Conditional versus unconditional forecasting with the New Area-Wide Model of the euro area

Christoffel, Kai and Coenen, Gunter and Warne, Anders

European Central Bank

July 2007

Conditional versus Unconditional Forecasting with the New Area-Wide Model of the Euro Area

Kai Christoffel, Günter Coenen and Anders Warne*,†

European Central Bank

10 July 2007

Preliminary and Incomplete Draft

Comments Welcome

Abstract

In this paper we examine conditional versus unconditional forecasting with a version of the New Area-Wide Model (NAWM) of the euro area designed for use in the context of the macroeconomic projection exercises at the European Central Bank (ECB). We first analyse the out-of-sample forecasting properties of the estimated model from 1999 to 2005 by comparing its unconditional forecasts with those obtained from a Bayesian VAR with a steady-state prior as well as naïve forecasts. Model-based forecasts that are conditioned on differing information sets are then studied and evaluated through, for instance, modesty statistics to assess the relevance of the Lucas critique. In contrast to other studies in the literature, we condition on a fairly large set of policy-relevant variables. Furthermore, we consider conditioning information that partially, albeit not fully determine the future path of the observed variables, but which restrict the channels through which they can be affected.

JEL Classification System: C11, C32, E32, E37

Keywords: DSGE modelling, open-economy macroeconomics, Bayesian inference, forecasting, euro area

* Corresponding author: Günter Coenen, Directorate General Research, European Central Bank, Kaiserstrasse 29, 60322 Frankfurt am Main, Germany, phone: +49-69-1344-7887, e-mail: gunter.coenen@ecb.int, homepage: http://www.guentercoenen.com.

† We appreciate comments and suggestions from Malin Adolfson, Frank Smets, Mattias Villani, Raf Wouters and participants in the 2007 SCE conference in Montréal. The opinions expressed are those of the authors and do not necessarily reflect views of the European Central Bank. Any remaining errors are the sole responsibility of the authors.

1. Introduction

Recent years have witnessed the development of a new generation of dynamic stochastic general equilibrium (DSGE) models that build on explicit micro-foundations with optimising agents. Major advances in estimation methodology allowed estimating variants of these models that are able to compete, in terms of data coherence, with more standard time-series models, such as vector autoregressions (VARs).[1] Accordingly, the new generation of DSGE models provides a framework that appears particularly suited for evaluating the consequences of alternative macroeconomic policies. More recently, increasing efforts have been undertaken to use these models also for forecasting purposes.[2]

In practice, forecasts at policy-making institutions are made conditional on a number of technical assumptions. In particular, institutional forecasts tend to be conditioned on a certain path for the nominal interest rate over the forecast horizon.[3] However, they are usually also conditioned on additional information, such as assumptions for the nominal exchange rate as well as fiscal and foreign developments, which may at least partially reflect advanced knowledge on the part of experts or market participants. To the extent that conditioning plays a crucial role in practical forecasting, incorporating conditioning information is deemed important for developing modern forecasting tools that are eventually to be used for forecasting purposes at policy-making institutions. Alternative methodological approaches for incorporating conditioning assumptions have been proposed for structural VARs by Waggoner and Zha (1999), Leeper and Zha (2003), and Robertson, Tallman and Whiteman (2005). These methods have been extended to DSGE models by Smets and Wouters (2004) and Adolfson, Laséen, Lindé and Villani (2005). But so far, empirical studies have largely focused on the role of conditioning on a path for the nominal interest rate alone, disregarding additional conditioning information.

In contrast to the existing studies, we examine model-based forecasts that are conditioned on a fairly large set of policy-relevant variables, namely nominal interest and exchange rates, but also fiscal and foreign variables. For our examination we utilise a version of the New

[1]See, among others, Smets and Wouters (2003, 2007), del Negro, Schorfheide, Smets and Wouters (2007), and Adolfson, Lindé, Laséen and Villani (2007).

[2]See Smets and Wouters (2004), Adolfson, Anderson, Lindé, Villani and Vredin (2005), Adolfson, Lindé and Villani (2007), and Edge, Kiley and Laforte (2006).

[3]Historically, the assumption of unchanged interest rates was widespread amongst central banks, whereas more recently assumptions based on market interest rates (see, e.g., the practices at the Bank of England) or an "own" interest rate path (see the Reserve Bank of New Zealand, Sveriges Riksbank and Norges Bank) have been more widely utilised. The European Central Bank employed the assumption of unchanged interest rates until March 2006 and has been using market-based interest rates since then.

Area-Wide Model (NAWM)—an estimated small open-economy model of the euro area that has been designed for use in the macroeconomic projection exercises at the European Central Bank (cf. Christoffel, Coenen and Warne, 2007).[4] In utilising the NAWM, we allow for conditioning information that partially, albeit not fully determines the future path of any particular endogenous variable, but which restricts the channels through which they can be affected. For instance, the nominal exchanges rate and foreign prices are part of our conditioning set, while domestic prices are not. The future path of the real exchange rate can thus only vary with shocks affecting domestic prices. To our knowledge, there is no study that looks at such a large, policy-relevant conditioning set or conditioning variables that only partially determine the path of the observed variables.

The conditional forecast approach we apply is based on recursively manipulating certain structural shocks to ensure that the observed variables are fully consistent with the conditioning information, following Leeper and Zha (1993) and Adolfson, Laséen, Lindé and Villani (2005). Clearly, such conditional forecast experiments may be subject to the Lucas (1976) critique since the shocks that need to be adjusted may behave very differently from what is assumed in the model and thereby give rise to changes in agents' beliefs about the model's structure. To assess the relevance of the Lucas critique, we evaluate the conditional forecasts through "modesty statistics", which were originally proposed by Leeper and Zha (2003) for structural VAR analysis as a simple metric for evaluating how *unusual* a conditional forecast of a variable is relative to an unconditional forecast. The underlying idea is to compare the shocks that are adjusted over the conditioning sample with values drawn from the estimated distribution of the shocks. If the behavior of the adjusted shocks over the conditioning sample is very different from that implied by the model, then the conditioning information need no longer be modest and instead be subject to the Lucas critique. Adolfson et al. (2005) extended Leeper and Zha's idea from structural VARs to DSGE models subsequently, also taking the multivariate nature of the underlying shock uncertainty into account.[5]

[4]While there exists a calibrated two-country version of the NAWM comprising the euro area and the United States (cf. Coenen, McAdam and Straub, 2007), the estimated version maintains the simplifying assumption that the euro area is a small open economy motivated by the fact that the ECB's macroeconomic projections are made conditional on assumptions regarding external developments. The development of the two versions of the NAWM builds extensively on the work by Smets and Wouters (2003) and Adolfson, Laséen, Lindé and Villani (2007), who estimated, respectively, a closed and a small-open economy model of the euro area using Bayesian techniques, and the advances made in developing the International Monetary Fund's calibrated Global Economy Model (GEM; cf. Bayoumi, Laxton and Pesenti, 2004) and the Federal Reserve Board's calibrated open-economy model named SIGMA (cf. Erceg, Guerrieri and Gust, 2005).

[5]Note that models which explicitly take expectations into account, such as DSGE models, seem more natural candidates for empirical studies of the Lucas critique than atheoretical time-series models, such as

In conducting our examination of conditional forecasting with the NAWM, we start by comparing the performance of the model's unconditional forecasts to those obtained from a Bayesian VAR and to different naïve forecasts. The comparison reveals that the NAWM performs favourably relative to the Bayesian VAR and the random walk, in particular in the case of real GDP growth and GDP inflation for horizons that extend beyond one year. We then show that conditioning on a possibly large set of policy-relevant variables helps to improve the NAWM's forecasting performance over some horizons, albeit not systematically. This is in line with our finding that the conditioning assumptions are modest in the sense of Leeper and Zha, at least as long as the multivariate nature of the shock uncertainty is taken into account. We finally study the probability of prediction events, such as the event that real GDP growth is negative for three consecutive quarters over the prediction horizon. In so doing, we identify a heightened probability of a recession in 2001. The recession signal is broadly similar across information sets, even though it is more pronounced when conditioning on (ex-post) foreign data.

The remainder of the paper is organised as follows. Section 2 outlines the theoretical specification of the NAWM, while Section 3 reports on our implementation of Bayesian inference methods and on our estimation results. Section 4 compares the performance of unconditional forecasts based on the NAWM against simple benchmarks. Section 5 examines conditional versus unconditional forecasting with the NAWM and assesses the modesty of the conditioning assumptions as well as prediction events. Section 6 concludes.

2. The New Area-Wide Model of the Euro Area

In this section, we outline the specification of the New Area-Wide Model (NAWM). Throughout, we maintain the simplifying assumption that the euro area is a small open economy. Within the domestic (i.e., the euro area) economy, there are four types of economic agents: households, firms, a fiscal authority, and a monetary authority. As regards firms, we distinguish between producers of tradable differentiated intermediate goods and producers of three non-tradable final goods: a private consumption good, a private investment good, and a public consumption good. In addition, there are foreign intermediate-good producers that sell their differentiated goods in domestic markets. International linkages

VARs, which do not distinguish between the intrinsic dynamics generated by the model structure and those generated by expectations.

arise from the trade of intermediate goods and international assets, allowing for limited exchange-rate pass-through and imperfect risk sharing.

In the following, we outline the behaviour of the different types of agents, formulate the aggregate resource constraint and state the law of motion for the domestic (net) holdings of foreign assets. In this context, we also define expressions for the trade balance and the terms of trade and derive an expression for export demand. To the extent needed, foreign variables and parameters are indexed with an asterisk, '*'.

2.1. Households

There is a continuum of households indexed by $h \in [0,1]$, the instantaneous utility of which depends on the level of consumption as well as hours worked. Each household accumulates physical capital, the services of which it rents out to firms, and buys and sells domestic government bonds as well as internationally traded bonds. This enables households to smooth their consumption profile in response to shocks. The households supply differentiated labour services to firms and act as wage setters in monopolistically competitive markets. As a consequence, each household is committed to supply sufficient labour services to satisfy firms' labour demand.

Preferences and Constraints: Each household h maximises its lifetime utility in a given period t by choosing purchases of the consumption good, $C_{h,t}$, purchases of the investment good, $I_{h,t}$, which determines next period's physical capital stock, $K_{h,t+1}$, the intensity with which the existing capital stock is utilised in production, $u_{h,t}$ and next period's (net) holdings of domestic government bonds and internationally traded foreign bonds, $B_{h,t+1}$ and $B^*_{h,t+1}$, respectively, given the following lifetime utility function:

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}\beta^k\left(\epsilon^C_{t+k}\ln\left(C_{h,t+k}-\kappa\, C_{t+k-1}\right)-\frac{\epsilon^N_{t+k}}{1+\zeta}\left(N_{h,t+k}\right)^{1+\zeta}\right)\right], \tag{1}$$

where β denotes the discount factor and ζ is the inverse of the Frisch elasticity of labour supply. The parameter κ measures the degree of external habit formation in consumption. Thus, the utility of household h depends positively on the difference between the current level of individual consumption, $C_{h,t}$, and the lagged economy-wide consumption level, C_{t-1}, and negatively on the number of hours worked, $N_{h,t}$. We will refer to ϵ^C_t and ϵ^N_t as consumption preference and labour-supply shocks, respectively.

Household h faces the following period-by-period budget constraint:

$$
\begin{aligned}
&(1+\tau_t^C)\, P_{C,t}\, C_{h,t} + P_{I,t}\, I_{h,t} \\
&\quad + (\epsilon_t^{RP} R_t)^{-1} B_{h,t+1} + ((1-\Gamma_{B^*}(s_{B^*,t+1};\epsilon_t^{RP^*}))\, R_t^*)^{-1} S_t\, B_{h,t+1}^* + \Xi_t + \Phi_{h,t} \\
&= (1-\tau_t^N-\tau_t^{W_h})\, W_{h,t}\, N_{h,t} + (1-\tau_t^K)\,(R_{K,t}\, u_{h,t} - \Gamma_u(u_{h,t})\, P_{I,t})\, K_{h,t} \\
&\quad + \tau_t^K\, \delta\, P_{I,t}\, K_{h,t} + (1-\tau_t^D)\, D_{h,t} - T_t + B_{h,t} + S_t\, B_{h,t}^*,
\end{aligned} \tag{2}
$$

where $P_{C,t}$ and $P_{I,t}$ are the prices of a unit of the private consumption good and the investment good, respectively. $N_{h,t}$ denotes the labour services provided to firms at wage rate $W_{h,t}$; $R_{K,t}$ indicates the rental rate for the effective capital services rented to firms, $u_{h,t}\, K_{h,t}$, and $D_{h,t}$ are the dividends paid by the household-owned firms. R_t and R_t^* denote the respective risk-less returns on domestic government bonds and internationally traded foreign bonds. The latter are denominated in foreign currency and, thus, their domestic value depends on the nominal exchange rate S_t (expressed in terms of units of home currency per unit of foreign currency).

As regards the provision of effective capital services, varying the intensity of utilising the physical capital stock, $u_{h,t}$, is subject to a proportional cost $\Gamma_u(u_{h,t})$ which is assumed to take the following form:

$$
\Gamma_u(u_{h,t}) \;=\; \gamma_{u,1}\,(u_{h,t}-1) + \frac{\gamma_{u,2}}{2}\,(u_{h,t}-1)^2 \tag{3}
$$

with $\gamma_{u,1}, \gamma_{u,2} > 0$.

The effective return on the risk-less domestic bonds depends on a financial intermediation premium, represented by the exogenous "risk" premium shock ϵ_t^{RP}, which drives a wedge between the interest rate controlled by the monetary authority and the return required by the household.[6] Similarly, when taking a position in the international bond market, the household encounters an external financial intermediation premium $\Gamma_{B^*}(s_{B^*,t+1};\epsilon_t^{RP^*})$ which depends on the economy-wide (net) holdings of internally traded foreign bonds expressed in domestic currency relative to domestic nominal output, $s_{B^*,t+1} = S_t\, B_{t+1}^*/P_{Y,t} Y_t$, and takes the form:

$$
\Gamma_{B^*}(s_{B^*,t+1};\epsilon_t^{RP^*}) \;=\; \gamma_{B^*}\left(\epsilon_t^{RP^*} \exp\left(\frac{S_t\, B_{t+1}^*}{P_{Y,t}\, Y_t}\right) - 1\right) \tag{4}
$$

with $\gamma_{B^*} < 0$.[7]

[6] See Smets and Wouters (2007) for further discussion.

[7] Note that we have used current nominal output and the current exchange rate to scale B_{t+1}^*, because the latter is a predetermined variable.

Here, the shock $\epsilon_t^{RP^*}$ represents the exogenous component of the external intermediation premium and will be referred to as external risk premium shock. This specification implies that, in the steady state, households have no incentive to hold foreign bonds and the economy's net foreign asset position is zero.[8] The incurred intermediation premium is rebated in a lump-sum manner, being indicated by Ξ_t.

The fiscal authority absorbs part of the household's gross income to finance its expenditure. In this context, τ_t^C denotes the consumption tax rate levied on the household's consumption purchases; and τ_t^N, τ_t^K and τ_t^D are the tax rates levied on the different sources of the household's income: wage income $W_{h,t}\, N_{h,t}$, rental capital income $R_{K,t}\, K_{h,t}$ and dividend income $D_{h,t}$.[9] Here, for simplicity, we assume that the utilisation cost of physical capital as well as physical capital depreciation, $\delta\, P_{I,t}\, K_{h,t}$, are exempted from taxation. $\tau_t^{W_h}$ is the additional pay-roll tax rate levied on wage income (representing the household's contribution to social security). The term T_t denotes lump-sum taxes.

Finally, it is assumed that each household h holds state-contingent securities, $\Phi_{h,t}$. These securities are traded amongst households and provide insurance against household-specific wage-income risk. This guarantees that the marginal utility of consumption out of wage income is identical across households.[10] As a result, all households will choose identical allocations in equilibrium.[11]

The capital stock owned by household h evolves according to the following capital accumulation equation:

$$K_{h,t+1} \quad = \quad (1-\delta)\, K_{h,t} + \epsilon_t^I\, (1 - \Gamma_I(I_{h,t}/I_{h,t-1}))\, I_{h,t}, \tag{5}$$

where δ is the depreciation rate, $\Gamma_I(I_{h,t}/I_{h,t-1})$ represents a generalised adjustment cost function formulated in terms of the (gross) rate of change in investment, $I_{h,t}/I_{h,t-1}$, and ϵ_t^I denotes an investment-specific technology shock. The adjustment cost function is assumed to take the following form:

$$\Gamma_I(I_{h,t}/I_{h,t-1}) \quad = \quad \frac{\gamma_I}{2}\left(\frac{I_{h,t}}{I_{h,t-1}} - g_z\right)^2 \tag{6}$$

with $\gamma_I > 0$. The term g_z denotes the rate of productivity growth in the economy's non-stochastic steady state.

[8]See Benigno (2001) for further discussion.

[9]For simplicity, it is assumed that dividends are taxed at the household level.

[10]The existence of state-contingent securities is assumed for analytical convenience and renders the model tractable under staggered wage setting when households are supplying differentiated labour services.

[11]This in turn guarantees that $C_{i,t} = C_{I,t}$ in equilibrium.

Choice of Allocations: Defining as $\Lambda_{h,t}/P_{C,t}$ and $\Lambda_{h,t}\,Q_{h,t}$ the Lagrange multipliers associated with the budget constraint (2) and the capital accumulation equation (5), respectively, the first-order conditions for maximising the household's lifetime utility function (1) with respect to $C_{h,t}$, $I_{h,t}$, $K_{h,t+1}$, $u_{h,t}$, $B_{h,t+1}$ and $B^*_{h,t+1}$ are given by:

$$\Lambda_{h,t} \;=\; \epsilon_t^C\,\frac{(C_{h,t}-\kappa\,C_{t-1})^{-1}}{1+\tau_t^C}, \tag{7}$$

$$\begin{aligned}\frac{P_{I,t}}{P_{C,t}} \;=\;& Q_{h,t}\,\epsilon_t^I\left(1-\Gamma_I(I_{h,t}/I_{h,t-1})-\Gamma_I'(I_{h,t}/I_{h,t-1})\,\frac{I_{h,t}}{I_{h,t-1}}\right) \\ &+\beta\,\mathrm{E}_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\,Q_{h,t+1}\,\epsilon_{t+1}^I\,\Gamma_I'(I_{h,t+1}/I_{h,t})\,\frac{I_{h,t+1}^2}{I_{h,t}^2}\right],\end{aligned} \tag{8}$$

$$\begin{aligned}Q_{h,t} \;=\;& \beta\,\mathrm{E}_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\Big((1-\delta)\,Q_{h,t+1}\right. \\ &\left.+(1-\tau_{t+1}^K)\,\frac{R_{K,t+1}}{P_{C,t+1}}\,u_{h,t+1}+\big(\tau_{t+1}^K\,\delta-(1-\tau_{t+1}^K)\,\Gamma_u(u_{h,t+1})\big)\,\frac{P_{I,t+1}}{P_{C,t+1}}\Big)\right],\end{aligned} \tag{9}$$

$$R_{K,t} \;=\; \Gamma_u'(u_{h,t})\,P_{I,t}, \tag{10}$$

$$\beta\,\epsilon_t^{RP}\,R_t\,\mathrm{E}_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\,\frac{P_{C,t}}{P_{C,t+1}}\right] \;=\; 1, \tag{11}$$

$$\beta\,(1-\Gamma_{B^*}(s_{B^*,t+1};\epsilon_t^{RP^*}))\,R_t^*\,\mathrm{E}_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\,\frac{P_{C,t}}{P_{C,t+1}}\,\frac{S_{t+1}}{S_t}\right] \;=\; 1. \tag{12}$$

Here, $\Lambda_{h,t}$ represents the shadow price of a unit of the consumption good; that is, the marginal utility of consumption out of income. Similarly, $Q_{h,t}$ measures the shadow price of a unit of the investment good; that is, Tobin's Q.[12]

In equilibrium, with all households choosing identical allocations, the combination of the first-order conditions with respect to the holdings of domestic and internationally traded bonds, (11) and (12), yields a risk-adjusted uncovered-interest-parity condition, reflecting the assumption that the return on internationally traded bonds is subject to an external financial intermediation premium.

Wage Setting: Each household h supplies its differentiated labour services $N_{h,t}$ in monopolistically competitive markets. There is sluggish wage adjustment due to staggered wage contracts à la Calvo (1983). Accordingly, household h receives permission to optimally reset its nominal wage contract $W_{h,t}$ in a given period t with probability $1-\xi_W$.

[12] Notice that the domestic risk premium shock, ϵ_t^{RP}, affects investment via Tobin's Q and helps to explain the co-movement of consumption and investment observed in the data. In contrast, the consumption preference shock, ϵ_t^C, moves consumption and investment in opposite directions.

All households that receive permission to reset their wage contracts in a given period t choose the same wage rate $\tilde{W}_t = \tilde{W}_{h,t}$. Those households which do not receive permission are allowed to adjust their wage contracts according to the following scheme:

$$W_{h,t} \;=\; g_{z,t}\, \Pi^{\dagger}_{C,t}\, W_{h,t-1}, \tag{13}$$

where $g_{z,t} = z_t/z_{t-1}$, with z_t representing trend labour productivity (see below), and $\Pi^{\dagger}_{C,t} = \Pi^{\chi_W}_{C,t-1}\bar{\Pi}^{1-\chi_W}_t$; that is, the nominal wage contracts are adjusted one-to-one with the (gross) rate of productivity growth and indexed to a geometric average of past (gross) consumer price inflation, $\Pi_{C,t-1} = P_{C,t-1}/P_{C,t-2}$, and the monetary authority's possibly time-varying (gross) inflation objective, $\bar{\Pi}_t$. Here, χ_W is an indexation parameter.

Each household h receiving permission to reset its wage contract in period t maximises its lifetime utility function (1) subject to its budget constraint (2), the demand for its differentiated labour services (the formal derivation of which we postpone until we consider the firms' problem in Section 2.2 below) and the wage-indexation scheme (13).

Hence, we obtain the following first-order condition characterising the households' optimal wage-setting decision:

$$\begin{aligned} \mathrm{E}_t \Bigg[\sum_{k=0}^{\infty} (\xi_W \beta)^k \Bigg(& \Lambda_{t+k}\,(1-\tau^N_{t+k}-\tau^{W_h}_{t+k})\, g_{z;t,t+k}\, \frac{\Pi^{\dagger}_{C;t,t+k}}{\Pi_{C;t,t+k}}\, \frac{\tilde{W}_t}{P_{C,t}} \\ & - \varphi^W_{t+k}\, \epsilon^N_{t+k}\, (N_{h,t+k})^{\zeta} \Bigg) N_{h,t+k} \Bigg] \;=\; 0, \end{aligned} \tag{14}$$

where Λ_{t+k} denotes the marginal utility out of income (equal across all households), $g_{z;t,t+k} = \prod_{s=1}^{k} g_{z,t+s}$, $\Pi^{\dagger}_{C;t,t+k} = \prod_{s=1}^{k} \Pi^{\chi_W}_{C,t+s-1}\bar{\Pi}^{1-\chi_W}_{t+s}$ and $\Pi_{C;t,t+k} = \prod_{s=1}^{k} \Pi_{C,t+s-1}$.

This expression states that in those labour markets in which wage contracts are re-optimised, the latter are set so as to equate the households' discounted sum of expected after-tax marginal revenues, expressed in consumption-based utility terms, Λ_{t+k}, to the discounted sum of expected marginal cost, expressed in terms of marginal disutility of labour, $\Delta_{h,t+k} = -N^{\zeta}_{h,t+k}$. In the absence of wage staggering ($\xi_W = 0$), the factor φ^W_t represents a possibly time-varying markup of the real after-tax wage charged over the households' marginal rate of substitution between consumption and leisure,

$$(1-\tau^N_t-\tau^{W_h}_t)\,\frac{\tilde{W}_t}{P_{C,t}} \;=\; -\,\varphi^W_t\,\epsilon^N_t\,\frac{\Delta_t}{\Lambda_t}, \tag{15}$$

reflecting the existence of monopoly power on the part of the households.[13]

[13]Note that, in this case, also the marginal disutility is equal across households; that is $\Delta_t = \Delta_{h,t}$.

Aggregate Wage Dynamics: With the continuum of households setting the wage contracts for their differentiated labour services according to equation (13) and equation (14), respectively, the aggregate wage index W_t evolves according to

$$W_t = \left(\xi_W \left(g_{z,t} \, \Pi^{\dagger}_{C,t} W_{t-1} \right)^{\frac{1}{1-\varphi^W_t}} + (1-\xi_W) \left(\tilde{W}_t \right)^{\frac{1}{1-\varphi^W_t}} \right)^{1-\varphi^W_t} . \tag{16}$$

2.2. Firms

There are two types of monopolistically competitive intermediate-good firms: A continuum of domestic intermediate-good firms indexed by $f \in [\,0,1\,]$ that produce differentiated outputs that are sold domestically or abroad, and a continuum of foreign intermediate-good firms indexed by $f^* \in [\,0,1\,]$ that produce differentiated outputs that are sold in domestic markets. In addition there is a set of three representative domestic firms, which combine the purchases of domestically-produced intermediate goods with purchases of imported intermediate goods into three distinct non-tradable final goods, namely a private consumption good, a private investment good and a public consumption good.

2.2.1. Domestic Intermediate-Good Firms

Technology: Each domestic intermediate-good firm f produces a differentiated intermediate good $Y_{f,t}$ with an increasing-returns-to-scale Cobb-Douglas technology that is subject to fixed costs of production, $z_t \, \psi$,

$$Y_{f,t} = \max \left[\varepsilon_t \, (K^s_{f,t})^{\alpha} \, (z_t \, N_{f,t})^{1-\alpha} - z_t \, \psi, \, 0 \right], \tag{17}$$

utilising as inputs homogenous capital services, $K^s_{f,t}$, that are rent from households in fully competitive markets, and an index of differentiated labour services, $N_{f,t}$, which combines household-specific varieties of labour that are supplied in monopolistically competitive markets,

$$N_{f,t} = \left(\int_0^1 \left(N^h_{f,t} \right)^{\frac{1}{\varphi^W_t}} dh \right)^{\varphi^W_t} , \tag{18}$$

where the possibly time-varying parameter $\varphi^W_t > 1$ is inversely related to the intratemporal elasticity of substitution between the differentiated labour services supplied by the households, $\eta_t = \varphi^W_t / (\varphi^W_t - 1) > 1$.[14]

[14] As shown above, the parameter φ^W_t has a natural interpretation as a markup in the household-specific labour market. In contrast, the exposition in Coenen, McAdam and Straub (2007) focuses on the intratemporal elasticity of substitution η which is assumed to be time invariant.

The variable ε_t represents a *transitory* technology shock that affects total-factor productivity, while the variable z_t denotes a *permanent* technology shock shifting the productivity of labour and introducing a unit root in the firm's output. Both shocks, and the fixed cost of production, are assumed to be identical across firms. The fixed cost is scaled by the permanent technology shock to guarantee that the fixed cost as a fraction of output do not vanish as output grows.[15]

Capital and Labour Inputs: Taking the rental cost of capital $R_{K,t}$ and the aggregate wage index W_t as given, the intermediate-good firm's optimal demand for capital and labour services must solve the problem of minimising total input cost $R_{K,t}\,K_{f,t} + (1+\tau_t^{W_f})\,W_t\,N_{f,t}$ subject to the technology constraint (17). Here, $\tau_t^{W_f}$ denotes the payroll tax rate levied on wage payments (representing the firms' contribution to social security).

Defining as $MC_{f,t}$ the Lagrange multiplier associated with the technology constraint (17), the first-order conditions of the firms' cost minimisation problem with respect to capital and labour inputs are given, respectively, by

$$\alpha\,\frac{Y_{f,t}+z_t\,\psi}{K_{f,t}^s}\,MC_{f,t} \quad = \quad R_{K,t}, \tag{19}$$

$$(1-\alpha)\,\frac{Y_{f,t}+z_t\psi}{N_{f,t}}\,MC_{f,t} \quad = \quad (1+\tau_t^{W_f})\,W_t, \tag{20}$$

or, more compactly,

$$\frac{\alpha}{1-\alpha}\,\frac{N_{f,t}}{K_{f,t}^s} \quad = \quad \frac{R_{K,t}}{(1+\tau_t^{W_f})\,W_t}. \tag{21}$$

Here, the Lagrange multiplier $MC_{f,t}$ measures the shadow price of varying the use of capital and labour services; that is, nominal marginal cost. We note that, since all firms f face the same input prices and since they all have access to the same production technology, nominal marginal cost $MC_{f,t}$ are identical across firms; that is, $MC_{f,t} = MC_t$ with

$$MC_t \quad = \quad \frac{1}{\varepsilon_t\,z_t^{1-\alpha}\alpha^\alpha(1-\alpha)^{1-\alpha}}\,(R_{K,t})^\alpha((1+\tau_t^{W_f})\,W_t)^{1-\alpha}. \tag{22}$$

With nominal wage contracts for differentiated labour services h being set in monopolistically competitive markets, firm f takes $W_{h,t}$ as given and chooses the optimal input of each labour variety h by minimising the total wage-related labour cost $\int_0^1 W_{h,t}\,N_{f,t}^h dh$, subject to the aggregation constraint (18).

[15]The parameter ψ will be chosen to ensure zero profits in steady state. This in turn guarantees that there is no incentive for other firms to enter the market in the long run.

The resulting demand for labour variety h is a function of the household-specific wage rate $W_{h,t}$ relative to the aggregate wage index W_t:

$$N_{f,t}^h = \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}} N_{f,t} \quad (23)$$

with $-\varphi_t^W/(\varphi_t^W-1)$ representing the wage elasticity of labour demand.

The wage index W_t can be obtained by substituting the labour index (18) into the labour demand schedule (23) and then integrating over the unit interval of households:

$$W_t = \left(\int_0^1 W_{h,t}^{\frac{1}{1-\varphi_t^W}} dh\right)^{1-\varphi_t^W}. \quad (24)$$

Aggregating over the continuum of firms f, we obtain the following aggregate demand for the labour services of a given household h:

$$N_t^h = \int_0^1 N_{f,t}^h \, df = \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}} N_t. \quad (25)$$

Price Setting: Each firm f sells its differentiated output $Y_{f,t}$ in both domestic and foreign markets under monopolistic competition. We assume that the firm charges different prices at home and abroad, setting prices in domestic (that is, producer) currency. In both markets, there is sluggish price adjustment due to staggered price contracts à la Calvo (1983). Accordingly, firm f receives permission to optimally reset prices in a given period t either with probability $1-\xi_H$ or with probability $1-\xi_X$, depending on whether the firm sells its differentiated output in the domestic or the foreign market.

Defining as $P_{H,f,t}$ the domestic price of good f and as $P_{X,f,t}$ its foreign price, all firms that receive permission to reset their price contracts in a given period t choose the same price $\tilde{P}_{H,t} = \tilde{P}_{H,f,t}$ and $\tilde{P}_{X,t} = \tilde{P}_{X,f,t}$, depending on the market of destination. Those firms which do not receive permission are allowed to adjust their prices according to the following schemes:

$$P_{H,f,t} = \Pi_{H,t-1}^{\chi_H} \bar{\Pi}_t^{1-\chi_H} P_{H,f,t-1}, \quad (26)$$

$$P_{X,f,t} = \Pi_{X,t-1}^{\chi_X} (\bar{\Pi}_t^*)^{1-\chi_H} P_{X,f,t-1}, \quad (27)$$

that is, the price contracts are indexed to a geometric average of past (gross) intermediate-good inflation, $\Pi_{H,t-1} = P_{H,t-1}/P_{H,t-2}$ and $\Pi_{X,t-1} = P_{X,t-1}/P_{X,t-2}$, and possibly time-varying (gross) inflation objectives of the domestic and foreign monetary authorities, $\bar{\Pi}_t$ and $\bar{\Pi}_t^*$, where χ_H and χ_X are indexation parameters.

Each firm f receiving permission to optimally reset its domestic and/or foreign price in period t maximises the discounted sum of its expected nominal profits,

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\xi_H^k \, D_{H,f,t+k} + \xi_X^k \, D_{X,f,t+k} \right) \right], \tag{28}$$

subject to the price-indexation schemes (26) and (27) and taking as given domestic and foreign demand for its differentiated output, $H_{f,t}$ and $X_{f,t}$ (to be derived below), where the stochastic discount factor $\Lambda_{t,t+k}$ can be obtained from the consumption Euler equation of the households, and

$$D_{H,f,t} \;=\; P_{H,f,t} \, H_{f,t} - MC_t \, H_{f,t}, \tag{29}$$

$$D_{X,f,t} \;=\; P_{X,f,t} \, X_{f,t} - MC_t \, X_{f,t} \tag{30}$$

are period-t nominal profits (net of fixed cost) yielded in the domestic and foreign markets, respectively, which are distributed as dividends to the households.[16] Hence, we obtain the following first-order condition characterising the firm's optimal pricing decision for its output sold in the domestic market:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} \xi_H^k \, \Lambda_{t,t+k} \left(\Pi^{\dagger}_{H,t,t+k} \, \tilde{P}_{H,t} - \varphi^H_{t+k} \, MC_{t+k} \right) \; H_{f,t+k} \right] \;=\; 0, \tag{31}$$

where we have substituted the indexation scheme (26), noting that $P_{H,f,t+k} = \Pi^{\dagger}_{H,t,t+k} \tilde{P}_{H,t}$ with $\Pi^{\dagger}_{H,t,t+k} = \prod_{s=1}^{k} \Pi^{\chi_H}_{H,t+s-1} \bar{\Pi}^{1-\chi_H}_{t+s}$.

This expression states that in those intermediate-good markets in which price contracts are re-optimised, the latter are set so as to equate the firms' discounted sum of expected revenues to the discounted sum of expected marginal cost. In the absence of price staggering ($\xi_H = 0$), the factor φ^H_t represents a possibly time-varying markup of the price charged in domestic markets over nominal marginal cost, reflecting the degree of monopoly power on the part of the intermediate-good firms.[17]

Similarly, we obtain the following first-order condition characterising the firm's optimal pricing decision for its output sold in the foreign market:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} \xi_X^k \, \Lambda_{t,t+k} \left(\Pi^{\dagger}_{X,t,t+k} \, \tilde{P}_{X,t} - \varphi^X_{t+k} \; MC_{t+k} \right) \; X_{f,t+k} \right] \;=\; 0, \tag{32}$$

[16] Note that we have made use of the first-order conditions (9) and (20) to derive the expressions for nominal profits.

[17] The markup depends on the intratemporal elasticity of substitution between the differentiated goods supplied by the intermediate-good firms to the domestic final-good firms, which in turn determines the final-good firms' price elasticity of demand for the differentiated intermediate goods.

where we have substituted the indexation scheme (27), noting that $P_{X,f,t+k} = \Pi^{\dagger}_{X,t,t+k}\tilde{P}_{X,t}$ with $\Pi^{\dagger}_{X,t,t+k} = \prod_{s=1}^{k} \Pi^{\chi_X}_{X,t+s-1}\bar{\Pi}^{1-\chi_X}$, assuming, for simplicity, that the foreign inflation objective is time invariant and equal to the domestic long-run inflation objective, $\bar{\Pi}^* = \bar{\Pi}$.

Aggregate Price Dynamics: With the continuum of intermediate-good firms f setting the price contracts for their differentiated products sold domestically, $P_{H,f,t}$, according to equation (26) and equation (31), respectively, the aggregate price index $P_{H,t}$ evolves according to

$$P_{H,t} = \left((1-\xi_H)(\tilde{P}_{H,t})^{\frac{1}{1-\varphi_t^H}} + \xi_H\left(\Pi^{\chi_H}_{H,t-1}\bar{\Pi}_t^{1-\chi_H}P_{H,t-1}\right)^{\frac{1}{1-\varphi_t^H}}\right)^{1-\varphi_t^H}. \tag{33}$$

A similar relationship holds for the aggregate index of price contracts set for the differentiated products sold abroad, $P_{X,t}$, with

$$P_{X,t} = \left((1-\xi_X)(\tilde{P}_{X,t})^{\frac{1}{1-\varphi_t^X}} + \xi_X\left(\Pi^{\chi_X}_{X,t-1}\bar{\Pi}^{1-\chi_X}P_{X,t-1}\right)^{\frac{1}{1-\varphi_t^X}}\right)^{1-\varphi_t^X}. \tag{34}$$

2.2.2. Foreign Intermediate-Good Firms

Each foreign intermediate-good firm f^* sells its differentiated good $Y^*_{f^*,t}$ domestically under monopolistic competition, setting the price in domestic (that is, local) currency, as in Betts and Devereux (1996). Again, there is sluggish price adjustment due to staggered price contracts à la Calvo. Accordingly, the foreign exporter receives permission to optimally reset its price in a given period t with probability $1-\xi^*$ and has access to the following indexation scheme with parameter χ^*:

$$P_{IM,f^*,t} = \Pi^{\chi^*}_{IM,t-1}\bar{\Pi}_t^{1-\chi^*}P_{IM,f^*,t-1}, \tag{35}$$

where $P_{IM,f^*,t} = P^*_{X,f^*,t}$ and $\Pi_{IM,t-1} = P_{IM,t-1}/P_{IM,t-2}$ with $P_{IM,t} = P^*_{X,t}$. Here, we have utilised the fact that, with foreign intermediate-good firms setting prices in domestic currency, the price of the intermediate good imported from abroad (the import price index of the home country) is equal to the price charged by the foreign exporter in the home country (the export price index of the foreign country).

Each foreign exporter f^* receiving permission to optimally reset its price in period t maximises the discounted sum of its expected nominal profits,

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\xi^*)^k\,\Lambda^*_{t,t+k}\,D^*_{f^*,t+k}\right], \tag{36}$$

subject to the price-indexation scheme and the domestic (import) demand for its differentiated output, $IM_{f^*,t} = X^*_{f^*}$ (to be derived below), where

$$D^*_{f^*,t} \;=\; P_{IM,f^*,t}\, IM_{f^*,t} - MC^*_t\, IM_{f^*,t} \tag{37}$$

with $MC^*_t = S_t\, P^*_{Y,t}$ representing the foreign exporter's nominal marginal cost.

Hence, we obtain the following first-order condition characterising the foreign exporter's optimal pricing decision for its output sold in the domestic market:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\xi^*)^k\, \Lambda^*_{t,t+k} \left(\Pi^{\dagger}_{IM,t,t+k}\, \tilde{P}_{IM,t} - \varphi^*_{t+k}\, MC^*_{t+k} \right)\, IM_{f^*,t+k} \right] \;=\; 0, \tag{38}$$

where we have substituted the indexation scheme (35), noting that $P_{IM,f^*,t+k} = \Pi^{\dagger}_{IM,t,t+k}$ $\tilde{P}_{IM,t}$ with $\Pi^{\dagger}_{IM,t,t+k} = \prod_{s=1}^{k} \Pi^{\chi^*}_{IM,t+s-1} \bar{\Pi}^{1-\chi^*}_{t+s}$.

The associated aggregate index of price contracts for the differentiated products sold in domestic markets, $P_{IM,t}$, evolves according to

$$P_{IM,t} \;=\; \left((1-\xi^*)(\tilde{P}_{IM,t})^{\frac{1}{1-\varphi^*_t}} + \xi^* \left(\Pi^{\chi^*}_{IM,t-1} \bar{\Pi}^{1-\chi^*}_t P_{IM,t-1} \right)^{\frac{1}{1-\varphi^*_t}} \right)^{1-\varphi^*_t}. \tag{39}$$

2.2.3. Final-Good Firms

There are three different types of final-good firms which combine the purchases of the domestically-produced intermediate goods with purchases of the imported intermediate goods into three distinct non-tradable final goods, namely a private consumption good, Q^C_t, a private investment good, Q^I_t, and a public consumption good, Q^G_t.

The representative firm producing the non-tradable final private consumption good, Q^C_t, combines purchases of a bundle of domestically-produced intermediate goods, H^C_t, with purchases of a bundle of imported foreign intermediate goods, IM^C_t, using a constant-returns-to-scale CES technology,

$$Q^C_t \;=\; \left(\nu_C^{\frac{1}{\mu_C}} \left(H^C_t\right)^{1-\frac{1}{\mu_C}} + (1-\nu_C)^{\frac{1}{\mu_C}} \left((1-\Gamma_{IM^C}(IM^C_t/Q^C_t;\epsilon^{IM}_t))\, IM^C_t \right)^{1-\frac{1}{\mu_C}} \right)^{\frac{\mu_C}{\mu_C-1}} \tag{40}$$

where $\mu_C > 1$ denotes the intratemporal elasticity of substitution between the distinct bundles of domestic and foreign intermediate goods, while the parameter ν_C measures the home bias in the production of the consumption good.

Notice that the final-good firm incurs a cost $\Gamma_{IM^C}(IM^C_t/Q^C_t;\epsilon^{IM}_t)$ when varying the use of the bundle of imported goods in producing the consumption good,

$$\Gamma_{IM^C}(IM^C_t/Q^C_t;\epsilon^{IM}_t) \;=\; \frac{\gamma^C_{IM}}{2} \left(\epsilon^{IM}_t \frac{IM^C_t/Q^C_t}{IM^C_{t-1}/Q^C_{t-1}} - 1 \right)^2 \tag{41}$$

with $\gamma_{IM}^C > 0$. As a result, the import share is relatively unresponsive in the short run to changes in the relative price of the bundle of imported goods, while the level of imports is permitted to jump in response to changes in overall demand.[18] We will refer to ϵ_t^{IM} as an import demand shock.

Defining as $H_{f,t}^C$ and $IM_{f^*,t}^C$ the use of the differentiated output produced by the domestic intermediate-good firm f and the differentiated output supplied by the foreign exporter f^*, respectively, we have

$$H_t^C = \left(\int_0^1 \left(H_{f,t}^C\right)^{\frac{1}{\varphi_t^H}} df \right)^{\varphi_t^H}, \tag{42}$$

$$IM_t^C = \left(\int_0^1 \left(IM_{f^*,t}^C\right)^{\frac{1}{\varphi_t^*}} df^* \right)^{\varphi_t^*}, \tag{43}$$

where the possibly time-varying parameters $\varphi_t^H, \varphi_t^* > 1$ are inversely related to the intratemporal elasticities of substitution between the differentiated outputs supplied by the domestic firms and the foreign exporters, respectively, with $\theta_t^H = \varphi_t^H/(\varphi_t^H - 1) > 1$ and $\theta_t^* = \varphi_t^*/(\varphi_t^* - 1) > 1$.[19]

With nominal prices for the differentiated goods f and f^* being set in monopolistically competitive markets, the final-good firm takes their prices $P_{H,f,t}$ and $P_{IM,f^*,t}$ as given and chooses the optimal use of the differentiated goods f and f^* by minimising the expenditure for the bundles of differentiated goods, $\int_0^1 P_{H,f,t}\, H_{f,t}^C\, df$ and $\int_0^1 P_{IM,f^*,t}\, IM_{f^*,t}^C\, df^*$, subject to the aggregation constraints (42) and (43). This yields the following demand functions for the differentiated goods f and f^*:

$$H_{f,t}^C = \left(\frac{P_{H,f,t}}{P_{H,t}} \right)^{-\frac{\varphi_t^H}{\varphi_t^H - 1}} H_t^C, \tag{44}$$

$$IM_{f^*,t}^C = \left(\frac{P_{IM,f^*,t}}{P_{IM,t}} \right)^{-\frac{\varphi_t^*}{\varphi_t^* - 1}} IM_t^C, \tag{45}$$

where

$$P_{H,t} = \left(\int_0^1 \left(P_{H,f,t}\right)^{\frac{1}{1-\varphi_t^H}} df \right)^{1-\varphi_t^H}, \tag{46}$$

$$P_{IM,t} = \left(\int_0^1 \left(P_{IM,f^*,t}\right)^{\frac{1}{1-\varphi_t^*}} df^* \right)^{1-\varphi_t^*} \tag{47}$$

[18] While our treatment of the adjustment cost as being external to the firm would formally involve assuming the existence of a large number of firms with appropriate changes in notation (see, e.g., Bayoumi, Laxton and Pesenti, 2004), we abstract from these changes for ease of exposition.

[19] The parameters φ_t^H and φ_t^* have a natural interpretation as markups in the markets for domestic and imported intermediate goods. In contrast, the exposition in Coenen, McAdam and Straub (2007) focuses on the intratemporal elasticities of substitution θ^H, θ^* which are assumed to be time invariant.

are the aggregate price indexes for the bundles of domestic and imported intermediate goods, respectively.

Next, taking the price indexes $P_{H,t}$ and $P_{IM,t}$ as given, the consumption-good firm chooses the combination of the domestic and foreign intermediate-good bundles H_t^C and IM_t^C that minimises $P_{H,t}\, H_t^C + P_{IM,t}\, IM_t^C$ subject to aggregation constraint (40). This yields the following demand functions for the intermediate-good bundles:

$$H_t^C = \nu_C \left(\frac{P_{H,t}}{P_{C,t}}\right)^{-\mu_C} Q_t^C, \tag{48}$$

$$IM_t^C = (1-\nu_C)\left(\frac{P_{IM,t}}{P_{C,t}\,\Gamma^{\dagger}_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM})}\right)^{-\mu_C} \frac{Q_t^C}{1-\Gamma_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM})}, \tag{49}$$

where

$$P_{C,t} = \left(\nu_C\,(P_{H,t})^{1-\mu_C} + (1-\nu_C)\left(\frac{P_{IM,t}}{\Gamma^{\dagger}_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM})}\right)^{1-\mu_C}\right)^{\frac{1}{1-\mu_C}} \tag{50}$$

is the price of a unit of the private consumption good and

$$\Gamma^{\dagger}_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM}) = 1-\Gamma_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM}) - \Gamma'_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM})\,IM_t^C. \tag{51}$$

The representative firm producing the non-tradable final private investment good, Q_t^I, is modelled in an analogous manner. Specifically, the firm combines its purchase of a bundle of domestically-produced intermediate goods, H_t^I, with the purchase of a bundle of imported foreign intermediate goods, IM_t^I, using a constant-returns-to-scale CES technology,

$$Q_t^I = \left(\nu_I^{\frac{1}{\mu_I}}\left(H_t^I\right)^{1-\frac{1}{\mu_I}} + (1-\nu_I)^{\frac{1}{\mu_I}}\left(\left(1-\Gamma_{IM^I}(IM_t^I/Q_t^I;\epsilon_t^{IM})\right)IM_t^I\right)^{1-\frac{1}{\mu_I}}\right)^{\frac{\mu_I}{\mu_I-1}}, \tag{52}$$

where $\mu_I > 1$ denotes the intratemporal elasticity of substitution between the distinct bundles of domestic and foreign intermediate inputs, while the possibly time-varying parameter $\nu_{I,t}$ measures the home bias in the production of the investment good.

All other variables related to the production of the investment good—import adjustment cost, $\Gamma_{IM^I}(IM_t^I/Q_t^I;\epsilon_t^{IM})$; the optimal demand for firm-specific and bundled domestic and foreign intermediate goods, $H_{f,t}^I$, H_t^I and $IM_{f^*,t}^I$, IM_t^I, respectively; as well as the price of a unit of the investment good, $P_{I,t}$—are defined or derived in a manner analogous to that for the consumption good.[20]

[20]Notice that even in the absence of import adjustment cost, the prices of the consumption and investment goods may differ due to differences in the import content.

In contrast, the non-tradable final public consumption good Q_t^G is assumed to be a composite made only of domestic intermediate goods; that is, $Q_t^G = H_t^G$ with

$$H_t^G = \left(\int_0^1 (H_{f,t}^G)^{\frac{1}{\varphi_t^H}} \, df \right)^{\varphi_t^H} . \tag{53}$$

Hence, the optimal demand for each domestic intermediate good f is given by

$$H_{f,t}^G = \left(\frac{P_{H,f,t}}{P_{H,t}} \right)^{-\frac{\varphi_t^H}{\varphi_t^H - 1}} H_t^G , \tag{54}$$

and the price of a unit of the public consumption good is $P_{G,t} = P_{H,t}$.

Aggregating across the three final-good firms, we obtain the following demand for domestic and foreign intermediate goods f and f^*, respectively:

$$H_{f,t} = H_{f,t}^C + H_{f,t}^I + H_{f,t}^G = \left(\frac{P_{H,f,t}}{P_{H,t}} \right)^{-\frac{\varphi_t^H}{\varphi_t^H - 1}} H_t , \tag{55}$$

$$IM_{f^*,t} = IM_{f^*,t}^C + IM_{f^*,t}^I = \left(\frac{P_{IM,f^*,t}}{P_{IM,t}} \right)^{-\frac{\varphi_t^*}{\varphi_t^* - 1}} IM_t , \tag{56}$$

where $H_t = H_t^C + H_t^I + H_t^G$ and $IM_t = IM_t^C + IM_t^I$.

2.3. Fiscal and Monetary Authorities

2.3.1. Fiscal Authority

The fiscal authority purchases the final public consumption good, G_t, issues bonds to refinance its outstanding debt, B_t, and raises both distortionary and lump-sum taxes. The fiscal authority's period-by-period budget constraint then has the following form:

$$\begin{aligned} P_{G,t}\, G_t + B_t &= \tau_t^C\, P_{C,t}\, C_t + (\tau_t^N + \tau_t^{W_h}) \int_0^1 W_{h,t}\, N_{h,t}\, dh + \tau_t^{W_f}\, W_t\, N_t \\ &\quad + \tau_t^K \left(R_{K,t}\, u_t - (\Gamma_u(u_t) + \delta)\, P_{I,t} \right) K_t + \tau_t^D\, D_t + T_t + R_t^{-1}\, B_{t+1} , \end{aligned} \tag{57}$$

where all quantities are expressed in economy-wide terms, except for the households' labour services and wages, $N_{h,t}$ and $W_{h,t}$, which are differentiated across households.

The purchases of the public consumption good G_t are assumed to evolve exogenously. As regards the evolution of the fiscal authority's outstanding debt B_t, we note that our model—in its current simplified specification—features "Ricardian equivalence". Hence, the particular time path of debt is irrelevant for the households' choice of allocations. For this reason and without loss of generality, we assume that lump-sum taxes close the fiscal authority's budget constraint each period. Finally, all distortionary tax rates τ_t^Z with $Z = C$, D, K, N, W_h and W_f are assumed to be set exogenously.

2.3.2. Monetary Authority

The monetary authority sets the nominal interest rate according to a simple log-linear interest-rate rule,

$$\begin{aligned}\widehat{r}_t &= \phi_R\,\widehat{r}_{t-1} + (1-\phi_R)\,\left(\widehat{\bar{\pi}}_t + \phi_\Pi\left(\widehat{\pi}_{C,t-1} - \widehat{\bar{\pi}}_t\right) + \phi_Y\,\widehat{y}_t\right) \\ &\qquad + \phi_{\Delta\Pi}\left(\widehat{\pi}_{C,t} - \widehat{\pi}_{C,t-1}\right) + \phi_{\Delta Y}\left(\widehat{y}_t - \widehat{y}_{t-1}\right) + \eta_t^R,\end{aligned} \tag{58}$$

where $\widehat{r}_t = \log(R_t/R)$ is the logarithmic deviation of the (gross) nominal interest rate from its steady-state value. Similarly, $\widehat{\pi}_{C,t} = \log(\Pi_{C,t}/\bar{\Pi})$ denotes the logarithmic deviation of (gross) quarter-on-quarter consumer price inflation $\Pi_{C,t} = P_{C,t}/P_{C,t-1}$ from the monetary authority's long-run inflation objective $\bar{\Pi}$, while $\widehat{\bar{\pi}}_t = \log(\bar{\Pi}_t/\bar{\Pi})$ represents the logarithmic deviation of the monetary authority's possibly time-varying inflation objective from its long-run value. Finally, $\widehat{y}_t = \widehat{Y_t/z_t}$ is the logarithmic deviation of aggregate output from trend output implied by the assumed unit-root technology (that is, the output gap), while η_t^R is a shock to the nominal interest rate.

2.4. Aggregate Resource Constraint and Net Foreign Assets

The model is closed by formulating the aggregate resource constraint and stating the law of motion for the domestic net foreign assets. In this context, it is convenient to define the trade balance and the terms of trade and to derive an expression for export demand.

2.4.1. Aggregate Resource Constraint

Imposing market-clearing conditions (see Christoffel, Coenen and Warne (2007) for details) implies the following aggregate resource constraint:

$$\begin{aligned}P_{Y,t}\,Y_t &= P_{H,t}\,H_t + P_{X,t}\,X_t \\ &= P_{C,t}\,C_t + P_{I,t}\,(I_t + \Gamma_u(u_t)\,K_t) + P_{G,t}\,G_t + P_{X,t}\,X_t \\ &\quad - P_{IM,t}\left(IM_t^C\,\frac{1-\Gamma_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM})}{\Gamma^\dagger_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM})} + IM_t^I\,\frac{1-\Gamma_{IM^I}(IM_t^I/Q_t^I;\epsilon_t^{IM})}{\Gamma^\dagger_{IM^I}(IM_t^I/Q_t^I;\epsilon_t^{IM})}\right).\end{aligned} \tag{59}$$

2.4.2. Net Foreign Assets

The domestic holdings of foreign bonds (that is, the domestic economy's net foreign assets, denominated in foreign currency) evolve according to

$$(R_t^*)^{-1}B_{t+1}^* = B_t^* + \frac{TB_t}{S_t}, \tag{60}$$

where

$$TB_t \quad = \quad P_{X,t}\, X_t - P_{IM,t}\, IM_t \tag{61}$$

is the domestic economy's trade balance, which is conveniently expressed as a share of domestic output, $s_{TB,t} \;=\; TB_t/P_{Y,t}Y_t$ (like the net foreign assets with $s_{B^*,t+1} \;=\; S_t\, B^*_{t+1}/P_{Y,t}Y_t$).[21]

The terms of trade (defined as the domestic price of imports relative to the price of exports in domestic currency) are given by:

$$ToT_t \quad = \quad \frac{P_{IM,t}}{P_{X,t}}. \tag{62}$$

Finally, the volume of exports X_t is determined by a demand equation similar in structure to the domestic import equation,

$$X_t \quad = \quad \nu^*_t \left(\frac{S_t\, P_{X,t}}{P^{c,*}_{X,t}\, \Gamma^\dagger_X(X_t/Y^{d,*}_t;\epsilon^X_t)} \right)^{-\mu^*} \frac{Y^{d,*}_t}{1-\Gamma_X(X_t/Y^{d,*}_t;\epsilon^X_t)}, \tag{63}$$

where ν^*_t is a possibly time-varying export share, which captures the foreign preference for domestic intermediate goods. The variable $P^{c,*}_{X,t}$ denotes the price of foreign competitors of domestic intermediate-good producers on the export side, $Y^{d,*}_t$ is a measure of foreign demand, and $\Gamma_X(X_t/Y^{d,*}_t;\epsilon^X_t)$ is an adjustment cost function given by

$$\Gamma_X(X_t/Y^{d,*}_t;\epsilon^X_t) \quad = \quad \frac{\gamma^*}{2} \left(\epsilon^X_t \, \frac{X_t/Y^{d,*}_t}{X_{t-1}/Y^{d,*}_{t-1}} - 1 \right)^2 \tag{64}$$

and

$$\Gamma^\dagger_X(X_t/Y^{d,*}_t;\epsilon^X_t) \quad = \quad 1-\Gamma_X(X_t/Y^{d,*}_t;\epsilon^X_t) - \Gamma'_X(X_t/Y^{d,*}_t;\epsilon^X_t)\, X_t. \tag{65}$$

3. Bayesian Estimation

We adopt the empirical approach outlined in Smets and Wouters (2003) and estimate our version of the NAWM employing Bayesian inference methods. This involves obtaining the posterior distribution of the model's parameters based on its log-linear state-space representation using the Kalman filter.[22, 23]

[21]Notice that the existence of a financial intermediation premium guarantees that, in the non-stochastic steady state, domestic holdings of internationally traded bonds are zero.

[22]For details on the derivation of the log-linear representation of the NAWM, see Christoffel and Coenen (2006).

[23]For all computations, we use YADA, a Matlab programme for Bayesian estimation and evaluation of DSGE models (cf. Warne, 2007).

In the following we briefly sketch the adopted approach and describe the data and the prior distributions used in its implementation. In this context, we also provide information on the structural shocks that we consider in the estimation and describe the calibration of those parameters that we keep fixed. We then present our estimation results.

3.1. Methodology

Employing Bayesian inference methods allows formalising the use of prior information obtained from earlier studies at both the micro and macro level in estimating the parameters of a possibly complex DSGE model. This seems particularly appealing in situations where the sample period of the data is relatively short, as is the case for the euro area. From a practical perspective, Bayesian inference may also help to alleviate the inherent numerical difficulties associated with solving the highly non-linear estimation problem.

Formally, let $p(\theta|m)$ denote the prior distribution of the vector $\theta \in \Theta$ with structural parameters for some model $m \in \mathcal{M}$, and let $p(\mathcal{Y}_T|\theta, m)$ denote the likelihood function for the observed data, $\mathcal{Y}_T = \{ y_1, \ldots, y_T \}$, conditional on parameter vector θ and model m. The joint posterior distribution of the parameter vector θ for model m is then obtained by combining the likelihood function for $\mathcal{Y}_T$ and the prior distribution of θ,

$$p(\theta|\mathcal{Y}_T, m) \quad \propto \quad p(\mathcal{Y}_T|\theta, m)\, p(\theta|m),$$

where "$\propto$" indicates proportionality.[24]

The posterior distribution is typically characterised by measures of central location, such as the mode or the mean, measures of dispersion, such as the standard deviation, or selected percentiles.

As discussed in Geweke (1999), Bayesian inference also provides a framework for comparing alternative and potentially misspecified models on the basis of their marginal likelihood. For a given model m the latter is obtained by integrating out the parameter vector θ,

$$p(\mathcal{Y}_T|m) \quad = \quad \int_{\theta\in\Theta} p(\mathcal{Y}_T|\theta, m)\, p(\theta|m)\, d\theta.$$

Thus, the marginal likelihood gives an indication of the overall likelihood of a model conditional on the observed data.

[24] As in Smets and Wouters (2003), and following Schorfheide (2000), we adopt a Monte-Carlo Markov-Chain (MCMC) sampling algorithm to determine the joint posterior distribution of the parameter vector θ. More specifically, we rely on the Random-Walk Metropolis-Hastings (RWMH) algorithm to obtain a large number of random draws from the posterior distribution of θ. The mode and a modified Hessian of the posterior distribution, the latter evaluated at the mode, are used to determine the initial proposal density for the RWMH algorithm. The posterior mode and the Hessian matrix are computed by standard numerical optimisation routines, namely Christopher Sims' optimiser csminwel.

3.2. Data and Shocks

3.2.1. Data

In estimating the NAWM, we use data on 17 key macroeconomic times series:

- real GDP (Y)
- private consumption (C)
- total investment (I)
- government consumption (G)
- extra-euro area exports (X)
- extra-euro area imports (IM)
- GDP deflator (P_Y)
- consumption deflator (P_C)
- extra import deflator (P_{IM})
- total employment (E)
- compensation of employees (W)
- nominal interest rate (R)
- nominal effective exchange rate (S)
- foreign competitors' prices ($P_X^{c,*}$)†
- foreign demand ($Y^{d,*}$)†
- foreign GDP deflator (P_Y^*)†
- foreign nominal interest rate (R^*)†

All time series are taken from an updated version of the AWM database (see Fagan, Henry and Mestre, 2001), except for the time series for extra-euro area trade data (both volumes and prices) which stem from internal ECB sources. The sample period ranges from 1985Q1 to 2005Q4 (using the period 1980Q2 to 1984Q4 as training sample). The times series marked with a dagger ('†') are modelled using a structural VAR, the estimated parameters of which are kept fixed throughout the estimation. Similarly, government consumption is assumed to follow an autoregressive (AR) process with fixed estimated parameters.

Prior to estimation, the following data transformations have been made:

- We measure real GDP, consumption, investment, extra-euro area exports and imports, the relevant deflators, wages and foreign demand in terms of quarter-on-quarter growth rates, approximated by the first difference of their logarithm.
- We remove excess mean growth, relative to real GDP, from extra-euro area exports and imports and foreign demand, to guarantee that these variables are commensurate with the balanced-growth-path property of the model.
- We take the logarithm of government consumption and remove a linear trend consistent with our assumptions of trend labour force growth of 0.8 percent and trend labour productivity growth of 1.2 percent, both growth rates being expressed at an annual rate.[25]

[25] That is, the model, which is implicitly defined in terms of per-capita variables, implies a trend growth rate of 2.0 percent per annum for all observed real variables.

- We take the logarithm of employment and remove a linear trend consistent with our assumption of trend labour-force growth of 0.8 percent at an annual rate, noting that, in the absence of a reliable measure of hours worked, we use data on employment in the estimation.[26]
- We construct a measure of the real effective exchange of the euro from the nominal effective exchange rate, the domestic GDP deflator and the foreign GDP deflator and then remove the mean.
- We deflate the competitors' price on the export side with the foreign GDP deflator and remove the existing linear trend.

The graphs of the transformed time series are depicted in **Figure 1**.

3.2.2. Shocks

Out of the total of 22 structural shocks incorporated in the outlined specification of the NAWM (including shocks to the inflation objective and the 6 tax rates), we employ a subset of 12 shocks, plus the 5 shocks of the autoregression and the structural VAR used to model the time series of government consumption and the foreign variables, respectively:[27]

- transitory technology shock (ε)
- permanent technology shock (g_z)
- domestic risk premium shock (ϵ^{RP})
- wage markup shock (φ^W)
- investment-specific techn. shock (ϵ^I)
- import demand shock (ϵ^{IM})
- price markup shock: domestic (φ^H)
- price markup shock: exports (φ^X)
- price markup shock: imports (φ^*)
- export preference shock (ν^*)
- interest rate shock (η^R)
- external risk premium shock (ϵ^{RP^*})
- shock to governm. consumption (η^G)
- shock to competitors' prices ($\eta^{P_X^{c,*}}$)
- shock to foreign demand ($\eta^{Y^{d,*}}$)
- shock to foreign inflation ($\eta^{\Pi_Y^*}$)
- shock to foreign interest rate (η^{R^*})

All shocks are assumed to follow first-order autoregressive processes, except for the price markup shocks, the interest rate shock and the shocks in the AR model for government

[26]We relate the employment variable to the unobserved hours-worked variable by an auxiliary equation following Smets and Wouters (2003),

$$\widehat{E}_t = \frac{\beta}{1+\beta}\,\mathrm{E}_t[\widehat{E}_{t+1}] + \frac{1}{1+\beta}\,\widehat{E}_{t-1} + \frac{(1-\beta\xi_E)\,(1-\xi_E)}{(1+\beta)\,\xi_E}\left(\widehat{N}_t - \widehat{E}_t\right),$$

where a hat ('$\widehat{\ }$') denotes the logarithmic deviation from trend in the case of employment and from the steady-state value in the case of hours worked. The parameter ξ_E determines the sensitivity of employment with respect to hours worked.

[27]That is, we do not include the consumption preference shock, ϵ^C, the labour supply shock, ϵ^N, and the export demand shock, ϵ^X, whereas the inflation objective and the tax rates are assumed to be constant.

consumption and the structural VAR model for the foreign variables, which are all assumed to be serially uncorrelated. In addition, we allow for measurement error in extra-euro area trade data (both volumes and prices) and in the data on real GDP and the GDP deflator, owing to prevailing problems regarding the measurement of the extra-euro area trade data and the consequences this may have for the measurement of GDP.[28]

3.3. Calibration and Prior Distributions

3.3.1. Calibration

We follow the literature and first set key steady-state ratios—including the ratios of the various nominal aggregate demand components over nominal GDP—equal to their empirical counterparts over our estimation sample. For example, the ratios of private consumption and total investment spending are set to 57.5 and 21 percent, respectively, while the export and import ratios are set equal to 16 percent. The trend growth rate in labour productivity g_z is calibrated to equal 1.2 percent per annum, while the long-run (net) inflation objective $\bar{\Pi} - 1$ is assumed to be 2.0 percent at an annualised rate. The discount factor β is then chosen to imply an annualised equilibrium real interest rate of 2.5 percent.

Further, we fix a number of additional parameters that are inherently difficult to identify empirically. This involves setting the capital share in production α to 0.3 and the depreciation rate δ to 0.025, as commonly assumed in the literature. The steady-state wage and price markups φ^W, φ^H, φ^X and φ^* are set uniformly to 0.20, broadly in line with empirical findings of studies conducted at the OECD (cf. Martins, Scarpetta and Pilat, 1996, and Jean and Nicoletti, 2002). Notice that we also set the inverse of the elasticity of labor supply to 2, reflecting the observation that most macro studies overstate the elasticity of labour supply. Regarding final-goods production, we choose values for the home-bias parameters ν_C and ν_I that allow the model to replicate the import content of consumption and investment spending—roughly 10 and 6 percent, expressed as ratios of nominal GDP—utilising information from input-output tables (cf. Statistics Netherlands, 2006). The intratemporal elasticities of substitution between domestic and imported intermediate goods μ_C and μ_I are uniformly set to 1.5, noting that, in the short run, these elasticities may vary depending on the size of the adjustment costs associated with changing the respective import content.

[28]We calibrate the standard deviations of the measurement errors for real GDP, the GDP deflator and the import price deflator such that the measurement errors explains less than five percent of their forecast-error variances, while we estimate the standard deviation of the measurement errors for trade volume data, σ_ω.

Finally, using data from OECD (2004) and Eurostat (2006), the tax rates on consumption purchases, labour and capital income and the contribution rates to social security are calibrated with $\tau^C = 18.3$, $\tau^N = 12.2$, $\tau^K = 0.30$, $\tau^{W_h} = 11.8$ and $\tau^{W_f} = 21.9$, respectively. Due to lack of reliable information the tax rate on dividend income τ^D is set to zero.

3.3.2. Prior Distributions

The left-hand columns in **Table 1** summarise our assumptions regarding the prior distribution of the 43 parameters that we estimate. The prior assumptions for most of the parameters of the domestic economy are similar to those chosen by Smets and Wouters (2003), while the prior assumptions for the open-economy parameters follow closely those in Adolfson et al. (2005).

For those parameters where theory implies boundedness between 0 and 1, a beta distribution is assumed as prior distribution. This group of parameters comprises the habit formation parameter, the Calvo and indexation parameters underlying the wage and price-setting decisions of households and firms, the degree of interest-rate smoothing in the interest-rate rule, the Calvo-style adjustment parameter in the employment equation and the degree of serial correlation of the structural shocks. Regarding the wage-setting decision of households and the price-setting decision of domestic firms selling their outputs at home, the prior means for both the Calvo and the indexation parameters are set to 0.75. In contrast, the prior means for the respective parameters of domestic firms selling abroad and for foreign exporters are set to 0.5, reflecting the higher volatility and lower persistence of import and export price inflation. The prior mean for the autoregressive coefficients of those shock processes featuring serial correlation is set to 0.85. Finally, the prior mean of the parameter determining the degree of interest-rate smoothing is set to 0.9.

The prior distributions for the remaining parameters of the interest-rate rule are modelled as normal distributions. The particular choice of these prior distributions follows Smets and Wouters (2003) and ensures determinacy of the model solution under the prior parametrisation. In particular, the means of the prior distributions equal 1.7 for the inflation response, 0.3 for the response to the change in inflation, 0.125 for the output gap response and 0.0625 for the response to the change in the output gap.

Those structural parameters that are only bounded from below are modelled using a gamma distribution. This group of parameters comprises the various adjustment cost parameters. Finally, the prior distribution for the standard deviations of the structural shocks follow inverse gamma distributions.

3.4. Estimation Results

The right-hand columns in **Table 1** report our benchmark estimation results for the NAWM.[29] The entries in the posterior-mode column give the values of the structural parameters obtained by maximising the posterior distribution with respect to these parameters. The next column shows the respective standard deviations. The remaining three columns report the mean, and the 5th and 95th percentile of the posterior distribution obtained from the Metropolis-Hastings sampling algorithm based on 250,000 draws.

The plots of the prior and posterior distributions in **Figure 2** give an indication of how informative the observed data are about the structural parameters. For those parameters where the posterior distribution turns out to be close to the prior distribution, the data are likely to be rather uninformative. Thus, Figure 2 suggests that the observed data provide additional information for all parameters, except for the inflation response in the interest-rate rule (ϕ_π), which is a frequent finding in the literature.

Regarding the price and wage-setting parameters, we observe that the estimated Calvo parameter for the domestic intermediate goods sold at home is rather high, which implies a rather low sensitivity of domestic inflation with respect to movements in aggregate marginal cost. This result is likely to depend on the chosen sample period. While our estimation is based on data ranging from 1985 to 2005, Smets and Wouters (2003) estimate their model on data from 1980 to 1999 and find a somewhat higher sensitivity. This is in line with empirical evidence that the slope coefficient of Phillips-curve relationships has been declining over recent years.[30] In contrast, the Calvo parameters for setting the price of intermediate goods sold abroad and for setting wages are noticeably lower. Similarly, the Calvo parameter governing the price-setting decisions of foreign exporters is found to be relatively low, causing a significant reduction in the degree of exchange-rate pass-through on the euro area's import side in the short to medium run.

In the NAWM, imports of foreign intermediate goods are used as inputs for the production of the final consumption and investment goods. The estimates of the adjustment

[29] Notice that in estimating our preferred specification, we restrict 4 out of 42 parameters, based on the marginal likelihood that we obtained for differing specifications (see Christoffel, Coenen and Warne (2007) for sensitivity analysis of the benchmark estimation results). In particular, in our preferred specification we do not allow for variable capital utilisation. Furthermore, we fix at their prior means the indexation parameters in the price-setting decisions of domestic and foreign exporters, as well as the sensitivity of the foreign intermediation premium to net foreign assets. These parameters are found to be inherently difficult to identify.

[30] In fact, as shown in Figure 3, recursively estimating the model over the years 1999 to 2005 indicates that the estimate of the Calvo parameter is increasing over this period, corroborating the empirical evidence of a flattening of the Phillips curve.

cost parameters associated with changing the import content differs substantially between the consumption and the investment good. Specifically, the estimated cost of changing the import content of the consumption good (γ_{IM^C}) is substantially higher than the cost associated with changing the import content of the investment good (γ_{IM^I}). Comparing posterior and prior distributions in Figure 2, we observe that this result is strongly driven by the data. Apparently, the relative smoothness of the consumption series implies that shocks affecting import quantities are mainly transmitted via adjustments in the import content of investment.

The estimation results for the parameters determining the real side of the domestic economy are broadly comparable to the results in Smets and Wouters (2003). However, we obtain a somewhat higher estimate for the degree of habit formation, even though the differences in the specification of the utility function dilute the direct comparability. Similarly, the estimated response coefficients of the interest-rate rule are broadly in line with the estimates in Smets and Wouters (2003). The estimated inflation response is safely above unity, ensuring determinacy of the equilibrium, while the response to the output gap is positive, albeit small. We also find supportive evidence for a relatively high degree of interest-rate smoothing.

4. Unconditional Forecasting

As pointed out by Adolfson, Lindé and Villani (2007), one important dimension for evaluating the empirical fit of a model in a policy environment is its out-of-sample forecasting performance. In this section we will examine the forecasting properties of the NAWM against a number of benchmarks. The purpose is not to find the best forecasting model, but to check if the unconditional forecasts generated by the NAWM are "reasonable"; i.e., to examine if the forecasting performance of the model is not considerably worse than that of the benchmarks.

In this regard we may evaluate the forecasting performance in a number of dimensions. First, we may consider standard univariate statistics such as mean forecast errors and root mean-squared forecast errors (RMSEs). Similarly, we may calculate multivariate statistics for point forecasts such as the log-determinant statistic and the trace statistic. An advantage of such statistics is that they take the multivariate nature of many forecasting situations into account. At the same time, they are sensitive to the performance across the most predictable dimensions; see, e.g., Adolfson, Lindé and Villani (2007). The multivariate statistics thus run the risk of being dominated by a specific variable which may be of little interest.[31] In this paper we focus on the univariate mean error and RMSE statistics, but we shall also report multivariate statistics for the variables we are primarily interested in.

4.1. Forecasting with the NAWM

Let $\theta \in \Theta$ be a vector with structural parameters for the log-linearised NAWM that we estimate. Given that a unique convergent solution exists at a particular value for the parameter vector, we can express the relationship between the model variables, defined as deviations from the steady state, and the parameters as a VAR system. Specifically, let η_t be a q-dimensional vector with i.i.d. standard normal structural shocks, ($\eta_t \sim N(0, I_q)$), while ξ_t is an r-dimensional vector of model variables for $t = 1, 2, \ldots, T$. The solution (reduced form) of the log-linearised NAWM can now be represented by:

$$\xi_t = F\xi_{t-1} + B\eta_t, \quad t = 1, \ldots, T, \tag{66}$$

where F and B are uniquely determined by θ. The observed variables are denoted by y_t, an n-dimensional vector, which is linked to the model variables ξ_t through the equation

$$y_t = A'x_t + H'\xi_t + w_t, \quad t = 1, \ldots, T. \tag{67}$$

The k-dimensional vector x_t is here assumed to be deterministic, while w_t is a vector of i.i.d. normal measurement errors with mean zero and covariance matrix R. The measurement errors and the shocks η_t are assumed to be independent, while the matrices A, H, and R are uniquely determined by θ.

The system in (66) and (67) is a state-space model with ξ_t being partially unobserved (if $r > n$) state variables. Equation (66) gives the state transition equation and (67) the measurement equation. Provided the number of measurement errors and structural shocks is large enough, we can calculate the likelihood function for the observed data $\mathcal{Y}_T = \{y_1, \ldots, y_T\}$ via the Kalman filter; see, e.g., Hamilton (1994) for details. The filter can also be used to estimate all unobserved variables in the model at the given value for θ.

Out-of-sample forecasts are calculated for the sample $T+1, \ldots, T+h$, and the model-consistent forecasts will be calculated as in Adolfson, Lindé and Villani (2007). The predictive distribution of $y_{T+1}, \ldots, y_{T+h}$ can be expressed as

$$p(y_{T+1}, \ldots, y_{T+h}|\mathcal{Y}_T) = \int p(y_{T+1}, \ldots, y_{T+h}|\theta, \mathcal{Y}_T)p(\theta|\mathcal{Y}_T)d\theta, \tag{68}$$

[31] Forecasting performance may also be evaluated from the perspective of forecast intervals and density forecasts more generally; see, e.g., Dawid (1982) and Geweke (1999), as well as the study by Adolfson, Lindé and Villani (2007).

where $p(\theta|\mathcal{Y}_T)$ is the posterior distribution of θ based on all available information at T. Since the integral in (68) cannot be evaluated analytically we use the numerical algorithm suggested by Adolfson, Lindé and Villani (2007). That is,

(1) Simulate θ from $p(\theta|\mathcal{Y}_T)$;

(2) Simulate the state variables at time T from $\xi_T \sim N(\xi_{T|T}, P_{T|T})$, where $\xi_{T|T}$ is the filter estimate of ξ_T and $P_{T|T}$ is the covariance matrix of ξ_T given θ and $\mathcal{Y}_T$;

(3) Simulate a path for the state variables from (66) using the simulated value for ξ_T as initial value and a sequence of simulated values for the structural shocks $\eta_{T+1}, \dots, \eta_{T+h}$ from $N(0, I_q)$;

(4) Simulate a sequence of measurement errors $w_{T+1}, \dots, w_{T+h}$ from $N(0, R)$ and compute the path for the observed variables $y_{T+1}, \dots, y_{T+h}$ using the measurement equation (67);

(5) Repeat steps 2-4 N_1 times for the same θ;

(6) Repeat steps 1-5 N_2 times.

The algorithm thus gives $N_1 \cdot N_2$ paths from the predictive distribution in (68). Point and interval forecasts can then be calculated in a straightforward manner, and in our case we use the mean as the point estimate and equal-tail 68 and 90 percent interval forecasts. The number of paths per parameter draw (N_1) and the number of parameters draws (N_2) are both set to 500 in the empirical exercises.

4.2. Forecasting with a Bayesian VAR

Bayesian vector autoregressions (BVARs) have long been considered a useful forecasting tool; see Litterman (1986). The BVAR we study is based on the parameterisation and prior studied by Villani (2005). That is, we consider a VAR model with a prior on the steady-state parameters, and a Minnesota-style prior on parameters on lags of the endogenous variables; see also Adolfson, Lindé and Villani (2007).

The VAR model for the p-dimensional covariance stationary vector z_t is given by:

$$z_t = \Psi d_t + \sum_{l=1}^{k} \Pi_l \big(z_{t-l} - \Psi d_{t-l}\big) + \varepsilon_t, \quad t = 1, \dots, T. \tag{69}$$

The d-dimensional vector d_t is deterministic, and the residuals ε_t are assumed to be i.i.d. normal with zero mean and positive definite covariance matrix Ω. The Π_l matrix is $p \times p$ for all lags, while Ψ is $p \times d$ and measures the expected value of x_t conditional on the parameters and other information available at $t = 0$.

One advantage with the parameterisation in (69) is, as pointed out by Villani (2005), that the steady-state (or mean) of the endogenous variables is directly parameterised via Ψ. For the standard parameterisation of a VAR model the parameters on the deterministic variables are written as $\Phi = (I_p - \sum_{l=1}^{k} \Pi_l)\Psi$ when $d_t = 1$. This makes it difficult to specify a prior on Φ which gives rise to a reasonable prior distribution on the steady-state. Moreover, when z_t is a subset of the observed variables used in the estimation of the NAWM, then we can directly form a prior on the steady state of z_t that is consistent with the steady-state prior for the NAWM as captured by a prior on A. This allows for a more balanced comparison between the models since they can share the same prior mean, or steady-state, for the variables that appear in both models. The steady-state in the NAWM is calibrated, while the steady-state prior covariance matrix is positive definite for the BVAR. Hence, some imbalance between the models remains for the steady-state parameters.[32]

Let $p(\Psi, \Pi, \Omega | \mathcal{Z}_T)$ denote the posterior density where $\Pi = [\Pi_1 \cdots \Pi_k]$ and $\mathcal{Z}_T = \{z_1, \ldots, z_T\}$. Simulation from this distribution is performed via Gibbs sampling for the three groups of parameters Ψ, Π, and Ω using the full conditional posteriors given by Villani (2005, Proposition 2.1). Out-of-sample forecasts for the BVAR are calculated for the sample $T+1, \ldots, T+h$, with the objective of estimating the prediction distribution $p(z_{T+1}, \ldots, z_{T+h} | \mathcal{Z}_T)$. The algorithm used for a BVAR was adapted to a multivariate setting by Villani (2001) from the univariate approach suggested by Thompson and Miller (1986). That is,

(1) Simulate (Ψ, Π, Ω) from $p(\Psi, \Pi, \Omega | \mathcal{Z}_T)$;

(2) Simulate residuals $\varepsilon_{T+1}, \ldots, \varepsilon_{T+h}$ from $N(0, \Omega)$ and calculate a path for the endogenous variables $z_{T+1}, \ldots, z_{T+h}$ using the VAR in (69);

(3) Repeat step 2 N_1 times for the same (Ψ, Π, Ω);

(4) Repeat steps 1-3 N_2 times.

Like for the NAWM we set $N_1 = N_2 = 500$ and calculate the mean from the predictive distribution as well as equal-tail 68 and 90 percent interval forecasts.

4.3. Results

In this section we will focus on a comparison between the NAWM, a BVAR, and two naïve forecasts regarding mean errors and root mean-squared errors (RMSEs). The naïve forecasts are given by a random walk (the last pre-forecast sample observation as the forecast) and

[32] Details on the BVAR model specification are given in Appendix A.

the pre-forecast sample mean (random walk with drift for the accumulated variable, e.g., the level of real GDP). The forecast sample is given by the period 1999Q1-2005Q4 and thus covers the period following the introduction of the euro. For both the BVAR and the NAWM the parameter estimates are updated annually with quarter 4 being the update period, while the forecasts are calculated out-of-sample. The point forecasts from the NAWM are given by the mean of the predictive distributions, while the point forecasts from the BVAR are currently given by the mean of the predictive distributions conditional on the posterior mode estimate of the parameters.

The mean errors for the four forecasts are displayed in **Figure 4**. Regarding year-on-year real GDP growth it is striking that all forecasts except those obtained from the NAWM have negative mean errors; i.e., these models tend to overpredict real GDP growth. The NAWM, on the other hand, tends to underpredict real GDP growth over the first year. For longer forecast horizons, the NAWM has smaller mean errors (in absolute terms) than the other models and, hence, it has a smaller bias. For shorter forecast horizons, the random walk has the smallest mean errors (in absolute terms).

Turning to GDP deflator inflation, we find that the NAWM, the BVAR, and the random walk underpredict this variable. The sample mean, on the other hand, greatly overpredicts inflation. Since the errors are very large (in absolute terms) we have dropped them from the graph. In addition, the mean errors are increasing over the forecast horizon.

Finally, we find that, on average, the nominal interest rate is overpredicted by all forecast models, with the NAWM performing worse than both the random walk and the BVAR. For the NAWM we also find that the forecast errors are increasing with the forecast horizon. As in the case of GDP deflator inflation, the mean errors from the sample mean are too large (in absolute terms) to fit into the figure.

The results on RMSEs are shown in **Figure 5** for year-on-year real GDP growth, year-on-year GDP deflator inflation, and the nominal interest rate. It can be seen here that the NAWM fares quite well in comparison to the BVAR and the naïve forecasts. The main exception is for real GDP growth where the sample mean does best from the 4th through the 8th quarter.[33]

In **Figure 6** we have plotted rolling mean predictions from the NAWM and the BVAR. Focusing on year-on-year real GDP growth, it is noteworthy that the NAWM is unable to

[33] It may be noted that the RMSEs of GDP deflator inflation and the nominal interest rate for the sample mean are much larger than those for the other forecasts.

predict the recovery at the very beginning of the forecast sample, while the BVAR fares better. In contrast, the 2000-2001 slow-down in economic activity and the subsequent recovery is well predicted by the NAWM, while the naïve benchmarks are, by construction, unable to capture this event. At the same time the length of the recovery that begins in 2002 is overestimated by the NAWM, while the BVAR roughly gets the length but not the height of the recovery right.

The poor performance for the BVAR regarding GDP deflator inflation is evident from Figure 6 where it on average under-predicts inflation. It appears that the choice of steady-state prior covariance matrix is not important for explaining this result. In particular, lowering the variance for the steady-state prior on domestic and foreign inflation does not affect the predictions from the BVAR to any great extent.[34]

We have plotted the mean predictions and equal-tail prediction intervals for the prediction sample beginning with 2001Q2 in **Figure 7**. The CEPR Business Cycle Dating Committee (2003) notes that the euro area has essentially stagnated since 2001Q1, even though the downturn thereafter is not regarded as a recession. It can be seen that euro area real GDP growth had already started to decline prior to early 2001 (the peak is located at 2000Q2). Based on our data the NAWM predicts that real GDP growth will continue to fall until early 2002 and thereafter pick up again. This actually corresponds very well with what happended. The mean predictions from the BVAR, in contrast, are fairly flat at the observed growth rate for 2001Q1, and therefore completely fail for this forecast period. For the GDP deflator inflation and the nominal interest rate predictions, the results from the NAWM and the BVAR are similar. It is worth noting that the prediction intervals are much tighter for the BVAR when it comes to the inflation predictions, especially for the longer horizons.

[Multivariate forecast performance measures based on the h-steps ahead MSE matrix will be added in the next draft.]

5. Conditional Forecasting with the NAWM

Conditional forecasting concerns forecasts of endogenous variables conditional on a certain path and length of path for some other endogenous variables; see, e.g., Waggoner and Zha (1999). In this section we will discuss conditional forecasting as we have implemented it for the NAWM. Specifically, it is assumed that the conditioning information satisfies *hard*

[34] We are currently considering alternative specifications of the prior on the parameters on lagged endogenous variables.

conditions (a particular path) rather than *soft conditions* (a range for the path). Following Leeper and Zha (2003) we also assume that selected shocks are manipulated to ensure that the conditioning information is met by the predictions; for alternative approaches, see Waggoner and Zha (1999), and Robertson, Tallman and Whiteman (2005).

The assumption that particular shocks can be picked is not restrictive for the majority of the conditioning variables that we will consider. The reason is that only a certain subset of the shocks can affect these variables. Specifically, the argument applies in particular to real government consumption and the four foreign variables, which are exogenous for the other observed variables. At the same, we will also condition on the nominal interest rate and the nominal exchange rate, where the Waggoner and Zha approach is a relevant alternative. Furthermore, in view of the Lucas (1976) critique we will evaluate the reasonableness of the conditioning assumptions by the *modesty* statistics developed by Leeper and Zha (2003) and Adolfson, Laséen, Lindé and Villani (2005).

5.1. Implementation of the Conditioning Assumptions

Let K_1 and K_2 be known $n \times q_m$ matrices with $n > q_m$ such that $\operatorname{rank}(K_1) = q_m$. Furthermore, consider the following relation:

$$c_{T+i} = K_1' y_{T+i} + \sum_{j=1}^{i-1} K_2' y_{T+i-j} + u_T, \quad i = 1, \ldots, g, \tag{70}$$

where the conditioning horizon g is less than or equal to the forecast horizon h.

The specification in equation (70) is general enough to satisfy our purposes. In the special case where $K_2 = 0$ and $u_T = 0$ the conditioning vector c_{T+i} is determined directly from y_{T+i}; e.g., from one particular observed variable. Although such a specification covers many interesting cases it does not allow us to handle the case when y_{T+i} includes the real exchange rate and the first differences of domestic and foreign prices, but where c_{T+i} is the nominal exchange rate. For example, let $p_{Y,t}$ and $p_{Y,t}^*$ denote the domestic and foreign GDP deflators, respectively, while s_t^n denotes the nominal exchange rate. We may then let K_1 be defined such that $K_1' y_{T+i} = (s_{T+i}^n + p_{Y,T+i}^* - p_{Y,T+i}) + \Delta p_{Y,T+i} - \Delta p_{Y,T+i}^*$, whereas $K_2' y_{T+i-j} = \Delta p_{Y,T+i-j} - \Delta p_{Y,T+i-j}^*$ and $u_T = p_{Y,T} - p_{Y,T}^*$.

To our knowledge the use of conditioning information that only partially restricts the future path of observed variables has not been considered previously in the literature. Moreover, the specification in (70) can easily be extended to handle first differences and annual differences of observed variables without complicating the algorithms provided below to any great extent.

To keep the values in c_{T+i} fixed over the given horizon we require that a subset of the structural shocks are adjusted to take on certain values. The selection of structural shocks is determined by the $q \times q_m$ matrix M, where $q > q_m$ and $\mathrm{rank}(M) = q_m$. Let $M_\perp$ be the $q \times (q - q_m)$ orthogonal matrix; i.e., $M_\perp' M = 0$. It now follows that $N = [M_\perp \; M]$ is a full rank $q \times q$ matrix, while

$$N'\eta_t = \begin{bmatrix} M_\perp' \\ M' \end{bmatrix} \eta_t = \begin{bmatrix} \eta_t^{(q-q_m)} \\ \eta_t^{(q_m)} \end{bmatrix}. \tag{71}$$

The shocks $\eta_t^{(q_m)}$ will be adjusted over the time interval $t = T+1, \ldots, T+g$ to ensure that (70) is met for all forecast paths of the observed variables over this time interval.

With $\bar{M} = M(M'M)^{-1}$ and $\bar{M}_\perp = M_\perp(M_\perp' M_\perp)^{-1}$ it is straightforward to show that the values for the structural shocks which guarantee that the conditioning path $c_{T+1}, \ldots, c_{T+g}$ in (70) is always met are:

$$\begin{aligned} \eta_{T+i}^{(q_m)} = \big(K_1' H' B \bar{M}\big)^{-1} \Bigg[& c_{T+i} - K_1' A' x_{T+i} - K_1' w_{T+i} - K_1' H' F \xi_{T+i-1}^{(q_m)} \\ & - K_1' H' B \bar{M}_\perp \eta_{T+i}^{(q-q_m)} - K_2' \sum_{j=1}^{i-1} y_{T+i-j}^{(q_m)} - u_T \Bigg], \quad i = 1, \ldots, g, \end{aligned} \tag{72}$$

while the states and the observed variables evolve according to:

$$\xi_{T+i}^{(q_m)} = F \xi_{T+i-1}^{(q_m)} + B \bar{M}_\perp \eta_{T+i}^{(q-q_m)} + B \bar{M} \eta_{T+i}^{(q_m)}, \quad i = 1, \ldots, g, \tag{73}$$

and

$$y_{T+i}^{(q_m)} = A' x_{T+i} + H' \xi_{T+i}^{(q_m)} + w_{T+i}, \quad i = 1, \ldots, g, \tag{74}$$

with $\xi_T^{(q_m)} = \xi_T$. It should be noted that the calculations involve the assumption that the matrix $K_1' H' B \bar{M}$ has full rank q_m. This is not a strong condition since $H'B$ must have rank n for the structural shocks to be uniquely determined from the data and the parameters, while q_m is always less than n.

For $i > g$ there are not any direct restrictions on the possible paths for the observed variables other than that the state vector at $T+g$ needs to be taken into account.

Estimation of the predictive distribution for $y_{T+1}, \ldots, y_{T+h}$, taking the conditioning assumptions into account, can now be achieved as follows:

(1) Simulate θ from $p(\theta|\mathcal{Y}_T)$;

(2) Simulate the state variables at time T from $\xi_T \sim N(\xi_{T|T}, P_{T|T})$, where $\xi_{T|T}$ is the filter estimate of ξ_T and $P_{T|T}$ is the covariance matrix of ξ_T given θ and $\mathcal{Y}_T$;

(3) Simulate a sequence of structural shocks $\eta_{T+1}, \ldots, \eta_{T+h}$ from $N(0, I_q)$ and a sequence of measurement errors $w_{T+1}, \ldots, w_{T+h}$ from $N(0, R)$;

(4) Calculate the restricted shocks $\eta_{T+i}^{(q_m)}$, states $\xi_{T+i}^{(q_m)}$, and the conditioning information consistent observed values $y_{T+i}^{(q_m)}$ for $i = 1, \ldots, g$ using equations (72)-(74);

(5) If $g < h$ calculate the remaining states $\xi_{T+g+1}, \ldots, \xi_{T+h}$ using the structural shocks $\eta_{T+g+1}, \ldots, \eta_{T+h}$ and the initial value for the states $\xi_{T+g}^{(q_m)}$ from the state equation (66), and the remaining observed variables $y_{T+g+1}, \ldots, y_{T+h}$ using the measurement equation (67);

(6) Repeat steps 2-5 N_1 times for the same θ;

(7) Repeat steps 1-6 N_2 times.

It should be pointed out that this algorithm assumes that the conditioning information is not informative about the unknown parameters. The approach suggested by Waggoner and Zha (1999) for VAR models is not subject to this technical weakness.

5.2. The Conditioning Information

Below we will study three nested conditioning information sets of increasing size. For all such sets the structural shocks that need to be fixed over the conditioning horizon are also specified. The latter is here equal to the forecast horizon; i.e., $g = h = 8$ quarters.

A: The nominal interest rate is assumed to be fixed at the last observed value over the conditioning horizon. That is, we condition on the assumption of unchanged interest rates, as in the ECB's macroeconomic projections over the relevant sample period. As in Adolfson, Laséen, Lindé and Villani (2005), the monetary policy shock is manipulated to ensure the assumed path.

B: In addition to A, the nominal exchange rate is fixed at the last observed value over the conditioning horizon. That is, we condition on a random walk for the nominal effective exchange rate, as opposed to determining the exchange rate by the uncovered interest parity condition implied by the log-linearised NAWM. The external risk premium shock is added to the set of shocks in $\eta^{(q_m)}$.

C: In addition to the variables and shocks in B, all 4 foreign variables and real government consumption are assumed to take on their ex-post realisations. The added shocks are the 4 foreign shocks and the government consumption shock.

The extra conditioning assumptions in C relative to B concern exogenous variables in the NAWM and, hence, the selection of additional shocks is natural. Moreover, this part of the

NAWM does not involve any parameters in θ. Hence, this subset of the conditioning set is indeed uninformative about θ. The choice of ex-post realisations is, however, potentially strong as it does not match a proper projection situation in real time.[35] Moreover, since parameters for the AR-process for government consumption as well as the SVAR system for the foreign variables is calibrated based on estimates obtained from the sample 1985Q1-2005Q4, the shocks needed to obtain the ex-post path for these variables will be equal to those found when the NAWM is estimated over the full sample. Hence, we expect these shocks to be modest by construction.

5.3. Comparisons with the Unconditional Forecasts

Before we turn our attention to the modesty tests it is interesting to compare the performance of the unconditional forecasts for the NAWM to the three sets of conditioning data. The mean forecast errors for year-on-year real GDP growth, year-on-real GDP deflator inflation and the nominal interest rate are displayed in **Figure 8**.

For real GDP growth we find that the conditioning information leads to smaller forecast errors (in absolute terms) until quarter 6. From that point on the unconditional forecasts have the smallest bias. While it is not surprising that using future data for real government consumption and the foreign variable can improve short-term real GDP forecasts, it is perhaps surprising that conditioning on a constant nominal interest rate leads to smaller short-term forecast errors. This is, however, most likely related to the somewhat worse performance of the NAWM relative to the random walk forecast for the nominal interest rate. In this context, it may also be recalled that the steady-state nominal interest rate in the NAWM is 4.5 percent, which is higher than the realised interest rate over most of the forecast sample.

Turning to GDP deflator inflation we find that the impact of the conditioning data is small for the mean errors. In particular, for conditioning set A (constant nominal interest rate) the mean errors are roughly the same as for the unconditional forecasts. Once the assumption of a constant nominal exchange rate is added, the mean errors tend to fall, especially for the longer horizons.

The RMSEs for the 4 forecast cases are shown in **Figure 9**. The RMSEs under conditioning are smaller for real GDP predictions than those for the unconditional forecasts for short-term forecasts. This is in line with the evidence we have reported above for the mean

[35] We will consider using real-time assumptions for the foreign variables and real government consumption instead of the ex-post realisations.

errors. Furthermore, the RMSEs for GDP deflator inflation are similar across the different conditioning assumptions. Again, conditioning set A leads to the smallest change relative to the unconditional forecasts.

In **Figure 10** we have plotted the mean for the unconditional predictive density along with the 68 percent prediction bands, as well as the mean predictions and the 68 percent prediction bands under the three conditioning sets. In the upper part of the Figure we find the results for year-on-year real GDP and in the lower part the evidence for year-on-year GDP deflator inflation. As in Figure 7 we focus on forecasts that begin in 2001Q2.

For GDP deflator inflation it is noteworthy that the conditioning information has only a minor impact on the predictions. For real GDP growth, however, the mean predictions as well as the equal-tail prediction bands shift downwards relative to the unconditional predictions. In particular, when all the conditioning assumptions are used (conditioning set C), then the prediction for 2002Q1 are shifted downward by roughly 0.5 percent.

Summarising our comparison between the unconditional and conditional forecasts with the NAWM, we have found that the impact of the conditioning assumptions is not extreme when it comes to the performance for the point forecasts. For real GDP growth the conditioning data improves the forecasts at the shorter horizons. This is especially true when conditioning set C is applied.

5.4. Modesty Statistics

Since the conditional forecasts rely on manipulating structural shocks that, by assumption, have zero mean and are uncorrelated with the history of the observed variables, there is no guarantee that the values that we obtain for the manipulated shocks will be consistent with this assumption. In other words, the experiment may be subject to the Lucas (1976) critique since the agents should be able to detect substantial changes in the behavior of the shocks. To assess the relevance of the Lucas critique we shall evaluate the conditional forecasts through *modesty* statistics developed by Leeper and Zha (2003) (LZ) for VAR models and further refined by Adolfson, Laséen, Lindé and Villani (2005) (ALLV) to DSGE models as well as to a multivariate setting.

The basic idea behind a modesty statistic is to compare the conditional forecasts with the unconditional forecasts, accounting for the forecast error variance under the unconditional forecast. If the conditioning assumptions are modest, then the resulting t-statistic should be standard normal. The statistic in LZ is formulated in terms of a VAR model and it is assumed that all shocks apart from those that are manipulated are set to zero over the forecast horizon. Moreover, all parameters and current and past values of the variables are treated as known since the economic agents within the model have no uncertainty about them. This statistic has been extended to DSGE (state-space) models by ALLV. They also note that it is more natural to take the uncertainty of all other shocks into account, especially when the other shocks account for the bulk of the forecast uncertainty of a variable. Apart from setting the modesty statistic in a more realistic environment, this extension allows for a multivariate treatment of the Lucas critique since the forecast error covariance matrix for all the observed variables will no longer be singular.

The question of how to treat the parameters when calculating the modesty tests, however, remains open. As mentioned above, the agents of the model are assumed to know the "true value" of θ, and ALLV uses the posterior mode estimate when computing the modesty statistics. This is *one* candidate, but we may just as well use some other point estimator such as the posterior mean or the median. This emphasises that from the econometrician's perspective the choice of parameter value is subject to uncertainty and there is no guarantee that different point estimators will give qualitatively similar results. Since the Bayesian framework allows us to account for parameter uncertainty we shall also utilise the posterior distribution. In this paper we will only consider extending the ALLV multivariate statistic, calculated as in their study but for different draws from the posterior. Like in ALLV we compare the multivariate statistic to a reference statistic that is calculated in the same way as the modesty statistic, except that the shocks that were originally manipulated are now drawn from their distribution. This gives us a total of $N_1 \cdot N_2$ values of each statistic and we calculate the tail probability by recording how many time the modesty statistic is less than or equal to the reference statistic divided by the number of times each statistic has been computed.

The modesty statistics when the parameters are given by the posterior mode estimates are shown in **Figure 11**. Rolling values of the univariate statistics (based on the posterior mode estimate of θ) for the 8-steps ahead forecasts are displayed in the first two figure columns, while the multivariate statistics (for the posterior mode estimate) are located in the third figure column. The evidence for the three conditioning sets is given in the figure rows. The vertical axis gives the tail probabilities of the statistics, while the horizontal axis gives the start date for the 8-steps ahead forecasts. Hence, the values for 2004Q1 concern the forecast that begins at that date and ends at 2005Q4.

It is striking that the tail probabilities for the univariate LZ statistics fluctuate considerably more than those for the univariate ALLV test. This is primarily due to the lower forecast error variances for the LZ statistics, where only the variances of the shocks that are manipulated are taken into account. A few large (but not necessarily unusually large) shocks may therefore greatly affect particular values of the LZ statistic. In the case of GDP deflator inflation, the LZ statistics signal that the conditioning assumptions in the sets A and B may not be regarded as modest over the 1999-2000 period. The univariate ALLV statistics, in contrast, yield high tail probabilities that fluctuate relatively little over the forecast sample. These latter results are supported by the multivariate statistics, which in particular under conditioning sets A and B are very stable at 50 percent.

One interpretation of the results for the univariate and multivariate ALLV statistics has already been touched upon. In Section 5.2 it was noted that the government consumption and foreign shocks are likely to be modest over the conditioning sample since the conditioning data are the ex-post realisations and the processes for these variables are calibrated based on estimates for the full sample. For conditioning set A we use the assumption that the nominal interest rate follows a random walk. When comparing the forecasting performance between the NAWM unconditionally and under conditioning set A we have found that the random walk assumption tends to improve the forecasts somewhat in terms of bias. However, the impact seems to be relatively small and the lack of modesty based on the ALLV statistics may therefore not be so surprising.

For conditioning set B we add the assumption that the nominal exchange rate follows a random walk. It is a well established empirical result for nominal exchange rate predictions that the random walk model is difficult to beat. If the forecasts of the nominal exchange rate from the NAWM are close to the random walk forecasts we expect the modesty statistics to be small. Once we add the conditioning assumptions in C, taking the above comments about these exogenous variables into account, the ALLV based modesty results may also be understood under this conditioning set.

[Multivariate modesty statistics based on draws from $p(\theta|\mathcal{Y}_T)$ to be added in the next draft.]

5.5. Prediction Event Probabilities

One important advantage of a Bayesian approach to forecasting is that we can calculate probabilities of certain events over the forecast sample. For example, we may want to know what the probability is that consumer price inflation at some point is greater than, say, 2 percent. Similarly, we may want to learn how big the probability of the economy going into a recession is. The algorithms for estimating the unconditional and conditional prediction distributions that are discussed in Sections 4 and 5 make it straightforward to calculate the probabilities of such prediction events.

In this section we are concerned with 5 different events for 3 observed variables. First, we define a recession as the case when year-on-year real GDP growth is negative for at least 3 consecutive quarters. Next, we wish to know what the probability is that year-on-year GDP deflator inflation lies between 0 and 2 percent, as well as the probability that it falls below 0 percent. Finally, the same two events are considered for year-on-year consumption deflator inflation.

The empirical results are summarised in **Figure 12**. The dates on the horizontal axis refer to the first forecast period. It is noteworthy that all predictive distributions give roughly the same prediction event probabilities over the forecast sample. The largest differences are found for the recession probabilities from late 2000 to early 2001. The most restrictive conditioning set (C) yields higher recession probabilities at the beginning of this period, suggesting that the ex-post realisation of the foreign variables are important indicators.

During this period the probabilities that year-on-year GDP deflator inflation lies between 0 and 2 percent is roughly 25 percent, while the probabilities of deflation is no more than 10 percent. Hence, the probability of GDP deflator inflation rising above 2 percent is regarded as quite high. For the year-on-year consumption deflator, the probability of inflation rising above 2 percent is somewhat lower, but nevertheless indicating a stronger upward than downward risk for price stability.

In connection with the close to zero nominal interest rates in Japan, there was heightened concern for deflationary risks also in the U.S. and in Europe. During the period 2003-2004 the deflation probabilities in the lower part of Figure 12 are slowly rising for both the GDP deflator and the consumption deflator series. The values reach about 20 percent for the GDP deflator and 30 percent for the consumption deflator. This may be compared with deflation probablities around 10 (15) percent for the GDP (consumption) deflator during the 2000-2001 period.

6. Conclusion

We examined conditional versus unconditional forecasting with a version of the NAWM, an estimated small open-economy model of the euro area designed for use in the macroeconomic projections at the ECB. In terms of forecasting performance, we showed that the NAWM fares quite well compared to a BVAR and the random walk, in particular in the case of real GDP growth and GDP inflation for horizons that extend beyond one year. We then demonstrated that conditioning the model-based forecasts on a large set of policy-relevant variables helps to improve the forecasting performance over some horizons, albeit not systematically. At the same time, we showed that conditioning on alternative information sets does not bias the forecasts. This is in line with our finding that the conditioning assumptions are modest in the sense of Leeper and Zha, at least as long as the multivariate nature of the underlying shock uncertainty is taken into account. As regards our analysis of certain prediction events, we identified a heightened probability of a recession in 2001 which is broadly similar across information sets, even though it is more pronounced when conditioning on (ex post) foreign data.

In the future we will examine the forecast performance on the basis of all observed variables used in the estimation of the NAWM. In this context we plan to compare the results for the NAWM with those obtained from larger BVARs including additional variables. Moreover, we shall consider replacing the ex-post realisations of government consumption and the foreign variables with real-time assumptions. Finally we plan to take parameter uncertainty into account for the modesty statistics.

References

Adolfson, M., M. K. Anderson, J. Lindé, M. Villani and A. Vredin, 2005, "Modern Forecasting Models in Action: Improving Macroeconomic Analysis at Central Banks", Sveriges Riksbank Working Paper Series, No. 188, September.

Adolfson, M., S. Laseén, J. Lindé and M. Villani, 2005, "Are Constant Interest Rate Forecasts Modest Policy Interventions? Evidence from a Dynamic Open-Economy Model", *International Finance*, 8, 509-544.

Adolfson, M., S. Laseén, J. Lindé and M. Villani, 2007, "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through", *Journal of International Economics*, 72, 481-511.

Adolfson, M., J. Lindé and M. Villani, 2007, "Forecasting Performance of an Open Economy DSGE Model", *Econometric Reviews*, 26, 289-328.

Bayoumi, T., D. Laxton and P. Pesenti, 2004, "Benefits and Spillovers of Greater Competition in Europe: A Macroeconomic Assessment", ECB Working Paper No. 341, European Central Bank, April.

Benigno, P., 2001, "Price Stability with Imperfect Financial Integration", mimeo, New York University, December.

Betts, C. and M. B. Devereux, 1996, "The Exchange Rate in a Model of Pricing-to-Market", *European Economic Review*, 40, 1007-1021.

Calvo, G. A., 1983, "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, 12, 383-398.

CEPR, 2003, *Findings of the Business Cycle Dating Committee of the Centre for Economic Policy Research*, September, available at http://cepr.org/data/dating.

Christoffel, K. and G. Coenen, 2006, "The Estimated Version of the NAWM: Technical Documentation", mimeo, European Central Bank, November.

Christoffel, K., G. Coenen and A. Warne, 2007, "The New Area-Wide Model (NAWM) for the Euro Area: Specification, Estimation Results and Properties", in progress, European Central Bank, June.

Coenen, G., P. McAdam and R. Straub, 2007, "Tax Reform and Labour-Market Performance in the Euro Area: A Simulation-Based Analysis Using the New Area-Wide Model", ECB Working Paper No. 747, April, forthcoming in the *Journal of Economic Dynamics and Control*.

Dawid, P., 1982, "The Well-Calibrated Bayesian", *Journal of the American Statistical Association*, 77, 605-610.

Del Negro, M., F. Schorfheide, F. Smets and R. Wouters, 2007, "On the Fit and Forecasting Performance of New-Keynesian Models", *Journal of Business & Economics Statistics*, 25, 123-143.

Edge, R., K. Kiley and J. P. Laforte, 2006, "A Comparison of Forecast Performance between Federal Reserve Staff Forecasts, Simple Reduced-Form Models, and a DSGE Model", paper presented at the 7th Workshop of the Euro Area Business Cycle Network (EABCN), August.

Erceg, C. J., L. Guerrieri and C. Gust, 2005, "SIGMA: A New Open Economy Model for Policy Analysis", International Finance Discussion Papers No. 835, Board of Governors of the Federal Reserve System, July.

Eurostat, 2006, "Structures of the Taxation Systems in the European Union, 1995-2004", European Commission, Brussels.

Fagan, G., J. Henry and R. Mestre, 2001, "An Area-Wide Model (AWM) for the Euro Area", ECB Working Paper No. 42, European Central Bank, January.

Geweke, J. F., 1999, "Using Simulation Methods for Bayesian Econometric Models: Inference, Developments and Communication", *Econometric Reviews*, 18, 1-73.

Hamilton, J. D., 1994, *Time Series Analysis*, Princeton University Press, Princeton.

Jean, S. and G. Nicoletti, 2002, "Product Market Regulation and Wage Premia in Europe and North America", OECD Economic Department Working Paper ECO/WKP(2002)4, Organisation for Economic Co-operation and Development.

Leeper, E. M. and T. Zha, 2003, "Modest Policy Interventions", *Journal of Monetary Economics*, 50, 1673-1700.

Litterman, R. B., 1986, "Forecasting with Bayesian Vector Autoregressions — Five Years of Experience", *Journal of Business & Economic Statistics*, 4, 25-38.

Lucas, Jr., R. E., 1976, "Econometric Policy Evaluation: A Critique", in: Carnegie-Rochester Series on Public Policy, Vol. 1, K. Brunner and A. H. Meltzer (eds.), North-Holland, Amsterdam, 19-46.

Martins, J. O., S. Scarpetta and D. Pilat, 1996, "Mark-up Pricing, Market Structure and the Business Cycle", *OECD Economic Studies*, 27, 71-106.

OECD, 2004, "Taxing Wages 2003-2004", OECD Statistics, Organisation for Economic Co-operation and Development.

Robertson, J. C., E. W. Tallman and C. H. Whiteman, 2005, "Forecasting Using Relative Entropy", *Journal of Money, Credit, and Banking*, 37, 383-401.

Schorfheide, F., 2000, "Loss-Function-Based Evaluation of DSGE Models", *Journal of Applied Econometrics*, 15, 645-670.

Smets, F. and R. Wouters, 2003, "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area", *Journal of the European Economic Association*, 1, 1123-1175.

Smets, F. and R. Wouters, 2004, "Forecasting with a Bayesian DSGE Model: An Application to the Euro Area", *Journal of Common Market Studies*, 42, 841-867.

Smets, F. and R. Wouters, 2007, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", ECB Working Paper No. 722, February, forthcoming in the *American Economic Review*.

Statistics Netherlands, 2006, "Attributing the Euro Area GDP Growth Rate to Final Demand Components", Report, July.

Thompson, P. A. and R. B. Miller, 1986, "Sampling the Future: A Bayesian Approach to Forecasting from Univariate Time Series Models", *Journal of Business & Economic Statistics*, 4, 427-436.

Villani, M., 2001, "Bayesian Prediction with Cointegrated Vector Autoregressions", *International Journal of Forecasting*, 17, 585-605.

Villani, M., 2005, "Inference in Vector Autoregressive Models with an Informative Prior on the Steady State", Sveriges Riksbank Working Paper Series No. 181, March.

Waggoner, D. F. and T. Zha, 1999, "Conditional Forecasts in Dynamic Multivariate Models", *Review of Economics and Statistics*, 81, 639-651.

Warne, A., 2007, "YADA Manual — Computational Details", mimeo, European Central Bank, June.

Appendix A: The Specification of the BVAR

As in Villani (2005) we assume that Ψ is a priori independent of Π_l and Ω with $\text{vec}(\Psi) \sim N(\mu_\psi, \Sigma_\psi)$ and Σ_ψ being positive definite. Regarding the parameters on lags of the endogenous variables we define $\Pi = [\Pi_1 \ \cdots \ \Pi_k]$ and assume that $\text{vec}(\Pi) \sim N(\mu_\pi, \Sigma_\pi)$. Finally, a diffuse prior on Ω is used as represented by the well-known form $p(\Omega) \propto |\Omega|^{-(p+1)/2}$.

To parameterise the prior on Π we assume that the prior mean of Π_l is zero for all $l \geq 2$. For the first lag all off-diagonal elements are assumed to be zero, while the diagonal elements are equal to λ_D when $z_{i,t}$ is a first differenced variable (e.g., real GDP growth), and given by λ_L when $z_{i,t}$ is a levels variables (e.g., the nominal interest rate). Regarding the parameterisation of Σ_π we use a Minnesota-style prior. Letting $\Pi_{ij,l}$ denote the element in row (equation) i and column (on variable) j for lag l, the matrix Σ_π is assumed to be diagonal with

$$\text{Var}\bigl(\Pi_{ij,l}\bigr) = \begin{cases} \dfrac{\lambda_o}{l^{\lambda_h}}, & \text{if } i = j, \\ \dfrac{\lambda_o \lambda_c \Omega_{ii}}{l^{\lambda_h} \Omega_{jj}}, & \text{otherwise.} \end{cases}$$

The parameter Ω_{ii} is simply the variance of the residual in equation i and, hence, the ratio Ω_{ii}/Ω_{jj} takes into account that variable i and variable j may have different scales.

Formally, this parameterisation is inconsistent with the prior being a marginal distribution since it depends on Ω. As is standard for the Minnesota-type of prior we deal with this by replacing the Ω_{ii} parameters with the maximum likelihood estimate. The hyperparameter $\lambda_o > 0$ gives the overall tightness of the prior around the mean, while $0 < \lambda_c < 1$ is the cross-equation tightness hyperparameter. Finally, the hyperparameter $\lambda_h > 0$ measures the harmonic lag decay.

In the empirical application the BVAR model has 7 variables that are taken from the observed variable set for NAWM. The variables we have selected are the same type of variables as were used by Adolfson, Anderson, Lindé, Villani and Vredin (2005) in their BVAR. They are: real GDP growth, GDP deflator inflation, nominal interest rate, real exchange rate, growth rate of real foreign demand, foreign GDP deflator inflation, and the foreign interest rate. For the steady state we let:

$$\mu_\psi' = \begin{bmatrix} 0.5 & 0.5 & 4.5 & 0 & 0.5 & 0.5 & 4.5 \end{bmatrix}, \quad \text{diag}(\Sigma_\psi)' = \begin{bmatrix} 1 & 1 & 5 & 20 & 1 & 1 & 5 \end{bmatrix},$$

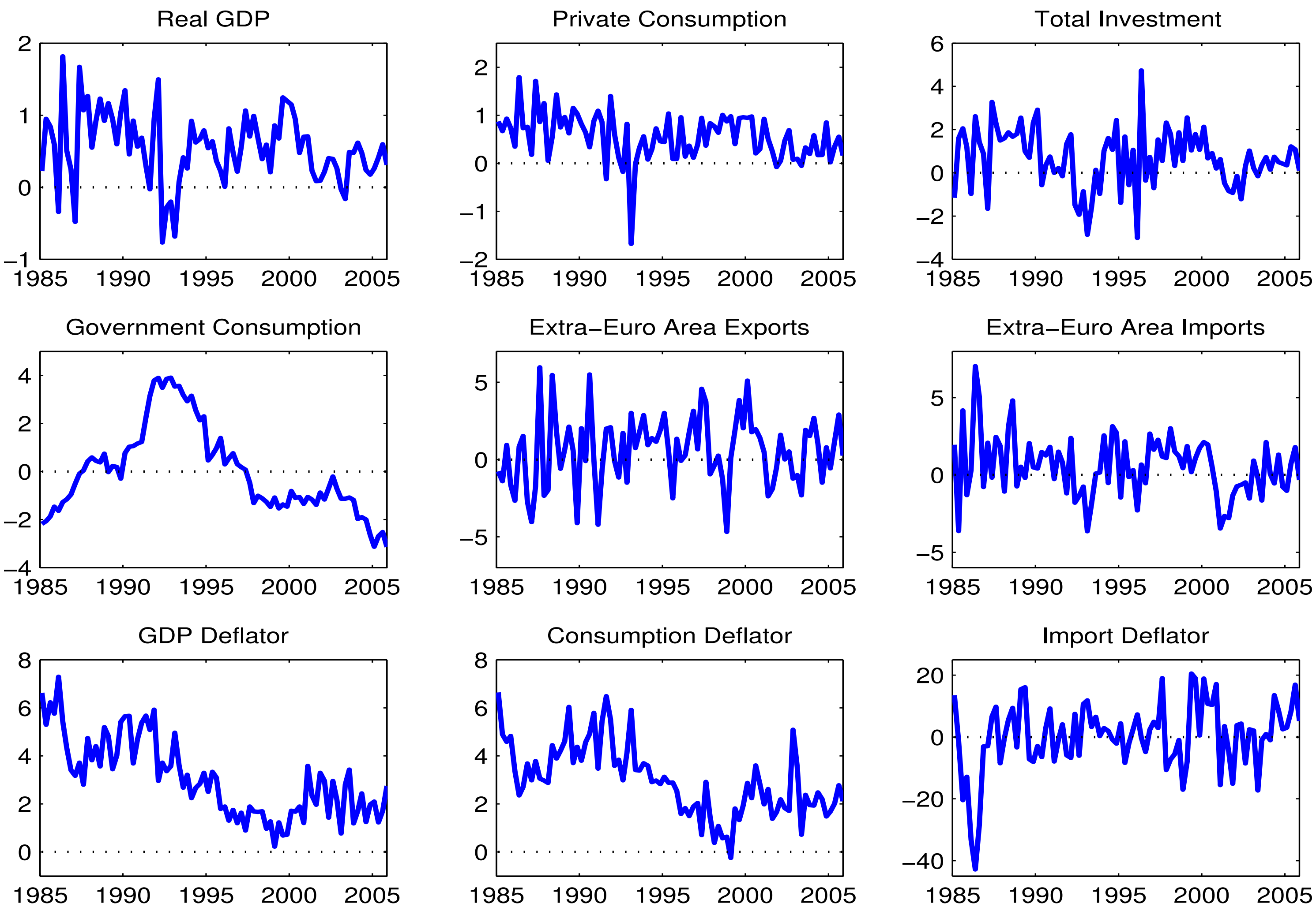
while all off-diagonal elements of Σ_ψ are zero. For real GDP growth and inflation the mean steady-state values concern the quarterly rates. The hyperparameters for the Π_l parameters are given by $\lambda_L = 0.9$, $\lambda_D = 0$, $\lambda_o = \lambda_c = 0.5$, while $\lambda_h = 1$. The variables viewed as first differenced variables are domestic real GDP growth and real foreign demand growth, while all other variables are viewed as levels variables. The lag order, k, is set to 4.

Table 1: Prior and Benchmark Posterior Distributions

Parameter		Prior distribution			Posterior distribution				
		type	mean	std/df	mode	std	mean	5%	95%
Preferences									
Habit formation	κ	beta	0.700	0.05	0.671	0.039	0.673	0.596	0.758
Wage and price setting									
Calvo parameter: wages	ξ_W	beta	0.750	0.05	0.841	0.026	0.838	0.775	0.893
Indexation: wages	χ_W	beta	0.750	0.15	0.340	0.115	0.312	0.151	0.510
Calvo parameter: domestic	ξ_H	beta	0.750	0.05	0.944	0.006	0.944	0.931	0.956
Indexation: domestic	χ_H	beta	0.750	0.15	0.660	0.058	0.684	0.561	0.811
Calvo parameter: exports	ξ_X	beta	0.500	0.10	0.815	0.031	0.812	0.743	0.870
Indexation: exports	χ_X	beta	0.500	0.15	0.500	—	—	—	—
Adjustment costs									
Investment	γ_I	gamma	4.000	0.75	5.834	0.678	5.883	4.562	7.322
Capital utilisation	$\gamma_{u,2}$	gamma	0.010	0.01	10^6	—	—	—	—
Import content: cons.	γ_{IM^C}	gamma	2.500	1.00	6.080	1.020	6.207	4.362	8.390
Import content: invest.	γ_{IM^I}	gamma	2.500	1.00	0.526	0.121	0.544	0.306	0.847
Financial intermediation	$\gamma_{B^\star}$	gamma	0.010	0.01	0.010	—	—	—	—
Foreign ex- and importers									
Calvo parameter: imports	$\xi^\star$	beta	0.500	0.10	0.778	0.031	0.774	0.700	0.841
Indexation: imports	$\chi^\star$	beta	0.500	0.15	0.500	—	—	—	—
Export price elasticity	$\mu^\star$	gamma	1.500	0.25	0.954	0.081	0.939	0.691	1.227
Export adjustment cost	$\gamma^\star$	gamma	2.500	1.00	1.321	0.183	1.309	0.859	1.846
Monetary policy									
Interest-rate smoothing	ϕ_R	beta	0.900	0.05	0.910	0.016	0.895	0.842	0.934
Resp. to inflation	ϕ_Π	normal	1.700	0.10	1.733	0.092	1.728	1.564	1.893
Resp. to inflation diff.	$\phi_{\Delta\Pi}$	normal	0.300	0.10	0.109	0.039	0.122	0.050	0.194
Resp. to output gap	ϕ_Y	normal	0.125	0.05	0.205	0.039	0.220	0.135	0.299
Resp. to output gap diff.	$\phi_{\Delta Y}$	normal	0.0625	0.05	0.111	0.025	0.093	0.027	0.153
Employment									
Calvo-style parameter	ξ_E	beta	0.500	0.15	0.859	0.007	0.859	0.834	0.883
Autoregressive coefficients									
Transitory techn. shock	ρ_ε	beta	0.850	0.10	0.904	0.017	0.902	0.849	0.945
Permanent techn. shock	ρ_{g_z}	beta	0.850	0.10	0.532	0.116	0.643	0.420	0.853
Risk premium shock: dom.	ρ_{RP}	beta	0.850	0.10	0.909	0.015	0.906	0.858	0.909
Wage markup shock	ρ_{φ^W}	beta	0.850	0.10	0.583	0.033	0.601	0.500	0.945
Inv.-spec. techn. shock	ρ_I	beta	0.850	0.10	0.594	0.053	0.563	0.375	0.744
Import demand shock	ρ_{IM}	beta	0.850	0.10	0.855	0.023	0.854	0.800	0.897
Export pref. shock	$\rho_{\nu^\star}$	beta	0.850	0.10	0.902	0.019	0.906	0.857	0.945
Risk premium shock: ext.	ρ_{RP^*}	beta	0.850	0.10	0.893	0.013	0.892	0.842	0.935
Standard deviations									
Transitory techn. shock	σ_ε	inv. gamma	0.857	2.00	1.024	0.079	1.094	0.801	1.482
Permanent techn. shock	σ_{g_z}	inv. gamma	0.245	2.00	0.205	0.026	0.197	0.139	0.257
Risk premium shock: dom.	σ_{RP}	inv. gamma	0.245	2.00	0.230	0.025	0.238	0.167	0.331
Wage markup shock	σ_{φ^W}	inv. gamma	0.184	2.00	0.111	0.009	0.109	0.086	0.136
Inv.-spec. techn. shock	σ_I	inv. gamma	0.245	2.00	0.459	0.037	0.481	0.377	0.597
Import demand shock	σ_{IM}	inv. gamma	0.245	2.00	5.327	0.421	5.441	4.480	6.626
Price markup shock: dom.	σ_{φ^H}	inv. gamma	0.184	2.00	0.176	0.014	0.179	0.154	0.209
Price markup shock: exp.	σ_{φ^X}	inv. gamma	0.367	2.00	1.396	0.123	1.421	1.186	1.699
Price markup shock: imp.	σ_{φ^*}	inv. gamma	0.367	2.00	2.288	0.193	2.329	1.946	2.789
Export pref. shock	σ_{ν^*}	inv. gamma	0.245	2.00	5.043	0.385	5.061	3.601	6.892
Interest rate shock	σ_R	inv. gamma	0.122	2.00	0.112	0.009	0.112	0.094	0.132
Risk premium shock: ext	σ_{RP^*}	inv. gamma	0.245	2.00	0.379	0.031	0.384	0.283	0.507
Measurement error	σ_ω	inv. gamma	0.245	2.00	1.105	0.090	1.115	0.977	1.275

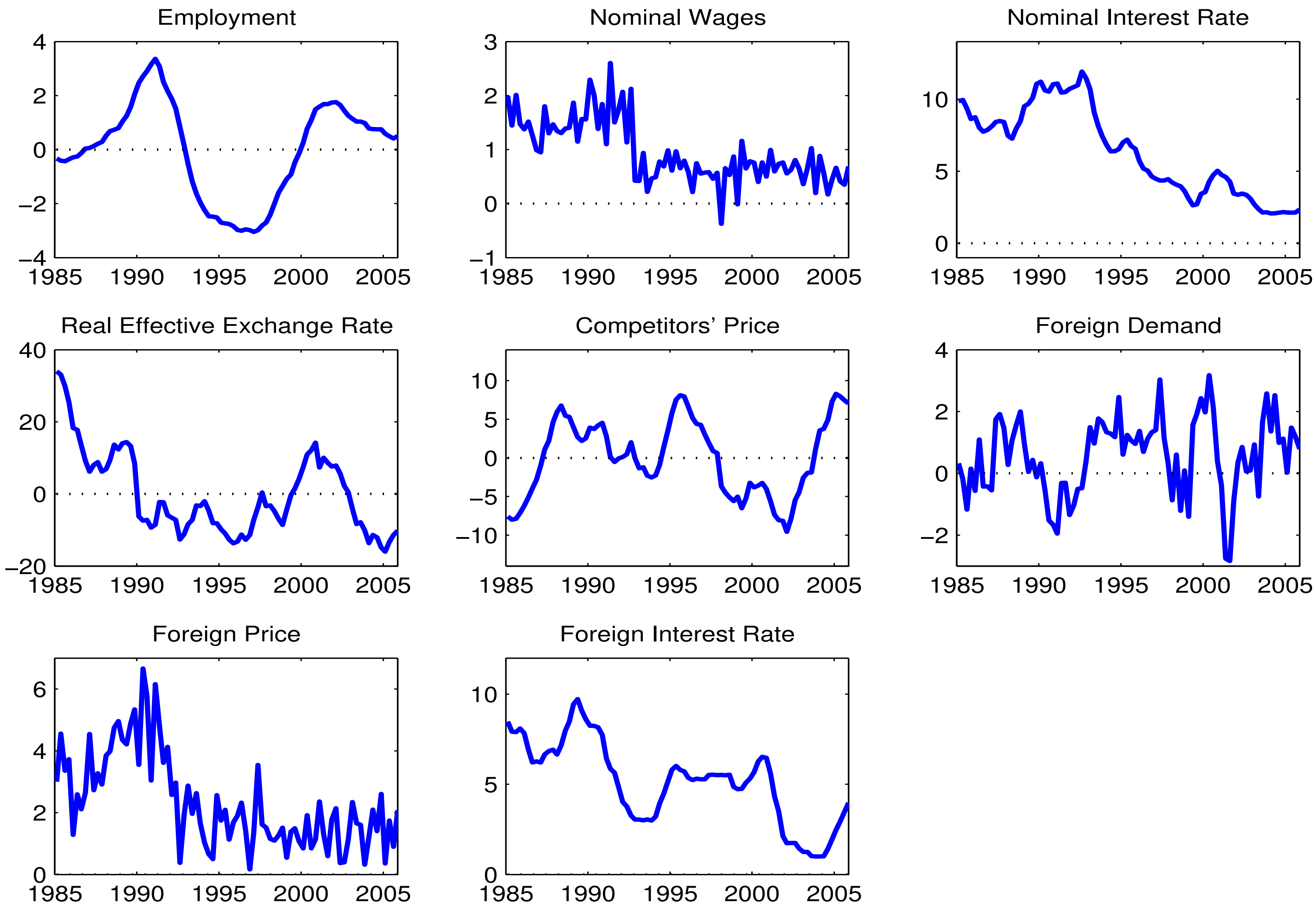
Note: This table provides information on the prior distributions as well as the benchmark posterior distributions for the NAWM.

FIGURE 1: The Data



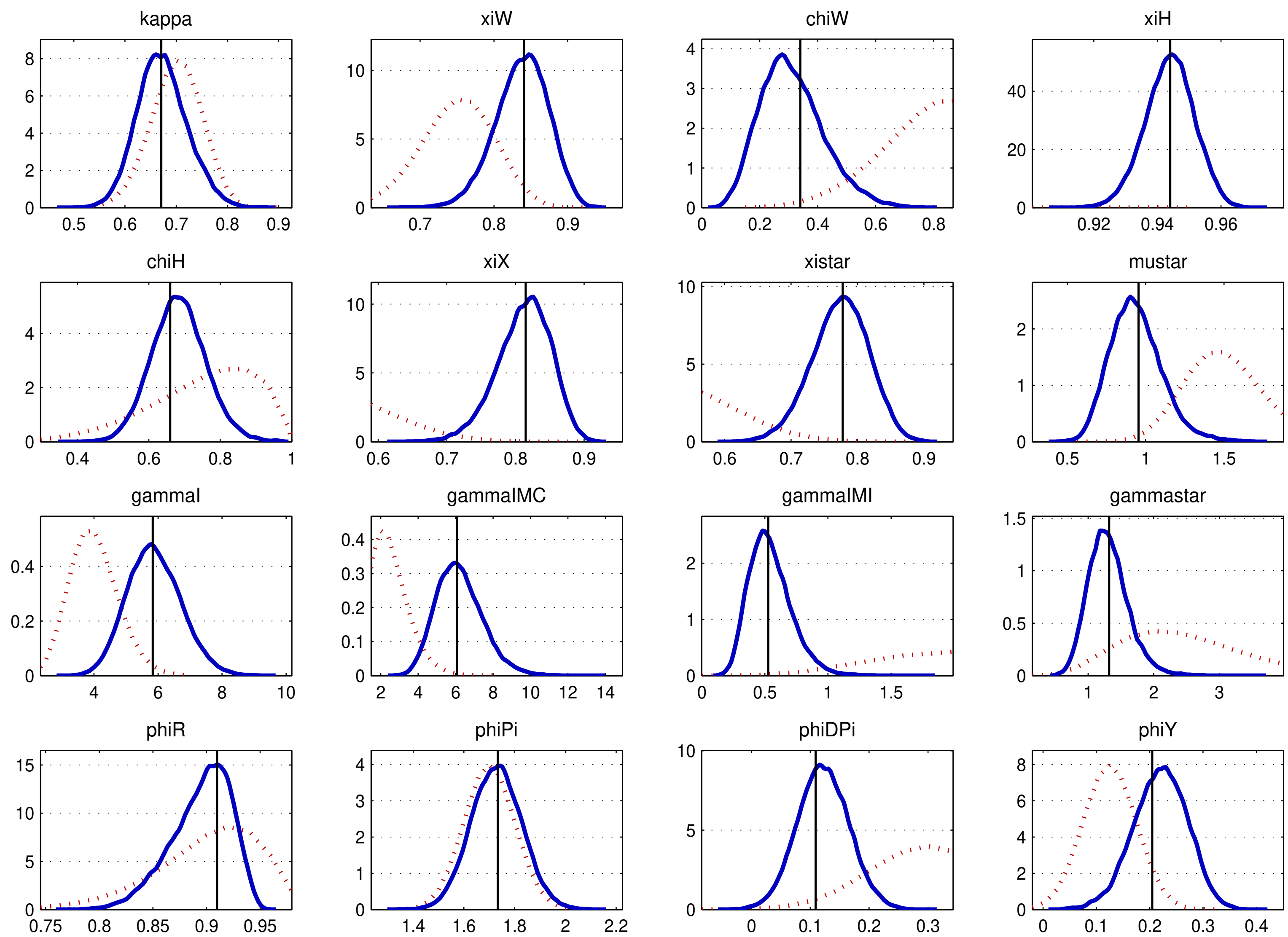
Note: This figure depicts the observed variables used in the estimation of the NAWM.

FIGURE 1: The Data (continued)



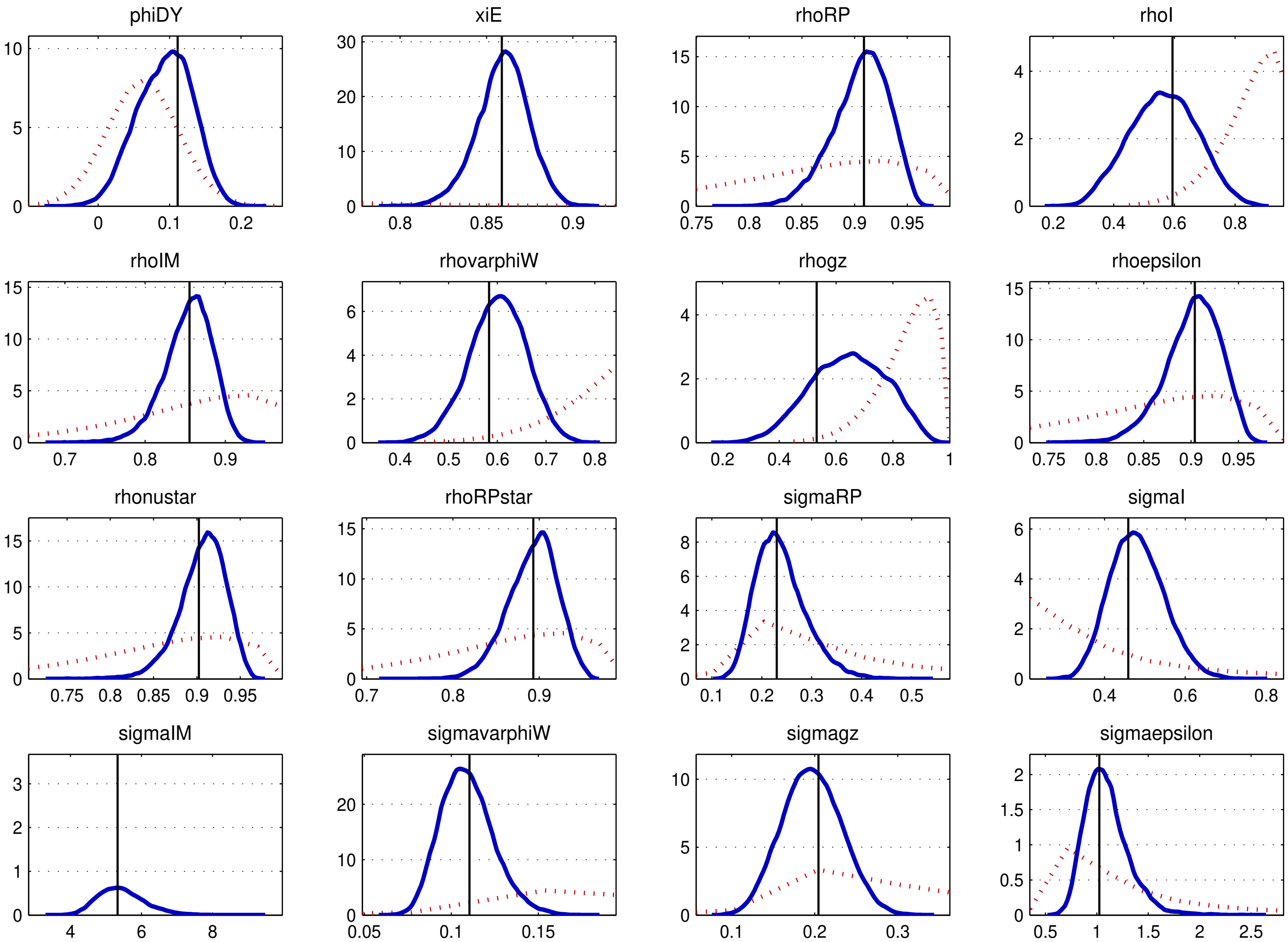
Note: See above.

Figure 2: Prior and Posterior Distributions of the Structural Parameters



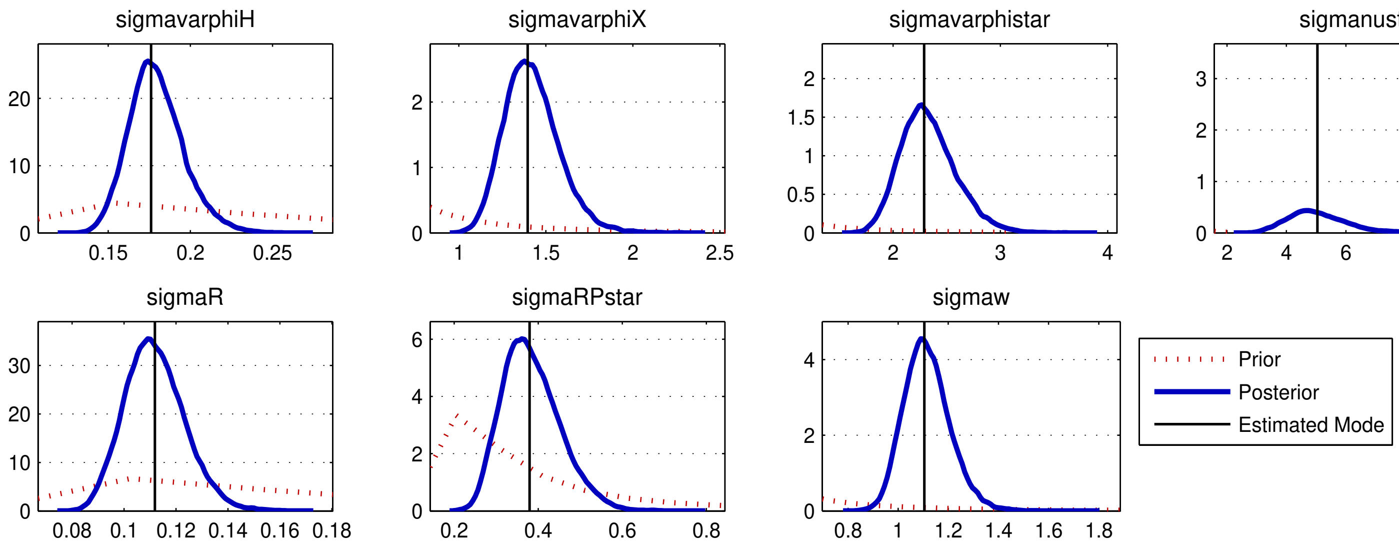
Note: For the baseline version of the NAWM, this figure depicts the posterior distributions of the model's structural parameters based on a Markov chain with 250,000 draws (blue bold lines) against their prior distributions (red dotted lines).

Figure 2: Prior and Posterior Distributions of the Structural Parameters (continued)



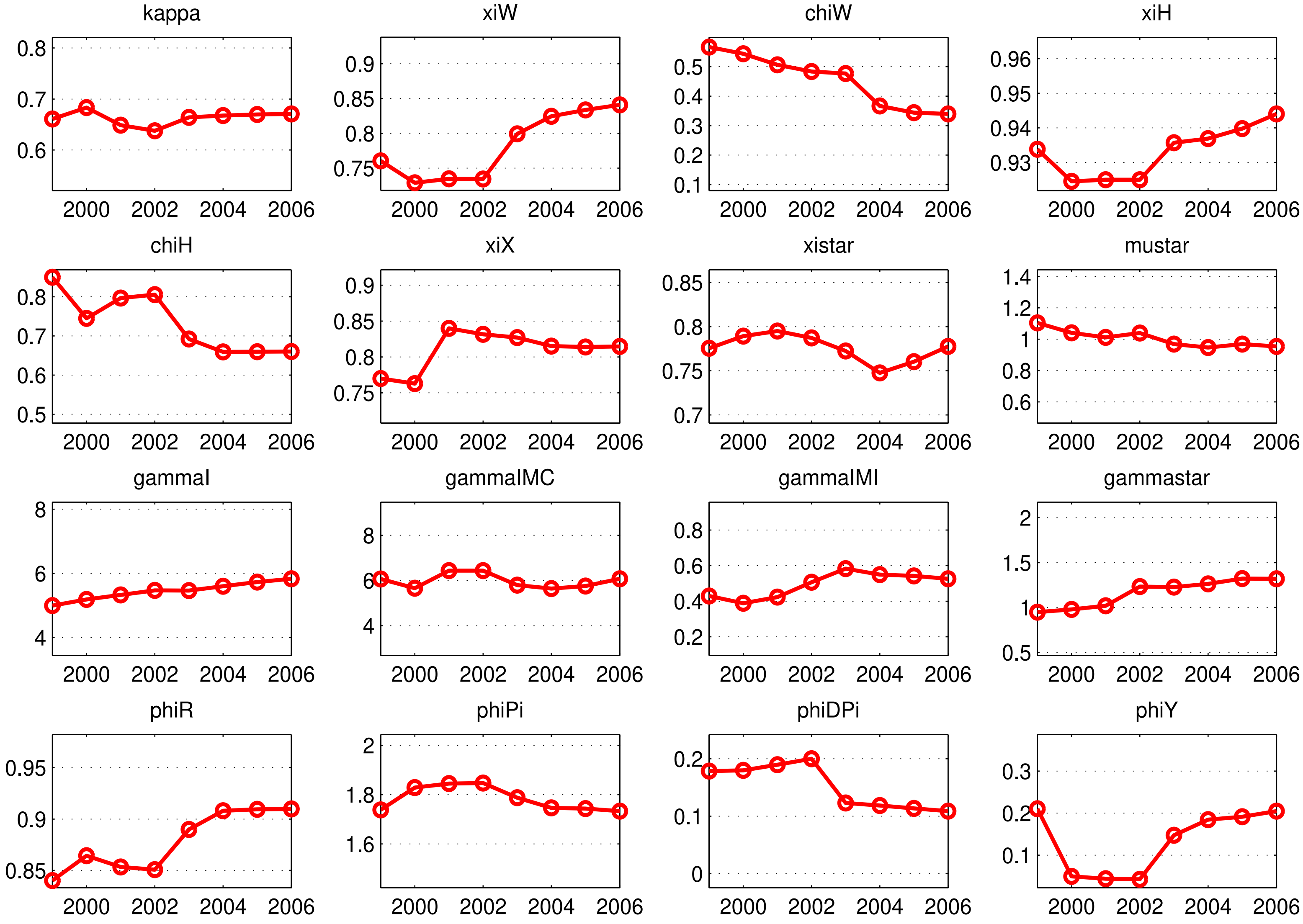
Note: See above.

Figure 2: Prior and Posterior Distributions of the Structural Parameters (continued)



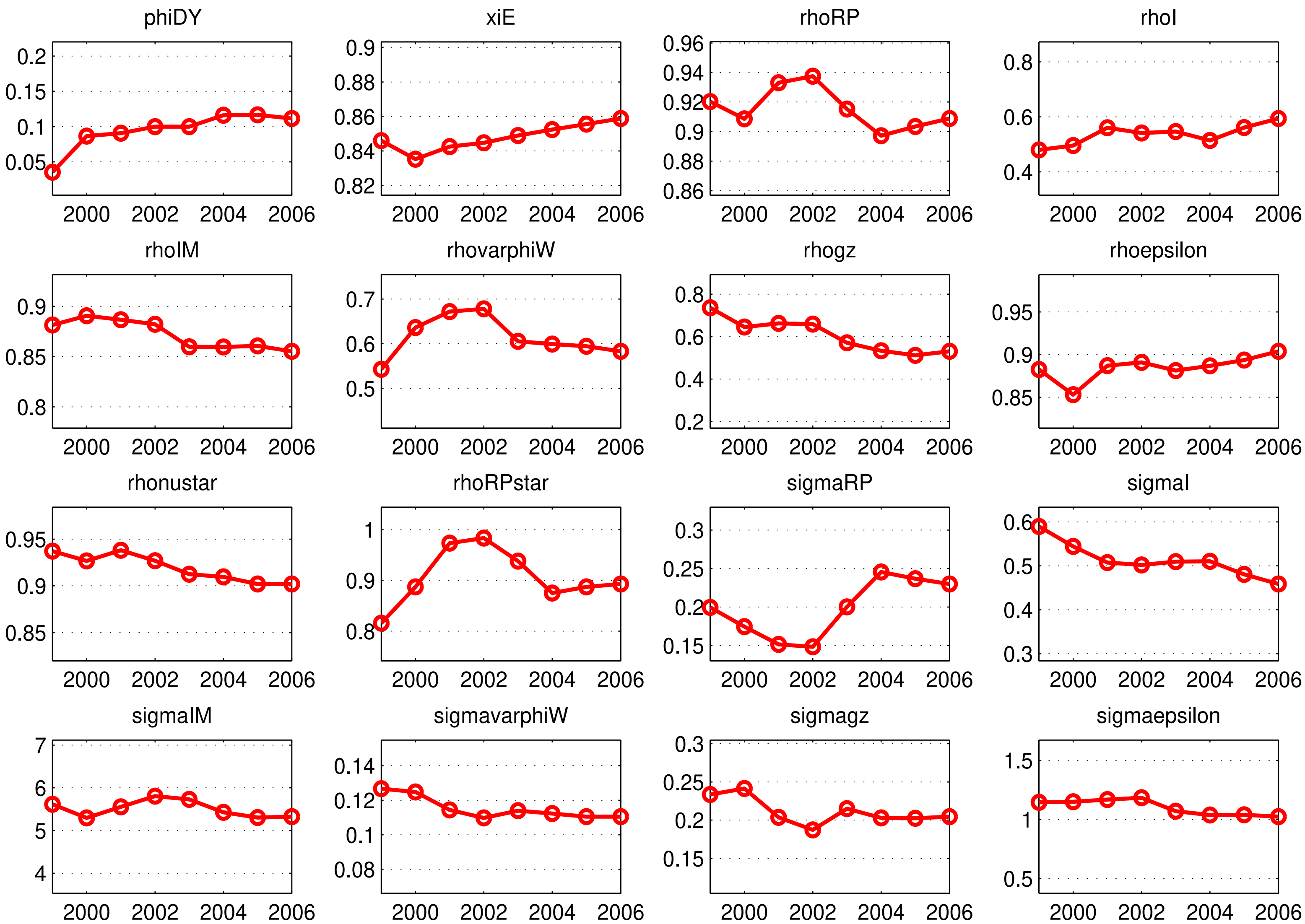
Note: See above.

Figure 3: Recursive Posterior Mode Estimates of the Structural Parameters



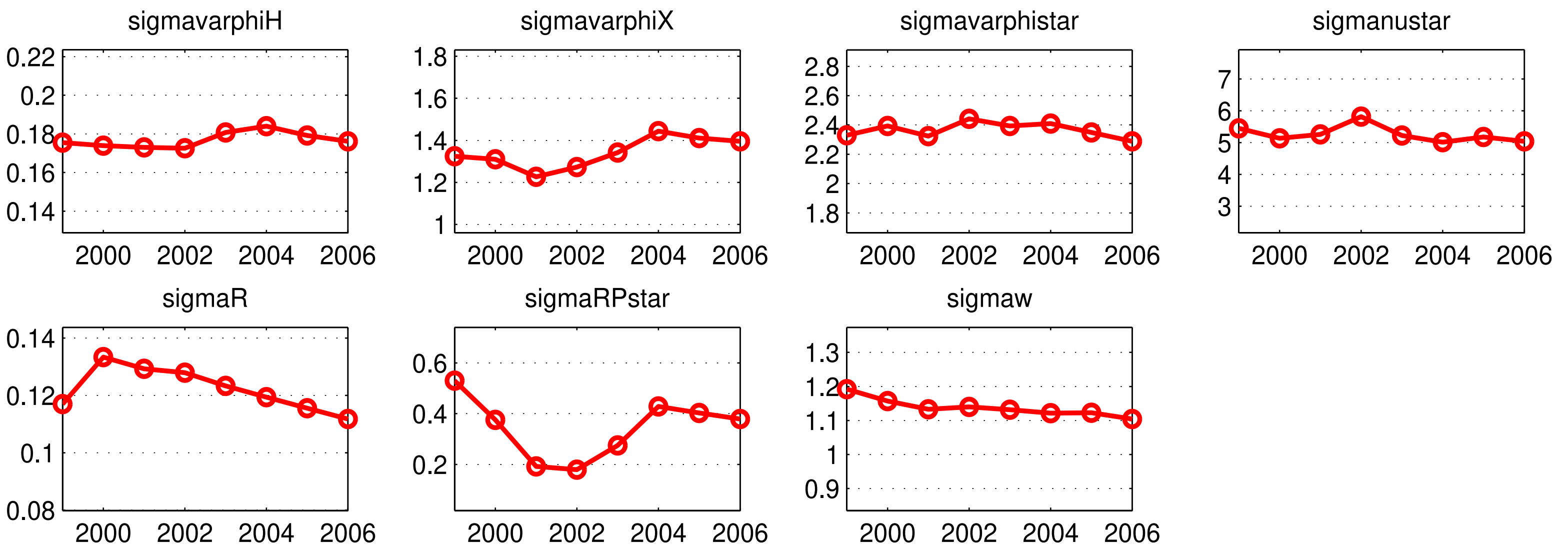
Note: For the baseline version of the NAWM, this figure depicts the posterior mode of the model's structural parameters estimated recursively with the estimation sample being gradually extended by a full year from 1998q4 to 2005q4.

FIGURE 3: Recursive Posterior Mode Estimates of the Structural Parameters (continued)



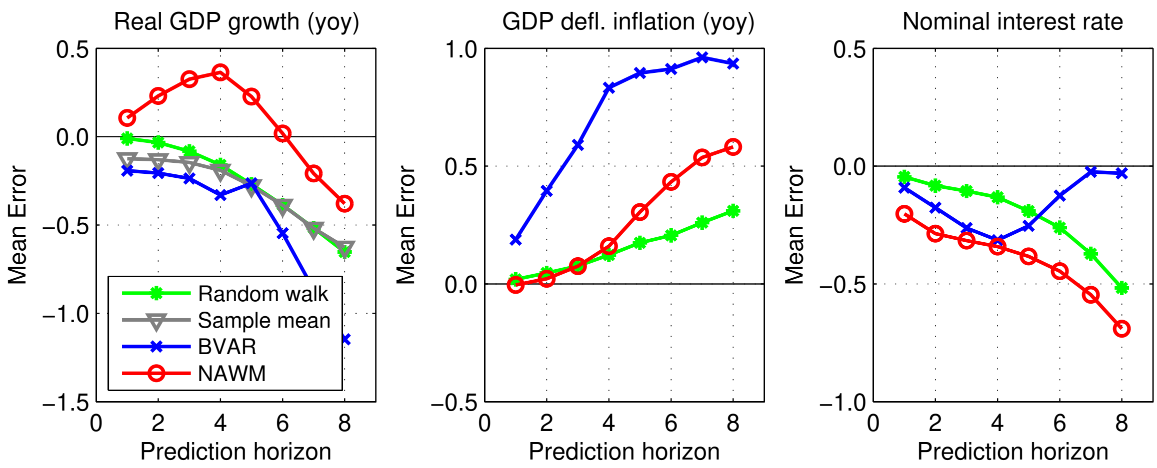
Note: See above.

Figure 3: Recursive Posterior Mode Estimates of the Structural Parameters (continued)



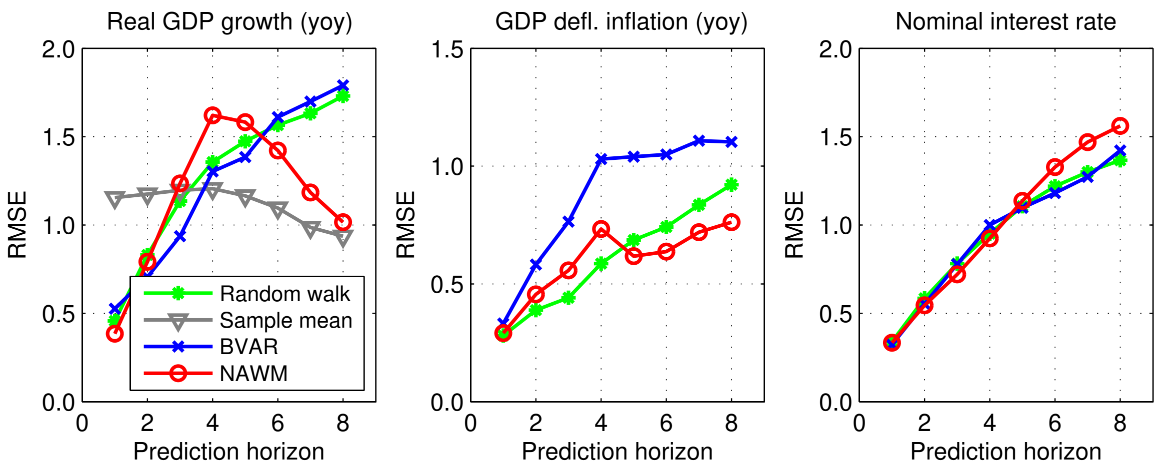
Note: See above.

Figure 4: Mean Errors of Unconditional Forecasts for the NAWM, a BVAR, and Two Naïve Forecasts



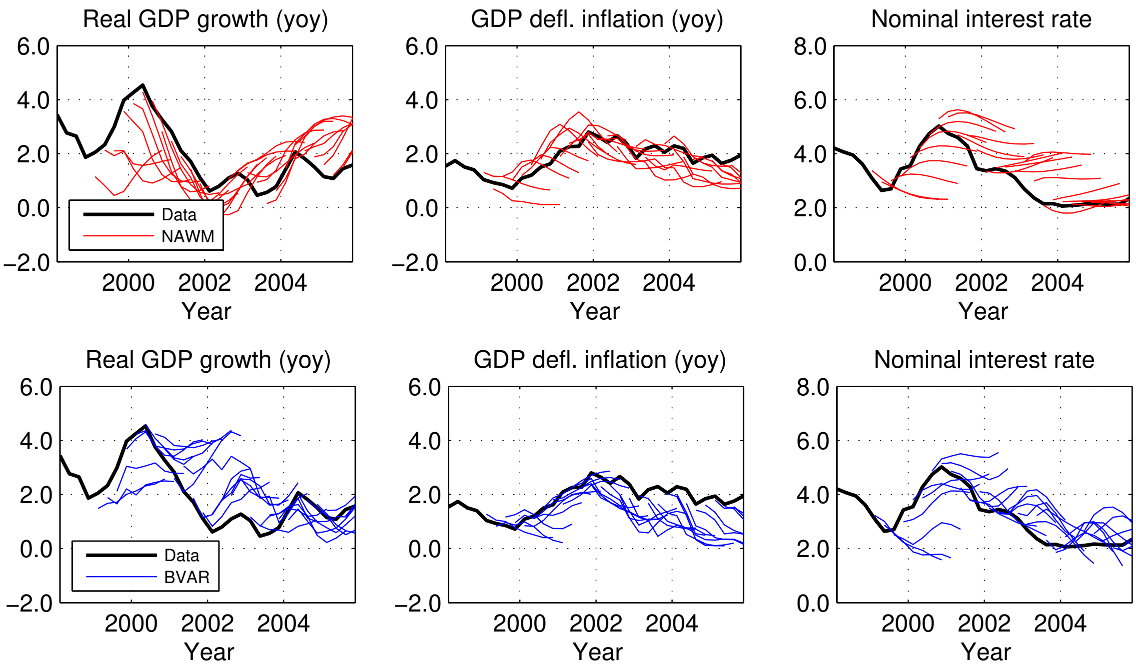
Note: For the NAWM, a BVAR with steady-state prior and two naïve forecasts (the random walk and the sample mean), this figure shows the mean errors (in percent) of unconditional 1-8 period-ahead forecasts for year-on-year real GDP growth, year-on-year GDP deflator inflation and the annual nominal interest rate. The forecasts have been computed recursively out of sample over the period 1999Q1-2005Q4, and the point forecasts for computing the mean errors are given by the means of the predictive densities.

Figure 5: Root Mean-Squared Errors of Unconditional Forecasts for the NAWM, a BVAR, and Two Naïve Forecasts



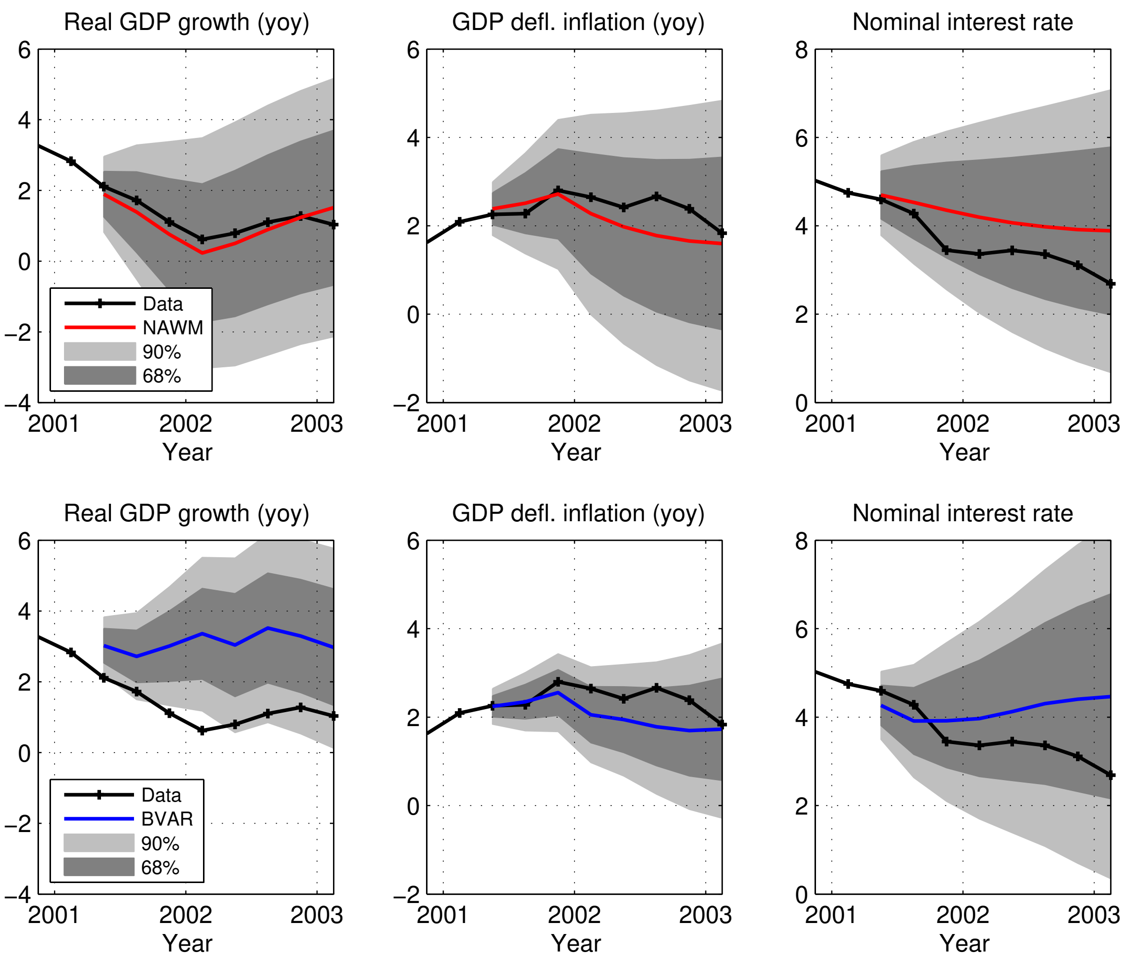
Note: For the NAWM, a BVAR with steady-state prior and two naïve forecasts (the random walk and the sample mean), this figure shows the root mean-squared errors (RMSEs, in percent) of unconditional 1-8 period-ahead forecasts for year-on-year real GDP growth, year-on-year GDP deflator inflation and the annual nominal interest rate. The forecasts have been computed recursively out of sample over the period 1999Q1-2005Q4, and the point forecasts for computing the RMSEs are given by the means of the predictive densities.

Figure 6: Mean Prediction Paths from the NAWM and a BVAR for Selected Variables



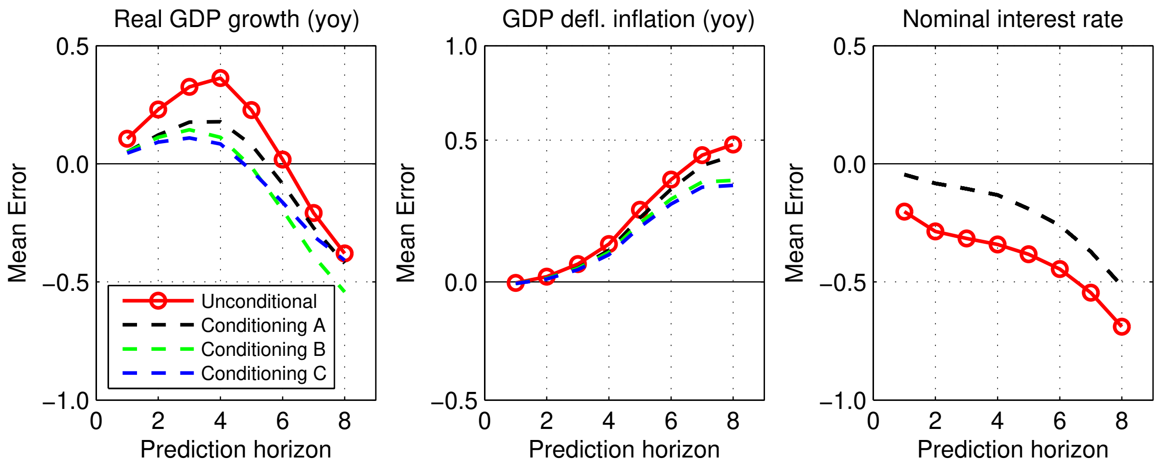
Note: For the NAWM and a BVAR with steady-state prior, this figure shows the mean prediction paths of unconditional 1-8 period-ahead forecasts for year-on-year real GDP growth, year-on-year GDP deflator inflation and the annual nominal interest rate. The forecasts have been computed recursively out of sample over the period 1999Q1-2005Q4.

FIGURE 7: Mean Predictions and Centred Prediction Intervals for the NAWM and a BVAR for the Prediction Sample 2001Q2-2003Q1



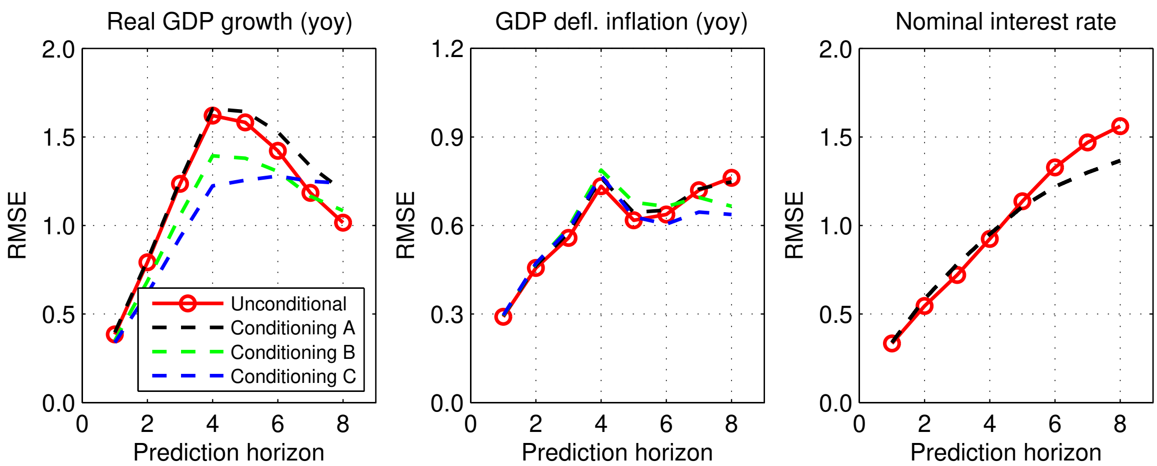
Note: For the NAWM and a BVAR with steady-state prior, this figure shows the unconditional mean predictions and the equal-tail 68 and 90 percent prediction intervals for year-on-year real GDP growth, year-on-year GDP deflator inflation and the annual nominal interest rate in the period 2001Q1 characterised by a slow-down in economic activity.

Figure 8: Mean Errors of Unconditional and Conditional Forecasts for the NAWM



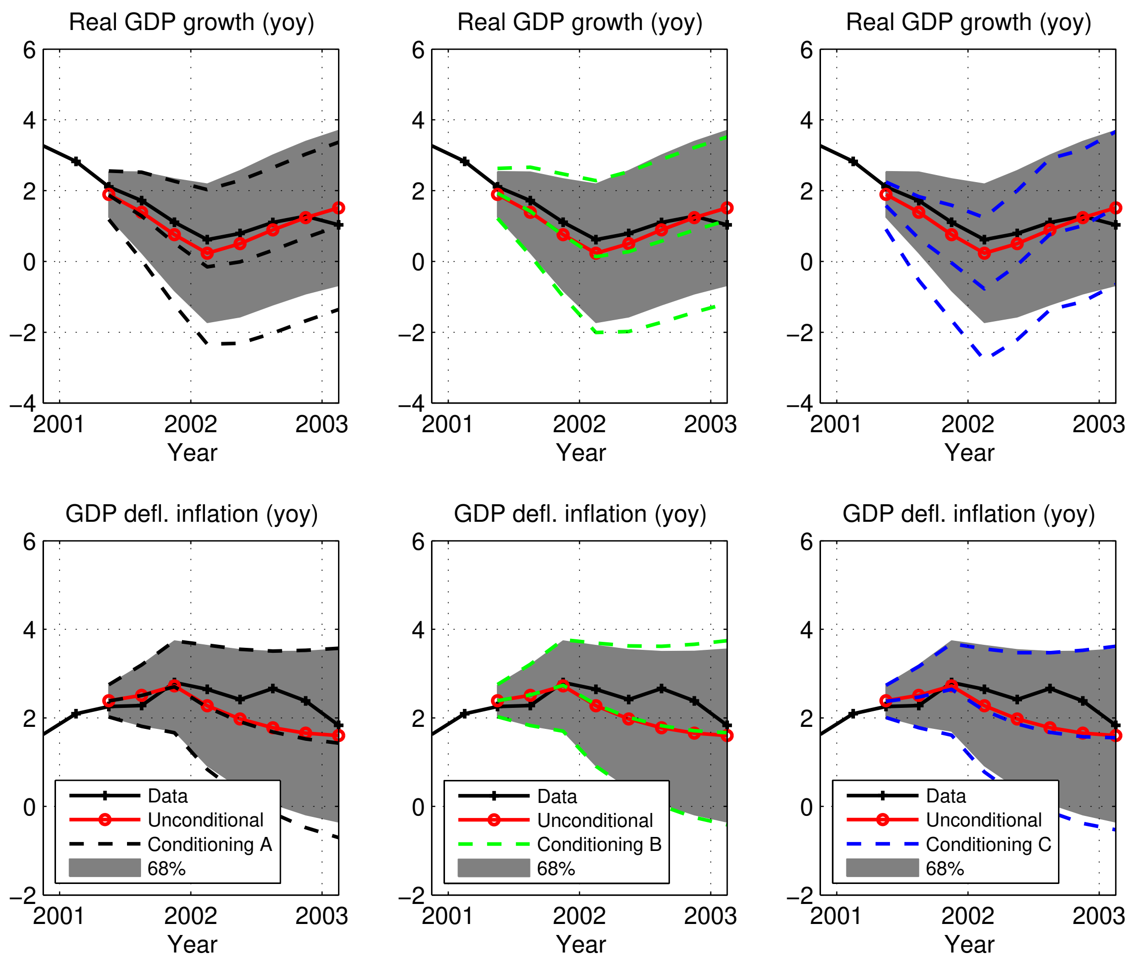
Note: For the NAWM, this figure compares the mean errors (in percent) of unconditional 1-8 period-ahead forecasts for year-on-year real GDP growth, year-on-year GDP deflator inflation and the annual nominal interest rate with those obtained under the alternative conditioning assumptions A to C. The forecasts have been computed recursively out of sample over the period 1999Q1-2005Q4, and the point forecasts for computing the mean errors are given by the means of the predictive densities.

Figure 9: Root Mean-Squared Errors of Unconditional and Conditional Forecasts for the NAWM



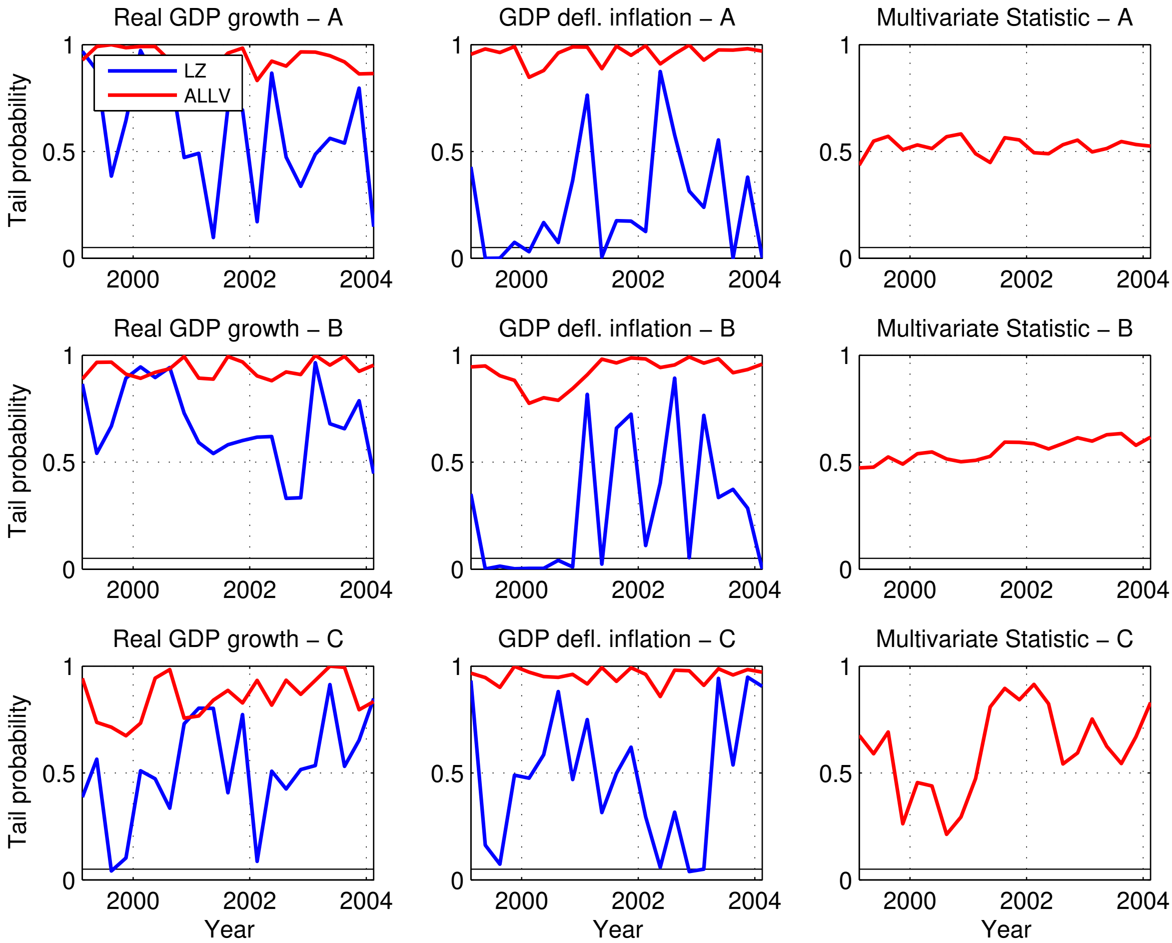
Note: For the NAWM, this figure compares the root mean-squared errors (RMSEs, in percent) of unconditional 1-8 period-ahead forecasts for year-on-year real GDP growth, year-on-year GDP deflator inflation and the annual nominal interest rate with those obtained under the alternative conditioning assumptions A to C. The forecasts have been computed recursively out of sample over the period 1999Q1-2005Q4, and the point forecasts for computing the RMSEs are given by the means of the predictive densities.

Figure 10: Unconditional and Conditional Mean Predictions and Centred Prediction Intervals for the NAWM for the Prediction Sample 2001Q2-2003Q1



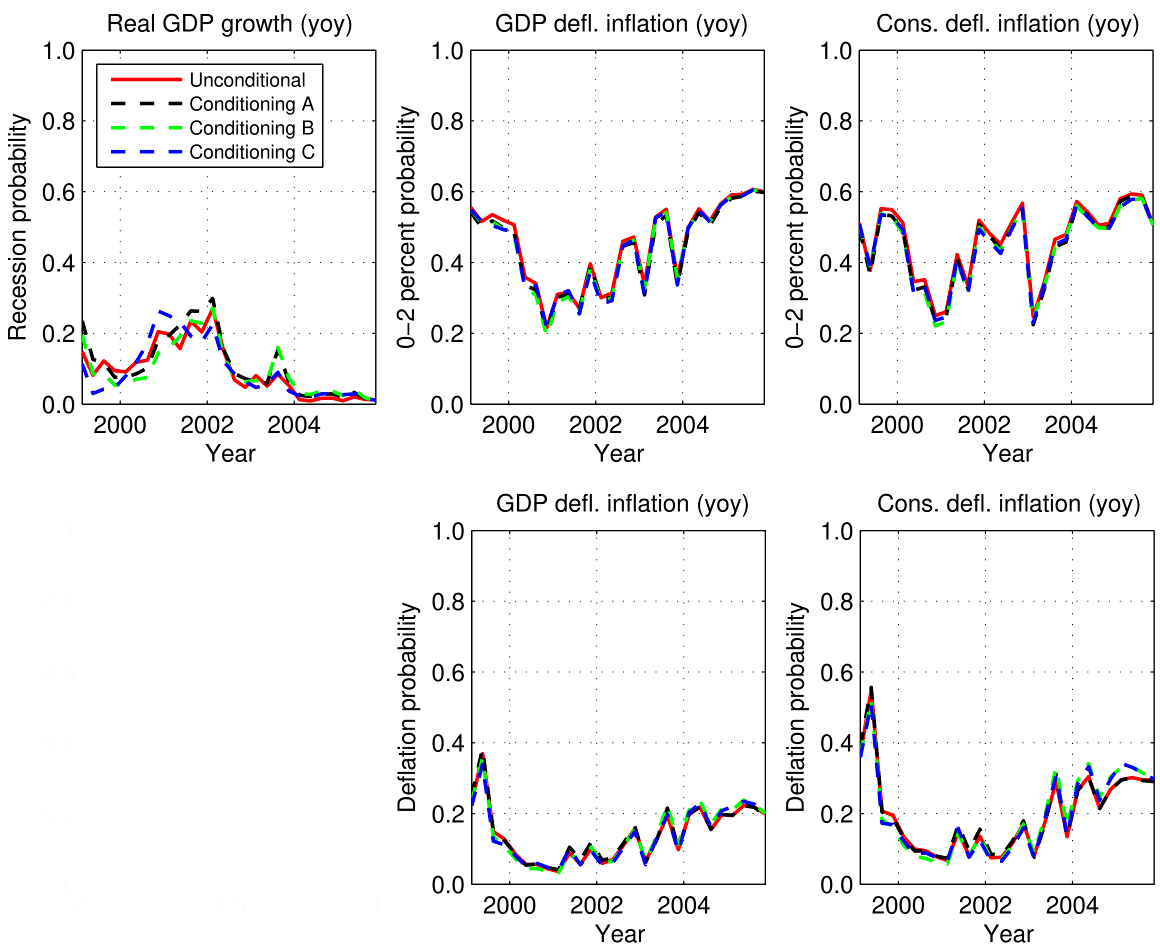
Note: For the NAWM, this figure compares the unconditional mean predictions and the equal-tail 68 percent prediction intervals for year-on-year real GDP growth, year-on-year GDP deflator inflation and the annual nominal interest rate in the period 2001Q1 characterised by a slow-down in economic activity with those obtained under the alternative conditioning assumptions A to C.

FIGURE 11: Tail Probabilities from Univariate and Multivariate Modesty Statistics of the Conditioning Information Sets for the NAWM



Note: For the NAWM and the three different conditioning information sets A to C, this figure shows the tail probabilities of three alternative modesty statistics for assessing the relevance of the Lucas critique: the univariate statistic proposed by Leeper and Zha (2003) (LZ) as well as the univariate and multivariate extensions proposed by Adolfson et al. (2005) (ALLV) taking into account the multivariate nature of the underlying shock uncertainty. The conditional forecasts have been computed recursively out of sample over the period 1999Q1-2005Q4, and the modesty statistics are evaluated at the posterior mode of the model parameters for a conditioning sample length of eight quarters.

Figure 12: Prediction Event Probabilities



Note: For the unconditional and conditional predictive distributions obtained for the NAWM over the horizon 1999Q1-2005Q4, this figures depicts the probabilities of certain prediction events for real GDP growth, GDP deflator inflation and consumer price inflation.