Government Commitment and Unemployment Insurance Over the Business Cycle

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Abstract

We investigate the role of government commitment to future policies in shaping unemployment insurance (UI) policy in a stochastic general equilibrium model of labor search and matching. Compared with the optimal (Ramsey) policy of a government with commitment, the policy under no commitment characterized by a Markov-perfect equilibrium has higher benefits and leads to higher unemployment rates in the steady state. We also find starkly different policy responses to a productivity shock or changes in unemployment. The differences arise because the Ramsey government can use an ex-ante committed policy to stimulate job search.

Keywords: Unemployment insurance, Commitment, Markov-perfect equilibrium, Business cycle
JEL classifications: E61, J64, J65, H21

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1 Introduction

A recent literature finds that the optimal (Ramsey) unemployment insurance (UI) policy is more generous at the start of a recession and less generous as the recession gets more severe to induce a recovery.[1] This pattern differs from the movement of benefits in the U.S. over the business cycle. In particular, benefits are more generous the deeper the recession gets as measured by unemployment.[2] In addition, Congress voted frequently on extending UI benefits from 2008 to 2013 during the Great Recession, which reflects the government's lack of commitment over UI policies. In this paper, we argue that when the government cannot commit to a prescribed path of benefit policies, it cannot use UI policy to induce recovery in recessions. Such a policy more closely resembles the U.S. policy. We characterize and compare the benefit policies with and without commitment and study its impact on the labor market.

The model integrates risk-averse workers and endogenous search intensity by unemployed workers into the Diamond-Mortensen-Pissarides framework, with business cycle driven by shocks to aggregate labor productivity. Three types of entities inhabit the model economy: workers, firms and government. Employed workers work for firms and get paid wages. Unemployed workers receive UI benefits and choose how much to search. Job search incurs utility cost, but also increases the probability that an unemployed worker finds a job. Firms matched with workers produce and pay workers wage, while unmatched firms post job vacancies at a fixed cost. Wages are determined through a Nash bargaining process.

Because the focus of the present paper is on the comparison between UI policies with and without government commitment, we abstract from the distinction between benefit level and the expected duration that an unemployed worker can receive benefits ("benefit duration") and instead use the benefit level alone to capture the generosity of UI policies. The government chooses UI benefits financed by a non-distortionary tax. More generous *future* UI benefits reduce unemployed worker's current search intensity. Through general equilibrium effects, higher benefits reduce firm's vacancy posting by increasing worker's outside option in the wage bargaining process.

We compare two economies. In the first economy, the optimal state-contingent UI policy is the solution to a Ramsey problem of the government, which takes competitive equilibrium conditions as constraints. Because the government can commit to future policies, it optimally chooses the level of UI benefits for all periods of time and for all possible realizations of productivity shocks. The Ramsey policy, however, is not time consistent. In particular, the current government would like to reduce promised future benefits *before* unemployed workers choose how much to search ("ex-ante policy"), and increase benefits *after* they have chosen job-search level ("ex-post policy").[3] Because

[1] See, for example, Jung and Kuester (2015) and Mitman and Rabinovich (2015).

[2] By generosity we refer loosely to either an increase in the benefit level received by unemployed workers or the potential duration that a benefit-eligible unemployed worker can stay on benefits.

[3] The ex-ante government incentive is to stimulate search by promising a low consumption level of unemployment. The ex-post incentive is to insure the unemployed workers who did not find a job against job-finding risk.

the government wants to implement different policies ex-ante and ex-post, such a policy is time inconsistent and cannot be implemented without government commitment.

In the second economy, we characterize and solve for the time-consistent UI policy. We use the concept of Markov-perfect equilibrium, à la Klein, Krusell and Ríos-Rull (2008). Each period the government chooses current policies to maximize current and future welfare, taking as given future government's policy rules. Because the Markov government only chooses policies for the current period, its policies differ from the Ramsey policies. First, it does not consider how its policies affect private-sector choices in the previous period, whereas the Ramsey government internalizes this effect. Second, the Markov government can only indirectly influence future polices through the states of the economy, while the Ramsey government predetermines a full sequence of state-contingent policies at time 0.

We calibrate the model to match the U.S. economy. Using these calibrated parameters, we then compute the Ramsey and the Markov policies. Overall, the Markov economy has more generous UI benefits, lower search intensity, lower job postings and higher unemployment rate than the Ramsey economy. This is not surprising, given that the Markov government has no commitment and chooses more generous benefits. This highlights the importance of commitment.

An important result concerns the dynamic responses of different governments. The Ramsey UI benefits decrease when current labor productivity is higher or unemployment rate is higher, whereas the Markov UI benefits increase when current labor productivity is higher but does not change much with respect to unemployment rate. The intuition is that the optimal (Ramsey) benefits are lower in states of the economy where the marginal social benefit of job creation is higher, because lower benefits encourage job search and vacancy posting. The marginal social benefit of job creation is higher when productivity is higher, because each worker-firm pair produces higher output; it is also higher when unemployment is higher, because the probability of filling a job is higher. The Markov government considers the previous period *bygone*, and so it does internalize these social gains. Instead, the Markov government increases UI benefits when productivity is high, because higher wages (increase in productivity shared between workers and firm) increase the gain from redistribution. When unemployment is higher, the incentive to encourage search is higher, but there is also a larger gain from providing insurance. These two effects cancel each other out, and the Markov benefits vary little with respect to unemployment rate.

Because of different policy responses, the Ramsey and Markov economies have very different dynamics. In response to a one-time negative productivity shock, the Ramsey government initially increases and then slowly reduces UI benefits to the pre-shock level. The initial rise is to help workers smooth consumption, while the subsequent fall creates incentive for search and job posting. The adverse impact of higher benefit on job creation in the initial periods is mitigated by the government's commitment to lower benefits in the future. In response to the same shock, the Markov government lowers benefits immediately, because the costs of financing benefits increase when productivity is low.

As the economy recovers, the Markov government gradually increases benefits to the pre-shock level. The richer dynamics of the optimal policy reflects the benefit of commitment—because the Ramsey government has commitment over future policies, it can use temporary changes in the UI policy to smooth consumption over the business cycle. As a result, the Ramsey economy experiences relatively fast recovery in unemployment, while the Markov economy undergoes a much slower recovery.

Several simplifying assumptions are made for tractability. First, neither workers nor government can save or borrow. Allowing workers to save will reduce the cyclical responses of both Ramsey and Markov governments as savings provide self-insurance. If government can save or borrow, there will be larger cyclical responses as the government is not constrained by budget constraints every period. Both assumptions, however, will not affect the comparison of the Ramsey and Markov policies. Second, the Markov government in our setup makes decision every period. Given the weekly frequency, it means that the government makes UI policy decisions every week in our setup, which is much more frequent than in reality. Changing model frequency to quarterly, however, will be a departure from the standard frequencies used in analysis of the labor market.

A review of the related literature and our contributions to the literature is given next.

1.1 Related literature

This paper is closely related to two strands of literature: the literature on UI and the literature on time-consistent public policy.

The literature on UI dates back to Mortensen (1977), who argues that unemployment insurance reduces job search effort by the unemployed. Since Mortensen, the majority of this literature takes one of two approaches: either studying the effects of actual UI policy or looking for an optimal policy. The present paper aims to bridge the two approaches by endogenizing government choice of UI policy. Here we take the stance that the government, when making UI policies, is unable to commit to future policies.

One of the classic empirical results in public finance is that social insurance programs such as UI reduce labor supply. Earlier works include Moffitt (1985) and Meyer (1990), who show that a 10% increase in unemployment benefits raises average unemployment durations by 4–8% in the U.S.Krueger and Meyer (2002) and Gruber (2007), for example, interpret this finding as evidence that UI has significant moral hazard costs. Our framework relies on a similar mechanism. When the *expected* payoff from unemployment is high relative to future wages, unemployed workers search less actively. Chodorow-Reich and Karabarbounis (2015) construct a time series of the opportunity cost of employment and find that the cost is procyclical and volatile over the business cycle.

More recently, Chetty (2008) explores an alternate explanation for the link between unemployment benefits and duration. He argues that unemployment benefits increase cash on hand for the unemployed, and thus reduces search intensity. This effect is stronger for unemployed workers with tighter liquidity constraints. Because this "liquidity effect" has the socially beneficial effect of correcting credit market failure, the truly optimal benefit level, as Chetty (2008) argues, should be higher than if such an effect were ignored. For tractability, the present paper abstracts from the credit market and therefore cannot directly control for the liquidity effect.

The literature on optimal UI has traditionally adopted a principal-agent framework (e.g., Hopenhayn and Nicolini 1997, Wang and Williamson 2002, Shimer and Werning 2007, 2008, and Golosov, Maziero and Menzio 2013). This framework allows moral hazard frictions to be characterized in a steady state, but it becomes intractable when extended to a business cycle environment. The literature typically shows that the optimal benefit should decline with the unemployment duration of an individual worker. For tractability, our paper abstracts from duration-dependent benefits. More recently, Mitman and Rabinovich (2015) study optimal benefits over the business cycle in a search-matching framework with endogenous unobservable search intensity. Jung and Kuester (2015) take a more general approach by studying the optimal mix of unemployment benefits, hiring subsidies, and layoff taxes in a recession. These papers assume that the government is able to commit to future policies. Although this assumption is innocuous and standard for normative analysis, such policies are time inconsistent and thus hardly implementable without government commitment. Our paper complements the literature by characterizing a time-consistent UI policy. Intuitively, the Ramsey government wants people to search hard in normal times when unemployment is low and the marginal return to search is high. To this end, the Ramsey government promises less generous benefits during recessions.

The current paper is also related to the literature on time-consistent public policy (see, e.g., Alesina and Tabellini 1990, Chari and Kehoe 2007, Battaglini and Coate 2008, and Yared 2010).[4] Methodologically, our paper follows Klein, Krusell and Ríos-Rull (2008) to characterize the Markov-perfect equilibrium of a dynamic game in terms of a generalized Euler equation (GEE). Whereas Klein, Krusell and Ríos-Rull (2008) focus on a deterministic economy, we are interested in how government policy responds to business cycle fluctuations. Recent applications of Markov-perfect equilibrium include Song, Storesletten and Zilibotti (2012), who study intergenerational conflict over debt in a politico-economic environment.

The rest of the paper proceeds as follows. Section 2 describes the model environment and defines the private-sector competitive equilibrium. Section 3 presents the government's problem first as an optimal policy problem, and then as part of a Markov-perfect equilibrium. We characterize the solutions and solve both government's problems in this section. Section 4 describes the calibration strategy. Section 5 presents the quantitative results. Section 6 concludes. We relegate all derivations and additional figures to the Appendix.

[4] See Klein and Ríos-Rull (2003) for a detailed review of the earlier literature.

2 Model

In this section, we describe the model environment and characterize the competitive equilibrium. Similar to Mitman and Rabinovich (2015), we use a Diamond-Mortensen-Pissarides framework with aggregate productivity shocks to model the private sector.[5]

2.1 Model environment

Time is discrete and infinite. The model is inhabited by a mass of infinitely lived workers and firms. The measure of workers is normalized to one. In any given period, a worker can be either employed or unemployed. Workers are risk-averse and maximize expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, s_t)$$

where $\mathbb{E}_0$ is the period-0 expectation factor, β is the time discount factor. Period utility $U(c, s)$ takes consumption of goods c and search intensity s as inputs. Utility is increasing in c and decreasing in s. Only unemployed workers supply positive search intensity, i.e. there is no on-the-job search. Each period, an employed worker gets paid wage from production. Wages are determined through a canonical bargaining process to be specified later in the section. An unemployed worker receives unemployment benefits b. In addition, an unemployed worker also produces h, which we take as the combined value of leisure, home production and welfare. There are no private insurance markets and workers cannot save or borrow.

Firms are risk neutral and maximize the expected discounted sum of profits, with the same discount factor β. A firm can be either matched to a worker (and producing) or vacant. A vacant firm posting a vacancy incurs a flow cost κ.

Unemployed workers and vacancies form new matches. Let u and v denote the measure of unemployed worker, and the measure of vacancies posted, respectively. Then the number of new matches formed in a period is given by the matching function $M(su, v)$, where the quantity su is the measure of efficiency units of search by the unemployed in the economy. The matching function exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and is bounded above by the number of potential matches : $M(su, v) \leq \min\{su, v\}$. The job-finding probabilities per efficiency unit of search intensity, f, and the job-filling probability per vacancy, q,

[5] Compared to Mitman and Rabinovich (2015), we allow all unemployed workers to receive benefits. We also use a different timing from them, because our timing highlights the effect of government commitment and is also a more conventional timing used in the search literature.

are functions of labor market tightness, $\theta = v/(su)$. More specifically,

$$\begin{aligned} f(\theta) &= \frac{M(su,v)}{su} = M(1,\theta) \\ q(\theta) &= \frac{M(su,v)}{v} = M\left(\frac{1}{\theta},1\right) \end{aligned}$$

Following the assumptions made on M, $f(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ. The job finding probability for an unemployed searching with intensity s is $sf(\theta)$. Existing matches are destroyed exogenously with constant job separation probability δ.

Only a matched pair of a worker and a firm can produce. Each matched pair produces z, where z is the aggregate labor productivity. z is constant $\bar{z}$ in the steady state, and time-varying z_t in the economy off steady-state.

2.2 Government

The government cannot borrow or lend; instead it balances budget each period. The government finances unemployment benefits b through a lump sum tax, τ, on all workers, both employed and unemployed.[6] The government budget constraint is

$$\tau = ub \tag{1}$$

The government decides on the generosity of the unemployment insurance program by varying benefit level, $b \geq 0$. Once a benefit level is determined, all unemployed workers receive the same benefit in that period. This way of modeling the unemployment insurance system is a simplification of the reality where not all unemployed workers receive benefits. This assumption is common in the literature, for example, Landais, Michaillat and Saez (2010) and Jung and Kuester (2015) both assume all unemployed receive benefits. The advantage of this setup is reduced computational complexity while still allowing the generosity of the unemployment insurance program to change. The unemployment benefits here can be thought of as compounding the potential duration and level of unemployment benefits.[7]

2.3 Timing

The timing of events within a period is illustrated in Figure 1 and is as follows. The economy enters a period t with a level of unemployment u. The aggregate shock z then realizes. (z,u) are the

[6] We experiment with alternative tax structure where either only employed workers pay tax, or only firms pay tax (in the form of a lump sum tax on profits). Results are not presented in the paper but are available upon request.

[7] Equivalently, it can be thought of as compounding benefit level and proportion of unemployed workers on benefit at any time.

Figure 1: Timing of events.

aggregate states of the economy. Government policies (b,τ) for the period are known to workers and firms.

Employed workers produce z and receive a bargained wage w. Unemployed workers produce h and receive benefits b. All workers pay a lump sum tax τ out of wage or benefit.

Given aggregate states and government policies for the period, unemployed workers choose search intensity s. At the same time, firms decide how many vacancies to post, at cost κ per vacancy. The aggregate search is then su, and the market tightness is equal to $\theta = v/(su)$. The fraction of unemployed workers who find jobs is $f(\theta)s$. At the same time, a fraction δ of the existing $1-u$ matches are exogenously destroyed. The law of motion of unemployed workers is

$$u' = \delta(1-u) + (1 - f(\theta)s)u \tag{2}$$

2.4 Workers

Denote by g the government policy (b,τ). Because we abstract from household savings and all unemployed workers receive unemployment benefits, a worker entering a period unemployed consumes $h+b$. He also chooses search intensity s. With probability $f(\theta(z,u;g))s$, he finds a job and starts working the following period. Let $V^e(z,u;g)$ and $V^u(z,u;g)$ be the values of an employed and an unemployed worker, respectively, with the beginning-of-period unemployment u and realized aggregate shock z, given government policy for that period $g=(b,\tau)$. An unemployed worker's optimization problem is

$$V^u(z,u;g) \quad = \quad \max_s \; U(c,s) + \beta f(\theta(z,u;g))s\mathbb{E}V^e(z',u';g') + \beta(1 - f(\theta(z,u;g))s)\mathbb{E}V^u(z',u';g') \tag{3}$$

A worker entering a period employed produces and consumes his wage w. With probability δ, he loses his job and becomes unemployed the following period. There is no intra-temporal search, so a newly separated worker remains unemployed for at least one period. The Bellman equation of an employed worker is then

$$V^e(z,u;g) \quad = \quad U(w(z,u;g),0) + \beta(1-\delta)\mathbb{E}V^e(z',u';g') + \beta\delta\mathbb{E}V^u(z',u';g') \tag{4}$$

Notice that market tightness θ and wage w are functions of the economy's states, (z,u). This is because they are objects determined in an equilibrium. As mentioned before, job separation rate δ

is taken to be constant through time.

2.5 Firms

In order to be matched with a worker and produce, a firm posts a vacancy.[8] A firm that posts a vacancy incurs a flow cost κ. With probability $q(\theta(z,u;g))$, a vacancy is filled and ready for production the following period. Let $J^u(z,u;g)$ be the value of an unmatched firm posting a vacancy. The Bellman equation of an unmatched firm is

$$J^u(z,u;g) = -\kappa + \beta q(\theta(z,u;g))\mathbb{E}J^e(z',u';g') + \beta(1-q(\theta(z,u;g)))\mathbb{E}J^u(z',u';g') \tag{5}$$

where $J^e(z,u;g)$ is the value of a matched firm. In equilibrium, under free-entry condition, the firm will post vacancies $v(z,u;g)$ until $J^u(z,u;g)=0$.

A matched firm receives output net of wages $z-w(z,u;g)$. With constant probability δ, a match is destroyed at the end of period. The Bellman equation of a matched firm is

$$J^e(z,u;g) = z-w(z,u;g)+\beta(1-\delta)\mathbb{E}J^e(z',u';g')+\beta\delta\mathbb{E}J^u(z',u';g') \tag{6}$$

2.6 Wage determination

Vacant jobs and unemployed workers are randomly matched each period according to the aggregate matching function $M(su,v)$. A realized match produces some economic rent that is shared between the firm and the worker through Nash bargaining. The assumption of Nash bargaining allows comparison with the literature. We assume that wages are set period by period, so equilibrium wages respond to the state of the economy.

Worker's surplus is the difference between the values of working at wage w and being unemployed and receiving benefit b. As a result, higher benefits increase worker's surplus, and tends to drive up bargained wage. Firm's surplus is the difference between the value of a match and that of running a vacancy. As explained before, vacant firm posts vacancies until its value is zero. Thus, the firm's outside option is zero in equilibrium.

In particular, wage is chosen to maximize a weighted product of the worker's surplus and the firm's surplus when the state of the economy is (z,u) and government policy is $g=(b,\tau)$. The worker-firm pair thus solves

$$\max_{w} \quad \Big(V^e(z,u;g)-V^u(z,u;g)\Big)^{\zeta}\Big(J^e(z,u;g)-J^u(z,u;g)\Big)^{1-\zeta} \tag{7}$$

where $\zeta\in(0,1)$ is the bargaining power of the worker. $V^e(z,u;g)-V^u(z,u;g)$ is the worker's surplus,

[8] The firms can be viewed as a representative firm with a collection of jobs and posts several vacancies.

and $J^e(z,u;g) - J^u(z,u;g)$ is the firm's surplus from the match. The solution to this bargaining problem, denoted $w(z,u;g)$, is a function of the economy's states.

2.7 Competitive equilibrium

DEFINITION 1. (competitive equilibrium) Given a policy $g = (b,\tau)$ and an initial condition (z^-, u^-), a competitive equilibrium consists of (z,u)- measurable functions for wages $w(z,u;g)$, worker's search intensity $s(z,u;g)$, market tightness $\theta(z,u;g)$, unemployment rate $u'(z,u;g)$, and value functions $V^e(z,u;g)$, $V^u(z,u;g)$, $J^e(z,u;g)$, $J^u(z,u;g)$ such that for all $(z,u;g)$

- the value functions satisfy the worker and firm Bellman equations (3)-(6)
- the search intensity s solves the unemployed worker's maximization problem of (3)
- the market tightness θ is consistent with the free-entry condition, $V^u(z,u;g) = 0$
- the wage w solves the maximization problem of (7)
- unemployment satisfies the law of motion equation (2)

2.8 Characterization

The competitive equilibrium can be characterized by three optimality conditions.[9] Appendix A contains derivation of the optimality conditions. In what follows, primes denote variables of the following period, and subscripts denote derivatives.

The optimal choice of search intensity s for the unemployed worker is characterized by

$$(8) \quad \frac{-U_s(h+b-\tau,s)}{f(\theta)} = \beta \mathbb{E}\left[U(w'-\tau',0) - U(h+b'-\tau',s') + (1-f(\theta')s'-\delta)\frac{-U_s(h+b'-\tau',s')}{f(\theta')}\right]$$

The worker's optimality condition states that the marginal cost (left-hand side) of increasing the job finding probability equals the marginal benefit (right-hand side). The marginal cost is the marginal disutility of search of the unemployed worker weighted by the aggregate job finding rate per efficiency unit of search. The marginal benefit is the sum of utility gain from being employed next period and the benefit of economizing on future search cost. A higher future benefit b' reduces the utility gain from being employed the next period, and thus lowers the marginal benefit of search today.

From firm's free-entry condition

$$(9) \quad \frac{\kappa}{q(\theta)} = \beta \mathbb{E}\left[z' - w' + (1-\delta)\frac{\kappa}{q(\theta')}\right]$$

[9] To economize on notation, we suppress the dependence on $(z,u;g)$. It should be understood throughout that the optimal decisions are functions with arguments $(z,u;g)$.

where the marginal cost (left-hand side) equals the marginal benefit (right-hand side) of a filled vacancy. The marginal cost is the flow cost of posting a vacancy weighted by the probability of filling that vacancy. The marginal benefit is the profits from employing a worker. Because a newly formed match does not become operational until the next period, the benefit from production only has components from the next period.

Finally, Nash bargaining implies a relationship between the worker's surplus from being employed and the firm's surplus from hiring a worker.

$$\frac{\left[U(w-\tau,0)-U(h+b-\tau,s)+(1-f(\theta)s-\delta)\frac{-U_s(h+b-\tau,s)}{f(\theta)}\right]/U_c(w-\tau,0)}{z-w+(1-\delta)\frac{\kappa}{q(\theta)}}=\frac{\zeta}{1-\zeta}$$

The left-hand side of the equation is the ratio of worker's to firm's surplus weighted by marginal utility of higher wage. The worker's surplus (top part) comes from utility gain of of being employed and reduced search cost (employed worker searches zero). Because workers are risk-averse, changes in wages have non-linear effect on his utility, as represented by $U_c(w-\tau,0)$. The firm's surplus (bottom part) derives from profit and reduced vacancy posting cost (producing firm posts zero vacancy). The right-hand side of the equation is the ratio of the worker's to firm's bargaining power. Equilibrium wage then equates the weighted ratio of worker/firm surplus to the ratio of their respective bargaining power.

Rearranging terms into a more compact condition for the equilibrium wage

(10)
$$\zeta U_c(w-\tau,0)\left[z-w+(1-\delta)\frac{\kappa}{q(\theta)}\right]=(1-\zeta)\left[U(w-\tau,0)-U(h+b-\tau,s)+(1-f(\theta)s-\delta)\frac{-U_s(h+b-\tau,s)}{f(\theta)}\right]$$

A higher future benefit b' lowers worker's surplus. Given the future Nash bargaining condition, next period's workers demand higher wage w', and thus firm's surplus is lower in equilibrium. The free-entry condition of (9) then implies a lower θ, and thus a lower job-finding rate per efficiency search unit $f(\theta)$.

A contemporaneous decrease in the aggregate labor productivity z reduces firm's surplus in (10) and as a result wages fall. Because current wage and productivity do not enter worker's and firm's optimality conditions (8)-(9), the contemporaneous fall in z does not directly affect search or job-finding rate. Now consider a fall in the expected future productivity z'. In this latter case, firm reduces vacancy posting since expected return to future production is lower. Worker reduces search intensity, both because fewer vacancies lead to lower per-search-unit job-finding probability, and because lower expected aggregate productivity implies lower expected future wages.

Because different z' lead to different equilibrium search and job-finding rate, the government will optimally tailor its unemployment benefit policy to current and future economic conditions. In the next section, we analyze how a benevolent government, under assumptions of commitment and non-commitment, designs benefit policy.

3 Government Policies

In this section, we describe how a Ramsey government and a Markov government choose their policies, respectively. To highlight the key difference between these two types of governments, we first illustrate the time inconsistency of the Ramsey policy using a simple example. We then describe the full Ramsey and Markov problems. We assume both governments are utilitarian planners, who maximize the expected value of the worker's utility. Both governments have the same policy instruments, which are unemployment benefit b and tax τ. However, the Ramsey government can commit to future policies, while the Markov government does not have the ability to commit. In the last subsection, we discuss the role of commitment.

3.1 A simple example to illustrate time inconsistency

Before describing the full Ramsey problem, we consider a simple example to illustrate the presence of time inconsistency in the Ramsey problem. There are two periods and a unit measure of workers. Workers search in the first period and consume in the second period. Assume no time discounting and firms. In the first period, $1-\bar{u}$ of workers are guaranteed a job in the second period. The remaining $\bar{u}$ workers choose whether to search, $s \in \{0,1\}$, for a job starting in the second period. Search incurs utility cost of $\bar{c}$. Worker's utility of consumption is given by $U(c)$.

If the worker searches ($s=1$), with probability f he finds a job and receives wage $\bar{w}=1$ in the second period; otherwise he receives unemployment benefit $b \le \bar{w}$. Optimally the worker searches is and only if $U(\bar{w}) - U(b) \ge \bar{c}/f$. The number of unemployed workers in the second period is $u = (1-fs)\bar{u}$.

Government in this economy chooses $b \in \{\bar{b}, \underline{b}\}$ at the beginning of period 1 to maximize average utility

$$\begin{aligned} W &= (1-u)U(\bar{w}) + uU(b) - \bar{u}\mathbb{1}_{s=1}\bar{c} \\ \text{subject to} \quad u &= (1-fs)\bar{u} \\ & s=1 \text{ iff } U(\bar{w}) - U(b) \ge \bar{c}/f \end{aligned}$$

Assume linear utility functions and $\bar{w} - \underline{b} \ge \bar{c}/f + [\bar{b} - \underline{b}]/f$ (Ass1) so that $b = \underline{b}$, $s=1$.[10] Essentially, the government is choosing between

$$\begin{aligned} W(b=\underline{b}, s=1) &= [1-(1-f)\bar{u}]\bar{w} + (1-f)\bar{u}\underline{b} - \bar{u}\bar{c} \\ W(b=\bar{b}, s=0) &= (1-\bar{u})\bar{w} + \bar{u}\bar{b} \end{aligned}$$

Then by assumption (Ass1) $W(b=\underline{b}, s=1) > W(b=\bar{b}, s=0)$, and the government optimally chooses

[10] For example, $\bar{w}=1$, $\bar{b}=0.3$, $\underline{b}=0$, $\bar{c}=0.1$,$f=0.5$.

$b^* = \underline{b}$, $s^* = 1$, $u^* = (1-f)\bar{u}$.

Now suppose the government can revise benefit after workers have chosen s. Then the *ex-post* optimal policy is $\hat{b} = \bar{b}$, with *ex-post* average utility given by $W(\hat{b} = \bar{b}, s = 1) = (1-u^*)\bar{w} + u^*\bar{b} - \bar{u}\bar{c} > W(b = \underline{b}, s = 1)$. The fact that there exists a better policy *ex-post* illustrates the time inconsistency in this setup; time inconsistency, in turn, means lack of commitment leads to different policy outcomes than an economy with government commitment.

3.2 Ramsey government

In this section, we set up the full Ramsey problem. The modeling of the Ramsey government is very similar to that in Mitman and Rabinovich (2015). Since the Ramsey government has commitment to all its future policies at the beginning of time, the government's decision problem is therefore to choose a sequence of unemployment benefits and taxes $\{b_t, \tau_t\}_{t=0}^{\infty}$ in order to maximize the worker's utility, taking into account how the private sector will respond to these policies. At time 0, the government decides on its policies for all future periods and for all possible realizations of shocks. The private sector takes government policies as given and follows the timing described in Section 2.

To reduce the number of policy instruments in the government's problem, we use the following function derived from the government's budget constraint to express tax

$$\mathcal{T}(u, b) := ub.$$

Then the government's problem can be equivalently written as one of choosing policies $\{b_t\}_{t=0}^{\infty}$, and allocation and prices $\{w_t, s_t, \theta_t, u_{t+1}\}_{t=0}^{\infty}$ to maximize utility subject to the government budget constraint and competitive equilibrium conditions.[11] Formally, the *government period return function* at time t is given by $R(u_t, b_t, w_t, s_t) = (1-u_t)U(w_t - \mathcal{T}(u_t, b_t), 0) + u_t U(h + b_t - \mathcal{T}(u_t, b_t), s_t)$.

Definition 2. (Ramsey policy) Given an initial unemployment rate u_0 and aggregate labor productivity z_0, the optimal government policy with commitment consists of a sequence of benefits and taxes $\{b_t\}_{t=0}^{\infty}$ that solves

$$\max_{\{b_t, w_t, s_t, \theta_t, u_{t+1}\}_{t=0}^{\infty}} \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t R(u_t, b_t, w_t, s_t)$$

over the set of all policies that satisfy the worker's law of motion equation (2) and the competitive equilibrium conditions (8)-(10), for all time t and aggregate shock $\{z_t\}_{t=0}^{\infty}$.

For easy exposition, we rewrite the competitive equilibrium conditions sequentially and use auxiliary functions $\tilde{\eta}_0$, $\tilde{\eta}_1$, $\tilde{\eta}_2$ and $\tilde{\eta}_3$ to denote the flow equation and the three private-sector optimality

[11] This is the primal approach to Ramsey problem.

conditions (8)-(10) respectively, [12]

$$\tilde{\eta}_0(u_t, s_t, \theta_t, u_{t+1}) = 0 \tag{11}$$

$$\tilde{\eta}_1(u_t, b_t, s_t, \theta_t, u_{t+1}, b_{t+1}, w_{t+1}, s_{t+1}, \theta_{t+1}) = 0 \tag{12}$$

$$\tilde{\eta}_2(\theta_t, z_{t+1}, w_{t+1}, \theta_{t+1}) = 0 \tag{13}$$

$$\tilde{\eta}_3(z_t, u_t, b_t, w_t, s_t, \theta_t) = 0 \tag{14}$$

where the three private-sector optimality conditions play the role of incentive constraints in the optimal policy problem, similar to the incentive constraints in a principal-agent setup, e.g. Hopenhayn and Nicolini (1997).

To derive a set of conditions that characterize the Ramsey policy, we let $\beta^t \pi^t \lambda_t$, $\beta^t \pi^t \mu_t$, $\beta^t \pi^t \gamma_t$ and $\beta^t \pi^t \nu_t$ be the Lagrange multipliers on (11)-(14), where π^t is the probability of a history realization $\{z_0, z_1, \ldots, z_t\}$ given an initial condition z_0. The optimal government policy can be characterized by the following government's first-order conditions with respect to b_t, w_t, s_t, θ_t and u_{t+1} for all time $t > 0$

$$\begin{aligned}
\mu_{t-1} \frac{\tilde{\eta}_{1b',t-1}}{\beta} + \mu_t \tilde{\eta}_{1b,t} + \nu_t \tilde{\eta}_{3b,t} &= R_{b,t} \\
\mu_{t-1} \frac{\tilde{\eta}_{1w',t-1}}{\beta} + \gamma_{t-1} \frac{\tilde{\eta}_{2w',t-1}}{\beta} + \nu_t \tilde{\eta}_{3w,t} &= R_{w,t} \\
\mu_{t-1} \frac{\tilde{\eta}_{1s',t-1}}{\beta} + \lambda_t \tilde{\eta}_{0s,t} + \mu_t \tilde{\eta}_{1s,t} + \nu_t \tilde{\eta}_{3s,t} &= R_{s,t} \\
\mu_{t-1} \frac{\tilde{\eta}_{1\theta',t-1}}{\beta} + \gamma_{t-1} \frac{\tilde{\eta}_{2\theta',t-1}}{\beta} + \lambda_t \tilde{\eta}_{0\theta,t} + \mu_t \tilde{\eta}_{1\theta,t} + \gamma_t \tilde{\eta}_{2\theta,t} + \nu_t \tilde{\eta}_{3\theta,t} &= 0 \\
\lambda_t \tilde{\eta}_{0u',t} + \mu_t \tilde{\eta}_{1u',t} &= \beta \mathbb{E}_t \{ R_{u,t+1} - \lambda_{t+1} \tilde{\eta}_{0u,t+1} \\
&\quad - \mu_{t+1} \tilde{\eta}_{1u,t+1} - \nu_{t+1} \tilde{\eta}_{3u,t+1} \}
\end{aligned} \tag{RAM}$$

where primes denote next period, and subscripts are derivatives.

The period-t solution is state dependent. It depends on the current productivity z_t and the beginning-of-period unemployment level u_t, as well as multipliers $(\mu_{t-1}, \gamma_{t-1})$. μ is the marginal value of relaxing the optimal search condition for the unemployed worker (8), and γ is the marginal value of relaxing the firm's equilibrium free-entry condition (9). The presence of μ_{t-1} and γ_{t-1} as states in the optimal policy captures commitment—the Ramsey government in period t has to deliver these marginal values, which it promised for the worker and firm in period $t-1$.

Note that commitment is assumed in the Ramsey case. If given the choice to break promise, the government will deviate from the sequence of policies prescribed by the government at time 0. The government of period t has an incentive to promise low future unemployment benefits to encourage search and vacancy posting, because, as explained in Section 2, current search and job-finding probability are higher when the future benefits are expected to be lower. But after employment outcome of

[12] See Appendix A for more details.

period t has realized, the government has an incentive to smooth workers' consumption by providing high benefits. This incentive to deviate from original plan is what constitutes time inconsistency in the Ramsey problem.

3.3 Markov government

In this section, we consider government policies that are time consistent. We use the concept of Markov-perfect equilibrium, similar to that in Klein, Krusell and Ríos-Rull (2008). By construction, the government policy in such an equilibrium is time consistent. As a result, lack of commitment does not make a difference in policy outcome.

Intuitively, one can think of the economy as having a sequence of governments, each lasting only one period. Each successive government only chooses current policy, taking future governments' policies as given. It neither considers how its policy affects previous periods, nor can it directly choose policies for future periods. Like Klein, Krusell and Ríos-Rull (2008), we focus on equilibria where government policy depends differentiably on the state of the economy.

The timing of events is illustrated in Figure 1. At the beginning of each period, the government chooses its benefit and tax policy for the current period. The private-sector agents (firms and workers) then move to choose its current period search, vacancy posting and wage, as described in Section 2. Because the economy consists of a mass of workers and firms, each of measure zero, the private-sector agents take future government policies as given.

The equilibrium described above can be equivalently stated as an equilibrium where the government chooses policy and private-sector allocations together given the state of the economy. The *government period return function* in this case is given by $R(u,b,w,s) = (1-u)U(w-\mathcal{T}(u,b),0) + uU(b-\mathcal{T}(u,b),s)$.

Definition 3. (Markov-perfect equilibrium) A Markov-perfect equilibrium consists of a value function G, government policy function Ψ, and private decision rules $\{W,S,\Theta,\Pi\}$ such that for all beginning-of-period unemployment u and aggregate productivity z, $b=\Psi(z,u)$, $w=W(z,u)$, $s=S(z,u)$, $\theta=\Theta(z,u)$ and $u'=\Pi(z,u)$ solve

$$\max_{b,w,s,\theta,u'} \quad R(u,b,w,s)+\beta\mathbb{E}G(z',u')$$

subject to

- the worker's law of motion

$$\eta_0(u,s,\theta,u') = u'-\delta(1-u)-[1-f(\theta)s]\,u \tag{15}$$

- the private-sector optimality conditions below

$$\eta_1(u,b,s,\theta,z',u';\Psi,W,S,\Theta) := \frac{-U_s(b-\mathcal{T}(u,b),s)}{f(\theta)}$$

$$-\beta E\left[U(W(z',u')-\mathcal{T}(u',\Psi(z',u')),0)-U(\Psi(z',u')-\mathcal{T}(u',\Psi(z',u')),S(z',u'))\right]$$

$$(16) \qquad -\beta E\left[(1-f(\Theta(z',\Psi(z',u')))S(z',u')-\delta)\frac{-U_s(\Psi(z',u')-\mathcal{T}(u',\Psi(z',u')),S(z',u'))}{f(\Theta(z',u'))}\right]=0$$

$$(17) \qquad \eta_2(\theta,z',u';W,\Theta) := \frac{\kappa}{q(\theta)}-\beta E\left[z'-W(z',u')+(1-\delta)\frac{\kappa}{q(\Theta(z',u'))}\right]=0$$

$$\eta_3(z,u,b,w,s,\theta) := \zeta U_c(w-\mathcal{T}(u,b),0)\left[z-w+(1-\delta)\frac{\kappa}{q(\theta)}\right]$$

$$(18) \qquad -(1-\zeta)\left[U(w-\mathcal{T}(u,b),0)-U(b-\mathcal{T}(u,b),s)+(1-f(\theta)s-\delta)\frac{-U_s(b-\mathcal{T}(u,b),s)}{f(\theta)}\right]=0$$

and

- the government value function satisfies the functional equation

$$(19) \qquad G(z,u) \equiv R(u,\Psi(z,u),W(z,u),S(z,u))+\beta\mathbb{E}G(z',\Pi(z,u))$$

Note that equation (19) reflects that future planners follow the policy rules $\{\Psi,W,S,\Theta,\Pi\}$. The Markovian assumption is reflected in the policy functions being time independent and only a function of the aggregate state. For ease of exposition, we have used the auxiliary functions η_0, η_1, η_2, η_3. Because policies are functions of aggregate productivity and unemployment, the auxiliary functions η_1 and η_2 are functions of z' and u'. In comparison, the Ramsey auxiliary functions $\tilde{\eta}_1$ and $\tilde{\eta}_2$ as defined in (12) and (13) are direct functions of next period decisions such as b', s'.

Let λ, μ, γ, ν be the Lagrange multipliers on (15)-(18), respectively. The benefit policy in a Markov equilibrium can be characterized by the following government's first-order conditions with respect to b, w, s, θ, and u':

$$\begin{aligned}
\mu\eta_{1b}+\nu\eta_{3b} &= R_b \\
\nu\eta_{3w} &= R_w \\
\lambda\eta_{0s}+\mu\eta_{1s}+\nu\eta_{3s} &= R_s \\
\lambda\eta_{0\theta}+\mu\eta_{1\theta}+\gamma\eta_{2\theta}+\nu\eta_{3\theta} &= 0 \\
\lambda\eta_{0u'}+\mu\eta_{1u'}+\gamma\eta_{2u'} &= \beta\mathbb{E}G'_u=\beta\mathbb{E}\left\{R'_u-\lambda'\eta'_{0u}-\mu'\eta'_{1u}-\nu'\eta'_{3u}\right\} \qquad (MAR)
\end{aligned}$$

where primes denote next period, and subscripts are derivatives. Note that because η_1 and η_2 contain functions of next period unemployment u' in the form of next period policy functions, derivatives of $\eta_{1u'}$ and $\eta_{2u'}$ contain policy function derivatives.

The Markov-perfect equilibrium is then characterized by a system of *functional* equations (1), (2), (16)-(18) and (MAR). An analytical characterization of the Markov-perfect equilibrium is not available. We solve for the equilibrium numerically using a standard cubic spline projection method to approximate the policy functions.

3.3.1 The Generalized Euler Equation

To build some intuition for how b is determined, we combine the government first-order conditions into a single equation that characterizes the Markov benefit policy[13]

$$
\begin{aligned}
&\left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w\right) \\
&+\mathbb{E}\left(-\frac{\eta_{1b}}{\eta_{1u'}}\right)\left\{\begin{array}{l} \left(-\frac{\eta_{0u'}}{\eta_{0s}}\right)\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w\right) - \frac{\eta_{3s}}{\eta_{3w}} R_w\right] \\ +\left(-\frac{\eta_{2u'}}{\eta_{2\theta}}\right)\left[-\frac{\eta_{1\theta}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w\right) - \frac{\eta_{0\theta}}{\eta_{0s}}\left(R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w\right) - \frac{\eta_{3s}}{\eta_{3w}} R_w\right) - \frac{\eta_{3\theta}}{\eta_{3w}} R_w\right] \end{array}\right\} \\
&+\beta\mathbb{E}\left(-\frac{\eta_{1b}}{\eta_{1u'}}\right)\left\{R'_u - \frac{\eta'_{0u}}{\eta'_{0s}}\left[R'_s - \frac{\eta'_{1s}}{\eta'_{1b}}\left(R'_b - \frac{\eta'_{3b}}{\eta'_{3w}} R'_w\right) - \frac{\eta'_{3s}}{\eta'_{3w}} R'_w\right] - \frac{\eta'_{3u}}{\eta'_{3w}} R'_w\right\} \\
&+\beta\mathbb{E} \underbrace{\left(-\frac{\eta_{1b}}{\eta_{1u'}}\right)\left(-\frac{\eta'_{1u}}{\eta'_{1b}}\right)}_{db'/db \text{ holding } \eta'_1=0,u'' \text{ unchanged}} \left(R'_b - \frac{\eta'_{3b}}{\eta'_{3w}} R'_w\right) = 0 \qquad (GEE)
\end{aligned}
$$

Because of the presence of policy function derivatives in auxiliary function derivatives, the above equation is also known as the Generalized Euler Equation or GEE. From the GEE, it is obvious any change in b has three trade-offs. First, the trade-off between consumption of unemployed and employed (first line). Second, the trade-off of current welfare through search and job posting (second and third lines) against future welfare through unemployment (fourth line). In particular, a higher b increases u', leading to lower s since b' is higher in expectation. Lower s increases today's welfare. At the same time, a higher u' reduces next period's welfare R both directly and indirectly through effects on search and wages. Lastly, the next period's trade-off between consumption of unemployed and employed (last line). The weight on the last term can be thought of as db'/db holding $\eta'_1 = 0$ and u'' unchanged The government determines current benefit by setting the net marginal value of b to zero.

Note that the GEE does not contain explicitly the derivative of Ψ; it appears indirectly in private-sector auxiliary function derivative $\eta_{1u'}$. This reflects an important point made earlier—the successive governments agree on a policy rule Ψ. The Markov government does not try to manipulate its successor through changing current b, hence the absence of directives of Ψ directly from the GEE. The fact that Ψ affects private-sector auxiliary function derivative captures the fact that how much a lower b increases private-sector search (and other decisions) depends on how the extra search will reduce next period unemployment. This makes the Markov government differ significantly from a government in a dynamic game setting. In that case, each successive government manipulates the next government to set a lower b than it chooses. Such a strategy leads to high consumption (high current b) and low future unemployment (low future b and hence high search).

[13] Appendix A contains two derivations of the GEE, one with policy functions defined as before, e.g. $S(z,u)$, and the other with policy functions such as $\tilde{S}(z,b,u)$. It can shown that the equilibria based on the two definitions are in fact equivalent.

3.4 The role of commitment

By comparing first-order conditions of the Ramsey government (RAM) and the Markov government (MAR), two key differences emerge.

First, the Markov optimality conditions do not contain promised marginal values from the previous period (Lagrange multipliers μ_{t-1} and γ_{t-1}) as the Ramsey conditions do. μ_{t-1} and γ_{t-1} are the marginal values to the unemployed workers and firms in period $t-1$, respectively. These marginal values are affected by expected policy and allocations in period t. For example, higher benefits in period t reduce expected gain from search and vacancy posting thus decreasing search and vacancy posting in period $t-1$. The presence of these marginal values as state variables in the Ramsey optimality conditions means that the Ramsey government needs to choose policies that can deliver these promises. In contrast, because the Markov government lacks commitment, it does not internalize how current policy affects incentives in the previous period, and thus does not deliver these promises.

The second difference is the presence of policy derivatives in the Markov auxiliary functions. In particular,

$$\eta_{1u'} \equiv \frac{\partial \eta_1}{\partial u'} + \underbrace{\frac{\partial \eta_1}{\partial b'}\Psi'_u + \frac{\partial \eta_1}{\partial w'}W'_u + \frac{\partial \eta_1}{\partial s'}S'_u + \frac{\partial \eta_1}{\partial \theta'}\Theta'_u}_{\text{disciplining effect}}$$

$$\eta_{2u'} \equiv \underbrace{\frac{\partial \eta_2}{\partial w'}W'_u + \frac{\partial \eta_2}{\partial \theta'}\Theta'_u}_{\text{disciplining effect}}$$

These policy derivatives illustrate how the current Markov government can induce the future governments to choose certain policy by changing the states of the economy. More specifically, the Markov government, by choosing current benefit level, affects search intensity in the current period[14], thus affecting next period's unemployment (and next period's state) holding other things constant. Next period's unemployment in turn influences how the government in the next period chooses its policy. This *disciplining effect* is captured by the presence of policy derivatives in the auxiliary functions. In contrast, because the Ramsey government can commit to future policies, its policy function is a sequence of states-contingent plans. In particular, the Ramsey government chooses in period 0 a sequence of state-contingent policies and allocations for all future periods.

4 Calibration

We describe our calibration strategy in this section. The model period is taken to be one week. We calibrate the model to match important features of the U.S. labor market, using a fixed unemployment

[14] The effect of benefit level on current period search works through non-separability of preference. Under our parametrization, lower benefit level increases current search intensity.

benefit schedule that is roughly in line with the current U.S. policy. Then in the next section, we study what the Ramsey and Markov policies should be in this model economy.[15]

The utility function is

$$U(c,s) = \frac{1}{1-\sigma}\left(\left[\frac{c}{v(s)}\right]^{1-\sigma} - 1\right)$$

where $v(s)$ is the cost of search. For $\sigma \neq 1$, this utility function represents a preference non-separable in c and s. We assume $v(\cdot)$ is a non-negative, strictly increasing and convex function, with the property that $v(0)$ is bounded and $v(0) > 0$. We choose the search cost function to be

$$v(s) = \exp\left(\frac{A}{1+\phi}[(1-s)^{-(1+\phi)} - 1] - (A-1)s\right)$$

This functional form is chosen to guarantee that s is strictly less than 1. In particular, for any $A > 0$, v exhibits positive and increasing marginal cost, $v_s(s) > 0$ and $v_{ss}(s) > 0$, $v(1) = v_s(1) = \infty$, and $v(0) = v_s(0) = 1 > 0$. With this functional form for search cost, when $\sigma = 1$, the utility function can be shown (using L'Hospital's Rule) to reduce to $\log c - \log v(s)$, which is utility function often used in the literature.[16] When $\sigma \neq 1$, the utility function features non-separability between consumption and search. This allows the Markov government to have disciplining power over its successor.

We adopt the matching function from Den Haan, Ramey and Watson (2000), which is also used in Hagedorn and Manovskii (2008) and Mitman and Rabinovich (2015)

$$M(su,v) = \frac{(su)v}{[(su)^{\chi} + v^{\chi}]^{1/\chi}}$$

Together with the search cost function, this matching function guarantees that both the job-finding rate, $f(\theta)s$ and the job-filling rate $q(\theta)$ are always strictly less than 1.

As in Shimer (2005), labor productivity z_t is taken to be average real output per person in the non-farm business sector. This measure is taken from the seasonally adjusted quarterly data constructed by the Bureau of Labor Statistics. We normalize the mean productivity to be $\bar{z} = 1$, and assume the shock to z follows an AR(1) process:

$$\log z_t = \rho \log z_{t-1} + \sigma_\epsilon \epsilon_t$$

where $\rho \in [0,1)$, $\sigma_\epsilon > 0$, and ϵ_t are i.i.d. standard normal random variables. The parameters are estimated to be $\rho = 0.9895$ and $\sigma_\epsilon = 0.0034$ at the weekly frequency.

It is well-known that without wage rigidity the search and matching model cannot easily generate the scale of cyclical fluctuations observed in reality.[17] We introduce wage rigidity in the form of

[15] This is also the strategy used by Mitman and Rabinovich (2015) and Jung and Kuester (2015).
[16] Section 5 contains a discussion on separable preference.
[17] See, for example, Shimer (2005), Hall (2005), and Hagedorn and Manovskii (2008) for more details on wage rigidity.

Table 1: Summary of Calibration

Parameter	Description	Value
β	Discount factor	$0.99^{1/12}$
h	Value of leisure	0.5
δ	Separation rate	0.008
κ	Vacancy posting cost	0.58
ρ	Persistence of productivity	0.9895
σ_ϵ	Std of innovation to productivity	0.0034
χ	Matching parameter	0.579
A	Disutility of search	0.089
ζ	Steady-state worker's bargaining power	0.604
ϵ_ζ	Degree of cyclicality of ζ	-1.25
$\bar{b}$	Unemployment benefit	0.4

countercyclical worker's bargaining power. Specifically, the worker's bargaining power is a function of aggregate productivity with the following functional form,[18]

$$\zeta(z) = \exp(\log \bar{\zeta} + \epsilon_\zeta \log z), \;\; \epsilon_\zeta < 0,$$

where $\bar{\zeta}$ represents the steady-state worker's bargaining power, and ϵ_ζ is the elasticity of bargaining power with respect to the aggregate productivity, effectively a measure of the degree of cyclicality of worker's bargaining power.

We set the discount factor $\beta = 0.99^{1/12}$, giving a quarterly discount factor of 0.99. The coefficient of relative risk aversion is $\sigma = 0.75$. Although this number is small compared to those used in the macro literature, we believe it is within reasonable range given the weekly frequency.[19] Hagedorn and Manovskii (2008) estimate weekly job separation rate to be 0.0081. They also estimate the costs of vacancy creation to be 58% of weekly labor productivity. Therefore, we set the job separation parameter $\delta = 0.0081$ and cost of vacancy posting $\kappa = 0.58$.

The value of non-market activity is taken to be $h = 0.5$, within the range of common values used in the search literature.[20] We set the search cost curvature parameter ϕ to 1.

This leaves us five parameters to be calibrated jointly: (1) the matching function parameter χ, (2) the level parameter of search cost A, (3) the worker's share of bargaining power in steady state $\bar{\zeta}$,

[18] Similar assumptions are common in the literature. Landais, Michaillat and Saez (2010) and Nakajima (2012) both directly specify $w(z) = \exp(\log \bar{w} + \epsilon_w z)$. Jung and Kuester (2015) use a cyclical bargaining power structure similar to ours in their benchmark calibration.

[19] Hopenhayn and Nicolini (1997) also set the relative risk aversion coefficient in the range of $0.5 - 0.75$ for a weekly frequency.

[20] Hall (2006) estimates a value of leisure relative to productivity at about 43%.

Table 2: Summary Statistics

Statistic		z	u'	v	v/u
Quarterly U.S. data 1951-2004					
Standard deviation		0.013	0.125	0.139	0.259
	z	1	-0.302	0.460	0.393
Correlation matrix	u'	-	1	-0.919	-0.977
	v	-	-	1	0.982
	v/u	-	-	-	1
Calibrated economy					
Standard deviation		0.013	0.125	0.131	0.221
	z	1	-0.875	0.796	0.928
Correlation matrix	u'	-	1	-0.611	-0.902
	v	-	-	1	0.887
	v/u	-	-	-	1

Note: Standard deviations and correlations are reported in log quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

(4) the degree of cyclicality of worker's bargaining power ϵ_ζ, and (5) the unemployment benefit. We simultaneously target five measured moments in the U.S. economy during 1951-2004: (1) the average weekly job-finding rate of workers 0.139, (2) the average weekly job-filling rate of firms 0.266, (3) the standard deviation of the unemployment rate 0.125, (4) the standard deviation of the vacancy-unemployment ratio 0.259, (5) the replacement rate of unemployment insurance (the ratio of benefit level to wage income when employed) 40 percent.[21] These data targets are directly measured in the U.S. data from 1951-2004.

The calibrated parameters are summarized in Table 1. Table 2 compares some labor market statistics in the U.S. economy and the calibrated economy. The calibrated model does a good job generating the relevant correlations. In particular, the model delivers negative correlation between unemployment and vacancy, thus preserving the Beveridge-curve relationship.

5 Quantitative Analysis

In this section, we describe the quantitative results of the model. In order to investigate how the economy behaves under the Ramsey policy and Markov policy, we bring the calibrated parameters

[21] We use a derivative-free algorithm for least-squares minimization to perform joint calibration. See Zhang, Conn and Scheinberg (2010) for details.

Table 3: Steady-State Statistics

Statistic	Ramsey	Markov
Benefit, b	0.120	0.454
Wages, w	0.975	0.982
Search, s	0.602	0.162
Vacancy, v	0.036	0.023
Unemployment, u	0.032	0.138
Replacement ratio(%)	12.3	46.2
Average consumption	0.960	0.916
Consumption equivalent welfare change(%)	–	4.84

Note: Steady states are computed using parameters calibrated to benchmark economy.

into the model under each policy.

5.1 Steady-state comparison

Table 3 compares steady states under the Ramsey and Markov policies. Not surprisingly, the Ramsey economy performs better than the Markov economy. The Markov government gives a much higher unemployment benefit than does the Ramsey government. The replacement ratio is 46% in the Markov economy, as opposed to 12% in the Ramsey economy. Although we do not calibrate to the Markov economy, its replacement ratio is much closer to the 40% replacement ratio as found in the U.S. economy. This suggests that non-commitment is a better description of the current U.S. policy. With higher benefit level, unemployed workers have less incentives to search. In fact, the Markov economy has much lower search intensity than the Ramsey economy.

Higher benefits give workers higher outside options, so wages are slightly higher in the Markov economy. Higher wages indicate lower profits for firms, and hence lower vacancy posting under the Markov policy. Lower search by workers and lower vacancy posting by firms lead to much higher unemployment level in the Markov economy. Therefore, output is lower and agents consume less in the Markov economy. In terms of welfare, the average consumption in the Markov economy has to increase by 4.84% to be equivalent to the Ramsey economy in steady state.

Table 3 highlights the importance of commitment. The government has two opposing incentives. One is insurance, through providing higher unemployment benefit to help workers smooth consumption. The other incentive is job creation, by giving lower benefits to encourage search and vacancy posting, thereby lowering unemployment and increasing output. Ideally, the government would like higher benefit in the current period and lower benefits in the following periods because, as discussed before, current search and job posting are mainly affected by expectations of future benefits. But when the government lacks the ability to commit to future policies—as in the case of the Markov economy and almost all governments in reality—it cannot make any credible promise of lower benefit in the future. As a result, such government consistently provides higher than optimal benefits and leads the economy into a state of high unemployment, low output and low welfare.

5.2 Policy functions

In this section, we present and compare Ramsey and Markov equilibrium policy functions solved using cubic spline projection method.

Figure 2 plots the Ramsey policy (left) and the Markov equilibrium policy (right) functions for benefit (top panels) and next period unemployment (bottom panels), holding productivity at the steady state level.[22] In each plot, the solid line represent policy function, and the dashed line indicates steady state unemployment rate.[23]

First, consider unemployment benefit (the top panels). The optimal unemployment benefit in the Ramsey case is decreasing in unemployment level, whereas the Markov benefit is only slightly decreasing in unemployment. One key difference between these two governments is that the Ramsey government internalizes the impact of its current policy on the actions of private sector in previous periods. When unemployment level is high, the marginal social benefit of job creation is higher, because the expected output gain of increasing vacancy positing is proportional to the number of unemployed workers. Thus the Ramsey government reduces unemployment benefit when unemployment is high, in order to induce more search and vacancy posting in the previous period.

In contrast, the Markov government considers the previous period *foregone* and hence does not internalize how previous period's expectation of current policy impacts the economy in the past. But it still has some incentive to decrease benefits when unemployment is high, so as to encourage search in the current period. At the same time, as more workers are unemployed, the government, with a utilitarian objective function, has a stronger motive to provide insurance and help smooth consumption. Overall, these two effects almost cancel each other out, and the Markov government only slightly decreases benefits when unemployment is high.

The bottom panels of Figure 2 plot the next period unemployment policy functions u' associated with the Ramsey policy (left) and the Markov policy (right). In both cases, the policy function is increasing in current unemployment and coincides with the 45-degree line once at the steady state. Notice that the slope of the Ramsey unemployment is flatter than that of the Markov unemployment. This is because the Ramsey government, by planning a sequence of policies at time 0, has more control over the economy, and thus can move the next period unemployment further away from

[22] The Ramsey policy function plots also hold promised marginal utilities μ_- and γ_- at their respective steady state level. Note that even though we solve Ramsey policies as functions, the solution to a Ramsey problem really should be understood as *sequences of variables* from $t = 0$ to $t = \infty$, given some initial state, (u_0, z_0) in this case.

[23] Appendix B contains other policy function plots, holding either unemployment or productivity at steady state.

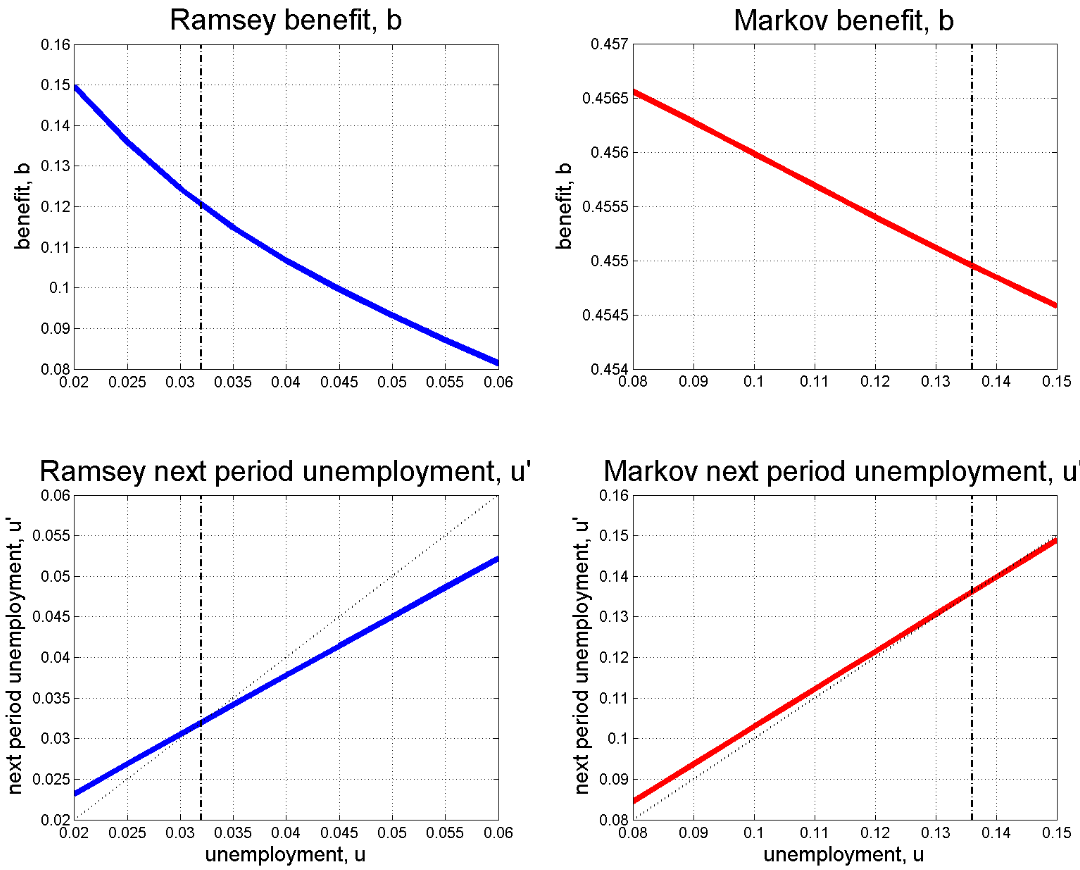

Figure 2: Ramsey (left) and Markov (right) benefit (top panels) and unemployment (bottom panels) policy functions holding productivity at steady state. In each plot, the solid line denotes policy function, and the dashed line indicates steady-state unemployment level. The bottom panels also plot the 45° line (the thin dotted).

current unemployment. The Markov government, in contrast, can only influence the next period economy through the *disciplining effect* on the next government, and thus has smaller power over the state of the economy.

Figure 3 plots the Ramsey (left) and the Markov equilibrium (right) benefit policy functions, holding unemployment at the steady state level. The Ramsey and Markov unemployment benefits are decreasing and increasing, respectively, in productivity. In other words, when productivity is low, optimal benefit is high whereas the Markov government provides low benefit. The difference comes again from the lack of commitment by the Markov government. From the perspective of the Ramsey government, the marginal social benefit of job creation is lower when productivity is low, since the output of each firm-worker pair is low. As a result, the marginal social cost (in the form of lower search and fewer vacancy postings) of unemployment benefit is low. So the Ramsey government provides high benefit.

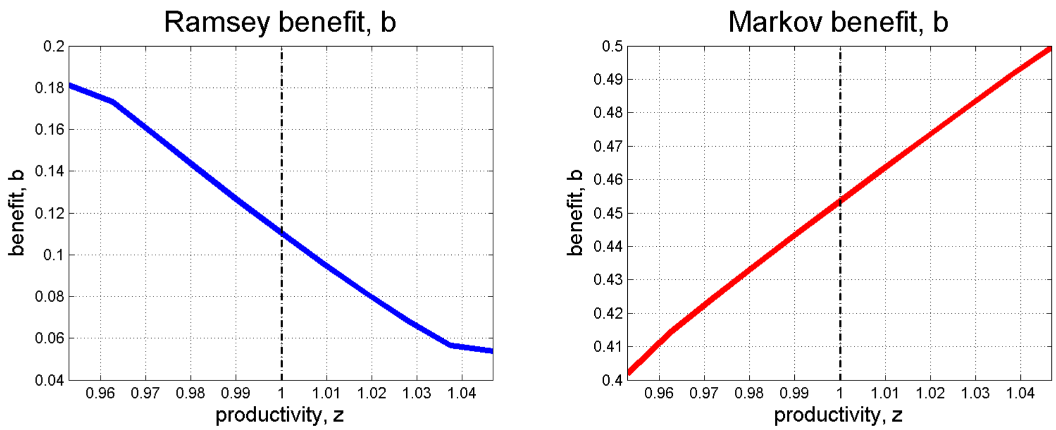

Figure 3: Ramsey (left) and Markov (right) benefit policy functions holding unemployment at steady state. In each plot, the solid line denotes policy function, and the dashed line indicates steady-state productivity level.

In contrast, the Markov government does not internalize the changing social marginal cost of benefits in the form of job creation. The Markov government weighs the welfare gain from redistribution against the financing cost of benefits. When productivity is low, output and the aggregate resource in the economy are low. As a result, the marginal cost of financing benefits is high, and so the Markov government provides low benefits. In addition, with persistent shocks, low productivity implies that future productivities are also likely to be low. With expectations of low future productivity, firms reduce vacancy posting. The Markov government reduces unemployment benefit to encourage current period search and vacancy posting,[24] thus increasing future aggregate resources.

5.3 Dynamics

To understand how the Ramsey and Markov economy behave over time, we simulate each economy. Figure 4 plots and compares the dynamic responses of key variables in the Ramsey and Markov economy, as well as the economy under the current policy in the baseline calibration, to a 1% drop in productivity. The optimal benefit level initially jumps up, then falls for about 30 weeks following the shock, and slowly reverts to its pre-shock level. Unemployment rises in response to the drop in productivity and continues rising for about 10 weeks before falling back to its pre-shock level. The Markov government, however, reduces benefits in response to lower productivity, and slowly raises benefits back to its pre-shock level. Because of the different initial responses in benefit policy, unemployment in the Markov economy also responds markedly differently compared to the Ramsey economy. Unemployment jumps up much more when the shock hits, then rises for about 25 weeks, and slowly falls back to pre-shock level.

[24] This effect works because preferences are non-separable in consumption and search intensity. Under our parameterization, the cross derivative of utility in benefit and search is negative. So when benefits are low, the marginal utility (cost) of search is high (low), and thus search is high. Higher search intensity increases the per-vacancy job-filling rate, so firms have more incentive to post vacancy.

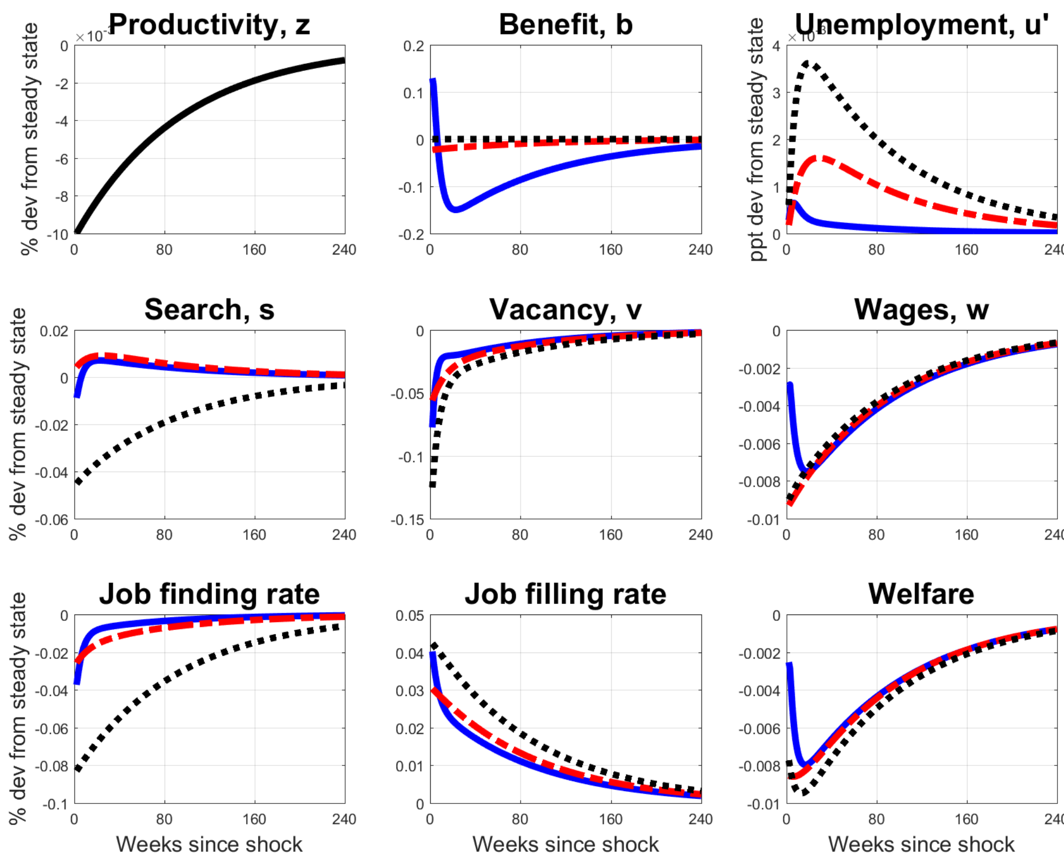

Figure 4: Ramsey (solid blue line), Markov (dashed red line) and U.S. policy (dotted grey line) responses to a 1% drop in productivity.

Such responses to a negative shock are consistent with the properties of the Ramsey and Markov policy functions. Immediately after the negative shock, productivity is low, and so is the social value of employment. As a result, the Ramsey government tolerates the rise in unemployment. The Markov government, not internalizing the changing social value of job creation, lowers benefits in response to lower aggregate output after the shock. As unemployment rises, the social benefit of creating more vacancies increases relative to the benefit of providing insurance, and the Ramsey government therefore cuts unemployment benefits to reduce unemployment. The Markov government, however, does not internalize the effect of benefits on job creation in the previous period; instead, after the initial drop, the Markov benefit rises as productivity rises (more resources to redistribute). Because the Markov government does not increase benefits immediately after the shock, unemployment does not peak until about 25 weeks after the shock, and peaks at a higher level, in deviation terms, than unemployment in the Ramsey economy.

Furthermore, the Markov economy features a much *slower* recovery in unemployment. The reason is as follows. We can rewrite equation (2), the law of motion of unemployment, as

$$\hat{u}' \approx [1 - f(\theta)s - \delta]\hat{u}$$

where $\hat{u} = u - \bar{u}$ is the deviation from the steady state. Due to lower search intensity and lower job posting, the job finding rate, $f(\theta)s$, is lower in the Markov economy than that in the Ramsey economy. It means that the unemployment deviation decays at a slower rate in the Markov economy. In other words, unemployment in the Markov economy recovers at a slower pace.

Turning to wages. Wages fall less, in percent deviation terms, in the Ramsey economy than they do in the Markov economy. This is because the initial rise in Ramsey benefits smooths the fall in wages through an increase in the worker's outside option. Wages also fall for a longer period—for about 15 weeks before picking up—under the optimal policy, whereas wages in the Markov economy dip upon impact, and rise monotonically back to their pre-shock level.

Overall, the dynamic responses of key variables under the current U.S. policy is much more similar to that under the Markov policy than that under the Ramsey policy. The implication is that lack of commitment is a better description of the current U.S. policy. For the current policy, we have assumed a constant unemployment benefit schedule. For the Markov policy, the benefit does not vary much, although the government is free to change it. The reason is that without commitment, the government is not able to flexibly change its policy so as to provide insurance and stimulate search together. Therefore, both the current and the Markov economies experience larger deviations in unemployment levels compared to the Ramsey economy.

The impulse response of the Ramsey economy in our paper is similar to that in Mitman and Rabinovich (2015) (henceforth MR). Both feature an initial rise in unemployment benefit followed by subsequent declines. However, this is different from the impulse response in Jung and Kuester (2015) (henceforth JK). First, the government in JK has more policy tools, namely, unemployment benefits, hiring subsidies, and layoff taxes. They find that in recessions, the Ramsey government relies mostly on hiring subsidies and layoff taxes, and the role of unemployment benefits is much smaller. Second, when JK restrict policy tools to only unemployment benefits, they find that benefit decreases during recession without having the initial rise as in our and MR's papers. The reason is due to endogenous separation in their model. Upon receiving a negative shock, the Ramsey government in JK reduces unemployment benefits, which slows down the separation between firms and workers. The government thus provides insurance by having less workers laid off. In our model, as well as in MR, we have exogenous separation. The initial rise in benefits serves to not only smooth consumption for the unemployed, but also smooth consumption for the employed by increasing wages. If we introduce more policy tools or endogenous separation as in JK, our conjecture is that the impulse responses would be similar.

Table 4 reports long-run characteristics of the Markov and Ramsey policies. Consistent with results from the simulation exercise in Figure 4, the Ramsey benefit policy is much more volatile

Table 4: Long-Run Characteristics of Markov and Ramsey Policies

Statistic	Markov policy	Ramsey policy
Mean	0.454	0.120
Standard deviation	0.031	0.276
Correlation with		
Productivity	0.998	0.719
Current unemployment	-0.522	-0.870

Note: Means are reported in levels. Standard deviations are reported in log quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

Table 5: Simulated Statistics under Markov and Ramsey Policies

Statistic	Productivity	Unemployment	Wages	Search	Vacancy
Markov policy					
Mean	1	0.138	0.982	0.162	0.024
Standard deviation	0.013	0.024	0.012	0.014	0.055
Ramsey policy					
Mean	1	0.032	0.975	0.602	0.036
Standard deviation	0.013	0.019	0.011	0.011	0.040

Note: Means are reported in levels. Standard deviations are reported in log quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

than the Markov policy. This is because upon receiving negative shocks, the Ramsey government raises benefit initially and then reduces it subsequently, in order to induce a faster recovery. This optimal policy is feasible, because the Ramsey government is able to commit to it. Notice that over the long-run, both policies are positively correlated with productivity and negatively correlated with current period unemployment (inherited from the end of previous period). The positive correlation of the Ramsey policy with productivity may seem peculiar given that the Ramsey benefit is decreasing in current productivity in Figure 3. This is because in Figure 3 we hold the current unemployment at its steady state to isolate the policy's response to productivity alone. In contrast, over the long-run both unemployment and productivity move, and in opposite directions.

Finally, Table 5 summarizes the moments of key variables of the Markov and Ramsey economies. Most variables have similar volatilities. The means of the variables are the same as Table 3.

5.4 Discussion

In this section, we discuss extensions to the main quantitative results and explore key assumptions.

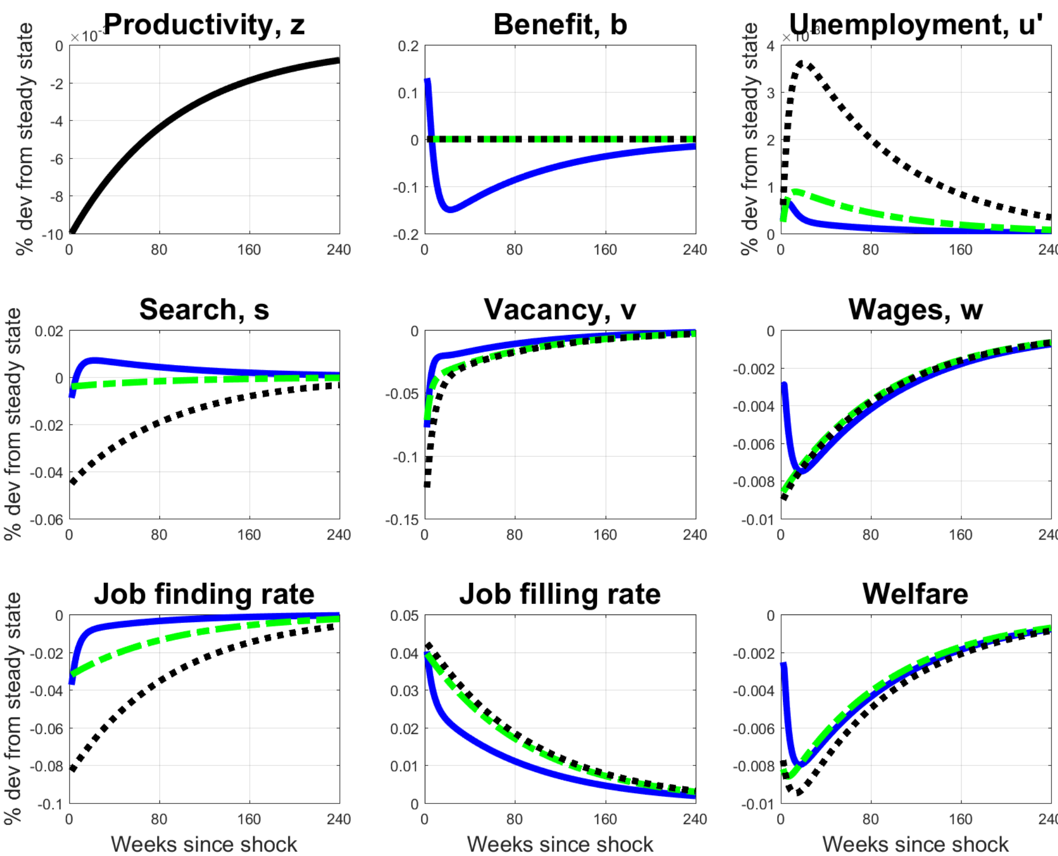

Figure 5: Ramsey (solid blue line) and U.S. policy (dotted grey line) and counterfactual policy (dashed green line) responses to a 1% drop in productivity.

5.4.1 The effect of cyclical policy (to be updated)

In the simulation exercise of Figure 5, the U.S. policy has higher steady state benefit than the Ramsey policy but does not have the cyclicality of the Ramsey policy. A natural question then is which of the two differences is more important in driving the difference in the underlying economy. To answer this question, we look at a counterfactual policy where the benefit policy has the same steady state level as the Ramsey policy (0.12 at the steady state), but has no cyclical variation. In Figure 5, the economy under this counterfactual policy is simulated together with the economies under the U.S. policy and the optimal policy. The resulting gap in unemployment between the U.S. policy and the counterfactual policy is much larger than the gap between the counterfactual and the optimal policy. This means that the effect of benefit level outweighs the effect of cyclicality in this calibration.

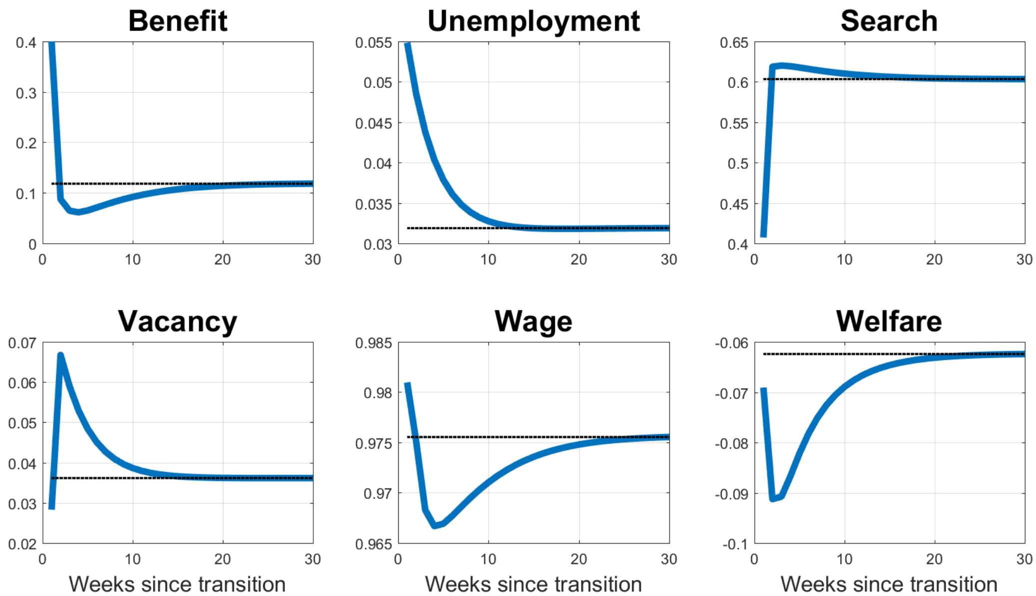

Figure 6: Transition from the U.S. economy to the Ramsey economy.

5.4.2 Transition analysis (to be updated)

In this section, we look at two scenarios where the government, starting at the U.S. policy, decides to follow the Ramsey policy or the Markov policy. We consider cases without any aggregate shocks to the economy. In the first scenario, we look at the transition from the U.S. policy to the Ramsey policy. In Figure 6, we assume the economy starts at period 1 with the U.S. policy at steady state. Starting at period 2, the government follows the optimal policy rule. Interestingly, the transition to a new steady state is not monotonic. For example, benefit at first falls below the steady state optimal benefit in response to a high unemployment level. As unemployment rate falls over the transition, driven by higher levels of search and vacancy postings, benefit level rises slightly. Unemployment initially drops quite fast, a result of drastic cut in benefit level. But as benefit starts rising, the fall in unemployment is slowed down by lower levels of search and vacancy postings. Even though the steady state welfare under the optimal policy is higher, over the transition, the economy initially suffers as welfare dips during the periods of low benefit and relatively high unemployment.

In the second scenario, we consider the transition from the U.S. policy to the Markov policy. In Figure 7, the economy starts at the steady state under the U.S. policy in period 1. Starting at period 2, the government follows the Markov policy rule. Again, the transition is not entirely monotonic. Benefit immediately increases above the steady state Markov benefit level. With higher benefit, both search and vacancy postings decrease, and as a result, unemployment increases. After several periods, benefit, search and vacancy postings start to decrease gradually, as unemployment rate continues to increase over the transition. Welfare increases initially because of the higher benefit

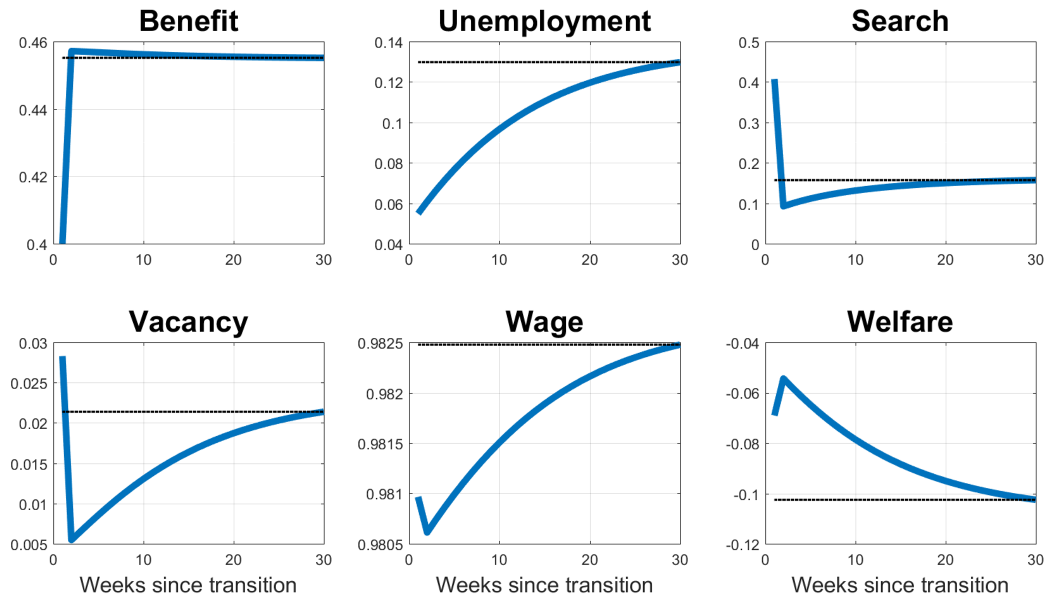

Figure 7: Transition from the U.S. economy to the Markov economy.

to the unemployed, but decreases to a much lower level over the transition due to the much higher unemployment rate. This shows that lack of commitment brings down welfare in the long run, despite a small gain in the short run.

5.4.3 Separable preferences

In the baseline calibration we allow for non-separable preferences. Under this specification, Markov and Ramsey government policies are starkly different both in the steady state and in response to a productivity shock. In this subsection, we relax the assumption of non-separable preference to look at the Markov policy with preferences separable in consumption and search. Here, the Markov government's optimality condition reduces to (the first line of GEE)

$$R_b + \underbrace{\left(-\frac{\eta_{3b}}{\eta_{3w}}\right)}_{=\partial w/\partial b|_{\eta_3=0}} R_w = 0 \tag{20}$$

Notice that this condition does not contain policy derivative. With separable preference, current benefit policy does not affect current period search and vacancy posting. As a result, the Markov government has no disciplining effect over future governments, and each government chooses benefit policy to maximize current period government return function ($dR/db = 0$). Effectively, the Markov benefit policy equates the marginal utility of worker and unemployed (the first term in (20)), taking into account how benefit affects equilibrium wage (the second term). In a way, each successive government behaves like "the last emperor".

Under the assumption of fixed wages (so the wage bargaining competitive equilibrium condition disappears), the Markov equilibrium with separable preference has an analytical solution.

Proposition 1. "The last emperor": Under the assumptions of separable utility and fixed wages $\bar{w}$, a Markov-perfect equilibrium is given by

$$b = \bar{w} - h, \quad s = 0, \quad u = 1, \quad \theta = q^{-1}\left(\frac{1-\beta(1-\delta)}{\beta(\bar{z}-\bar{w})}\kappa\right).$$

Proof. The proof is straightforward. With fixed wage, (20) reduces to $R_b = 0$, or equivalently,

$$U_c(h + b - \tau) = U_c(\bar{w} - \tau)$$

which, given strict monotonicity of preference in consumption, entails $h + b = \bar{w}$. Since unemployed workers receive the same consumption as employed workers, it follows that $s = 0$ and $u = 1$. When wages are fixed, steady-state market tightness is also fixed. □

5.4.4 Continuity of Markov-perfect equilibrium

As in the previous literature on dynamic games, we cannot prove general existence or uniqueness results for the Markov-perfect equilibrium. But with fixed wages, we can show the continuity of Markov equilibrium policy rules. Figure 8 shows that the Markov equilibria with non-separable preference converge monotonically and smoothly to the equilibrium with separable preference as $\sigma \to 1$. The figure plots the Markov equilibrium steady-state benefit (left) and unemployment (right) for economies with relative risk aversion σ ranging from 0.6 to 1, holding all other parameters as for the case of flexible-wage and given in Table 1. Wages are fixed at $\bar{w} = 0.982$, the steady-state level in the baseline flexible-wage Markov equilibrium. Circles indicate the 35 values of σ for which the Markov equilibrium is computed numerically. The values for $\sigma = 1$ correspond to the equilibrium computed analytically in Proposition 1. At $\sigma = 1$, the equilibrium features high benefit and high unemployment. As σ increases toward 1, both benefit and unemployment rise monotonically and converge smoothly toward the analytical equilibrium.

6 Conclusion

This paper studies how a welfare-maximizing government chooses UI benefit policy when the government can and cannot commit to future policies. We use the Ramsey policy to describe government policy with commitment, and the Markov-perfect equilibrium for a government without commitment.

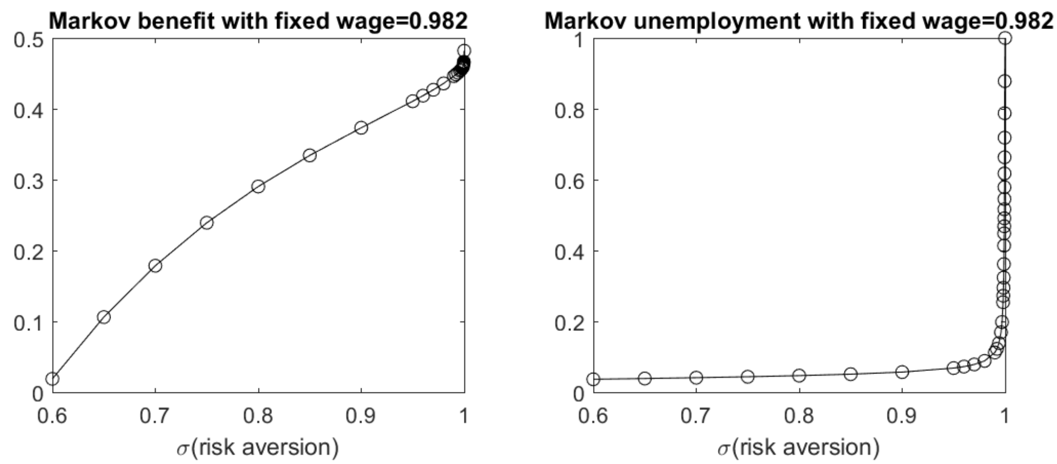

Figure 8: Continuity of Markov-perfect equilibrium. Markov equilibrium steady-state benefit (left) and unemployment (right) for economies with relative risk aversion σ ranging from 0.6 to 1. Wages are fixed at $\bar{w} = 0.982$. All other parameters follow Table 1. Circles indicate the 35 values of σ for which the Markov equilibrium is computed numerically. The values for $\sigma = 1$ correspond to the equilibrium computed analytically in Proposition 1.

The Markov equilibrium has higher benefits and higher unemployment rate than the Ramsey economy. Over the business cycle, the Ramsey policy—which is optimal—raises benefits at the beginning of a recession, and gradually decreases benefits over time. In contrast, the Markov policy—which is time consistent—reduces benefits at the onset of a recession, and slowly increases benefits as the economy recovers. Our findings thus highlight the importance of commitment when designing the optimal unemployment insurance policy over the business cycle.

A Derivations

A.1 Derivation of private sector optimality conditions

Solving unemployed person's problem by taking derivative with respect to s

$$\frac{-U_s(h+b-\tau,s)}{f(\theta)} = \beta\mathbb{E}[V^e(z',u') - V^u(z',u')] \tag{21}$$

Using worker's bellman equations

$$V^e(z,u) - V^u(z,u) = U(w-\tau,0) - U(h+b-\tau,s) + \beta(1-f(\theta)s-\delta)\mathbb{E}[V^e(z',u') - V^u(z',u')]$$

Combining the two equations

$$V^e(z,u) - V^u(z,u) = U(w-\tau,0) - U(h+b-\tau,s) + (1-f(\theta)s-\delta)\frac{-U_s(h+b-\tau,s)}{f(\theta)} \tag{22}$$

Update one period, take expectations and substitute into (21)

$$\frac{-U_s(h+b-\tau,s)}{f(\theta)} = \beta\mathbb{E}\left[U(w'-\tau',0) - U(h+b'-\tau',s') + (1-f(\theta')s'-\delta)\frac{-U_s(h+b'-\tau',s')}{f(\theta')}\right]$$

From unmatched firm's value function, assuming free entry, i.e. $J^u(z,u) = 0$

$$\frac{\kappa}{q(\theta)} = \beta\mathbb{E}J^e(z',u') \tag{23}$$

Then firm's value function can be rewritten as

$$J^e(z,u) = z - w + (1-\delta)\frac{\kappa}{q(\theta)} \tag{24}$$

Update one period, take expectations and substitute into (23)

$$\frac{\kappa}{q(\theta)} = \beta\mathbb{E}\left[z' - w' + (1-\delta)\frac{\kappa}{q(\theta')}\right]$$

Take first-order condition of the Nash bargaining problem (7) with respect to w

$$\zeta U_c(w-\tau,0)\Big[J^e(z,u) - J^u(z,u)\Big] \quad = \quad (1-\zeta)\Big[V^e(z,u) - V^u(z,u)\Big]$$

Substitute in (22) and (24)

$$\zeta U_c(w-\tau,0)\Big[z - w + (1-\delta)\frac{\kappa}{q(\theta)}\Big] \quad = \quad (1-\zeta)\Big[U(w-\tau,0) - U(h+b-\tau,s) + (1-f(\theta)s-\delta)\frac{-U_s(h+b-\tau,s)}{f(\theta)}\Big]$$

A.2 Definition of auxiliary functions in the Ramsey problem

$$
\begin{aligned}
\tilde{\eta}_1(u_t,b_t,s_t,\theta_t,u_{t+1},b_{t+1},\tau_{t+1},w_{t+1},s_{t+1},\theta_{t+1}) &= \frac{-U_s(h+b_t-\mathcal{T}(u_t,b_t),s_t)}{f(\theta_t)} \\
&\quad -\beta\mathbb{E}_t\Big[U(w_{t+1}-\tau_{t+1},0)-U(h+b_{t+1}-\mathcal{T}(u_{t+1},b_{t+1}),s_{t+1})\ldots \\
&\quad \ldots.+(1-f(\theta_{t+1})s_{t+1}-\delta)\frac{-U_s(h+b_{t+1}-\mathcal{T}(u_{t+1},b_{t+1}),s_{t+1})}{f(\theta_{t+1})}\Big]
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\eta}_2(\theta_t,z_{t+1},w_{t+1},\theta_{t+1}) &= \frac{\kappa}{q(\theta_t)}-\beta\mathbb{E}_t\left[z_{t+1}-w_{t+1}+(1-\delta)\frac{\kappa}{q(\theta_{t+1})}\right] \\
\tilde{\eta}_3(z_t,u_t,b_t,w_t,s_t,\theta_t) &= \zeta U_c(w_t-\tau_t,0)\left[z_t-w_t+(1-\delta)\frac{\kappa}{q(\theta_t)}\right] \\
&\quad -(1-\zeta)\Big[U(w_t-\mathcal{T}(u_t,b_t),0)-U(h+b_t-\mathcal{T}(u_t,b_t),s_t)\ldots \\
&\quad \ldots.+(1-f(\theta_t)s_t-\delta)\frac{-U_s(h+b_t-\mathcal{T}(u_t,b_t),s_t)}{f(\theta_t)}\Big] \\
\tilde{\eta}_0(u_t,s_t,\theta_t,u_{t+1}) &= u_{t+1}-\delta(1-u_t)-(1-f(\theta)s)u_t
\end{aligned}
$$

A.3 Derivation of Markov GEE

Throughout this section, we drop the dependence of functions on productivity shock z to economize on notation. Combine government first-order conditions,

$$
\begin{aligned}
&\frac{1}{\eta_{0s}}\left[R_s-\frac{\eta_{1s}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)-\frac{\eta_{3s}}{\eta_{3w}}R_w\right]+\frac{\eta_{1u'}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right) \\
&-\frac{\eta_{2u'}}{\eta_{2\theta}}\left[\frac{\eta_{0\theta}}{\eta_{0s}}\left(R_s-\frac{\eta_{1s}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)-\frac{\eta_{3s}}{\eta_{3w}}R_w\right)+\frac{\eta_{1\theta}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)+\frac{\eta_{3\theta}}{\eta_{3w}}R_w\right] \\
=\; &\beta\Omega'_u \qquad (FOC)
\end{aligned}
$$

Rewrite Bellman equation in shorthand

$$\Omega(u) \;=\; R(u,\Psi(u),W(u),S(u))+\beta\Omega(\Pi(u))$$

Taking derivative of Bellman equation with respect to u

$$\Omega_u \;=\; R_u+R_b\Psi_u+R_wW_u+R_sS_u+\beta\Omega'_u\Pi_u \qquad (ENV)$$

Combine FOC and ENV to eliminate $\beta\Omega'_u$

$$
\begin{aligned}
\Omega_u &= R_u+R_b\Psi_u+R_wW_u+R_sS_u \\
&\quad +\Pi_u\left\{\frac{1}{\eta_{0s}}\left[R_s-\frac{\eta_{1s}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)-\frac{\eta_{3s}}{\eta_{3w}}R_w\right]+\frac{\eta_{1u'}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)\right\} \\
&\quad -\Pi_u\left\{\frac{\eta_{2u'}}{\eta_{2\theta}}\left[\frac{\eta_{0\theta}}{\eta_{0s}}\left(R_s-\frac{\eta_{1s}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)-\frac{\eta_{3s}}{\eta_{3w}}R_w\right)+\frac{\eta_{1\theta}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)+\frac{\eta_{3\theta}}{\eta_{3w}}R_w\right]\right\}
\end{aligned}
\tag{25}
$$

Differentiate η_1 and η_2 with respect to u

$$\begin{aligned} 0 &= \eta_{1u} + \eta_{1b}\Psi_u + \eta_{1s}S_u + \eta_{1\theta}\Theta_u + \eta_{1u'}\Pi_u \\ 0 &= \eta_{2\theta}\Theta_u + \eta_{2u'}\Pi_u \end{aligned}$$

Re-arrange

$$\begin{aligned} \frac{\eta_{1u'}}{\eta_{1b}}\Pi_u &= -\frac{\eta_{1u}}{\eta_{1b}} - \Psi_u - \frac{\eta_{1s}}{\eta_{1b}}S_u - \frac{\eta_{1\theta}}{\eta_{1b}}\Theta_u \\ \frac{\eta_{2u'}}{\eta_{2\theta}}\Pi_u &= -\Theta_u \end{aligned}$$

Substitute into (25)

$$\begin{aligned} \Omega_u &= R_u + R_b\Psi_u + R_wW_u + R_sS_u \\ &\quad +\Pi_u\frac{1}{\eta_{0s}}\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right] \\ &\quad -\left(\frac{\eta_{1u}}{\eta_{1b}} + \Psi_u + \frac{\eta_{1s}}{\eta_{1b}}S_u + \frac{\eta_{1\theta}}{\eta_{1b}}\Theta_u\right)\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) \\ &\quad +\Theta_u\left[\frac{\eta_{0\theta}}{\eta_{0s}}\left(R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right) + \frac{\eta_{1\theta}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) + \frac{\eta_{3\theta}}{\eta_{3w}}R_w\right] \\ &= R_u + \left[\frac{1}{\eta_{0s}}\Pi_u + S_u + \frac{f_\theta(\theta)s}{f(\theta)}\Theta_u\right]\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right)\right] - \frac{\eta_{1u}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) \\ &\quad + \left[W_u + \frac{\eta_{3b}}{\eta_{3w}}\Psi_u + \frac{\eta_{3\theta}}{\eta_{3w}}\Theta_u - \frac{\eta_{3s}}{\eta_{3w}}\left(\frac{1}{\eta_{\eta_{0s}}}\Pi_u + \frac{f_\theta(\theta)s}{f(\theta)}\Theta_u\right)\right]R_w \end{aligned} \tag{26}$$

Given the worker flow equation

$$\Pi(u) = \delta(1-u) + [1 - f(\Theta(u))S(u)]u$$

Differentiate with respect to u

$$\frac{1}{uf(\theta)}\Pi_u + S_u + \frac{f_\theta(\theta)s}{f(\theta)}\Theta_u = \frac{1-\delta-f(\theta)s}{uf(\theta)}$$

Given $\eta_3\left[u, \Psi(u), W(u), S(u), \Theta(u)\right] = 0$, differentiate with respect to u

$$W_u + \frac{\eta_{3b}}{\eta_{3w}}\Psi_u + \frac{\eta_{3\theta}}{\eta_{3w}}\Theta_u = -\frac{\eta_{3u}}{\eta_{3w}} - \frac{\eta_{3s}}{\eta_{3w}}S_u$$

Substitute into (26)

$$\Omega_u = R_u - \frac{\eta_{3u}}{\eta_{3w}}R_w + \frac{1-\delta-f(\theta)s}{uf(\theta)}\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right] - \frac{\eta_{1u}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right)$$

Update and substitute into FOC, we get the GEE

$$\underbrace{\frac{1}{uf(\theta)}\left[R_s-\frac{\eta_{1s}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)-\frac{\eta_{3s}}{\eta_{3w}}R_w\right]}_{\lambda\eta_{0u'}}+\underbrace{\frac{\eta_{1u'}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)}_{\mu\eta_{1u'}}$$

$$\underbrace{-\frac{\eta_{2u'}}{\eta_{2\theta}}\left[\frac{f_\theta(\theta)s}{f(\theta)}\left(R_s-\frac{\eta_{1s}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)-\frac{\eta_{3s}}{\eta_{3w}}R_w\right)+\frac{\eta_{1\theta}}{\eta_{1b}}\left(R_b-\frac{\eta_{3b}}{\eta_{3w}}R_w\right)+\frac{\eta_{3\theta}}{\eta_{3w}}R_w\right]}_{\gamma\eta_{2u'}}$$

$$(27)\quad = \quad \beta R'_u+\beta\underbrace{\frac{1-\delta-f(\theta')s'}{u'f(\theta')}\left[R'_s-\frac{\eta'_{1s}}{\eta'_{1b}}\left(R'_b-\frac{\eta'_{3b}}{\eta'_{3w}}R'_w\right)-\frac{\eta'_{3s}}{\eta'_{3w}}R'_w\right]}_{-\lambda'\eta'_{0u}}-\beta\underbrace{\frac{\eta'_{1u}}{\eta'_{1b}}\left(R'_b-\frac{\eta'_{3b}}{\eta'_{3w}}R'_w\right)}_{\mu'\eta'_{1u}}-\beta\underbrace{\frac{\eta'_{3u}}{\eta'_{3w}}R'_w}_{\nu'\eta'_{3u}}$$

where $\tilde{\eta}_{0,t}(u_t,s_t,\theta_t,u_{t+1}) := u_{t+1}-\delta(1-u_t)-(1-f(\theta_t)s_t)u_t$. Re-arrange to get the GEE in the text.

A.4 Alternative (and equivalent) definition of Markov-perfect equilibrium

This section provides an alternate and equivalent definition for the Markov-perfect equilibrium where the government chooses b only and the private sector acts optimally. The derivation of Markov GEE for this definition is given after the definition.

Definition 4. (Markov-perfect equilibrium) A Markov-perfect equilibrium consists of a value function $\Omega(u)$, government's policy function $\Psi(u)$, and private decision rules $\tilde{W}(u,b)$ $\tilde{S}(u,b)$, $\tilde{\Theta}(u,b)$ and $\tilde{\Pi}(u,b)$ solving

- for all u

$$\Psi(u) \quad \in \quad \arg\max_b R\left(u,b,\tilde{W}(u,b),\tilde{S}(u,b)\right)+\beta\Omega\left(\tilde{\Pi}(u,b)\right)$$

- for all u and b

$$\begin{aligned}
(28)\qquad \tilde{\Pi}(u,b) &= \delta(1-u)+\left[1-f(\tilde{\Theta}(u,b))\tilde{S}(u,b)\right]u \\
(29)\qquad 0 &= \eta_1\left(u,b,\tilde{S}(u,b),\tilde{\Theta}(u,b),\tilde{\Pi}(u,b)\right) \\
(30)\qquad 0 &= \eta_2\left(\tilde{\Theta}(u,b),\tilde{\Pi}(u,b)\right) \\
(31)\qquad 0 &= \eta_3\left(u,b,\tilde{W}(u,b),\tilde{S}(u,b),\tilde{\Theta}(u,b)\right)
\end{aligned}$$

- for all u

$$\Omega(u) \quad \equiv \quad R\left(u,\Psi(u),\tilde{W}(u,\Psi(u)),\tilde{S}(u,\Psi(u))\right)+\beta\Omega\left(\tilde{\Pi}(u,\Psi(u))\right)$$

i.e. the government moves first, choosing b and τ, then private sector moves according to (28)-(31). First-order condition of the government's problem, suppressing functional arguments, is given by

$$(32)\qquad R_b+R_w\tilde{W}_b+R_s\tilde{S}_b+\beta\Omega'_u\tilde{\Pi}_b=0.$$

Differentiating Bellman equation with respect to u

$$\Omega_u \quad = \quad R_u + R_b \Psi_u + R_w \left[\tilde{W}_u + \tilde{W}_b \Psi_u\right] + R_s \left[\tilde{S}_u + \tilde{S}_b \Psi_u\right] + \beta \Omega'_u \left[\tilde{\Pi}_u + \tilde{\Pi}_b \Psi_u\right] = 0. \tag{33}$$

Substitute expression for $\beta\Omega'_u$ from (32) into (33)

$$\Omega_u \quad = \quad R_u + R_b \Psi_u + R_w \left[\tilde{W}_u + \tilde{W}_b \Psi_u\right] + R_s \left[\tilde{S}_u + \tilde{S}_b \Psi_u\right] - \frac{R_b + R_w \tilde{W}_b + R_s \tilde{S}_b}{\tilde{\Pi}_b} \left[\tilde{\Pi}_u + \tilde{\Pi}_b \Psi_u\right].$$

Update one period and substitute into (32)

$$0 \quad = \quad R_b + R_w \tilde{W}_b + R_s \tilde{S}_b + \beta \tilde{\Pi}_b \left\{ R'_u + R'_b \Psi'_u + R'_w \left[\tilde{W}'_u + \tilde{W}'_b \Psi'_u\right] + R'_s \left[\tilde{S}'_u + \tilde{S}'_b \Psi'_u\right] - \frac{R'_b + R'_w \tilde{W}'_b + R'_s \tilde{S}'_b}{\tilde{\Pi}'_b} \left[\tilde{\Pi}'_u + \tilde{\Pi}'_b \Psi'_u\right] \right\}$$

Re-arrange to get the GEE

$$0 = \underbrace{\left[R_b + \mathbb{E}\tilde{W}_b R_w + \mathbb{E}\tilde{S}_b R_s\right]}_{\text{effect of } db \text{ holding } u'} + \beta \mathbb{E} \tilde{\Pi}_b \underbrace{\left[R'_u + \tilde{W}'_u R'_w + \tilde{S}'_u R'_s\right]}_{\text{effect of } du' \text{ holding } u''} + \beta \mathbb{E} \underbrace{\tilde{\Pi}_b \left(-\frac{\tilde{\Pi}'_u}{\tilde{\Pi}'_b}\right)}_{db'/db \text{ holding } u''} \underbrace{\left[R'_b + \tilde{W}'_b R'_w + \tilde{S}'_b R'_s\right]}_{\text{effect of } db' \text{ holding } u''} \tag{34}$$

where the functions with tilde are transformations of the ones without, e.g. $S(z,u) \equiv \tilde{S}(z,b(u),u)$ and $S_u = \tilde{S}_u + \tilde{S}_b \Psi_u$. Because of the presence of policy function derivatives such as $\tilde{S}_u$ and $\tilde{S}_b$, the above equation is also known as the Generalized Euler Equation or GEE. From the GEE, it is obvious any change in b has three effects. First, it affects the contemporaneous wages and search and thus both directly and indirectly changes the value of current government return function. Second, through changing u', it changes next period's unemployment, wages and search, thus changing next period's value. Last, it also has an effect on next period's value through its effect on next period benefit b'. The government determines current benefit by setting the net marginal value of b to zero.

B Additional Figures

B.1 Additional policy function plots over unemployment and over productivity shock

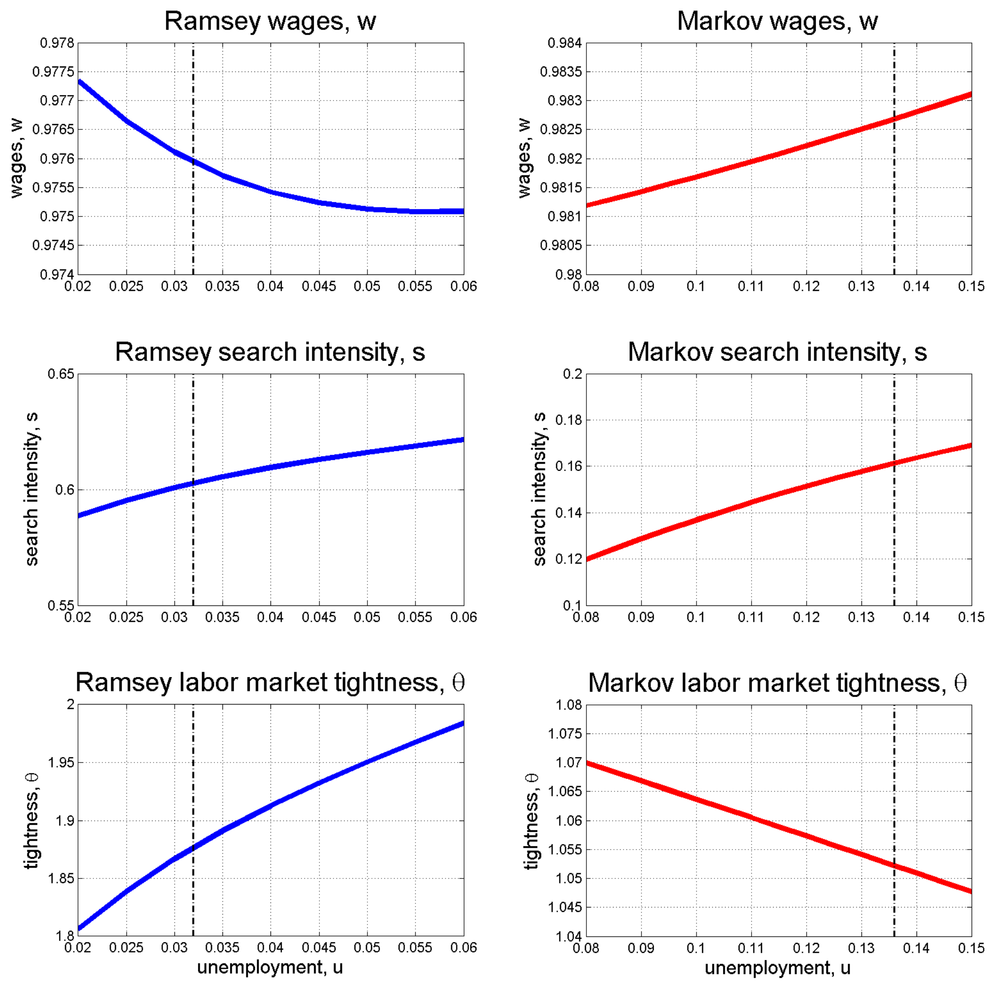

Figure 9: Ramsey (left) and Markov (right) wage (top panel), search intensity (middle panel) and market tightness (bottom panel) policy functions holding productivity at steady state. In each plot, the solid line denotes policy function, and the dashed line indicates steady state unemployment level.

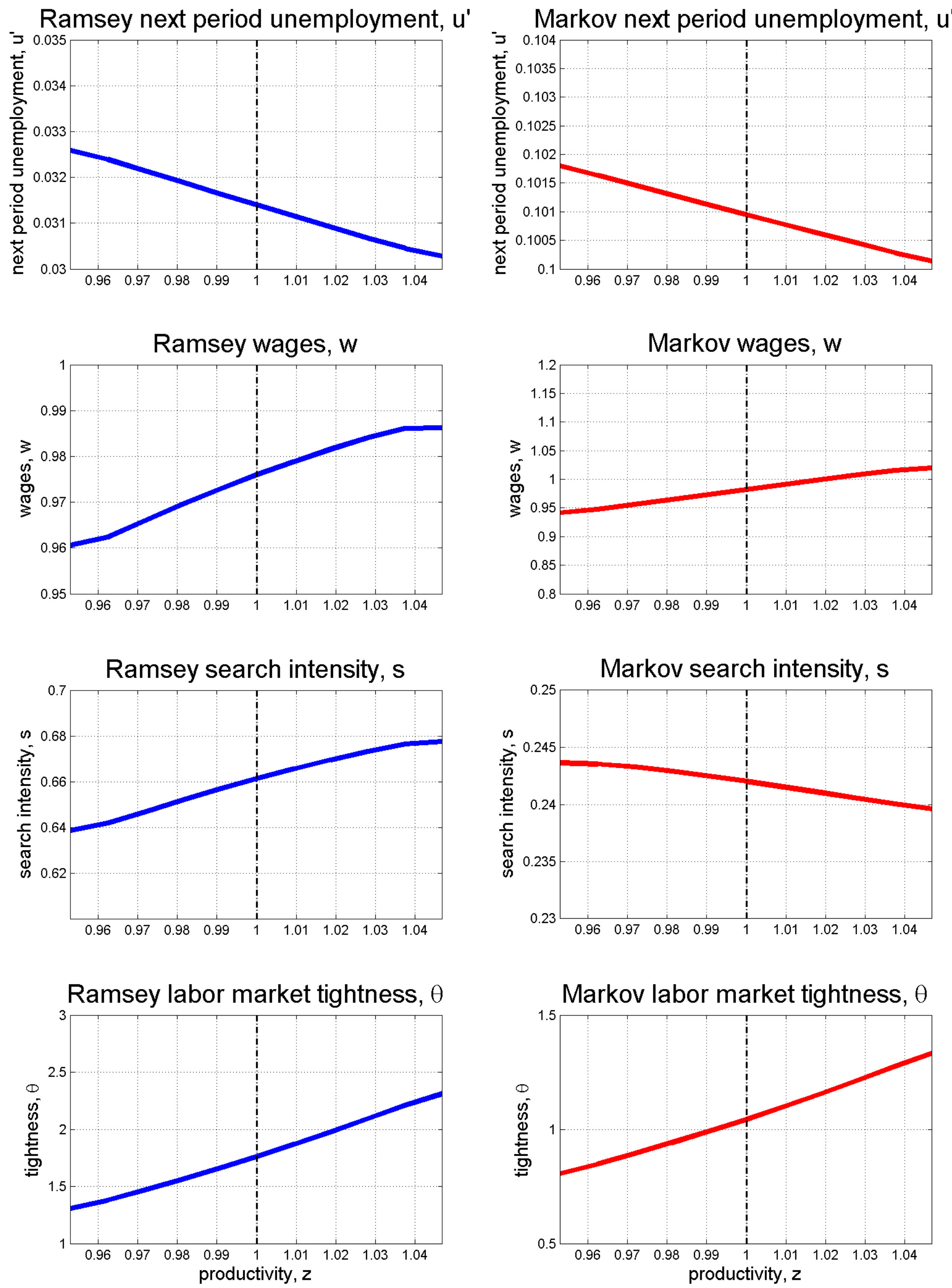

Figure 10: Ramsey (left) and Markov (right) unemployment, wage, search intensity and market tightness policy functions holding unemployment at steady state. In each plot, the solid line denotes policy function, and the dashed line indicates steady state productivity level.

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