Cost-benefit analysis of transport improvements in the presence of spillovers, matching and an income tax

Jonas Eliasson and Mogens Fosgerau

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Jonas Eliasson, KTH Royal Institute of Technology, Sweden, jonas.eliasson@abe.kth.se
Mogens Fosgerau, Technical University of Denmark, mfos@dtu.dk

Abstract

This paper addresses the problem of measuring the welfare benefits of a transport improvement. We formulate and analyze a rich spatial model that allows for spillovers, matching and income tax, in a setting with multiple work and residential locations and very general worker heterogeneity. The conventional consumer surplus captures part of the benefits and is calculated based on predictions of changes in travel demand and transport costs. The issue is to determine which so-called wider impacts to add to this. We find that adding the change in total output as a wider impact leads to double-counting of benefits. The output change due to spillovers should be added, while the output change due to matching is already partly included in the consumer surplus. These results are useful for applied cost-benefit analysis of transport policies.

JEL: R4, D6, H4

Keywords: Agglomeration; spillovers; matching; cost-benefit analysis; transport policy

1. Introduction

1.1 Motivation

Conventional cost-benefit analysis of transport projects relies on the consumer surplus on transport markets to capture all the benefits of transport improvements. This approach is valid provided there are no imperfections on secondary markets (Jara-Díaz, 1986; Kidokoro, 2004, 2006; Mohring, 1993). It is also an extremely applicable approach as the analysis can be based simply on a traffic forecast with no need for complicated economic modeling. The practicality of this approach is a main reason that transport policy, in contrast to many other policy areas, is routinely subjected to economic evaluation around the world.

There is, however, a growing realization that secondary market imperfections may have significant impacts that are relevant for the economic evaluation of transport projects. In the last decade or two, the importance of labor and product market imperfections has been increasingly appreciated both among researchers and practitioners. The UK SACTRA report (SACTRA, 1999) was a milestone in this development, as was the subsequent high-profile case study of the London Crossrail project (Worsley, 2011). The term “wider impacts” (sometimes “wider economic benefits”) arose as part of the UK work and refers to the welfare effects of a transport improvement that are additional to the change in the consumer surplus. The issue of wider impacts is then how to modify a conventional cost-benefit analysis to take into account secondary market imperfections.

In an influential paper, Venables (2007) set up a classical monocentric city model where productivity in the CBD depends on the total employment there. The city is open such that a transport cost reduction increases urban employment. This leads to the first wider impact, which is increased tax revenues arising since workers receive higher wages in the city than outside. The second wider impact is that the increased employment in the CBD increases productivity through agglomeration effects. The result by Venables that the entire increase in production should be added to the conventional user benefits made its way into the Crossrail
study, and subsequently into the influential UK Transport Appraisal Guidelines ("WebTag") (Department for Transport, 2014).

Agglomeration effects are fundamental drivers behind urbanization and economic growth. The degree of agglomeration at a location may be measured through the concept of accessibility, used to summarize the spatial availability of opportunities while taking transport costs into account. The mechanisms through which accessibility can increase productivity are summarized by Duranton and Puga (2004) in the phrase “sharing, matching and learning”. In brief, sharing refers to the sharing of specialized inputs, matching refers to the matching of workers to employment opportunities, while learning refers to the process where workers and firms learn from each other. All three mechanisms are facilitated by agglomeration, but our understanding of the relative importance of the three mechanisms is limited (Melo & Graham, 2014; Puga, 2010).

From the transportation perspective there is an important distinction between sharing and learning on the one hand and matching on the other. If a transport improvement leads to better accessibility of workers to jobs and therefore to better matches with higher wages, the after-tax part of this effect is internal to the workers’ choice of job and will then be reflected in the consumer surplus in the transport market. In contrast, the effects of improved accessibility through sharing and learning are generally external to the workers’ commute decisions. Matching effects must therefore be treated differently from sharing and learning effects in a transport cost-benefit analysis.

In Venables (2007), all employment is located in the CBD and agglomeration is measured simply as total employment in the city. There is then no way for transport improvements to affect the degree of agglomeration except through the size of the city. The dependency of productivity on total employment may be thought of as describing sharing and learning effects. We refer in this paper to the sum of these effects as spillovers. Matching is, however, not captured in Venables’ setup with only one work location. The Venables model is therefore also unable to predict, e.g., situations where transport improvements attract workers to low productivity locations and thereby cause negative wider impacts.

The aim of the current paper is similar to that of Venables (2007), but we use a setup that allows for matching as well as a description of accessibility that incorporates transport costs. We consider a finite number of work and residential locations in an arbitrary spatial arrangement, which may be taken to represent a city or indeed a whole country. The residential locations of workers is fixed while the worker choice of workplace is endogenous, described by a general random utility model. A worker’s choice of workplace is influenced by transport costs as well as the wage, net of income tax, at each work location. The wage at each work location is in turn the product of a productivity that is specific to each combination of workers and jobs and a local wage rate (per productivity unit) at each job that depends on the accessibility from that workplace to workers at other work locations. The accessibility is the sum of employment at each work location weighted by a decreasing function of job-to-job transport cost (reflecting the impedance of information and interaction between workers). In this way, accessibility and hence the local wage rate (per productivity unit) reflects the spatial distribution of workplaces as well as transport costs.

An additional feature of the present model is that it allows unemployment to exist and to be endogenous. The model is sufficiently general that one work location can be interpreted as unemployment. The local productivity would be zero such that unemployment yields zero wage while commuting costs to unemployment would be zero.

We find two sources of wider impacts of a transport improvement in this model. As in Venables’ model, the change in total output is important but we must decompose this change into two parts: the change in local wage rates holding local employment constant, and the change in local employment holding local wage rates (per productivity unit) constant.
The first part of the change in output, the change in local wage rates, is due to changes in job-to-job accessibility and may be thought of as representing spillovers, i.e. sharing and learning effects. All of this should be counted as a wider impact, part of which accrues to workers and part of which is tax revenue.

The second part directly reflects the workers’ choice of work locations and it is then a matching effect. The tax share of this is tax revenue and should be counted as a wider impact. The remainder is part of the consumer surplus and should therefore not be counted as a wider impact.

This contrasts directly with Venables (2007), who counts all of the increase in total output as a wider impact. This issue is important in applied transport policy analysis. Several countries, and notably the UK, have amended their CBA guidelines to include wider economic impacts, including the entire increase in production as a wider benefit that is added to the change in consumer surplus. The present results show that this practice entails double counting of benefits when the increase in total wages is partly due to matching.

The wider impacts in the present model may be positive or negative, depending on whether the transport cost reduction induces workers to shift employment toward high- or low-productivity locations. Again, this is an effect that cannot be represented in the monocentric city model used by Venables.

In return for allowing multiple work locations and thereby a matching effect, we have to assume that residential locations are fixed. We cannot then obtain the effect present in Venables (2007) whereby an urban transport improvement attracts workers from elsewhere. However, as we have a very general representation of space, we can take our model to represent a whole country, which makes the issue of variable population size fairly moot. Still, our assumption of fixed residential location is a constraint that could be lifted in future work.

1.2 Previous literature

The positive relationship between productivity and city size (an easily available proxy for accessibility) was pointed out already by Smith (1776) and Marshall (1890). Starrett’s impossibility theorem (Starrett, 1978; Ottaviano & Thisse, 2004) showed that agglomeration effects are necessary to explain the existence of large cities. A large number of studies have confirmed the correlation between high accessibility and high productivity (Rosenthal & Strange, 2004). Early studies used city size (population or number of jobs) as a proxy for accessibility, while more recent studies have used measures of economic density (Graham, 2007a; Graham & van Dender, 2011) or accessibility measures derived from transport models (Anderstig, Berglund, Eliasson, & Andersson, 2016; Norman, Börjesson, & Anderstig, 2017).

Recent studies have also shown that not all of the observed correlation between accessibility and productivity is causal; sorting and self-selection among workers and firms also play a role (Combes, Duranton, & Gobillon, 2008; Graham, Melo, Jiwattanakulpaisarn, & Noland, 2010). Still, effects of accessibility on productivity persist even after controlling for sorting (Börjesson, Isacsson, Andersson, & Anderstig, 2015; Maré & Graham, 2013). Of particular interest is a recent and careful study by Melo and Graham (2014), showing that matching effects are indeed an important part of agglomeration effects, even after controlling for endogeneity and selection.

Kanemoto (2013a) extends the work of Venables (2007) by considering imperfect competition, examining whether the results remain valid when monopolistic competition with differentiated products provides the microfoundation of agglomeration economies. Kanemoto (2013b) summarizes and compares earlier results for such CBA rules, i.e. situations with price distortions associated with imperfect competition. These models do not include worker/firm matching as a source of agglomeration benefits, however.

Calthrop et al. (2010) consider the implications of tax distortions and interactions with the labor market for transport project cost-benefit analysis, while Fosgerau and Pilegaard (2008) consider
the implications of search unemployment. Search unemployment is related to worker-to-job accessibility, which means there is some conceptual overlap with the matching effect considered here.

1.3 Layout

Section 2 presents our theoretical model for which we are able to derive the welfare consequences of a transport improvement in terms of consumer surplus, a spillover effect and a matching effect. Section 3 presents some additional analytical results for a simplified version of the model where spillovers are purely local. Here we establish conditions that ensure unique existence of equilibrium, a result that informs about when a transport improvement can be expected to increase or decrease total output, and conditions that ensure that an income tax increase will reduce total output. Section 4 illustrates the model with a series of simulations. Finally, Section 5 concludes.

2. Model formulation and basic result

2.1 Model formulation

Workers are divided into a finite number of types \(i = 1, \ldots, I\). The types distinguish workers by residential location as well as other characteristics such as education and experience. There are \(N_i\) workers of type \(i\), treated as a continuum, and the total population \(N = \sum_i N_i > 1\). The type of each worker, including the residential location, is fixed, which means that the model does not allow workers to move residence.

There is also a finite number of job types \(j = 1, \ldots, J\). The job types distinguish jobs by location and by other characteristics such as industry and specific job characteristics. The generalized commuting cost for type \(i\) workers to type \(j\) jobs is non-negative and denoted by \(c_{ij}\). This allows workers of different types to have different commuting costs, even if they have the same residential location. Residential and work locations with the same physical location could have commuting cost of zero.

A productivity level \(q_{ij} \geq 0\) is associated to each worker-job match. There is an endogenous wage rate per productivity unit \(\gamma_j > 0\) at each location, taken as given by workers, such that a worker \(i\) in job \(j\) receives a gross wage of \(\gamma_j q_{ij}\). This income is taxed at the exogenous rate \(\tau\). Finally, a worker \(i\) in job \(j\) receives an idiosyncratic money-metric utility shock \(\epsilon_{ij}\). For each type \(i\), the vector with elements \(\epsilon_{ij}, j = 1, \ldots, J\) follows an absolutely continuous multivariate distribution with finite mean and a density that is everywhere positive. These shocks need not have mean zero, such that the model may accommodate that not all types of workers are equally suited for all types of jobs.

A worker \(i\) working at job \(j\) then obtains random utility

\[
u_{ij} = (1 - \tau)\gamma_j q_{ij} - c_{ij} + \epsilon_{ij}.
\]

He/she chooses job type to maximize this random utility and then chooses job \(j\) with probability

\[
P_{ij} = P\left(u_{ij} = \max_j u_{ij'}\right).
\]

This means that the model comprises a matching effect, whereby it is more worthwhile for high productivity workers to incur higher transport costs in order to reach a work location with higher wage rate. The model then comprises a mechanism that induces high productivity workers to commute longer, ceteris paribus, in accordance with European empirical evidence (Carra, Mulalic, Fosgerau, & Barthelemy, 2016).
The equations (1) and (2) describe a standard additive random utility model (McFadden, 1981). We shall make use of a few general results for such models; they can be found in (Fosgerau, McFadden, & Bierlaire, 2013).

Denote $G(m; i) = E \max_j \{m_j + u_{ij} \}$. This is the expected maximum utility for workers of type $i$, viewed as a function of location shifts $m = (m_1, ..., m_J)$, which serve to facilitate differentiation of $G$; the location shift vector is zero in the model and we shall suppress it in the notation, writing just $G(i)$. As proved by Fosgerau et al. (2013), $G$ is a convex function of $m$ with partial derivatives at $m = 0$ satisfying $G_{ij}(i) = P_{ij}$ and the hessian of $G(i)$ is positive semidefinite. Moreover, the second partial derivatives satisfy

$$
G_{jj}(i) \geq 0 \\
G_{jk}(i) \leq 0, j \neq k \\
G_{jj}(i) = - \sum_{k \neq j} G_{jk}(i).
$$

(3)

We assume that $G_{jj}(i)$ is bounded by a constant. This condition is satisfied by the multinomial logit model, where $G_{jj}(i) = P_{ij}(1 - P_{ij}) \leq 1/4$. The condition bounds how quickly the work location choice probabilities can change.

Note that we allow considerable generality by allowing the distribution of the random utility components $e_{ij}$ to vary by worker type and by not restricting the distributions in any way except for the mild regularity condition just stated.

The number of commuters of type $i$ to $j$ is

$$D_{ij} = N_i P_{ij}(q)$$

(4)

and the effective labor supply at job $j$ is

$$L_j = \sum_i q_{ij} D_{ij}.$$  

(5)

Let $d_{jk}$ be the elements of a matrix of non-negative transport costs from job $j$ to job $k$. The commuting costs $e_{ij}$ and the job-to-job transport costs $d_{jk}$ are different in general but may be derived from the same underlying transport network, such that a change in the transport network has impact on both kinds of transport costs.\(^1\) We also define a decay function $w$ that is decreasing and differentiable with $w(0) = 1$ and $w(\infty) = 0$. Then we define the job-to-job accessibility at job $j$ as a weighted sum of local effective labor supply where the weights are inversely related to the job-to-job transport cost through the decay function:

$$A_j = \sum_k w(d_{jk}) L_k.$$  

(6)

The job-to-job accessibility captures the effect of spillovers (sharing and learning) on the local wage rate (per productivity unit), which is given by

$$y_j = A_j^{\gamma_j},$$

(7)

\(^1\) It should be pointed out, however, that while decades of research have provided a good understanding of how generalized commuting costs depend on monetary costs, travel time components and several other factors, much less is known about job-to-job transport costs. In our model, they are interpreted as the impedance of contact, interaction and information between workers at different job locations. Understanding commuting costs is comparatively easy since commuting patterns are observable, allowing researchers to study how generalized commuting costs are made up of monetary costs, travel time components and so on. Understanding, for example, the impedance of information flow or worker interactions is clearly much harder, since the spillovers that we model here are not observable in the same simple way.
where $\eta_j \geq 0$ is a local parameter that determines the returns to job-to-job accessibility at location $j$. When $\eta_j = 0$, the local wage rate is constant and equal to 1.

Production takes place with labor as the only input and the value of production is returned to workers. Then the output from job $j$ is

$$Y_j = Y_j L_j.$$  \hfill (8)

The total production from all jobs is denoted $Y = \sum_j Y_j$ and then the total revenue from the income tax is $\tau Y$.

Welfare is measured as the expected utility of all workers plus the income tax revenue.

$$W = \sum_i N_i G(i) + \tau Y.$$  \hfill (9)

The model may seem restrictive in that it appears to require all workers to actually work. But the model can in fact accommodate some workers being unemployed. This can be achieved by designating, say, job type 0 as unemployment and setting $q_{i0} = 0$. The transport costs to job 0 can be set to zero or to some negative transport cost that can represent an unemployment benefit.

2.2 The welfare consequences of a transport cost reduction

Transport costs enter the model both as commuting costs $c_{ij}$ and as job-to-job transport costs $d_{jk}$. A change to the transport network will affect both kinds of transport costs. It is however convenient to consider them separately. We begin with the commuting costs, considering without loss of generality a reduction in just one commuting cost.

Theorem 1. The welfare effect of a commuting cost reduction is

$$-\frac{\partial W}{\partial c_{11}} = D_{11} - \tau \frac{\partial Y}{\partial c_{11}} - (1 - \tau) \sum_j \frac{\partial y_j}{\partial c_{11}} L_j =$$

$$= D_{11} - \sum_j \frac{\partial y_j}{\partial c_{11}} L_j - \tau \sum_j y_j \frac{\partial L_j}{\partial c_{11}}.$$  \hfill (10)

The first row decomposes the welfare effect into consumer surplus, increased tax revenues, and the after-tax part of increased wage rates due to spillovers. The second row decomposes the welfare effect into consumer surplus, increased wage rates due to spillovers, and increased tax revenues from matching effects.

Proof.

$$-\frac{\partial W}{\partial c_{11}} = -\sum_i N_i \frac{\partial G(i)}{\partial c_{11}} - \tau \frac{\partial Y}{\partial c_{11}} = [\text{use } G(i) = p_{ij}] =$$

$$= -\sum_{ij} N_i p_{ij} \frac{\partial p_{ij}}{\partial c_{11}} - \tau \frac{\partial Y}{\partial c_{11}} =$$

$$= N_i p_{ij} - \sum_{ij} N_i p_{ij} q_{ij} (1 - \tau) \frac{\partial y_j}{\partial c_{11}} - \tau \frac{\partial Y}{\partial c_{11}} =$$

$$= D_{11} - (1 - \tau) \sum_j \frac{\partial y_j}{\partial c_{11}} L_j - \tau \frac{\partial Y}{\partial c_{11}} =$$

$$= D_{11} - (1 - \tau) \sum_j \frac{\partial y_j}{\partial c_{11}} L_j - \tau \sum_j y_j \frac{\partial L_j}{\partial c_{11}} =$$

$$= D_{11} - \sum_j \frac{\partial y_j}{\partial c_{11}} L_j - \tau \sum_j y_j \frac{\partial L_j}{\partial c_{11}}.$$  \hfill (11)
This is a very intuitive result. Let us talk about the representation in the first row of (10) first. The first term is the marginal change in the consumer surplus and comprises the direct effect on consumers of reduced transport costs and changed work location choices. The second term is the increase in tax revenues. The third term is the wage increase, net of taxes, that follows the commuting cost reduction; this component accrues to workers but is external to their commute decision as each individual worker has no impact on wages.

The second row of (10) is also informative as it splits the welfare effect into the direct effect, spillovers and matching. The first term is still the marginal change in the consumer surplus. The second term is the benefit due to changes in wage rates (per productivity unit), holding work location choices constant, such that the changes act only through changes in job-to-job accessibility; the second term thus captures the effect of spillovers. The third term is the change in tax revenues due to changed job choices. It then captures the part of the matching effect that is external to the workers’ commuting decision. The net-of-tax effect on wages of changed job choices is already captured in the consumer surplus.

As has been noted, \( \gamma_j \) is constant and equal to 1 if \( \eta_j = 0 \). If all \( \eta_j = 0 \), then the spillover effect is zero and only the matching effect remains. If \( \eta_j > 0 \), the spillover effect for each job type \( j \) is equal to the relative change in accessibility multiplied by the output and the spillover parameter for that zone: \( \frac{\partial \gamma_j}{\partial c_{11}} L_j = \eta_j \frac{\partial A_j}{\partial c_{11}} \).

The effect of a change in a job-to-job transport cost is slightly different, since these affect workers’ job choices only through the wage rates, which are taken as given by workers. Without loss of generality, we consider a change in job-to-job transport cost \( d_{11} \).

**Theorem 2.** The welfare effect of a job-to-job transport cost reduction is

\[
- \frac{\partial W}{\partial d_{11}} = \tau \frac{\partial Y}{\partial d_{11}} - (1 - \tau) \sum_j \frac{\partial \gamma_j}{\partial d_{11}} L_j = - \sum_j \frac{\partial \gamma_j}{\partial d_{11}} L_j - \tau \sum_j \frac{\partial L_j}{\partial d_{11}} \gamma_j \]  

(12)

The first row decomposes the welfare effect into increased tax revenues and the after-tax part of increased wage rates due to spillovers. The second row decomposes the welfare effect into increased wage rates due to spillovers and increased tax revenues from matching effects.

**Proof.**

Similar to proof of Theorem 1.

In the first row of (10), the first term is the total increase in tax revenues, and the second term is the after-tax part of wage rate changes, which accrue to workers. In the second row, the first term is the spillover effect, capturing benefits to workers of wage rate changes through changes in job-to-job accessibility. Again, this is zero if \( \eta_j \) are equal to zero such that there is no spillover effect. The second term is the matching effect, capturing the change in tax revenues due to changed job choices holding wage rates constant.

A marginal change in the transport network will lead to changes in both commuting costs \( c_{ij} \) and job-to-job transport cost \( d_{jk} \) and the effects must then be added to obtain the full welfare impact.

In conventional transport CBA, only the consumer surplus from a transport improvement is included. Theorems 1 and 2 show that it is always correct to add the change in total tax revenues.
revenues, regardless of the source or mechanism generating the increase in tax revenues. They may be sourced either from reduced commuting costs or from reduced job-to-job transport costs, or both; they may be generated by spillovers or matching or both; either way, the entire increase in tax revenues should be added to the CBA. For the after-tax part of an output increase, however, the analyst needs to distinguish between output effects generated by spillovers and matching. Only the former should be added to the CBA, since the after-tax benefits from matching is already included in the consumer surplus. However, few (if any) empirical estimates of the relationship between accessibility and economic output make this distinction: the contributions from matching and spillovers are confounded.

Section 4 below presents some simulation results that provide an impression of the magnitude of the effects involved. The simulation also illustrates the confounding of spillovers and matching in the output/accessibility relationships that can be observed at aggregate levels. Before that, we provide some analytical results for a simplified version of the model.

3. Analysis of simplified model

The representation of the spillover effect through job-to-job accessibility makes the derivation of analytical results a quite daunting task that involves much complex and not very transparent mathematical notation. We therefore undertake some analysis of the model under the simplification that \( w(0) = 1 \) and \( w(d) = 0 \) for \( d > 0 \), such that \( A_j = L_j \) and \( Y = \sum_j L_j^{1+\eta_j} \). Under this simplification, only jobs of type \( j \) contribute spillover effects to jobs of type \( j \) and job-to-job transport costs play no role. It is intuitive to think of \( j \) as a specific industry cluster. We can think of no essential reason why the results that we will derive for the simplified model should not also apply to the full model.

3.1 Existence and uniqueness of equilibrium

When all \( \eta_j = 0 \), the existence and uniqueness of equilibrium is trivial since wage rates are constant in this case. This is no longer the case when \( \eta_j > 0 \), since then the wage rate at job \( j \) is determined by the number of workers there, while the number of workers at each job is determined by the set of wage rates. Equilibrium still exists as shown in the following theorem, but may not be unique. It is easy to construct an example where there are multiple equilibria: if there are just two ex ante identical work locations, then equilibrium employment may concentrate in either location or be evenly split among them. The following theorem shows however that equilibrium is necessarily unique provided \( \eta_j \) are sufficiently small.

**Theorem 3.** Equilibrium always exists. Equilibrium exists uniquely when \( \eta_j \) are sufficiently small.

**Proof.** Consider the mapping \( \Gamma \), which takes a wage rate vector \( \gamma = (\gamma_1, ..., \gamma_f) \) into a new wage rate vector \( \Gamma(\gamma) \), with components given by \( \Gamma^{(j)} = \gamma_j = L_j^{\eta_j} \) where \( L_j = \sum_i q_{ij} G_j^{(i)} N_i \) is regarded as a function of vector \( \gamma \).

A fixed point for \( \Gamma \) is an equilibrium in the model. We will show that a fixed point always exists and that it is unique when \( \eta_j \) are small by applying the Schauder and Banach fixed point theorems.

First, we shall compute the Jacobian of \( \Gamma \): it has elements

\[
\Gamma_{ik}^{(j)}(\gamma) = \eta_j L_j^{\eta_j-1} \left( 1 - \tau \right) \sum_i q_{ij} q_{ik} G_{jk}^{(i)} N_i.
\]  

(13)

Then \( \Gamma \) is continuously differentiable. Its domain is a compact convex set, since the wage rate at any job is bounded by the wage rate that would result if the whole population worked at that location. Hence the domain also contains the range of \( \Gamma \). This is sufficient for the Schauder fixed point theorem which establishes that an equilibrium exists.
Using (3),

$$
\sum_k |\Gamma_k^{ij}(\gamma)| = \eta_j(1 - \tau)\gamma_j L_j^{-1} \sum_i q_{ij} q_{ik} 2G_{ij}(0)N_i. \tag{14}
$$

By the regularity conditions imposed on \( G \) and \( f \), the sum on the RHS in (14) is uniformly bounded. We then only need to consider \( \gamma_j L_j^{-1} \). But \( \gamma_j \leq N_j^{max} \), where \( N = \sum_i N_i > 1 \) is the total population in the model. Finally, we need a lower bound for \( L_j \): this exists since \( P_{ij} > 0 \) everywhere and transport costs and wages are bounded. Combining these bounds, we find that \( \sum_k |\Gamma_k^{ij}(\gamma)| \leq \eta_j K \) for some \( K > 0 \). Hence the norm of the Jacobian of \( \Gamma \) is smaller than 1 when \( \eta_j \) are sufficiently small. Then the Banach fixed point theorem applies and this completes the proof. \( \blacksquare \)

### 3.2 The production consequences of a transport improvement

It is important to understand when a transport improvement can be expected to increase total output. As we shall see, it can even be the case that a transport improvement will decrease total output. Moreover, a change in output translates directly into the welfare effects in (10) and (12) since we may decompose an output change into spillover and matching effects by

$$
\partial \gamma \partial c_{ij} = \sum_j \gamma_j L_j + \sum_j \gamma_j \frac{L_j}{c_{ij}}, \tag{15}
$$

with a similar decomposition applying for a change in job-to-job transport costs.

It is therefore important to understand when a transport improvement will increase or decrease total output. In general, we expect a transport improvement affecting high-productivity jobs to lead to an increase in total production and conversely for a transport improvement affecting low-productivity jobs. This phenomenon requires the matching mechanism, since if local effective labor supplies \( L_j \) are held constant then a transport cost reduction can only increase job-to-job accessibility and thereby increase local wage rates.

In order to show that a transport improvement may decrease total output, we consider a simplified version of the model where worker-job specific productivities factor into worker and job-specific productivity effects \( q_{ij} = q_i a_j \) and where the strength of the spillover effect is the same for all jobs, i.e. \( \eta_j = \eta \). The separability of \( q_{ij} \) implies that

$$
\sum_j \frac{L_j}{a_j} = \sum_i q_i P_{ij} N_i = \sum_i q_i N_i,
$$

which is constant, so this implies

$$
a_1^{-1} \frac{\partial L_1}{\partial c_{11}} = - \sum_{j>1} a_1^{-j} \frac{\partial L_j}{\partial c_{11}}, \tag{16}
$$

which can be used to rewrite the effect on total output of a change in commuting cost \( c_{11} \):

$$
\frac{\partial \gamma}{\partial c_{11}} = \frac{\partial}{\partial c_{11}} \sum_j L_j^{1+\eta} = (1 + \eta) \sum_j L_j^n a_j a_j^{-1} \frac{\partial L_j}{\partial c_{11}} = [\text{use (15)}]
$$

$$
= (1 + \eta) \sum_{j>1} (\gamma_j a_j - \gamma_1 a_1) \frac{\partial L_j}{\partial c_{11}}. \tag{17}
$$

As to the signs of the derivatives of \( L_j \), we have \( \frac{\partial L_j}{\partial c_{11}} > 0 \) and \( \frac{\partial L_j}{\partial c_{11}} < 0 \) for all \( j > 1 \). This is because the first-order effect of a reduction in \( c_{11} \) is an increase in \( D_{11} \), which increases \( L_1 \) and reduces all other \( L_j \), which in turn causes the wage rate to increase (weakly) at work location 1 and to decrease (weakly) at all other locations. This effect is amplified, since the change in wage rates
increases commuting to location 1 and decreases commuting to all other locations. The sign of (17) is still ambiguous in general, but can be signed when work location 1 has the smallest or largest productivity. When $\gamma_1 a_1 < \gamma_j a_j, \forall j > 1$, then $-\frac{\partial Y}{\partial c_{11}} < 0$ and conversely when $\gamma_1 a_1 > \gamma_j a_j, \forall j > 1$. This implies that a transport cost reduction for commuting to the least productive work location will decrease total production, while a transport cost reduction for commuting to most productive work location will increase total production.

In the general case, without the simplifying assumptions made above in this subsection, we expect the same result to hold except in extreme cases where many workers are each more productive in jobs with low wage rates. In general, a transport cost reduction may then be expected to increase (decrease) total production if it improves commuter accessibility to the more (less) productive work locations.

3.3 The production consequences of a change in the income tax rate

The next theorem establishes conditions that ensure the intuitive result that an increase in the income tax rate will cause a decrease in production.

**Theorem 4.** If all $\eta_j = 0$ then $\frac{\partial Y}{\partial \tau} \leq 0$. If the Hessian of each $G(i)$ is positive definite, then $\frac{\partial Y}{\partial \tau} < 0$ for $\eta_j$ sufficiently close to zero.

**Proof.** We begin with the effect on the wage rate at each work location, using $Y_j = \frac{\eta_j}{1 + \eta_j L_j^{-1}}$ to express that in terms of the effect on the output at location $j$:

$$\frac{\partial Y_j}{\partial \tau} = \frac{\eta_j}{1 + \eta_j L_j^{-1}} \frac{\partial Y_j}{\partial \tau}.$$  (18)

From (18), observe that all $\eta_j = 0$ implies that $\frac{\partial Y_j}{\partial \tau} = 0$. In that case,

$$\frac{\partial Y}{\partial \tau} = \sum_{ij} y_j q_{ij} \frac{\partial G_j(i)}{\partial \tau} N_i$$

$$= - \sum_i \left( \sum_{jk} y_j q_{ij} G_{jk}(i) \gamma_k q_{ik} \right) N_i.$$

The first conclusion that $\frac{\partial Y}{\partial \tau} \leq 0$ follows since the matrix with entries $G_{jk}(i)$ is positive semidefinite. If it is positive definite then the second conclusion that $\frac{\partial Y}{\partial \tau} < 0$ follows when all $\eta_j = 0$. Then by continuity, the conclusion also follows for small $\eta_j > 0$. $\square$

4. Simulation results

In this section, we illustrate our results using a simulation setup of the model.

Consider a linear city with 21 zones, spaced 6 kms apart. The intrazonal trip distance is 3 kms. Each zone contains both workers and workplaces. Travel speed is 40 km/h door-to-door, the travel cost is 0.2 €/km and the value of time is 10 €/hour, which gives a generalized travel cost (both ways) to a workplace zone at distance $dist$ of $c = \left( 0.2 + \frac{10}{40} \right) \cdot 2 \cdot dist$. Idiosyncratic utilities $\epsilon$ are extreme value type I (Gumbel) with dispersion parameter $\mu = 0.4$.

The productivity of a worker of type $i$ at location $j$ is $q_{ij}$. Index $i$ denotes a combination of location zone and worker type (education, idiosyncratic skills etc). The productivities $q_{ij}$ are drawn from a uniform distribution $(130 - \Delta q, 130 + \Delta q)$ €/day. The spread in productivities $\Delta q$
is varied in the simulations around a value of $\Delta q = 50$. In the simulation, we assume 1000 worker types per zone, meaning that we draw 1000 $q_{ij}$ per residential zone $i$. The tax rate is $\tau = 0.4$.

The decay function $w(d_{jk})$ in the job-to-job accessibility measure is taken to be $w(d_{jk}) = (d_{jk})^{-\alpha}$. The decay parameter $\alpha$ is varied in the simulations around a value of $\alpha = 1$ (taken from estimations in Graham (2007)). Job-to-job transport costs are assumed to be equal to commuting transport costs except that intra-zonal transport costs are normalized to 1. The local spillover parameter $\eta$ is taken to be independent of location $j$ and is varied in the simulations around a value of $\eta = 0.01$.

The parameters are chosen such that the output elasticity, transport cost elasticities and average trip length are in line with typical empirical evidence (see Table 1).

Table 1. Elasticities of the calibrated model compared to reference values from the literature.

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Simulation model</th>
<th>Reference value(s) from literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total VKT wrt. travel cost</td>
<td>0.23</td>
<td>0.23 (Bastian, Börjesson, &amp; Eliasson, 2016) (value for Sweden)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.22 (Small &amp; Van Dender, 2007)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.26 (de Jong &amp; Gunn, 2001)</td>
</tr>
<tr>
<td>Total VKT wrt. travel time</td>
<td>0.29</td>
<td>0.29 (de Jong &amp; Gunn, 2001)</td>
</tr>
<tr>
<td>Total VKT wrt. generalized travel cost</td>
<td>0.53</td>
<td>Implied by the elasticities above together with average speed, travel cost per km and the valuation of travel time (taken from Börjesson and Eliasson (2014)),</td>
</tr>
<tr>
<td>Total output wrt. accessibility</td>
<td>0.04</td>
<td>0.06 (Ciccone &amp; Hall, 1996)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05 (Ciccone, 2002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.03-0.05 (Anderstig et al., 2016)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.02-0.10; typical values according to Graham and van Dender (2011)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.04-0.11; typical values according to Venables (2007)</td>
</tr>
</tbody>
</table>

The equilibrium in the simulation model was calculated by computing commuting patterns given the wage rates $\gamma_j$ in each zone, then calculating new wage rates given these commuting patterns, and iterating until convergence. Different starting points for $\gamma_j$ were tested to check for multiple equilibria, but no such were found.

4.1 Spatial structure

Figure 1 below illustrates simulation results varying the spillover parameter $\eta$ between 0 and 0.02. The left panel shows the number of workers per zone, while the right panel shows the average output per worker in each zone. Since we are simulating a random distribution of productivities, the number of workers per zone becomes somewhat irregular, but the tendency that workers concentrate in central zones is clear.
We observe that workers tend to concentrate in the central zones, even when $\eta = 0$. This happens since job-worker productivity levels $q_{ij}$ vary randomly, while average transport costs are lowest to central locations, which means that more workers will find their optimal work location in a central zone. We thus see that matching alone, even in the absence of spillover effects, will cause jobs to concentrate in central zones and wages to be higher there.

A positive value of the spillover parameter $\eta$ reinforces this matching effect by increasing the wage rate in the central zones. This induces some workers to change their work location to the central zones, which further increases the wage rate in the central zones. In the $\eta = 0$ scenario, the highest zonal average output per worker is 2.2% higher than the lowest zonal average. This ratio increases to 2.5% when $\eta = 0.01$ and to 2.9% when $\eta = 0.02$.

### 4.2 Output changes due to matching and spillovers

As explained above, a change in transport costs affects total output through two mechanisms, matching and spillovers. The strength of the matching effect is determined by the heterogeneity of the productivity of job-worker combinations, controlled in the simulation by the parameter $\Delta q$. With much heterogeneity there is much scope for a reduction in transport costs to induce workers to commute to locations where they are more productive.

The strength of the spillover effect is determined by the size of the parameter $\eta$. The spillover effect is zero if $\eta = 0$, but even then a reduction in transport costs will improve matching and hence increase total output.

Table 2 and Table 3 show some consequences of varying the spillover parameter and the degree of heterogeneity of individual productivities. We find as expected that the elasticity of output with respect to a proportional change in generalized transport costs increases as the spillover parameter increases and as the degree of heterogeneity of productivity increases. The same output elasticity can then result at many combinations of the level of spillovers and heterogeneity.

#### Table 2. Variation in local spillover $\eta$ (with constant $\Delta q = 50$)

<table>
<thead>
<tr>
<th></th>
<th>$\eta = 0$</th>
<th>$\eta = 0.01$</th>
<th>$\eta = 0.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of output</td>
<td>-0.032</td>
<td>-0.039</td>
<td>-0.047</td>
</tr>
<tr>
<td>w.r.t. travel cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity of VKT</td>
<td>-0.53</td>
<td>-0.52</td>
<td>-0.53</td>
</tr>
<tr>
<td>w.r.t. generalized</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>travel cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average commuting</td>
<td>8.9</td>
<td>9.3</td>
<td>9.7</td>
</tr>
<tr>
<td>distance (km)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3. Variation in productivity heterogeneity $\Delta q$ (with constant $\eta = 0.01$)

<table>
<thead>
<tr>
<th></th>
<th>$\Delta q = 0$</th>
<th>$\Delta q = 30$</th>
<th>$\Delta q = 50$</th>
<th>$\Delta q = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of output w.r.t. generalized travel cost</td>
<td>-0.012</td>
<td>-0.037</td>
<td>-0.039</td>
<td>-0.049</td>
</tr>
<tr>
<td>Elasticity of VKT w.r.t. generalized travel cost</td>
<td>-1.21</td>
<td>-0.58</td>
<td>-0.52</td>
<td>-0.44</td>
</tr>
<tr>
<td>Average commuting distance (km)</td>
<td>5.1</td>
<td>7.8</td>
<td>9.3</td>
<td>11.9</td>
</tr>
</tbody>
</table>

4.3 The social benefits of a transport cost reduction

Consider a project that reduces all generalized transport costs by 10%. Table 4 shows the associated welfare gain for some combinations of $\eta$ and $\Delta q$ that yield roughly the same output elasticities. Benefits are divided into three parts: change in consumer surplus (calculated by the rule-of-a-half), increased tax revenues due to matching, and spillover benefits. The latter two components are the ones that are omitted from conventional transport CBA, so the sum of these is presented in the column “total wider benefits”. All wider benefits are shown as relative additions to the consumer surplus.

Table 4. Consumer surplus and wider benefits of a 10% transport cost reduction, for different parameter combinations.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$\Delta q$</th>
<th>Elasticity of output w.r.t. transport cost</th>
<th>Change in consumer surplus</th>
<th>Increased tax revenues due to matching (in % of CS)</th>
<th>Spillover benefits (in % of CS)</th>
<th>Total wider benefits (in % of CS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>0.038</td>
<td>20976</td>
<td>27%</td>
<td>0%</td>
<td>27%</td>
</tr>
<tr>
<td>0.01</td>
<td>50</td>
<td>0.039</td>
<td>16873</td>
<td>27%</td>
<td>21%</td>
<td>48%</td>
</tr>
<tr>
<td>0.02</td>
<td>10</td>
<td>0.040</td>
<td>9863</td>
<td>22%</td>
<td>65%</td>
<td>87%</td>
</tr>
<tr>
<td>0.03</td>
<td>0</td>
<td>0.038</td>
<td>7469</td>
<td>1%</td>
<td>139%</td>
<td>140%</td>
</tr>
</tbody>
</table>

Since the matching effect is partly internalized by commuters while the spillover effect is external, the wider benefits of a project generating a change in economic output will be different depending on how much of the output change is due to spillovers or matching. Table 4 illustrates this: although the output elasticity is about the same in all four scenarios, the size of the wider benefits relative to the conventional consumer surplus varies substantially, since the output effect may be generated completely by matching (row 1), completely by spillovers (row 4) or by a combination (rows 2 and 3).

4.4 Confounding of matching and spillovers

So we see that different combinations of $\eta$ and $\Delta q$ may lead to the same output elasticity but will imply very different welfare implications of a transport cost reduction. In table 4, the size of the wider economic effect relative to the change in consumer surplus varies by more than a factor 4, depending on whether the increase in output is due to matching or spillovers. This shows that it is crucial that empirical studies of agglomeration effects take both matching and spillovers into account and that they are properly able to distinguish matching effects from spillovers. Our reading of the literature suggests that studies that manage to do that are rare.

Assume for the sake of example that the spillover parameter $\eta$ is constant across zones. Production is then
\[ Y_j = \bar{q}_j N_j \left( \sum_k \bar{q}_k N_k w(d_{jk}) \right)^\eta, \]  

(19)

where \( \bar{q}_j \) is the average productivity of workers in zone \( j \). Adding noise terms and assuming for simplicity that the decay function \( w(d_{jk}) \) is known, it is possible in principle to estimate \( \eta \) through the regression

\[ \ln \frac{Y_j}{N_j} = \ln \bar{q}_j + \eta \ln \left( \sum_k \bar{q}_k N_k w(d_{jk}) \right) + \xi_j. \]  

(20)

This requires, however, that the average productivities \( \bar{q}_j \) are observable, which may be challenging in practice. An analyst who is unable to observe \( \bar{q}_j \) might instead estimate \( \beta \) in the following simplified regression:

\[ \ln \frac{Y_j}{N_j} = c + \beta \ln \left( \sum_k N_k w(d_{jk}) \right) + \xi_j. \]  

(21)

This specification has been used in the economic literature (Graham, 2007, Rice et al, 2006) and the accessibility measure (multiplied by the parameter \( \beta \)) is often called the effective density. The parameter \( \beta \) measures the elasticity of output per worker with respect to effective density.

It is clear that an estimate of \( \beta \) is not an estimate of \( \eta \). From Table 4 it is also clear that the same correlation between transport costs and output can be generated from a range of combinations of \( \eta \) and \( \Delta \). But Table 4 also shows that it matters very much for the assessment of the wider benefits of a transport cost reduction whether these are due to matching or spillovers. It is then potentially quite misleading if an estimate of \( \beta \) is misinterpreted as an estimate of a pure spillover effect.

4.5 Job-to-job accessibility: the decay function

The size of the spillover effect depends on the decay function, regulated by the distance decay parameter \( \alpha \). The higher \( \alpha \) is, the faster is the decay and the more local is the job-to-job spillover effect. This is illustrated in Table 5 (with \( \eta = 0.01, \Delta q = 50 \)).

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>Elasticity of output w.r.t. gen. transport cost</th>
<th>Elasticity of VKT w.r.t. gen. transport cost</th>
<th>Change in consumer surplus</th>
<th>Incr. tax revenues from matching (in % of CS)</th>
<th>Spillover benefits (in % of CS)</th>
<th>Total wider benefits (in % of CS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.039</td>
<td>-0.52</td>
<td>16 873</td>
<td>27%</td>
<td>21%</td>
<td>48%</td>
</tr>
<tr>
<td>2</td>
<td>0.037</td>
<td>-0.52</td>
<td>16 814</td>
<td>26%</td>
<td>16%</td>
<td>42%</td>
</tr>
<tr>
<td>4</td>
<td>0.031</td>
<td>-0.52</td>
<td>16 797</td>
<td>26%</td>
<td>3%</td>
<td>29%</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.030</td>
<td>-0.52</td>
<td>16 796</td>
<td>26%</td>
<td>1%</td>
<td>27%</td>
</tr>
</tbody>
</table>

Note that the VKT elasticity, the consumer surplus and the tax revenues caused by matching effects are virtually unaffected by the value of the decay parameter \( \alpha \). The spillover effect and hence the size of the wider benefits, however, depend strongly on \( \alpha \). For high \( \alpha \) values, spillover benefits are only generated by workers choosing other job locations, i.e. relocation effects. For lower \( \alpha \) values, spillovers also increase when job-to-job transport costs decrease.
As pointed out earlier, measuring job-to-job transport costs and how spillover effects decrease with such costs is difficult, and our understanding of them is still limited compared to our understanding of commuting costs.

4.6 Comparing different transport improvements

As was pointed out in section 3.2, total output may increase or decrease depending on where transport costs change. Output will decrease if workers are attracted to less productive zones, and vice versa. Moreover, two projects with similar effects in terms of consumer surplus may have quite different total benefits when wider impacts are taken into account. The next example illustrates this.

Consider two projects, A and B, chosen to yield similar conventional consumer surpluses. In project A, transport costs on the links between the central zones 9-13 are reduced by 60%. In project B, transport costs on the links between the peripheral zones 1-3 and 19-21 are reduced by 100% (intra-zonal costs still apply, though). Table 6 below shows aggregate benefits of these projects (assuming parameters $\eta = 0.01$ and $\Delta q = 50$).

Table 6. Consumer surplus and wider benefits of two reductions of transport costs.

<table>
<thead>
<tr>
<th>Project</th>
<th>Change in consumer surplus</th>
<th>Incr. tax revenues from matching (in % of CS)</th>
<th>Spillover benefits (in % of CS)</th>
<th>Total wider benefits (in % of CS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (reduced cost on central links)</td>
<td>19 276</td>
<td>37%</td>
<td>$26%$</td>
<td>64%</td>
</tr>
<tr>
<td>B (reduced cost on peripheral links)</td>
<td>19 096</td>
<td>8%</td>
<td>31%</td>
<td>40%</td>
</tr>
</tbody>
</table>

The two projects yield similar consumer surplus, and would hence yield similar benefits in a conventional CBA. Their wider economic benefits are different, however. Project A reduces transport costs to the most productive zones, so the aggregate output effect is large. Project B, on the other hand, reduces transport costs to the least productive zones, so the wider impacts are smaller. Another difference is that project A generates much larger matching benefits, since transport costs to already productive zones are reduced. Project B, on the other hand, generates larger spillover benefits, since job-to-job accessibility in zones with relatively low accessibility is improved.

The important lesson here is that accounting for wider impact may show that projects which seem equivalent in a conventional CBA may in fact yield quite different benefits when wider impacts are taken into account. Taking matching and spillovers into account may then affect the ranking of projects.

5. Concluding remarks

This paper has analyzed the welfare effects of transport improvements within a quite general framework that incorporates matching, spillovers and an income tax. It seems worthwhile to summarize the main results:

A transport project leads to wider economic impacts beyond the change in consumer surplus through matching and through spillovers. The wider economic impacts of projects may differ even if they yield the same change in consumer surplus, so accounting for wider impacts may be important when ranking projects in terms of cost efficiency.

The matching effect relate to the matching of workers to jobs, holding wages constant at each possible worker-job combination. The matching effect is partly internalized by workers in their choice of job. But changes among jobs paying different wages leads to a change in income tax revenues, which should be accounted for in the welfare calculus. The change in income tax
revenues due to matching resulting from a reduction in transport costs may be positive or negative.

The spillover effect works through wage changes, holding matches constant. The spillover effect is entirely external to the commuting decisions of workers and should be added to the consumer surplus in its entirety.

Adding the entire output change resulting from a transport project to a conventional CBA will hence result in double-counting if some of the output change is due to matching effects. Only increased tax revenues and the after-tax part of the output change that is due to spillovers should be added. As illustrated in the simulations, the size of the wider benefits may differ substantially even if the output elasticity is the same, depending on whether the change in output is due to matching, spillovers or a combination. The problem of econometrically distinguishing between matching and spillover contribution to output effects is not trivial, however, since matching and spillover effects are both contribute to generating correlation between confounded agglomeration and output.

Our results differ from the influential paper by Venables (2007). The fundamental difference is that Venables does not allow for matching effects, and hence all agglomeration effects in his model are external to workers. The conclusions from Venables (2007) are applied in the UK cost-benefit guidelines for transport project appraisal (WebTag) and these could be updated in the light of the current findings.

It was noted already by Forsyth (1980) that the change in tax revenues should be included in the cost-benefit analysis of a transport improvement. This insight has been slow to make its way into practice. A difficulty has been calculating the change in output following a transport improvement. However, empirical estimates of the causal impact of accessibility improvements on wages or more generally on value added are now becoming available (Börjesson et al., 2015; Combes & Gobillon, 2014; Graham, 2007a, 2007b; Norman et al., 2017; Rice, Venables, & Patacchini, 2006).

The current research agenda is by no means exhausted, of course. Perhaps the most pressing issue from the perspective of this paper is to distinguish matching and spillover effects empirically and to find ways of predicting these effects that can be applied in cost-benefit analysis of transport projects and policies.

6. References


