License and entry strategies for outside innovator in duopoly

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Abstract

In Proposition 4 of Kamien and Tauman (1986), assuming linear demand and cost functions with fixed fee licensing it was argued that for the outside innovating firm under oligopoly when the number of firms is small (or very large), strategy to enter the market with license of its cost-reducing technology to the incumbent firm (entry with license strategy) is more profitable than strategy to license its technology to the incumbent firm without entering the market (license without entry strategy). However, their result depends on their definition of license fee, and it is inappropriate if the innovating firm can enter the market. If we adopt an alternative more appropriate definition based on the threat by entry of the innovating firm, license without entry strategy is more profitable in the case of linear demand and cost functions. Also we investigate the problem in the case of quadratic cost functions in which entry with license strategy may be optimal. Further we will show that the optimal strategies for the innovating firm when license fees are determined under the assumption that the licensor takes all benefit of new technology and its optimal strategies when license fees are determined according to Nash bargaining solution are the same.

Keywords: entry, license, duopoly, cost-reducing innovation, innovating firm, incumbent firm

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1 Introduction

In Proposition 4 of Kamien and Tauman (1986), assuming linear demand and cost functions with fixed fee licensing it was argued that for the outside innovating firm under oligopoly when the number of firms is small (or very large), strategy to enter the market and at the same time license its cost-reducing technology to the incumbent firm (entry with license strategy) is more profitable than strategy to license its technology to the incumbent firm without entering the market (license without entry strategy). However, their result depends on their definition of license fee. Interpreting their analysis in a duopoly model, they defined the license fee in the case of license without entry by the difference between the profit of the incumbent firm in that case and its monopoly profit before entry and license. It is inappropriate when the innovator can enter the market on the ground of game theoretic view point. If the negotiation between the innovating firm and the incumbent firm about the license fee breaks down, that is, the offered license fee is refused by the incumbent firm, the innovating firm can punish the incumbent firm by entering the market without license. The innovating firm may use this threat if and only if it is a credible threat. When the innovating firm does not enter nor sell a license, its profit is zero; however, when it enters the market without license, its profit is positive. Therefore, such a threat is credible. Then, even if the innovating firm does not enter the market, the incumbent firm must pay the difference between its profit when it uses the new technology and its profit when the innovating firm enters without license as a license fee.

In this paper we examine a choice of strategies for the outside innovating firm under duopoly to license its new cost-reducing technology with a fixed fee to the incumbent firm without entry, or to enter the market without license, or to enter the market with license. At present the incumbent firm is a monopolist, and if the innovating firm enters, the market becomes a duopoly. Using an alternative definition of a license fee based on above considerations we will show the result which is converse to the result in Kamien and Tauman (1986) that license without entry strategy is optimal for the innovator, and entry with license strategy is never optimal in the case of linear demand and cost functions. Also we will consider a case of quadratic cost functions in which entry with license strategy can be optimal.

In more detail we will show the following results. When the cost functions are linear, if the innovation is non-drastic, license without entry strategy is the optimal strategy for the innovating firm; if the innovation is drastic, both of entry without license and license without entry strategies are the optimal strategies. When the cost functions are quadratic, so long as the innovation is not so drastic, entry with license strategy is the optimal strategy for the innovating firm; if the innovation is very drastic, license without entry strategy is the optimal strategy.

Further we consider two cases about determination of license fees. The first is a case such that the licensor takes all of the profit increase due to adoption of the new technology. We call this case *the licensor takes all case*. The second is a case such that license fees are determined according to Nash bargaining solution. We call this case *the Nash bargaining solution case*. We will show that the same results hold in both cases, that is, determination of the optimal strategies for the innovating firm in the licensor takes all case and that in the Nash bargaining solution case are the same.

In the next section we review some related studies including Kamien and Tauman (1986). In Section 3 we present the model. In Section 4 we study the licensor takes all case, and in

Section 5 we consider the Nash bargaining solution case.

2 Some related studies

We mention some references about technology adoption or R&D investment under imperfect competition including Kamien and Tauman (1986). Lots of researches focus on the relation between technology licensor and licensee. The difference of means of contracts which are royalties, up-front fixed fees, the combinations of these two and auction are well discussed (Katz and Shapiro (1985)). Also, the previous works analyze the difference of whether the licensor have the production capacity, which is externally given, or not. Kamien and Tauman (1986) shows that if the licensor does not have production capacity, fixed fee is better than royalty and it is also better for consumers. Kamien and Tauman (2002) shows that outside innovator prefers auction but industry incumbent prefers royalty. This topic under Stackelberg oligopoly is discussed in Kabiraj (2004) when the licensor does not have production capacity, and discussed in Wang and Yang (2004) when the licensor has production capacity.

Sen and Tauman (2007) compared the license system in detail when the licensor is an outsider and that when the licensor is an incumbent firm using the combination of royalties and fixed fees. However, the existence of production capacity is given externally and it does not analyze the choice of entry. Therefore, the optimal strategies of outside innovator who can use the entry as threat are not discussed enough. About the strategies of new entrant to the market, Duchene, Sen and Serfes (2015) pay attention to the future entrants with old technology, and argues that low license fee can be used to deter entry of potential entrants. But, it is assumed that the firm with new technology is incumbent and its choice of entry is not analyzed. Also, Chen (2016) analyzed the model of endogenous market structure determined by the potential entrant with the old technology and shows that the licensor uses the fixed fee and zero royalty in both incumbent and outside innovator cases which is exogenously given. In this paper we consider process innovation, that is, cost-reducing innovation. On the other hand, Hattori and Tanaka (2016b) analyzed license and entry strategies of an innovator about product innovation, that is, introduction of higher quality good in vertically differentiated industry, and has shown similar results.

Other topics are analyzed as follows. A Cournot oligopoly with fixed fee under cost asymmetry is analyzed in La Manna (1993). He shows that if technologies can be replicated perfectly, a lower-cost firm always has the incentive to transfer its technology and hence a Cournot-Nash equilibrium cannot be fully asymmetric, but there exists no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyses bargaining between licensor with no production capacity and oligopolistic firms. In recent research, market structure and technology improvement is analyzed. Boone (2001) and Matsumura et. al. (2013) find a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) shows that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may make more social welfare than Bertrand competition under differentiated goods market. In Hattori and Tanaka (2014) and Hattori and Tanaka (2016a) adoption of new technology in Cournot duopoly and Stackelberg duopoly is analyzed. Rebolledo and Sandonís (2012) presented analyses about the effectiveness of R&D subsidies in an oligopolistic model in the cases of international R&D competition and cooperation.

3 The model

There are two firms Firm A and B. Firm A is an innovating firm and Firm B is an incumbent firm. Although at present only Firm B produces a good and Firm A is an outside innovator, after introducing the new technology, Firm A may also produce the same good. It has a superior new technology and can produce the good at lower cost than Firm B.

Firm A have three options. One option is to enter the market without license to Firm B, the second option is to license its superior technology to Firm B, and the third option is to enter the market with license to Firm B. If the innovating firm enters, the market becomes a duopoly.

Let p be the price and X be the total supply of the good. The inverse demand function of the good is written as

$$p = a - X.$$

a is a positive constant. Let x_A and x_B be the outputs of Firm A and B.

About determination of license fees we consider two cases. The first is a case where the licensor takes all of the profit increase due to adoption of new technology. We call this case *the licensor takes all case*. The second is a case where license fees are determined according to Nash bargaining solution. We call this case *the Nash bargaining solution case*. We will show that the same results hold in both cases, that is, determination of the optimal strategy for the innovating firm in the licensor takes all case and that in the Nash bargaining solution case are the same.

About cost functions of the firms we consider two cases. One is a case of linear cost functions, and the other is a case of quadratic cost functions. In the linear cost functions case, the cost functions of Firm A and B are $c_A x_A$ and $c_B x_B$, where c_A and c_B are positive constants such that $c_A < c_B$. In the quadratic cost functions case the cost functions of Firm A and B are $c_A x_A^2$ and $c_B x_B^2$, where c_A and c_B are positive constants such that $c_A < c_B$.

We define drasticity of innovation for each case of cost functions.

Linear cost functions case

1. The innovation is non-drastic if $c_A > 2c_B - a$.
2. The innovation is drastic if $c_A \leq 2c_B - a$. In this case the output of the incumbent firm is zero when the innovating firm enters the market without license.

Quadratic cost functions case

1. The innovation is not so drastic if $c_A > \frac{\sqrt{2}-1}{2}$.
2. The innovation is very drastic if $c_A \leq \frac{\sqrt{2}-1}{2}$.

4 Licensor takes all case

Suppose that the licensor takes all of the increase in the profit of Firm B due to adoption of the new technology.

4.1 Linear cost functions

First we investigate a case of linear cost functions.

4.1.1 Case A: entry without license

Suppose that Firm A enters the market without license to Firm B. The inverse demand function in this case is written as

$$p = a - x_A - x_B.$$

The profits of Firm A and B are

$$\pi_A = (a - x_A - x_B)x_A - c_A x_A,$$

$$\pi_B = (a - x_A - x_B)x_B - c_B x_B.$$

We assume Cournot type behavior of the firms. The conditions for profit maximization of Firm A and B are

$$a - 2x_A - x_B - c_A = 0,$$

$$a - x_A - 2x_B - c_B = 0.$$

1. If the innovation is non-drastic ($c_A > 2c_B - a$), then $x_B > 0$, and the equilibrium outputs, price and profits are

$$x_A = \frac{a - 2c_A + c_B}{3},\ x_B = \frac{a - 2c_B + c_A}{3},$$

$$p = \frac{a + c_A + c_B}{3},$$

$$\pi_A = \frac{(a - 2c_A + c_B)^2}{9},\ \pi_B = \frac{(a - 2c_B + c_A)^2}{9}.$$

2. If the innovation is drastic ($c_A \leq 2c_B - a$), then $x_B = 0$, that is, Firm B drops out from the market, and the equilibrium output of Firm A, the price and the profits of the firms are

$$x_A = \frac{a - c_A}{2},\ p = \frac{a + c_A}{2},$$

$$\pi_A = \frac{(a - c_A)^2}{4},\ \pi_B = 0.$$

   Firm A becomes a monopolist.

Denote the profits of Firm A and B in this case by π_A^e and π_B^e.

4.1.2 Case B: license without entry

Next suppose that Firm A licenses its technology to Firm B at a fixed license fee, and does not enter the market. Denote the fixed license fee by L.

The inverse demand function is

$$p = a - x_B.$$

The profit of Firm B is

$$\pi_B = (a - x_B)x_B - c_A x_B - L.$$

Firm B can produce the good at the marginal cost $c_A < c_B$. The equilibrium output, price and profit are

$$x_B = \frac{a - c_A}{2},\ p = \frac{a + c_A}{2},\ \pi_B = \frac{(a - c_A)^2}{4} - L.$$

If the negotiation between the innovating firm and the incumbent firm about the license fee breaks down, the innovating firm can enter the market without license. When the innovating firm does not enter nor sell a license, its profit is zero; however, when it enters the market without license, its profit is positive. Therefore, such a threat is credible, and the incumbent firm must pay the difference between its profit excluding the license fee in this case and its profit in the previous entry without license case as a license fee.

1. If the innovation is non-drastic ($c_A > 2c_B - a$), the license fee is equal to

$$L = \frac{(a - c_A)^2}{4} - \frac{(a - 2c_B + c_A)^2}{9} = \frac{(a + 4c_B - 5c_A)(5a - 4c_B - c_A)}{36}.$$

2. If the innovation is drastic ($c_A \le 2c_B - a$), the license fee is equal to

$$L = \frac{(a - c_A)^2}{4} - 0 = \frac{(a - c_A)^2}{4}.$$

Denote the profit of Firm B and the license fee in this case by π_B^l and L^l.

4.1.3 Case C: entry with license

Suppose that Firm A enters the market and at the same time licenses its technology to Firm B at a fixed license fee. The inverse demand function is

$$p = a - x_A - x_B.$$

The profits of Firm A and B are

$$\pi_A = (a - x_A - x_B)x_A - c_A x_A,$$

$$\pi_B = (a - x_A - x_B)x_B - c_A x_B - L.$$

L is the license fee. The conditions for profit maximization of Firm A and B are

$$a - 2x_A - x_B - c_A = 0,$$

$$a - x_A - 2x_B - c_A = 0.$$

The equilibrium outputs, price and profits are

$$x_A = \frac{a - c_A}{3},\ x_B = \frac{a - c_A}{3},\ p = \frac{a + 2c_A}{3},$$

$$\pi_A = \frac{(a - c_A)^2}{9},\ \pi_B = \frac{(a - c_A)^2}{9} - L.$$

Similarly to the previous case if the negotiation between the innovating firm and the incumbent firm about the license fee breaks down, the innovating firm can enter the market without license. The incumbent firm must pay the difference between its profit excluding the license fee in this case and its profit in the entry without license case as a license fee.

1. If $c_A > 2c_B - a$, the license fee is equal to

   $$L = \frac{(a - c_A)^2}{9} - \frac{(a - 2c_B + c_A)^2}{9} = \frac{4(a - c_B)(c_B - c_A)}{9}.$$

   The total profit of Firm A is the sum of the license fee and its profit as a firm in the duopoly. It is equal to

   $$\frac{(a - c_A)^2}{9} + \frac{4(a - c_B)(c_B - c_A)}{9} = \frac{a^2 - 6ac_A + c_A^2 + 4ac_B + 4c_Ac_B - 4c_B^2}{9}.$$

2. If $c_A \leq 2c_B - a$, the license fee is equal to

   $$L = \frac{(a - c_A)^2}{9} - 0 = \frac{(a - c_A)^2}{9}.$$

   The total profit of Firm A is

   $$\frac{(a - c_A)^2}{9} + \frac{(a - c_A)^2}{9} = \frac{2(a - c_A)^2}{9}.$$

Denote the profits of Firm A and B, and the license fee in this case by π_A^{el}, π_B^{el} and L^{el}.

4.1.4 The optimal strategy for the innovator

1. First assume $c_A > 2c_B - a$. Let us compare the profit of Firm A in Case B (license without entry) and its profit in Case C (entry with license). Then, we get

   $$\begin{aligned} L^l - (L^{el} + \pi_A^{el}) =& \frac{(a + 4c_B - 5c_A)(5a - 4c_B - c_A)}{36} \\ &- \frac{a^2 - 6ac_A + c_A^2 + 4ac_B + 4c_Ac_B - 4c_B^2}{9} \\ =& \frac{(a - c_A)^2}{36} > 0. \end{aligned}$$

Thus, license without entry strategy is more profitable than entry with license strategy for the innovating firm. Next let us compare the profit of Firm A in Case B and its profit in Case A (entry without license). Then,

$$\begin{aligned} L^l - \pi_A^e =& \frac{(a+4c_B-5c_A)(5a-4c_B-c_A)}{36} - \frac{(a-2c_A+c_B)^2}{9} \\ =& \frac{(a-2c_B+c_A)(a-11c_A+10c_B)}{36}. \end{aligned}$$

Since $a > 2c_B - c_A$ and

$$a - 11c_A + 10c_B = a - 2c_B + c_A + 12(c_B - c_A) > 0,$$

we have $L^l - \pi_A^e > 0$. Thus, license without entry strategy is more profitable than entry without license strategy for the innovating firm. Therefore, license without entry strategy is the optimal strategy for the innovating firm.

2. Assume $c_A \leq 2c_B - a$. Then, we obtain

$$L^l - (L^{el} + \pi_A^{el}) = \frac{(a-c_A)^2}{4} - \frac{2(a-c_A)^2}{9} = \frac{(a-c_A)^2}{36} > 0,$$

and

$$L^l - \pi_A^e = \frac{(a-c_A)^2}{4} - \frac{(a-c_A)^2}{4} = 0.$$

Thus, both of license without entry strategy and entry without license strategy are the optimal strategies for the innovating firm. Summarizing the results, we get the following proposition

Proposition 1. *In the linear cost functions case, if the innovation is non-drastic ($c_A > 2c_B - a$), license without entry strategy is the optimal strategy for the innovating firm. On the other hand, if the innovation is drastic ($c_A \leq 2c_B - a$), both of license without entry strategy and entry without license strategy are its optimal strategies.*

This result is converse to the result in Proposition 4 of Kamien and Tauman (1986). In the definition by Kamien and Tauman (1986) the license fee in the case of license without entry is equal to the profit of the incumbent firm in that case and its profit before license and entry. However, by our definition the license fee in that case is equal to the profit of the incumbent firm in that case and its profit when the innovating firm enters the market without license. Therefore, the license fee in that case under the alternative definition is larger than that under the definition by Kamien and Tauman (1986), and so license without entry strategy can be optimal.

Example

Assume $a = 18,\ c_B = 10$. The relationships among the profits of Firm A in three cases and the value of c_A are depicted in Fig. 1.

In the domain $0 \leq c_A \leq 2$, π_A^e and L^l coincide.

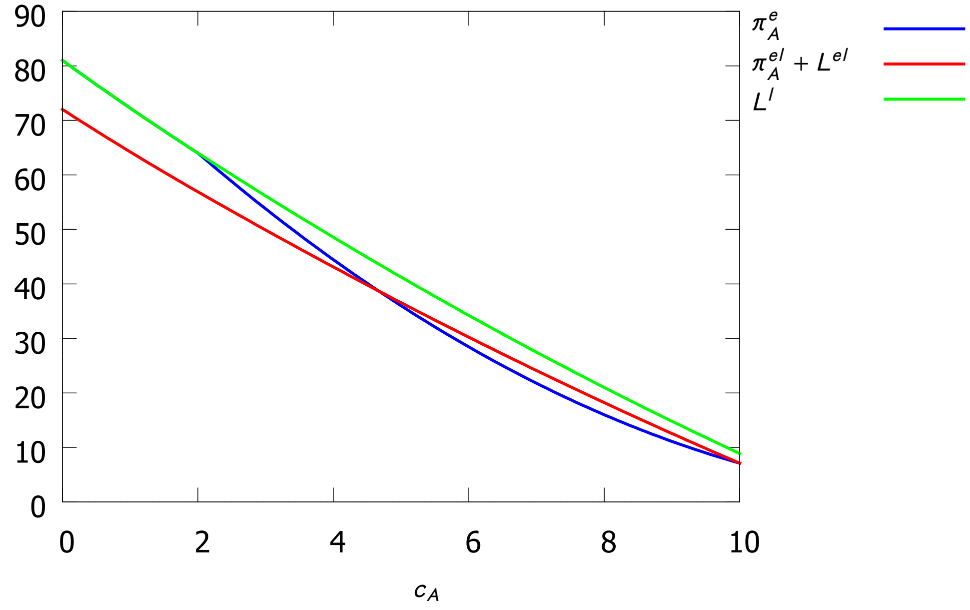

Figure 1: Example of the linear cost functions case

4.2 Quadratic cost functions

In this section we investigate the case of quadratic cost functions.

4.2.1 Case A: entry without license

Suppose that Firm A enters the market. The inverse demand function is written as

$$p = a - x_A - x_B.$$

The profits of Firm A and B are

$$\pi_A = (a - x_A - x_B)x_A - c_A x_A^2,$$

$$\pi_B = (a - x_A - x_B)x_B - c_B x_B^2.$$

The conditions for profit maximization of Firm A and B are

$$a - 2x_A - x_B - 2c_A x_A = 0,$$

$$a - x_A - 2x_B - 2c_B x_B = 0.$$

The equilibrium values of the outputs, price and profits are

$$x_A = \frac{(1 + 2c_B)a}{3 + 4c_A + 4c_B + 4c_A c_B},\ x_B = \frac{(1 + 2c_A)a}{3 + 4c_A + 4c_B + 4c_A c_B},$$

$$p = \frac{(1 + 2c_A)(1 + 2c_B)a}{3 + 4c_A + 4c_B + 4c_A c_B},$$

$$\pi_A = \frac{(1+c_A)(1+2c_B)^2a^2}{(3+4c_A+4c_B+4c_Ac_B)^2},\ \pi_B = \frac{(1+c_B)(1+2c_A)^2a^2}{(3+4c_A+4c_B+4c_Ac_B)^2}.$$

Denote the profits of Firm A and B in this case by π_A^e and π_B^e.

4.2.2 Case B: license without entry

Next suppose that Firm A licenses its technology to Firm B at a fixed license fee, and does not enter the market. Denote the fixed license fee by L.

The inverse demand function is

$$p = a - x_B.$$

The profit of Firm B is

$$\pi_B = (a - x_B)x_B - c_Ax_B^2 - L.$$

The equilibrium output, price and profit are

$$x_B = \frac{a}{2(1+c_A)},\ p = \frac{(1+2c_A)a}{2(1+c_A)},\ \pi_B = \frac{a^2}{4(1+c_A)} - L.$$

If the negotiation between the innovating firm and the incumbent firm about the license fee breaks down, the innovating firm can enter the market without license. Therefore, the incumbent firm must pay the difference between its profit excluding the license fee in this case and its profit in the previous entry without license case as a license fee. Thus, the license fee is

$$\begin{aligned}
L =& \frac{a^2}{4(1+c_A)} - \frac{(1+c_B)(1+2c_A)^2a^2}{(3+4c_A+4c_B+4c_Ac_B)^2}\\
=& \frac{(16c_A^2c_B^2 + 32c_Ac_B^2 + 16c_B^2 - 16c_A^3c_B + 36c_Ac_B + 20c_B - 16c_A^3 - 16c_A^2 + 4c_A + 5)a^2}{4(1+c_A)(3+4c_A+4c_B+4c_Ac_B)^2}.
\end{aligned}$$

Denote the profit of Firm B and the license fee in this case by π_B^l and L^l.

4.2.3 Case C: entry with license

Suppose that Firm A enters the market and at the same time licenses its technology to Firm B at a fixed license fee. The inverse demand function is

$$p = a - x_A - x_B.$$

The profits of Firm A and B are

$$\pi_A = (a - x_A - x_B)x_A - c_Ax_A^2,$$

$$\pi_B = (a - x_A - x_B)x_B - c_Ax_B^2 - L.$$

The conditions for profit maximization of Firm A and B are

$$a - 2x_A - x_B - 2c_Ax_A = 0,$$

$$a - x_A - 2x_B - 2c_A x_B = 0.$$

The equilibrium outputs, price and profits are

$$x_A = \frac{a}{3+2c_A},\ x_B = \frac{a}{3+2c_A},\ p = \frac{(1+2c_A)a}{3+2c_A},$$

$$\pi_A = \frac{(1+c_A)^2 a^2}{(3+2c_A)^2},\ \pi_B = \frac{(1+c_A)^2 a^2}{(3+2c_A)^2} - L.$$

Similarly to the previous case if the negotiation between the innovating firm and the incumbent firm about the license fee breaks down, the innovating firm can enter the market without license. The incumbent firm must pay the difference between its profit excluding the license fee in this case and its profit in the entry without license case as a license fee. Thus, the license fee is

$$\begin{aligned}L =&\frac{(1+c_A)^2 a^2}{(3+2c_A)^2} - \frac{(1+c_B)(1+2c_A)^2 a^2}{(3+4c_A+4c_B+4c_Ac_B)^2}\\ =&\frac{(c_B-c_A)(16c_A^3c_B+48c_A^2c_B+48c_Ac_B+16c_B+16c_A^3+48c_A^2+48c_A+15)a^2}{(3+2c_A)^2(3+4c_A+4c_B+4c_Ac_B)^2}.\end{aligned}$$

The total profit of Firm A in this case is the sum of the license fee and its profit as a firm in the duopoly. It is

$$\frac{(1+c_A)^2 a^2}{(3+2c_A)^2} + L = \frac{A}{(3+2c_A)^2(3+4c_A+4c_B+4c_Ac_B)^2},$$

where

$$\begin{aligned}A =&(32c_A^3c_B^2+96c_A^2c_B^2+96c_Ac_B^2+32c_B^2-16c_A^4c_B+88c_A^2c_B+112c_Ac_B+39c_B\\ &-16c_A^4-32c_A^3-8c_A^2+18c_A+9)a^2.\end{aligned}$$

Denote the profits of Firm A and B, and the license fee in this case by π_A^{el}, π_B^{el} and L^{el}.

4.2.4 The optimal strategy for the innovator

Let us compare the profit of Firm A in Case C (entry with license) and its profit in Case B (license without entry). Then, we get

$$\begin{aligned}(\pi_A^{el}+L^{el}) - L^l =& \frac{A}{(3+2c_A)^2(3+4c_A+4c_B+4c_Ac_B)^2}\\ &-\frac{(16c_A^2c_B^2+32c_Ac_B^2+16c_B^2-16c_A^3c_B+36c_Ac_B+20c_B-16c_A^3-16c_A^2+4c_A+5)a^2}{4(1+c_A)(3+4c_A+4c_B+4c_Ac_B)^2}\\ =&\frac{(4c_A^2+4c_A-1)a^2}{4(1+c_A)(3+2c_A)^2},\end{aligned}$$

where A is the same as that in Case C. This is positive when $c_A > \frac{\sqrt{2}-1}{2} \approx 0.207$ irrespective of the values of a and c_B. Thus, so long as the innovation is not so drastic, in the quadratic cost functions case entry with license strategy is more profitable than license without entry strategy. However, if $c_A < \frac{\sqrt{2}-1}{2}$, license without entry strategy is more profitable than entry with license strategy. If $c_A = \frac{\sqrt{2}-1}{2}$, they are indifferent.

Let us compare the profit of Firm A in Case B and its profit in Case A (entry without license). Then, we have

$$\begin{aligned}
&L^l - \pi_A^e \\
&= \frac{(16c_A^2c_B^2 + 32c_Ac_B^2 + 16c_B^2 - 16c_A^3c_B + 36c_Ac_B + 20c_B - 16c_A^3 - 16c_A^2 + 4c_A + 5)a^2}{4(1 + c_A)(3 + 4c_A + 4c_B + 4c_Ac_B)^2} \\
&- \frac{(1 + c_A)(1 + 2c_B)^2a^2}{(3 + 4c_A + 4c_B + 4c_Ac_B)^2} \\
&= -\frac{(1 + 2c_A)(8c_A^2c_B + 4c_Ac_B - 4c_B + 8c_A^2 + 6c_A - 1)a^2}{4(1 + c_A)(3 + 4c_A + 4c_B + 4c_Ac_B)^2}.
\end{aligned}$$

This is positive for $0 < c_A \leq \frac{\sqrt{36c_B^2+52c_B+17}-2c_B-3}{8(1+c_B)}$. $\frac{\sqrt{36c_B^2+52c_B+17}-2c_B-3}{8(1+c_B)}$ is larger than $\frac{\sqrt{2}-1}{2}$ for $c_B > \frac{\sqrt{2}-1}{2}$.

Let us compare the profit of Firm A in Case C and its profit in Case A. Then,

$$\begin{aligned}
(\pi_A^{el} + L^{el}) - \pi_A^e = &\frac{A}{(3 + 2c_A)^2(3 + 4c_A + 4c_B + 4c_Ac_B)^2} - \frac{(1 + c_A)(1 + 2c_B)^2)a^2}{3 + 4c_A + 4c_B + 4c_Ac_B)^2} \\
&= \frac{(c_B - c_A)(16c_A^3c_B + 32c_A^2c_B + 12c_Ac_B - 4c_B + 16c_A^3 + 36c_A^2 + 24c_A + 3)a^2}{(3 + 2c_A)^2(3 + 4c_A + 4c_B + 4c_Ac_B)^2}.
\end{aligned}$$

This is positive except for very small value of c_A, and we can verify that it is positive for $\frac{\sqrt{2}-1}{2} \leq c_A < c_B$. We have $c_B - c_A > 0$ and $(3 + 2c_A)^2(3 + 4c_A + 4c_B + 4c_Ac_B)^2 > 0$. Let

$$\phi = 16c_A^3c_B + 32c_A^2c_B + 12c_Ac_B - 4c_B + 16c_A^3 + 36c_A^2 + 24c_A + 3.$$

Differentiating this with respect to c_A yields

$$\frac{d\phi}{dc_A} = 4(12c_A^2c_B + 16c_Ac_B + 3c_B + 12c_A^2 + 18c_A + 6) > 0.$$

We have

$$\phi_{c_A=\frac{\sqrt{2}-1}{2}} = 4(\sqrt{2} + 1) > 0.$$

Thus, ϕ is positive for $\frac{\sqrt{2}-1}{2} \leq c_A < c_B$.

Therefore, we obtain the following results.

Proposition 2. *In the case of quadratic cost functions so long as the innovation is not so drastic ($c_A > \frac{\sqrt{2}-1}{2}$), entry with license strategy is the optimal strategy for the innovating firm. However, if the innovation is very drastic ($c_A < \frac{\sqrt{2}-1}{2}$), license without entry strategy is the optimal strategy. If $c_A = \frac{\sqrt{2}-1}{2}$, both of them are the optimal strategies.*

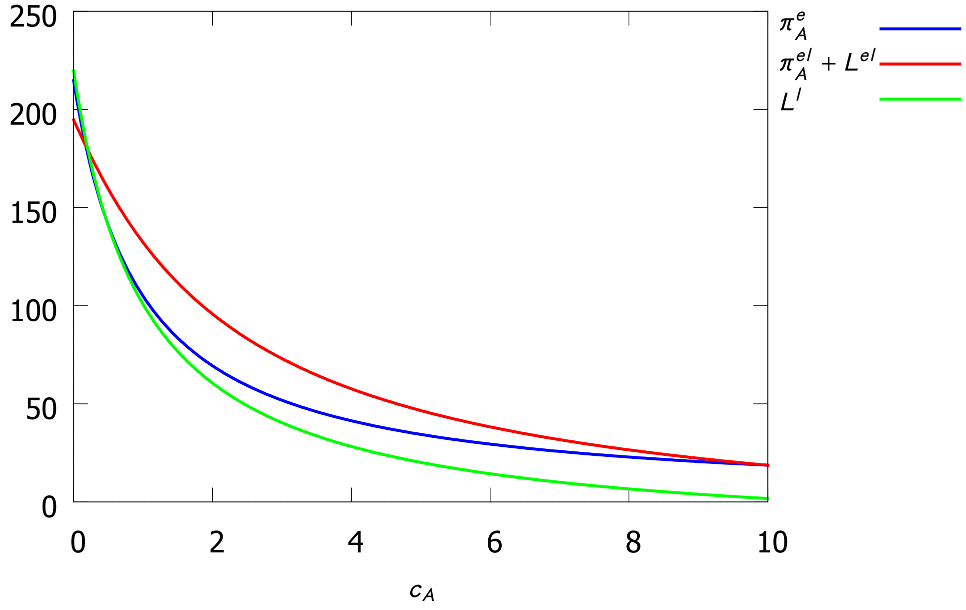

Figure 2: Example of the quadratic cost functions case 1

When the cost functions of the firms are quadratic, their marginal costs are increasing with respect to the outputs, and then large outputs are unprofitable than small outputs. Therefore, entry with license strategy is optimal for the innovating firm so long as the innovation is not so drastic. However, the innovation is very drastic, the marginal cost of a firm which adopts the new technology is small even if the cost functions are quadratic. Thus, in that case the optimal strategy for the innovating firm is license without entry strategy.

A note on the case where $c_B \leq \frac{\sqrt{2}-1}{2}$

When $c_B \leq \frac{\sqrt{2}-1}{2}$, we can verify $c_A < c_B \leq \frac{\sqrt{36c_B^2+52c_B+17}-2c_B-3}{8(1+c_B)}$. Then, since $L^l > \pi_A^e$ and $L^l > \pi_A^e + L^{el}$, license without entry strategy is optimal for Firm A.

Example

Assume $a = 30$, $c_B = 10$. The relationship among the profits of Firm A in three cases and the value of c_A are depicted in Fig. 2 and 3. Fig. 3 magnifies the part where the value of c_A is small.

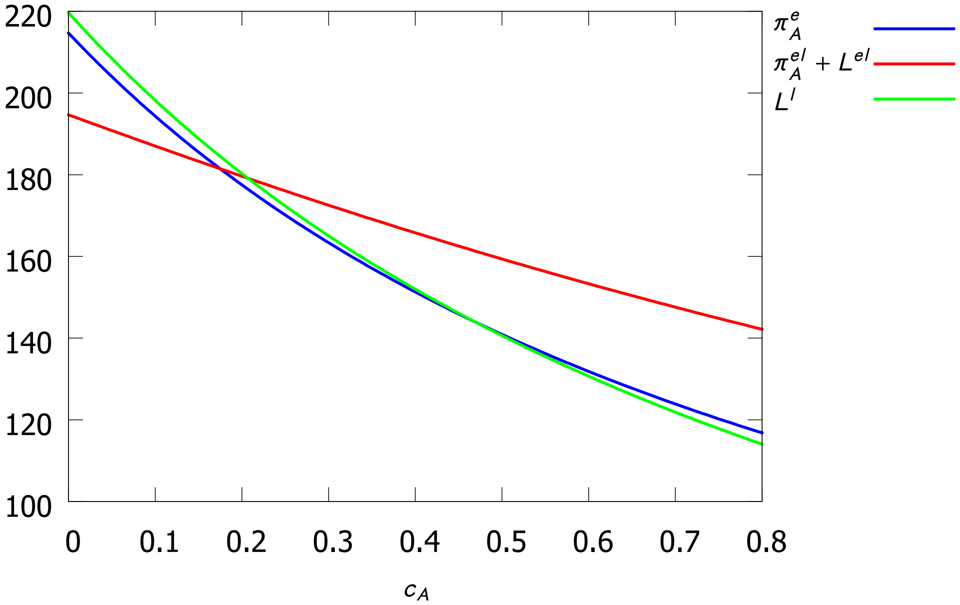

Figure 3: Example of the quadratic cost functions case 2

5 The Nash bargaining solution case

We consider the results when license fees are determined according to the Nash bargaining solution. The arguments hold for both linear and quadratic cost functions cases[1]. We use the following notations.

π_A^e, π_B^e:	profits of Firm A and B in Case A
φ_B^l:	profit of Firm B in Case B before paying the license fee
$\tilde{L}^l$, $\tilde{L}^{el}$:	license fees in Case B and Case C
$\pi_B^l (= \varphi_B^l - \tilde{L}^l)$:	profit of Firm B in Case B after paying the license fee
φ_B^{el}:	profit of Firm B in Case C before paying the license fee
$\pi_B^{el} (= \varphi_B^{el} - \tilde{L}^{el})$:	profit of Firm B in Case C after paying the license fee
π_A^{el}:	profit of Firm A in Case C

The values of them other than the license fees in this case are equal to those in the licensor takes all case. The payoff of Firm A in the license without entry case is $\tilde{L}^l$, and it is $\pi_A^{el} + \tilde{L}^{el}$ in the entry with license case.

The threat point of the negotiation is (π_A^e, π_B^e). In the license without entry case the problem of maximization of the Nash product is written as follows.

$$\max_{\tilde{L}^l} (\tilde{L}^l - \pi_A^e)(\pi_B^l - \pi_B^e) \text{ subject to } \tilde{L}^l + \pi_B^l = \varphi_B^l.$$

The condition for the solution is

$$\tilde{L}^l - \pi_A^e = \pi_B^l - \pi_B^e.$$

[1]Watanabe and Muto (2008) also analyzed bargaining among licensor and licensees. However, they used the concepts of core and bargaining set, and did not used Nash bargaining solution.

We get

$$\tilde{L}^l = \frac{\varphi_B^l + \pi_A^e - \pi_B^e}{2}.$$

In the entry with license case the problem of maximization of the Nash product is written as follows.

$$\max_{\tilde{L}^{el}} (\pi_A^{el} + \tilde{L}^{el} - \pi_A^e)(\pi_B^{el} - \pi_B^e) \text{ subject to } \tilde{L}^{el} + \pi_B^{el} = \varphi_B^{el}.$$

The condition for the solution is

$$\pi_A^{el} + \tilde{L}^{el} - \pi_A^e = \pi_B^{el} - \pi_B^e.$$

We get

$$\tilde{L}^{el} = \frac{\varphi_B^{el} - \pi_B^e + \pi_A^e - \pi_A^{el}}{2}.$$

and

$$\pi_A^{el} + \tilde{L}^{el} = \frac{\varphi_B^{el} - \pi_B^e + \pi_A^e + \pi_A^{el}}{2}.$$

1. Comparison between the payoff of Firm A in Case B and its payoff in Case A yields

$$\tilde{L}^l - \pi_A^e = \frac{\varphi_B^l - \pi_A^e - \pi_B^e}{2}. \tag{1}$$

2. Comparison between the payoff of Firm A in Case C and its payoff in Case A yields

$$\pi_A^{el} + \tilde{L}^{el} - \pi_A^e = \frac{\varphi_B^{el} - \pi_B^e + \pi_A^{el} - \pi_A^e}{2}. \tag{2}$$

3. Comparison between the payoff of Firm A in Case C and its payoff in Case B yields

$$\pi_A^{el} + \tilde{L}^{el} - \tilde{L}^l = \frac{\varphi_B^{el} - \pi_B^e + \pi_A^e + \pi_A^{el}}{2} - \frac{\varphi_B^l + \pi_A^e - \pi_B^e}{2} = \frac{\varphi_B^{el} + \pi_A^{el} - \varphi_B^l}{2}. \tag{3}$$

On the other hand, since $L^l = \varphi_B^l - \pi_B^e$ in the licensor takes all case, we obtain

$$L^l - \pi_A^e = \varphi_B^l - \pi_A^e - \pi_B^e.$$

It is exactly twice of (1). Since $L^{el} = \varphi_B^{el} - \pi_B^e$,

$$\pi_A^{el} + L^{el} - \pi_A^e = \varphi_B^{el} - \pi_B^e + \pi_A^{el} - \pi_A^e.$$

It is exactly twice of (2). And we obtain

$$\pi_A^{el} + L^{el} - L^l = \pi_A^{el} + \varphi_B^{el} - \pi_B^e - \varphi_B^l + \pi_B^e = \pi_A^{el} + \varphi_B^{el} - \varphi_B^l.$$

It is exactly twice of (3). Therefore, we get the following result.

Proposition 3. *The determination of the optimal strategy for the innovating firm in the Nash bargaining solution case is the same as that in the licensor takes all case, and Proposition 1 and 2 are robust for determination of license fees according to the Nash bargaining solution.*

Now we compare the license fees in the Nash bargaining solution case and those in the licensor takes all case. Since $L^l - \pi_A^e = 2(\tilde{L}^l - \pi_A^e)$, we have

$$L^l - \tilde{L}^l = \tilde{L}^l - \pi_A^e.$$

Therefore, if $L^l > \pi_A^e$ and $\tilde{L}^l > \pi_A^e$, L^l is larger than $\tilde{L}^l$, and if $L^l < \pi_A^e$ and $\tilde{L}^l < \pi_A^e$, L^l is smaller than $\tilde{L}^l$.

Also, since $\pi_A^{el} + L^{el} - \pi_A^e = 2(\pi_A^{el} + \tilde{L}^{el} - \pi_A^e)$, we have

$$L^{el} - \tilde{L}^{el} = \pi_A^{el} + \tilde{L}^{el} - \pi_A^e.$$

Therefore, if $\pi_A^{el} + L^{el} > \pi_A^e$ and $\pi_A^{el} + \tilde{L}^{el} > \pi_A^e$, L^{el} is larger than $\tilde{L}^{el}$, and if $\pi_A^{el} + L^{el} < \pi_A^e$ and $\pi_A^{el} + \tilde{L}^{el} < \pi_A^e$, L^{el} is smaller than $\tilde{L}^{el}$.

We have shown the following results.

Proposition 4.

1. *If license without entry strategy is more (or less) profitable for the innovating firm than entry without license strategy, the license fee in Case B in the licensor takes all case is larger (or smaller) than the license fee in the Nash bargaining solution case.*

2. *If entry with license strategy is more (or less) profitable for the innovating firm than entry without license strategy, the license fee in Case C in the licensor takes all case is larger (or smaller) than the license fee in the Nash bargaining solution case.*

6 Concluding Remarks

We have presented an alternative definition of license fee for new technology developed by an outside innovator under duopoly when the outside innovator can enter the market with or without license, replacing the definition by Kamien and Tauman (1986). Our definition of license fee is based on the threat to the incumbent firm by entry of the innovating firm when the incumbent firm does not buy a license in the case of license without entry. We have shown that in the linear demand and cost functions case although the innovator's optimal strategy according to the definition by Kamien and Tauman (1986) is entry with license strategy, it is license without entry strategy according to the alternative definition. Also we have presented results when the cost functions of the firms are quadratic. In that case entry with license strategy may be optimal. Further we have shown that the optimal strategy for the innovator when the license fees are determined under the assumption that the licensor takes all benefit of new technology and its optimal strategy when the license fees are determined according to the Nash bargaining solution are the same.

In the future research we want to study the problem under oligopoly, that is, the case with more than one incumbent firms, and government's policy to promote or prevent license or entry by the innovating firm. Under oligopoly, credibility of threats is more subtle problem than under duopoly.

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