeta)$ for a loan with LTV z'. Therefore $\lambda = 0$ holds, as is evident in expression (6). If there is a bailout (and banks anticipate so), they charge $\rho(z';\hat{\eta},\zeta)$, and they make a loss (if there were no bailout) of $Ph'z'[\rho(z';\bar{\eta},\zeta) - \rho(z';\hat{\eta},\zeta)]$ for an agent with (h', z'). However, when the expectations are fulfilled and a bailout occurs, these losses are completely covered by the government. Observe that all banks actually make zero profits/losses *ex post*, i.e. they are made whole after a bailout occurs in a crisis equilibrium. In this sense our definition of equilibrium losses reflects the "pre-bailout" situation in the economy.

2.6 Equilibrium under Quasi-Linear Preferences

In this section, to obtain a sharper characterization, we assume that the instantaneous utility function is quasi-linear in consumption, i.e. $u(c, h) = c + v(h)$, where $v(h)$ is strictly increasing and strictly concave, and satisfies $\lim_{h \downarrow 0} v'(h) = \infty$ and for some $h < \infty, v'(h) < 1$. To rule out the possibility that the non-negativity constraint on c is ever binding, we assume that $v'(y) < 1$. For our quantitative results we will consider both this case and a limited set of results with more standard (Cobb-Douglas) preferences, to argue that the quasi-linear specification provides tractability without significantly affecting the main results.

Under the assumption of quasi-linearity, the choice of LTV ratio and housing is *independent* of the wealth level. We make the following observations that follow from quasi-linearity:

- Agents are effectively risk-neutral, therefore only an interest rate r that satisfies $\beta(1 + r) = 1$ can be supported in equilibrium. If $\beta(1 + r) > 1$, no finite a can satisfy the Euler equation, and if $\beta(1 + r) < 1$, $a = 0$ must hold, both of which violate capital market clearing condition.

- Absent any subsidies or foreclosure costs (that is, $\eta = \gamma = 0$), when presented with the opportunity to borrow, agents are indifferent between choosing any LTV level $z \in [0, 1]$. When a baseline subsidy of $\eta \in (0, 1]$ for conforming loans $z \leq \zeta$ is introduced, agents strictly prefer borrowing up to the limit ζ. By a continuity argument, this is also true when foreclosure costs γ are positive but small.

- Since the moving and refinancing shocks are i.i.d. and preferences are quasi-linear, all movers demand the same amount of housing $\bar{h}$. In addition, since quality shocks are i.i.d. and $\mathbb{E}(x) = 1$, the expected value of the quality of housing always equals $\bar{h}$ between two consecutive moving shocks. Market clearing for housing implies $\bar{h} = 1$ must hold, i.e. in equilibrium, every mover demands unit housing.

- Again, thanks to the absence of selection, the home price index is independent of the distribution of households, and in principle, only depends on how much a representative mover is willing to pay for housing. This will be clarified further in the home price calculation below.

In the Appendix, we show that, under the additional assumption that foreclosure costs are positive but small (relative to the baseline subsidy), and imposing the equilibrium condition $\beta(1+r) = 1$, we have the following recursive expression that holds in equilibrium:

$$\tilde{V}(h,z) = -Ph(1-z) + v(h) + \beta Ph\mathbb{E}\Big(\max\{x-z,0\} - \rho(.)z\Big) + \beta\Bigg(m\tilde{V}(1,\zeta) \quad (9)$$
$$+ (1-m)f\mathbb{E}\tilde{V}(hx,\zeta) + (1-m)(1-f)\mathbb{E}\tilde{V}(hx,\min\{\zeta,(1-\theta)\min\{1,\frac{z}{x}\}\})\Bigg)$$

where $\tilde{V}(h,z)$ represents the value of holding housing stock h and a debt with LTV z *after* housing and LTV decisions are made.[22] To derive this expression, we use the property that value function (4) is quasi-linear in total resources I, a property inherited from the quasi-linearity of the instantaneous utility $u(c,h')$,

2.7 Computing the Home Price Index

Since movers solve the problem $\max_{h'\geq 0, z'\in[0,1]} \tilde{V}(h',z')$ and it is optimal for them to choose $h' = 1$ and $z' = \zeta$, home price index P must solve $\frac{\partial \tilde{V}(h',z';P)}{\partial h'}\Big|_{(h',z')=(1,\zeta)} \equiv$

[22]Note that this is in contrast to the value function in expression (4), where value $V_i(h,z)$ is defined prior to the decisions on housing.

$\tilde{V}_1(1, \zeta; P) = 0$. Differentiating the recursive expression above, we obtain

$$\tilde{V}_1(h,z) = -P(1-z) + v'(h) + \beta P \mathbb{E}\big(\max\{x-z,0\} - \rho(.)z\big) \\ + \beta\Bigg((1-m)f\mathbb{E}[\tilde{V}_1(hx,\zeta)x] + (1-m)(1-f)\mathbb{E}[\tilde{V}_1(hx, \min\{\zeta, (1-\theta)\min\{1, \frac{z}{x}\}\})x]\Bigg)$$

Multiply both sides by h and let $W(h,z) \equiv \tilde{V}_1(h,z)h$ to obtain

$$W(h,z) = -Ph(1-z) + v'(h)h + \beta Ph\mathbb{E}\big(\max\{x-z,0\} - \rho(.)z\big) \\ + \beta\Bigg((1-m)f\mathbb{E}W(hx,\zeta) + (1-m)(1-f)\mathbb{E}W(hx, \min\{\zeta, (1-\theta)\min\{1, \frac{z}{x}\}\})\Bigg) \tag{10}$$

Observe that $V_1(1,\zeta) = 0$ if and only if $W(1,\zeta) = 0$. This motivates our computational procedure to find the equilibrium price. We solve equation (10) using recursive methods for $W(h,z;P)$ and update P until $W(1,\zeta;P) = 0$ is satisfied.

3 Dynamic Analysis: Impact of Relaxing the Conforming Loan Limit

Having characterized stationary equilibrium, we now undertake a dynamic analysis in which the conforming loan limits are relaxed. Before turning to the details, we can more transparently illustrate the model's price mechanism in a highly simplified example.

3.1 Understanding the Price Effect: A Two-Period Example

There are two periods, $t = 0, 1$. There will be no dynamics, of course, and we take government policy at the initial period as given (rather than triggered by economic conditions), but the example will very cleanly show how expectations of a bailout inflates asset prices. In particular, despite its simplicity the example yields precisely the same pricing formula for housing as we will find below in the fully dynamic model.

To highlight the price mechanism, we make the following assumptions:

- We dispense with capital and production, and specify an endowment economy with a linear storage technology

- As in the full model, there are two assets:
  - "Housing" h that provides direct utility, can be used to secure mortgage debt, but has non-diversifiable idiosyncratic risk x
  - A fungible asset a that can be consumed as c or exchanged for h
- Preferences are also linear as well, so households are risk-neutral. Equilibrium prices are assumed to leave households indifferent to any non-negative c and h that satisfy their budget constraints.
- Households are endowed at $t = 0$ with some (possibly heterogeneous) quantity of the tradable asset a_0 and one unit of housing h_0.
- The yield on storing a is $r = 1/\beta - 1$; h has a terminal value $h/(1-\beta)$.[23]
- We set default costs $\gamma = 0$.

The timing is as follows: At $t = 0$ households choose a_1, implying an allocation between c_0 and c_1, they choose h_1 at price P_1, and financing zP_1h_1. At $t = 1$, h_1 is hit with the shock $x \geq 0$, which is stochastic with $\mathbb{E}(x) = 1$. Households with negative equity default, all others repay their loans, and all choose between c_1 and h_2. This implies a price per unit of housing in the second period $P_2 = \frac{1}{1-\beta}$.

We retain the essential features of the dynamic model: Households allocate their resources a_0 and P_1h_0 between the risk-free asset a_1, consumption c_0, and housing h_1. They can borrow at $t = 0$ using h_1 as collateral. Banks take deposits of a_1 and promise a guaranteed interest rate of r, and make secured loans to households at an interest rate that gives them an expected return of r. Households default if and only if the value of their house falls below the principle owed on their loan, in which case the bank takes the value of the house. Government subsidizes a fraction $\eta \geq 0$ of all losses due to default, sets an LTV limit of ζ, and taxes the consumers lump-sum ($T \geq 0$) to finance these subsidies.

The households' problem is

$$\max_{\{c_0, c_1, a_1, z, h_1, h_2\}} c_0 + h_1 + \beta\mathbb{E}\{c_1 + h_2/(1-\beta)\}$$

[23]Assuming this terminal value is a convenient normalization to make our results comparable to those of our benchmark infinite-horizon model. It represents the value of a perpetuity that provides one unit of consumption each period.

subject to

$$c_1, c_2, a_2, h_2 \geq 0, z \in [0,1]$$

$$c_0 + P_1 h_1 (1-z) + a_1 \leq a_0 + P_1 h_0$$

$$c_1 + P_2 h_2 \leq a_1(1+r) - \rho(z;\eta) z P_1 h_1 + P_1 h_1 \big(\max\{x \frac{P_2}{P_1} - z, 0\} \big) - T$$

given $a_0, h_0 = 1$. The zero-profit condition requires that interest rate ρ satisfy

$$\rho(z;\eta) z = rz + (1-\eta) \int_{\underline{x}}^{\frac{P_1}{P_2} z} \big(z - \frac{P_2}{P_1} x\big) dG(x)$$

The terminal value for h_2 implies that $P_2 = 1/(1-\beta)$. It is straightforward to show (from the first-order condition for h_1) that P_1 given η and z satisfies

$$P_1 = \frac{1}{1-\beta} \left(1 + \beta\eta \int_{\underline{x}}^{\frac{P_1}{P_2} z} \big(\frac{P_1}{P_2} z - x \big) dG(x) \right)$$

With no subsidy, i.e. $\eta = 0$, $P_1 = 1/(1-\beta)$, the same as P_2.

We can define $\hat{p} \equiv \frac{P_1}{1/(1-\beta)} = (1-\beta) P_1$ as the distortion in house prices at $t = 1$ due to the underpricing of risk. When $\eta > 0$, it is easy to show that agents will borrow as much as possible up to $z = \zeta$, and we can express $\hat{p}$ as follows:[24]

$$\hat{p} = 1 + \beta\eta \int_{\underline{x}}^{\zeta\hat{p}} [\zeta\hat{p} - x] dG(x)$$

Though $\hat{p}$ appears on both sides of the equation, this expression is easy to interpret: How much an agent is willing to pay for housing (P_1) is inflated above the fundamental $1/(1-\beta)$, because only a share $(1-\eta)$ of the losses due to default is factored into the mortgage interest payment. In other words, borrowers do not have to fully compensate the lender for the capital loss incurred in the states of the world in which they default (represented by the integral), because a share η of this is loss is covered by the government.

It should be noted that $\hat{p}$ is theoretically unbounded: as β, η, and ζ all approach

[24]We leave the verification of this claim to the readers, we prove $z = \zeta$ within the context of our baseline model in the appendix. It is easy to show that the derivative with respect to z is always positive, because as long as loans are subsidized, there is an incentive to borrow an extra dollar against housing and invest it in the risk-free asset.

one, the limiting value of $\hat{p}$ is the upper bound of the support of x.

This result can be easily extended to the case in which lenders incur a loss of a share γ of the loan principle:

$$\hat{p} = 1 + \beta\eta \int_{\underline{x}}^{\zeta\hat{p}} \left[\zeta\hat{p} - (\frac{1-\eta}{\eta}\gamma + 1)x\right] dG(x) \tag{11}$$

In this case agents will not borrow at all if $\eta = 0$, but will still borrow up to the limit ζ if γ is sufficiently small relative to η.

We shall see that the exact same pricing equation shows up in the infinite-horizon version of the model with quasi-linear preferences, and it holds "approximately" under more general, convex preferences (in our case a Cobb-Douglas aggregator and CRRA utility). In fact, we find that the solution $\hat{p}$ for the above equation is quantitatively indistinguishable from the ultimate price inflation we find in any of the specifications we used in our quantitative analysis. This suggests that our quantitative findings with respect to house prices are robust. Next, we turn to the details of the dynamic analysis of our infinite horizon benchmark model.

3.2 Infinite-Horizon Model

In our fully-dynamic model, we assume that the economy starts at its steady-state, and an unanticipated relaxation of the conforming loan limit ζ occurs in period zero. This can be thought of as GSEs purchasing higher-risk mortgages either directly or indirectly, perhaps due to lax oversight, or government policies aimed at expanding home ownership. We do not model why the limit is increased, just as we do not model why the government is tempted to bail out large financial institutions, but take both propensities as given. Although the increase in ζ is treated as an ex ante zero-probability exogenous event, it can easily be generalized to a probabilistic change in a Markov switching process. Similarly, we assume that with the policy change resulting in a crisis, the government reverts to the baseline policy with probability one, though this could be generalized as well.

In order to slow down the adoption of such mortgages, we assume that only movers are able to obtain them. Although we do not have explicit transactions costs, the idea is that since they are already obtaining financing for a new house, it is a natural point at which they could easily obtain a high-LTV mortgage. We further assume that once someone has obtained a high-LTV mortgage, they continue to be able to do so when refinancing. In other words, refinancers are able to get a new conforming mortgage at the same LTV as their original mortgage, or up to the current conforming limit, whichever is lower.[25]

Assumption 3 *Households become eligible for subsidized high-LTV loans when they move, and they remain eligible until there is a change in policy.*

The role this assumption plays is that even if the high conforming loan limit presents a systemic risk, a crisis cannot occur immediately unless moving probability m is very large. Essentially, the measure of agents who are "eligible" for loans with the new conforming loan limit builds up over time. A critical mass of these agents must be present for any bank to trigger a bailout.

A justification for this assumption and for the inertial frictions we introduced earlier is illustrated in figure 5. The first panel exhibits the well-known rapid increase in outstanding mortgage debt as a share of GDP over the boom period. Interestingly, as Justiniano, Primiceri, and Tambalotti (2015) first pointed out, mortgage debt as a share of market value of real estate in the second panel is very stable over the same period. It only jumps up significantly when house prices turn down beginning in 2006. This evidence suggests that (i) over the boom period, increases in debt do not reflect a significant increase in loan-to-value ratios; rather they are consequence of households borrowing more against rising home prices; and, (ii) the debt-to-housing value ratio only goes up just prior to the crisis as a consequence of the huge drop in home prices. We will show in the following sections that our model predictions are consistent with this evidence due to the frictions we introduce and the assumption we make above: The increase in home prices will be driven by movers, a small share of households who are eligible for high-LTV mortgages and who actively transact in the housing market, therefore average LTV ratio in the economy does not increase by much. Instead, it will jump up significantly over the crisis period due to the crash in home prices. In effect,

[25] The specifics of these frictions are for concreteness and simplicity. What is essential is only that the opportunity to obtain a high-LTV loan spreads slowly, whether from inertia, lack of awareness, or lack of availability.

our "movers" act as speculators who drive up house prices for everyone, but only until the inevitable crash.

Many features of the steady state also hold over the transition. For instance, $\beta(1+r)=1$ must hold period by period. So does the property that the home price index is determined by the movers and does not depend on the distribution of (h,z). This implies, among other things, if agents do not anticipate any further policy changes in the economy, home price moves to its new steady-state level *immediately* in period zero. For our purposes, the more interesting case is one in which the new conforming loan limit $\hat{\zeta}$ is high enough to trigger a bailout sometime in the future. In this more interesting case, because the bailout is presumed to be followed by an enforcement of a stricter conforming loan limit $\bar{\zeta}$ (i.e. agents anticipate a policy change), despite the fact that prices do not depend on distribution of households, home prices follow a non-trivial path, which must be solved for explicitly.

To characterize prices over the transition, we use the following recursive expression $W_t(h,z)$, which is derived from the steady-state version (10). Since only movers price housing, $W_t(h,z)$ represents the first-order necessary condition for an *eligible household*, i.e. a household who moved in some period $\tau \in \{0,1,\ldots,t\}$.

$$
\begin{aligned}
W_t(h,z) = {} & -P_t h(1-z) + v'(h)h + \beta P_t h \mathbb{E}\Big(\max\{\frac{P_{t+1}}{P_t}x - z, 0\} - \rho_t(.;\eta_{t+1})z\Big) \\
& + \beta\Bigg((1-m)f\mathbb{E}W_{t+1}(hx,\zeta_{t+1}) \\
& + (1-m)(1-f)\mathbb{E}W_{t+1}(hx,\min\{\zeta_{t+1},(1-\theta)\min\{1,\frac{zP_t}{xP_{t+1}}\}\})\Bigg)
\end{aligned}
\tag{12}
$$

where ζ_t denotes the time-specific LTV limit for the conforming loans. In our particular case, if there is a bailout anticipated in some period T (which is a variable whose value is determined as part of an equilibrium), $\zeta_t = \hat{\zeta}$ for $0 \leq t < T$ and $\zeta_t = \bar{\zeta}$ for $t \geq T$ must be satisfied where $\hat{\zeta} > \bar{\zeta}$ denotes the elevated LTV limit in place from the onset of the boom. Similarly, since the bailout occurs in period T, the effective subsidy that is factored into the mortgage interest rate $\rho_t(.)$ will depend on the baseline level of guarantees $\eta_{t+1} = \bar{\eta}$ for $t \neq T-1$, and $\rho_{T-1}(.)$ will depend on the elevated bailout guarantee $\eta_T = \hat{\eta} > \bar{\eta}$.

Just as we did for the steady-state case, to solve for the prices over transition, we use expression (12). Since prices do not depend on the distribution, the equilibrium price drops immediately to the steady-state price for $\zeta = \bar{\zeta}$ in the bailout period T. Denote this steady-state price as P^{ss}. Given P_{t+1}, price P_t must satisfy the necessary condition $W_t(1, \zeta_t) = 0$ since a mover in period t demands $h = 1$ and $z = \zeta_t$. Using the fact that $P_T = P^{ss}$, we can solve for the prices by backward induction.

How is the equilibrium T determined? The procedure of finding T involves repeating the calculation above for different values of T and checking whether there is a bank that has a profitable deviation over the given transition. To be more precise, we say that T is an equilibrium bailout period, if no single bank has an incentive to offer different mortgage rates over $t \in \{0, 1, 2..., T-2\}$. A potential profitable deviation would be one that involves offering lower mortgage rates, capturing the entire mortgage market, driving the aggregate losses above λ^* and enjoying the subsidy (a profit, for instance if the rate is $\varepsilon > 0$ above the zero-profit value). This is not a straightforward calculation because banks are not atomistic agents and they internalize the impact of their choice of mortgage interest rates on the home price index as well as on expectations. Hence, for each candidate $t \in \{0, 1, 2..., T-2\}$, we need to calculate a sequence of off-equilibrium prices resulting from the deviation in which *maximal* potential losses (which we call λ^o_{t+1}) are incurred in $t+1$, and check if λ^o_{t+1} in fact does not exceed λ^* in period $t+1$ with the given sequence of off-equilibrium prices. Note that if $\lambda^o_{t+1} > \lambda^*$ for some t, then the crisis would have in fact occurred in period $t+1 < T$, so we can dismiss T as part of a bailout equilibrium.

The assumption of quasi-linear preferences simplifies this calculation significantly. Here is a summary of the computational procedure: Fix $T > 0$

1. Let $(P_0, P_1, ..., P_T)$ denote the sequence of equilibrium prices with the expectation of a bailout in period T. Calculate them using (12) as explained above in detail. Then, for each $t \in \{0, 1, 2..., T-2\}$,

2. Consider the case in which a bank unilaterally offers mortgages at rate $\rho_t(.; \hat{\eta})$, the competitive rate that would prevail if there is a bailout next period, while all other banks are driven out of the competition. This choice *maximizes* aggregate losses, rationalizing the intent of the deviation, while

still retaining the possibility of breaking even *ex post* if the government were to bail it out.

3. This deviation will naturally lead to an off-equilibrium price sequence, which we denote $\mathbf{P}_t^o \equiv (P_t^o, P_{t+1}^o...)$. If there is a bailout next period due to this deviation, $P_{t+j}^o = P^{ss}$ for all $j \geq 1$ thanks to quasi-linearity, and due to our assumption that tight limits are reinstated forever after a bailout. Hence, this deviation would have a non-trivial impact only on P_t^o, and this price can be computed in a straightforward manner using expression (12). In fact, under quasi-linearity, it can be shown that $P_t^o = P_{T-1}$, the equilibrium home price index right before the presumed crisis date.[26]

4. Given $P_t = P_t^o$ and $P_{t+1} = P^{ss}$ and the distribution μ_t, aggregate losses/GDP ratio for the following period, λ_{t+1}^o can be calculated in a straightforward manner using its definition.

5. Check if $\lambda_{t+1}^o < \lambda^*$. If this condition holds, the bank would not find it profitable to deviate in period t, because the highest aggregate losses a deviation can achieve does not exceed the threshold to trigger a bailout.

If the final condition is satisfied for all t over the transition, we verify that T is an equilibrium bailout period.

3.3 Calibration

To calibrate the friction parameters m and f, we rely on statistics from before the boom. Deng, Quigley, and Van Order (2000) find that approximately 50 percent of mortgages are repaid (either because of moving or refinancing) within 10 years. Venti and Wise (1989) find that approximately four percent of homeowners move each year. These facts suggest values of $m = 0.04$ and $f = 0.033$. Of course in reality these hazards, especially the prepayment rates, are not constant or independent of duration, but for our purposes, the assumption of constant hazard rates is tractable and seems relatively innocuous. For robustness we also computed the

[26]This algorithm is difficult to implement under more general preferences, because the home price index is a non-trivial function of distribution of households. This greatly complicates the computation of off-equilibrium price sequences.

solution to the model for the case $m = 1$, i.e. with no moving friction. While of course there were difference in some dimensions, the price response was very similar (and under quasi-linearity, identical) to what we find with $m = 0.04$.

We set θ, the rate at which non-refinanced mortgages must be paid down each year, at 0.033, to reflect the typical repayment of principal for a 30-year mortgage. Of course this rate is not constant for a self-amortizing mortgage, but again the assumption of a constant rate is made for tractability's sake.

For its flexibility, we use a Kumaraswamy distribution for the idiosyncratic shock x. This distribution has 4 parameters: The lower bound $\underline{x}$, upper bound $\bar{x}$, and two shape parameters $a, b > 0$), making it effectively as flexible as the Beta distribution, but with the advantage of having a closed-form density and c.d.f. In its standard form the c.d.f. $\hat{G}$ and density $\hat{g}$ are

$$\begin{aligned} \hat{G}(x) &= 1 - (1 - x^a)^b \\ \hat{g}(x) &= abx^{a-1}(1 - x^a)^{b-1} \end{aligned}$$

for $x \in [0, 1]$. For our purposes we will consider the generalized distribution with a change of variables so that the support of the distribution is $[\underline{x}, \bar{x}]$, where $0 \leq \underline{x} < \bar{x} < \infty$.[27]

For the baseline calibration, we fix $\bar{x} = 1.4$ and choose the shape parameters $(a, b, \underline{x})$ jointly to target $\mathbb{E}(x) = 1$, the standard deviation σ_x and the equilibrium annual default probability. The literature provides conflicting annual volatility estimates based on different data sets. OFHEO reported annualized volatility estimates quarterly for each state separately between 1996-2000. These estimates ranged from 0.08 to 0.12. We think that these estimates should be taken as a conservative lower bound since aggregate volatility should be higher than regional volatilities. Flavin and Yamashita (2002) estimate an annual volatility of around 0.15 based on data at the national level, and based on the lack of correlation with

[27]This makes the distribution and density functions

$$\begin{aligned} G(x) &= 1 - \left(1 - \left(\frac{x - \underline{x}}{\Delta}\right)^a\right)^b \\ g(x) &= \frac{ab}{\Delta}\left(\frac{x - \underline{x}}{\Delta}\right)^{a-1}\left(1 - \left(\frac{x - \underline{x}}{\Delta}\right)^a\right)^{b-1} \end{aligned}$$

where $\Delta \equiv \bar{x} - \underline{x}$.

returns on T-Bills, Stocks, and Bonds, argue that this volatility is almost entirely associated with idiosyncratic risk. This value is also consistent with the estimates reported by Case and Shiller (1989).[28] Based on this evidence, we target an annual volatility of $\sigma_x = 0.15$. While our calibration cannot of course pin down all of the parameters of the distribution, we choose the parameters values $a = 1.329$, $b = 2.232$, $\underline{x} = 0.743$ jointly to match $\mathbb{E}(x) = 1$, $\sigma_x = 0.15$ and a baseline steady state default rate of $d = 0.02$. The latter is based on data from the Mortgage Bankers Association (MBA), which reports quarterly FHA foreclosure starts as a percentage of outstanding insured loans. This rate was fairly stable around 2% between 1990 and 2000.[29] Jeske, Krueger, and Mitman (2013) also use 2 percent as their baseline default rate.

As mentioned earlier, researchers have found that the foreclosure "discount" is about 22 percent. In our model much of this would be explained by selection, meaning that foreclosed houses are those that have had adverse x shocks. We instead base our choice of γ on studies of the direct costs of foreclosures, excluding those that amount to pure transfers such as the inability of the lender to collect mortgage payments during the process. For example, Cutts and Merrill (2008) document costs that suggest these dead-weight losses in the vicinity of 3 to 5 percent of the home's value, so we set $\gamma = 0.03$. The model's predictions are not sensitive to this choice.

For preferences, in the quasi-linear case with $u = c + v(h)$ we assume $v(h) = \nu\frac{h^{1-\mu}}{1-\mu}$ with $\mu = 2$ (though the results are not at all sensitive to this parameter). With Cobb-Douglas utility we have $u(c, h) = c^{1-\psi}h^{\psi}$. Expenditure share parameters $\nu = 0.18$ and $\psi = 0.18$ are chosen to match average expenditure shares on housing from NIPA tables between 1959 and 2015, which is approximately 18% in the post-war era.[30]

The choice of a value for λ^* presents a challenge, as the concept of "potential losses" makes it intrinsically difficult to measure, given that actions by the govern-

[28]Quoting Case and Shiller (1989), "Individual housing prices are like many individual corporate stock prices in the large standard deviation of annual percentage change, close to 15 percent a year for individual housing prices."

[29]For details, see the report by Pinto (2011) who compiled these data from MBA sources. Fannie Mae and Freddie Mac reported somewhat lower delinquency rates prior to 2005.

[30]The value of ν equals 0.18 just by coincidence, we set its value so that in equilibrium, housing expenditure share $P^{ss}r/(P^{ss}r+C^{ss})$ equals its counterpart from data. Note that for the transacting agents $h = 1$ and the implicit rents equal $P^{ss}r$.

ment taken to avoid such losses imply that they may never occur. Moreover, given the contagious nature of financial crises, losses may multiply into different asset classes. Also, however it is measured, it must be an upper bound, since we only know what has been observed to trigger a bailout; the actual trigger point could be lower. In any case, there have been numerous attempts to quantify this "impulse" that hit the financial system. As a starting point we rely on the discussion in the report by Financial Crisis Inquiry Commission (2011) that suggests numbers in the vicinity of \$1 trillion to \$1.5 trillion (p. 228): \$80 billion from the GSE's mortgage-backed securities, "as much as \$170 billion" in other mortgage assets held by banks and the GSEs, \$500 billion from (presumably private-label) mortgage-backed securities and collateralized debt obligations (CDOs), and "another \$655 billion in write-downs on commercial mortgage-backed securities, CLOs, leveraged loans, and other loans and securities...." Accordingly we set $\lambda^* = 0.08$ as a share of GDP.

Finally, there is the choice of $\bar{\eta}$, the baseline subsidy, and $\hat{\eta}$, the bailout parameter. A rough upper bound on this baseline η can be seen from the difference between mortgage rates on conforming and non-conforming loans, the latter being ineligible for purchase and securitization by the GSEs. Passmore, Sherlund, and Burgess (2005) find a differential of seven basis points between conforming and non-conforming mortgages, after controlling for other risk factors, and 4.5 basis points for loans not exceeding 80 percent LTV. In our model, this implies a baseline $\bar{\eta}$ of 0.15. Table 1 summarizes the calibration. This turns out to be large enough to offset the disincentive to borrow due to foreclosure costs, so that agents choose to borrow up to the limit.

As for $\hat{\eta}$, the earlier discussion of bailouts for highly levered financial institutions suggests a number very close to one. With this in mind we set $\hat{\eta} = 0.99$, which would suffice to bail out debt-holders and not completely wipe out equity holders.

3.4 Results with No Frictions ($\mathbf{m = 1}$)

To understand the magnitude of the price effects, it is illuminating to look at the case where everyone moves every period. It turns out that in this special case, the steady-state price as well as price inflation can be expressed in a relatively simple form.

Assume further that before the increase in the conforming loan limit, $\zeta = \bar{\zeta}$. Using expression (10), $W(1, \bar{\zeta}) = 0$, and letting $m = 1$, we obtain the following expression for the steady-state price

$$P^{ss}(\bar{\zeta}) = \frac{v'(1)}{(1 - \bar{\zeta}) - \beta \mathbb{E}\big(\max\{x - \bar{\zeta}, 0\} - \rho(\bar{\zeta}; \bar{\eta})\bar{\zeta}\big)} \tag{13}$$

It is easy to verify that in the limiting case of $\bar{\eta} = \gamma = 0$ this price simply equals $v'(1)/(1 - \beta)$.

Suppose as part of an equilibrium, a bailout occurs in period T after the LTV limits relax to some $\bar{\zeta}$ in period zero. Next we ask the following question: How much does the price go up right before the bailout? Since price reverts back immediately to $P^{ss}(\bar{\zeta})$ in period T, we can use expression (12) to compute P_{T-1}. More specifically we have

$$P_{T-1} = \frac{v'(1)}{(1 - \hat{\zeta}) - \beta \mathbb{E}\big(\max\{\frac{P^{ss}}{P_{T-1}} x - \hat{\zeta}, 0\} - \rho(\frac{P_{T-1}}{P^{ss}}\hat{\zeta}; \hat{\eta})\hat{\zeta}\big)}$$

This expression reflects the fact that a bailout occurs in period T where the effective subsidy equals η^*. Let $\hat{p} = \frac{P_{T-1}}{P^{ss}}$ represent the price "inflation" due to bailout. Dividing the two expressions above yields an implicit expression in $\hat{p}$.

$$\hat{p} = \frac{(1 - \bar{\zeta}) - \beta \mathbb{E}\big(\max\{x - \bar{\zeta}, 0\} - \rho(\bar{\zeta}; \hat{\eta})\bar{\zeta}\big)}{(1 - \hat{\zeta}) - \beta \mathbb{E}\big(\max\{\frac{1}{\hat{p}} x - \hat{\zeta}, 0\} - \rho(\hat{p}\hat{\zeta}; \hat{\eta})\hat{\zeta}\big)}$$

This expression becomes even simpler if we assume either $\bar{\eta} = 0$, or that $\bar{\zeta} \leq \underline{x}$. Either of these assumptions results in a baseline with a zero default rate. This is a useful benchmark because our baseline calibration features a very low default rate (in line with the data from pre-boom era), and consequently results in a quantitatively very similar steady state. Under these assumptions, and imposing the zero-profit condition (5) for $\rho(.)$, we can show that $P^{ss}(\underline{x}) = \frac{v'(1)}{1-\beta}$, and house price inflation in the bailout state with $\eta = \hat{\eta}$ satisfies

$$\hat{p} = 1 + \beta \eta \int_{\underline{x}}^{\hat{\zeta}\hat{p}} \big[\hat{\zeta}\hat{p} - \big(\frac{1 - \hat{\eta}}{\hat{\eta}}\gamma + 1\big)x\big] dG$$

Note that this pricing equation is *identical* to the pricing equation (11) we found in our two period example. Clearly, in equilibrium, the default rate (and foreclosure rate) in the crisis equals $d = G(\hat{\zeta}\hat{p})$. Note that for this to be an equilibrium, equilibrium losses λ must exceed the threshold value λ^* that warrants a bailout—otherwise η would remain at the baseline $\bar{\eta}$. It can be shown that

$$\lambda = \frac{v'(1)}{1-\beta} \frac{(\hat{\eta} - \bar{\eta}) \int_{\underline{x}}^{\hat{\zeta}\hat{p}} \left[\hat{\zeta}\hat{p} - (1-\gamma)x\right] dG}{Y}$$

Observe that when $m = 1$, either a bailout occurs in period $T = 1$, one period after the LTV limit is relaxed, or it never does, depending on the level of $\hat{\zeta}$. The reason is that the price effect characterized above is independent of time. If, the time-independent condition $\lambda \geq \lambda^*$ is satisfied, firms move in period 0, offer loans at a highly subsidized rate (reflecting the expectation of the bailout), drive the aggregate losses above λ^* (since every consumer can move) and trigger a bailout, consistent with the initial expectations. On the other hand if $\lambda < \lambda^*$, no bank (or group of banks) can gain by offering highly subsidized ($\eta = \hat{\eta}$) mortgage interest rates because they cannot drive the default rate above the threshold rule. In this less interesting case, the steady-state price moves up permanently in period 0 to the level $P^{ss}(\hat{\zeta})$ and stays there forever.

With no frictions it is feasible also to consider more realistic preferences. Figure 6 depicts the steady state distributions of housing and LTV in the case with Cobb-Douglas preferences. In this case the convexity of preferences gives rise to a wide distribution of both variables in the population. This is because the idiosyncratic shocks to housing value act as permanent wealth shocks, so that each household's c and h respond to its history of x shocks. (By contrast, in the quasi-linear case, h is essentially independent of the shocks and only varies because of frictions.)

Figure 7 depicts the response of the housing price P, the default rate, and LTV to a relaxation of the LTV limit from 0.8 to 0.99. Because there are no inertial frictions, everything happens at once: P jumps by about 17 percent, average LTV jumps from about 0.4 up to 0.99, and the default rate jumps from the low baseline of about 2 percent up to over 80 percent.

Before adding frictions, we can compare no-frictions results with Cobb-Douglas and quasi-linear preferences. The point of this is to show that the magnitude of the price response is similar in both cases, so that when we add frictions and focus only on the quasi-linear case for tractability, we have some confidence that quasi-linearity is not playing a crucial role. Figure 8 compares the price response in the two cases. We see that the response is similar in magnitude, but in the Cobb-Douglas case the price response exhibits greater volatility, rising higher than under quasi-linear preferences, and then falling below the steady state level during the crisis before recovering. Thus if anything our reliance on quasi-linear preferences in the next section may understate the price impact.

3.5 Results with Frictions

We now add inertial frictions to the model, so that our calibrated quantitative analysis will yield more realistic dynamics. They are associated with moving or refinancing, and have the effect of prolonging the boom over many periods, and thus postponing the crisis/bailout after conforming loan limits are relaxed.

The upper panel of Figure 9 depicts the steady state distribution of housing in the model with quasi-linear preferences and inertial frictions. (Note that the model does not determine a particular distribution of c and a.) The spike at $h = 1$ represents the choices of movers, while the rest of the distribution results from the fact that at any point in time $1 - m$ of households do not move, and their effective housing evolves over time from the x shocks. The lower panel displays the distribution of LTV. Again the spike at 0.8 reflects the common choice of both movers and refinancers to borrow up to the conforming loan limit because of the modest subsidy. The rest of the distribution is the consequence of the friction that only $1 - m - f$ of the population can reset their borrowing.

Figure 10 illustrates the dynamic response of the economy for the benchmark calibration. It is worth repeating that in this model the LTV ratio proxies for all risk characteristics, so an increase in the conforming limit is a metaphor for any kind of relaxation of lending standards. We see that the frictions yield a sustained boom period because of the slow adjustment of borrowing behavior. The lower right panel shows that overall average LTV rises only modestly, from approximately 0.55 up to 0.58. This is consistent with the evidence in figure 5 described earlier: While there were many new mortgages that were very risky (whether in terms of LTV, FICO scores, or other characteristics), the aggregate ratio did not change dramatically, as increased borrowing was largely accompanied for most of this period by price appreciation. Upper left panels depict the potential losses/GDP explained in our dynamic analysis section, i.e. the maximum losses that a bank would have incurred had it taken over the whole market by offering risk-free mortgage rates. This variable goes up over time as more and more household become eligible for high LTV loans, and ultimately transcends the critical loss threshold $\lambda^* = 0.08$, at which point a crisis/bailout occurs.

Figures 11, and 12 depict alternative equilibria with $\lambda^* = 0.07$ and $\lambda^* = 0.09$ respectively. Clearly, the price impact of the relaxation of the LTV limit is the same for all three cases, and the only difference is the timing of the crisis in equilibrium. In the benchmark calibration, a crash occurs in period $T^* = 12$, and for the other two cases, it occurs in periods $T^* = 7$ and $T^* = 21$ respectively.

Figures 13, 14, 15, and 16 depict the dynamic response of various economic variables of interest to a relaxation of borrowing standards (an increase in the conforming limit from 0.8 to 1) at $t = 0$ under the baseline parameters and various alternatives: Lower $\bar{\eta}$ (0.05 instead of 0.15); lower $\hat{\eta}$ (0.95 versus 0.99); higher θ (0.05 versus 0.033); and lower σ (0.12 versus 0.15). Obviously, given the benchmark threshold value of λ^*, these parameter changes also affect the equilibrium T^*. To facilitate an accurate comparison of variables with the benchmark economy, for all these cases, we adjust λ^* so that a crisis occurs in period $T^* = 12$ like in the benchmark case. It should be noted that in the absence of any inertial frictions (that is, $m = f = 1$), the impact of this relaxation of credit standards would be an immediate jump in the house price of exactly the peak jump (about 18 percent) in the case with frictions, which would then immediately precipitate a crisis.

Of course the primary interest is in the price effect, shown in the upper left panel. The initial impact on price is modest, as the crisis is years away, and only a small fraction of borrowers takes advantage of the relaxed credit environment. They pay more for housing primarily because of the larger effective subsidy implied by higher LTV loans, even if $\bar{\eta}$ is unchanged for the time being. As this process continues, however, the price increases accelerate and the accumulating leverage and default risk drive the economy towards a crisis in which aggregate losses pass the threshold that induces a bailout.

The ultimate price effect of approximately 18 percent is very robust to a variety of parameter assumptions. For example, the price jumps by approximately the same 18 percent in the frictionless case ($m = 1$), the only difference being that the jump occurs immediately upon the change in lending standards, and the crisis occurs one period later.[31]

While the welfare impact is not our primary focus, in the model the cost of the crisis is limited to the increase in foreclosure costs. While these can be substantial, the assumption of a fixed housing stock limits other channels of impact. As mentioned above, we also do not model any benefits from increased home ownership or increased liquidity from relaxation of credit constraints. Of course such "benefits" from the crisis would be transitory.

We have argued that the crisis outcome requires both a relaxed LTV limit and the implicit inability to let large financial institutions fail. It is clear from our earlier analysis that tight credit limits keep losses from exceeding the threshold that trigger bailouts. We also need to show that relaxed credit limits would not, in the absence of bailout expectations, result in inflated house prices. In terms of our policy parameters, this is equivalent to assuming that $\lambda^* = 1$—that is, the threshold for losses that trigger bailouts is so high that they never occur.

We carry out this exercise both in the quasi-linear model and the Cobb-Douglas specification. In the quasi-linear case, relaxing the LTV limit leads to a one-time permanent jump in in home price index of only about 4.5%. In magnitude this is equal to the initial jump in home prices in period-0 for the benchmark calibration under the bailout equilibrium in figure 10, and is much smaller than the overall price increase over the boom. The initial jump in the benchmark dynamics is almost entirely driven by the movers who borrow up to the now-higher borrowing limit and implicitly enjoy the baseline subsidy $\bar{\eta}$. As explained earlier, the bailout (and therefore the bailout subsidy) is too far ahead in the future to have much impact on the movers' pricing of the asset in period 0 due to discounting. In our counterfactual exercise, the price goes up precisely due to the same effect. The price increase is permanent in the counterfactual, because there is no expectation of a bailout and a policy change, and it is constant due to quasi-linearity. Financial institutions suffer no losses, i.e. $\lambda_t = 0$ for all periods t, because mortgages are priced based on the actual default risk involved, net of the small benchmark sub-

[31] In our discussion we speak of prices, but the results apply to price/rent ratios as well, since in the model rents are essentially constant–precisely constant in the quasi-linear case, and changing very little with Cobb-Douglas preferences.

sidy. The burden to the taxpayers of the baseline subsidy is less than 1% of labor income in the new steady-state distribution with higher LTV limit, even though all movers and re-financers borrow up to the high LTV limit.

In the Cobb-Douglas specification, the price impact of increasing the LTV limit with no bailout prospects is actually *negative*, though the difference is too small to be of any economic significance (less than 0.1%). The reason prices are not bid up in the Cobb-Douglas specification in this counterfactual is the convexity of preferences. Because the baseline subsidy is small, high-LTV mortgages are only attractive to wealth-poor agents. These agents do not make much of an impact on overall demand. As a consequence, the overall price effect is negative because of now-higher dead-weight losses associated with the higher incidence of default. Since these losses are paid in terms of consumption goods, the price of consumption is bid up relative to housing, and this leads to a lower housing price. The impact of the higher LTV limit on the tax burden is even smaller than in the quasi-linear case.

This confirms the model's implication that if the government could have pre-committed not to bail out large financial institutions, the price impact of introducing high-LTV loans would have been small and possibly negative. The banks would have mostly (except for the modest baseline subsidy) internalized the higher default costs and made no losses in expectation, and the tax burden resulting from the subsidy would have been small, despite an increase in the incidence of defaults.

Finally, while we do not explicitly model a rental market, we can say something about the implicit rental price for homes. It is apparent that with quasi-linear preferences, the rental price will be constant at $v'(1)$. Any variations in P are entirely due to changes in expectations regarding the net subsidy η given foreclosure costs, which gets capitalized into the asset price. Consequently our findings regarding house prices also characterize the behavior of price/rent ratios.[32]

[32]With more general preferences the implicit rental price is not so clearly defined as in the quasi-linear case, because of heterogeneity in the marginal rate of substitution arising from incomplete markets. Even so, it should be clear that the distortions we describe below affect only house prices and not rents by any reasonable definition. The implicit subsidy of Too Big to Fail is clearly one related to ownership, not renting.

4 Conclusion

It is widely believed that a relaxation of lending standards, through a rapid expansion of the subprime market and availability of high-LTV loans, was the dominant force that paved the way to the financial crisis of 2008. Many observers also cite "Too Big to Fail" (i.e. the government's unwillingness to allow large financial institutions to fail or incur enormous losses) as a factor in those institutions' increasing leverage—both their own and those of their clients. The main contribution of this paper is to directly link these phenomena both to each other and to a quantitatively large endogenous boom and bust in house prices and price/rent ratios. That is, credit limits in our model are not arbitrary frictions, but a welfare-improving response to the inability of government to allow large financial institutions to fail, and their relaxation has major adverse consequences. This contrasts with many models in the literature in which credit limits are imposed or removed arbitrarily and are actually welfare-reducing when in place.

At a normative level, our counterfactual exercise suggests that it is the combination of the government's unwillingness to let large financial institutions fail together with lax regulation (or underestimation) of portfolio risk that produce the potential for a crisis. Either of these alone results in modest damage. Absent expectation of bailouts, lax regulation of mortgage risk results in higher default rates, but because much of the cost is internalized, there is negligible spillover into asset prices. At the same time, our baseline case shows that adequate regulation of risk-taking prevents the expectation of bailouts from distorting asset prices.

Taking the model more literally, an alternative to the direct supervision of risk would be a market share limitation on financial institutions, including GSE's such Fannie Mae and Freddie Mac. The high-default "bad equilibrium" involves firms becoming large by undermining credit standards, thereby generating a sort of "race to the bottom" in credit quality. Size limitations could lead those institutions to plausibly expect a response to aggregate adverse outcomes more like that seen during the Savings & Loan crisis, when hundreds of small institutions were allowed to fail, thereby reducing the risk-taking that would lead to such an outcome. In addition, recent work by Neuberg et al. (2016) suggests that it may be possible to reduce market expectations of bailouts by offering instead the prospect of "bail-ins," in which governments intervene but impose losses on creditors.

In short, if the government cannot help but protect too-big-to-fail lenders in the event of a crisis, it is essential that this policy be coupled either with strict controls on leverage and other sources of credit risk, or with a mechanism that induces proper pricing of risk, at least for those large firms. With such mechanisms in place, the equilibrium that results in a crisis and subsequent bailout is eliminated. Alternatively, a government that is unable or unwilling to restrict high-risk activities by large institutions, should either find a way to bind itself not to bail out failing institutions, or, alternatively, limit the size of institutions in terms of market share.

Our main technical contribution is to illustrate that a model with homogeneous and rational beliefs can generate asset price movements that appear to deviate substantially from fundamentals over a number of periods. These deviations are not "bubbles" in the standard sense of that term. House prices are distorted by an implicit subsidy that we presume to be unsustainable, and hence responsible for a boom-bust cycle. These findings provide an alternative (not necessarily mutually exclusive) to the view that beliefs were irrational or otherwise non-standard and heterogeneous. Although some aspects of the model may appear too simplified to warrant quantitative conclusions, we argue that those conclusions are robust, depending to a great extent only on the cross-sectional distribution of house price changes, and very little on the details of preferences or default behavior. Trading frictions were shown to affect the timing but not the magnitude of our results on house prices and defaults.

We also find that a boom in house prices can be detrimental to welfare. Since housing is used as collateral for borrowing, an increase in its price would effectively act to loosen credit constraints in the economy. However, we show that when these price increases are driven by the policy and market failures depicted in our model, the distorting effects are potentially large and costly. This prediction contrasts with some of the literature that suggests, either directly or indirectly, that there could be "welfare-improving booms" in the real-estate market.

While our analysis captures many characteristics of the mortgage and housing markets which we believe played an important role in the crisis, we have abstracted from some potentially important aspects of the market. We have demonstrated the robustness of our key findings, but there are several extensions that would add realism to the model and enable it to replicate a broader set of facts.

First, aggregate shocks (aside from the policy shock) would yield a more realistic boom and bust, insofar as a persistent favorable shock that results in aggregate growth might help prolong a boom and create realistic uncertainty about the timing of a collapse. The bust and crisis could be triggered by an adverse aggregate shock, thus having uncertain timing, in contrast to the perfect foresight in our model. Uncertainty with regard to government behavior, such as the threshold for bailouts, would have a similar impact. Second, we do not allow for home construction. A persistent boom, such as we find with inertial frictions, could stimulate new home construction, which might temper the rise in house prices. On the other hand, the resulting overhang at the time of the crash would result in a bigger decline in prices, and likely increase the welfare costs of the policy change.[33] Finally, we think it is feasible add demographics (for example with an overlapping generations framework), and to add an explicit rental market, so agents can choose between owning and renting. Extensions along these lines will make the model more realistic without significantly altering the main results.

[33]We estimate that the U.S. housing stock was only about 1.5 percent above trend at the peak of the boom in 2007. This contrasts with Spain, for example, where according to Bardhan, Edelstein, and Kroll (2012), between 2000-2009, 5 million new homes were built, relative to a stock of 20 million, during the same time period.

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Table 1: Parameter Values-Baseline Calibration

Variable	Baseline Value	Value in Boom/Target
	Policy Variables	
Loan-to-Value Limit (ζ)	0.800	1.000
Subsidy/Bailout Rate (η)	0.150	0.990
Threshold Agg. Losses/GDP (λ^*)	0.080	
	Fixed Parameters	
Discount Rate (β)	0.960	$r = 0.04$
Default Cost (γ)	0.030	Foreclosure costs 3%
Mortgage Paydown Rate (θ)	0.033	Average on 30-year mortgage
Shock Distribution Parameter (a)	1.329	2% default rate
Shock Distribution Parameter (b)	2.232	$\mathbb{E}(x) = 1$
Shock Distribution Parameter ($\underline{x}$)	0.743	$\sigma_x = 0.15$
Shock Distribution Parameter ($\bar{x}$)	1.400	
Moving Hazard (m)	0.040	4% annual homeowner moving rate
Refinancing Hazard (f)	0.033	50% ten-year prepayment rate
Housing Share for Quasi-Linear (ν)	0.181	Average expenditure share (NIPA)
Housing Share for Cobb-Douglas (ψ)	0.180	Average expenditure share (NIPA)

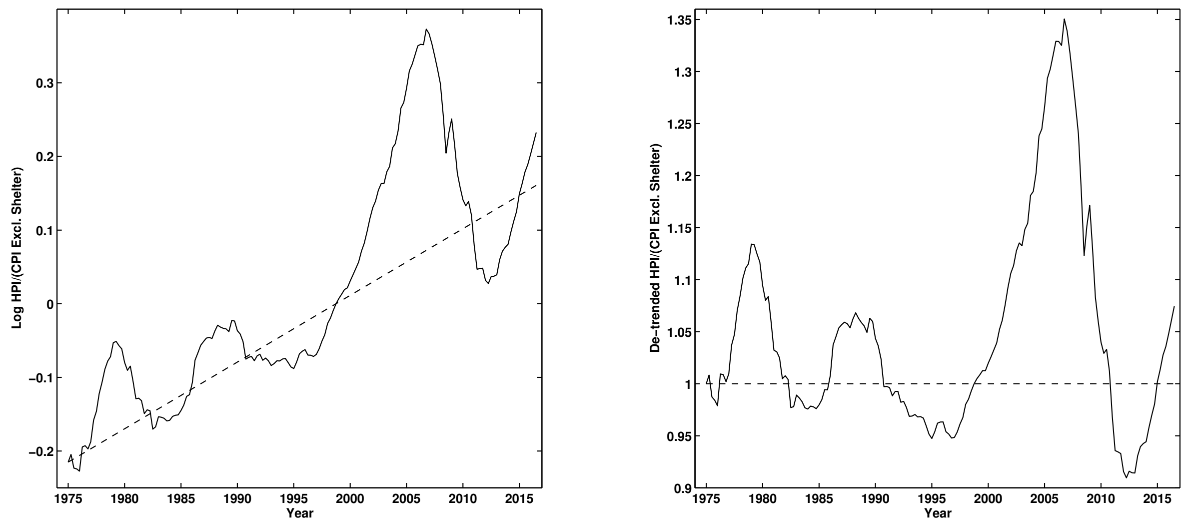

Figure 1: Real House Price Index

Note.– Ratio of FHFA House Price Index (All Transactions) to the CPI (All Urban): All less Shelter. Left Panel-Log Scale (1998Q4=0). Right panel-Levels relative to trend (1998Q4=1).

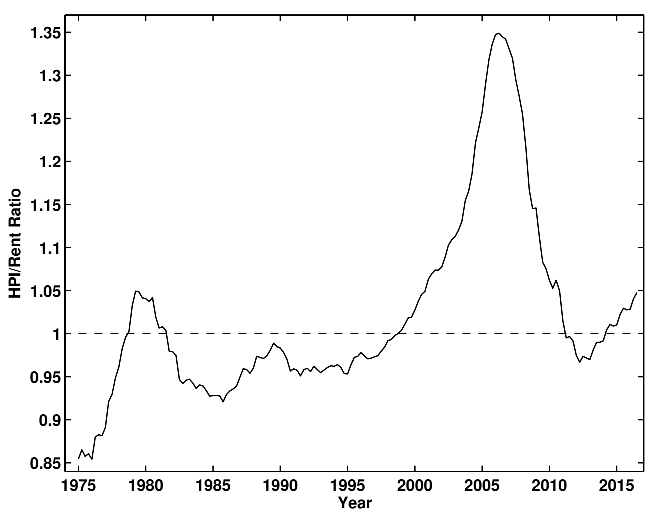

Figure 2: House Price/Rent Index.

Note.– Ratio of FHFA House Price Index (All Transactions) to the CPI (All Urban): Rent of Primary Residence. (1998Q4=1)

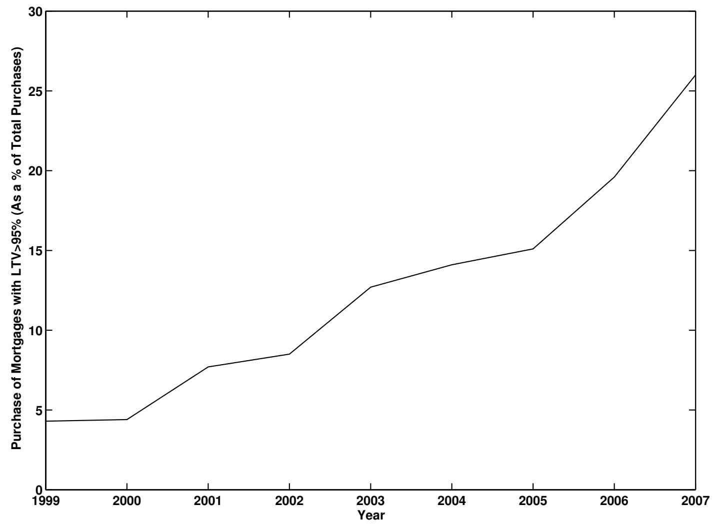

Figure 3: Fannie Mae High-LTV Mortgage Purchases

Source: Department of Housing and Urban Development

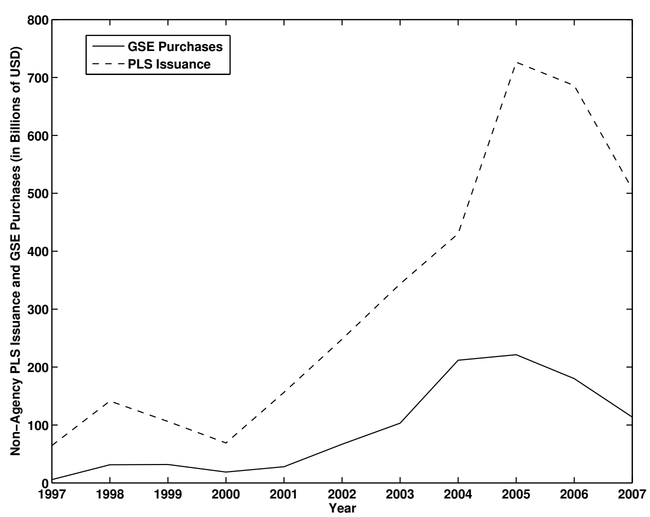

Figure 4: GSE Investments in Private Label Securitizations

Source: OFHEO, Report to Congress, 2008

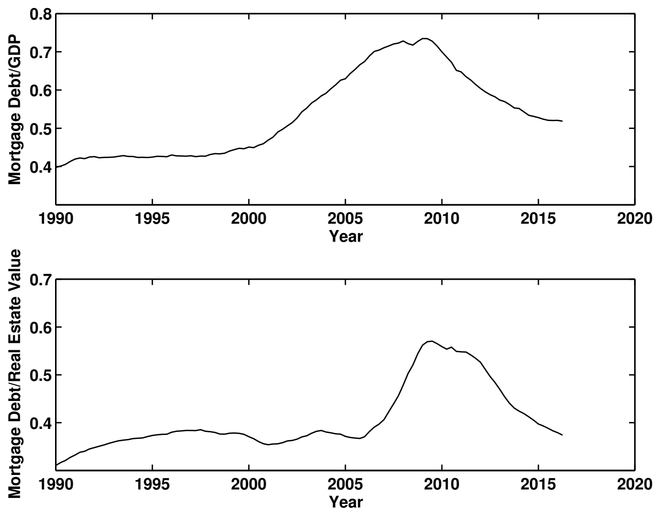

Figure 5: Comparison of Outstanding Mortgage Debt-to-GDP ratio and Outstanding Mortgage Debt-to-Real Estate Value

Source: Federal Reserve Economic Database (FRED) and Flow of Funds

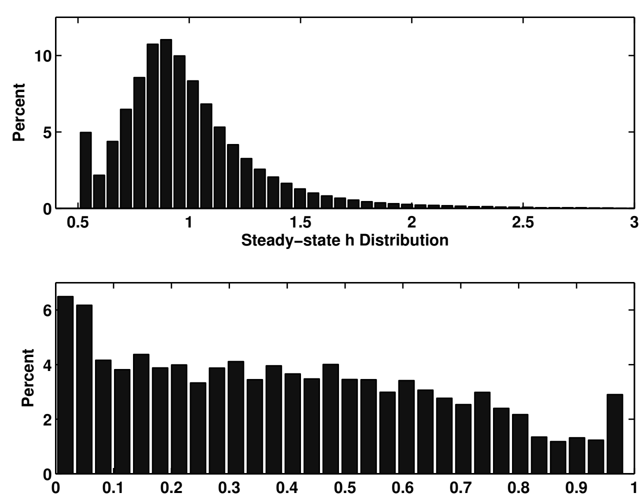

Figure 6: Stationary Distribution: Frictionless with Cobb-Douglas Utility

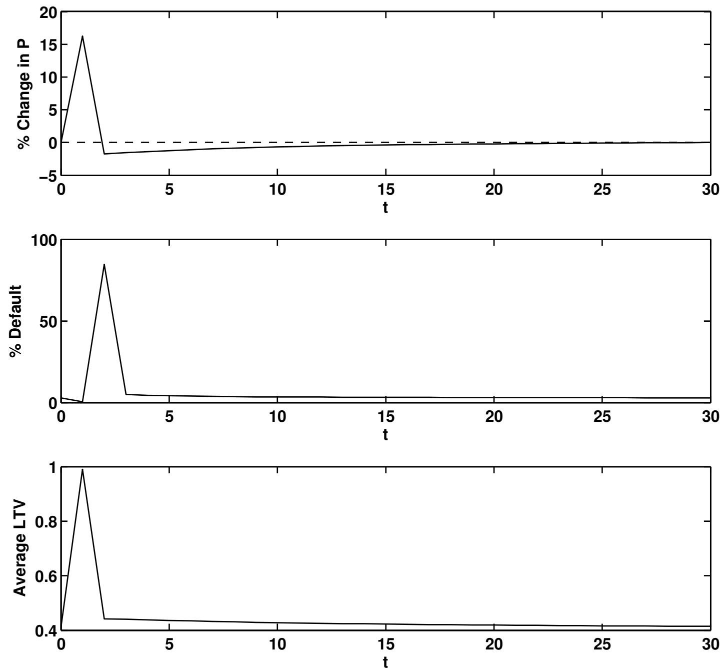

Figure 7: Dynamic Response: Frictionless with Cobb-Douglas Utility

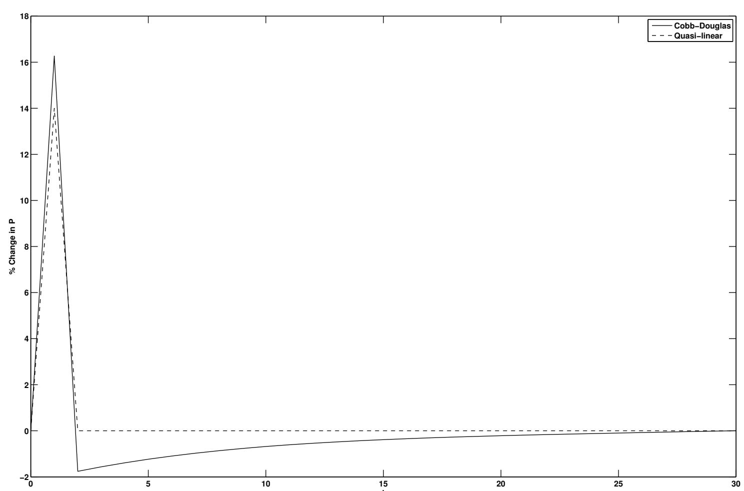

Figure 8: Price Responses: Cobb-Douglas vs. Quasi-linear Utility

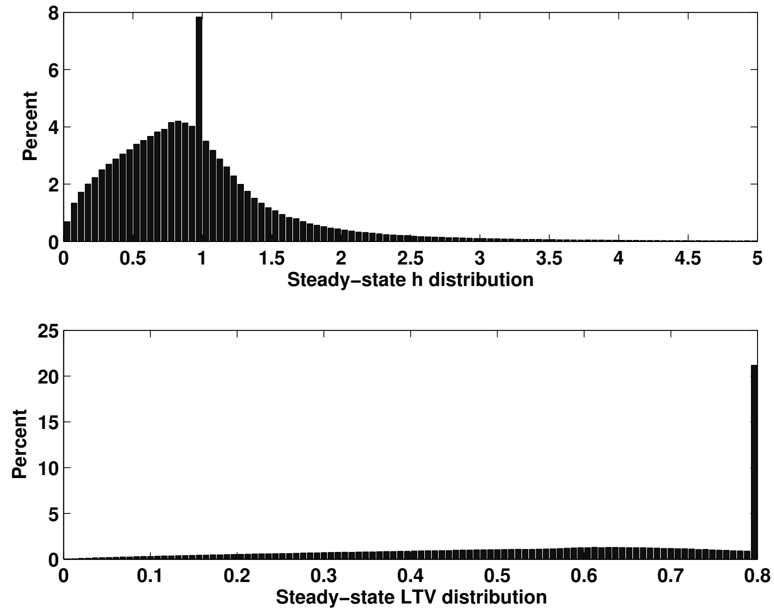

Figure 9: Steady State Distributions with Frictions in the Quasi-Linear Case

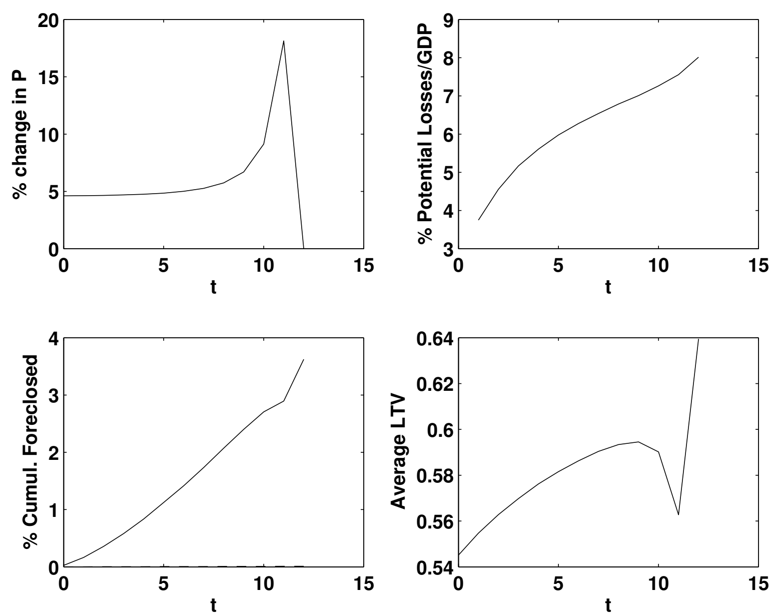

Figure 10: Dynamic Response: Baseline with Quasi-linear Utility

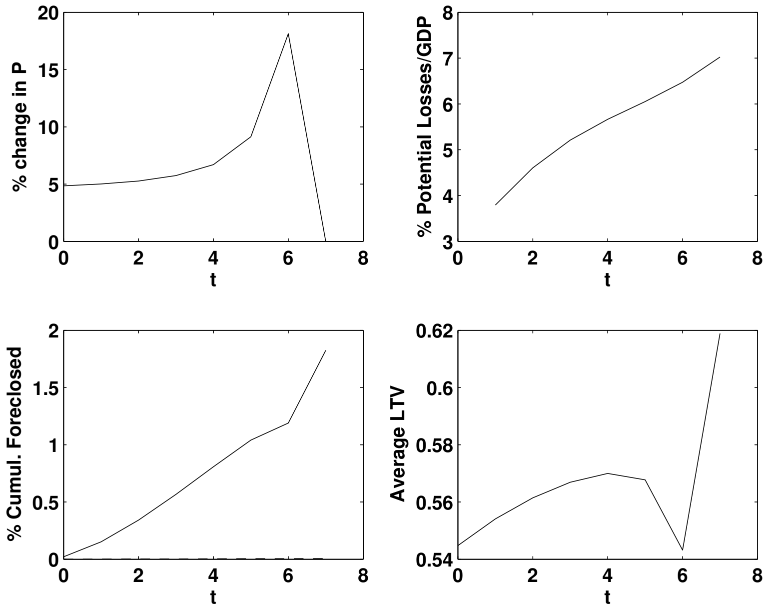

Figure 11: Dynamic Response: Equilibrium with $\lambda^* = 0.07$

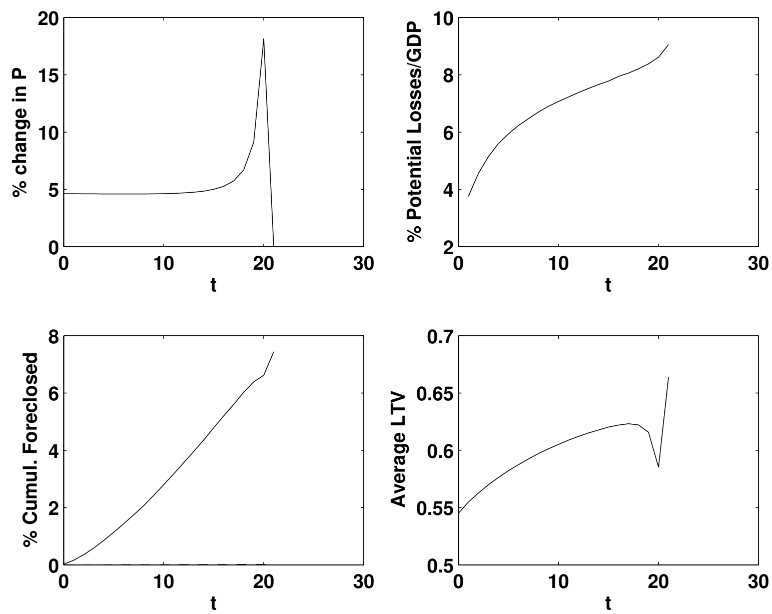

Figure 12: Dynamic Response: Equilibrium with $\lambda^* = 0.09$

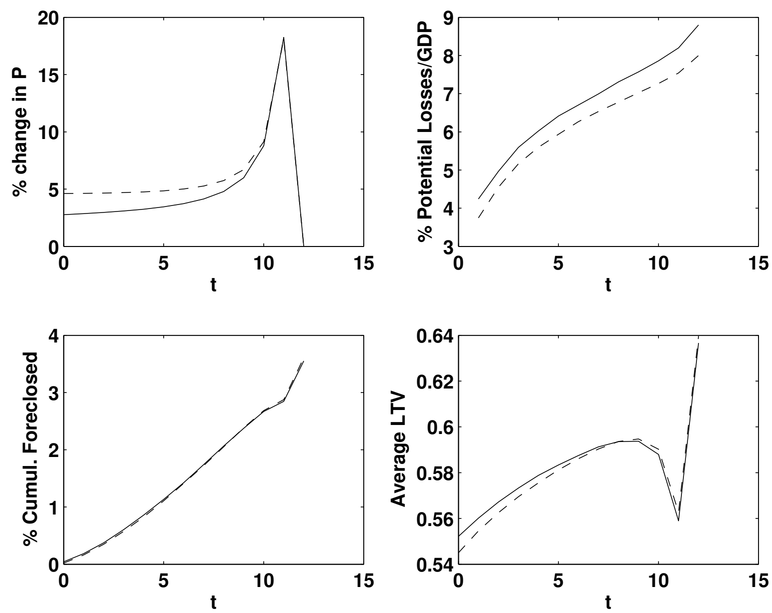

Figure 13: Dynamic Response: $\bar{\eta} = 0.05$ vs. Baseline $\bar{\eta} = 0.15$ (dotted)

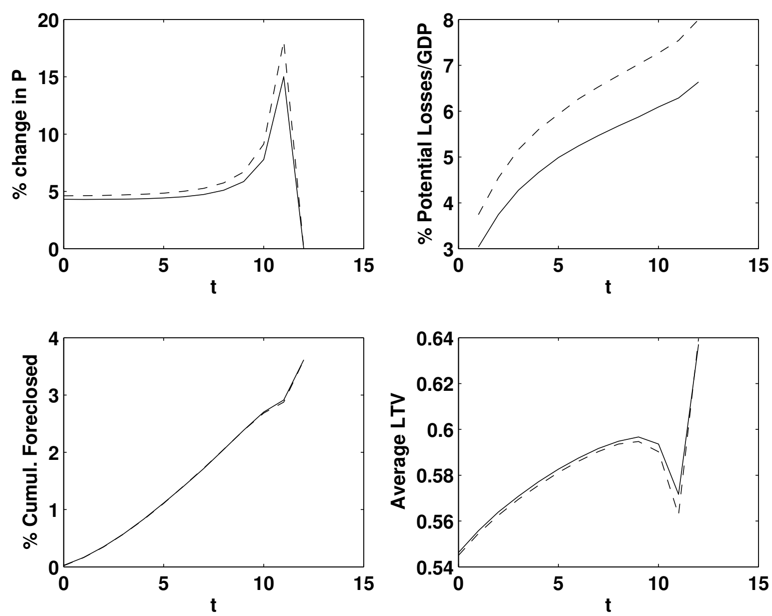

Figure 14: Dynamic Response: $\hat{\eta} = 0.95$ vs. Baseline $\hat{\eta} = 1$ (dotted)

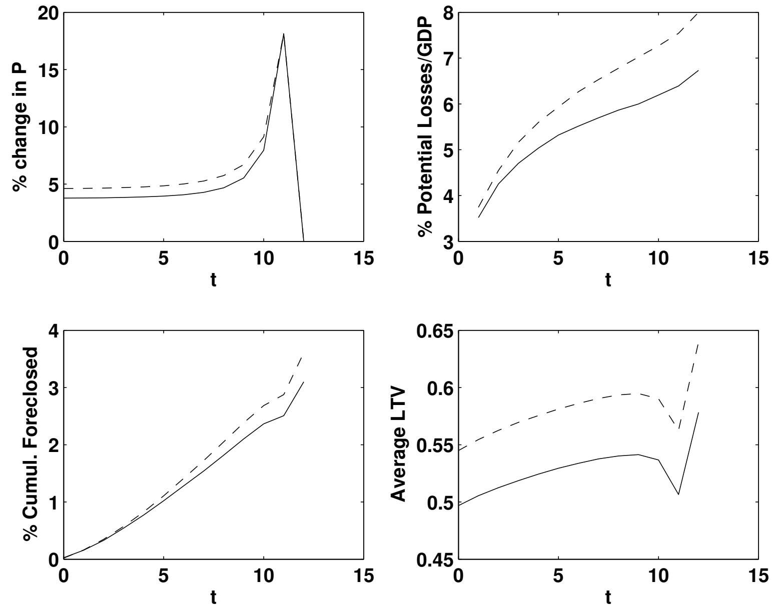

Figure 15: Dynamic Response: $\theta = 0.05$ vs. Baseline $\theta = 0.033$ (dotted)

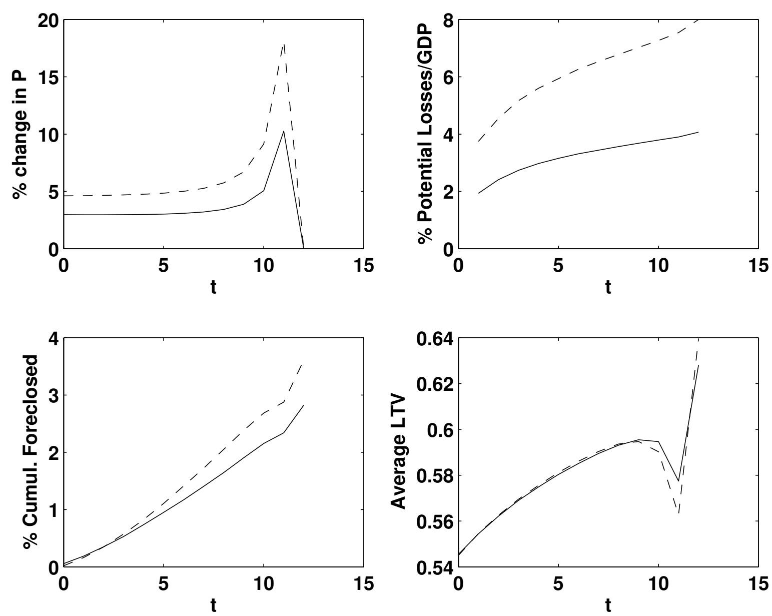

Figure 16: Dynamic Response: $\sigma = 0.12$ vs. Baseline $\sigma = 0.15$ (dotted)

Appendices

A Technical Results on Quasi-Linear Case

For all results that follow, assume that utility function takes the form $u(c_t, h_{t+1}) = c_t + v(h_{t+1})$ where function $v(.)$ is strictly increasing, strictly concave, differentiable, and satisfies $\lim_{h\downarrow 0} v'(h) = \infty$.

A.1 Choice of LTV for borrowers

Consider an agent of type $i \in \{m, f\}$, maximizing objective function (1) subject to the set of constraints (3). In this section, we demonstrate that this agent borrows up to the conforming loan limit when subsidy η is sufficiently large compared to the foreclosure cost γ. For the choice of $z \in [0, 1]$, the derivative of the objective function with respect to z_{t+1} equals

$$Ph_{t+1} + \beta Ph_{t+1}\frac{\partial}{\partial z_{t+1}}\Big(\mathbb{E}(\max\{0, x_{t+1} - z_{t+1}\}) - \rho(z_{t+1}; \eta, \zeta)z_{t+1}\Big)$$

Using expression (5) for $\rho(.)$, differentiating the second expression, and simplifying, we obtain

$$\begin{cases} Ph_{t+1}\big(1 + \beta(-(1+r) - \gamma m z_{t+1} g(z_{t+1})(1-\eta) + \eta G(z_{t+1}))\big) & z_{t+1} \leq \zeta \\ Ph_{t+1}\big(1 + \beta(-(1+r) - \gamma m z_{t+1} g(z_{t+1})\big) & z_{t+1} > \zeta \end{cases}$$

Imposing the equilibrium condition $\beta(1 + r) = 1$, this simplifies further:

$$\begin{cases} Ph_{t+1}\big(\eta G(z_{t+1}) - (1-\eta)\gamma m z_{t+1} g(z_{t+1})\big) & z_{t+1} \leq \zeta \\ -Ph_{t+1}\gamma m z_{t+1} g(z_{t+1}) & z_{t+1} > \zeta \end{cases}$$

Under our assumptions on the pdf and cdf functions $g(x)$ and $G(x)$, we can make the following observations:

1. When $\gamma = \eta = 0$, the derivative expressions are identically zero over all choices of $z_{t+1} \in [0, 1]$. Hence, in this case, *agent is indifferent between any choice of LTV*.

2. When $\gamma > 0$ and $\eta = 0$, these expressions are negative for all $z_{t+1} \in (\underline{x}, 1]$. We conclude that in this case, *agent does not engage in risky borrowing,* and we can assume without loss of generality, that $z_{t+1} = \underline{x}$.

3. When $\gamma = 0$ and $\eta > 0$, increasing z_{t+1} beyond $\underline{x}$ up to and including ζ improves the objective, as the derivative is strictly positive. The discontinuity of the derivative at $z_{t+1} = \zeta$ does not alter the analysis, since the derivative is negative only for $z > \zeta$. In this case, *agent optimally borrows up to the conforming loan limit*, i.e. $z_{t+1} = \zeta$.

Thus the borrowing decision takes extreme values (corner solutions) that depend on the relative magnitudes of η and γ. Since these functions are continuous in both of these parameters, these observations suggest that item 3 would still hold true when η is "large" and γ is "small". In our numerical analysis, for all cases we cover, we also verify these results numerically.

A.2 Irrelevance of risk-free assets and labor income for housing and LTV decisions

In this section, we illustrate that with quasi-linear preferences we can safely ignore the choice of a_{t+1} and after-tax earnings $\bar{w}$ for the sake of analyzing housing and LTV decisions. More specifically, under the equilibrium interest rate that satisfies $\beta(1 + r) = 1$ and quasi-linearity, the objective can be written in such a way that does not involve the choice of these variables.

Using the constraint (3) to eliminate c_t from the objective (1), we obtain the following equivalent sequential problem

$$\max_{h_{t+1}, z_{t+1}, a_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big(I_t - P h_{t+1}(1 - z_{t+1}) - a_{t+1} + v(h_{t+1}) \big)$$

subject to

$$\begin{aligned} I_{t+1} &\equiv I_{t+1}(h_{t+1}, z_{t+1}, x_{t+1}, a_{t+1}) \\ &= \bar{w} + a_{t+1}(1 + r) + P h_{t+1} \big(\max\{0, x_{t+1} - z_{t+1}\} - \rho(z_{t+1}) z_{t+1} \big) \text{ for all } t \geq 0 \end{aligned}$$

$$c_t, h_{t+1}, a_{t+1} \geq 0, \text{ and } z_{t+1} \in [0, 1]$$

$$h_{t+1} = h_t x_t \text{ for types } \{f, n\}$$

$$z_{t+1} \leq (1-\theta)\min\{1, \frac{z_t}{x_t}\} \text{ for type } n$$

given $I_0 > 0$, $h_0 > 0$, $z_0 \geq 0$, $x_0 \in [\underline{x}, \bar{x}]$.

Under the assumption that the non-negativity constraints on c_t and a_{t+1} never bind[34], we observe that

1. The present value of labor earnings $\bar{w} > 0$ enters additively, and therefore can be omitted without affecting the optimal policies.

2. Since $\beta(1+r) = 1$, all terms involving a_{t+1} for all $t \geq 0$ cancel out from the objective.

3. The expected value $\mathbb{E}_t I_{t+1} = P h_{t+1} \mathbb{E}\big(\max\{0, x_{t+1} - z_{t+1}\} - \rho(z_{t+1}) z_{t+1}\big)$, conditional on having chosen (h_{t+1}, z_{t+1}), is independent of the type realization in $t+1$, where the expectation is taken over x_{t+1}.

Using these results and moving terms across time periods, the objective function can be simplified further,

$$\max_{h_{t+1}, z_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big(-P h_{t+1}(1 - z_{t+1}) + v(h_{t+1}) + \beta P h_{t+1}\big(\max\{0, x_{t+1} - z_{t+1}\} - \rho(z_{t+1}) z_{t+1}\big)\Big)$$

subject to

$$c_t, h_{t+1} \geq 0, \text{ and } z_{t+1} \in [0, 1]$$

$$h_{t+1} = h_t x_t \text{ for types } \{f, n\}$$

$$z_{t+1} \leq (1-\theta)\min\{1, \frac{z_t}{x_t}\} \text{ for type } n$$

given $h_0 > 0$, $z_0 \geq 0$, $x_0 \in [\underline{x}, \bar{x}]$.

A.3 Derivation of the Bellman Equation

A rigorous proof of equivalence of recursive and sequential representation of this problem is beyond the scope of this paper. However, with standard arguments,

[34] We conjecture that the non-negativity constraints will not bind provided $\bar{w}$ is sufficiently large, and in practice do not observe any cases where they do bind in our simulations.

one can show that if a solution to the sequential problem in the previous section exists, than it must satisfy the following recursive version

$$\tilde{V}(h,z) = -Ph(1-z) + v(h) + \beta Ph\mathbb{E}\big(\max\{x-z,0\} - \rho(.)z\big) + \beta\Big(m\mathbb{E}(\max_{(h',z')\in\Gamma_m}\tilde{V}(h',z'))$$

$$+ (1-m)f\mathbb{E}(\max_{z'\in\Gamma_f}\tilde{V}(hx,z')) + (1-m)(1-f)\mathbb{E}(\max_{z'\in\Gamma_n(z,x)}\tilde{V}(hx,z'))\Big)$$

where $\Gamma_m \equiv \{(h',z')|h' > 0, z' \in [0,1]\}$ denotes the choice set for a mover, $\Gamma_f \equiv \{z'|z' \in [0,1]\}$ denotes the choice set for a refinancer, and $\Gamma_n(x,z) \equiv \{z'|z' \in [0,(1-\theta)\min\{1,\frac{z}{x}\}]\}$ denotes the choice set for other agents. Function $\tilde{V}(h,z)$ represents the value of the household *after* shock x is realized, and after default, as well as (h,z) decisions have been made.

In the first part of this appendix, we have shown that a mover and a refinancer always borrow up to the conforming loan limit. Moreover, at an equilibrium, every mover demands $h = 1$. Now consider a type n agent: When presented with a choice of LTV limit in $z' \in [0,(1-\theta)\min\{1,\frac{z}{x}\}]$, this agent would opt for an LTV limit as high as possible, up to the conforming loan limit for the same reason as the other types of agents. Hence the choice would satisfy $z' = \min\{\zeta,(1-\theta)\min\{1,\frac{z}{x}\}\}$. Putting all these pieces together, we obtain the value function in expression (9).