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Abstract

We add arbitraging middlemen -- investors who attempt to profit from buying low and selling high -- to a canonical housing market search model. Flipping tends to take place in sluggish and tight, but not in moderate, markets. To follow is the possibility of multiple equilibria. In one equilibrium, most, if not all, transactions are intermediated, resulting in rapid turnover, a high vacancy rate, and high housing prices. In another equilibrium, few houses are bought and sold by middlemen. Turnover is slow, few houses are vacant, and prices are moderate. Moreover, flippers can enter and exit en masse in response to the smallest interest rate shock. The housing market can then be intrinsically unstable even when all flippers are akin to the arbitraging middlemen in classical finance theory. In speeding up turnover, the flipping that takes place in a sluggish and illiquid market tends to be socially beneficial. The flipping that takes place in a tight and liquid market can be wasteful as the efficiency gain from any faster turnover is unlikely to be large enough to offset the loss from more houses being left vacant in the hands of flippers. Based on our calibrated model, which matches several stylized facts of the U.S. housing market, we show that the housing price response to interest rate change is very non-linear, suggesting cautions to policy attempt to "stabilize" the housing market through monetary policy.

Key words: Search and matching, housing market, liquidity, flippers and speculators, financing and bargaining advantage.

JEL classifications: D83, R30, G12

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1 Introduction

In many housing markets, the purchases of owner-occupied houses by investors who attempt to profit from buying low and selling high rather than for occupation are commonplace. For a long time, anecdotal evidence abounds as to how the presence of these investors, who are popularly known as flippers in the U.S., in the housing market can be widespread. More recently, empirical studies have began to systematically document the extent to which transactions in the housing market are motivated by buying and selling for short-term gains and how these activities are correlated with the housing price cycle. In particular, Haughwout et al. (2011) report that the share of all new purchase mortgages in the U.S. taken out by investors – individuals who hold two or more first-lien mortgages – was as high as 25% on average during the early to mid 2000s. At the peak of the housing market boom in 2006, the figures reached 35% for the whole of the U.S. and 45% for the"bubble states". Depken et al. (2009) report that for the same period, on average, 13.7% of housing market transactions were for houses sold again within the first two years of purchase in the metropolitan Las Vegas area. At the peak in 2005, it reached a high of 25%. Bayer et al. (2011) report that for five counties in the LA metropolitan area, over 15% of all homes purchased near the peak of the housing market boom in 2003-2005 were re-sold within two years. Even in the cold period in the 1990s, the percentage remained above 5%.

Arguably, the central questions on flipping in the housing market are how it may contribute to housing price volatility and whether it serves any useful purpose. In this paper, we study a housing market search model along the lines of Arnold (1989) and Wheaton (1990) in which houses are demanded by flippers in addition to end-user households to address the two questions.

In our model, the end-user households are liquidity constrained to the extent that each cannot hold more than one house at a time. In this case, a household which desires to move because the old house is no longer a good match must first sell it before the household can buy a new house. The primary advantage of flippers in our model is that, collectively, they can hold as many houses as the market needs them to do so. A mismatched and liquidity-constrained household can then sell the old house quickly to flippers to be able to buy a new house sooner. Thus, first of all, the flippers in our model are akin to the arbitraging middlemen in illiquid markets in classical finance theory, who help speed up turnover and improve liquidity in the market. Such liquidity services, not surprisingly, are in greatest demand in an otherwise sluggish and illiquid market in which it can take a long time for mismatched households to sell houses themselves.

Often times, investors in the housing market are cash-rich investors as well as experienced flippers. As cash-rich investors, they tend to have lower opportunity costs than others in holding vacant houses.[1] As experienced flippers, they should

[1] Helbert et al. (2013) report that many investors in the housing market are all-cash buyers.

be more adept at bargaining than many end-user households, who only buy and sell houses infrequently.[2] To capture such financing and bargaining advantages, in our model, we allow for the possibility that the flippers finance real estate investment at a lower cost and possess a greater bargaining strength than others. With these advantages, flippers sell houses at a relatively high price, in which case mismatched households can be better off letting flippers sell on their behalf, irrespective of how quickly they can sell houses themselves, if the flippers are not buying houses from them at too big a discount. In a tight and liquid market where houses are sold quickly, there can only be a small "bid-ask" spread in house flipping. Then, flippers must be buying houses from mismatched households at a similarly high price if they are selling those houses later on at a high price. The main novelty in our analysis is that we find that mismatched households may find it attractive to sell to flippers not just in sluggish and illiquid markets, but also in tight and liquid markets for the especially high price they receive from selling to flippers in such markets where the flipper-sellers' advantages are passed onto household-sellers to the fullest extent possible.

Because flippers can also thrive in a tight and liquid market while the market tends to be tight and liquid when flipping is prevalent, there can be multiple equilibria in our model. In one equilibrium, most, if not all, transactions are intermediated, resulting in rapid turnover and high housing prices. In another equilibrium, few houses are bought and sold by flippers. Turnover is slow and prices are moderate.

With the multiplicity of equilibrium, wide swings in price and transaction can happen without any underlying changes in housing supply, preference, and interest rate. Moreover, in our model, flippers can enter and exit en masse in response to the smallest interest rate shock. Then, on top of the usual effect of interest rates on asset prices, any such shocks can have a significant indirect impact on housing prices through their influences on the activities of flippers. In all, we find that even in the entire absence of any kind of extrinsic uncertainty and less-than-fully rational agents, flipping can still contribute to housing price volatility. A natural question to follow up is how important such a channel of volatility can be. Our quantitative analysis indicates that housing prices can differ by up to 23 percent across steady-state equilibria and vary by 26 percent in response to a seemingly unimportant interest rate shock when the model is calibrated to several observable characteristics of the U.S. housing market.

While more houses are being flipped and remain vacant in the hands of flippers, more households have to seek shelter in rental housing. In the model housing market, flipping can be excessive if the efficiency gains from the faster turnover fall short of the rental expenditures households incur during which houses are being flipped and left vacant. In an equilibrium where mismatched households are selling to flippers primarily for the high price flippers offer, they do not tend to be better off than if

[2] Bayer et al. (2011) document the prevalence of experienced flippers in the housing market and show that they often buy houses at lower prices and sell them at higher prices than others.

there were no flipping, for later on, the households will be buying new houses at a similarly high price. In this case, there is little to gain from flipping to offset its cost and the strategic complementary that gives rise to flipping in tight markets can be a form of market failure.

Our model has a number of readily testable implications. First, it trivially predicts a positive cross-section relation between housing prices and Time-On-The-Market (TOM) – mismatched homeowners can either sell quickly to flippers at a discount or to wait for a better offer from an end-user buyer to arrive – which agrees with the evidence reported in Merlo and Ortalo-Magne (2004), Leung et al. (2002) and Genesove and Mayer (1997), among others.[3]

An important goal of the recent housing market search and matching literature is to understand the positive time-series correlation between housing prices and sales and the negative correlation between the two and the average TOM.[4] In our model, across steady-state equilibria, a positive relation between prices and sales and a negative relation between the two and the average TOM also hold – in the equilibrium in which more houses are sold to flippers, prices and sales are both higher, whereas houses on average stay on the market for a shorter period of time. More importantly, our analysis adds a new twist to the time-series empirics of the housing market, which is that vacancies should increase together with prices and sales if the increase in sales is due to more houses sold to flippers, who will just leave them vacant until they are sold to the eventual end users.

Insofar as the flippers in our model act as middlemen between the original homeowners and the eventual end-user buyers, this paper contributes to the literature on middlemen in search and matching pioneered by Rubinstein and Wolinsky (1987). Previously, it was argued that middlemen could survive by developing reputations as sellers of high quality goods (Li, 1998), by holding a large inventory of differentiated products to make shopping less costly for others (Johri and Leach, 2002; Shevchenko, 2004; Smith, 2004), by raising the matching rate in case matching is subject to increasing returns (Masters, 2007), and by lowering distance-related trade costs for others (Tse, 2011). This paper studies the role of middlemen in the provision of market liquidity and the effects of any financing and bargaining advantages that middlemen may possess on the nature of equilibrium.

Among papers in the housing market search and matching literature, perhaps Moen and Nenov (2014) is closest to ours in that they study how decisions made by mismatched households, like those in our model, can lead to multiplicity. In their model, households are permitted to hold up to two houses, but holding two houses is

[3] Albrecht et al. (2007) emphasize another aspect of the results reported in Merlo and Ortalo-Magne (2004), which is that downward price revisions are increasingly likely when a house spends more and more time on the market.

[4] Stein (1995), who explains how the down-payment requirement plays a crucial role in amplifying shocks, is an early non-search-theoretic explanation for the positive relation between prices and sales. Hort (2000) and Leung et al. (2003), among others, provide recent evidence. Kwok and Tse (2006) show that the same relation holds in the cross section.

a particularly costly state, and so is holding no house. The strategic complementary in their model is that when other mismatched households are choosing to buy a new home first before selling, there will be a tight market, in which an individual mismatched household is also better off to buy first before selling. To do otherwise can leave the household in the costly state of holding no house for an extended period of time. Conversely, when other households choose selling first, there will be a slow market, in which it is optimal for an individual household to also choose selling first to avoid being stuck in the costly state of holding two houses for a long time. Like their model, decisions made by mismatched households in ours impart on market tightness, through which the strategic complementary works. Unlike their model in which prices do not appear to play any role in the strategic complementary, how prices vary with market tightness is crucial to the strategic complementary in ours. It is this last mechanism through which our model gives rise to the implication that housing prices are positively correlated with sales and vacancies but negatively correlated with TOM in the time series.

A simple model of housing market flippers as middlemen is also in Bayer et al. (2011). The model though is partial equilibrium in nature and cannot be used to answer many of the questions we ask in this paper. Intermediaries who buy up mismatched houses from households and then sell them on their behalf are also present in the model of the interaction of the frictional housing and labor markets of Head and Lloyd-Ellis (2012). But there the assumption is merely a simplifying assumption and the presence of these agents in the given setting appears inconsequential. Analyzes of how middlemen may serve to improve liquidity in a frictional market also include Gavazza (2012) and Lagos et al. (2011). These studies do not allow end-user households a choice of whether to deal with the middlemen and for the multiplicity of equilibrium like we do though. Multiple equilibria in a search and matching model with middlemen can also exist in Watanabe (2010). The multiplicity in that model is due to the assumption that the intermediation technology is subject to increasing returns to scale, whereas the multiplicity in ours arises from a particular strategic complementary. Moreover, only one of the two steady-state equilibria in that model is stable, whereas there can be more than one stable steady-state equilibria in ours.

The next section presents the model, the detailed analysis of which follows in Section 3. In section 4, we study the planner's problem of optimal flipping in the model housing market. Section 5 explores various empirical implications of the model. In section 6, we test the time-series implications of our model as pertain to especially the behavior of the vacancy rate. In Section 7, we calibrate the model to several observable characteristics of the U.S. housing market to assess the amount of volatility that the model can generate. Section 8, which draws on results we present in a technical note (Leung and Tse, 2016a) to accompany the paper, discusses several extensions of the model. Section 9 concludes. All proofs are relegated to the Appendix. For brevity, we restrict attention to analyzing steady-state equilibria in this paper. A second companion technical note (Leung and Tse, 2016b) covers the analysis of the dynamics for the special case in which all agents possess the same bargaining power.[5]

2 Model

2.1 Basics

There is a continuum of measure one risk-neutral households, each of whom discounts the future at the rate r. There are two types of housing: owner-occupied, the supply of which is perfectly inelastic at $H < 1$ and rental, which is supplied perfectly elastically for a rental payment of q per time unit. A household staying in a matched owner-occupied house enjoys a flow utility of $\upsilon > 0$, whereas a household either in a mismatched house or in rental housing none. A household-house match breaks up exogenously at a Poisson arrival rate δ, after which the household may continue to stay in the house but it no longer enjoys the flow utility υ. In the meantime, the household may choose to sell the old house and search out a new match. An important assumption is that households are liquidity constrained to the extent that each can hold at most one house at a time. Then, a mismatched homeowner must first sell the old house and move to rental housing before she can buy a new house. Our qualitative results should hold as long as there is a limit, not necessarily one, on the number of houses a household can own at a time.[6] The one-house-limit assumption simplifies the analysis considerably.

The end-user market Households buy and sell houses in a frictional market in which the flow of matches falls out from a concave and CRS matching function $M(B, S)$, with B and S denoting, respectively, the measures of buyers and sellers in the market. Let $\theta = B/S$ denote market tightness. Then, the rate at which a seller finds a buyer is

$$\eta(\theta) \equiv \frac{M(B, S)}{S} = M(\theta, 1),$$

whereas the buyer's matching rate is $\mu(\theta) = \eta(\theta)/\theta$. Given that M is increasing and concave in B and S,

$$\frac{\partial \eta}{\partial \theta} > 0, \qquad \frac{\partial \mu}{\partial \theta} < 0.$$

We impose the usual regularity conditions on M for

$$\lim_{\theta \to 0} \eta(\theta) = \lim_{\theta \to \infty} \mu(\theta) = 0, \qquad \lim_{\theta \to \infty} \eta(\theta) = \lim_{\theta \to 0} \mu(\theta) = \infty.$$

[5] Not for publication, available for download in http://www.sef.hku.hk/~tsechung/index.htm

[6] In Section 8 and in Leung and Tse (2016a), we explain how this is the case when households can hold up to two houses at a time.

The Walrasian investment market Instead of waiting out an end-user buyer to come along, a mismatched household may sell the old house right away in a Walrasian market to specialist investors – agents who do not live in the houses they have bought but rather attempt to profit from buying low and selling high. Because homogeneous flippers do not gain by buying and selling houses among themselves, the risk-neutral flippers may only sell in the end-user market and will succeed in doing so at the same rate $\eta(\theta)$ that any household-seller does in the market. We assume that flippers discount the future at the same rate r that end-user households do but allow for the possibility that they finance real estate investment at a different rate r_F. In the competitive investment market, prices adjust to eliminate any excess returns on real estate investment.

Of course, it is hard to envisage that there are two distinct markets – one frictional and one Walrasian – for a household-seller to choose between in reality. The assumption is but a convenient modelling fiction for where the seller chooses between selling right away at a discount to investors and waiting for a better offer from an end-user buyer to arrive.

The assumption of a Walrasian investment market is, by all means, a simplifying assumption. A more general assumption is to model the market as a frictional market too, but possibly one in which the frictions are less severe than in the end-user market. If flippers are entirely motivated by arbitrage considerations and do not care if the houses to be purchased are good matches for their own occupation, search should be a much less serious problem. Moreover, if flippers enter the housing market and search at a zero entry cost, in equilibrium, the market will be populated by an infinite mass of flipper-buyers, in which case, any household-sellers who bother to search for an investor to trade with will be able to find one instantaneously for any usual matching function. In this way, a Walrasian investment market can be thought of as the limit of a frictional market in which the costs of entry and search for flippers tend to zero. In Section 8 below, we discuss how our qualitative results should hold in the general case in which the market is populated by only a finite measure of flipper-buyers who meet household-sellers in a frictional market.

2.2 Stocks and flows

Accounting identities At any one time, a household can either be staying in a matched house, in a mismatched house, or in rental housing. Let n_M, n_U, and n_R denote the measures of households in the respective states, which must sum to the unit measure of households in the market; i.e.,

$$n_M + n_U + n_R = 1. \tag{1}$$

Each owner-occupied house must be held either by an end-user household or by a flipper. Hence,

$$n_M + n_U + n_F = H, \tag{2}$$

where n_F denotes both the measures of active flippers and houses held by these agents.

If each household can hold at most one house at any moment, the only buyers in the end-user market are households in rental housing; i.e.,

$$B = n_R. \tag{3}$$

On the other hand, sellers in the market include both mismatched homeowners and flippers, so that

$$S = n_U + n_F. \tag{4}$$

Housing market flows In each unit of time, the inflows into matched owner-occupied housing are comprised of the successful buyers among all households in rental housing $(\mu(\theta) n_R)$, whereas the outflows are comprised of those who become mismatched in the same time period (δn_M). In the steady state,

$$\dot{n_M} = 0 \Rightarrow \mu(\theta) n_R = \delta n_M. \tag{5}$$

Households' whose matches just break up may choose to sell their old houses right away to flippers in the investment market or to wait out a buyer to arrive in the end-user market. Let α denote the (endogenously determined) fraction of mismatched households who sell in the investment market and $1-\alpha$ the fraction who sell in the end-user market. In each time unit then, the measure of mismatched homeowners selling in the end-user market increases by $(1-\alpha)\delta n_M$, whereas the exits are comprised of the successful sellers $(\eta(\theta) n_U)$ in the meantime. In the steady state,

$$\dot{n_U} = 0 \Rightarrow (1-\alpha)\delta n_M = \eta(\theta) n_U. \tag{6}$$

It can be shown that (5) and (6), together with $\theta = \frac{n_R}{n_U + n_F}$ and $\mu = \frac{\eta}{\theta}$, imply that

$$\alpha\delta n_M + \eta(\theta) n_U = \mu(\theta) n_R,$$

$$\alpha\delta n_M = \eta(\theta) n_F,$$

which say that, respectively, the flows into and out of rental housing are equal $\left(\dot{n_R} = 0\right)$ and the measure of houses bought by flippers are matched by the measure sold $\left(\dot{n_F} = 0\right)$. Figure 1 depicts the flows of households into and out of the three states.

(figure 1 about here)

2.3 Market tightness

Solving (1)-(6), we can show the following.

Lemma 1

a. *At* $\alpha = 0$, $n_F = 0$.

b. *As* α *increases from 0 toward 1,*

$$\frac{\partial n_F}{\partial \alpha} > 0, \quad \frac{\partial n_U}{\partial \alpha} < 0, \quad \frac{\partial n_R}{\partial \alpha} > 0, \quad \frac{\partial n_M}{\partial \alpha} > 0.$$

c. *At* $\alpha = 1$, $n_U = 0$.

Parts (a) and (c) of the Lemma say that, respectively, at one extreme, when no mismatched households are selling to flippers, there cannot be any vacant houses in the hands of these agents, and at the other extreme, when all households sell to flippers immediately after becoming mismatched, there cannot be any household remaining in a mismatched house in the steady state. In general, by part (b), as α increases from 0 toward 1, there are more houses in the hands of flippers and fewer in the hands of mismatched homeowners.

What is less obvious in the Lemma is that as α increases, there would also be more households in rental housing, as well as being matched in the steady state. Intuitively, when flippers hold a greater fraction of the owner-occupied housing stock, fewer households can stay in owner-occupied houses and therefore more must be accommodated in rental housing. In the meantime, there are fewer mismatched households spending any time at all selling their old houses in the end-user market before initiating search for a new match. Then, there should only be more matched households in the steady state.

When more mismatched households sell to flippers right away, there are not only more buyers in the end-user market but also fewer sellers as well.[7] This is because as houses are sold faster where there are more buyers, there can only be a smaller stock of houses left for sale in the steady state. The two tendencies should then conspire to give rise to a tighter market with a larger

$$\theta = \frac{B}{S} = \frac{n_R}{n_U + n_F}.$$

Lemma 2 *The stock-flow equations* (1)-(6) *can be combined to yield a single equation,*

$$\delta + \eta(\theta)(1 - H) - (1 - \alpha + \theta) H\delta = 0, \tag{7}$$

[7] By (2), $S = n_U + n_F = H - n_M$, which is decreasing in α given that $\partial n_M / \partial \alpha > 0$.

in α and θ, from which an implicit function $\theta = \theta_S(\alpha)$ for $\alpha \in [0,1]$, can be defined, and that $\partial\theta_S/\partial\alpha > 0$. Both the lower and upper bounds, given by, respectively, $\theta_S(0)$ and $\theta_S(1)$, are strictly positive and finite for any $H \in (0,1)$. For each α, $\theta(\alpha)$ is decreasing in H, and that $\lim_{H\to 1}\theta(0) = 0$.

The last part of the Lemma says that a larger housing stock tends to give rise to a slower end-user market. Other things equal, there will be more houses for sale with a larger housing stock and as houses are sold more slowly as a result, there can only be fewer mismatched households entering the end-user market as buyers. Altogether then, market tightness θ falls. Indeed, if there is one house for each household ($H = 1$) and if no mismatched households are selling to flippers at all ($\alpha = 0$), no household-sellers can rid themselves of the old mismatched houses to start searching for a new house in which case, with $B = n_R = 0$, market tightness falls to zero.

2.4 Asset values and housing prices

Lemmas 1 and 2 above describe how the measures of households in the three states, the stock of houses in the hands of flippers, and the market tightness depends on α. The value of α – for the fraction of mismatched households selling to flippers – depends on the households' comparison of the payoffs of selling in the investment versus the end-user markets.

Asset values for households A household staying in a mismatched house and trying to sell it to an end user has asset value V_U satisfying,

$$rV_U = \eta(\theta)(V_R + p_H - V_U), \tag{8}$$

where p_H denotes the price the household expects to receive for selling the house to an end user and V_R the value of being in rental housing. Under (8), the mismatched homeowner is entirely preoccupied with disposing the old house while she makes no attempt to search for a new match. This is due to the assumption that a household cannot hold more than one house at a time. Once the household manages to sell the old house and only then, it moves to rental housing and enters the end-user market as a would-be buyer, who may eventually be buying either from a mismatched homeowner at price p_H or from a flipper at a price we denote as p_{FS}.

Lemma 3 *In the steady state, the fraction of flipper-sellers among all sellers in the end-user market is equal to the fraction of mismatched households selling to flippers in the first place; i.e.,*

$$\frac{n_F}{n_F + n_U} = \alpha.$$

In this case,

$$rV_R = -q + \mu(\theta)(V_M - (\alpha p_{FS} + (1-\alpha)p_H) - V_R), \tag{9}$$

where q is the exogenously given flow rental payment and V_M the value of staying in a matched house, given by

$$rV_M = \upsilon + \delta \left(\max \left\{ V_R + p_{FB}, V_U \right\} - V_M \right). \tag{10}$$

The owner-occupier enjoys the flow utility υ while being matched. The match will be broken, however, with probability δ, after which the household may sell the house right away in the investment market at a price we denote as p_{FB} and switch to rental housing immediately thereafter. Alternatively, the household can continue to stay in the house while trying to sell it to an end user. Note that under (10) and (8), the mismatched household has one chance only to sell the old house in the investment market, at the moment the match is broken. Those who forfeit this one-time opportunity must wait out a buyer in the end-user market to arrive. The restriction is without loss of generality though in a steady-state equilibrium, in which the asset values and housing prices stay unchanging over time.[8]

Asset values for flippers If a flipper expects to receive p_{FS} for selling a house in the end-user market and has previously paid p_{FB} for it in the investment market, the flipper has asset value V_F satisfying,

$$rV_F = -r_F p_{FB} + \eta(\theta) \left(p_{FS} - p_{FB} - V_F \right),$$

where r_F is the flipper's cost of funds. With free entry in house flipping, $V_F = 0$ and therefore,

$$p_{FB} = \frac{\eta(\theta)}{\eta(\theta) + r_F} p_{FS}. \tag{11}$$

Bargaining When a household-buyer in rental housing is matched with a flipper-seller, the division of surplus in the bargaining satisfies

$$\beta_F \left(V_M - p_{FS} - V_R \right) = \left(1 - \beta_F \right) \left(p_{FS} - p_{FB} \right), \tag{12}$$

where β_F denotes the flipper-seller's share of the match surplus. When the same buyer is matched with a household-seller in a mismatched house, the division of surplus in the bargaining satisfies

$$\beta_H \left(V_M - p_H - V_R \right) = \left(1 - \beta_H \right) \left(V_R + p_H - V_U \right), \tag{13}$$

where β_H denotes the household-seller's share of the match surplus. If flippers are agents specializing in buying and selling, it is most reasonable to assume that $\beta_F \geq \beta_H$.

[8]Equation (9) assumes that the rental household is better off buying a house either from a mismatched homeower or a flipper rather than continuing to stay in rental housing. Equations (10) and (8) assume that the mismatched household is better off selling the old house either in the investment or end-user market rather than just staying in the mismatched house. Lemma 14 in the Appendix verifies that all this holds in equilibrium.

2.5 Households' optimization

Write

$$D(\theta, \alpha) \equiv V_R + p_{FB} - V_U, \tag{14}$$

as the difference in payoff for a mismatched household between selling in the investment market ($V_R + p_{FB}$) and in the end-user market (V_U). If $D(\theta, \alpha) > 0$ (< 0), for all $\alpha \in [0, 1]$, the household strictly prefers to sell in the investment (end-user) market at the given θ no matter what others choose to do. For certain θ, there may exist some $\alpha_D(\theta) \in [0, 1]$ such that $D(\theta, \alpha_D(\theta)) = 0$, in which case equilibrium requires a fraction $\alpha_D(\theta)$ of mismatched households selling in the investment market and the rest selling in the end-user market. In sum, we can define a relation,

$$\alpha_O(\theta) = \begin{cases} 1 & D(\theta, 1) \geq 0 \\ \alpha_D(\theta) & D(\theta, \alpha_D(\theta)) = 0 \\ 0 & D(\theta, 0) \leq 0 \end{cases},$$

between market tightness θ and the fraction α of mismatched households selling in the investment market from the households' optimization.

2.6 Equilibrium

We now have two steady-state relations between α and θ: the $\theta_S(\alpha)$ function in Lemma 2 from the stock-flow equations and the $\alpha_O(\theta)$ relation from mismatched households' optimization. A steady-state equilibrium is any $\{\alpha, \theta\}$ pair that simultaneously satisfies the two relations.[9]

3 Analysis

3.1 Flippers' advantages and the nature of equilibrium

To proceed with the analysis of equilibrium, we begin with the characterization of the $D(\theta, \alpha)$ function that underlies the $\alpha_O(\theta)$ relation. By (11)-(14),

$$D(\theta, \alpha) = (1 - \beta_H)^{-1} \left(\frac{\beta_H r_F + \beta_F \eta(\theta)}{\beta_F \eta(\theta)} p_{FB} - p_H \right). \tag{15}$$

For $\beta_H \neq 1$, (15) has the same sign as

$$p_{FB} - p_H \frac{\eta(\theta) \beta_F}{\eta(\theta) \beta_F + r_F \beta_H}. \tag{16}$$

Recall that a mismatched household receives p_{FB} right away if it sells in the investment market, whereas it will receive p_H at some uncertain future date if it offers the

[9]Proposition 6 in the Appendix establishes the existence of equilibrium.

house for sale in the end-user market. By (16), the comparison in (14) is likened to a comparison between the instantaneous reward p_{FB} and an appropriately discounted future reward p_H of selling the house in the two markets.

In a tighter end-user market with a larger θ, houses, on average, are sold faster in the market. Then, p_H in (16) should be discounted less heavily in the comparison of the payoffs between selling in the two markets. Indeed, for given p_{FB} and p_H, the expression in (16) is decreasing in θ, whereby households find selling in the investment market less attractive when they can sell their mismatched houses faster in the tighter end-user market.

Both p_{FB} and p_H, however, can also depend on θ. How $D(\theta, \alpha)$ behaves as a function of θ can only be resolved by also checking how the two prices vary with θ. Solving p_{FB} and p_H, together with those of the various asset values, from (8)-(13) and substituting in the solutions to (15), $D(\theta, \alpha)$ is seen to have the same sign as

$$\widehat{D}(\theta, \alpha) \equiv \left(\beta_F \frac{r}{r_F} - \beta_H - z\beta_H\right) \eta(\theta) + (1 - \beta_H - \alpha(\beta_F - \beta_H)) \mu(\theta) - (\delta + r) z, \tag{17}$$

where $z = q/\upsilon$.[10]

3.1.1 Inventory advantage

It is straightforward to verify that $\widehat{D}(\theta, \alpha)$ is indeed everywhere decreasing in θ for

$$r_F \geq \frac{\beta_F}{\beta_H} \frac{r}{1+z} \equiv \overline{r}. \tag{18}$$

Lemma 4 *For $r_F \geq \overline{r}$, $\alpha_O(\theta)$ is everywhere non-increasing, given by,*

$$\alpha_O(\theta) = \begin{cases} 1 & \theta \leq \theta_1^d \\ \alpha_D(\theta) & \theta \in \left(\theta_1^d, \theta_0^d\right) \\ 0 & \theta \geq \theta_0^d \end{cases},$$

where $\theta_1^d < \theta_0^d$ are defined by, respectively, $\widehat{D}\left(\theta_1^d, 1\right) = 0$ and $\widehat{D}\left(\theta_0^d, 0\right) = 0$, and that $\partial \alpha_D(\theta) / \partial \theta < 0$.[11]

(figure 2 about here)

[10]The solutions are presented in Lemma 13 in the Appendix. There are two sets of prices and asset values, one derived under the assumption that $D \leq 0$ and the other $D \geq 0$. In either case, D is seen to have the same sign as $\widehat{D}$ in (17).

[11]The superscript "d" denotes the θ is at where $\widehat{D}$ is decreasing in θ. Likewise, the superscript "u" is used later on to denote the θ is at where $\widehat{D}$ is increasing in θ.

Panel A of Figure 2 illustrates an example of the $\alpha_O(\theta)$ function in the Lemma, under which mismatched households only prefer selling in the investment market ($\alpha > 0$) for small θ to avoid the possibly lengthy wait in selling in a slow end-user market. The condition of the Lemma, by (18), is met for $r_F = r$ and $\beta_F = \beta_H$. This means that in the absence of any financing or bargaining advantage over end-user households, flippers may survive only when the end-user market is relatively slow and illiquid (small θ) and on the basis of helping mismatched households overcome the liquidity problems that they face in the market.

To pin down the equilibrium α and θ, we invert $\theta_S(\alpha)$ in Lemma 2 to define $\alpha_S \equiv \theta_S^{-1}$, whereby $\alpha_S : [\theta_S(0), \theta_S(1)] \to [0,1]$. In the three panels of Figure 2, we superimpose a different $\alpha_S(\theta)$ onto the same $\alpha_O(\theta)$. Given a strictly increasing $\alpha_S(\theta)$ and a non-increasing $\alpha_O(\theta)$, equilibrium is guaranteed unique. In Panel A, at $\theta = \theta_S(0)$, $\alpha_O(\theta) = 0$; then $\{\alpha, \theta\} = \{0, \theta_S(0)\}$ is the unique steady-state equilibrium. In this equilibrium, which we refer to as the no-intermediation equilibrium, all sales and purchases are between two end users. In Panel C, at $\theta = \theta_S(1)$, $\alpha_O(\theta) = 1$; then $\{\alpha, \theta\} = \{1, \theta_S(1)\}$ is the unique steady-state equilibrium. In this equilibrium, which we refer to as the fully-intermediated equilibrium, all transactions are intermediated by flippers. In between, there can be equilibria at where $\widehat{D}(\theta_S(\alpha), \alpha) = 0$, as illustrated in Panel B. In such partially-intermediated equilibria, with mismatched households indifferent between selling in the investment and the end-user markets, a fraction, but only a fraction, of all transactions are intermediated.

Lemma 5 *For each $r_F \geq \overline{r}$, there exists some $H(r_F) < 1$, such that $\alpha > 0$ in equilibrium if and only if $H \in (H(r_F), 1]$, and that $\partial H(r_F)/\partial r_F > 0$.*

The Lemma says that we can divide H-r_F space, for each $r_F \geq \overline{r}$, into two subsets bordered by an upward-sloping $H(r_F)$, whereby $\alpha > 0$ ($\alpha = 0$) in the unique equilibrium for $\{H, r_F\}$ to the right (left) side of the border, as illustrated in Figure 3. Intuitively, when flippers are obliged to finance real estate investment at a larger r_F, they can only afford to pay a lower price. Households then still find it attractive to sell to flippers when selling in the end-user market is becoming more difficult. By Lemma 2, as the housing stock H increases, there will be a more sluggish end-user market in which houses are sold more slowly. In this way, as r_F increases, the indifference condition $\widehat{D} = 0$ holds only for larger and larger H. Perhaps of particular interest is that the Lemma implies that $\widehat{D} > 0$ can hold even for an arbitrarily large r_F if there is also a large enough H. That is, there can be a liquidity provision role for flippers even when they are handicapped by a very large financing cost if, in the meantime, there is a large enough housing stock to make selling in the end-user market very difficult for mismatched households.

(figure 3 about here)

3.1.2 Financing/Bargaining advantage

For all $r_F > 0$, $\widehat{D}(\theta, \alpha)$ in (17) is at first decreasing in θ – there is less of a liquidity-provision role for flippers as the end-user market gets tighter and more liquid. For $r_F < \overline{r}$, however, which holds for r_F sufficiently below r and/or for β_F sufficiently above β_H,[12] $\widehat{D}(\theta, \alpha)$ is eventually increasing in θ. Altogether, $\widehat{D}(\theta, \alpha)$ is U-shaped if moreover $\partial\widehat{D}(\theta, \alpha)/\partial\theta$ changes sign just once, which is guaranteed to be the case under a fairly weak condition on $\eta(\theta)$ – a condition we assume holds in the following.[13] That is, if the flipper-seller possesses some sufficiently large financing and/or bargaining advantages over the household-seller, there is a force, other than for liquidity reason, that makes selling in the investment market attractive for mismatched households, and such a force strengthens as the market gets tighter.

Where $r_F < r$, the flipper-seller has a lower opportunity cost of holding the vacant house and therefore a more favorable outside option in bargaining than the household-seller. Where $\beta_F > \beta_H$, the flipper-seller can extract a greater share of the match surplus than the household-seller can. With one or both advantages, the flipper-seller would be able to bargain for a price p_{FS} above the price p_H that the mismatched homeowner can bargain for herself in the sale to the same end-user buyer, other things being equal. In a tighter and more liquid market, by (11), the "bid-ask" spread in house flipping narrows as houses are sold more quickly. Then, as the market tightens, if the flipper is able to sell a house at a relatively high p_{FS} in the end-user market, the flipper must be paying a more than proportionately higher p_{FB} for the house earlier in the investment market. In all, for $r_F < \overline{r}$, mismatched households may also find selling to flippers attractive for the especially high price flippers offer when the market is tight and liquid as the flipper-seller's advantages are passed onto the household-seller to the fullest extent possible in such a market.

Lemma 6 *There exist some r' and r'', with $0 < r' < r'' \leq \overline{r}$, such that*

a. *For $r_F \in (0, r']$,*

$$\alpha_O(\theta) = 1 \text{ for } \theta \geq 0.$$

b. *For $r_F \in (r', r'')$,*

$$\alpha_O(\theta) = \begin{cases} 1 & \theta \leq \theta_1^d \\ \alpha_D(\theta) & \theta \in \left(\theta_1^d, \theta_1^u\right) \\ 1 & \theta \geq \theta_1^u \end{cases},$$

[12]Given that the condition $r_F < \overline{r}$, by (18), is a condition on by how much r_F/β_F is below r/β_H, it appears that it may be more illuminating to state results in terms of the comparison of the two ratios instead of stating results in terms of the level of r_F. The conditions to follow in Lemma 6, however, cannot be stated just in terms of one and/or both ratios. For coherence, we think it is least confusing to state conditions throughout in terms of the level of r_F.

[13]The condition is $2\frac{\partial\eta}{\partial\theta}\left(\eta(\theta) - \theta\frac{\partial\eta}{\partial\theta}\right) + \theta\frac{\partial^2\eta}{\partial\theta^2}\eta(\theta) \leq 0$, which is guaranteed to hold if $\eta(\theta)$ is isoelastic.

where $\theta_1^d < \theta_1^u$ *are defined by* $\widehat{D}\left(\theta_1^d, 1\right) = 0$ *and* $\widehat{D}\left(\theta_1^u, 1\right) = 0$ *over where* $\widehat{D}(\theta, \alpha)$ *is decreasing and increasing in* θ*, respectively. Here,* $\alpha_D(\theta)$ *is first decreasing, reaching a minimum above zero, and then increasing toward 1.*

c. *For* $r_F \in [r'', \overline{r})$,

$$\alpha_O(\theta) = \begin{cases} 1 & \theta \leq \theta_1^d \\ \alpha_D(\theta) & \theta \in \left(\theta_1^d, \theta_0^d\right) \\ 0 & \theta \in \left[\theta_0^d, \theta_0^u\right] \\ \alpha_D(\theta) & \theta \in \left(\theta_0^u, \theta_1^u\right) \\ 1 & \theta \geq \theta_1^u \end{cases},$$

where $\theta_1^d < \theta_0^d < \theta_0^u < \theta_1^u$ *are defined by, respectively,* $\widehat{D}\left(\theta_1^d, 1\right) = 0$ *and* $\widehat{D}\left(\theta_0^d, 0\right) = 0$*, over where* $\widehat{D}(\theta, \alpha)$ *is decreasing in* θ*, and* $\widehat{D}\left(\theta_0^u, 0\right) = 0$ *and* $\widehat{D}\left(\theta_1^u, 1\right) = 0$ *over where* $\widehat{D}(\theta, \alpha)$ *is increasing in* θ*. For* $\theta \in \left(\theta_1^d, \theta_0^d\right)$*,* $\partial\alpha_D(\theta)/\partial\theta < 0$*, whereas for* $\theta \in \left(\theta_0^u, \theta_1^u\right)$*,* $\partial\alpha_D(\theta)/\partial\theta > 0$*.*

Part (a) of the Lemma covers the situation in which the U-shaped $\widehat{D}(\theta, \alpha)$ function stays above zero for all α and θ, meaning that the mismatched household's payoff of selling in the investment market exceeds that of in the end-user market at any market tightness. Then, with $\alpha_O(\theta) = 1$ for all θ, as illustrated in Panel A of Figure 4, the unique equilibrium must be at $\alpha = 1$ for any values of H, as indicated in Figure 4. In this case, flippers' financing/bargaining advantage is so overwhelming that the unique equilibrium must be a fully-intermediated equilibrium.

(figure 4 about here)

Lemma 6(b) covers the situation for r_F above a first threshold r' but below a second threshold r'', whereby $\alpha_O(\theta)$, though can fall below one, never falls all the way down to zero for any θ, as illustrated in Panel B of Figure 4, so that any equilibrium must be at where $\alpha > 0$, as indicated in Figure 3. In this case, flippers' financing/bargaining advantage, though remains operative, only suffices to allow them to offer a price attractive enough to lure all mismatched households to sell in the investment market when the end-user market is sufficiently tight.

As r_F reaches and rises above the r'' threshold, Lemma 6(c) says that $\alpha_O(\theta)$ now does fall to zero over a given interval $\left[\theta_0^d, \theta_0^u\right]$ of market tightness, as illustrated in Panel C of Figure 4.

Lemma 7 *For each* $r_F \in [r'', \overline{r})$*, a no-intermediation equilibrium with* $\alpha = 0$ *exists if and only if* $H \in \left[\underline{H}(r_F), \overline{H}(r_F)\right]$*, for some* $\underline{H}(r_F)$ *and* $\overline{H}(r_F)$*, where* $0 < \underline{H}(r_F) \leq \overline{H}(r_F) < 1$*, and that* $\partial\underline{H}(r_F)/\partial r_F < 0$ *and* $\partial\overline{H}(r_F)/\partial r_F > 0$*. Otherwise, any equilibrium must either be a partially- or fully-intermediated equilibrium with* $\alpha > 0$*.*

Lemma 7 says that we can divide the H-r_F space for $r_F \in [r'', \overline{r})$ in Figure 3 into three subsets with the middle, but only the middle, subset made up of all $\{H, r_F\}$ for which there exists a no-intermediation equilibrium. Conversely, for small H, with which the end-user market tends to be sufficiently tight and liquid to cause flippers to pass onto their advantages to household-sellers to a large enough extent, and for large H, with which the end-user market tends to be sufficiently slow and illiquid to present enough difficulties for households to sell houses themselves, any equilibrium must involve at least a fraction of households selling in the investment market. That the border in Figure 3 dividing the second and the third subsets is upward-sloping has the same interpretation as to why the border in the Figure for $r_F \geq \overline{r}$ is upward-sloping: an increase in r_F hinders flippers' ability to offer a price attractive enough to allow them to carry out their liquidity-provision role so that flipping can continue to take place only when the end-user market is more illiquid due to a larger housing stock, whereas the border dividing the first and second subsets slopes down as the weakened financing advantage of flippers caused by a larger r_F must be compensated by more of the advantage being passed onto households as resulting from the market becoming tighter due to a smaller housing stock for households to still find selling to flippers advantageous.

3.1.3 Summing up

In the model housing market, flippers possess two advantages over end-user households: (1) inventory advantage and (2) financing/bargaining advantage. For the smallest r_F and largest β_F, the financing/bargaining advantage suffices to enable flippers to offer a good enough price to lure all mismatched households to sell in the investment market at any level of market tightness in the end-user market resulting from any level of housing supply. As the advantage weakens, flippers may only survive in sluggish markets on the basis of helping mismatched households overcome the liquidity problems and in tight markets on the basis of offering a very attractive price. Finally, when the financing/bargaining advantage weakens further and disappears altogether, flippers may only survive on the basis of providing liquidity services. In this case, equilibrium is guaranteed unique and may involve flipping only for a relatively sluggish market resulting from a large housing supply.

3.2 Multiplicity

In Panels B and C of Figure 4, where $\alpha_O(\theta)$ becomes an increasing function over a range of θ, there can well be multiple equilibria given that $\alpha_S(\theta)$ is upward sloping throughout. Figures 5 and 6 illustrate two examples of multiplicity. In both examples, there are as many as three equilibria. Full intermediation, with $\alpha = 1$, is equilibrium in the examples since at $\theta = \theta_S(1)$, flippers will be paying a price attractive enough to lure all mismatched households to sell in the investment market, while if all mismatched houses are sold in the investment market right away, the rapid turnover will indeed give rise to a tight market with $\theta = \theta_S(1)$. In a slower market, flippers will no longer be paying a p_{FB} attractive enough to lure all mismatched households to sell in the investment market. But precisely because fewer or none at all mismatched houses are sold in the investment market, a relatively sluggish end-user market will emerge from the slower turnover. As a result, a smaller α and a smaller θ is also equilibrium in Figures 5 and 6.

(figures 5, 6 about here)

Consider a small perturbation from the middle equilibrium in Figures 5 and 6 that knocks the $\{\alpha, \theta\}$ pair off to the right of the $\alpha_O(\theta)$ function. Then, $\widehat{D}(\theta, \alpha) > 0$ since $\partial \widehat{D}/\partial \theta > 0$ for $\theta \geq \theta_0^u$, after which all mismatched households will find it better to sell in the investment market. The increase in turnover will raise θ further. Eventually the market should settle at the $\alpha = 1$ equilibrium. Conversely, a perturbation that knocks the $\{\alpha, \theta\}$ pair off to the left of $\alpha_O(\theta)$ function from the middle equilibrium in Figures 5 and 6 should send the market to a smaller α equilibrium. In general, an equilibrium at where $\alpha_O(\theta)$ is increasing should be unstable. By analogous arguments, the other equilibria in the two examples should be locally stable.[14] Hence, there are not just multiple steady-state equilibria but also multiple locally *stable* steady-state equilibria.

A necessary condition for multiplicity, by Lemmas 4 and 6, is that $r_F \in (r', \overline{r})$ over which $\alpha_O(\theta)$ becomes increasing over a range of θ. The two examples above suggest that for $r_F \in [r'', \overline{r})$, suffice for the existence of multiple equilibria is that

$$\theta_S(0) < \theta_0^u < \theta_1^u \leq \theta_S(1), \tag{19}$$

where the first inequality ensures the existence of at least one $\alpha < 1$ equilibrium and the last inequality the existence of the $\alpha = 1$ equilibrium. Lemma 15 in the Appendix presents a set of conditions that guarantee that (19) holds.

With multiple steady-state equilibria, how much flipping takes place in the market can be fickle, especially when the equilibrium the market happens to be in is unstable. In general, where there are multiple equilibria, any seemingly unimportant shock can dislocate the market from one equilibrium and move it to another, causing catastrophic changes in flippers' market share, turnover, and sales. To follow such discrete changes in flipping can be significant fluctuations in housing price, a subject we shall address in Section 6. And then in Section 7, we will calibrate the model to several observable characteristics of the U.S. housing market to assess the quantitative importance of such a channel of volatility. But next, we should first study the problem of optimal flipping in the model housing market.

[14] A more rigorous local stability analysis is in Leung and Tse (2016b).

4 Efficiency

The planner, who is subject to the same trading and financing frictions that agents in the model housing market face, chooses the fraction of mismatched households using the services of flippers to maximize the utility flows over time that households derive from matched owner-occupied housing net of the rental expenditures incurred; i.e.,

$$\max_{\alpha} \left\{ \int_0^{\infty} e^{-rt} \left(n_M \upsilon - n_R q \right) dt \right\}, \tag{20}$$

subject to (1)-(4), the equations of motions for n_M and n_U given by the differences between the LHS and RHS of (5) and (6), respectively, and some given initial conditions for the two state variables.[15]

Lemma 8 *In the steady state, the planner chooses*

$$\alpha = \begin{cases} 1 & E\left(\theta_S\left(1\right)\right) \geq 0 \\ \alpha_E & E\left(\theta_S\left(\alpha_E\right)\right) = 0 \\ 0 & E\left(\theta_S\left(0\right)\right) \leq 0 \end{cases}, \tag{21}$$

where

$$E\left(\theta\right) = \frac{\partial \eta}{\partial \theta} - \left(r + \delta + \eta\left(\theta\right) - \theta \frac{\partial \eta}{\partial \theta} \right) z. \tag{22}$$

The function $E\left(\theta\right)$, which can be thought of as the planner's incentives to have mismatched households selling to flippers,[16] starts off equal to positive infinity at $\theta = 0$, is everywhere decreasing in θ, and ends up equal to some negative value as $\theta \to \infty$. Then, with $\partial \theta_S\left(\alpha\right) / \partial \alpha > 0$, one and only one of the three cases in (21) applies for a given parameter configuration. In particular, the third line implies that the necessary and sufficient condition for any flipping to be optimal is that $E\left(\theta_S\left(0\right)\right) > 0$, which holds for small $\theta_S\left(0\right)$ with $E\left(\theta\right)$ a decreasing function. Other things equal, a small $\theta_S\left(0\right)$ will arise out of a large housing supply, with which there will be a sluggish and illiquid market. Conversely, in a tight and liquid market with a large $\theta_S\left(0\right)$ due to a small housing supply, there will be rapid turnover no matter what, in which case there is little to gain from any increase in turnover to offset the additional resources incurred in the provision of rental housing while flipping takes place. In comparing households' private incentives to sell to flippers as given by $\widehat{D}$ in

[15] The equations of motion for n_R and n_F do not constitute independent restrictions given (1)-(4) and the equations of motions for n_M and n_U as the two equations can be shown to be implied by the former set of equations. Likewise, the initial values for n_R and n_F are not free variables but are restricted by the initial conditions for n_M and n_U and (1) and (2).

[16] Fornally, $E\left(\theta\right)$ denotes the sign of the steady-state shadow value of n_R in the planner's optimization. When it is positive, the objective function in (20) increases in value as the planner sends one more mismatched household to rental housing by making the household sell to a flipper.

(17) with the planner's incentives as given by E in (22) in Proposition 1 below, we find that such excessive flipping is indeed possible in equilibrium.

Proposition 1

a. *Equilibrium is efficient if*

$$\beta_H = 1 - \frac{\theta}{\eta(\theta)} \frac{\partial \eta}{\partial \theta} \equiv \beta^e,$$

$$\beta_F = \left(1 - \frac{\theta}{\eta(\theta)} \frac{\partial \eta}{\partial \theta}\right) \frac{\theta - \alpha}{\frac{r}{r_F}\theta - \alpha} \equiv \beta_F^e$$

at the optimum θ.

b. $\widehat{D}(\theta, \alpha) > E(\theta)$ for θ at which

$$\frac{\partial \widehat{D}}{\partial \theta} > 0$$

and

$$\frac{\partial \eta}{\partial \theta} \frac{1}{\eta(\theta)} \leq 2 \left[\left(\frac{\overline{r}}{r_F} - 1 \right) (1 + z) z \beta_H \right]^{1/2}. \tag{23}$$

In case $r_F = r$, Proposition 1(a) says that $\beta_F^e = \beta^e$ and the Hosios (1990) condition holds exactly. The congestion externalities and the effects of imperfect appropriability in the present model just cancel out and efficiency is obtained when the buyer's share of the match surplus, whoever the buyer is matched with, is just equal to the elasticity of the matching function with respect to the measure of buyers. For $r_F < r$, $\beta_F^e < \beta^e$ though. This means that in particular, if $r_F < r$ but $\beta_F = \beta_H = \beta^e$, the price that flippers offer will be too attractive to give rise to excessive incentives for mismatched households to sell in the investment market.

The conditions in part (b) are general sufficient conditions for excessive incentives for flipping. If equilibrium is at where $\partial \widehat{D} / \partial \theta > 0$, households sell to flippers primarily for the high price in the investment market. The RHS of (23) is a positive real number for $r_F < \overline{r}$, which will exceed the LHS of the condition for large θ given the concavity of $\eta(\theta)$. The Proposition can then be interpreted to say that in a tight and liquid market, possibly arising from a small housing supply, there is little efficiency gain from flipping even though mismatched households may find the high price in the investment market attractive, which in itself does not contribute to efficiency since households would be paying a similarly high price to buy new houses later on when they are selling at a high price in the investment market earlier.

Given that efficiency does not always increase with α and θ and that the market incentives to sell to flippers can be suboptimal or excessive, there is no reason to expect that when there exist multiple equilibria, the more active equilibria are necessarily more efficient. Most of all, the equilibria cannot be pareto ranked under all circumstances. Specifically, in a steady-state equilibrium where $\widehat{D}(\theta,\alpha)=0$, asset values for matched and mismatched homeowners and renters are given by, respectively,[17]

$$V_M = \frac{(r+\beta_H \eta(\theta))\,\upsilon}{r(r+\delta+\beta_H\eta(\theta))},$$

$$V_U = \frac{\beta_H \eta(\theta)\,\upsilon}{r(r+\delta+\beta_H\eta(\theta))},$$

$$V_R = \frac{(\beta_H r_F - \beta_F r)\,\eta(\theta)\,\upsilon}{r_F r(r+\delta+\beta_H\eta)},$$

It is straightforward to verify that both V_M and V_U are increasing in θ. Any homeowners – matched or mismatched – benefit from the higher housing prices in a tighter market. But the asset value for households in rental housing V_R is decreasing in θ if $\beta_H r_F < \beta_F r$, which is a necessary condition for the multiplicity of equilibrium (equation (18) and Figure 3). In this case, would-be buyers are made worse off by the higher housing prices in the tighter market. In the comparison between two steady-state equilibria both at where $\widehat{D}(\theta,\alpha)=0$, homeowners are better off whereas renters are worse off in the larger θ equilibrium than in the smaller θ equilibrium. Any two such equilibria cannot then be pareto ranked. The same conclusion can be shown to carry over to comparisons between a $\widehat{D}(\theta,\alpha)>0$ equilibrium and a $\widehat{D}(\theta,\alpha)=0$ equilibrium and between a $\widehat{D}(\theta,\alpha)=0$ equilibrium and a $\widehat{D}(\theta,\alpha)<0$ equilibrium.

5 Empirical implications

In this section, we shall explore several empirical implications of the model. To this end, we begin with characterizing how the model housing market's vacancy rate, trading volume, and the turnover of houses and households vary with α.

5.1 Vacancy, trading volume, TOM, and TBM

In the model housing market, the entire stock of vacant house is comprised of houses held by flippers. With a given housing stock, the vacancy rate is simply equal to n_F/H. A direct corollary of Lemma 1(b) is that:

Lemma 9 *In the steady state, the vacancy rate for owner-occupied houses is increasing in α.*

[17] The asset values and housing prices referred to hereinafter are special cases of those in Lemma 13 in the Appendix. In particular, the equations for V_M and V_U are from (42) and (43), respectively, whereas the equation for V_R is from (41), evaluated at $D(\theta,\alpha)=0$.

Housing market transactions per time unit in the model are comprised of (i) $\alpha\delta n_M$ houses households sell to flippers, (ii) $\eta(\theta)\, n_F$ houses flippers sell to households, and (iii) $\eta(\theta)\, n_U$ houses sold by one household to another, adding up to an aggregate transaction volume,

$$TV = \alpha\delta n_M + \eta(\theta)\, n_F + \eta(\theta)\, n_U. \tag{24}$$

Lemma 10 *In the steady state, TV is increasing in α.*

Given that houses sold in the investment market are on the market for a vanishingly small time interval and houses sold in the end-user market for a length of time equal to $1/\eta(\theta)$ on average, we may define the model's average TOM as

$$\frac{\alpha\delta n_M}{TV} \times 0 + \frac{\eta(\theta)(n_F + n_U)}{TV} \times \frac{1}{\eta(\theta)}. \tag{25}$$

Lemma 11 *In the steady state, on average, TOM is decreasing in α.*

TOM is a measure of the turnover of houses for sale. A more household-centric measure of turnover is the length of time a household (rather than a house) has to stay unmatched. We define what we call Time-Between-Matches (TBM) as the sum of two spells: (1) the time it takes for a household to sell the old house, and (2) the time it takes to find a new match afterwards. While the first spell (TOM) on average is shorter with an increase in α, the second is longer as the increase in θ to accompany the increase in α causes the buyer's matching rate to fall. The old house is sold more quickly. But it also takes longer on average to find a new match in a market with more buyers and fewer sellers. To examine which effect dominates, write the model's average TBM as

$$\alpha\frac{1}{\mu(\theta)} + (1-\alpha)\left(\frac{1}{\eta(\theta)} + \frac{1}{\mu(\theta)}\right). \tag{26}$$

where $1/\mu$ is the average TBM for households who sell in the investment market[18] and $1/\eta + 1/\mu$ for households who sell in the end-user market.[19]

Lemma 12 *In the steady state, on average, TBM is decreasing in α.*

Lemma 12 may be taken as the dual of Lemma 1(b) ($\partial n_M/\partial\alpha > 0$). When matched households are more numerous in the steady state, on average, they must be spending less time between matches.

[18] The household sells the old house instantaneously. Given a buyer's matching rate μ, the average TBM is then $1/\mu$.

[19] Let t_1 denote the time it takes the household to sell the old house in the end-user market and $t_2 - t_1$ the time it takes the household to find a new match after the old house is sold. Then the household's TBM is just t_2. On average, $E[t_2] = \int_0^\infty \eta e^{-\eta t_1}\left(\int_{t_1}^\infty t_2 \mu e^{-\mu(t_2 - t_1)} dt_2\right) dt_1 = 1/\eta + 1/\mu$.

5.2 Housing prices

No-intermediation equilibrium In the no-intermediation equilibrium, all housing market transactions are between pairs of end-user households at price[20]

$$p_H = \frac{\beta_H (\eta(\theta) + r) - (1 - \beta_H) \mu(\theta)}{(r + \delta + \beta_H \eta(\theta)) r} \upsilon + \frac{q}{r}, \tag{27}$$

evaluated at $\theta = \theta_S(0)$.

Fully-intermediated equilibrium In the fully-intermediated equilibrium, all houses are first sold from mismatched households to flippers at price

$$p_{FB} = \frac{\beta_F \eta(\theta) (\upsilon + q)}{(r_F + \beta_F \eta(\theta)) r + (\delta + (1 - \beta_F) \mu(\theta)) r_F}, \tag{28}$$

in the investment market and then at price

$$p_{FS} = \frac{\beta_F (\eta(\theta) + r_F) (\upsilon + q)}{(r_F + \beta_F \eta(\theta)) r + (\delta + (1 - \beta_F) \mu(\theta)) r_F}, \tag{29}$$

from flippers to end-user households in the end-user market, both evaluated at $\theta = \theta_S(1)$. With houses sold by households to flippers on the market for a vanishingly small time interval and houses sold by flippers to households for, on average, $1/\eta(\theta) > 0$ units of time, prices and TOM in the model housing market, as in the real-world housing market, are positively correlated in the cross section, given that by (28) and (29), $p_{FB} < p_{FS}$. Besides, with $p_{FB} < p_{FS}$, the model trivially predicts that houses bought by flippers are at lower prices than are houses bought by non-flippers. Both Depken et al. (2009) and Bayer et al. (2011) find evidences of such flipper-buy discounts in their respective hedonic price regressions.

Partially-intermediated equilibrium In a steady-state equilibrium in which mismatched households sell in both the investment and end-user markets, in addition to the two prices

$$p_{FB} = \frac{\beta_F \eta(\theta)}{r_F (r + \delta + \beta_H \eta(\theta))} \upsilon, \tag{30}$$

$$p_{FS} = \frac{\beta_F \eta(\theta) + \beta_F r_F}{r_F (r + \delta + \beta_H \eta(\theta))} \upsilon, \tag{31}$$

for transactions between a flipper and an end-user household, there will also be transactions between two end-user households, carried out at price

$$p_H = \frac{\beta_F \eta(\theta) + \beta_H r_F}{r_F (r + \delta + \beta_H \eta(\theta))} \upsilon. \tag{32}$$

[20] Equation (27) is from (38) evaluated at $\alpha = 0$; (28) and (29) are from (46) and (45), respectively, evaluated at $\alpha = 1$; (30), (31), and (32) are from (40), (39), and (38), respectively, all evaluated at $D(\theta, \alpha) = 0$.

For $\beta_F > \beta_H$, $p_{FB} < p_H < p_{FS}$. Just as in the fully-intermediated equilibrium, a positive relation between prices and TOM holds in the cross section and houses bought by flippers are at lower prices. Moreover, here houses sold by flippers are sold at a premium over houses sold by one end-user household to another. Such flipper-sell premiums are also found to exist in Depken et al. (2009) and Bayer et al. (2011).

Across equilibria Across steady-state equilibria, θ is largest in the equilibrium where flippers are most numerous. Then, prices should be highest in the given equilibrium where the competition among buyers is most intense.

Proposition 2 *Across steady-state equilibria in case there exist multiple equilibria, housing prices in both the end-user and investment markets are highest in the equilibrium with the tightest market and lowest in the equilibrium with the most sluggish market.*

5.3 Correlations among prices, TV, vacancy, TOM, and TBM

Now, a direct corollary of Proposition 2 and Lemmas 9-12 is that:

Proposition 3 *Across steady-state equilibria in case there exist multiple equilibria, prices, TV, and vacancies increase or decrease together from one to another equilibrium, whereas the average TOM and TBM move with the former set of variables in the opposite direction.*

Interest rate shocks In a typical asset pricing model, the price of an asset falls when the interest rate goes up. The same tends to hold in the present model. Specifically, in the no-intermediation equilibrium, an increase in r, by (27), leads to a lower p_H for sufficiently large $\theta_S(0)$ and/or q. Similarly, in the fully-intermediated equilibrium, by (28) and (29), respectively, both p_{FB} and p_{FS} are decreasing in r_F. But in either equilibrium, with α remaining fixed at 0 or 1, market tightness, vacancies, turnover, and sales are all invariant to the respective interest rate shocks.

In a partially-intermediated equilibrium, prices in the end-user market, p_{FS} and p_H, as well as in the investment market p_{FB}, are decreasing in r_F, just as they are in the fully-intermediated equilibrium. Housing prices in a partially-intermediated equilibrium, however, can also vary to follow any movements in θ triggered by the given interest rate shock – when the market becomes tighter in particular, prices are also higher. Hence, if a given positive (negative) shock to r_F should cause α and therefore θ to decrease (increase), there will be lower (higher) housing prices to follow because of a direct negative (positive) effect and of an indirect effect due to the exit (entry) of flippers. When the two effects work in the same direction, the interest rate shock can cause significantly more housing price volatility than in a model that only allows for the usual effect of interest rates on asset prices.

A positive shock to r_F need not cause α and θ to fall though. In case there exist multiple equilibria, the shock can possibly dislocate the market from a given equilibrium and send it to another equilibrium. In case the direct effect of an interest rate shock and the indirect effect via the movements in θ affect housing prices differently, in what direction housing prices will move cannot be unambiguously read off from (30)-(32). To proceed, we solve $\widehat{D}(\theta, \alpha) = 0$ for r_F and substitute the result into (30)-(32), respectively,

$$p_{FB} = \frac{\beta_H \eta(\theta) + ((\beta_F - \beta_H)\alpha_S(\theta) - (1-\beta_H))\mu(\theta)}{r(r+\delta+\beta_H\eta(\theta))}\upsilon + \frac{q}{r}, \tag{33}$$

$$p_{FS} = \frac{\beta_H \eta(\theta) + \beta_F r + ((\beta_F - \beta_H)\alpha_S(\theta) - (1-\beta_H))\mu(\theta)}{r(r+\delta+\beta_H\eta(\theta))}\upsilon + \frac{q}{r}, \tag{34}$$

$$p_{H} = \frac{\beta_H \eta(\theta) + \beta_H r + ((\beta_F - \beta_H)\alpha_S(\theta) - (1-\beta_H))\mu(\theta)}{r(r+\delta+\beta_H\eta(\theta))}\upsilon + \frac{q}{r}. \tag{35}$$

The three expressions are independent of r_F – whatever effects a given change in r_F will have on housing prices are subsumed through the effects of the change in θ that follows the change in r_F obtained from holding $\widehat{D}(\theta, \alpha) = 0$. To evaluate the effects of r_F on housing prices is to simply check how these three expressions behave as functions of θ.

Proposition 4 *Across steady-state equilibria and holding $\widehat{D}(\theta, \alpha) = 0$, a shock to r_F, whether positive or negative, will cause housing prices to increase (decrease), as long as to follow the interest rate shock are increases (decreases) in α and θ.*

By Proposition 4, the indirect effect of an interest rate shock on housing prices through the entry and exit of flippers and then in market tightness always dominates the direct effect shall the two be of opposite directions. A surprising implication is that housing prices can actually go up in response to an increase in flippers' cost of financing, if to follow the higher interest rate is also a heightened presence of flippers in the market. In any case, a direct corollary of Lemmas 9-12 and Proposition 4 is that:

Proposition 5 *Across steady-state equilibria and holding $\widehat{D}(\theta, \alpha) = 0$, a shock to r_F will cause housing prices, TV, and vacancies to move in the same direction, whereas the average TOM and TBM will move in the opposite direction.*

In the above, we have restricted attention to analyzing how changes in r_F alone may affect housing prices. It turns out that many of the implications continue to hold for equiproportionate increases or decreases in r and r_F. Proposition 7 in the Appendix contains the details.

6 Time-series Relations among Housing price, TV, and Vacancy

By Propositions 3, 5, and 7 in the Appendix, any movement from one to another steady-state equilibrium would involve housing prices, TV, and vacancies all going up or down together. The positive time-series relation between housing prices and the volume of transaction is well known and numerous models have been constructed to account for it. In Kranier (2001), for instance, a positive but temporary preference shock can give rise to higher prices and a greater volume of transaction, whereas Diaz and Jerez's (2013) analysis implies that an adverse shock to construction will shorten TOM, and may possibly lead to higher prices and a greater volume of transaction. The paper by Ngai and Tenreyro (2014) studies the comovement in prices and sales over the seasonal cycle and they argue that increasing returns in the matching technology play a key role in generating such cycles.

Unique to our analysis is that vacancies should also move in the same direction with prices and TV. In contrast, in both the Kranier and the Diaz and Jerez's models, the increase in sales should be accompanied by a decline in vacancy – given that when a house is sold, it is sold to an end-user, who will immediately occupy it, vacancies must decline, or at least remain unchanged. In Ngai and Tenreyro's model, households are assumed to move out of their old houses and into rental housing immediately after they become mismatched. Then, any and all houses on the market are vacant houses and given the assumed increasing-returns-to-scale matching technology, vacancies rise and fall with prices and the volume of transaction in the seasonal cycle. A mismatched household in their model, however, could well have stayed in the old house and avoided rental housing and the payment thereof until it has successfully sold the old house. In this alternative setup, the stock of vacant houses only includes houses held by people who have bought new houses before they manage to sell their old ones. Then, it is no longer clear that vacancies must rise and fall with prices and the volume of transaction in the seasonal cycle of Ngai and Tenreyro.

Figure 7 depicts the familiar positive housing price-transaction volume correlation for the U.S. for the 1981Q1 to 2011Q3 time period.[21] The usual housing market search model predicts that vacancies should decline in the housing market boom in the late 1990s to the mid 2000s and rise thereafter when the market collapses around 2007. Figures 8 and 9 show that any decline in vacancy is not apparent in the boom.[22] In

[21] Housing Price is defined as the nominal house price, which is the transaction-based house price index from OFHEO (http://www.fhfa.gov), divided by the CPI, from the Federal Reserve Bank at St. Louis. We set Housing Price at 1981Q1 equal to 100. Transaction is measured by the *quarterly* sales in single-family homes, apartment condos, and co-ops, normed by the stock of such units. The sales data are from the *Real Estate Outlook* by the National Assoication of Realtors, complied by Moody's Analytics. The housing stock is defined as the sum of owner-occupied units and vacant and for-sale-only units. The data are from the Bureau of Census's CPS/HVS Series H-111 available at http://www.census.gov/housing/hvs/data/histtabs.html.

[22] Vacancy rate is obtained by dividing the number of vacant and for-sale-only housing units by

fact, if there is any comovement between vacancies on the one hand and prices and sales on the other hand in the run-up to the peak of the housing market boom in 2006, vacancies appear to have risen along with prices and the volume of transaction. In a literal interpretation of our model, vacancies should fall very significantly to follow the market collapse since 2007. Apparently, the decline in vacancy in Figures 8 and 9 in the post-2007 period is modest, compared to the increase in the boom years. Two forces absent in our analysis – the massive amount of bank foreclosures and unsold new constructions in the market bust – may have accounted for the slow decline in vacancy since 2007.

(figures 7, 8, 9 about here)

In a more systematic analysis, we first verify that in the 1981Q1 to 2011Q3 sample period, all three variables are $I(1)$ at conventional significance levels. Next, we test for cointegration. Assuming the absence of any time trends and intercepts in the cointegrating equations, both the Trace test and the Max-eigenvalue test indicate two such equations, whose normalized forms read

$$\text{Price} - 6045.51 \times \text{Vacancy} = 0,$$

$$\text{Transaction} - 0.74 \times \text{Vacancy} = 0,$$

which together imply that the three variables do tend to move in the same direction from one to another long-run equilibrium over time.[23]

7 Quantitative predictions on volatility

Given the possible multiplicity of equilibrium and that an interest rate shock may have important effects on the extent of intermediation, the model can be consistent with a volatile housing market. The question remains as to how important quantitatively such channels of volatility can be. In this section, we calibrate the model to several

the housing stock as defined in the prevous note.

[23]With other time trend and intercept assumptions, either one or both of the tests suggest that there exist only one or as many as three cointegrating equations. In a single cointegrating equation with non-zero coefficients for all three variables, at least two of the three coefficients must be of the same sign. Then, the two variables concerned must move in opposite directions across long-run equilibria. With as many cointegrating equations as the number of variables, there exist definite long-run values for the three variables, which rules out the possibility of the system moving from one to another long-run equilibrium altogether. Restricting a priori to two cointegrating equations in the estimation, however, we always obtain two equations whose coefficients have the same signs as those in the system above whatever the trend and intercept assumptions are. Then, any long-run movements of the three variables must be in the same direction.

observable characteristics of the U.S. housing market and study by how much housing prices can fluctuate across steady-state equilibria and in response to interest rate shocks.

To begin, we take a time unit in the model to be a quarter of a year and assume a Cobb-Douglas matching function with which $\eta(\theta) = a\theta^b$ for some $a > 0$ and $b \in (0, 1)$. We set a priori the mismatch rate $\delta = 0.014$ to calibrate a two-year mobility rate of 11.4% for owner-occupiers reported in Ferreira et al. (2010) and $b = 0.84$, which is the elasticity of the seller's matching hazard with respect to the buyer-seller ratio reported in Genesove and Han (2012). Next, the parameters a and H and the share of mismatched households selling to flippers α are chosen to calibrate:

1. a quarterly transaction rate of owner-occupied houses of 1.78%
2. a vacancy rate of owner-occupied houses of 1.84%
3. the share of houses bought by flippers among all transactions of owner-occupied houses equal to 19%

The first two targets are, respectively, the average quarterly transaction rate and the average vacancy rate for the period 2000Q1-2006Q4, calculated from our dataset for the plots in Figures 7~9 and the estimations in Section 6. Estimates of the share of houses bought by flippers come from two sources: the 25% investors' share of all new purchase mortgages in the whole of the U.S. in Haughwout et al. (2011) in the 2000Q1-2006Q4 time period and the 13.7% housing market transactions share for houses sold again within the first two years of purchase in the metropolitan Las Vegas area in Depken et al. (2009) in the same time period. Because an investor in Haughwout et al. (2011) may intend to hold the house as a long-term investment, the 25% share is probably an overestimate of the true flippers' share. Because not all houses bought for short-term flips can actually be sold within two years, the 13.7% share in Depken et al. (2009) is probably an underestimate of the true flippers's share. Our 19% target is obtained by taking a simple average of the two estimates.

Given the targets, denoted as x_i, $i = 1, 2, 3$, respectively, we choose a, H, and α to

$$\min \left\{ \sum_{i=1}^{3} \left(\frac{x_i - \widehat{x}_i}{x_i} \right)^2 \right\},$$

where the $\widehat{x}_i$'s are the model's calibrated values of the corresponding targets[24], yielding $a = 0.085$, $H = 0.865$, and $\alpha = 0.25$ at which the calibrated values of the three targets are reported in the second column of Table 1.

[24] The $\widehat{x}$s are equal to TV/H, n_F/H, and $\alpha\delta n_M/TV$ for the model's transaction rate, vacancy rate, and the flippers' transaction share, respectively. The minimization is carried out via a grid search with a grid size of 0.005 for each of a, H, and α, subject to $a \leq 1.2$ and $H \geq 0.6$. The first constraint is for expediency in the grid search and is not binding. Given that H in the model is the stock of owner-occupied houses relative to the population of households demanding such housing, anything near the bound of the second constraint is probably unreasonable.

Table 1: Calibration Targets and Calibrated Model Values

	Targeted value	Calibrated value
Transaction rate (quarterly)	0.0178	0.016
Vacancy rate	0.0184	0.018
Flippers' share in transactions	0.19	0.19

Thus far in the calibration, we have effectively identified $\alpha_S = 0.25$ as equilibrium. For equilibrium to be indeed at $\alpha = 25$, we need to pick the values for $\{q, \upsilon, \beta_H, \beta_F, r, r_F\}$ to force $\alpha_O = 0.25$ as well. Since only the ratio $z = q/\upsilon$, but not the levels of the two parameters, matters for the value of α_O and the comparison of prices, we first normalize $\upsilon = 1$. We then obtain an estimate of z (or equivalently q) equal to 1.43 from the results in Anenberg and Bayer (2013). The details are in Appendix 10.2. Next, we set $r = 0.02$ for an annual rate of 8% to match the usual 30-year fixed-rate mortgage rate. Lastly, for the lack of any obvious empirical counterpart, we set the household-seller's bargaining strength $\beta_H = 0.5$. Then, for each of $\beta_F = 0.5$, 0.6, 0.65, 0.7, and 0.8, we look for the value of r_F at which $\alpha_O = 0.25$. The results are shown in Table 2.[25,26]

For the last two pairs of β_F and r_F in Table 2, the $\alpha = 0.25$ equilibrium is the unique equilibrium. For the first three pairs, there are two other equilibria each beside the $\alpha = 0.25$ equilibrium. Table 3 reports the prices in these equilibria. For instance, for $\beta_F = 0.5$ and $r_F = 3.1\%$ per annum, the three equilibria are at $\alpha = 0$, 0.25, and 1, respectively.[27] The price $\overline{p}$, on the next row, is the average of p_H, p_{FB}, and p_{FS}, weighted by the shares of transactions taking place at the respective prices, with $\overline{p}$ in the smallest-α equilibrium set equal to 1. Evidently, the volatility arising from the multiplicity is non-trivial, with average prices differing by up to 23% across the equilibria.

Table 2: Calibrated β_F and r_F for $\alpha_O = 0.25$

[25] A r_F below r by a few percentage points can make sense if flippers, but not end-user households, tend to be all-cash investors. Herbert et al. (2013) report that the majority of investors acquiring foreclosures are indeed all-cash buyers. Even though no comparable evidence is available for other properties, it would not be surprising that cash is often used too. Moreover, investors may also make use of mortgages with zero initial or negative amortization, short interest rate reset periods, or low introductory teaser interest rates. Such mortgages obviously are ideal for flippers who plan to sell quickly for short-term gains. Amromin et al. (2012) find that borrowers who take out such "complex" mortages are usually high income individuals with good credit scores. Foote et al. (2012) find that periods of interest rate resets do not tend to trigger significant increases in defaults, consistent with the finding of Amromin et al. that the borrowers of such mortgages are sophisticated investors.

[26] Notice that the model does not require $r_F < r$ for flippers to survive or for the multiplicity of equilibria. For smaller values for z, we can force α_O to be equal to 0.25 for much larger r_F.

[27] At $\alpha = 1$, in the steady state, one half of all sales are purchases made by flippers. This is just about equal to the peak investor share in the "bubble states" reported in Haughwout et al. (2011).

β_F	0.5	0.6	0.65	0.7	0.8
r_F	0.0077	0.0091	0.0099	0.0106	0.012
r_F (annual basis)	3.1%	3.7%	4%	4.23%	4.8%

Table 3: Multiple Equilibria

β_F r_F (annual basis)	0.5 3.1%			0.6 3.7%			0.65 4%		
α	0	0.25	1	0	0.25	1	0.25	0.71	1
$\overline{p}$	1	1.1	1.2	1	1.11	1.23	1	1.08	1.11

Table 4: Housing Prices and Interest Rates, $\beta_F = 0.7$

r_F (annual basis)	3.5%	4.21%	4.22%	4.23%	4.27%	6%
$\alpha = 0$					0.89	0.89
$\alpha = 0.25$				1		
$\alpha = 0.63$			1.07			
$\alpha = 1$	1.13	1.12				

To study the response of housing prices to interest rate shocks, we report in Table 4 average housing prices $\overline{p}$ for various small deviations of r_F from a benchmark of $r_F = 4.23\%$ and $\beta_F = 0.7$ at which equilibrium is unique at the calibrated value of $\alpha = 0.25$. Fixing $\beta_F = 0.7$, for all values of r_F under consideration, equilibrium remains unique. The entries in the table are normed by the average equilibrium price at the benchmark r_F. Here, housing prices hardly move to follow a given interest rate shock if the shock has not caused any changes in equilibrium α. But when the given interest rate shock does cause α to change significantly, it also leads to significant changes in housing prices. Specifically, a decline in r_F from 6% per annum to 4.27% per annum causes no noticeable change in $\overline{p}$ when the given movement in r_F has no effect on α. But a further decline in r_F from 4.27% per annum to 4.21% per annum now causes $\overline{p}$ to increase by 26% as α rises from 0 to 1 in the meantime. Thereafter, $\overline{p}$ remains essentially unchanged from any additional decline in r_F as α has already reached the upper bound of 1. All this suggests that the response of housing prices to interest rate shocks can appear erratic and unpredictable. Before a given threshold r_F is reached, the response is at most moderate. When r_F crosses the threshold to trigger the entry of flippers, the housing market can become significantly tighter and housing prices significantly higher as a result.

8 Extensions

The model we studied in this paper is, by all means, a very special model. In Leung and Tse (2016a), we study how the major results of the paper may be affected under competitive search, a two-house-limit liquidity constraint for households, allowing for investors choosing between short-term flips and long-term investments and letting mismatched households sell to housing market intermediaries right before buying a new house. Below are the summaries of the findings.

8.1 Competitive search

In our model, the multiplicity of equilibrium arises out of flipper-sellers passing onto household-sellers their advantages in bargaining to the fullest extent possible in a tight and liquid market. A natural question to ask is whether the multiplicity is special to price determination by bargaining as we have assumed or whether similar conclusions hold under the alternative assumption of price determination via competitive search.

In the competitive search version of our model, based on Mortensen and Wright (2002), the end-user market is segmented into submarkets, each of which is controlled by a competitive market maker, who charges entry fees for other agents for buying and selling in his submarket in return for regulating the transaction price and market tightness at some pre-specified levels. The household-buyers, household-sellers and flipper-sellers choose which submarket – among the options available – to enter into to maximize the respective expected returns of buying and selling. In this setting, we find that in equilibrium,

$$\alpha = \begin{cases} 1 & C\left(\theta_S\left(1\right)\right) \geq 0 \\ \alpha_C & C\left(\theta_S\left(\alpha_C\right)\right) = 0 \\ 0 & C\left(\theta_S\left(0\right)\right) \leq 0 \end{cases},$$

where

$$C\left(\theta\right) = \frac{\partial \eta}{\partial \theta} + \left(\eta\left(\theta\right) - \theta\frac{\partial \eta}{\partial \theta}\right)\left(\frac{r}{r_F} - \left(1+z\right)\right) - z\left(\delta + r\right),$$

can be thought of as the incentives of mismatched households to sell in the investment market under competitive search – the counterpart to $\widehat{D}\left(\theta, \alpha\right)$ in (17) under bargaining and $E\left(\theta\right)$ in (22) for efficiency.

First notice that if $r_F = r$, $C\left(\theta\right) = E\left(\theta\right)$ and hence, as is well known, competitive search is efficient. For $r_F \neq r$, however, $C\left(\theta\right) \neq E\left(\theta\right)$. Most importantly for our purpose, for

$$r_F < \frac{r}{1+z},$$

$C\left(\theta\right)$, like $\widehat{D}\left(\theta, \alpha\right)$ for $r_F < \overline{r}$, is U-shaped, first decreasing but eventually increasing. This, of course, opens up the possibility of multiplicity. Indeed, this necessary condition for multiplicity is the same necessary condition for multiplicity in Lemma 6 and

Figure 3 for $\beta_F = \beta_H$. Thus, as long as flippers possess a sufficiently large financing advantage, they can attract mismatched households to sell in the investment market when the end-user market is particularly tight, in addition to when the end-user market is particularly sluggish, whether prices in the end-user market are determined by bargaining or via competitive search.

8.2 Two-house-limit liquidity constraint

In some version of the housing market search model, most notably in Wheaton (1990), there is not any rental housing but rather a mismatched household must stay in its old house while searching for a new match, and then the old house will be put up for sale only after the household has found a new house to move into. In this environment, a household will be holding as many as two mismatched houses if the household is hit by a moving shock again before it is able to sell the previously mismatched house. If there is a two-house-limit liquidity constraint, the household will then be prevented from entering the end-user market as a buyer until it is able to sell one of the two mismatched houses that it is now holding. A liquidity provision role for flippers arises. Indeed, a household may wish to sell to flippers right after it is moving to a new house from the old mismatched house. By doing so, the household will never be holding two houses at any moment in time. In all, we find that when more households sell to flippers, either right after finding new houses to move into or right after being hit by two successive moving shocks, there will be a tighter end-user market.

In this setup, flippers, like those in the present model, may be able to lure households to sell in the investment market not just when the end-user market is sluggish but also when it is tight if they possess a large enough financing/bargaining advantage. In this model though, there is not any relationship between the activities of flippers and the vacancy rate since the latter is simply equal to the difference between the housing stock and the population of households, in the entire absence of rental housing. Furthermore, unlike the present model, for efficiency, all households should use the services of flippers since nobody would be incurring any rental expenditures during which flipping takes place. And in case there exist multiple equilibria, a more active equilibrium should pareto dominate a less active one, with any household owning at least one house at any moment.

8.3 Short-term flips versus long-term investment

In reality, there can be two strategies for housing market investments – short-term flip versus long-term investment in which an investor holds the house for an extended period of time, earning the rental revenue in the interim and in anticipation for a certain capital gain in the medium to long term. In a summary of case studies of four metropolitan areas in the U.S., Herbert et al. (2013) report that both investment strategies were commonly adopted by housing market investors in the wake of the collapse of the housing market in the U.S. in 2007. In the early phase when prices appeared to have reached the lowest level, most investments were found to be short-term flips, whereas in the latter phase when the market appeared to have stabilized, most investments were found to be medium- to long-term investments.

In the present model, the supply of rental housing is assumed perfectly elastic at some exogenously given rental. We could have chosen to assume the arguably more realistic setting in which the stock is exogenously given while the market rental is determined in equilibrium. All the same, underlying either setting is the presumption that the rental and owner-occupied housing stocks are two separate stocks. By all means, a richer analysis would allow for the same housing stock to serve as both rental and owner-occupied housing. In this revised model, just as in the present model, mismatched households choose between selling in the investment market or offering their houses for sale in the end-user market. Unlike the present model, the specialist investors in the revised model may choose to offer their properties for sale and/or for rent. It turns out that in this setup, the only kind of steady state is one in which at least a fraction of mismatched households choose to sell to investors while investors choose to offer their properties both for sale and for rent. Any such steady state, however, can be equilibrium only under selected values of the housing stock and investors' cost of financing, just as a fully- or partially-intermediated equilibrium exists only for some subset of the parameter space in the present model. When the conditions are not met, no steady-state equilibrium exists in the revised model. This result is perhaps highly suggestive for in reality, housing market investors do predominantly choose whether to flip or to invest long term during different phases of the housing price cycle as reported in Herbert et al. (2013). No matter, to analyze a model that allows for investors choosing between the two investment strategies, it becomes imperative to study the full dynamics. Undoubtedly, this can be a very fruitful exercise towards a fuller understanding of the dynamics of housing market investment but is best to be left for future research.[28]

8.4 Selling to housing market intermediaries right before buying a new house

We have in this paper assumed that a mismatched household must either sell to a flipper or to an end-user household before it can start looking for a new house. Strictly speaking, given the assumption of instantaneous sale in the investment market, the household can choose to stay in the old house while searching for a new one and then sell the old house to a flipper only right before the household buys the new house.[29]

[28] Indeed, Head et al. (2014) have explored a number of interesting implications of a model that allows households and real estate developers a choice between the two strategies. They do not allow for specialist investors in their model like we do though.

[29] The households who move within a given city in Head and Lloyd-Ellis (2012) can do just that. There, households do not face any liquidity constraints and the assumption of having housing market

This means that, a one-house-limit liquidity constraint notwithstanding, mismatched households should be able to enter the end-user market as buyers before selling their old houses.

Given that all mismatched houses are for sale in the end-user market, whoever their owners are, had we allowed for any and all mismatched households to enter the end-user market as buyers right away, the tightness in the end-user market would not depend in any way on how many mismatched households choose to sell to flippers in the first instance, if any mismatched households choose to do that at all. In this alternative setting, market tightness depends solely on the housing stock, among other factors, and is completely isomorphic to price determination in equilibrium. Moreover, flippers' liquidity provision role is wholly fulfilled so long as they are in the market ready to buy up any houses would-be buyers need to sell just before buying. Then, unless flippers possess some large enough financing/bargaining advantage, α must be just equal to zero in equilibrium. In sum, equilibrium is guaranteed unique and any sale to flippers that takes place at the moment households first becoming mismatched must arise out of flippers' financing/bargaining advantage.

True, with a frictionless investment market in place, our assumption that mismatched households must rid themselves of their old houses first before entering the end-user market as buyers is ad hoc. We could have included some additional technical details to better justify the assumption.[30] But we think it is more constructive to simply note that the assumption is a well-motivated assumption. In reality, the investment market for the housing asset is by no means completely free of frictions. Sales in the market are certainly not instantaneous. If a household is not able to sell the old house quickly enough to pay off the mortgage for the house, it can face considerable difficulties in getting a mortgage for the new house. A more realistic and arguably more rigorous analysis is to model the investment market as a frictional market too to formally motivate the assumption that households must first sell before they enter the market as buyers.

Of course, the questions of whether and how our qualitative results would survive in the more general setting of a frictional investment market remain. Suppose in particular, there is but a finite measure of flipper-buyers in a frictional investment market whom mismatched households meet randomly. Where there is not a conscious choice of selling in which market for households, the asset value of a matched household simplifies to,

$$rV_M = \upsilon + \delta\left(V_U - V_M\right).$$

Once becoming mismatched, a household is a seller in both the end-user and the

intermediaries is merely a simplifying assumption to facilitate analysis.

[30]For example, consider a discrete time version of the model. Say a period is divided into two subperiods where the investment market is open in the first subperiod only and the search market is open next in the second subperiod. In Leung and Tse (2016a), we propose three other alternative justifications.

investment markets, in which case V_U satisfies,

$$rV_U = \eta(\theta) \times \max\{V_R + p_H - V_U, 0\} + \eta_I(\theta_I) \times \max\{V_R + p_{FB} - V_U, 0\}, \quad (36)$$

where η_I denotes the rate at which the household-seller meets a flipper-buyer in a frictional investment market and θ_I the ratio of flipper-buyers to household-sellers in the market. On the other side of the investment market, a flipper-buyer meets a household-seller at the rate $\mu_I(\theta_I) = \eta_I(\theta_I)/\theta_I$, whereby the flipper's asset value V_{FB} satisfies,

$$rV_{FB} = -c + \mu_I(\theta_I) \times \max\{V_{FS} - p_{FB} - V_{FB}, 0\},$$

with $c \geq 0$ denoting the flow search cost of the flipper and V_{FS}, given by

$$r_F V_{FS} = \eta(\theta)(p_{FS} - V_{FS}), \quad (37)$$

the asset value of a flipper-seller in the end-user market, or what is the same thing the value of a vacant house to a flipper.

If flippers enter the market at an entry cost of k, in equilibrium $V_{FB} \leq k$. Like prices in the end-user market, the price in the investment market p_{FB} is determined by Nash bargaining. Our model is a special case of this more general model for which $\theta_I \to \infty$ as c and k tend to zero.

In this model, there should always be a positive surplus in a household-seller and household-buyer match to result in a positive $V_R + p_H - V_U$ in (36) since flippers must be buying at a lower price p_{FB} than end-user households do at p_H to make any profit at all from house flipping. Then, the household-seller should always find it optimal to sell to an end user should he be lucky to meet one before he ever meets any flipper-buyer. Whether or not there is a non-negative surplus in a match between a household-seller and a flipper-buyer can depend on θ, the tightness in the end-user market, in much the same way whether or not household-sellers prefer to sell in the Walrasian investment market in our model depends on θ. Specifically, other things equal, there can be a non-negative surplus in a household-seller and flipper-buyer match to result in a non-negative $V_R + p_{FB} - V_U$ in (36) only when the outside option of the seller V_U is relatively unfavorable due to a sluggish end-user market. Absent any financing/bargaining advantage on the part of flippers then, flipping should only take place in slow markets. In case flipper-sellers do possess some sufficiently large financing/bargaining advantage, they tend to sell houses at a relatively high p_{FS} and by (37), the high p_{FS} should be followed by a disportionately high V_{FS} in a tight end-user market in which vacant houses are quickly sold. In turn, if flippers place a relatively high valuation on vacant houses, there tend to be a large surplus in a household-seller and flipper-buyer match to result in a non-negative $V_R + p_{FB} - V_U$ in (36). But precisely because all or at least a fraction of end users sell to flippers, the end-user market tightens. In all, results similar to Lemmas 4-7 of the paper, which form the core of our qualitative results, should hold in the generalization to a frictional investment market.

9 Concluding remarks

Housing market flippers can be arbitraging middlemen like those we model in this paper, intermediaries who survive on the basis of superior information, or momentum traders blindly chasing the market trend. Our analysis suggests a number of empirical implications to distinguish between these theories of flipping. First and foremost, if flippers are predominantly momentum traders instead of specialist middlemen, there should not be any significant flipper-buy discounts and flipper-sell premiums. But such discounts and premiums do exist and they are sizeable, as reported in Depken et al. (2009) and Bayer et al. (2011). Second, in a housing market boom fed by the entry of momentum traders, there can and usually will be sales and purchases among flippers. While in a more elaborate theory of intermediation as in Wright and Wong (2014), this can also happen. But this does not seem like a robust implication of a theory of middlemen in the housing market. Third, a theory of housing price speculation should imply that prices should stay at the peak for at most a short while and then fall right afterwards. In our model, the market can conceivably move from a low-price to a high-price steady-state equilibrium and then just stays at the new equilibrium for any length of time. Relatedly, speculators should only buy in an up market, whereas in our model, there can be multiple locally stable steady-state equilibria involving flipping. Indeed, in our model, the gross returns to flipping, p_{FS}/p_{FB}, by (11), is actually higher in a lower-price less active equilibrium than in a higher-price more active equilibrium. Lastly, if housing market middlemen are mostly "market edge" investors who survive on the basis of informational advantage rather than investors who have more flexible and less costly financing like those in our model, they should use ordinary mortgages as much as end-user households do. Herbert et al. (2013) report that there was little evidence of bank lending to investors acquiring foreclosed properties in the wake of the 2007 housing market meltdown in the U.S. Instead, the great majority of acquisitions were bought with cash. In addition, in the housing market boom in the U.S. before 2007, there is ample evidence, as we remarked in note 27, that investors took advantage of "complex mortgages" more than ordinary households did. Other than observations on the choices of financing, the two theories may also be distinguished by what market conditions under which flipping takes place. In our model, flipping tends to occur in particularly tight as well as particularly sluggish markets – a prediction that is hard to envisage to come from a theory of housing market intermediaries who survive on the basis of superior information.

In the U.S., house flipping is thought to often involve renovating before selling rather than simply buying and then putting up the house for sale right after. In this line of thinking, the returns to flipping are more about the returns to the renovations investment than the returns to holding the houses on behalf of the liquidity-constrained owners. The question then is what prevents the original owners themselves from earning the returns on the investment. A not implausible explanation is that many original owners lack the access to capital to undertake the investment, just as the original owners in our model lack the access to capital to hold more than one house at a time. Thus, at a deeper level, our model is not just a model of buy-and-sell flips but should also encompass, with suitable modifications, buy-renovate-sell flips.

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A Appendix

A.1 Lemma and Proposition

Lemma 13 *If* $\max\{V_R + p_{FB}, V_U\} = V_U$,

$$\begin{aligned} p_H &= \{(\beta_H(\eta+r)(r_F+\beta_F\eta) - (1-\beta_H)((1-\alpha)\beta_F\eta + (1-\alpha\beta_F)r_F)\mu)\upsilon \\ &\quad + (r+\delta+\beta_H\eta)(r_F+\beta_F\eta)q\}/G_U, \end{aligned} \tag{38}$$

$$p_{FS} = \beta_F(\eta+r_F)\frac{(r+\beta_H\eta-\mu(1-\alpha)(1-\beta_H))\upsilon + (r+\delta+\beta_H\eta)q}{G_U}, \tag{39}$$

$$p_{FB} = \beta_F\eta\frac{(r+\eta\beta_H-\mu(1-\alpha)(1-\beta_H))\upsilon + (r+\delta+\eta\beta_H)q}{G_U}, \tag{40}$$

$$\begin{aligned} V_R &= \{((1-\alpha)(1-\beta_H)\beta_F\eta r_H + (1-\beta_F)\eta\beta_H\alpha r_F + (1-\beta_H-\alpha(\beta_F-\beta_H)) \\ &\quad \times r r_F)\mu\upsilon - (r+\delta+\beta_H\eta)(r_F+\beta_F\eta)rq\}/(rG_U), \end{aligned} \tag{41}$$

$$V_M = \frac{(r+\beta_H\eta)\upsilon}{r(r+\delta+\beta_H\eta)}, \tag{42}$$

$$V_U = \frac{\eta\beta_H\upsilon}{r(r+\delta+\beta_H\eta)}, \tag{43}$$

where

$$G_U = (r+\delta+\eta\beta_H)((r_F+\eta\beta_F)r + (1-\beta_F)r_F\mu\alpha).$$

If $\max\{V_R + p_{FB}, V_U\} = V_R + p_{FB}$,

$$\begin{aligned} p_H &= \{(\beta_H(\eta+r)(r_F+\beta_F\eta) - \mu(1-\beta_H)(r_F+(1-\alpha)\beta_F\eta - \alpha\beta_F r_F))\upsilon \\ &\quad + ((1-\beta_H)\delta r_F + (r+\beta_H\eta)(r_F+\beta_F\eta))q\}/G_F, \end{aligned} \tag{44}$$

$$p_{FS} = \frac{\beta_F(\eta+r_F)((r+\beta_H\eta-(1-\beta_H)(1-\alpha)\mu)\upsilon + (r+\beta_H\eta)q)}{G_F}, \tag{45}$$

$$p_{FB} = \frac{\beta_F\eta((\beta_H\eta+r-\mu(1-\alpha)(1-\beta_H))\upsilon + (\beta_H\eta+r)q)}{G_F}, \tag{46}$$

$$\begin{aligned} V_R &= \{(((1-\alpha)\beta_F\eta + r_F)(1-\beta_H)r + (1-\beta_F)\beta_H\eta\alpha r_F - (\beta_F-\beta_H)\alpha r_F r) \\ &\quad \times\mu\upsilon - (r+\eta\beta_H)(r_F r + \delta r_F + \beta_F\eta r_H)q\}/(rG_F), \end{aligned} \tag{47}$$

$$V_M = \frac{(r+\beta_H\eta)(((r_F+\beta_F\eta)r + (1-\beta_F)r_F\mu\alpha)\upsilon - r_F\delta q)}{rG_F}, \tag{48}$$

$$V_U = \frac{\eta\beta_H(((r_F+\beta_F\eta)r + (1-\beta_F)r_F\mu\alpha)\upsilon - r_F\delta q)}{rG_F}, \tag{49}$$

where

$$G_F = (r+\beta_H\eta)(r_F r + \delta r_F + \beta_F\eta r + (1-\beta_F)r_F\mu\alpha) - (1-\beta_H)(1-\alpha)\mu\delta r_F.$$

Lemma 14

a. *Write*

$$S_{RF} = V_M - V_R - p_{FB},$$
$$S_{RU} = V_M - V_U,$$

respectively, as the match surpluses in matches between a buyer in rental housing and a flipper-seller and a mismatched household-seller. If $D(\theta, \alpha) \geq 0$*,* $S_{RF} \geq 0$*. If* $D(\theta, \alpha) \leq 0$*,* $S_{RU} \geq 0$*.*

b. $\max\{V_R + p_{FB}, V_U\} \geq 0$.

Lemma 15 *Assume that* $\eta(\theta) = a\theta^b$ *for some* $a > 0$ and $b \in (0, 1)$*. The first inequality of* (50) *below is the condition for* $r_F > r''$*, whereas the second inequality and* (51) *together guarantee that* $\theta_1^u < 1$ *and* $r_F < \overline{r}$*.*

$$\frac{r\beta_F}{\left(\left(\frac{b}{1-\beta_H}\right)^b (\delta + r)\frac{z}{a}\right)^{\frac{1}{1-b}} (1-b) + (1+z)\beta_H} < r_F \tag{50}$$
$$< \frac{r\beta_F}{(\delta + r)\frac{z}{a} - (1-\beta_F) + (1+z)\beta_H}.$$

$$(\delta + r)\frac{z}{a} \geq (1 - \beta_F), \tag{51}$$

The two conditions above and that H *be sufficiently close to 1 guarantee that* (19) *holds.*

Proposition 6 *Equilibrium exists for all* $\{r, r_F, \upsilon, q, \delta, \beta_F, \beta_H, H\}$ *tuple.*

Proposition 7

a. *In the fully-intermediated equilibrium, equiproportionate increases in* r *and* r_F *lower housing prices. The same effect is felt in the no-intermediation equilibrium for sufficiently large* $\theta_S(0)$ and/or q.

b. *In a partially-intermediated equilibrium,*

   i. *equiproportionate increases in* r *and* r_F*, holding* θ *fixed, lower housing prices;*

   ii. *across steady-state equilibria and holding* $\widehat{D}(\theta, \alpha) = 0$*, equiproportionate changes in* r *and* r_F*, whether positive or otherwise, cause* p_{FB} *to increase (decrease) as long as to follow the interest rate shocks are increases (decreases) in* θ *and* α *for* $\beta_F = \beta_H = 1/2$ and $r/r_F \in [0, 1+z]$*; the same effect is felt on* p_{FS} and p_H *for* r/r_F *in neighborhoods of* $r/r_F = 0$*, 1, and* $1 + z$*.*

A.2 Calibrating $z = q/\upsilon$

In the model housing market, the flow payoffs for matched owner-occupiers, mismatched owner-occupiers, and buyers in rental housing are equal to υ, 0, and $-q$, respectively. In this case then, the difference between the flow payoffs of matched and mismatched owner-occupiers is equal to υ, and that between mismatched owner-occupiers and buyers is q. Anenberg and Bayer (2013) report estimates on

1. mean flow payoff for matched owner-occupiers 0.0273
2. flow payoff for owner-occupiers mismatched with their old houses 0.024, comprising 30% of all mismatched households
3. flow payoff for owner-occupiers mismatched with the metro area 0.0014, comprising 70% of all mismatched households

while normalizing the flow payoff of buyers to 0. We may thus equate $\upsilon = 0.0273 - 0.024 = 0.00033$ and $q = 0.024$ so that $z = 8$ for households who are mismatched with their old houses and $\upsilon = 0.0273 - 0.0014 = 0.0259$ and $q = 0.0014$ so that $z = 0.054$ for households who are mismatched with the metro area. Taking a weighted average of the two estimates gives a value for z equal to 2.49. Alternatively, one may take the weighted average first before taking the ratio: $\upsilon = 0.0273 - 0.03 \times 0.024 - 0.7 \times 0.0014$ and $q = 0.03 \times 0.024 + 0.7 \times 0.0014$, so that $z = 0.43$. Since it is not clear in the context of our model which method is conceptually better than the other, we resort to taking a simple average of 2.49 and 0.43 to obtain a value of 1.43 for z.

A.3 Proofs

Proof of Lemmas 1 and 2 Solve (1)-(6) for

$$n_M = \frac{\eta H}{\eta + \delta}, \tag{52}$$

$$n_F = \frac{\alpha \delta H}{\delta + \eta}, \tag{53}$$

$$n_U = \frac{(1-\alpha)\,\delta H}{\delta + \eta}, \tag{54}$$

$$n_R = \frac{\eta\,(1-H) - (1-\alpha)\,\delta H + \delta}{\delta + \eta}. \tag{55}$$

Equation (7) in Lemma 2 is obtained by substituting (53)-(55) into

$$\theta = \frac{n_R}{n_U + n_F}.$$

The LHS of (7) is equal to $(1 - (1-\alpha)\,H)\,\delta > 0$ at $\theta = 0$ but is negative for arbitrarily large θ given the concavity of η. A solution is guaranteed to exist. Differentiating with respect to θ yields

$$\frac{\partial \eta}{\partial \theta}\,(1-H) - H\delta,$$

which is positive for small θ but negative otherwise. The solution to (7) must then be unique, and that the LHS is decreasing in θ at where it vanishes. Given that the LHS of the equation is increasing in α, $\partial\theta_S/\partial\alpha > 0$. The lower and upper bounds $\theta_S(0)$ and $\theta_S(1)$ are given by the respective solutions to

$$\delta + \eta(1-H) - (1+\theta) H\delta = 0, \tag{56}$$

$$\delta + \eta(1-H) - \theta H\delta = 0, \tag{57}$$

both of which are seen to be positive and finite. That $\partial\theta_S/\partial H < 0$ holds as the LHS of (7) is decreasing in H. That $\lim_{H\to 1}\theta_S(0) = 0$ falls out from (57).

Solve (7) for

$$\alpha = \frac{\theta\delta H - (1-H)(\eta+\delta)}{\delta H}, \tag{58}$$

and substitute into (53)-(55), respectively,

$$n_F = \frac{\theta\delta H - (1-H)(\eta+\delta)}{\delta+\eta}, \tag{59}$$

$$n_U = \frac{\eta(1-H) + \delta(1-\theta H)}{\eta+\delta}, \tag{60}$$

$$n_R = \frac{\theta\delta H}{\eta+\delta}. \tag{61}$$

The comparative statics Lemma 1 can be obtained by differentiating (52) and (59)-(61), respectively, with respect to θ, and then noting that $\partial\theta_S/\partial\alpha > 0$. The boundary values for n_F and n_U are obtained for setting $\alpha = 0$ and $\alpha = 1$ in (53) and (54), respectively.

Proof of Lemma 3 Substituting from (53) and (54) into $\frac{n_F}{n_F+n_U}$ yields the result of the Lemma.

Proof of Lemmas 4-7 and the construction of Figure 3 Given that $\lim_{\theta\to 0}\eta = 0$ and $\lim_{\theta\to\infty}\mu = 0$,

$$\lim_{\theta\to 0}\widehat{D} = (1-\beta_H - \alpha(\beta_F - \beta_H))\mu - (\delta+r)z = \infty,$$

$$\lim_{\theta\to\infty}\widehat{D} = \left(\beta_F\frac{r}{r_F} - \beta_H - z\beta_H\right)\eta - (\delta+r)z. \tag{62}$$

Differentiating,

$$\frac{\partial\widehat{D}}{\partial\theta} = \left(\beta_F\frac{r}{r_F} - \beta_H - z\beta_H\right)\frac{\partial\eta}{\partial\theta} + (1-\beta_H - \alpha(\beta_F-\beta_H))\frac{\partial\mu}{\partial\theta}. \tag{63}$$

For $r_F \geq \overline{r}$, the expressions in both (62) and (63) are negative for sure. In this case, as a function of θ, $\widehat{D}$ starts out equal to positive infinity and falls continuously below zero. Then, there exist some θ_1^d and θ_0^d that satisfy $\widehat{D}\left(\theta_1^d, 1\right) = 0$ and $\widehat{D}\left(\theta_0^d, 0\right) = 0$, respectively. Given that $\widehat{D}$ is decreasing in θ and in α,

1. $\theta_1^d < \theta_0^d$,

2. for any $\alpha \in [0,1]$, $\widehat{D}(\theta,\alpha) \geq \widehat{D}(\theta,1) > 0$ for $\theta < \theta_1^d$,

3. for any $\alpha \in [0,1]$, $\widehat{D}(\theta,\alpha) \leq \widehat{D}(\theta,0) < 0$ for $\theta > \theta_0^d$,

4. for $\theta \in \left(\theta_1^d, \theta_0^d\right)$, $\widehat{D}(\theta,\alpha) = 0$ holds at some $\alpha = \alpha_D(\theta)$, where $\partial\alpha_D(\theta)/\partial\theta < 0$.

This completes the proof of Lemma 4.

For $r_F \geq \overline{r}$, by the three Panels of Figure 2, there will be flipping in the unique equilibrium if and only if

$$\theta_S(0) < \theta_0^d. \tag{64}$$

By (56), $\theta_S(0)$ is decreasing in H, with limiting values

$$\lim_{H\to 0}\theta_S(0) = \infty \qquad \lim_{H\to 1}\theta_S(0) = 0.$$

By (17), θ_0^d satisfies

$$\left(\beta_F\frac{r}{r_F} - \beta_H(1+z)\right)\eta\left(\theta_0^d\right) + (1-\beta_H)\,\mu\left(\theta_0^d\right) - (\delta+r)\,z = 0,$$

whereby it is decreasing in r_F. The limiting values

$$\lim_{r_F\to\infty}\theta_0^d = \overleftarrow{\theta}{}_0^d \qquad \lim_{r_F\to\overline{r}}\theta_0^d = \overrightarrow{\theta}{}_0^d,$$

are the respective solutions to

$$-\beta_H(1+z)\,\eta\left(\overleftarrow{\theta}{}_0^d\right) + (1-\beta_H)\,\mu\left(\overleftarrow{\theta}{}_0^d\right) - (\delta+r)\,z = 0,$$

$$(1-\beta_H)\,\mu\left(\overrightarrow{\theta}{}_0^d\right) - (\delta+r)\,z = 0,$$

both of which are positive and finite. Then, condition (64) cannot be met for H close to 0 but must be satisfied for H close to 1. With $\theta_S(0)$ decreasing in H and θ_0^d decreasing in r_F, the combinations of H and r_F under which (64) holds as an equality define an upward-sloping relation $H(r_F)$ between the two parameters. This proves Lemma 5 and explains the part of Figure 3 for $r_F \geq \overline{r}$.

For $r_F < \overline{r}$, $\lim_{\theta\to\infty}\widehat{D}$ in (62) is equal to positive infinity, whereas if the condition in note 14 is met, $\partial\widehat{D}/\partial\theta$ in (63) is at first negative, reaches zero, and becomes positive thereafter. In this case, $\widehat{D}$, as a function of θ, is U-shaped and starts off and ends up equal to positive infinity.

Write

$$\widehat{D}_{\min}(\alpha; r_F) \equiv \min_{\theta}\widehat{D}(\theta,\alpha),$$

where by the Envelope Theorem, $\partial\widehat{D}_{\min}/\partial\alpha < 0$ and $\partial\widehat{D}_{\min}/\partial r_F < 0$. Notice also that $\lim_{r_F\to 0}\widehat{D}_{\min}(\alpha; r_F) = \infty$ and $\lim_{r_F\to\overline{r}}\widehat{D}_{\min}(\alpha; r_F) = -\infty$. Then, there exist some r' and r'' that satisfy $\widehat{D}_{\min}(1; r') = 0$ and $\widehat{D}_{\min}(0; r'') = 0$, respectively, where $0 < r' < r'' < \overline{r}$.

For $r_F \leq r'$, $\widehat{D}_{\min}(1; r_F) \geq 0$. Given that $\widehat{D}(\theta, \alpha) \geq \widehat{D}_{\min}(\alpha; r_F)$, $\widehat{D}(\theta, \alpha) > 0$ for all $\alpha < 1$. This proves Lemma 6(a) and explains the part of Figure 3 for $r_F \leq r'$.

For $r_F \in (r', r'')$, $\widehat{D}_{\min}(1; r_F) < 0$. Hence, there exists two θ, namely θ_1^d and θ_1^u at which $\widehat{D}(\theta, 1) = 0$, over where $\partial\widehat{D}/\partial\theta < 0$ and $\partial\widehat{D}/\partial\theta > 0$, respectively. Within the interval (θ_1^d, θ_1^u), the exists some $\alpha_D(\theta) \in (0, 1)$ such that $\widehat{D}(\theta, \alpha_D(\theta)) = 0$. For $\theta \notin [\theta_1^d, \theta_1^u]$, $\widehat{D}(\theta, 1) > 0$. Moreover, since $\widehat{D}_{\min}(0; r_F) > 0$ for $r_F < r''$, $\alpha_O(\theta) > 0$ for all θ. This proves Lemma 6(b) and explains the part of Figure 3 for $r_F \in (r', r'')$.

At $r_F = r''$, there is one θ at which $\widehat{D}(\theta, 0) = 0$. Call this θ^*. For $\theta \neq \theta^*$, $\widehat{D}(\theta, 0) > 0$. Then, an $\alpha = 0$ equilibrium exists if and only if $\theta_S(0) = \theta^*$. Given that as H varies from 0 to 1, $\theta_S(0)$ spans the entire positive real line, there exists one and only one H at which $\theta_S(0) = \theta^*$. This explains how the U-shaped border in Figure 3 is tangent to the $r_F = r''$ line.

For $r_F \in [r'', \overline{r})$, $\widehat{D}_{\min}(\alpha, r_F) < 0$. Then, there are two θ, namely θ_0^d and θ_0^u at which $\widehat{D}(\theta, 0) = 0$, over where $\partial D/\partial\theta < 0$ and $\partial D/\partial\theta > 0$, respectively. Within the interval (θ_0^d, θ_0^u), $\widehat{D}(\theta, 0) < 0$. As r_F increases from r'' to $\overline{r}$, given that $\partial\widehat{D}/\partial r_F < 0$, and that $\partial\widehat{D}/\partial\theta < 0$ at $\theta = \theta_0^d$ and $\partial\widehat{D}/\partial\theta > 0$ at $\theta = \theta_0^d$, the interval (θ_0^d, θ_0^u) expands, whereby $\partial\theta_0^d/\partial r_F < 0$ and $\partial\theta_0^u/\partial r_F > 0$. This proves Lemma 6(c).

As to the construction of the U-shaped border in Figure 3, first notice that in this case, an $\alpha = 0$ equilibrium exists if and only if

$$\theta_S(0) \in [\theta_0^d, \theta_0^u].$$

Given that $\partial\theta_0^d/\partial r_F < 0$ and $\partial\theta_S(0)/\partial H < 0$, the condition $\theta_S(0) = \theta_0^d$ defines an upward-sloping border $\overline{H}(r_F)$ in the H-r_F space. Given the continuity of $\widehat{D}$, in the limit as $r_F \to \overline{r}$, θ_0^d tends to the same θ_0^d at which $r_F = \overline{r}$ just hold. Then, there is the same H that keeps $\theta_S(0) = \theta_0^d$ at $r_F = \overline{r}$. On the other hand, with $\partial\theta_0^u/\partial r_F > 0$, the condition $\theta_S(0) = \theta_0^u$ defines a downward-sloping border $\underline{H}(r_F)$ in the H-r_F space.

Proof of Lemma 8 Only two of the four equations of motion constitute independent restrictions. By utilizing (1)-(4), we can reduce the system to that of two state variables, the equations of motion of which are given by, respectively,

$$\dot{n}_M = \eta(H - n_M) - \delta n_M, \tag{65}$$

$$\dot{n}_R = \alpha\delta n_M - \eta(n_R - (1 - H)), \tag{66}$$

and an equation for θ given by

$$\theta = \frac{n_R}{H - n_M}. \tag{67}$$

Write the Hamiltonian of maximizing (20) subject to (65)-(67) as

$$H = e^{-rt}(n_M v - n_R q) + \Gamma_{n_M}(\eta(H - n_M) - \delta n_M) + \Gamma_{n_R}(\alpha\delta n_M - \eta(n_R - (1 - H))), \tag{68}$$

where Γ_{n_M} and Γ_{n_R} are the respective co-states for n_M and n_R. Then, we have

$$\frac{\partial H}{\partial \alpha} = \Gamma_{n_R} \delta n_M, \tag{69}$$

$$\dot{\Gamma}_{n_M} = -e^{-rt}\upsilon - \Gamma_{n_M}\left(-\eta - \delta + \theta \frac{\partial \eta}{\partial \theta}\right) - \Gamma_{n_R}\left(\alpha\delta - \theta \frac{\partial \eta}{\partial \theta} \frac{n_R - (1-H)}{H - n_M}\right), \tag{70}$$

$$\dot{\Gamma}_{n_R} = e^{-rt} q - \Gamma_{n_M} \frac{\partial \eta}{\partial \theta} + \Gamma_{n_R}\left(\eta + \frac{\partial \eta}{\partial \theta} \frac{n_R - (1-H)}{H - n_M}\right). \tag{71}$$

The last two equations imply that the co-states must be growing at the rate $-r$ in the steady state. We can then write

$$\Gamma_{n_M} = \widetilde{\Gamma}_{n_M} e^{-rt}, \tag{72}$$

$$\Gamma_{n_R} = \widetilde{\Gamma}_{n_R} e^{-rt}, \tag{73}$$

for some $\widetilde{\Gamma}_{n_M}$ and $\widetilde{\Gamma}_{n_R}$ that are stationary over time. Substitute (72), (73) and the steady-state values of n_M and n_R from (52) and (55), respectively, into (70) and (71),

$$r\widetilde{\Gamma}_{n_M} = \upsilon - \widetilde{\Gamma}_{n_M}\left(\eta + \delta - \theta \frac{\partial \eta}{\partial \theta}\right) + \widetilde{\Gamma}_{n_R} \alpha \left(\delta - \theta \frac{\partial \eta}{\partial \theta}\right),$$

$$r\widetilde{\Gamma}_{n_R} = -q + \widetilde{\Gamma}_{n_M} \frac{\partial \eta}{\partial \theta} - \widetilde{\Gamma}_{n_R}\left(\eta + \frac{\partial \eta}{\partial \theta}\alpha\right),$$

the solutions of which are

$$\widetilde{\Gamma}_{n_M} = \upsilon \frac{\left(r + \eta + \frac{\partial \eta}{\partial \theta}\alpha\right)\upsilon - \alpha\left(\delta - \frac{\partial \eta}{\partial \theta}\theta\right) z}{\left(r + \eta - \frac{\partial \eta}{\partial \theta}\theta + \frac{\partial \eta}{\partial \theta}\alpha + \delta\right)(r + \eta)},$$

$$\widetilde{\Gamma}_{n_R} = \upsilon \frac{\frac{\partial \eta}{\partial \theta} - \left(r + \delta + \eta - \theta\frac{\partial \eta}{\partial \theta}\right) z}{\left(r + \eta - \theta\frac{\partial \eta}{\partial \theta} + \frac{\partial \eta}{\partial \theta}\alpha + \delta\right)(r + \eta)}.$$

By (69), the optimal value for α depends on the sign of $\widetilde{\Gamma}_{n_R}$ above. This explains $E(\theta)$ in (22).

Proof of Lemma 10 By (56),

$$\frac{1-H}{\delta H} = \frac{\theta_S(0)}{\delta + \eta(\theta_S(0))} \leq \frac{\theta}{\delta + \eta(\theta)}, \tag{74}$$

since $\theta \geq \theta_S(0)$. Substituting (2) into (24) and then from (58) and (52),

$$TV = \alpha\delta n_M + (H - n_M)\eta = \frac{\eta \delta H}{\delta + \eta}(1 + \theta) - \eta(1 - H). \tag{75}$$

Differentiating and simplifying,

$$\begin{aligned}\frac{\partial TV}{\partial \theta} &= \delta H\left(\frac{\partial \eta}{\partial \theta}\frac{\delta(1+\theta)}{(\delta+\eta)^2}-\frac{\partial \eta}{\partial \theta}\frac{1-H}{\delta H}+\frac{\eta}{\eta+\delta}\right)\\ &\geq \delta H\left(\frac{\partial \eta}{\partial \theta}\frac{\delta(1+\theta)}{(\delta+\eta)^2}-\frac{\theta\partial\eta/\partial\theta}{\delta+\eta}+\frac{\eta}{\eta+\delta}\right)>0,\end{aligned}$$

where the first inequality is by (74) and the second inequality by the concavity of η. But then $\partial\theta_S/\partial\alpha>0$; hence $\partial TV/\partial\alpha>0$.

Proof of Lemma 11 Substituting from (2), (75), and (52) and simplifying, (25) becomes

$$\left((1+\theta)\eta-\frac{1-H}{\delta H}\eta(\delta+\eta)\right)^{-1}.$$

Differentiating with respect to θ yields an expression having the same sign as

$$\begin{aligned}-\left(\eta+\theta\frac{\partial\eta}{\partial\theta}-\frac{1-H}{\delta H}\frac{\partial\eta}{\partial\theta}(\delta+2\eta)\right) &< -\left(\eta+\theta\frac{\partial\eta}{\partial\theta}-\frac{\theta}{\delta+\eta}\frac{\partial\eta}{\partial\theta}(\delta+2\eta)\right)=\\ -\left(\frac{\eta}{\delta+\eta}\left(\delta+\eta-\theta\frac{\partial\eta}{\partial\theta}\right)\right) &< 0,\end{aligned}$$

where the first inequality is by (74) and the second by the concavity of η. The Lemma follows given that $\partial\theta_S/\partial\alpha>0$.

Proof of Lemma 12 Substituting from (5), (6), and then (1), (26) becomes

$$\frac{1}{\mu}+\frac{1-\alpha}{\eta}=\frac{1-n_M}{\delta n_M},$$

a decreasing function of n_M. But where $\partial n_M/\partial\alpha>0$, there must be a smaller average TBM.

Proof of Lemma 13 Setting $\max\{V_R+p_{FB},V_U\}=V_U$ in (10) and solving (8)-(13) for the three prices and three asset values yield the solutions in the first part of the Lemma. Setting $\max\{V_R+p_{FB},V_U\}=V_R+p_{FB}$ before solving (8)-(13) yield the solutions in the second part.

Proof of Lemma 14 In case $D>0$, by the second part of Lemma 13, S_{RF} has the same sign as

$$(1+z)(\beta_H\eta+r)>0.$$

If $D=0$, by the first part of Lemma 13, S_{RF} has the same sign as

$$r+(1+z)\beta_H\eta-\mu(1-\alpha)(1-\beta_H)+(\delta+r)z. \tag{76}$$

But if $D = 0$, by (7),

$$\left(\beta_F \frac{r}{r_F} - \beta_H - z\beta_H\right)\eta + (1 - \beta_H - \alpha(\beta_F - \beta_H))\mu = (\delta + r) z.$$

Substituting into (76) for $(\delta + r) z$,

$$\frac{(r_F + \beta_F \eta) r + (1 - \beta_F) r_F \mu \alpha}{r_F} > 0.$$

If $D \leq 0$, by the first part of Lemma 13,

$$S_{RU} = \frac{v}{r + \delta + \beta_H \eta} > 0.$$

This completes the proof of Part (a) of the Lemma.

For Part (b), if $D \geq 0$, the condition to check is

$$V_R + p_{FB} \geq 0.$$

By the second part of Lemma 13, the condition becomes

$$\frac{r}{r_F}\beta_F \eta + (1 - \beta_F)\mu - (\delta + r) z \geq 0,$$

which holds whenever $\widehat{D} \geq 0$. If $D \leq 0$, we want to show that

$$V_U \geq 0,$$

which holds by the first part of Lemma 13.

Proof of Lemma 15 By (17), θ_1^u satisfies,

$$\left(\beta_F \frac{r}{r_F} - (1 + z)\beta_H\right)\eta(\theta_1^u) + (1 - \beta_F)\mu(\theta_1^u) - (\delta + r) z = 0,$$

at where the above is increasing in θ. Let $\eta(\theta) = a\theta^b$. Then, $\theta_1^u < 1$ if

$$\beta_F \frac{r}{r_F} - (1 + z)\beta_H + 1 - \beta_F - (\delta + r)\frac{z}{a} > 0.$$

The second inequality of (50) follows under (51), which serves to ensure that $r_F < \overline{r}$. By the proof of Lemmas 4-7, r'' satisfies,

$$\min_{\theta}\left\{\left(\beta_F \frac{r}{r''} - (1 + z)\beta_H\right)\eta(\theta) + (1 - \beta_H)\mu(\theta) - (\delta + r) z\right\} = 0.$$

Evaluating the above and solving for r'' yield the far left term in (50).

With $\theta_S(1)$ implicitly defined by (57), it can be shown that $\lim_{H\to 1}\theta_S(1) = 1$, whereas by Lemma 2, $\lim_{H\to 1}\theta_S(0) = 0$. Under (51) and (50), there exists a strictly positive θ_0^u. Then, the first inequality of (19) would hold for H sufficiently close to 1. Under the same conditions, the last inequality of (19) holds too with $\theta_1^u < 1$ but $\lim_{H\to 1}\theta_S(1) = 1$.

Proof of Proposition 1 Part (a) is obtained by setting $\widehat{D}(\theta,\alpha)$ and $E(\theta)$, given by (17) and (22) respectively, equal to each other and solving for β_H and β_F. For part (b), if $\partial\widehat{D}(\theta,\alpha)/\partial\theta > 0$,

$$(1-\beta_H-\alpha(\beta_F-\beta_H)) > \left(\beta_F\frac{r}{r_F}-\beta_H-z\beta_H\right)\frac{\frac{\partial\eta}{\partial\theta}\theta^2}{\eta-\theta\frac{\partial\eta}{\partial\theta}} > 0.$$

Then,

$$\begin{aligned}
\widehat{D}(\theta,\alpha)-E_S(\theta) &= \left(\beta_F\frac{r}{r_F}-\beta_H-z\beta_H\right)\eta+(1-\beta_H-\alpha(\beta_F-\beta_H))\frac{\eta}{\theta} \\
&\quad -\frac{\partial\eta}{\partial\theta}+\left(\eta-\theta\frac{\partial\eta}{\partial\theta}\right)z. \\
&> \left(\beta_F\frac{r}{r_F}-\beta_H-z\beta_H\right)\left(\frac{1}{1-\frac{\partial\eta}{\partial\theta}\frac{\theta}{\eta}}\right)+\left(1-\frac{\theta}{\eta}\frac{\partial\eta}{\partial\theta}\right)z-\frac{\partial\eta}{\partial\theta}\frac{1}{\eta} \\
&> 2\left[\left(\beta_F\frac{r}{r_F}-\beta_H-z\beta_H\right)z\right]^{1/2}-\frac{\partial\eta}{\partial\theta}\frac{1}{\eta}.
\end{aligned}$$

The second inequality comes from minimizing

$$\frac{\beta_F\frac{r}{r_F}-\beta_H-z\beta_H}{\varepsilon}+z\varepsilon$$

with respect to ε. Rewriting the last line gives the condition in the Proposition.

Proof of Proposition 2 We begin with showing end-user market price p_{FS} in the full-intermediation equilibrium, given by (29), is higher than p_{FS} in a partially-intermediated equilibrium, given by (31). Now, at where $\theta=\theta_1^u$, $D(\theta,1)=0$, the two p_{FS} are by construction equal. Second, with p_{FS} in the first equation increasing in θ by the concavity of η and the p_{FS} in the second equation increasing in θ for $r_F<\overline{r}$, which is a necessary condition for multiplicity, p_{FS} in the full-intermediation equilibrium must exceed p_{FS} in the partially-intermediated equilibrium, since in this case $\theta_S(1)\geq\theta_1^u$ whereas p_{FS} in the partially-intermediated equilibrium has a $\theta\leq\theta_1^u$. Lastly, with $p_{FS}>p_H$ in the partially-intermediated equilibrium, the single end-user market price in the full-intermediation equilibrium exceeds the two end-user market prices in the partially-intermediated equilibrium. Next, in a comparison between p_{FS} in two partially-intermediated equilibria, given that p_{FS} in (31) is increasing in θ, there must be higher p_{FS} in the larger θ equilibrium. The same ranking applies to the two p_H, given that p_H in (32) is similarly increasing in θ in case $r_F<\overline{r}$. The final comparison is between p_H in a partially-intermediated equilibrium and p_H in the no-intermediation equilibrium, given by (27). At where $\theta=\theta_0^u$, $D(\theta,0)=0$, the two p_H are by construction equal. With the first p_H known to be increasing in θ, the proof is completed by noting that p_H, given by (27), is likewise is increasing in θ given the concavity of η. This completes the proof that end-user market housing prices across steady-state equilibria can be ranked by the value of θ. Given that $p_{FB}=\frac{\eta}{\eta+r_F}p_{FS}$, investment market housing prices are ranked in the same order as in end-user market housing prices.

Proof of Proposition 4 By differentiating (33)-(35) with respect to θ and noting that $\alpha_S \leq 1$ and $\partial \alpha_S / \partial \theta > 0$.

Proof of Proposition 6 Define $\Phi(\alpha) \equiv \alpha_O(\theta_S(\alpha))$, a continuous function mapping $[0,1]$ into itself. A steady-state equilibrium is any fixed point of Φ. By Brouwer's Fixed Point Theorem, a continuous function mapping the unit interval into itself must possess a fixed point.

Proof of Proposition 7 Write $R = r/r_F$, which remains constant amid any equiproportionate changes in r and r_F. Substituting $r_F = rR^{-1}$ into (28) and (29) and differentiating proves the first part of (a). In a no-intermediation equilibrium, p_H is given by (27), which is independent of r_F but decreasing in r for sufficiently small $\theta_S(0)$ and/or q. This proves the second part of (a). For (b), substituting $r_F = rR^{-1}$ into (30)-(32), respectively, yields,

$$p_{FB} = \frac{\beta_F \eta}{rR^{-1}(r + \delta + \beta_H \eta)} \upsilon, \tag{77}$$

$$p_{FS} = \frac{\beta_F \eta + \beta_F r R^{-1}}{rR^{-1}(r + \delta + \beta_H \eta)} \upsilon, \tag{78}$$

$$p_H = \frac{\beta_F \eta + \beta_H r R^{-1}}{rR^{-1}(r + \delta + \beta_H \eta)} \upsilon. \tag{79}$$

all of which are decreasing in r. Solving $\widehat{D} = 0$ from (17) for

$$r = \frac{(\beta_F R - \beta_H - z\beta_H)\eta + (1 - \beta_H - \alpha(\beta_F - \beta_H))\mu}{z} - \delta, \tag{80}$$

and substituting into (77)-(79), respectively, gives

$$p_{FB} = \frac{R\beta_F \eta q^2}{G}, \tag{81}$$

$$p_{FS} = \frac{((\beta_F R - \beta_H)\eta + (1 - \beta_H - \alpha(\beta_F - \beta_H))\mu)\beta_F \upsilon - (\eta\beta_H + \delta - \eta R)\beta_F q}{G} q, \tag{82}$$

$$p_H = \frac{((\beta_F R - \beta_H)\eta + (1 - \beta_H - \alpha(\beta_F - \beta_H))\mu)\beta_H \upsilon - (\beta_H(\eta\beta_H + \delta) - \beta_F \eta R)q}{G} q, \tag{83}$$

where

$$\begin{aligned} G &= (((\beta_H - \beta_F R)\eta - (1 - \beta_H - \alpha(\beta_F - \beta_H))\mu)\upsilon + (\delta + \beta_H \eta)q) \\ &\quad \times ((\beta_H - \beta_F R)\eta - (1 - \beta_H - \alpha(\beta_F - \beta_H))\mu). \end{aligned} \tag{84}$$

Differentiating (81) with respect to θ, evaluating the resulting expression at $\beta_F = \beta_H = 1/2$ yields an expression whose sign is given by that of

$$\eta(R - 1 + 1/\theta - z) - 2\delta z - (R - 1 + 1/\theta)\left(\theta^2 \frac{\partial \eta}{\partial \theta}(R - 1 + 1/\theta - z) - \eta\right).$$

The expression is strictly positive at $R = 0$ and $R = 1 + z$ if the RHS of (80) at $\beta_F = \beta_H = 1/2$ is positive. And then differentiating twice with respect to R yields

$$-2\theta^2 \frac{\partial \eta}{\partial \theta} < 0.$$

Thus, p_{FB} in (81) must be increasing in θ for $R \in [0, 1 + z]$. For p_{FS} and p_H, differentiating (82) and (83) with respect to θ and evaluating at $\beta_F = \beta_H = 1/2$ and $R = 0$, 1, and $1 + z$, respectively, all yield a strictly positive expression as long as the RHS of (80) is positive at $\beta_F = \beta_H = 1/2$. Then, p_{FS} and p_H in (82) and (83) must be increasing in θ for R in neighborhoods of 0, 1, and $1 + z$.

Figures

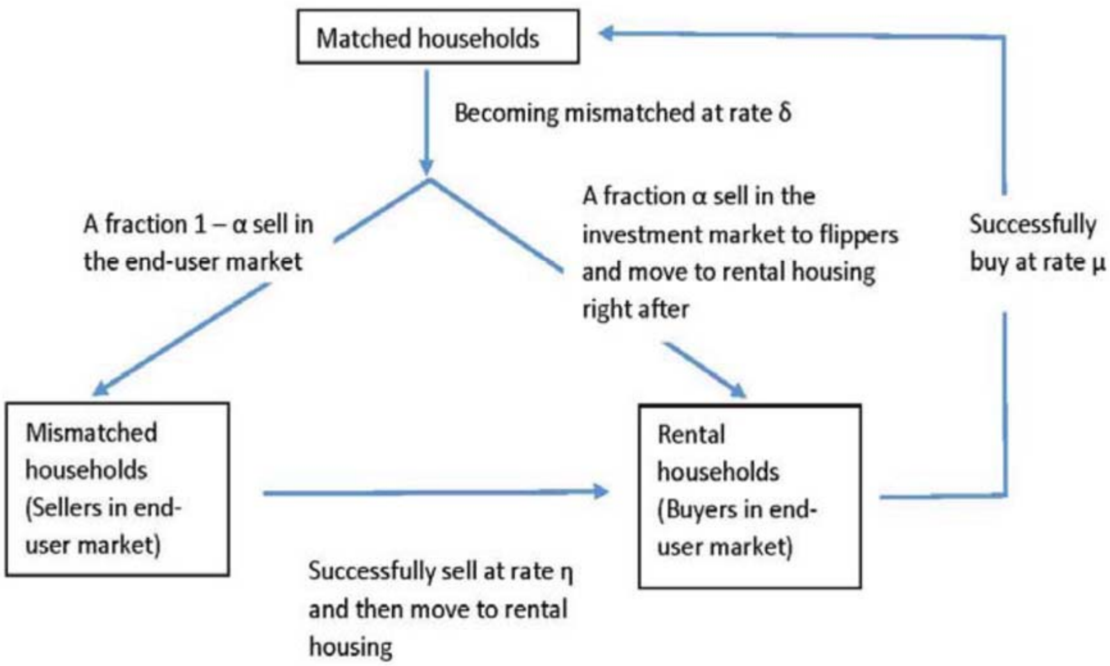

Figure 1: The flows of households into and out of the three states

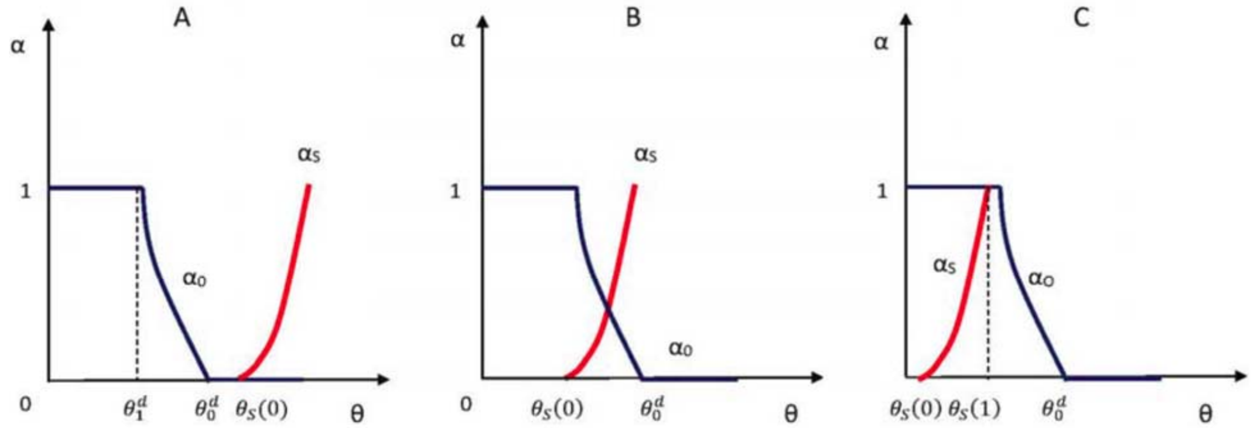

Figure 2: The unique equilibrium for $r_F \geq \overline{r}$

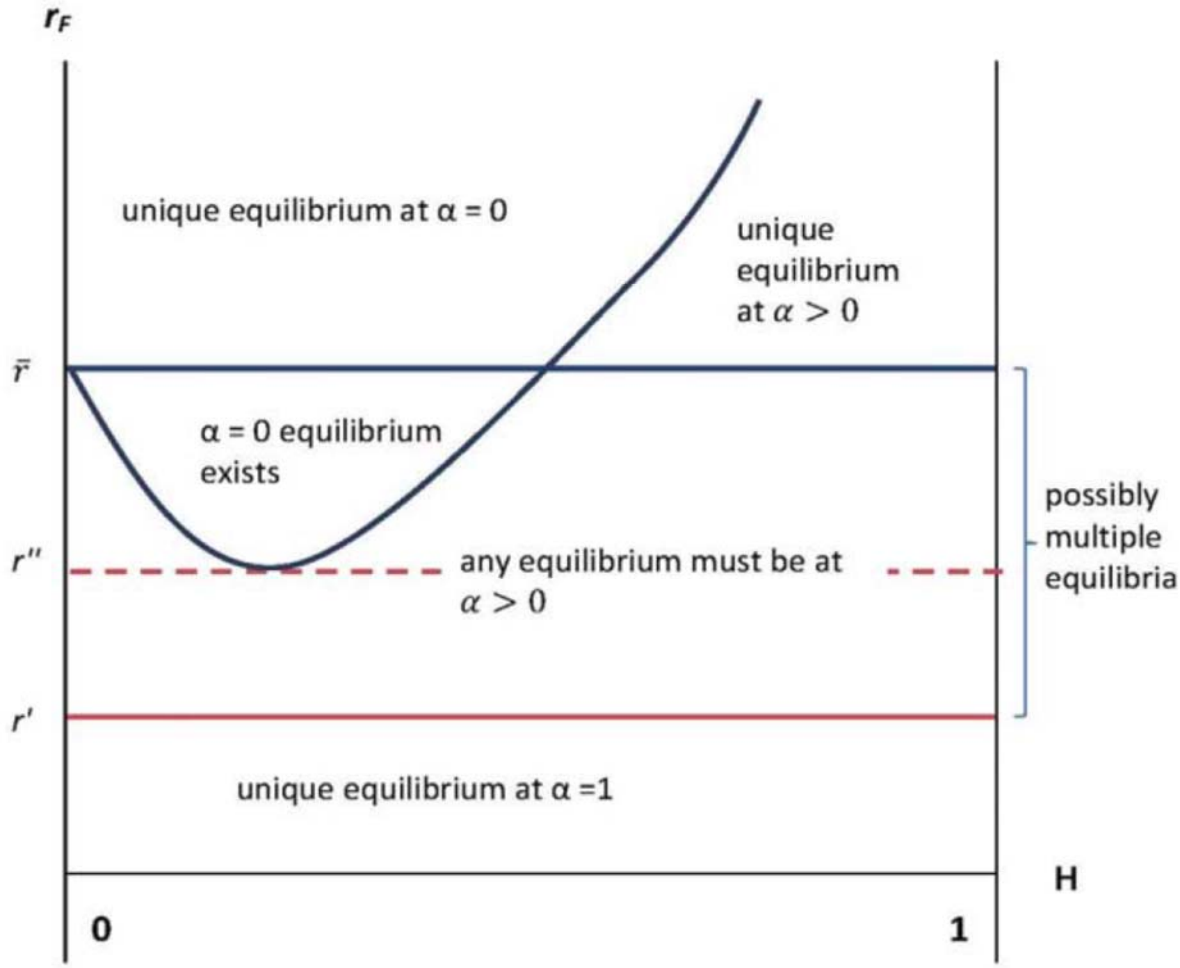

Figure 3: The nature of equilibrium

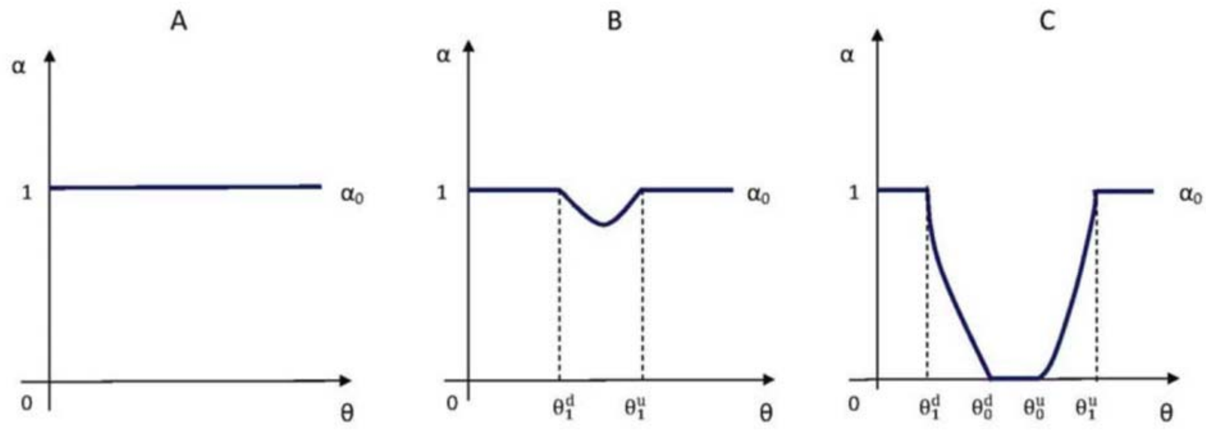

Figure 4: The $\alpha_O(\theta)$ function : $r_F < \overline{r}$

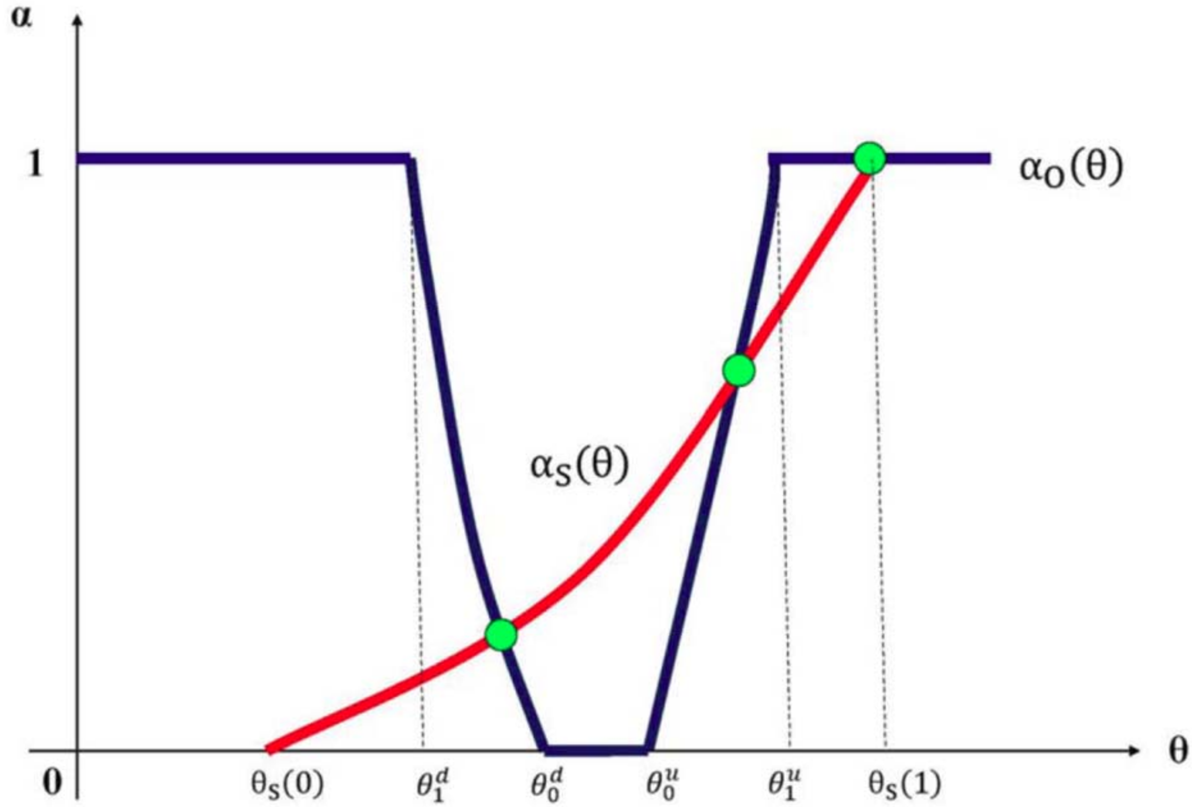

Figure 5: Multiple Equilibria: $\theta_0^d > \theta_S\,(0)$

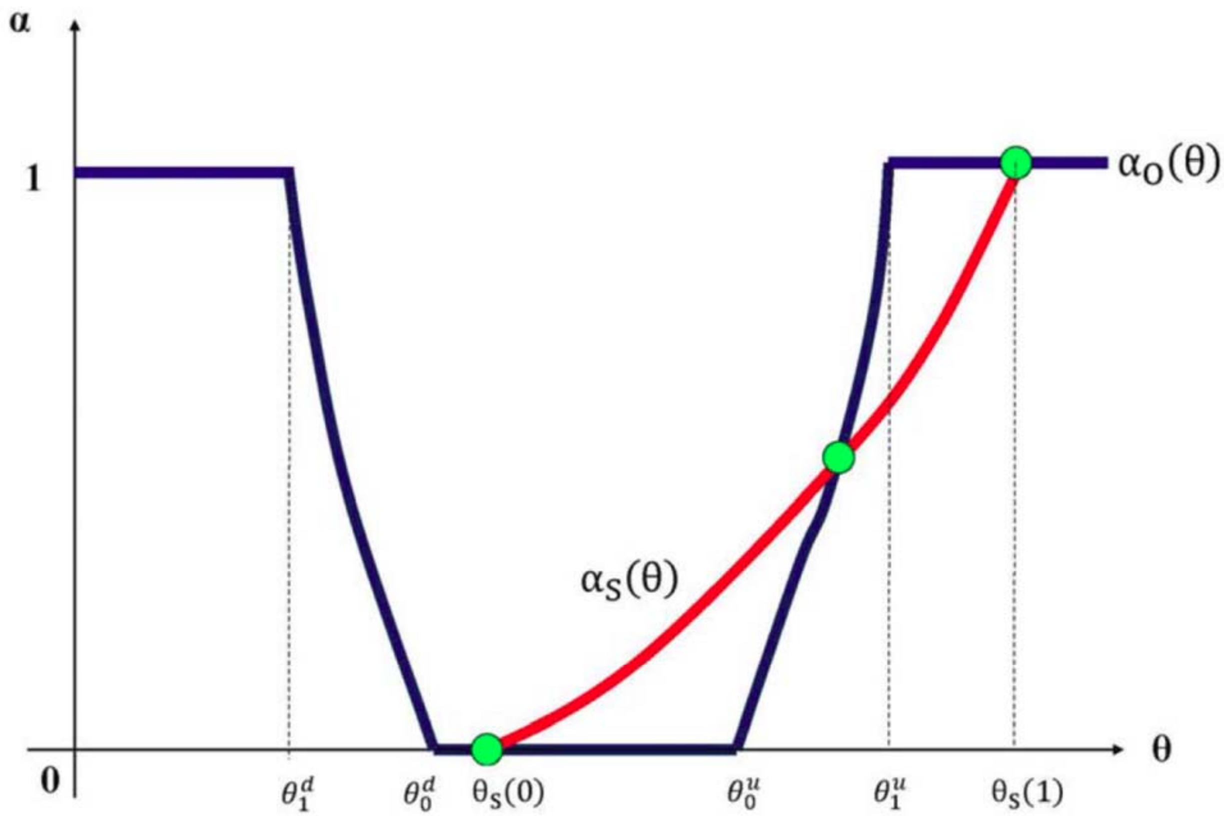

Figure 6: Mutliple Equilibria: $\theta_S\left(0\right) > \theta_0^d$

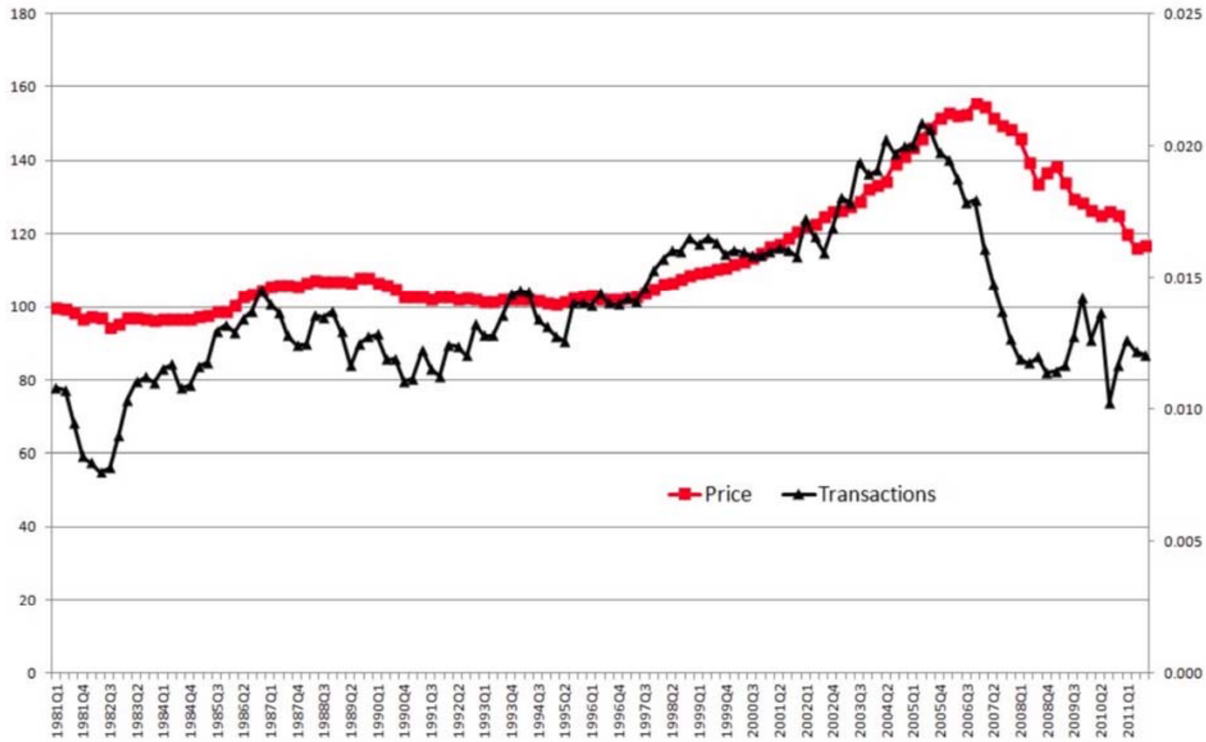

Figure 7: Price and Transactions

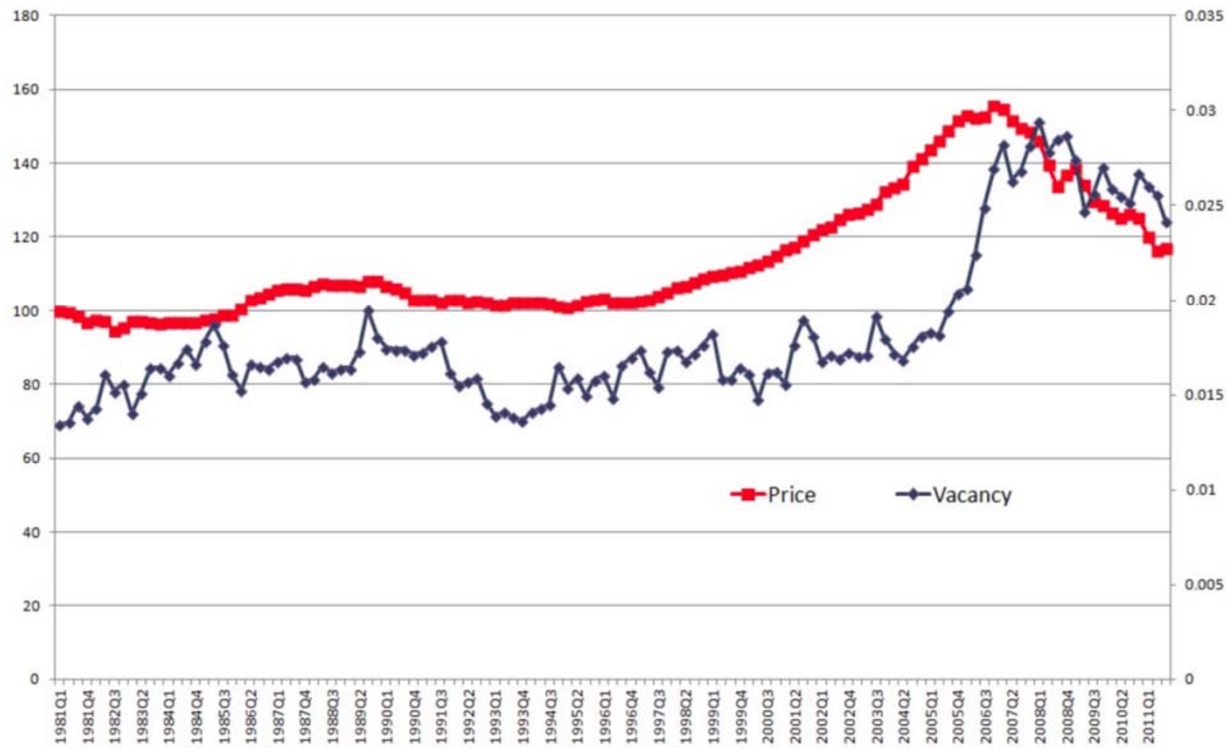

Figure 8: Price and Vacancy

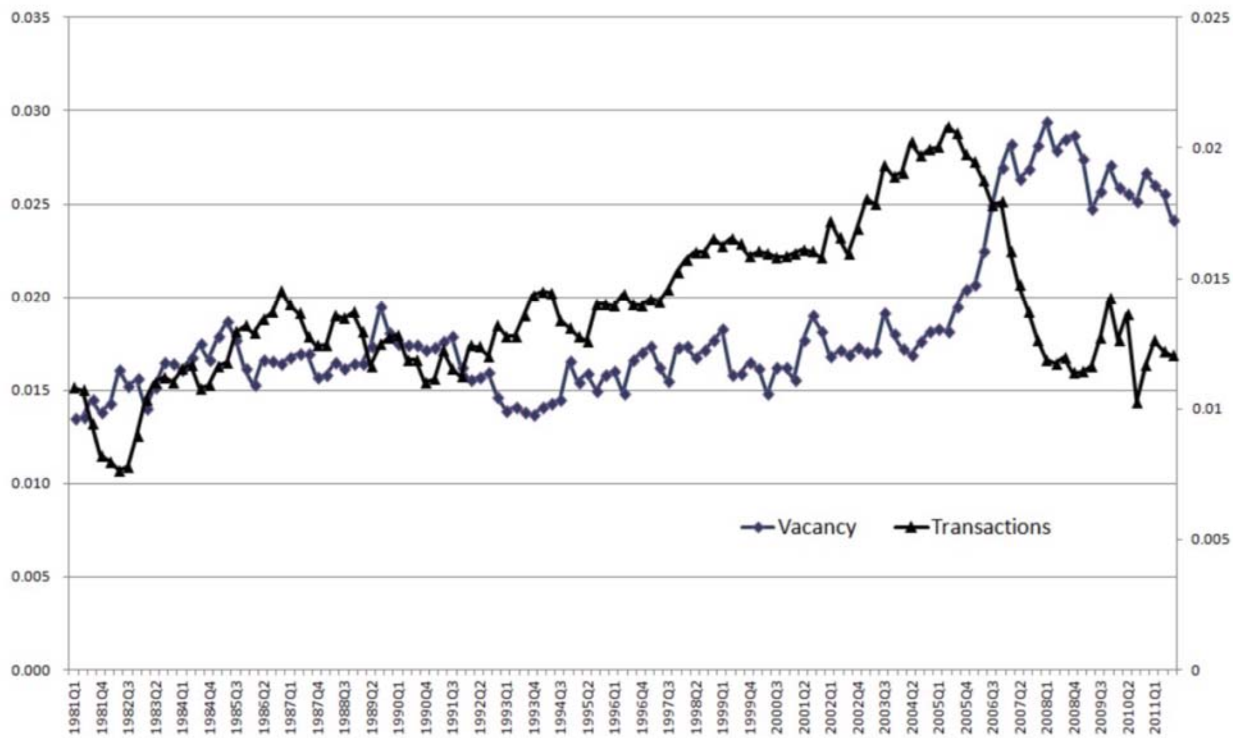

Figure 9: Transactions and Vacancy