Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation

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Abstract

In the United States, rising energy efficiency, rather than the use of less carbon-intensive energy sources, has driven the decline in the carbon intensity of output. Thus, understanding how environmental policy will affect energy efficiency should be a primary concern for climate change mitigation. In this paper, I evaluate the effect of environmental taxes on energy use in the United States. To do so, I construct a putty-clay model of directed technical change that matches several key features of the data on U.S. energy use. The model builds upon the standard Cobb-Douglas approach used in climate change economics in two ways. First, it allows the elasticity of substitution between energy and non-energy inputs to differ in the short and long run. Second, it allows for endogenous and directed technical change. In the absence of climate policy, the new putty-clay model of directed technical change and the standard Cobb-Douglas approach have identical predictions for long-run energy use. The reactions to climate policies, however, differ substantially. In particular, the new putty-clay model of directed technical change suggests that a 6.9-fold energy tax in 2055 is necessary to achieve policy goals consistent with the 2016 Paris Agreement and that such a tax would lead to 6.8% lower consumption when compared to a world without taxes. By contrast, the standard Cobb-Douglas approach suggests that a 4.7-fold tax rate in 2055 is sufficient, which leads to a 2% decrease in consumption. Thus, compared to the standard approach, the new model predicts that greater taxation and more forgone consumption are necessary to achieve environmental policy goals.

Keywords Energy, Climate Change, Directed Technical Change, Growth

JEL Classification Codes O30, O44, H23

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1 Introduction

To combat global climate change, it is crucial to understand how carbon emissions will respond to policy interventions, and energy efficiency will be an important component of this reaction. Indeed, rising energy efficiency, rather than the use of less carbon intensive energy sources, has been the major force behind the decline in the carbon intensity of output in the United States over the last 40 years (Raupach et al., 2007; Nordhaus, 2013). Thus, energy efficiency will be a major factor in any future approach to mitigating climate change.

Integrated assessment models (IAMs) are the standard tool in climate change economics. They combine models of the economy and climate to calculate optimal carbon taxes. The leading models in this literature frequently treat energy as an input in a Cobb-Douglas aggregate production function (Nordhaus and Boyer, 2003; Golosov et al., 2014).[1] Despite the significant insights gained from the IAMs, there are two restrictive assumptions in this approach to modeling energy. First, in response to changes in energy prices, the Cobb-Douglas approach allows smooth substitution between capital and labor, which is at odds with short-run features of the U.S data (Pindyck and Rotemberg, 1983; Hassler et al., 2012, 2016b). This suggests that the standard approach may not fully capture the effect of new environmental taxes, which will raise the effective price of energy. Second, technological change is exogenous and undirected in the standard model. A substantial literature, however, suggests that improvements in energy-specific technology will play a pivotal in combating climate change and that environmentally-friendly research investments respond to economic incentives (e.g., Popp et al., 2010; Acemoglu et al., 2012).

In this paper, I construct a putty-clay model of directed technical change that matches several key features of the data on U.S. energy use. In particular, the model developed in this paper captures the short-run dynamics between energy prices and energy use, and it incorporates technological progress. In the model, each piece of capital requires a fixed amount of energy to operate at full potential. Technical change, however, can alter the lower this input requirement in the next vintage of the capital good, or it can increase the ability of the next vintage to produce final output.[2,3] When energy prices rise, the energy expenditure share of output will increase in the short run, but

[1]This is particularly relevant to the literature building on the standard neoclassical growth model. Another strand of the climate change literature uses large computable general equilibrium models. Of particular relevance to the current paper are analyses using the EPPA (Morris et al., 2012) or Imaclim (Crassous et al., 2006) models, each of which has elements of putty-clay production.

[2]Capital good producers turn raw capital, 'putty,' into a capital good with certain technological characteristics, including energy efficiency. While energy efficiency can be improved by research and development, there is no substitution between energy and non-energy inputs once the capital good in constructed, capturing the rigid 'clay' properties of installed capital.

[3]The literature on putty-clay production functions has a long history (e.g., Johansen, 1959; Solow, 1962; Cass and Stiglitz, 1969; Calvo, 1976). Of particular relevance is work by Atkeson and Kehoe (1999) who investigate the role of putty-clay production in explaining the patterns of substitution between energy and non-energy inputs in production. The older literature on putty-clay models focuses on choosing a type of capital from an existing distribution. The current paper focuses on how the cutting-edge of technology, which is embodied in capital goods, evolves over time.

firms will have an increased incentive to improve the energy efficiency of new capital goods, driving the expenditure share back down.[4]

Rather than importing the seminal directed technical change model developed by Acemoglu (1998, 2002, 2007), I take a new approach in which innovation occurs in different characteristics of capital goods, not in different sectors. In other words, energy efficiency occurs when capital goods require less energy to run, not when the energy sector becomes more efficient at turning primary energy (e.g., coal) into final-use energy (e.g., electricity). This modeling choice is motivated by data from the United States, where reductions in the energy intensity of output have been driven by decreases in final-use energy intensity. This theoretical innovation significantly alters the underlying incentives for research and development.

In the absence of climate policy, the new model and the standard Cobb-Douglas approach have identical predictions for long-run energy use. The putty-clay model of directed technical change, however, predicts significantly different reactions to climate policy. In particular, I apply energy taxes necessary to achieve environmental policy goals laid out in international agreements.[5] The new putty-clay model of directed technical change suggests that a 6.9-fold energy tax in 2055 is necessary to achieve policy goals consistent with the 2016 Paris Agreement and that such a tax would lead to 6.8% lower consumption when compared to a world without taxes.[6] By contrast, the standard Cobb-Douglas approach suggests that a 4.7-fold tax rate in 2055 is sufficient, which leads to a 2% decrease in consumption. When applying the same tax rate to both models, the new putty-clay model of directed technical change predicts 20% greater cumulative energy use over the next century. Thus, compared to the standard approach, the new model predicts that greater taxation and more forgone consumption are necessary to achieve environmental policy goals.

While the focus of the paper is measuring the impact of climate policies on energy use, the model also has predictions for the long-run sustainability of economic growth in a world with non-renewable resources. In particular, the model predicts that, despite the low elasticity of substitution between energy and non-energy inputs in production, consumption and output per capita can continue to grow at current rates indefinitely, even as energy extraction costs tend towards infinity. This occurs because, in the long run, improvements in energy efficiency exactly offset increases in

[4] As discussed in the next section, this modeling approach draws insight from Hassler et al. (2012, 2016b), who provide econometric evidence that a putty-clay model of directed technical change could fit patterns of substitution in U.S. energy use and investigate the implication of these forces for long-run economic growth in a social planner's model with finite energy resources and an aggregate production.

[5] In particular, I simulate taxes needed to reduce energy use to 60% of 2005 levels by the year 2055. This is consistent with goals laid out in the recent Paris Agreement, which suggests that the United States adopt policies consistent with a 80% reduction in carbon emission by 2050. Thus, I examine a case where half of the required reduction in carbon emissions comes from reductions in energy use. The goals are outlined in the Intended Nationally Determined Contribution (INDC) submitted by the United States to the United Nations Framework Convention on Climate Change (UNFCC), which is available at: `http://www4.unfccc.int/submissions/INDC/Published%20Documents/United%20States%20of%20America/1/U.S.%20Cover%20Note%20INDC%20and%20Accompanying%20Information.pdf`.

[6] This analysis abstracts from the consumption losses caused by climate change. As a result, the 'forgone' consumption is a measure of the economic cost of implementing environmentally friendly policies, which would need to be balanced again damages from environmental degradation to calculate optimal policy. In this paper, I take a different approach and examine the policies necessary to achieve stated, rather than optimal, goals for environmental policy.

extraction costs, leading to a constant long-run energy expenditure share of output. This result is more optimistic than those in the existing literature (Krautkraemer, 1998; Hassler et al., 2012, 2016b; Peretto and Valente, 2015).

The rest of this paper is structured as follows. Section 2 discusses the related literature. Section 3 discusses the empirical motivation underlying the theoretical underpinning of the paper. The model is presented in Section 4 and the calibration in section 5. Section 6 reports the results of the quantitative analysis, and Section 7 concludes.

2 Related Literature

As described above, this paper contributes to the literature on climate change economics that takes a Cobb-Douglas approach to energy modeling in IAMs. This paper is also closely related to a growing literature demonstrating that directed technical change (DTC) has important implications for environmental policy. These studies generally focus on clean versus dirty sources of energy, rather than energy efficiency. Acemoglu et al. (2012) demonstrate the role that DTC can play in preventing environmental disasters and emphasize the elasticity of substitution between clean and dirty production methods. The model in this paper bears more resemblance to an 'alternate' approach they mention where firms can invest in quality improvements or carbon abatement, where the latter only occurs in the presence of carbon taxes. Peretto (2008) and Gans (2012) also conduct an analyses where policy interventions affect how technological change is directed between production and abatement activities. Aghion et al. (2016) provide a directed technical change model of clean and dirty innovation in the automotive industry that includes an intra-product decision about energy efficiency. I build on these earlier works by constructing a new model of directed technical change, focusing on energy efficiency, quantitatively investigating the macroeconomic effects of prominent environmental policies, and comparing the results to the standard approach taken in IAMs.[7]

Two recent papers extend the standard DTC model to quantitative investigation of macroeconomic policy (Fried, 2015; Acemoglu et al., 2016). Both focus on the issue of clean versus dirty energy sources, rather than energy efficiency. Methodologically, this paper is closer to the approach taken by Fried (2015), who accounts for energy efficiency by calibrating growth in clean energy to overall de-carbonization of the economy, which includes overall energy efficiency as well as shifts towards the clean energy sector. In this way, the current paper builds on her work by explicitly investigating energy efficiency as a separate source of innovation that is complementary with other inputs, using a new underlying model of DTC, and comparing the results to the standard approach taken in climate change economics.

This paper is also closely related to the pioneering paper of Smulders and De Nooij (2003) who apply the original DTC model directly to energy efficiency and use it to analyze the effects of exogenous changes in energy availability. André and Smulders (2014) extend this analysis to incorporate

[7] A related and influential literature looks at induced, but not directed, technical change and its implications for climate policy. These models tend to focus on social planner problems. Key contributions in this literature include Goulder and Schneider (1999), Goulder and Mathai (2000), Sue Wing (2003), and Popp (2004).

Hotelling forces and examine the role of changes in extraction technology. Other recent advances in the relatively small energy efficiency and DTC literature include studies by Lemoine (2015), who examines how changes in resource-specific energy efficiency can lead to energy transitions, and Van der Meijden and Smulders (2014), who investigate the dual effects of expectations and directed technical change in energy efficiency. I build upon this energy efficiency literature by constructing a new model of DTC that can recreate key data patterns, using it to quantitatively evaluate prominent environmental policies, and comparing the results to the Cobb-Douglas approach.

The new putty-clay model of directed technical change builds on the aggregate model of energy use developed by Hassler et al. (2012, 2016b). The current model differs from their work in two key respects. First, rather than focusing on a social planner model and an aggregate production function, I construct a decentralized model with incentives for innovation. Second, I consider the case of infinite potential supplies of energy and increasing extraction costs, whereas Hassler et al. (2012, 2016b) investigate the optimal depletion of a fixed resource. These methodological differences add realism, allowing me to investigate the impacts of climate change mitigation policy, which is the primary goal of this study. The decentralized model is important to account for inter-temporal externalities and to capture the difference between primary and final-use energy. The potentially infinite supply of energy captures the role of coal in fossil fuel energy use.[8] Moreover, as prices rise, new methods of resource extraction will become feasible, expanding the supply of available energy sources.[9]

While the primary goal of this study is to evaluate the effect of taxation on energy use, I also arrive at significantly different long-run predictions for consumption growth and the energy expenditure share when compared to Hassler et al. (2012, 2016b). In particular, I find that the data over the last several decades is consistent with a balanced growth path in the decentralized model, suggesting that current trends can continue indefinitely.[10] On the other hand, the current data is inconsistent with the balanced growth path in a model where a social planner optimally depletes a finite energy resource. Thus, Hassler et al. (2012, 2016b) find that the energy share must increase significantly and consumption growth must decrease in order to converge to a balanced growth path in the long run.

It is also important to note that the DTC literature is supported by microeconomic studies that investigate the presence of directed technical change. Newell et al. (1999) and Jaffe et al. (2003) demonstrate that the energy efficiency of energy-intensive consumer durables (air conditioners and gas water heaters) responds to changes in prices and government regulations, providing evidence for the existence of directed technical change. Similarly, Popp (2002) finds that energy efficiency

[8]Coal is predicted to to be the primary driver of global carbon emissions and is available in abundant supply (Golosov et al., 2014; Hassler et al., 2016a).

[9]For example, the United States Geological Survey (USGS) recently estimated that there were 565 billion barrels of oil (BBO) of undiscovered conventional crude oil reserves in the world (excluding the United States), but 513 BBO of undiscovered and harder-to-extract 'unconventional' reserves, such as oil shale and oil sands, in Venezuela alone (Schenk et al., 2012).

[10]This is also consistent with the standard Cobb-Douglas approach to energy model within climate change economics (Nordhaus and Boyer, 2003; Golosov et al., 2014).

innovation, as measured by patents, responds to changes in energy prices. He looks at both innovations in the energy sector and in the energy efficiency characteristics of other capital goods. More recently Dechezleprêtre et al. (2011) and Calel and Dechezlepretre (2016) find that patents for 'low carbon' technologies, which include more energy efficient and less carbon intensive innovations, respond to both energy prices and public policies designed specifically to address climate change. Aghion et al. (2016) find that government policies have a strong effect on energy efficient research in the automotive sector.

3 Empirical Motivation

In this section, I discuss a number of patterns in the data that motivate the theoretical choices made in this paper. In particular, I present evidence that a) declines in the final-use energy intensity of output drive of reductions in the carbon intensity of output, b) there is very low short-run elasticity of substitution between energy and non-energy inputs, and c) there is no long-run trend in the energy expenditure share of final output.

To analyze the determinants of the carbon intensity of output, I consider the following decomposition:

$$\frac{CO_2}{Y} = \frac{CO_2}{E_p} \cdot \frac{E_p}{E_f} \cdot \frac{E_f}{Y}, \tag{1}$$

where CO_2 is yearly carbon emissions, Y is gross domestic product, E_p is primary energy use (e.g., coal, oil), and E_f is final-use energy consumption (e.g., electricity, gasoline). The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, captures substitution between clean and dirty sources of energy (e.g., coal versus solar). The efficiency of the energy sector, which transforms primary energy into final-use energy, is captured by $\frac{E_p}{E_f}$. For example, the ratio decreases when power plants become more efficient at transforming coal into electricity. The final-use energy intensity of output, $\frac{E_f}{Y}$, measures the quantity of final-use energy used in production and consumption. For example, the ratio decreases when manufacturing firms use less electricity to produce the same quantity of goods.

The results of this decomposition are presented in figure 1, which plots the carbon intensity of output and each component from equation (1) for the United States from 1971-2011. Data are normalized to 1971 values. Energy and carbon dioxide data are from the International Energy Agency (IEA).[11] Real GDP data are from the U.S. Bureau of Economic Analysis (BEA).[12] The carbon intensity of output fell over 60% during this time period, and this decline is matched almost exactly by the decline in the final-use energy intensity of output. Thus, the results demonstrate the primary importance of $\frac{E_f}{Y}$ in understanding how the economy will react to climate change mitigation policy. The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, declined approximately 10% over this period. While this is a significant improvement for environmental outcomes, it is relatively

[11]See 'IEA Headline Energy Data' at `http://www.iea.org/statistics/topics/energybalances/`.

[12]See Section 1 of the NIPA tables at: `https://www.bea.gov//national/nipaweb/DownSS2.asp`.

small compared to the overall improvements in the carbon intensity of output. Finally, the efficiency of the energy transformation sector, as measured by the inverse of $\frac{E_p}{E_f}$, actually declined roughly 15% over this period, indicating that it offset the environmental benefits achieved elsewhere. This result is driven by differences in the efficiency of transformation across different sources of primary energy, rather than technological regress. The results reject the notion that improvements in the carbon or energy intensity of output have been driven by technological improvements in the energy transformation sector.

Motivated by this evidence, I construct a model that focuses on the final-use energy intensity of output. This creates a significant break with existing work. Existing macroeconomic research on directed technical change and climate change focuses on clean versus dirty sources of energy and abstracts from energy efficiency (Acemoglu et al., 2012; Fried, 2015; Acemoglu et al., 2016). Transition to cleaner energy sources will undoubtedly be an important component of any approach to mitigate climate change, but the historical data strongly suggest that improved energy efficiency will be a pivotal aspect of any policy response. At the same time, applying the seminal DTC model of Acemoglu (1998, 2002, 2007) to the question of energy efficiency would require focusing on the efficiency of the energy sector (e.g., Smulders and De Nooij, 2003; André and Smulders, 2014). Thus, I construct a new model where energy efficiency is driven by the energy requirements of capital goods, rather than the productivity of the energy transformation sector. This theoretical innovation significantly alters the underlying incentives for research and development.[13]

Figure 2 plots an index of real fossil fuel prices, the expenditure share of fossil fuel energy, and total fossil fuel energy use in the United States from 1971-2011. Energy use and price data are from the U.S. Energy Information Agency.[14] The sample is restricted to fossil fuels due to limitation on the price data, and a very similar graph serves as the motivation for Hassler et al. (2012). Output is again from the BEA. The data indicate that expenditure, but not total fossil fuel energy use, reacts to short-term price fluctuations, suggesting that there is very low short-run substitution between energy and non-energy inputs. At the same time, there is no trend in the energy expenditure share of output, suggesting a constant long-run level in the absence of shocks. The model in this paper will match both of these facts. Hassler et al. (2012, 2016b) provide a formal maximum likelihood estimate of the elasticity of substitution between energy and non-energy inputs using this data. They find an elasticity of substitution very close to zero. For the purposes of this paper, I will treat the elasticity as exactly zero and use a Leontief production structure, which allows for the

[13]Of course, not all improvements in energy efficiency need to driven by technical change. In particular, sectoral reallocation could explain aggregate changes in energy use. Recent papers addressing this question find that technological change, rather than sectoral reallocation, is the key driver of falling energy intensity over this period (Wing and Eckaus, 2007; Wing, 2008). The same studies suggest that, prior to 1970, sectoral reallocation with the primary driver of falling energy intensity. Thus, this paper will focus on the post-1970 period. A shortcoming of this approach is that the model will not be able to explain trends in energy efficiency at earlier points in time. Such a model would likely need to combine structural change with technological improvements in energy efficiency.

[14]See table 3.1 'Fossil fuel production prices, 1949-2011' and table 1.3 'Primary energy consumption estimates by source, 1949-2012' at `http://www.eia.gov/totalenergy/data/annual/`.

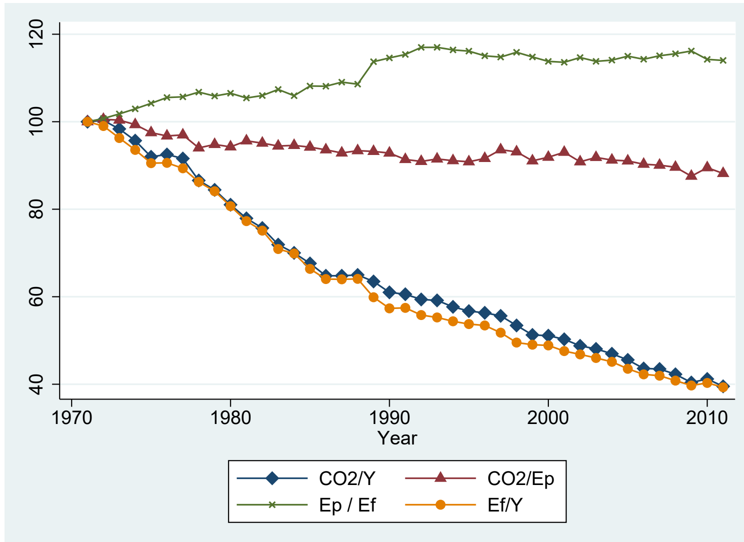

Figure 1: This figure decomposes the decline in the carbon intensity of output. CO_2 is yearly carbon emissions, Y is GDP, E_p is primary energy, and E_f is final-use energy. This figure demonstrates that the fall in the carbon intensity of output $\frac{CO_2}{Y}$ has been driven by decreases in final-use energy intensity of output $\frac{E_f}{Y}$, rather than the use of cleaner energy sources, $\frac{CO_2}{E_p}$, or a more efficient energy transformation sector, $\frac{E_p}{E_f}$. Data are from the International Energy Agency (IEA) and the Bureau of Economic Analysis (BEA). All values are normalized to 1971 levels.

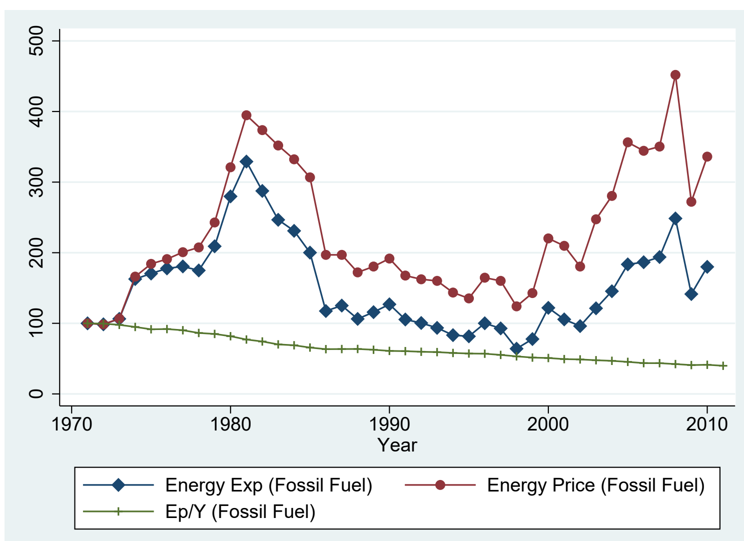

Figure 2: This figure demonstrates that short-run movements in energy prices affect short-run expenditures, but have very little affect on short-run energy use. At the same time, there is no trend in the energy expenditure share of output. Only fossil fuels are considered due to limitations in price data. Data are taken from the Bureau of Economic Analysis and the Energy Information Agency. All values are normalized to 1971 levels.

construction of a tractable putty-clay model. They also find that energy efficiency increases after prices rise, suggesting a putty-clay model of the type investigated here.[15]

The trendless expenditure share of energy in figure 2, in combination with the analysis of Hassler et al. (2012, 2016b), serves as the motivation for the Cobb-Douglas production function in IAMs (Golosov et al., 2014; Barrage, 2014). At the same time, the analysis by Hassler et al. (2012, 2016b) suggests that the long-run energy expenditure share – which will eventually be constant must be significantly higher than the current level. The model developed in this paper will bridge the gap between these two approaches. It yields a constant energy expenditure share that matches the current level, while simultaneously replicating both short- and long-run patterns of substitution.

4 Model

4.1 Structure

4.1.1 Final Good Production

The production structure of the model extends the standard DTC production function to account for energy use. To match the extremely low short-run elasticity of substitution between energy and non-energy inputs (see figure 3), I use a Leontief structure

$$Q_t = \int_0^1 min[A_{N,t}(i)X_t(i)^\alpha L_t^{1-\alpha}, A_{E,t}(i)E_t(i)]\ di, \tag{2}$$

$$s.t.\quad A_{E,t}(i)E_t(i) \leq A_{N,t}(i)X_t(i)^\alpha L_t^{1-\alpha}\ \forall i, \tag{3}$$

where Q_t is gross output at time t, L_t is the aggregate (and inelastic) labor supply, $A_{N,t}(i)$ is the the quality of capital good i, $X_t(i)$ is the quantity of capital good i, $A_{E,t}(i)$ is the energy efficiency of capital good i, and $E_t(i)$ is the amount of energy devoted to operating capital good i. Several components of the production function warrant further discussion. As in the standard endogenous growth production function, output is generated by a Cobb-Douglas combination of aggregate labor, L_t, and a series of production process, each of which uses a different capital good, $X_t(i)$. Unlike the endogenous growth literature, each production process also requires energy to run. Thus, the usual capital-labor composite measures the potential output that can be created using each production process, and the actual level of output depends on the amount of energy devoted to each process, $E_t(i)$. The notion of potential output is captured by constraint (3). Each capital good i has two distinct technological characteristics. The quality of the capital good, $A_{N,t}(i)$, measures its ability

[15] As demonstrated in figure 2, the price of energy in the United States has had an upward trend since 1970. Thus, this paper will treat energy prices as continually increasing. Once again, this is a good match for post-1970 data, but not for U.S. data in the preceding two decades, where energy price actually declined. The long-run increase in prices is consistent with theoretical work based on the Hotelling problem or increasing extraction costs (Hotelling, 1931; Pindyck, 1978), as well as empirical work suggesting a U-shaped pattern in long-run energy prices (Pindyck, 1978, 1999; Hamilton, 2012). Thus, there is strong reason to believe that, in the long-run, energy prices will have an upward trend (Hamilton, 2008).

to produce output, and the energy efficiency of the capital good, $A_{E,t}(i)$, lowers the amount of energy needed to operate the production process at full potential.[16]

4.1.2 Energy Sector

Energy is available in infinite supply, but is subject to to increasing extraction costs. Models with increasing extraction costs have a long history in energy economics (e.g., Pindyck, 1978; Livernois and Uhler, 1987). This extraction cost is paid in final goods, and energy is provided by a perfectly competitive sector. Recent research suggests that most new production comes from the exploitation of new geographic areas, rather than improved technology applied to existing sources of energy (Hamilton, 2012). Thus, increasing search costs for new sources of energy and the increased difficulty in extracting energy from harder to access locations are likely to be primary drivers of the increase in energy prices. As in Golosov et al. (2014), the treatment of energy sources as infinite in potential supply captures the extreme abundance of coal, which is predicted to be the major driver of climate change (Hassler et al., 2016a). The infinite supply of energy at increasing extraction costs also captures the existence of 'unconventional' energy reserves, such as shale oil and oil sands, which have high extraction costs, but are available in abundant supplies (Schenk et al., 2012).

The marginal cost of extraction, which will also be equal to the price due to the perfectly competitive nature of the sector, is given by

$$p_{E,t} \quad = \quad \xi \bar{E}_{t-1}^{\iota}, \tag{4}$$

where $\bar{E}_{t-1}$ is total energy ever extracted at the start of the period. The law of motion for the stock of extracted energy is given by

$$\bar{E}_t = E_{t-1} + \bar{E}_{t-1}. \tag{5}$$

Intuitively, at the beginning of each period, energy producers search for new sources of energy to exploit, the cost of which is determined by total amount of energy ever extracted. This is consistent with recent evidence from the oil industry, where drilling, but not within-well production, responds to changes in prices (Anderson et al., 2014).[17]

[16]Consistent with the econometric literature on energy use, energy requirements depend both on the amount of capital and the amount of labor being used in the production process (Van der Werf, 2008; Hassler et al., 2012, 2016b). Second, consistent with both the econometric and DTC literatures, improvements in non-energy technology, $A_N(i)$, raise energy requirements (Smulders and De Nooij, 2003; Van der Werf, 2008; Hassler et al., 2012, 2016b; Fried, 2015).

[17]A primary goal of this paper is to compare the results of the putty-clay model to the standard Cobb-Douglas approach used in IAMs. Since IAMs examine worldwide outcomes, it is crucial to consider the equilibrium effect of policy on energy prices. Hence, the comparison between models is most accurate when considering endogenous prices. At the same time, I also use the model to investigate the affect of policies pursued in the United States. In this case, endogenous energy prices can be motivated in two ways. First, it is possible to think of the United States as a closed economy, which has obvious limitations considering the global nature of energy sector. Alternatively, one can imagine the policies being applied on a worldwide level with the United States making up a constant fraction of total energy. To ensure that the key qualitative results of the paper are not driven by this assumption, I also consider

4.1.3 Final Output

Final output is given by gross production less total energy extraction costs, which are equal to energy expenditures by the final good producer. As long as equation (3) holds with equality,[18] final output is given by

$$Y_t \;=\; L_t^{1-\alpha}\int_0^1 A_{N,t}(i)\big[1-\frac{p_{E,t}}{A_{E,t}(i)}\big]X_t(i)^{\alpha}\,di. \tag{6}$$

This formulation helps illuminate the continuity between the production function used here and the standard approach in endogenous growth models. Output has the classic Cobb-Douglas form with aggregate labor interacting with a continuum of perfectly substitutable types of capital. As in the endogenous growth literature, this structure maintains tractability in the putty-clay model, despite the Leontief nature of production.

Final output can either be consumed or saved for next period. In the empirical application, each period will be ten years. Following existing literature, I assume complete depreciation between periods (Golosov et al., 2014). Thus, market clearing in final goods implies

$$Y_t = C_t + K_{t+1} = L_t w_t + r_t K_t + \Pi_t + p_t^R + T_t, \tag{7}$$

where K_t is aggregate capital, Π_t is total profits, p_t^R is total payments to R&D inputs (discussed in the next section), and T_t is total government revenue, which is distributed lump sum to consumers.

4.1.4 Capital Goods and Research

Each type of capital good is produced by a single profit-maximizing monopolist. This monopolist also undertakes R&D activities to improve the characteristics of the machine, $A_{N,t}(i)$ and $A_{E,t}(i)$. There are no entrants in the model.[19] For both types of technological change, I adopt the common specification:

$$A_{J,t}(i) \;=\; \big[1+\eta_J R_{J,t}(i)R_{J,t}^{-\lambda}\big]A_{J,t-1}, \quad J\in\{N,E\}, \tag{8}$$

where $R_{J,t}(i)$ is R&D inputs assigned to characteristic J by firm i in period t, $R_{J,t}\equiv\int_0^1 R_{J,t}(i)di$, and $A_{J,t-1}\equiv max\{A_{J,t-1}(i)\}$. In other words, R&D builds on aggregate knowledge, $A_{J,t-1}$, and current period within-firm research allocations, $R_{J,t}(i)$, but is also subject to congestion effects $R_{J,t}^{-\lambda}$

the opposite extreme of exogenous energy prices, which implicitly treats the United States as a small open economy taking unilateral policy actions. In this case, energy prices will increase at a constant exogenous rate.

[18] To ensure that equation (3) holds with equality, it is sufficient, but not necessary, to assume that capital fully depreciates after each period. If capital fully depreciates, then in equilibrium forward looking consumers will never 'over-invest' in capital and drive its return to zero. This assumption will be maintained in the empirical analysis, which uses a time period of ten years, and is also employed in Golosov et al. (2014).

[19] A recent analysis by Garcia-Macia et al. (2015) suggests that most innovation is due to incumbents improving existing lines of goods. Moreover, since innovation will occur with certainty, incumbents are not at a disadvantage relative to entrants as in Schumpeterian growth theory (e.g., Aghion and Howitt, 1992).

caused by duplicated research effort. When the period ends, patents expire and the best technology becomes available to all firms.

There are a unit mass of R&D inputs, yielding

$$R_{N,t} + R_{E,t} = 1 \; \forall t. \tag{9}$$

This is consistent with both existing literature on DTC and the environment (Acemoglu et al., 2012; Fried, 2015) and the social planner model provided by (Hassler et al., 2012, 2016b).[20] I assume that the investment price is fixed at unity. Thus, market clearing implies that

$$\int_0^1 X_t(i)di = K_t, \tag{10}$$

where K_t is aggregate capital.

4.1.5 Consumer Problem

The consumer side of the problem is standard. In particular, the representative household chooses a path of consumption to maximize

$$\{c_t\}_{t=0}^{\infty} = max \sum_{t=0}^{\infty} \beta^t L_t \frac{c_t^{1-\theta}}{1-\theta}, \tag{11}$$

where $c_t = C_t/L_t$. The representative household takes prices and technology as given. Population growth is given exogenously by:

$$L_{t+1} = (1+n)L_t. \tag{12}$$

4.2 Analysis

As demonstrated in Appendix 8.2, the first order conditions for the final good producer yield the following inverse demand functions:

$$p_{X,t}(i) = \alpha A_{N,t}(i)\big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big] L_t^{1-\alpha} X_t(i)^{\alpha-1}, \tag{13}$$

$$w_t = (1-\alpha) A_{N,t}(i)\big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big] L_t^{-\alpha} X_t(i)^{\alpha}. \tag{14}$$

[20] Often, models of directed technical change refer to the fixed set of research inputs as scientists (Acemoglu et al., 2012; Fried, 2015). This would be applicable here, though generating the standard Euler equation would require the representative household to ignore scientist welfare (in the environmental literature, directed technical change and capital are generally not included simultaneously). This would be a close approximation to a more inclusive utility function as long as scientists made up a very small portion of the overall population. Another simple solution would be to define utility over total consumption. With log preferences, which will be used in the empirical section, both the putty-clay model with directed technical and the Cobb-Douglas model would yield the standard Euler equation. For simplicity, I simply refer to research inputs, which could be scientists, research labs, etc.

The intuition for the result is straightforward. The final good producer demands capital goods until marginal revenue is equal to marginal cost. Unlike the usual endogenous growth model, marginal revenue is equal to marginal product minus the cost of energy needed to operate capital goods. Consider the case where the final good producer is already operating at a point where $A_{N,t}(i)L_t^{1-\alpha}X_t(i)^\alpha = A_{E,t}(i)E_t(i)$. If the final good producer purchases more capital, he receives no increase in output unless there is a corresponding increase in energy purchased. The final good producer realizes this when making optimal decisions and adjusts demand for capital accordingly. This iso-elastic form for inverse demand maintains the tractability of the model.

Monopolist providers of capital goods must decide on optimal production levels and optimal research allocations. See appendix 8.3 for a formal derivation of the monopolists' behavior. As usual, monopolists set price equal to a constant markup over unit costs. Since capital goods must be rented from consumers, the unit cost is given by the rental rate, r_t. Thus, monopolist optimization yields

$$p_{X,t}(i) = \frac{1}{\alpha}r_t, \tag{15}$$

$$X_t(i) = \alpha^{\frac{2}{1-\alpha}} r_t^{\frac{1}{\alpha-1}} A_{N,t}(i)^{\frac{1}{1-\alpha}} L_t\big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}, \tag{16}$$

$$\bar{\pi}_{X,t}(i) = (\frac{1}{\alpha} - 1)\alpha^{\frac{2}{1-\alpha}} r_t^{\frac{\alpha}{\alpha-1}} A_{N,t}(i)^{\frac{1}{1-\alpha}} L_t\big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}, \tag{17}$$

where $\bar{\pi}_{X,t}(i)$ is production profits (i.e., profits excluding research costs) of the monopolist.

To understand research dynamics, it is helpful to look at the relative prices for research inputs,

$$\frac{p^R_{E,t}(i)}{p^R_{N,t}(i)} = \frac{p_{E,t}A_{N,t}(i)}{A_{E,t}(i)^2\big[1 - (\frac{p_{E,t}}{A_{E,t}(i)})\big]} \frac{\eta_E R_{E,t}^{-\lambda} A_{E,t-1}}{\eta_N R_{N,t}^{-\lambda} A_{N,t-1}}, \tag{18}$$

where $p^R_{J,t}(i)$ is the rent paid to research inputs used by firm i to improve technological characteristic J at time t. Their are several forces affecting on the returns to R&D investment. First, increases in the price on energy increase the relative return to investing in energy efficiency. Second, the return to investing in a particular type of R&D is increasing in the efficiency of research in that sector. Research efficiency, in turn, depends on inherent productivity, η_J, accumulated knowledge, $A_{J,t-1}$, and the amount of congestion, $R_{J,t}^{-\lambda}$. Since energy and non-energy inputs are complements in production, increases in $A_{N,t}(i)$ raise the return to investing in $A_{E,t}(i)$ and vice versa. These effects, however, are asymmetric. To maximize profits, monopolists balance two forces that drive demand for their products: 'output-increasing' technological progress, $A_{N,t}(i)$, and 'cost-saving' technological progress, $A_{E,t}(i)$. The asymmetry occurs because energy efficiency, $A_{E,t}(i)$, has a negative and concave effect of the effective cost of energy, $\frac{p_{E,t}}{A_{E,t}(i)}$. Conversely, proportional increases in $A_{N,t}(i)$ always lead to proportional increases in output.

In the usual DTC model, this analysis would demonstrate the role of market size and price effects in research incentives. As demonstrated in equation (18), however, aggregate inputs do not affect R&D decisions in this model. In other words, market size effects play no role in this model. This occurs because innovators are not deciding what market to enter and, instead, are deciding how to augment different characteristics of their goods. Moreover, the price effects in this model differ from the standard approach. In particular, if monopolists were entering different industries, they would not be motivated by how energy efficiency improvements affect the demand for capital goods in the non-energy sector. Thus, the price effects that drive the decisions in this model would be externalities in the standard approach. Thus, the modeling strategy developed here significantly affects the incentives for research and development when compared to the standard approach. This new theoretical approach is motivated by the fact that declines in the final-use energy intensity of output have been the primary driver of decreases in the carbon intensity of output (see figure 1 and the corresponding discussion in Section 3).

Given that all firms use common technology at the start of the period, they make identical R&D decisions and, as a result, they end the period with identical technology. Moreover, there is a unit mass of monopolists. Thus, $R_{J,t}(i) = R_{J,t}\ \forall i, J, t$. The optimal research allocations are given by the implicit solution to (19) and (20),

$$\frac{1+\eta_N(1-R_{E,t})^{1-\lambda}}{\eta_N(1-R_{E,t})^{-\lambda}} = \frac{1+\eta_E R_{E,t}^{1-\lambda}}{\eta_E R_{E,t}^{-\lambda}}\Big[\frac{A_{E,t-1}}{p_{E,t}}(1+\eta_E R_{E,t}^{1-\lambda})-1\Big], \tag{19}$$

$$R_{N,t} = 1-R_{E,t}. \tag{20}$$

Equation (19) can also be rewritten as

$$R_{E,t} = \frac{\sqrt{\frac{p_{E,t}}{A_{E,t-1}}}\sqrt{\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N(1-R_{E,t})^{-\lambda}}+\eta_E R_{E,t}^{-\lambda}+1}-1}{\eta_E R_{E,t}^{-\lambda}}, \tag{21}$$

which readily highlights the simple closed form solution in the special case where $\lambda = 0$.

The consumer problem yields

$$\Big(\frac{c_t}{c_{t+1}}\Big)^{-\theta} = \frac{\beta r_{t+1}}{(1+n)}. \tag{22}$$

Noting that all monopolists make the same decisions and that there is a unit mass of monopolists, the real interest rate is given by

$$r_t = \alpha^2 A_{N,t}\Big[1-\frac{p_{E,t}}{A_{E,t}}\Big]L_t^{1-\alpha}K_t^{\alpha-1}, \tag{23}$$

where the market clearing condition from equation (10) has been applied.

4.3 Equilibrium

DEFINITION 1: A *competitive equilibrium* is a sequence of prices, $\{w_t, p_{X,t}, r_t, p_t^R, p_{E,t}\}_{t=0}^{\infty}$, allocations, $\{C_t, K_t, L_t, E_t, R_{N,t}, R_{E,t}\}_{t=0}^{\infty}$, and technology levels $\{A_{N,t}, A_{E,t}\}_{t=0}^{\infty}$, such that each of the following conditions holds $\forall t$:

- Optimal research allocations solve (19) and (20).
- The law of motion for technology solves (18), noting that all monopolists make identical decisions.
- Consumer behavior follows the Euler equation, (22).
- Factor prices are given by (4), (45), (14), and (23), noting that all monopolists make identical decisions.
- The economy obeys laws of motion for total extracted energy, (5), and population, (12).
- The economy obeys market clearing conditions for capital goods, (10), and final goods, (7).
- Initial Conditions $A_{J,0}$ for $J \in [E, N]$, K_0, L_0, and $\bar{E}_0$ are given.

4.4 Balanced Growth Path

DEFINITION *2*: A *balanced growth path* occurs when final output, technology, and consumption grow at constant rates.

On a balanced growth path (BGP), research allocations remain fixed. Considering equations (19) and (20), this implies that $\frac{A_{E,t-1}}{p_{E,t}}$ is constant. Intuitively, this occurs because of the non-linear relationship between energy efficiency, $A_{E,t}$, and the cost of energy per unit of output, $\frac{p_{E,t}}{A_{E,t}}$. When energy prices increase, monopolists have greater incentive to invest in energy efficient technology, but this incentive dissipates as technology improves. As a result, on a BGP both energy prices and energy efficient technology grow at the same constant rate, g_E^*.[21] Thus, the increasing price of energy is exactly offset by improvements in energy efficiency.

DEFINITION *3*: The *energy share of expenditure* (E_{Share}) is total resources paid to energy producers as a fraction of final output, $\frac{p_{E,t}E_t}{Y_t}$.

[21]For the price of energy to grow at a constant rate, energy use must also grow at a constant rate, which will occur on the BGP.

Given that energy prices and energy efficient technology grow at the same rate on the BGP, it is straightforward to show that the energy share of expenditure is constant:

$$E_{share} = \frac{p_{E,t}/A_{E,t}}{1 - p_{E,t}/A_{E,t}} \tag{24}$$

$$= \frac{\frac{p_{E,t}}{A_{E,t-1}(1+g_E^*)}}{1 - \frac{p_{E,t}}{A_{E,t-1}(1+g_E^*)}}. \tag{25}$$

Thus, despite the Leontief nature of production, the model still delivers a constant long-run energy expenditure share. The constant relationship between energy efficiency and the price of energy also demonstrates the remaining properties of the balanced growth path, which match those of the standard neoclassical growth model with monopolistic competition. To see this, note that

$$TFP_t \equiv \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}} \tag{26}$$

$$= A_{N,t}\big[1 - \frac{p_{E,t}}{A_{E,t}}\big]. \tag{27}$$

Thus, on the BGP, TFP grows at a constant rate, g_N^*. Since the consumer problem is standard, the model now reduces to the neoclassical growth model with monopolistic competition. Thus, the putty-clay model with directed technical change has the usual BGP properties.

Proposition 1: On a *balanced growth path*, each of the following holds true:

1. Output per worker and consumption per worker grow at a constant rate, $g_R^* = (1+g_N^*)^{\frac{1}{1-\alpha}} - 1$.
2. Total output and the capital stock grow at a constant rate, $(1+g_R^*)(1+n)-1$, which implies that the capital-output ratio is fixed.
3. The real interest rate, r_t, is constant.
4. The expenditure shares of energy, capital, and labor are all constant.
5. Energy use grows at rate $g_M^* = \frac{1+g_N^*}{1+g_E^*}[(1+g_N^*)(1+n)]^{\alpha}(1+n)^{(1-\alpha)} - 1$, which may be positive or negative.

Proof. The intuition follows from the preceding discussion, and a formal proof is provided in section 8.5.[22] □

4.5 Comparison to Cobb-Douglas

As mentioned in the introduction, the standard approach in climate change economics is to treat energy as a Cobb-Douglas component of the aggregate production function (Nordhaus and Boyer,

[22] In all quantitative applications, the economy converges to the BGP. I have not shown formally that the economy necessarily converges to the BGP from any initial condition.

2003; Golosov et al., 2014). In this case, the energy expenditure share is always constant. Thus, the Cobb-Douglas approach can match long-run elasticity of substitution between energy and non-energy inputs, but cannot match the near-zero short-run elasticity of substitution. By contrast, the short-run elasticity of substitution in the putty-clay model developed here is exactly zero. Since climate change economics is inherently concerned with long-run policy questions, it has been posited that IAMs may still provide accurate predictions about the reaction of energy use to public policy interventions over the relevant time frame, even if they cannot match short-run responses (Golosov et al., 2014). A key component of the quantitative section will be to compare the outcomes of the Cobb-Douglas model to that of the putty-clay model developed in this paper. In the current section, I derive a few key properties of the Cobb-Douglas model to facilitate this comparison.

The standard Cobb-Douglas production function is given by:

$$Q_t^{CD} = A_t^{CD} K_t^{\alpha} E_t^{\nu} L_t^{1-\alpha-\nu},$$

where A_t^{CD} grows at an exogenous rate g_{CD}. Since energy extraction costs $p_{E,t}$ units of the final good, final output is given by

$$Y_t^{CD} = (1-\nu) A_t^{CD} K_t^{\alpha} E_t^{\nu} L_t^{1-\alpha-\nu}.$$

As a result, the energy expenditure share under Cobb-Douglas is given by

$$E_{share}^{CD} = \frac{\nu}{1-\nu}.$$

Thus, even on the transition path or in response to new energy taxes, the energy share of expenditure is constant, which is inconsistent with the data from the United States (see figure 2). When a new tax raises the effective price of energy, therefore, energy use will immediately decrease enough to fully offset the increase in price, leaving the expenditure share unchanged. In the putty-clay model with directed technical change, by contrast, it may take several decades for improvements in energy efficiency to fully offset an increase in energy prices. Moreover, the long-run energy expenditure share in the putty-clay model with directed technical change may react to a new tax regime, especially in the case of continually increasing taxes. The quantitative importance of these differences is of fundamental importance for climate change economics and will be investigated in section 6.[23]

[23] In appendix section 8.6, I explain the calibration procedure for Cobb-Douglas and describe the balanced growth path. I calibrate both models so that they have identical predictions for output and energy use in the absence of environmental taxes. Due to other differences between the models, especially the difference in market structure – monopolistic competition in the putty-clay model with directed technical change and perfect competition in the Cobb-Douglas model – predictions for interest rates and levels (though not growth rates) of consumption and capital differ between the models. Given that incentives for innovation are an important part of the difference between the two models, I maintain these differences in the quantitative analysis.

5 Calibration

5.1 External Parameters

The model is solved in 10 year periods. As discussed above, the consumer side of the problem is standard. Thus, I take several parameters from the existing literature. In particular, I follow Golosov et al. (2014) and set $\alpha = .35$, $\delta = 1$, $\theta = 1$, and $\beta = .860$.[24]

I take trend growth rates and the average energy expenditure share from the data. As discussed above, I use data from 1971-2011. Energy use and energy price data are from the U.S. Energy Information Agency. Prices are only available for fossil fuel energy. Thus, I also only use fossil fuel energy in the analysis. As discussed above, this is a good fit the energy sector in the model, which is motivated by increasing extraction costs in fossil fuel energy sources. Gross Domestic Product data are from the BEA.

Following the structure of the model, I calculate gross output, Q_t, as final output, Y_t, plus energy expenditure. I measure $A_{E,t} = E_t/Q_t$, yielding $g_E^* = .2608$ on the BGP (2.35% annual growth). Assuming that, on average, the economy is on the BGP during the sample period, g_N can be measured as the growth rate of output per capita. This yields $g_N^* = .2606$ on the BGP. The average energy expenditure share in the data is 3.3%, which I take to be the balanced growth level.

Below, I calibrate the R&D sector of the model to match key BGP moments. The BGP is uninformative about research congestion, λ, which measures the trade-off between advances in overall productivity and energy efficiency. As a base value, I take $\lambda = .21$ from Fried (2015), who also captures the congestion of moving scientists from energy-related research to general purpose research, making it a natural starting point for quantitative exercises presented here. I will also consider cases where $\lambda \in [0, .11, .31]$ for robustness.

5.2 R&D Calibration

The key R&D parameters remaining to be calibrated are the inherent efficiencies of each sector, η_N and η_E. To calibrate them, it is also necessary to solve for R_E^*. To start, I re-write the research arbitrage equation in terms of observables,

$$\frac{1+g_E^*}{1+g_N^*} \quad = \quad E_{share}^* \frac{\eta_E}{\eta_N} \Big(\frac{R_E^*}{1-R_E^*}\Big)^{-\lambda}. \tag{28}$$

This equation has a natural interpretation. Monopolists must trade-off the relevant benefits and costs of investing in the two types of technology. E_{share} is a summary measure of the incentive to invest in energy efficiency that fully captures the relative benefits of improving each type of technology. When the energy share of expenditure is higher, monopolists have greater incentive to

[24] I normalize $TFP_0 = E_0 = L_0 = 10$. This normalization simply sets the units of the analysis and has no effect on the quantitative results of the model. I also assume that the economy is on the BGP at time $t = 0$. Given the other parameters in the model, this yields $Y_0 = 84.96$, $K_0 = 6.28$, $p_{E,0} = .28$, $A_{E,0} = 8.72$, $A_{N,0} = 10.33$. These normalizations set the scale for energy sector parameters, ξ and $\bar{E}_1$.

invest in energy efficiency. The remaining terms capture the relative costs, i.e. relative research efficiencies, of investing in the two types of technology. The term $\frac{\eta_E}{\eta_N}$ captures the inherent productivities of the two sectors, while $\left(\frac{R_E^*}{1-R_E^*}\right)^{-\lambda}$ captures the differences in efficiencies due to the differing levels of congestion.

In the data, $g_E^* \approx g_N^*$. Since the measured energy expenditure share is low (3.3%), the relative efficiency of energy efficiency research must be high. Moreover, a substantial fraction of this relative efficiency must come from the inherent productivities. This is true because total productivity growth in each type of technology is an increasing function of R&D inputs devoted to that sector. If the difference in marginal research efficiencies was due only to congestion, then the growth rate of energy efficiency technology, g_E^*, would have to be much smaller than the growth rate of output-increasing technology, g_N^*. Thus, the data strongly suggest that the inherent productivity of energy efficiency research is significantly higher than the efficiency of other types of research.[25]

To complete the R&D calibration, I add the following two equations:

$$g_E^* = \eta_E (R_E^*)^{1-\lambda}, \tag{29}$$

$$g_N^* = \eta_N (1 - R_E^*)^{1-\lambda}. \tag{30}$$

These equations ensure that levels of technological progress match their values in the data, thereby quantifying the intuition given above. As expected, η_E is significantly greater than η_N. The exact values for all of the parameters are provided in table 1.

5.3 Energy Sector Calibration

To calibrate the remaining parameters for the energy sector, I start by noting that, on the BGP, energy use grows at a constant rate, g_M^*. The most important parameter for the energy sector is ι, which captures the rate at which growth in energy use translates into growth in energy prices,

$$\iota = \frac{ln(1 + g_E^*)}{ln(1 + g_M^*)}. \tag{31}$$

In the model, energy taxes will lower energy use and the price energy. This will have the general equilibrium effect of lowering the incentive for energy efficient research and increasing the demand for capital. The size of these effects depends directly on ι.

Next, to ensure that the economy starts in a steady state, it must be the case the total extracted energy grows at a constant rate. Thus, we can calculate the initial level of extracted energy as:

$$\bar{E}_{-1} = g_M^* / E_0, \tag{32}$$

[25] From an environmental perspective, this seems like a very promising result – improvements in energy efficiency can occur with only small amounts of labor reallocation. Despite this optimistic result, the putty-clay model with directed technical change suggests that much less energy is saved in response to new taxes, when compared to the standard Cobb-Douglas approach.

Table 1: Parameters

Parameter	Value	Description	Source
α	.35	Capital Share of Income	PWT
δ	1	Depreciation	Golosov *et al.* (2014)
β	.860	Discount Factor	Golosov *et al.* (2014)
θ	1	Inter-temporal substitution	Golosov *et al.* (2014)
λ	0.21	Research congestion	Fried (2015)
η_E	3.96	Research efficiency	Calibrated
η_N	0.27	Research efficiency	Calibrated
ι	1.72	Energy cost growth	Calibrated
ξ	0.01	Energy cost scale	Calibrated
$\bar{E}_{-1}$	16.53	Initial extracted energy	Calibrated

where $\bar{E}_{-1}$ is the total energy used on the last period before the energy taxes are announced. As noted above, the specific level of $\bar{E}_{-1}$ is uninformative and simply reflects the scale chosen for E_0. Finally, ξ is a scale parameter calibrated to the starting price,

$$\xi = \frac{p_{E,0}}{\bar{E}_{-1}^{\iota}}. \tag{33}$$

5.4 Solving the Model

Conditional on the price of energy, the model can separated into three pieces: the R&D allocations, the standard consumer problem from the neoclassical growth model with monopolistic competition, and the energy sector. The fact that innovation occurs in different characteristics of capital goods, rather than in different sectors, facilitates the solution of the model. In particular, equations (19) and (20) demonstrate that, conditional on the price of energy, the R&D allocations and technology growth rates can be solved independently of the consumer problem. To find the equilibrium, then, I employ the following steps:[26]

1. Guess a vector of energy prices.
2. Solve for productivity paths and R&D allocations using equations (8), (19) and (20), noting that all monopolists make identical research decisions.
3. Solve the neoclassical growth model conditional on the path of productivities using equations (64) - (70) in appendix section 8.5.
4. Back out implied energy use and energy prices using equations (2), (4), and (5). This takes advantage of the fact that (3) holds with equality in all periods.

[26] In all quantitative applications, this procedure is sufficient to find a competitive equilibrium. I have not shown that such a procedure must converge to an equilibrium, or that the competitive equilibrium is unique. In all cases, I use the BGP in the absence of energy taxes to generate the initial guess of energy prices.

5. Check if the initial guess and resulting prices are the same. If they are, then consumers have made optimal decisions taking all future prices as given and the economy is in equilibrium.
6. If the economy is not in equilibrium, start from (1) with a convex combination of initial guess and resulting prices.

6 Quantitative Results

In this section, I examine the effect of energy taxes in the putty-clay model of directed technical change and compare the results to those in the standard Cobb-Douglas model. The time period in the model is ten years. All future policies are announced in the initial period, which I take as 2005 to match the stated objectives of international climate agreements. All policies take effect in 2015. The gap between the announcement and implementation of the policy allows one round of endogenous and directed technical change to occur before comparing the outcomes across the two models. If the policy were unexpected, the final good producer in the Cobb-Douglas model could react, whereas there would be no adjustment in the putty-clay model with directed technical change due to the Leontief structure. Thus, this approach lessens the difference between the two models by not considering the very short run.

6.1 Long-Run Sustainability

Before comparing the effects of policy between the two models, I briefly consider the implications for sustainable economic growth in the putty-clay model with directed technical change. The model predicts that consumption can continue to grow at its current rate, even as extraction costs tend towards infinity in the long run. This result creates a significant difference with the work of Hassler et al. (2012, 2016b), who are focused on the use of oil and the social planner's problem. The intuition for the difference is straightforward. In both cases, effective (technology-inclusive) energy and non-energy inputs must grow and the same rate on the balanced growth path, i.e, $A_{E,t}E_t = A_{N,t}K_t^{\alpha}L_t^{1-\alpha}$. Moreover, total inputs available for research are fixed. In the social planner solution with finite energy, total energy use must decrease along the BGP. Currently, however, energy use in increasing. Thus, in the model of Hassler et al. (2012, 2016b) we cannot currently be on the BGP, and in the long run, some research effort must be moved from non-energy research into energy research to maintain the equality of growth rates between effective energy and non-energy inputs. This implies that the growth rate of general purpose technological progress and consumption must fall.

The model examined in this paper has an infinite supply of energy that can be extracted at increasing cost. Moreover, I examine the outcome in the decentralized economy. In this case, there is no restriction that energy use must be decreasing in the long-run, implying that recent trends can be continued indefinitely. Figure 2 indicates that the long-run average of the energy expenditure share has been roughly constant (at 3.3%) in the United States over the last 40 years, and I take this as evidence that this is approximately the BGP level of the energy expenditure share. This implies, in turn, that growth rates of output and consumption can continue indefinitely on their current paths. In this way, the results presented here build upon Hassler et al. (2012, 2016b) by considering how the existence of unlimited energy supplies (even with extraction costs tending towards infinity) and consideration of the decentralized equilibrium affect long-run predictions regarding consumption growth. This result is consistent with the Cobb-Douglas approach, which also assumes that the energy expenditure share will remain and its current level (Golosov et al., 2014; Barrage, 2014).

6.2 Energy Taxes

To best understand the quantitative impacts of the new model of energy use developed in this paper, it is necessary to consider a realistic path of future energy taxes. Under the recent Paris Agreement on climate change, the United States aims to adopt policies consistent with a 80% reduction in carbon emissions by the year 2050, when compared to 2005 levels. I apply taxes such that half of this gain, a 40% reduction, comes from reductions in energy use.[27] The evidence in figure 1 suggests that energy efficiency has been responsible for well more than half of past decreases in the carbon intensity of output. I consider a path of proportional energy taxes that grow at a constant rate,

$$\tau_t = 1 \cdot (1 + g_\tau)^{\frac{t-2005}{10}}. \tag{34}$$

To achieve the environmental goals given above, the putty-clay model with directed technical change requires $g_\tau = .47$, implying that heavy energy taxation is necessary to achieve environmental policy goals outlined in prominent international agreements. In particular, this yields a 6.9-fold tax rate in 2055. All taxes are rebated to consumers in a lump sum fashion.

Figure 3 presents the outcomes of the model under the path of proportional energy taxes outlined above. In particular, it demonstrates the paths of energy use, output, TFP, consumption, and capital from 2005 to 2115. All outcomes are given as a fraction of the baseline scenario, which has zero energy taxation. As expected, energy taxes simultaneously increase the energy expenditure share and decrease energy use. In other words, monopolists have increased incentive to invest in energy efficiency, but this incentive is insufficient to improve energy efficiency enough to fully offset the increase in the price of energy. In this way, it is already apparent that the results will differ from those in the Cobb-Douglas model.

By 2055, the economy experiences an 6.8% decrease in consumption and 4.3% decrease in TFP relative to the baseline. Energy use plummets to 9.0% of baseline by the end of the century. At the same time, consumption decreases by 10.9% and TFP is 6.6% lower than in the baseline scenario without energy taxes. Discounted back 100 years, this lost consumption will have a very small impact of the current-day utility of the representative household. Within climate change economics, however, there is a spirited debate as to whether the discount rate held by individual

[27]Since the model is solved in ten year periods, I choose taxes such that the 40% reduction occurs by 2055.

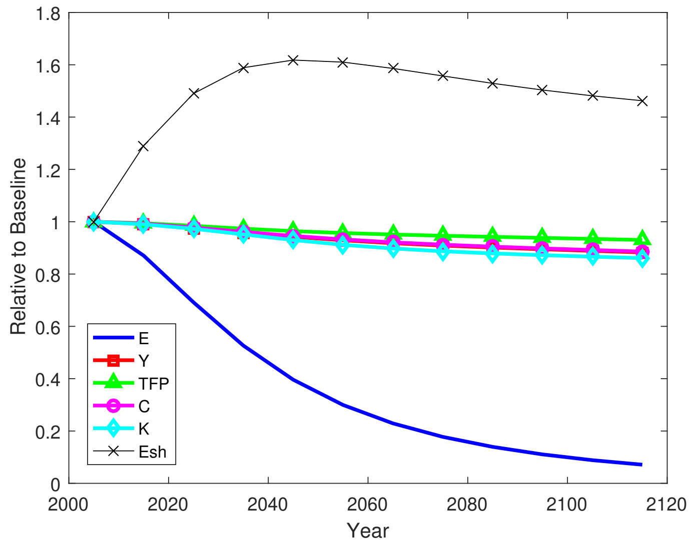

Figure 3: This table demonstrates the effect of energy taxes in the putty-clay model with directed technical change. Energy taxes are proportional to the price of energy and grow at a constant rate: $\tau_t = 1 \cdot (1+g_\tau)^{\frac{t-2005}{10}}$, with $g_\tau = .47$. This level of taxation achieves a 40% reduction in energy in by 2055, compared to 2005 levels. All taxes are rebated to consumers in a lump sum fashion. All outcomes in the figure are given as a fraction of the outcomes in the baseline scenario, which has no energy taxation.

consumers is appropriate for social welfare calculations (Nordhaus, 2007; Stern, 2013; Barrage, 2016). Given that consumption losses are back-loaded, discount rate choices would have significant effects on welfare in this setting.

Figure 4 repeats the analysis for the standard Cobb-Douglas model with exogenous technological progress. Once again, all outcomes are given relative to the baseline scenario with no energy taxes. The effect of policy in the Cobb-Douglas approach differs considerably from the putty-clay model with directed technical change. In this case, $g_\tau = .36$ is sufficient to achieve a 40% reduction in energy use by 2055, and $\tau_{2055} = 4.65$. To achieve the environmental policy priories, consumption decreases by 2.0% in 2055 and 3.7% by the end of the century, relative to a 'business as usual' case without taxes. By the end of the century, energy use is 13% of baseline levels.

As expected, the energy share of expenditure is essentially unchanged in the Cobb-Douglas model.[28] Thus, energy use decreases by enough to fully offset the increase in energy prices. This can be seen in how quickly the Cobb-Douglas model responds to new taxes. In 2015, energy use decreases by almost 30% compared to the baseline, in comparison to a 10% decrease in the putty-clay model with directed technical change. This occurs even though the tax rate is lower in the Cobb-Douglas model.

[28]The slight decrease in the energy expenditure share is due to the lump sum tax rebates. The expenditure share of energy in gross output is constant, but after taxes are implemented, a proportion of energy expenditure is rebated to consumers.

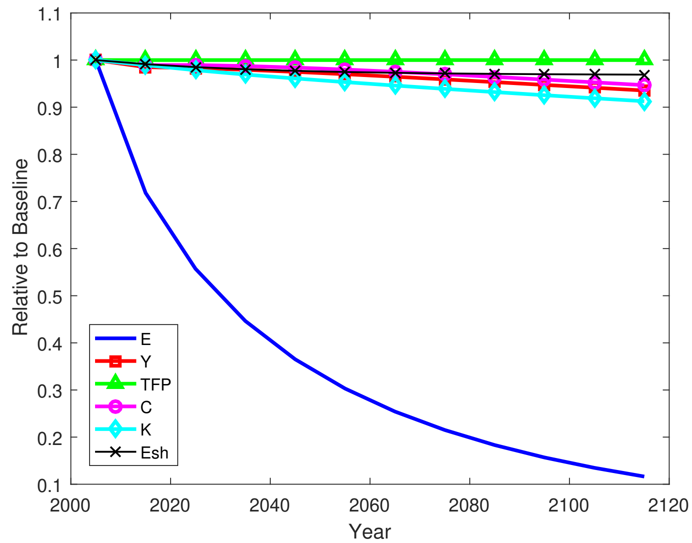

Figure 4: This table demonstrates the effect of energy taxes in the standard Cobb-Douglas model with exogenous technological progress. Energy taxes are proportional to the price of energy and grow at a constant rate: $\tau_t = 1 \cdot (1 + g_\tau)^{\frac{t-2005}{10}}$, with $g_\tau = .36$. This level of taxation achieves a 40% reduction in energy in by 2055, compared to 2005 levels. All taxes are rebated to consumers in a lump sum fashion. All outcomes in the figure are given as a fraction of the outcomes in the baseline scenario, which has zero energy taxation.

Figure 5 provides a direct comparison of energy use and consumption in the two models when applying the same path of energy taxes, specifically those necessary to achieve environmental policy priorities in the Cobb-Douglas model. Thus, the analysis quantifies the error that would occur if policy was designed with the Cobb-Douglas model, but the true economy was putty-clay with directed technical change. Energy use is measured as a fraction of the 2005 level, and consumption is measured relative to the baseline.[29]

When applying the requisite taxes from the Cobb-Douglas model to the putty-clay model with directed technical change, energy use in 2055 declines by 24% when compared to 2005 levels, missing the environmental target 16 percentage points. At the same time, forgone consumption is 2.7 percentage points higher than would be expected by policy-makers using the Cobb-Douglas model. Despite the goals of policy, what matters for overall environmental conditions is the cumulative difference in energy use, which is given by the area between the two energy use curves. Over the course of the coming century, cumulative energy use is 20% higher in the putty-clay model with directed technical change. These results further illuminate the important differences between the two models and demonstrate that policy designed for the Cobb-Douglas model would yield drastically different outcomes in a world more closely resembling the putty-clay model with directed technical change.

[29]Given the difference in market structure, the baseline level of consumption, but not the growth rate of consumption, differs in the two models.

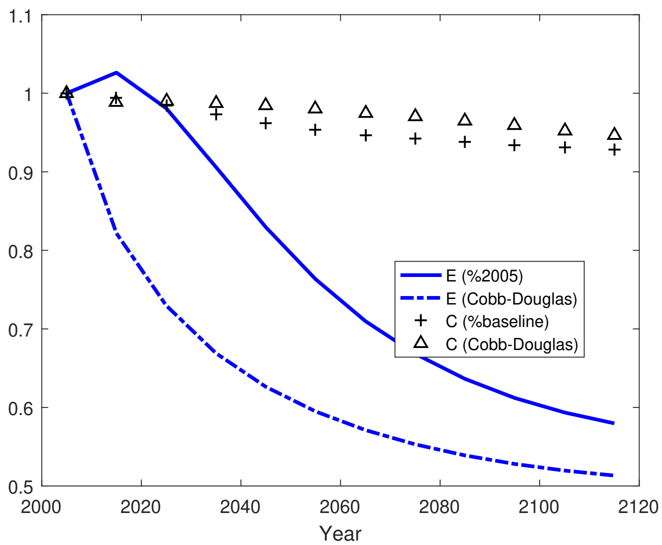

Figure 5: This table demonstrates the difference between the putty-clay model of directed technical change and the standard Cobb-Douglas model with exogenous technological progress. Energy taxes are proportional to the price of energy and grow at a constant rate: $\tau_t = 1 \cdot (1 + g_\tau)^{\frac{t-2015}{10}}$, with $g_\tau = .36$. In the Cobb-Douglas model with exogenous technical change, this level of taxation achieves a 40% reduction in energy in by 2055, compared to 2005 levels. All taxes are rebated to consumers in a lump sum fashion. Energy use is measured as a fraction of 2005 levels. Consumption is measured relative to the baseline, which does not include energy taxes. The baseline level of consumption differs in the two models.

In appendix section 8.7, I demonstrate the robustness of these core results to alternate scenarios. In particular, I consider alternate values of the congestion parameter, λ, and the results when energy prices increase at an exogenous rate. All robustness exercises are conducted with the same path of energy taxes to facilitate comparison between the results. Also, I re-calibrate the putty-clay model for each new value of λ.

Decreasing research congestion yields more similar results between the two models by allowing for faster energy reductions in the putty-clay model. Even in the extreme case of zero research congestion, however, the difference between the models is large (see figure 7). When prices are exogenous, smaller amounts of taxation are needed to meet environmental policy goals. In particular, $g_\tau = .4$ in the putty-clay model with directed technical change. This result is intuitive since endogenous price movements mitigate the effect of taxes. The results are presented in figures 12–14.

7 Conclusion

In this paper, I build a tractable putty-clay model of directed technical change and use it to analyze the effect of environmental policies on energy use in the United States. The model matches several key data patterns that cannot be explained by the standard Cobb-Douglas approach used in climate change economics. The results suggest that large taxes are necessary to achieve environmental goals laid out in international agreements. In particular, the new putty-clay model of directed technical change suggests that a 6.9-fold energy tax in 2055 is necessary to achieve policy goals consistent with the 2016 Paris Agreement and that such a tax would lead to 6.8% lower consumption when compared to a world without taxes. By contrast, the standard Cobb-Douglas approach suggests that a 4.7-fold tax rate in 2055 is sufficient, which leads to a 2% decrease in consumption. When applying the same tax rate to both models, the new putty-clay model of directed technical change predicts 20% more energy use over the next century. Thus, compared to the standard approach, the new model predicts that greater taxation and more forgone consumption are necessary to achieve environmental policy goals.

There are several possible extensions to the analysis presented here that would provide important insights into environmental policy questions. The most direct extension would entail adding a third margin of technological investment in clean versus dirty technology. In this case, it would be possible to gain a more complete understanding of the effect of carbon taxes on emissions. Combined with a model of the carbon cycle, such an analysis could yield important updates to existing estimates of optimal carbon taxes. It would also allow for the comparison of important second-best policies, such as subsidies for renewable energy – which would limit the incentive to improve energy efficiency – and energy taxes or efficiency mandates, which provide no incentive to invest in clean energy sources.

Another extension would be to expand the geographic scope. The analysis presented here focuses on a single economy, but there are important implications for a multi-region world. In particular, existing analyses with exogenous technological progress suggest that unilateral policy actions among rich countries will have small impacts on overall carbon emissions (Nordhaus, 2010). In a world with endogenous technological progress and diffusion or trade, however, unilateral policies would improve worldwide energy efficiency, leading to greater environmental benefit. This magnifies the difference with the standard Cobb-Douglas approach, where substitution of capital for energy in one country would have no direct impact on other countries. The positive implications of these international spillovers could potentially outweigh the more pessimistic conclusions about the reaction of energy use to taxation that result from considering the putty-clay model with directed technical change.

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8 Appendix

8.1 Microfoundation

In this section, I provide a simple microfoundation for the aggregate production function, (2), which highlights the continuity with the existing DTC literature. Consider the following equation,

$$\begin{aligned} Y_t &= L_t^{1-\alpha} \int_0^1 A_{N,t}(i) X_t(i)^\alpha \frac{E_t(i)}{R_t(i)} di && (35) \\ s.t. &\quad E(i) \leq R(i), && (36) \end{aligned}$$

where L_t is the aggregate (and inelastic) labor supply, $A_{N,t}(i)$ is the the quality of capital good i, $X_t(i)$ is the quantity of capital good i, $R_t(i)$ is the amount of energy required to run capital good i at full capacity, and $E_t(i)$ is the amount of actual energy used to run capital good i.

It is easiest to start by comparing this equation to the standard production function used in DTC models (and, more generally, in many endogenous growth models): $Y_t = L_t^{1-\alpha} \int_0^1 A_{N,t}(i) X_t(i)^\alpha di$. Here, final production is the combination of a set of processes, each of which combines aggregate labor, L, with a specific capital good, $X_t(i)$. The effectiveness of each process is determined by the quality of the capital good, $A_{N,t}(i)$. Each of these processes is perfectly substitutable with the others, though each is used in equilibrium because of diminishing returns. To this standard approach, I add energy requirements. In particular, I assume that each piece of capital requires a specific amount of energy, $R_t(i)$, to run at full capacity. If the amount of energy, $E_t(i)$, devoted to process i is less than $R_t(i)$, then the final goods producer receives less than the full benefit of that process. In particular, if the final good producer allocates, say, 80% of the required energy, i.e. $E_t(i)/R_t(i) = .8$, then it receives 80% of full capacity output.

To actually work with the model, it is necessary to assign a functional form to the energy requirement function, $R(i)$. Consider the following specification:

$$R_t(i) = A_{N,t}(i) L_t^{1-\alpha} X_t(i)^\alpha \frac{1}{A_{E,t}(i)}, \qquad (37)$$

where $A_{E,t}(i)$ is a measure of energy efficiency. There are several key things to note about this function. First, consistent with the econometric literature on energy use, energy requirements depend both on the amount of capital and the amount of labor being used in the production process (Van der Werf, 2008; Hassler et al., 2012). Second, consistent with both the econometric and DTC literatures, improvements in non-energy technology, $A_{N,t}(i)$, raise energy requirements (Smulders and De Nooij, 2003; Van der Werf, 2008; Hassler et al., 2012, 2016b; Fried, 2015). In appendix section 8.2, I solve the final goods producer problems and demonstrate that the two production functions, (2) and (35), are equivalent.

8.2 Final Good Producer Problem

In this section, I show derive the inverse demand functions (13) and (14) and demonstrate that the basic production function (2) is equivalent to the microfounded version in appendix 8.1. Consider the maximization of (35) subject to (36) with $\lambda_t(i)$ as the Lagrange multiplier attached to capital good i,

$$\mathcal{L} = L_t^{1-\alpha}\int_0^1 A_{N,t}(i)X_t(i)^\alpha \frac{E_t(i)}{R_t(i)}di - w_tL_t - \int_0^1 \; p_{X,t}(i)X_t(i)\; di - p_{E,t}\int_0^1 \; E_t(i)\; di \\ - \int_0^1 \lambda_t(i)\big[E_t(i) - R_t(i)\big]di. \quad (38)$$

Complementary slackness implies

$$\lambda_t(i)\big[E_t(i) - R_t(i)\big] = 0 \; \forall i. \quad (39)$$

Substituting $R_t(i)$ from (37) yields

$$\mathcal{L} = L_t^{1-\alpha}\int_0^1 A_{E,t}(i)E_t(i)di - w_tL_t - \int_0^1 \; p_{X,t}(i)X_t(i)\; di - p_{E,t}\int_0^1 \; E_t(i)\; di \\ - \int_0^1 \lambda_t(i)\big[E_t(i) - A_{N,t}(i)L_t^{1-\alpha}X_t(i)^\alpha \frac{1}{A_{E,t}(i)}\big]di. \quad (40)$$

Now, complementary slackness implies

$$\lambda_t(i)\big[E_t(i) - A_{N,t}(i)L_t^{1-\alpha}X_t(i)^\alpha \frac{1}{A_{E,t}(i)}\big] = 0 \; \forall i. \quad (41)$$

The solution to equations (40) and (41) yields the optimal behavior of the final goods producer. Importantly, this is exactly the same problem that arises from maximizing (2) subject to (3). This can be seen by multiplying and dividing both (41) and the last term of (40) by $A_E(i)$ and redefining the Lagrange multiplier appropriately. Thus, the two production functions are equivalent.

I focus on the case where (3) holds with equality. For this to be true, it is sufficient, but not necessary, to assume that $\delta = 1$, as noted in the main text. Consider the first order conditions of the final good producer,

$$\begin{aligned} (\frac{\partial \mathcal{L}}{\partial E_t(i)}): \quad \lambda_t(i) &= 1 - \frac{p_{E,t}}{A_{E,t}(i)}, && (42) \\ (\frac{\partial \mathcal{L}}{\partial X_t(i)}): \quad \lambda_t(i) &= \frac{p_{X,t}(i)}{\alpha A_{N,t}(i)L_t^{1-\alpha}X_t(i)^{\alpha-1}}, && (43) \\ (\frac{\partial \mathcal{L}}{\partial L_t}): \quad \lambda_t(i) &= \frac{w_t}{(1-\alpha)A_{N,t}(i)L_t^{-\alpha}X_t(i)^\alpha}. && (44) \end{aligned}$$

Substituting (43) and (44) into (42), respectively, and multiplying through yields

$$p_{X,t}(i) = \alpha A_{N,t}(i)\big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big] L_t^{1-\alpha} X_t(i)^{\alpha-1}, \tag{45}$$

$$w_t = (1-\alpha) A_{N,t}(i)\big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big] L_t^{-\alpha} X_t(i)^{\alpha}. \tag{46}$$

Thus, we have arrived at equation (13) and (14) from the text. They key result here is that inverse demand is iso-elastic, which allows for the usual simple closed forms. Moreover, the expenditure shares of all factors will be constant.

8.3 Monopolist Problem

The monopolist maximizes profits subject to demand and research productivity constraints:

$$max\ \pi_{X,t}(i) = p_{X,t}(i) X_t(i) - r_t X(i) - p^R_{E,t} R_E(i) - p^R_{N,t} R_N(i), \tag{47}$$

$$\tag{48}$$

subject to

$$p_{X,t}(i) = \alpha A_{N,t}(i)\big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big] L_t^{1-\alpha} X_t(i)^{\alpha-1}, \tag{49}$$

$$A_{J,t}(i) = \big[1 + \eta_J R_{J,t}(i) R_{J,t}^{-\lambda}\big] A_{J,t-1}, \quad J \in \{N, E\}, \tag{50}$$

$$R_{J,t}(i) \in [0,1], \quad J \in \{N, E\}. \tag{51}$$

First, substitute (49) into (47) and take the first order condition with respect to $X(i)$. Constraints (50) and (51) are independent of the production level, $X_t(i)$. Hence, we get the standard first order conditions and results,

$$r_t = \alpha^2 A_{N,t}(i)\big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big] L_t^{1-\alpha} X_t(i)^{\alpha-1}, \tag{52}$$

$$X_t(i) = \alpha^{\frac{2}{1-\alpha}} r^{\frac{-1}{1-\alpha}} A_{N,t}(i)^{\frac{1}{1-\alpha}} L_t \big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}, \tag{53}$$

$$p_{X,t}(i) = \frac{1}{\alpha} r_t. \tag{54}$$

Next, to find optimal profits, we can re-write the monopolist problem after substituting in results we have found so far:

$$max\ \pi_{X,t}(i) = \tilde{\alpha} r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}(i)^{\frac{1}{1-\alpha}} L_t \big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}} - p^R_{E,t} R_{E,t}(i) - p^R_{N,t} R_{N,t}(i) \tag{55}$$

subject to

$$A_{J,t}(i) \;=\; \big[1+\eta_J R_{J,t}(i) R_{J,t}^{-\lambda}\big] A_{J,t-1}, \quad J \in \{N,E\}, \tag{56}$$

$$R_{J,t}(i) \;\in\; [0,1], \quad J \in \{N,E\}, \tag{57}$$

where $\tilde{\alpha} = (\frac{1}{\alpha}-1)\alpha^{\frac{2}{1-\alpha}}$. The first order conditions for technology levels and research scientist allocations yields

$$p^R_{N,t} \;=\; \psi A_{N,t}^{\frac{\alpha}{1-\alpha}} \big[1-\frac{p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}} \eta_N R_{N,t}^{-\lambda} A_{N,t-1}, \tag{58}$$

$$p^R_{E,t} \;=\; \psi A_{N,t}^{\frac{1}{1-\alpha}} p_{E,t} A_{E,t}(i)^{-2} \big[1-\frac{p_{E,t}}{A_{E,t}(i)}\big]^{\frac{\alpha}{1-\alpha}} \eta_E R_{E,t}^{-\lambda} A_{E,t-1}, \tag{59}$$

where $\psi = \frac{\tilde{\alpha}}{1-\alpha} r_t^{\frac{-\alpha}{1-\alpha}} L_t$ is common to both terms. In the next section, I shown the optimal research allocations resulting from these first order conditions. Taking ratios of these first order conditions yields (18) in the main text.

8.4 R&D Allocations

In this section, I derive the optimal research allocations given in equations (19), (20), and (21). First, note that $R_{J,t}(i) = R_{J,t}\ \forall i,t$. This occurs because all monpolists make identical decisions and there is a unit mass of monopolists. This also implies that $A_{J,t}(i) = A_{J,t}\ \forall i,t$. Also, factor mobility ensures that $p^R_{E,t} = p^R_{N,t}\ \forall t$. Thus, equation (18) can then be re-written as

$$\frac{1}{\eta_E R_E^{-\lambda}} \frac{A_{E,t}}{A_{E,t-1}} \big[\frac{A_E}{p_{E,t}} - 1\big] \;=\; \frac{A_{N,t}}{A_{N,t-1}} \frac{1}{\eta_N R_N^{-\lambda}}. \tag{60}$$

Replacing growth rates and technology levels with the values given by (8) yields (19) and applying the resource constraint (9) yields (20). I now continue to derive (21). Multiplying by the denominator on the left hand side and distributing the left-hand side terms outside the bracket and then reversing the sides of the equation yields

$$\big[(1+\eta_E R_E^{1-\lambda})^2 \frac{A_{E,t-1}}{p_{E,t}} - (1+\eta_E R_E^{1-\lambda})\big] = \frac{\eta_E R_E^{-\lambda}}{\eta_N (1-R_E)^{-\lambda}} \big(1+\eta_N (1-R_E)^{1-\lambda}\big). \tag{61}$$

Now, isolating the term including the energy price yields

$$[(1+\eta_E R_E^{1-\lambda})^2 \frac{A_{E,t-1}}{p_{E,t}} = \frac{\eta_E R_E^{-\lambda}}{\eta_N (1-R_E)^{-\lambda}} + \eta_E R_E^{-\lambda} + 1. \tag{62}$$

Now, (21) can be derived by multiplying through by $\frac{p_{E,t}}{A_{E,t-1}}$, taking the square root of both sides, subtracting one, and dividing by $\eta_E R_E^{-\lambda}$.

8.5 Solving the Model

In this section, I solve the consumer portion of the model in intensive form. This simultaneously demonstrates the conditions listed in Proposition 1 and demonstrates how to solve the model computationally as discussed in section 5.4. As described in that section, I can take the path of productivities as given for this exercise. This portion of the model is almost equivalent to a standard neoclassical growth model. The only differences are a) the interest rate must be adjusted for monopolistic competition and taxes, and b) the growth rate of TFP may not be constant.

Let τ_t be the proportional energy tax applied at time t. For any variable Z_t, I define:

$$z_t \equiv \frac{Z_t}{L_t A_{R,t}}, \tag{63}$$

where $A_{R,t} = TFP_t^{\frac{1}{1-\alpha}}$ and $TFP = A_{N,t}\big[1 - \frac{p_{E,t}}{A_{E,t}}\big]$. Applying (6), (7), and (10), this yields

$$y_t = k_t^{\alpha}, \tag{64}$$

$$k_{t+1} = \frac{y_t - c_t}{(1+g_{r,t+1})(1+n)}, \tag{65}$$

where $1 + g_{r,t} = \frac{A_{R,t}}{A_{R,t-1}} = (1 + g_{TFP,t})^{\frac{1}{1-\alpha}}$. Moreover, the Euler equation yields

$$\big(\frac{c_{t+1}}{c_t}\big) = \frac{\beta r_{t+1}}{(1+g_{r,t+1})(1+n)}, \tag{66}$$

where I have taken advantage of the fact that $\theta = 1$.

Finally, when considering the interest rate, it is also important to keep track of the energy tax rate, τ_t. Let $\tilde{A}_{R,t} = A_{N,t}\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\big]$ be TFP adjusted for energy taxes. Then, from equation (16),

$$\begin{aligned} r_t &= \alpha^2\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\big] A_{N,t} K_t^{\alpha-1} L_t^{1-\alpha} && (67)\\ &= \alpha^2\big(\frac{K_t}{\tilde{A}_{R,t} L_t}\big)^{\alpha-1} && (68)\\ &= \alpha^2\big(\frac{A_{R,t}}{\tilde{A}_{R,t}}\big)^{\alpha-1}\big(\frac{K_t}{A_{R,t} L_t}\big)^{\alpha-1} && (69)\\ &= \tilde{\tau}_t \alpha^2 k_t^{\alpha-1}, && (70) \end{aligned}$$

where $\tilde{\tau}_t \equiv \big(\frac{A_{R,t}}{\tilde{A}_{R,t}}\big)^{\alpha-1}$ is the interest rate wedge caused by the introduction of energy taxes.

Thus, the solution to the model is given by (64), (65) (66) and (70), noting that $g_{R,t}$ and $\tilde{\tau}_t$ are determined by the research allocations and can be taken as exogenous for this part of the solution. As described above, this is just the standard neoclassical growth model with a few additions. The α^2 term in (70) is the standard adjustment for monopolistic competition, $\tilde{\tau}_t$ is the wedge in the interest rate caused by carbon taxes, and $g_{R,t}$ may not be constant due to endogenous research allocations.

To find the BGP, I set $\tau = 1$. This refers to a 'business as usual' scenario with no new energy taxes, though a BGP exists with any fixed τ. As discussed in the main text, $g_{TFP} = g_N^*$ on the BGP because $\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]$ is fixed. Thus, $g_R^* = (1 + g_N^*)^{\frac{1}{1-\alpha}} - 1$. This yields

$$\bar{r} = \frac{(1 + g_R^*)(1 + n)}{\beta}, \tag{71}$$

$$\bar{k} = (\frac{\bar{r}}{\alpha^2})^{\frac{1}{\alpha-1}}, \tag{72}$$

$$\bar{y} = \bar{k}^{\alpha}, \tag{73}$$

$$\bar{c} = \bar{y} - (1 + g_R^*)(1 + n)\bar{k}. \tag{74}$$

Thus, r_t is constant, Y_t/L_t and C_t/L_t grow at rate g_R^*, and Y_t and K_t grow at rate $g_Y^* = (1 + g_R^*)(1 + n) - 1$.

At any point in time, energy use is given by

$$E_t = \frac{A_{N,t}}{A_{E,t}} K_t^{\alpha} L_t^{1-\alpha}. \tag{75}$$

On the BGP, therefore, the growth rate of energy is given by

$$g_M^* = \left(\frac{(1 + g_N^*)}{(1 + g_E^*)}(1 + g_Y^*)^{\alpha}(1 + n)^{1-\alpha}\right) - 1. \tag{76}$$

To find the expenditure shares, I apply all of the market clearing conditions to the factor price equations. To start, from equation (14) note that

$$w_t L_t = (1 - \alpha) A_{N,t}[1 - \frac{p_{E,t}}{A_{E,t}}] K^{\alpha} L^{1-\alpha} = (1 - \alpha) Y_t. \tag{77}$$

Next, from (23) and (16),

$$r_t K_t = \alpha^2 A_{N,t}[1 - \frac{p_{E,t}}{A_{E,t}}] K^{\alpha} L^{1-\alpha} = \alpha^2 Y_t. \tag{78}$$

The remaining share, $(1 - \alpha - \alpha^2) Y_t$, is the pre-R&D spending production profits of the monopolists. This can be further divided into pure profits and payments to research inputs. All research inputs are hired at the same rate. By equation (58), total payments to research inputs is given by

$$p_t^R = (\frac{1}{\alpha} - 1)\frac{r_t X_t}{A_{N,t}} \eta_N R_N^{-\lambda} A_{N,t-1} \tag{79}$$

$$= (\frac{1}{\alpha} - 1) \cdot \frac{\eta_N (R_N^*)^{-\lambda}}{1 + g_N^*} \cdot \alpha^2 Y_t, \tag{80}$$

noting that there is a unit mass of research inputs. The remaining share of final output is paid to monopolists as profits.

8.6 The Cobb-Douglas Model

In this section, I derive the BGP results for the Cobb-Douglas model and describe the calibration procedure. Let τ_t be the proportional energy tax applied at time t. To start, I note that, due to perfect competition, aggregate energy use is given by

$$E_t = (\frac{\nu}{\tau_t p_{E,t}})^{\frac{1}{1-\nu}} (A_t^{CD})^{\frac{1}{1-\nu}} K_t^{\frac{\alpha}{1-\nu}} L_t^{\frac{1-\alpha-\nu}{1-\nu}}. \tag{81}$$

This, in turn, yields

$$\begin{aligned} Q_t &= (\frac{\nu}{p_{E,t} \cdot \tau_t})^{\frac{\nu}{1-\nu}} (A_t^{CD})^{\frac{1}{1-\nu}} K_t^{\frac{\alpha}{1-\nu}} L_t^{\frac{(1-\alpha-\nu)}{1-\nu}}, && (82) \\ Y_t &= (1 - \frac{\nu}{\tau})Q_t. && (83) \end{aligned}$$

To find the BGP, I assume $\tau_t = 1$ and consider the 'business as usual' scenario without any new energy taxes. The BGP exists for any fixed tax. I define

$$z_t = \frac{Z_t}{L_t (A_t^{CD})^{\frac{1}{1-\alpha-\nu}} (\tau_t \cdot p_{E,t})^{\frac{-\nu}{1-\nu-\alpha}}}, \tag{84}$$

for any variable Z_t. This notation is specific to appendix section 8.6.

The Euler equation is the same as in the putty-clay case. In intensive form,

$$\frac{c_{t+1}}{c_t} = \frac{\beta r_{t+1}}{(1+g_{CD})^{\frac{1}{1-\nu-\alpha}} (1+\tilde{g}_{P,t+1})^{\frac{-\nu}{1-\nu-\alpha}} (1+n)}, \tag{85}$$

where $1 + \tilde{g}_{P,t+1} = (1 + g_{\tau,t+1})(1 + g_{P,t+1})$ and $1 + g_{\tau,t} = \frac{\tau_t}{\tau_{t-1}}$. The rest of the dynamics are given by

$$\begin{aligned} k_{t+1} &= \frac{y_t - c_t}{(1+g_{CD,t+1})^{\frac{1}{1-\nu-\alpha}} (1+\tilde{g}_{P,t+1})^{\frac{-\nu}{1-\nu-\alpha}} (1+n)}, && (86) \\ y_t &= (1 - \frac{\nu}{\tau}) k_t^{\frac{\alpha}{1-\nu}}, && (87) \\ r_t &= \alpha k_t^{\frac{\alpha-(1-\nu)}{1-\nu}}. && (88) \end{aligned}$$

Thus, on the initial BGP, where energy prices grow at a constant rate, g_P^*, and energy taxes are constant,

$$\begin{aligned} \bar{r} &= \frac{(1+g_{CD}^*)^{\frac{1}{1-\nu-\alpha}} (1+g_P^*)^{\frac{-\nu}{1-\nu-\alpha}} (1+n)}{\beta}, && (89) \\ \bar{k} &= (\bar{r}/\alpha)^{\frac{1-\nu}{\alpha-(1-\nu)}}, && (90) \\ \bar{y} &= (1-\nu)\bar{k}^{\frac{\alpha}{1-\nu}}, && (91) \\ \bar{c} &= \bar{y} - (1+g_{CD}^*)^{\frac{1}{1-\nu-\alpha}} (1+g_P^*)^{\frac{-\nu}{1-\nu-\alpha}} (1+n)\bar{k}. && (92) \end{aligned}$$

As a result, r_t is constant, Y_t/L_t and C_t/L_t grow at rate $(g_R^*)^{CD} = (1+g_{CD}^*)^{\frac{1}{1-\nu-\alpha}}(1+g_P^*)^{\frac{-\nu}{1-\nu-\alpha}} - 1$, and Y_t and K_t grow at rate $g_Y^{CD} = (1+g_R^*)^{CD}(1+n) - 1$.

I calibrate the model to the BGP using the same data as employed for the putty-clay model, leading to observationally equivalent paths for output and energy use. To match the energy expenditure share, I set

$$\frac{\nu}{1-\nu} = 3.3\% \ \Rightarrow \nu = .032. \tag{93}$$

All that remains is to ensure that total output grows at the same rate in the two models, which implies that energy use will also grow at the same rate. Since the energy sector is equivalent in the two models, this further implies that the price of energy will grow at the same rate. Thus, I set $(g_R^*)^{CD} = g_R^*$, where the later comes from the putty-clay model in section 8.5. This implies that

$$g_R^* = (1+g_{CD}^*)^{\frac{1}{1-\nu-\alpha}}(1+g_P^*)^{\frac{-\nu}{1-\nu-\alpha}} - 1 \ \Rightarrow \tag{94}$$

$$g_{CD}^* = (1+g_R^*)^{1-\alpha-\nu}(1+g_E^*)^{\nu} - 1. \tag{95}$$

The calibration yields $g_{CD}^* = .42$, which corresponds to an annual growth rate of 3.5%. The growth rate of TFP is higher in the Cobb-Douglas case because it needs to overcome the drag of rising energy prices to achieve the same BGP rates of growth in consumption and output.

8.7 Robustness Exercises

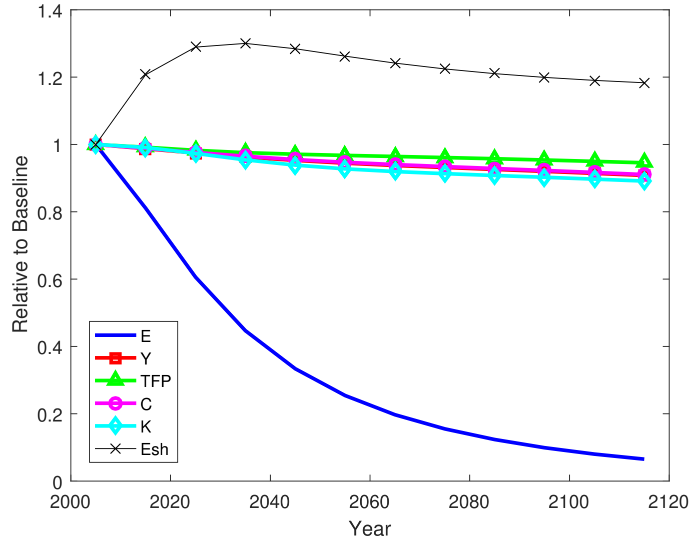

Figure 6: Robustness for table 3 with $\lambda = 0$.

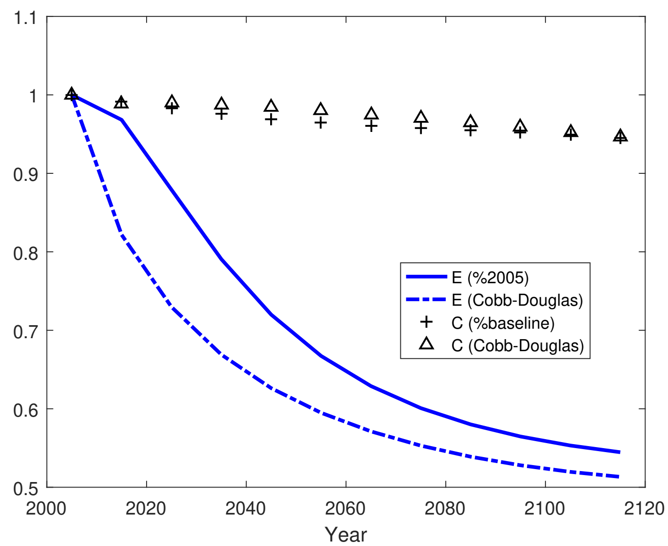

Figure 7: Robustness for table 5 with $\lambda = 0$.

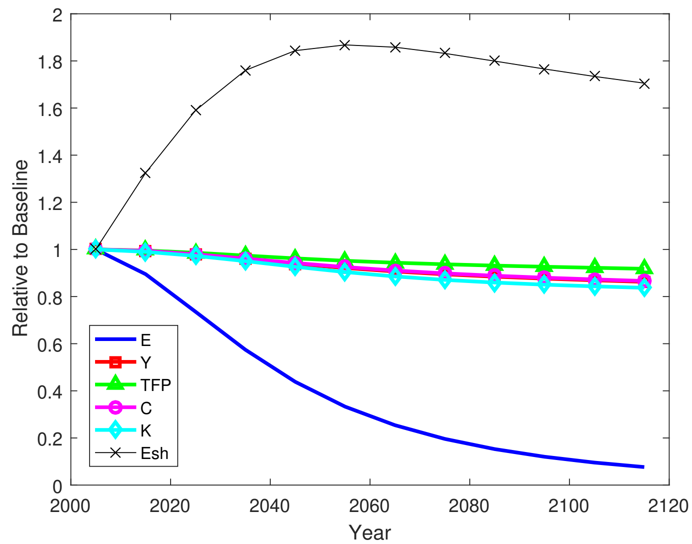

Figure 8: Robustness for table 3 with $\lambda = .31$.

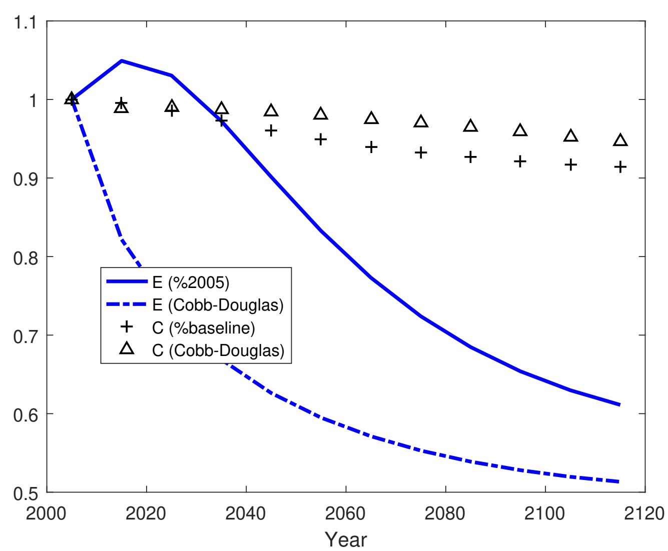

Figure 9: Robustness for table 5 with $\lambda = .31$.

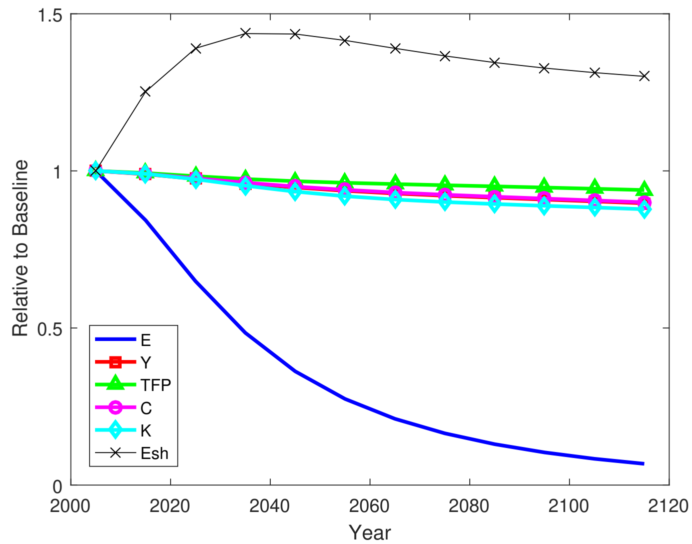

Figure 10: Robustness for table 3 with $\lambda = .11$.

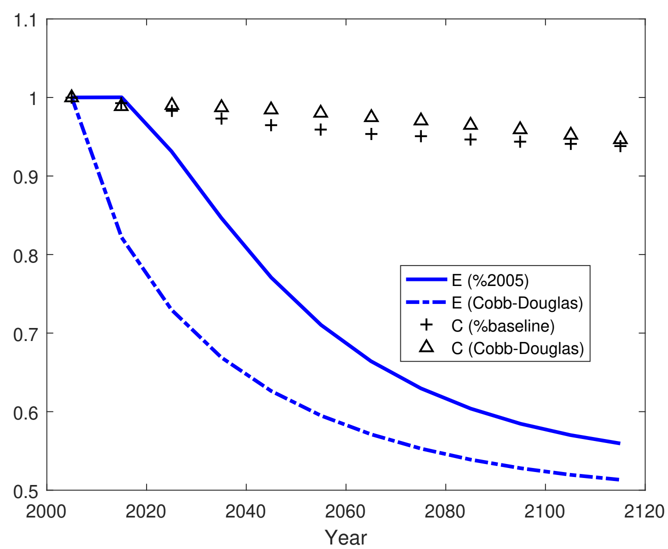

Figure 11: Robustness for table 5 with $\lambda = .11$.

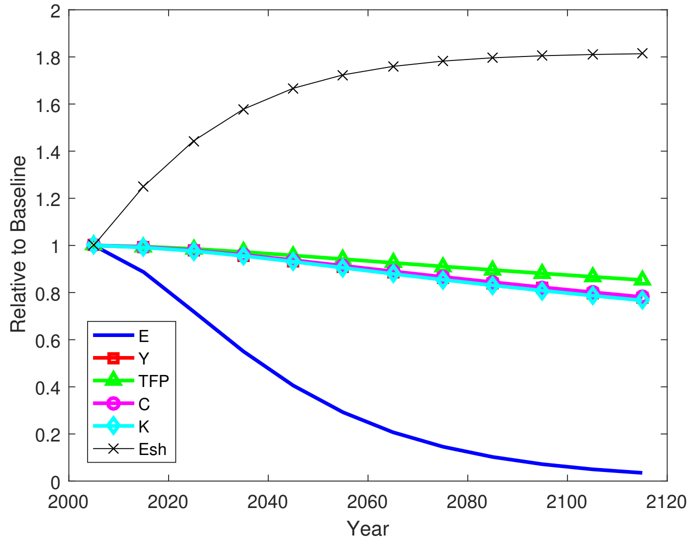

Figure 12: Robustness for table 3 with exogenous energy prices. In this case, $g_\tau = .4$.

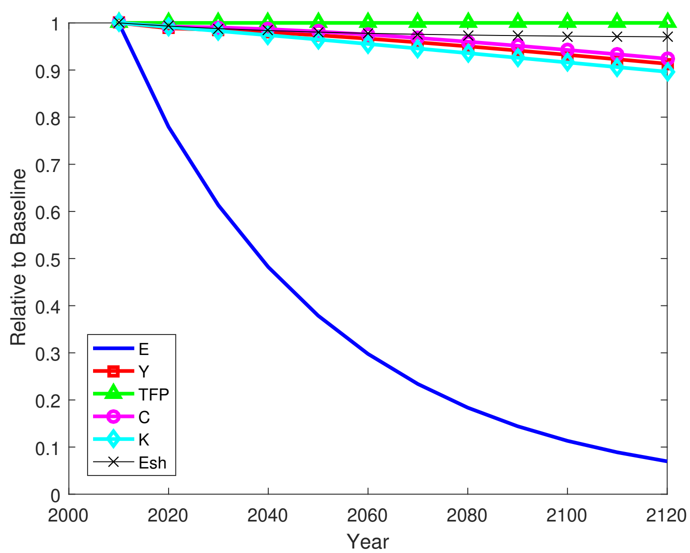

Figure 13: Robustness for table 4 with exogenous energy prices. In this case, $g_\tau = .26$.

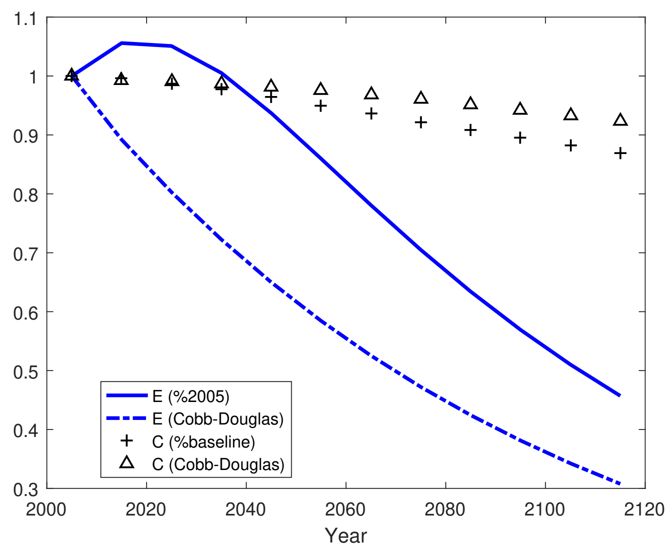

Figure 14: Robustness for table 5 with exogenous energy prices. In this case, $g_\tau = .26$.