Factor Endowment, the Choice of Technology, and the Volume of Trade

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Abstract

This paper studies impacts of factor endowment on international trade in a general equilibrium model in which firms choose their technologies endogenously. Though countries only differ in factor endowment *ex ante*, countries may also differ in their chosen technologies. If industries choose different capital-labor intensities in equilibrium, the Heckscher-Ohlin theorem, factor price equalization theorem, the Rybczynski theorem, and the Stolper-Samuelson theorem hold. If industries choose the same capital-labor intensity in equilibrium, the volume of trade is zero. None of the four theorems applies.

Keywords: Choice of technology, factor endowment, factor price equalization, Heckscher-Ohlin model, volume of trade

JEL Classification Numbers: F10, O33

1. Introduction

The Heckscher-Ohlin (HO) paradigm was originated by Heckscher (1919) and elaborated by his student Ohlin (1933), an eminent scholar and politician. A formal presentation of the two-factor, two-goods model is provided by Samuelson (1948, 1949) and the multiple-factor and multiple-goods version of the model (the HOV model) is studied by Vanek (1968). The HO theorem argues that a country will export the product that uses its relatively abundant factor more intensively. Starting with Leontief (1953), this hypothesis has been subjected to extensive empirical testing. Overall, empirical performance of the model is unsatisfactory. Staiger (1988, p. 129) views that the bulk of the empirical evidence suggests that factor content of trade as a linear function of national and world endowment is not an empirically reliable description of the pattern of international trade. Similar opinions have been expressed by Maskus (1985), Bowen et al. (1987), and Trefler (1995). In Trefler (1995), it is also found that the volume of trade is much lower than the level predicted by the HO theorem.

The HO paradigm has some charming features. First, the HO model is intuitively appealing. For example, Kuwait exports oil mainly because it is well endowed with oil.

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Hong Kong SAR imports apples because it is not endowed with the type of climate to produce apples. Second, the HO paradigm is versatile. In the Ricardian model, since labor is the only factor of production, income distribution effects of trade are absent. As stressed by Heckscher (1919), the HO framework is built to address the income distribution effects of trade. The four main theorems of the HO paradigm: the HO theorem (Heckscher 1919, Ohlin 1933), the factor price equalization theorem (Heckscher 1919, Samuelson 1948, 1949), the Stolper-Samuelson theorem (Stolper and Samuelson 1941), and the Rybczynski theorem (Rybczynski 1955) address various issues, such as the impact of tariffs on factor returns. The HO model is also rich in policy implications. As the HO model is a very important vehicle for studies in international trade, the inconsistency between theoretical studies and empirical evidence thus poses a theoretically significant and policy relevant question: can the HO framework be reformulated to be consistent with empirical evidence?

One key assumption in the HO model is that countries employ the same production technologies. This assumption is controversial as scholars are concerned with the empirical validity of this assumption. While Heckscher (1919) is more willing to assume that countries have the same technologies, Ohlin (1933) stresses differences in technologies among countries as a cause of international trade.[2] Samuelson (1948, p. 181, 1949, p. 195) is cautious about this assumption even though he makes this assumption explicitly.[3] Samuelson (1951-1952, p. 121) even views this assumption may have the impact of "explaining nothing and possibly obscuring a great deal." Empirical studies such as Bowen et al. (1987), Trefler (1993, 1995), Davis and Weinstein (2001), and Schott (2003) have consistently revealed that by allowing differences in countries' technologies, the performance of the HO model is improved.

One possible way to save the HO model is to drop the assumption of identical production technologies. However, scholars may have reservations about using differences in technologies together with differences in factor endowments to explain the pattern of

[2] Heckscher (1919, p. 280) is aware that a tacitly made assumption in his paper is that "the same technique is used to produce a given commodity in different countries". For Ohlin (1933, p. 101), he writes "many important articles are produced in various countries by means of widely different technical processes."

[3] With the assumption of identical technologies between countries, Heckscher (1919) and Samuelson (1948, 1949) find that international trade leads to equalization of factor returns. As Ohlin (1933) views that different technologies are relevant, his opinion is that trade will lead to partial rather than full equalization of factor returns.

trade. By assuming that countries have the same production technologies, the HO model tries to isolate the impact of different factor endowments on the pattern of international trade. Assuming differences in both factor endowments and technologies to explain the pattern of international trade deviates from this spirit.

This paper studies the impact of factor endowment on international trade in a two-sector general equilibrium model in which firms choose their production technologies endogenously. In this paper, similar to the HO model, the only difference between the two countries is their endowments of factors of production. It is assumed that different countries have access to the same *set* of technologies. In each country, given the prices of factors of production, firms choose their technologies. These technologies reflect the possibility that there is some degree of substitution between capital and labor. A firm's choice of technology is affected by the prices of factors of production, which reflect the endowments of factors.

The optimal choice of technologies leads to two possibilities. In the first case, the two sectors have the same capital-labor ratio in equilibrium. None of the four theorems of the HO model applies. In this case, though countries differ in their factor endowments, the volume of trade of final goods is zero as the price ratio of final goods is the same in both countries. Thus, one contribution of this paper is that it provides an explanation to Trefler's (1995) observation of "missing trade." The intuition behind this case is that different factor endowments between countries are totally absorbed by different technologies, rather than by different price ratios of final goods. In the second case that the two industries choose different factor intensities in equilibrium, it is shown that the four theorems of the HO model are valid. Though countries only differ in factor endowment *ex ante*, countries will also differ in their production technologies as countries choose different technologies in equilibrium, regardless of whether industries choose the same factor intensity or not. Thus, another contribution of this paper is that it shows technology is a channel through which endowment differences affect the pattern of trade. With this indirect channel, the factor content of trade may not be a linear function of national and world factor endowment.

Whether the two sectors choose the same factor intensity in equilibrium depends on the specification of production technologies of the two sectors. The two sectors are more likely to have the same factor intensity if they have similar degrees of substitution between capital and labor. The degree of substitution of an industry can be measured by empirical studies.

Following Zhou (2004), the key assumption in this paper is that there are many different technologies to produce the same product. Casual observation supports the empirical validity of this assumption. An example is the technology for the production of agricultural goods. In a developing country such as China, labor is used intensively in the production of agricultural goods. In a developed country such as Canada, the production of agricultural goods relies more on capital inputs, such as harvest machines. Though China and Canada have access to the same set of production technologies, they choose different technologies in equilibrium. Given China's large surplus of workers and low wage rate, though harvest machines are available in China, they are not adopted as it is more profitable to use labor more intensively. Similarly, given the large amount of accumulated capital and high wage rates, though agricultural goods in Canada could be produced by mainly employing labor, it is more profitable to use capital more intensively.

In this paper, compared to a country with a lower capital-labor intensity, a country with a higher capital-labor intensity substitutes labor by capital in production. The discussion of the substitution between capital and labor on international trade goes back at least to Heckscher (1919). Arrow et al. (1961) argue that this type of substitution is very important in various fields of economics. Minhas (1962) formally explores the implications of this type of substitution in international trade. Impact of the choice of technology is also studied at Zhou (2007a, b). The innovation of this paper is that it connects the substitution between capital and labor to the fundamental endowment of factors. Thus, by employing a simple general equilibrium model, a third contribution of this paper is that it introduces a firm's endogenous choice of technology to the study of the impacts of factor endowments on international trade.

The rest of the paper is organized as follows. This paper allows the production function to be either of the constant returns to scale type or the increasing returns to scale type. The basic model assumes perfect competition as Sections 2-4 study the case that the production functions have constant returns to scale. Section 2 sets up the basic framework. In Section 3, as industries choose the same factor intensity in equilibrium, the four theorems of the HO model do not apply. In Section 4, industries choose different factor intensities in equilibrium. The four theorems of the HO model are shown to be valid. Imperfect competition is studied in Section 5 in which the production functions exhibit increasing returns to scale. Regardless of whether the production functions exhibit constant or increasing returns to scale, it is shown that the relative price of final goods can be independent of a country's endowment of factors of production. As a result, trade generated by differences in factor endowments is zero. Section 6 concludes.

2. Constant Returns to Scale Production Technologies

In this section, it is assumed that the production functions exhibit constant returns to scale. There are two countries: home and foreign. Only the home country is studied as the analysis for the foreign country is similar.

Capital and labor are the two factors of production. There are two goods: clothing (c) and food (f). For $i = c, f$, the price of product i is denoted by p_i. Consumers in the two countries have the same preferences. The only difference between the two countries is that they have different ratios of capital to labor. Let the home country's endowment of capital and labor be K and L, respectively. Let r denote the rental price of a unit of capital service and w denote the wage rate.

It is assumed that countries have access to the same *set* of production technologies. To produce each product, there is a continuum of fixed coefficient technologies. Different production technologies correspond to different combinations of capital and labor. For $i = c, f$, let n_i denote the level of technology for a firm producing product i. To produce each unit of product i, the quantity of capital needed is $k_i(n_i)$ and the quantity of labor needed is $l_i(n_i)$. Thus, the constant marginal cost of producing product i is $rk_i(n_i) + wl_i(n_i)$. To capture the idea that capital and labor are substitutable in production, it is assumed that when n_i increases, $k_i(n_i)$ decreases and $l_i(n_i)$ increases. That is, $k_i' < 0$ and $l_i' > 0$.

A firm's profit of producing each unit of product i is $p_i - k_i(n_i)r - l_i(n_i)w$.[4] A firm takes the price of its product, the wage rate, and the interest rate as given and chooses the level of technology optimally to maximize its profit. For a firm producing clothing, the first order condition for its optimal choice of technology is

$$k_c'(n_c)r + l_c'(n_c)w = 0. \tag{1a}$$

Similarly, for a firm producing food, the optimal choice of technology leads to

$$k_f'(n_f)r + l_f'(n_f)w = 0. \tag{1b}$$

Equations (1a) and (1b) show that a firm's choice of technology is affected by the relative price of capital to labor. In equilibrium, returns to factors are affected by factor endowments. Thus, a firm's choice of technology is ultimately determined by factor endowments.

From (1a) and (1b), the second order condition for profit maximization requires that

$$k_i''(n_i)r + l_i''(n_i)w > 0, \text{ for } i = c, f.$$

It is assumed that $l_i'' \geq 0$ and $k_i'' \geq 0$. Also, l_i'' and k_i'' are not equal to zero at the same time. Thus, the second order condition is satisfied. Plugging the values of w and r from equations (1a) and (1b) into the second order condition leads to

$$\frac{l_i''k_i'}{l_i'} - k_i'' < 0, \text{ for } i = c, f. \tag{2}$$

As prices of factors are flexible, in equilibrium all factors will be fully employed. Let X_i denote the total industry output of product i. The total demand for labor is $X_c l_c + X_f l_f$. The total supply of labor is L. Full employment of labor requires that

$$X_c l_c + X_f l_f = L. \tag{3a}$$

Full employment of capital requires that

$$X_c k_c + X_f k_f = K. \tag{3b}$$

Zero profits require that a firms' cost of production equals the price it receives:

$$k_c r + l_c w = p_c. \tag{4a}$$

$$k_f r + l_f w = p_f. \tag{4b}$$

[4] With constant returns to scale in production, a firm's output is indeterminate. But this is not essential in this paper.

It is assumed that consumers have homothetic preferences. With this assumption, demand for goods is not affected by the distribution of income. Let γ and δ denote positive constants. Let q_i denote a consumer's consumption of product i. A consumer's utility function is given by $\gamma q_c^{\frac{\delta}{1-\delta}} + (1-\gamma) q_f^{\frac{\delta}{1-\delta}}$. For this type of utility functions, it is well known that a consumer's utility maximization leads to a fixed percentage of income spent on each product. The total income in this economy is $wL + rK$. As γ percent of the income is spent on clothing, total demand for clothing is $\gamma(wL + rK)$. The total value of supply of clothing is $p_c X_c$. Similarly, $1-\gamma$ percentage of total income is spent on food and the supply of food is $p_f X_f$. Goods market equilibrium requires that

$$\gamma(wL + rK) = p_c X_c, \tag{5a}$$

$$(1-\gamma)(wL + rK) = p_f X_f. \tag{5b}$$

Equations (1a), (1b), and (3a)-(5b) form a system of eight equations defining eight variables n_c, n_f, X_c, X_f, p_c, p_f, w, and r. An equilibrium is a set of variables n_c, n_f, X_c, X_f, p_c, p_f, w, and r satisfying equations (1a), (1b), and (3a)-(5b).

The price of clothing is normalized to 1:

$$p_c \equiv 1 \; . \tag{6}$$

With this normalization, the price of food p_f also measures the price ratio of the two goods.

Following Samuelson (1949), it is assumed that there is no factor-intensity reversal. [5] A sufficient condition for this assumption to be valid is as follows. For $i = c, f$, the elasticity of substitution between capital and labor in industry i is defined as $\sigma_i \equiv \dfrac{dk_i / k_i}{dl_i / l_i} = \dfrac{k_i' l_i}{l_i' k_i}$. Let k_i^j (l_i^j) denote industry i's capital (labor) input for factor prices at level j. Suppose the clothing industry is more capital intensive when the wage rate is w^1 and the interest rate is r^1 and the clothing industry is less capital intensive when the wage rate is w^2 and the interest rate is r^2. That is,

[5] Some discussions of this concept include Robinson (1956), Minhas (1962), and Wong (1990). Wong provides sufficient conditions to rule out factor intensity reversal in a multi-factor, two-good economy.

$$\frac{k_c^1}{l_c^1} > \frac{k_f^1}{l_f^1}, \tag{7a}$$

$$\frac{k_c^2}{l_c^2} < \frac{k_f^2}{l_f^2}. \tag{7b}$$

As equations (1a) and (1b) hold, inequalities (7a) and (7b) lead to

$$\sigma_c^1 > \sigma_f^1, \tag{8a}$$

$$\sigma_c^2 < \sigma_f^2. \tag{8b}$$

For constant elasticity of substitution functions, it is impossible for (8a) and (8b) to hold at the same time. Thus, for this type of production functions, factor intensity reversal will not occur. One example of this type of production function is that $l_i(n_i) = n_i$, and $k_i(n_i) = (n_i)^{(\rho-1)/\rho}$, with $\rho \in (0,1)$. In this case, $\delta_i = \frac{\rho - 1}{\rho}$.

Equations (1a) and (1b) lead to

$$\frac{k_c'}{l_c'} = \frac{k_f'}{l_f'}. \tag{9}$$

Equation (9) defines an implicit relationship between n_c and n_f. Depending on the explicit functional forms of k_c, l_c, k_f, and l_f, the restriction imposed by equation (9) may lead to two cases. In the first case, the two industries have the same capital-labor ratio in equilibrium,

$$\frac{k_c}{l_c} = \frac{k_f}{l_f}. \tag{10}$$

An example of this type of technologies is that

$$k_c = \frac{1}{n_c},\ l_c = n_c,\ k_f = \frac{\eta}{n_f},\ \text{and } l_f = n_f. \tag{11}$$

In (11), η is a positive constant. Plugging (11) into (9) leads to $n_f = \eta^{1/2} n_c$. This leads to (10).

In the second case, the two industries have different capital-labor ratios in equilibrium, $\frac{k_c}{l_c} \neq \frac{k_f}{l_f}$. An example of this type of technologies is that

$$k_c = \frac{1}{n_c},\ l_c = (n_c)^2,\ k_f = \frac{1}{\sqrt{n_f}},\ \text{and}\ l_f = \sqrt{n_f}\ . \tag{12}$$

From (11) and (12), industries are more likely to choose different factor intensities when the degrees of substitution between capital and labor in different industries are very different. The degree of substitution of an industry can be measured by empirical studies, such as the one conducted in Arrow et al. (1961).

In the following, the case that industries choose the same factor intensity and the case that industries choose different factor intensities are studied sequentially.

3. Industries with the Same Factor Intensity

This section focuses on the case that the optimal choice of technologies leads the two industries to have the same level of capital-labor intensity. For (3a), (3b), and (10) to be consistent, it is needed that

$$\frac{l_c}{k_c} = \frac{l_f}{k_f} = \frac{L}{K}. \tag{13}$$

The following lemma studies the impact of factor endowments on a firm's choice of technology.

Lemma 1. *When industries choose the same factor intensity in equilibrium, an increase in the endowment of capital increases the capital-labor ratio in both industries.*

Proof. It is clear from (13) and the assumptions that $k_i' < 0$ and $l_i' > 0$. QED

From Lemma 1, a country with a higher capital-labor ratio use technologies employing capital more intensively in every industry.

The following lemma studies how the returns to factors are affected by factor endowments.

Lemma 2. *The wage rate is positively related to the amount of capital and negatively related to the amount of labor in a country. The interest rate is negatively related to the amount of capital and positively related to the amount of labor in a country.*

Proof. Equations (1a), (4a), and (6) lead to $r = \dfrac{1}{k_c - l_c \dfrac{k_c'}{l_c'}}$. For the denominator of the right-hand side of this expression, from (2), it is clear that

$$\frac{d\left(k_c - l_c \dfrac{k_c'}{l_c'}\right)}{dn_c} = \frac{l_c}{l_c'}\left(\frac{k_c' l_c''}{l_c'} - k_c''\right) < 0 .$$

Thus, $\dfrac{dr}{dn_c} > 0$. Combining this result with Lemma 1, it is clear that $\dfrac{dr}{dK} = \dfrac{dr}{dn_c}\dfrac{dn_c}{dK} < 0$.

Equations (1a), (4a), and (6) can also be employed to yield $w = \dfrac{1}{l_c - k_c \dfrac{l_c'}{k_c'}}$. For the denominator of the right-hand side of this expression, from (2), it is clear that

$$\frac{d\left(l_c - k_c \dfrac{l_c'}{k_c'}\right)}{dn_c} = \frac{k_c}{k_c'}\left(\frac{l_c' k_c''}{k_c'} - l_c''\right) > 0 .$$

Thus, $\dfrac{dw}{dn_c} < 0$. Combining this result with Lemma 1, it is clear that $\dfrac{dw}{dK} = \dfrac{dw}{dn_c}\dfrac{dn_c}{dK} > 0$.

QED

From (1a), (1b), and (3a)-(5b), $X_c = \gamma \dfrac{L}{l_c}$, and $X_f = (1-\gamma)\dfrac{L}{l_f}$. As X_c and X_f move in the same direction as L changes, the output of both goods increases when the amount of factor endowment increases.

Manipulation of the system of equations (1a), (1b), and (3a)-(5b) leads to the following three equations (14a)-(14c) defining three variables p_f, n_c, and n_f as functions of exogenously given variables:

$$V_1 \equiv p_f l_c - l_f = 0, \tag{14a}$$

$$V_2 \equiv l_c - \frac{L}{K} k_c = 0, \tag{14b}$$

$$V_3 \equiv l_f - \frac{L}{K}k_f = 0\,.[6] \tag{14c}$$

The validity of the four theorems of the HO model depends on the assumption that industries have different factor intensities. In the following, the four theorems of the HO model are shown to be invalid as industries choose the same factor intensity in equilibrium. First, Proposition 1 shows that the HO theorem does not apply.

Proposition 1. *If industries choose the same capital-labor intensity in equilibrium, the price ratio of final goods is independent of the endowment ratio.*

Proof. Total differentiation of the system of equations (14a)-(14c) leads to

$$\begin{pmatrix} \frac{\partial V_1}{\partial p_f} & \frac{\partial V_1}{\partial n_c} & \frac{\partial V_1}{\partial n_f} \\ 0 & \frac{\partial V_2}{\partial n_c} & 0 \\ 0 & 0 & \frac{\partial V_3}{\partial n_f} \end{pmatrix} \begin{pmatrix} dp_f \\ dn_c \\ dn_f \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{\partial V_2}{\partial K} \\ -\frac{\partial V_3}{\partial K} \end{pmatrix} dK\,. \tag{15}$$

For Δ_A as the determinant of the coefficient matrix of (15), it can be shown that

$$\Delta_A = l_c\left(l_c' - \frac{L}{K}k_c'\right)\left(l_f' - \frac{L}{K}k_f'\right) > 0\,.$$

Define

$$\Delta_1 \equiv \frac{\partial V_1}{\partial n_f}\frac{\partial V_2}{\partial n_c}\frac{\partial V_3}{\partial K} + \frac{\partial V_1}{\partial n_c}\frac{\partial V_2}{\partial K}\frac{\partial V_3}{\partial n_f}\,.$$

From (14a)-(14c), it can be shown that

$$\Delta_1 = \frac{L}{K^2}p_f k_c l_c'\left(l_f' - \frac{L}{K}k_f'\right) - \frac{L}{K^2}k_f l_f'\left(l_c' - \frac{L}{K}k_c'\right)$$

$$= \frac{l_c' l_f' k_c L}{K^2}\left(1 - \frac{L}{K}\frac{k_f'}{l_f'}\right)\left(p_f - \frac{l_f}{l_c}\right).$$

[6] Equation (14a) comes from (4a), (4b), (6), and (10a). Equations (14b) and (14c) come from (13).

From (14a), $\Delta_1 = 0$. Application of Cramer's rule on the system (15) leads to $\frac{dp_f}{dK} = \frac{\Delta_1}{\Delta_A} = 0$. QED

From Proposition 1, though countries have different ratios of factor endowments, they have the same price ratio of final goods. Difference in endowments is totally absorbed by the choice of different technologies. Countries will not trade final goods as the price ratio of final goods is the same in both countries. In this case, the volume of trade generated by differences in factor endowments is zero. This provides an explanation to Trefler's observation (1995) that the volume of trade is much lower than the level predicted by the HO model.

Proposition 1 is a formal presentation of Heckscher's claim (1919, p. 278) that "a (further) indispensable condition is that the proportions in which the factors of production are combined should not be the same for one commodity as for another. In the absence of this (second) condition, the price of one commodity, compared with the price of another would remain the same in all countries regardless of differences in relative factor prices."

Second, it is clear that the factor price equalization theorem does not hold in this case. The reason is that there is no trade of final goods to equalize different factor returns between countries.

Third, the Stolper-Samuelson theorem does not apply if the two sectors choose the same factor intensity. Stolper and Samuelson (1941) argue that the impact of international trade comes from a change in the price ratio of final goods. By treating the prices of final goods as exogenous parameters and conducting comparative static analysis, the impact of international trade on factor returns can be studied. The invalidity of the Stolper-Samuelson theorem is clear from the system of equations (1a), (1b), and (3a)-(4b). From (1a), (1b), and (4b), $r = \frac{p_f}{k_f - l_f \frac{k_f'}{l_f'}}$, and $w = \frac{p_f}{l_f - k_f \frac{l_f'}{k_f'}}$. As k_f and l_f are determined by (13) and not affected by p_f, it is clear that r and w move in the same direction as p_f changes.

Fourth and finally, the Rybczynski theorem does not apply. This theorem shows that if the prices of final goods are exogenous, the output of the capital-intensive goods increases and the output of the labor-intensive goods decreases when the endowment of capital increases. If prices of final goods are exogenous and industries choose the same factor intensity, the output of the two final goods is undetermined.

4. Industries with Different Factor Intensities

This section focuses on the case that the optimal choice of technologies leads the two industries to have different capital-labor intensities in equilibrium.

Manipulation of equations (3a) and (3b) leads to the following expression of the output of the two sectors:

$$X_c = \frac{Lk_f - Kl_f}{l_c k_f - k_c l_f}, \tag{16a}$$

$$X_f = \frac{Kl_c - Lk_c}{l_c k_f - k_c l_f}. \tag{16b}$$

Manipulation of equations (4a) and (4b) leads to the following expression of the returns to factors:

$$w = \frac{p_c k_f - p_f k_c}{l_c k_f - k_c l_f}, \tag{17a}$$

$$r = \frac{p_f l_c - p_c l_f}{l_c k_f - k_c l_f}. \tag{17b}$$

Equations (1a), (1b), and (3a)-(5b) can be simplified into the following system of three equations:

$$H_1 \equiv k_c' l_f' - k_f' l_c' = 0, \tag{18a}$$

$$H_2 \equiv \gamma\, p_f (Kl_c - Lk_c) - (1-\gamma)(Lk_f - Kl_f) = 0, \tag{18b}$$

$$H_3 \equiv k_c'(p_f l_c - p_c l_f) + l_c'(p_c k_f - p_f k_c) = 0.^7 \tag{18c}$$

From (18c), it can be shown that $\frac{\partial H_3}{\partial n_f} = 0$. Total differentiation of the system of equations (18a)-(18c) with respect to p_f, n_c, n_f, and K leads to

7 Equation (18a) comes from equations (1a) and (1b). Equation (18b) comes from equations (5a), (5b), (16a), and (16b). Equation (18c) comes from equations (1a), (1b), (17a), and (17b).

$$\begin{pmatrix} 0 & \frac{\partial H_1}{\partial n_c} & \frac{\partial H_1}{\partial n_f} \\ \frac{\partial H_2}{\partial p_f} & \frac{\partial H_2}{\partial n_c} & \frac{\partial H_2}{\partial n_f} \\ \frac{\partial H_3}{\partial p_f} & \frac{\partial H_3}{\partial n_c} & 0 \end{pmatrix} \begin{pmatrix} dp_f \\ dn_c \\ dn_f \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{\partial H_2}{\partial K} \\ 0 \end{pmatrix} dK . \tag{19}$$

Let Δ_B denote the determinant of the coefficient matrix of (19). For stability, Δ_B is assumed to be negative. The following proposition shows that a firm's technology is determined by factor endowments.

Proposition 2. *When industries choose different capital-labor intensities in equilibrium, an increase in the endowment of capital increases the capital-labor ratio in both industries.*

Proof. Application of Cramer's rule on the system (19) leads to

$$\frac{dn_c}{dK} = \frac{\partial H_1}{\partial n_f} \frac{\partial H_2}{\partial K} \frac{\partial H_3}{\partial p_f} \bigg/ \Delta_B ,$$

$$\frac{dn_f}{dK} = -\frac{\partial H_1}{\partial n_c} \frac{\partial H_2}{\partial K} \frac{\partial H_3}{\partial p_f} \bigg/ \Delta_B .$$

Partial differentiation of equations (18a)-(18c) yields

$$\frac{\partial H_1}{\partial n_c} = k_c'' l_f' - k_f' l_c'' > 0 , \tag{20}$$

$$\frac{\partial H_1}{\partial n_f} = l_c' \left(\frac{l_f'' k_c'}{l_c'} - k_f'' \right) < 0 , \tag{21}$$

$$\frac{\partial H_2}{\partial K} = \gamma\, p_f l_c + (1-\gamma) l_f > 0 , \tag{22}$$

$$\frac{\partial H_3}{\partial p_f} = l_c k_c' - k_c l_c' < 0 . \tag{23}$$

From (20)-(23), it is clear that $\frac{dn_c}{dK} < 0$, and $\frac{dn_f}{dK} < 0$. QED

Proposition 2 shows that countries will have different input coefficients if they have different ratios of factor endowments. Lemma 1 and Proposition 2 together show that countries also differ in technologies if their factor endowment are different, no matter whether in equilibrium industries choose the same factor intensity or not. This result is consistent with the empirical research of Bowen et al. (1987) as they find that factor input matrices between different countries are different.

In the following, the four theorems of the HO model are shown to be valid. First, Proposition 3 is a modified version of the HO theorem with technologies chosen endogenously. It shows that $dp_f / dK < 0$ if and only if $k_f / l_f > k_c / l_c$.

Proposition 3 (HO Theorem). *When the two industries have different factor intensities, an increase in the endowment of capital decreases the price ratio of the product using capital more intensively.*

Proof. Partial differentiation of equation (18c) yields

$$\frac{\partial H_3}{\partial n_c} = (rk_c'' + wl_c'')(l_c k_f - k_c l_f) . \tag{24}$$

For $\Delta_2 \equiv -\frac{\partial H_1}{\partial n_f}\frac{\partial H_2}{\partial K}\frac{\partial H_3}{\partial n_c}$, from (21), (22), and (24), it can be shown that

$$\Delta_2 = -l_c'\left(\frac{l_f'' k_c'}{l_c'} - k_f''\right)(\gamma p_f l_c + (1-\gamma) l_f)(rk_c'' + wl_c'')(l_c k_f - k_c l_f) .$$

The sign of Δ_2 is the same as the sign of $l_c k_f - k_c l_f$. If $l_c k_f - k_c l_f > 0$, or $\frac{k_f}{l_f} > \frac{k_c}{l_c}$, the clothing industry is less capital intensive and $\Delta_2 > 0$. Thus, $\frac{dp_f}{dK} = \frac{\Delta_2}{\Delta_B} < 0$. Similarly, if $\frac{k_f}{l_f} < \frac{k_c}{l_c}$, $\frac{dp_f}{dK} > 0$. QED

In this case that industries have different factor intensities, with the opening of international trade, the country with a higher capital-labor ratio will export the product using capital more intensively.

Second, Proposition 4 shows that the factor price equalization theorem holds.

Proposition 4 (Factor Price Equalization Theorem). *If industries choose different factor intensities in equilibrium, international trade leads to the equalization of the wage rate and the interest rate.*

Proof. Equations (18a) and (18c) define the level of technologies as functions of prices of final goods. As trade leads to the equalization of prices of final goods, countries will adopt the same technologies. From (1a) and (1b), same technologies lead to the same wage rate and the same interest rate. QED

From the proof of Proposition 4, trade not only leads to a convergence of factor returns, but also to a convergence of technologies used in different countries. The proof of Proposition 4 depends on the assumption that trade will lead to the same prices of final goods all over the world. Due to transportation costs and other trade impediments, equalization of prices of final goods may not occur in reality.

For the rest of this section, the prices of final goods are treated as exogenous parameters. For exogenously given prices, equations (1a), (1b), (3a), (3b), (4a), and (4b) define a set of six variables.

Third, Proposition 5 shows that the result in Stolper and Samuelson (1941) is valid for endogenous technologies.

Proposition 5 (Stolper-Samuelson Theorem). *Suppose food is the capital-intensive product. If the price of food increases, the interest rate increases and the wage rate decreases.*

Proof. Manipulation of equation (17b) yields

$$H_4 \equiv (k_c l_f - k_f l_c) r - (p_c l_f - p_f l_c) = 0 . \tag{25}$$

Equations H_1, H_3, and H_4 can be differentiated to get

$$\begin{pmatrix} \frac{\partial H_1}{\partial n_c} & \frac{\partial H_1}{\partial n_f} & 0 \\ \frac{\partial H_3}{\partial n_c} & 0 & 0 \\ \frac{\partial H_4}{\partial n_c} & \frac{\partial H_4}{\partial n_f} & \frac{\partial H_4}{\partial r} \end{pmatrix} \begin{pmatrix} dn_c \\ dn_f \\ dr \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{\partial H_3}{\partial p_f} \\ -\frac{\partial H_4}{\partial p_f} \end{pmatrix} dp_f \,. \tag{26}$$

For the coefficient of the determinant matrix of (26), $\Delta_C = -\frac{\partial H_1}{\partial n_f}\frac{\partial H_3}{\partial n_c}\frac{\partial H_4}{\partial r}$. Partial differentiation of equation (25) yields $\frac{\partial H_4}{\partial n_c} = 0$, $\frac{\partial H_4}{\partial n_f} = 0$, and $\frac{\partial H_4}{\partial p_f} = l_c$. Application of Cramer's rule on the system (26) leads to

$$\frac{dr}{dp_f} = -\frac{\partial H_4 / \partial p_f}{\partial H_4 / \partial r} = -\frac{l_c}{k_c l_f - k_f l_c} \,.$$

Thus, $\frac{dr}{dp_f} > 0$ if and only if $k_c l_f - k_f l_c < 0$, or $\frac{k_c}{l_c} < \frac{k_f}{l_f}$. Similarly, $\frac{dw}{dp_f} < 0$ if and only if $\frac{k_c}{l_c} < \frac{k_f}{l_f}$. QED

From (26), it can be shown that

$$\frac{dn_c}{dp_f} = -\frac{\partial H_3 / \partial p_f}{\partial H_3 / \partial n_c} = \frac{k_c' l_c - l_c' k_c}{(rk_c'' + wl_c'')(l_c k_f - k_c l_f)} \,. \tag{27}$$

Since $k_c' l_c - l_c' k_c < 0$ and $rk_c'' + wl_c'' > 0$, from (27), $dn_c / dp_f > 0$ if and only if $l_c k_f - k_c l_f < 0$. Similarly, $dn_f / dp_f > 0$ if and only if $l_c k_f - k_c l_f > 0$. Thus, if food is the capital-intensive product, when the price of food increases, the capital-labor intensity for the food sector increases and the intensity of the clothing sector decreases.

Fourth and finally, Proposition 6 shows that the Rybczynski theorem is valid for endogenous technologies.

Proposition 6 (Rybczynski Theorem). *An increase in the amount of capital increases the output of clothing and decreases the output of food if the clothing industry is more capital-intensive.*

Proof. Manipulation of equation (16a) yields

$$H_5 \equiv (k_c l_f - k_f l_c) X_c - (K l_f - L k_f) = 0 .$$

Differentiation of H_1, H_3, and H_5 leads to

$$\begin{pmatrix} \frac{\partial H_1}{\partial n_c} & \frac{\partial H_1}{\partial n_f} & 0 \\ \frac{\partial H_3}{\partial n_c} & 0 & 0 \\ \frac{\partial H_5}{\partial n_c} & \frac{\partial H_5}{\partial n_f} & \frac{\partial H_5}{\partial X_c} \end{pmatrix} \begin{pmatrix} dn_c \\ dn_f \\ dX \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\frac{\partial H_5}{\partial K} \end{pmatrix} dK . \tag{28}$$

For the determinant of the coefficient matrix of (28), $\Delta_D \equiv -\frac{\partial H_1}{\partial n_f}\frac{\partial H_3}{\partial n_c}\frac{\partial H_5}{\partial X_c}$. For $\Delta_3 \equiv \frac{\partial H_1}{\partial n_f}\frac{\partial H_3}{\partial n_c}\frac{\partial H_5}{\partial K}$, application of Cramer's rule on the system (28) leads to

$$\frac{dX_c}{dK} = \frac{\Delta_3}{\Delta_D} = \frac{l_f}{k_c l_f - k_f l_c} .$$

Thus, $dX_c / dK > 0$ if and only if $k_c l_f - k_f l_c > 0$. Similarly, $dX_f / dK < 0$ if and only if $k_c l_f - k_f l_c > 0$. QED

5. Increasing Returns to Scale Production Technologies

In this section, the production functions exhibit increasing returns to scale arising from fixed costs of production. For Ohlin (1933), increasing returns to scale is a very important source for countries to engage in international trade. When there are two products, if countries specialize in producing one product and trade, some fixed cost of production can be saved. This benefit is absent here as it is assumed that both countries produce both products. The main purpose of this section is to show that under increasing returns to scale, differences in factor endowments may not lead to differences in the price ratio of final goods.

With the existence of fixed costs of production, the number of firms producing each product is small.[8] In this case, a firm may have market power in the factor market. To avoid this, instead of two goods, it is assumed that there are two groups of goods. In each group, there is a continuum of goods. The utility function is modified correspondingly to

$$\left(\int_0^1 q_c(\omega))^{\frac{\delta-1}{\delta}} d\omega\right)^{\frac{\gamma\delta}{\delta-1}} \left(\int_0^1 q_m(v))^{\frac{\delta-1}{\delta}} dv\right)^{\frac{\delta(1-\gamma)}{\delta-1}} . \tag{29}$$

It is assumed that all goods in the same group are symmetric in terms of costs of production. As there is a continuum of goods, though a firm may have market power in the output market, it does not have market power in the factor market. With this assumption, each group of goods can be viewed as one product. To produce a product, both fixed and marginal costs are needed. To simplify the analysis, it is assumed that only capital is used in the fixed cost of production and only labor is used for the marginal cost of production.[9]

For $i = c, f$, a firm's fixed cost is $\alpha_i(n_i)$ units of capital and its marginal cost is $\beta_i(n_i)$ units of labor. When n_i increases, $\alpha_i(n_i)$ decreases and $\beta_i(n_i)$ increases. This assumption captures the idea that there is some degree of substitution between capital and labor in production. If a technology uses a lot of machines, the fixed cost of capital is high. However, the marginal cost of labor is low.[10]

Let x_i denote the quantity of production for a firm producing product i. Since a firm's total revenue is $p_i x_i$ and its total cost is $\alpha(n_i)r + \beta(n_i)x_i w$, its profit is $p_i x_i - \alpha_i r - \beta_i x_i w$. A firm chooses the level of technology optimally to maximize its profit. Taking first order condition with respect to n_i yields

$$-\alpha_i' r - \beta_i' x_i w = 0 \text{, for } i = c, f .^{11} \tag{30}$$

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[8] When firms have market power in the product market, the opening of trade may be beneficial since it increases the degree of competition (Brander, 1981).

[9] This assumption is more appropriate compared to the assumption that capital is only related to marginal cost of production and labor is only related to the fixed cost of production. Capital is embodied in equipments. In their empirical research, De Long and Summers (1991) show that each extra percent of GDP invested in equipment leads to an increase in GDP growth of one third of a percentage point per year over the period1960-1985. Equipments may be more appropriately modeled as a fixed cost rather than a marginal cost of production.

[10] Zhou (2004) has a detailed discussion of the motivation of this type of assumptions.

[11] It is assumed that the second order condition $-\alpha_i'' r - \beta_i'' x_i w < 0$ is satisfied. For the cost function specified in (42), it can be checked that this assumption is valid.

A firm also chooses the quantity of production optimally. It is assumed that firms producing the same product engage in Cournot competition. Taking first order condition with respect to x_i yields $p_i + x_i \frac{\partial p_i}{\partial x_i} - \beta_i w = 0$. Let m_i denote the number of firms producing product i. For the utility function (29), a consumer's utility maximization leads to the result that the elasticity of demand faced by a firm is $-\delta m_i$.[12] Plugging this elasticity into the first order condition with respect to output leads to

$$p_i\left(1 - \frac{1}{\delta m_i}\right) = \beta_i w \text{, for } i = c, f. \tag{31}$$

In equilibrium, a firm makes a profit of zero. This requirement leads to

$$p_i x_i - \alpha_i r - \beta_i x_i w = 0 \text{, for } i = c, f. \tag{32}$$

Each firm producing clothing demands $\beta_c x_c$ units of labor and the total demand for labor from the clothing industry is $m_c \beta_c x_c$. Similarly, the total demand for labor from the food industry is $m_f \beta_f x_f$. The total supply for labor in the economy is L. Clearance of labor market requires that

$$m_c \beta_c x_c + m_f \beta_f x_f = L. \tag{33}$$

The demand for capital from the clothing sector is $m_c \alpha_c$. The demand for capital from the food sector is $m_f \alpha_f$. The total supply of capital is K. Clearance of capital market requires that

$$m_c \alpha_c + m_f \alpha_f = K. \tag{34}$$

The total demand for clothing is $\gamma(wL + rK)$ and the total demand for food is $(1-\gamma)(wL + rK)$. The total supply of product i is $p_i m_i x_i$. Goods market equilibrium requires that

$$\gamma(wL + rK) = p_c m_c x_c, \tag{35a}$$

$$(1-\gamma)(wL + rK) = p_f m_f x_f. \tag{35b}$$

[12] See Zhou (2006) for a detailed derivation of this type of formula.

Equations (30) and (31)-(35b) are a system of ten equations defining ten variables n_c, n_f, x_c, x_f, p_c, p_f, m_c, m_f, w, and r. The following proposition shows that a difference in factor endowments may not lead to a difference in the price ratio of final goods.

Proposition 7. *For the increasing returns to scale production technologies, the price ratio of the final goods is independent of the endowment ratio.*

Proof. For symmetry in this section, let the interest rate rather than the price of clothing be normalized to unity. That is, $r \equiv 1$. For $i = c, f$, from (31), it can be shown that

$$m_i = \frac{p_i}{\delta(p_i - \beta_i w)}. \tag{36}$$

Equation (32) yields

$$x_i = \frac{\alpha_i}{p_i - \beta_i w}. \tag{37}$$

By plugging equation (37) into (30), it can be shown that

$$p_i = \frac{(\beta_i \alpha_i' - \beta_i' \alpha_i)}{\alpha_i'} w. \tag{38}$$

Plugging (37) and (38) into (34) yields

$$\frac{\beta_c' \alpha_c - \beta_c \alpha_c'}{\delta \beta_c'} + \frac{\beta_f' \alpha_f - \beta_f \alpha_f'}{\delta \beta_f'} - K = 0. \tag{39}$$

Dividing (35a) by (35b) and plugging in (36)-(38) leads to

$$\frac{\gamma}{1-\gamma} = \left(\frac{(\beta_c \alpha_c' - \beta_c' \alpha_c) \beta_f'}{(\beta_f \alpha_f' - \beta_f' \alpha_f) \beta_c'} \right)^2 \frac{\alpha_f}{\alpha_c}. \tag{40}$$

Equations (39) and (40) define n_c and n_f as functions of exogenous variables. As L does not appear in any of the equations, a firm's choice of technology is not affected by L. Thus, it is not affected by the factor endowment ratio.

From (40), the price ratio of final goods is given by

$$\frac{p_c}{p_f} = \frac{(\beta_c \alpha_c' - \beta_c' \alpha_c)}{(\beta_f \alpha_f' - \beta_f' \alpha_f)} \frac{\alpha_f'}{\alpha_c}. \tag{41}$$

From (41), the price ratio is determined by the level of technologies only. Thus, the price ratio is not affected by the endowment ratio. QED

The intuition behind Proposition 7 can be obtained by inspection of the system of equations (39) and (40). Labor endowment does not enter equation (39) (which is a transformation of equation (33)) since the number of firms in each industry is determined by this industry's technology only. Labor endowment also does not enter equation (40) (which is a result of dividing (35a) by (35b)). From the right-hand side of (35a) and (35b), labor endowment may affect the price ratio either through its impact on the wage rate or through the number of firms in an industry. Though labor endowment affects the wage rate, with a homothetic utility function, the impact of wage rate on price ratio that works from the demand side cancels out as a result of dividing (35a) by (35b). From the supply side, the wage rate affects the output as each industry's output is inversely related to the wage rate (through (30)). However, as a result of dividing (35a) by (35b), impact of labor endowment through the wage rate and thus output also cancels out. As a result, labor endowment does not affect the price ratio of final goods.

One example of this independence of price ratio of final goods on factor endowment is as follows. For μ and τ denoting positive constants, the technologies are specified as

$$\alpha_c = \mu(n_c)^{1/2},\ \beta_c = \tau(n_c)^{-1/2},\ \alpha_f = \theta\mu(n_f)^{1/2}, \text{ and } \beta_f = \tau(n_f)^{-1/2}. \tag{42}$$

Plugging (42) into the system of equations (30) and (31)-(35b) leads to $p_f / p_c = \theta\gamma/(1-\gamma)$. As the price ratio is not affected by endowments of factors of production, countries may not trade final goods as the price ratio of final goods is the same in both countries.

6. Conclusion

This paper studies the impact of factor endowments on international trade in a two-sector model in which firms choose their production technologies endogenously. Though countries differ only in their factor endowments *ex ante*, they may also differ in their chosen technologies *ex post*. If industries choose different capital-labor intensities in equilibrium, the HO theorem, the factor price equalization theorem, the Stolper-Samuelson theorem, and the Rybczynski theorem hold. If industries choose the same capital-labor intensity in equilibrium, the volume of trade of final goods is zero. None of the four theorems is valid in this case.

This paper has employed some special functional forms to demonstrate the results. The essence of this paper that countries' differences in factor endowments can be embodied not only through different ratios of prices of final goods but also through different technologies should be robust in a general background.

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