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Abstract

Investment is a function of the present value of expected future output. A change in the regime of foreign inflows can boost these expectations, so that investment propensities exceed savings. Even so, a pricing rule exists that ensures stability and maximizes expected profits. A macro dynamic system results in which there are two classes of equilibria, with high (low) capacity utilization associated with lower (higher) mark-ups. There are unique classes of adjustment paths approaching these equilibria, and medium-run growth cycles occur due to switches between these. Expectations can jump to either equilibrium, causing an endogenous amplification of shocks. In the case of a shock to foreign inflows supportive macroeconomic policies that tie the domestic to the world interest rate are required to achieve the highest feasible growth path.

Key words: Animal spirits; multiple equilibria; growth cycles; pricing rule; stability

JEL Classification: E2, E32, F36, O23

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1. Introduction

In a small open economy investment can differ from domestic savings because of foreign inflows. Underdevelopment can be understood as a low level equilibrium trap set by history and past choices. A useful question to ask is: can foreign inflows help a developing country escape from history? Foreign inflows are sustainable if they lead to a rise in investment, growth and exports. Investment is a function of expectations. Foreign inflows can help expectations of the future to differ rationally from the past. In this paper we make precise this process, and its potential drawbacks.

If there is excess capacity, output is demand determined; that is, net investment determines current output. But investment itself is forward looking, determined by the discounted present value of expected future output (this is Tobin's q). Current savings will fall if expected future output rises, and consumption is smoothed over the horizon. Or savings as a proportion of income will stay constant, if capital market imperfections prevent borrowing for consumption. We show that while investment is correctly a function of marginal q on adjustment paths, it can be written in terms of average q, or current output, but the coefficients would vary.

Therefore there can exist periods when the response of investment to current output exceeds that of savings. From the simple Keynesian multiplier this should lead to instability. But if firms follow a pricing rule such that the profit share varies counter-cyclically, stability is restored. As the variance of profits is lowered, the rule turns out to maximize expected profits for risk-averse firms. The rule makes a large response in induced demand and forward looking expectations consistent with aggregate stability, and this is its fundamental justification. Adjustments such as cutting prices in a recession will only worsen the recession as investment falls, similar to the Mundell-Tobin effect, and will not lead to better equilibria.

As the mark-up is also the profit share it naturally enters both the savings and investment decisions. The macro dynamic system that results, switches between two classes of equilibria, with high (low) capacity utilization associated with lower (higher) mark-ups. There are unique classes of adjustment paths approaching these equilibria, and medium-run growth cycles occur due to switches between these. Expectations are rational in the sense of converging to a value that is unique with respect to a set of exogenous conditions. The stock of capital varies over the time taken to reach equilibrium; therefore the medium-run is the relevant time frame of analysis. There is no unique steady state, and excess capacity can persist even in the long-run.

The small counter-cyclical variations in the medium-run mark-up imply that in response to shocks, expectations of demand can jump to either set of equilibria, causing a stable endogenous amplification of the shocks. Quantity adjustments can dominate price adjustments. The medium-run growth cycles contribute towards understanding a pattern of growth where fuller utilization and expansion of capacity occurs together. They give excess capacity a productive role to play. There is waste in the system, but deep recessions are prevented and periods of accelerated growth are feasible. Policy can ensure a longer stay on the higher growth paths.

Fundamental changes in the nature of capital flows in the nineties make them important as potential shocks that can stimulate a shift to "good" or high growth equilibria. But our model demonstrates that this is so only under the correct policies. If, for example, structural reforms lead to higher expected future output, investment will rise and foreign inflows will finance it. But if policy makers use the inflows to build up reserves and raise interest rates, there will be expectations of depreciation, investment will fall and the economy will shift to a path approaching lower growth equilibrium. With mobile short-term capital flows, dangers of cumulative deflation are increased (Goyal 1997, 1998).

If agents are able to foresee a unique market clearing equilibrium they should take actions to cause the economy to reach it. But here, even though expectations are forward looking moving to the better equilibrium requires both favourable shocks and correct policy responses.

The instability of multiplier accelerator models is well-known. There have been many attempts to moderate the knife-edge property of the Harrod-Domar growth model. Flaschel (1994) demonstrates such instability in the wage-price dynamics of an IS-LM model and mentions that a general analysis of the conditions for stability has not yet been developed.

Historical lock-in and malfunctioning institutions have been blamed for macro fluctuations and underdevelopment traps. Hysteresis and path dependence have become familiar terms in the macro literature. Krugman (1991) had a model in which the economy could be trapped in a low level equilibrium given by history, or escape from it. The escape would require coordination of expectations. Instead of history or expectations, as in Krugman's model, we have history and expectations working together. History has developed the rule that allows a freer role to expectations. The latter can amplify supply shocks and lead to a switch in growth paths. Supply shocks can trigger a change in expectations and therefore in the parameters of the investment function. The mark-up rule ensures stability in the presence of these parameter changes and therefore maximizes expected profits over the firm's horizon. Rational expectations converge to unique saddle-stable adjustment paths. But the paths switch between non market-clearing multiple equilibria. Therefore the economy is not supply driven and demand continues to play a major role.

Over the last decade, a number of models, although derived from strict micro foundations, had multiple equilibria that provided explanations for Keynes type non-market clearing, and for aggregate demand effects. Diamond, 1982 was a seminal paper. There has been some analysis of the dynamic structure of such models (Krugman, 1991). But models in this tradition are limited to very specific theoretical micro structures. Our model, even though it is theoretical and stylized, is capable of explaining macro aggregative data. It was tested and calibrated using time-series for the Indian economy (Goyal1994a, b).

The structure of the paper is as follows: in the next section we present the model, derive the investment function, and describe model-dynamics. Section III systematically explores the dynamic flow and derives the mark-up rule to complete the dynamic system. Section IV examines how foreign inflows can instigate parameter changes that amplify shocks. Section V concludes, and is followed by Appendix A, which calibrates the model for India while Appendix B collects the proofs.

2. The Model

All wages and a fixed proportion of profit income are consumed. Firms maximize expected profits or the value of the firm, over an infinite horizon. The economy is small and open so that the real rate of interest is linked to the world rate. World inflation is zero and the nominal exchange rate is fixed. Therefore a change in the domestic price level changes the real exchange rate. Because of perfect global capital mobility and arbitrage, a shock to domestic interest rates must be associated with an expected devaluation. Since there are no other credit instruments and the focus is on the medium-run, investment must be financed by internal and sustainable external savings. The domestic real interest rate will rise, both, if foreign inflows fall short of the difference between ex-ante investment and domestic savings, or if they exceed the gap but stimulate sterilization and tight money policies. In equilibrium foreign inflows are determined by ex-ante investment demand, but are subject to stochastic shocks. Quantities are normalized by capital and u = output/capital also serves as an index of capacity utilization[1]. The output-capital ratio, u, is constant in any equilibrium, because output and capital then grow at the same rate. The rate differs across equilibria, and u itself varies on adjustment paths. Although the economy starts from a position of unemployed labour and excess capacity, it is perpetuated by the dynamics of the system.

The aggregate demand equals aggregate income payments identity is reduced to the saving investment equality, or goods market equilibrium condition:

$$f^i + s_p\,\pi = i \tag{1}$$

Where s_p the propensity to save out of profit income π, investment is i, and f_i is foreign inflows net of interest payments. This equals net imports or imports m minus exports, x, giving the external balance (2):

$$f^i = m - x \tag{2}$$

All small letters refer to real variables deflated by the domestic price level, P. The nominal exchange rate multiplied by the foreign price level is normalized to unity so that 1/P is the real exchange rate. The real domestic interest rate is r and w the real wage rate. A superscript dash on any variable denotes its time derivative. The flow equations refer to a single period, but the subscript t is dropped as a notational simplification.

Aggregate output is distributed between wage and profit income. Substituting for the output capital ratio $u = y/k$, and $r_k = \pi/k$ in (3) leads to equation (4), that τ is the profit share[2] and equation (5) that the rental rate on capital, r_k is given by τu.

$$Y \equiv WL + P\,\pi \tag{3}$$

$$\tau = 1 - \frac{W}{PF_L} = \frac{\pi}{y} \tag{4}$$

[1] We are concerned with a period beyond the very short-run, so that capital cannot be regarded as fixed. As growth occurs, the economy is growing in scale so that some normalization is required. Capital stock is conventionally chosen, but it implies the judgment that given other variables, (7) is more stable as a ratio to K and, investment will double with a doubling of scale measured by K (see Marglin and Bhaduri, 1988).

[2] The mark-up is most frequently defined as a charge on wage costs, $P = wb(1 + m)$. The mark up in the text is linked to m, by $\tau = m/(1 + m)$, so that both would increase or decrease together.

$$r_K = \tau\, u \tag{5}$$

Given the fixed propensity to save out of profits, savings normalized by capital, are:

$$\frac{s}{k} = s_p\, \tau\, u + e_s + f^i = s\,(\tau, u; e_s, f^i) \tag{6}$$

The generic form of the shocks e_i (for investment) and e_s (for savings) is autoregressive with a decaying influence of the lagged autoregressive term, and a random component. While f^i is endogenous, the random component of foreign inflows is included in e_s. Inflows in excess of i-s would be absorbed in foreign exchange reserves.

An investment function (7) with a similar functional form is derived in the section below, from intertemporal optimization by a representative firm, in the presence of excess capacity.

$$\frac{i}{k} = i\,(q, u; e_i) = i_1\;\; \tau u + i_2 u + e_i \tag{7}$$

The Investment Function

In steady-state equilibrium investment can be written as a function of average q or τ and u because the latter two are constant. But along adjustment paths both are varying. Optimal investment can continue to be written as a function of τ and u, but the coefficients of the variables would change.

In making its investment decision the firm maximizes V(0), the value of the firm, subject to quadratic adjustment costs of investment. V(0) is the current discounted value of future cash flow. In our simple model it is the discounted share of profit, with investment costs as well as wage costs now subtracted from output. The rate of discount r equals the world interest rate plus any expected devaluation. The time path of $\{r_t, W_t\}$ from zero to infinity, and technology, are known to the firm. Per unit of investment h(.) is required to transform goods into capital. Since there is excess capacity and investment will increase capacity, the firm also chooses capacity utilization, and the latter affects both output and depreciation. Demand and capital stock are endogenized. The per capita production function f(..) has the usual properties, except that it has capacity utilization as an additional argument. k is the per capita capital stock and dk/dt is therefore gross investment, i. Depreciation, D, is a homogenous function of capacity utilization, CU, and capital stock, K. Per capita capacity utilization $cu_1 = CU\backslash L$ and $cu = CU\backslash K$. The objective of the firm is to maximize:

$$V(0) = \int_0^\infty \left(f(K_t, cu_{1t}) - i_t\left[1 + h\left(\frac{i_t}{K_t}\right)\right] - w_t\right) \rho_t dt \tag{8}$$

subject to:

$$K_t' = i_t - D(cu_t, 1) \tag{9}$$

Where:

$$\rho_t = e^{\left(-\int_0^\infty r_v\, dv\right)} \tag{10}$$

If the exchange rate is expected to depreciate, domestic interest rates would rise above world rates.

Let *x* = *i/k*. We setup the present value Hamiltonian, H, using *u* as a proxy for capacity utilization:

$$H = [f(k_t, u_t) - i_t \left[1 + h\left(\frac{i_t}{k_t}\right)\right] - w_t + q_t(i_t - D(u_t, 1)]\rho_t\, dt \tag{11}$$

the first order conditions are:

$$H_i: 1 + h(x_t) + x_t\, h_x\,(x_t) = q_t \tag{12}$$

$$\frac{dq_t\ \rho_t}{dt} = -H_k: q'_t = \left(r_t + D(u_t, 1)\right)'$$

$$q_t - [f_k\,(K_t, u_t) + x_t^2\, h_x\,(x_t)] \tag{13}$$

$$lim_{t\to\infty}\, q_t\, K_t\, \rho_t = 0 \tag{14}$$

equation (12) states that the marginal cost of investing must equal the shadow value of installed capital. If there were no adjustment costs then the market value of the firm would rise by *q* for one additional unit of investment. But more than a unit of investment is required to compensate for adjustment costs. The equation can be inverted to give investment as a function of *q*, as in Tobin's theory[3]. Equation (13) can be solved subject to the transversality condition (14) to give

$$q_t = \int_0^\infty ([x_s^2\, h_x(x_s) + f_k(K_s, u_s)])\rho_t\, ds \tag{15}$$

The shadow price *q* is just the present value of the marginal product of capital. This is marginal *q*. If the firm is a price taker in its output market and the production and adjustment cost function are linearly homogenous, marginal *q* equals q_a (average *q*). Consider an

[3] Hayashi (1982), showed that the modified Neoclassical theory of investment, and Tobin's q theory are identical. The former obtains the investment decision from the maximization of present discounted value of net cash flows subject to costs of installing-new investment goods, by a firm. The latter has investment as a function of the ratio of the market value of new additional investment goods to their replacement cost.

equilibrium with τ, u and r constant, and the firm a price taker. Then the following relationship holds:

$$q_a = \frac{V(0)}{PK} = \frac{\tau Y}{PK} = \frac{\tau\, u}{r} \tag{16}$$

If investment can be written as a function of q_a it can be written as a function of τu. But q is the firm's decision variable, and it is only in equilibrium, with τ, u and r constant[4], that q_a equals q. In that case:

$$q = q_a = \int_0^\infty E(\tau_t\, u_t) exp^{-rt}\, dt \tag{17}$$

so that,

$$q = \frac{\tau u}{r} \tag{18}$$

or the present value of profits net of adjustment costs, normalized by capital stock. If investment is written as a function of τ and u, therefore, the coefficients of these variables will change as forward looking expectations jump to a different equilibrium. This is worked out in Section IV. Second, capacity utilization is also a decision variable. Therefore q alone is insufficient to allow the firm to make its gross investment decisions. By rewriting equation (10), noting that net investment is a function of q, and the depreciation function is linearly homogenous, we get,

$$i_t = \Psi\,(\mathrm{q}) + \chi(\mathrm{u_t}) \;\; \chi > 0, \chi' > 0 \tag{19}$$

where χ is equivalent to D(*cu*, 1), from the linear homogeneity of the depreciation function and u is used as a proxy for *cu*. This formulation depends on the assumption that the path of prices and wages is given. Doing the maximization for capacity utilization would give a reduced form where it was a function of the price variables. However, given q, investment rises with utilization (Motahar 1992).

This detailed maximization exercise shows that investment functions such as (7) are not *ad hoc*. They are compatible with full intertemporal optimization, and forward looking expectations; but have time varying parameters. The investment function would shift in a way that makes precise the concept of ‘animal spirits’.

[4] For a small open economy with a credibly fixed exchange rate r would be exogenous at the unique equilibrium determined by equality of r to the world real interest rate. At other equilibria it can differ by the expected depreciation of the exchange rate, itself determined by expected future output. These variations in r are therefore reflected in the coefficients of the investment function.

The Dynamic System

Excess demand in the goods market leads to a rise in the output-capital ratio, u. Therefore, using equation (6) and (7) for savings and investment, adjustment in the goods market occurs as follows:

$$u' = f\left(\frac{i}{k} - \frac{s}{k}\right) = f(\tau, u; e) \tag{20}$$

Where $e = e_i - e_s$. In equilibrium f(.) = 0. The partial derivatives can take the values:

$$f(0,0) \lesseqgtr 0, f_\tau \lesseqgtr 0, f_u > f_\tau, f_{\tau u} \lesseqgtr f_{uu} = 0$$

Where subscripts refer to partial derivatives. Either the upper row of inequalities holds or the lower holds. The system can switch from one set to the other. If $f_u > 0$, it implies Keynesian multiplier instability. If the coefficient of investment is a function of expected output, as in the Harrod-Domar model, there can be periods when investment propensities exceed savings and $f_u > 0$. But unlike in the Harrod-Domar model expectations are forward looking, they jump to differing equilibria. A large rise in induced net demand, in response to a policy, an external, or a technology shock, could cause such a switch in the signs of the partial derivatives. But for an induced rise in consumption or exports to impact on output beyond the short-run it must lead to a rise in investment in the first period, and capacity in the second.

The restrictions on the second order partial derivatives follow since τ and u enter equation (20) multiplicatively. This creates the non-linearity in the equation. The value of f_u exceeds that of f_τ because of the effect of excess capacity on investment. The response of investment and therefore output is greater to a rise in u than to a rise in τ.

We hypothesize that the firm follows a mark-up rule. This allows high induced expenditure yet imposes stability. Therefore there exists a non-linear combination of u and τ such that the firm would not want to change the mark-up. We call this function the no change in mark-up or $\tau' = 0$ function.

$$g(u, \tau; z) = 0$$

Or

$$\tau' = g(u, \tau; z) \tag{21}$$

Shocks to market structure, such as the degree of monopoly, are captured by z. The partial derivatives of equation (21) define the τ' function. They can be derived such that the dynamic system comprising equations (20) and (21) are stable. This is done in the next section, by a systematic examination of the dynamic flow. The function g(.) together with its partial derivatives constitutes a mark-up rule that is evolutionarily stable because it maximizes expected profits over the horizon.

The simultaneous adjustment of τ and u along medium-run dynamic adjustment trajectories, moves the system towards a full medium-run goods market equilibrium. Rest of the points of the dynamic system generate medium-run multiple equilibria where ex-ante goods market demand is equal to supply, and the ratios τ and u are constant. Debt is sustainable on these rest points, since investment and therefore capital inflows, adjust to future expected output. Along the adjustment trajectories ex-post equality of investment and savings is achieved by rationing or credit expansion. Appendix A presents the specific functional forms of equations (20) and (21) that were used in simulations for the Indian economy.

3. Deriving the Mark-Up Rule from the Dynamic Flow

First, we briefly characterize the dynamic flow. As the linearization on which the local dynamics depends is not affected by additive shocks, for the time being we assume that the shocks are not occurring.

$$x' = h(x) \tag{22}$$

$$x = \begin{bmatrix} u \\ \tau \end{bmatrix}$$

x is a vector. The dynamic flow $\Phi_t: U \rightarrow R^2$, $\Phi_t(x) = \Phi(x, t)$ is a smooth function defined for all x in U and t, and Φ satisfies:

$$U = [\underline{\tau}, \overline{\tau}], [\underline{u}, \overline{u}] \tag{23}$$

As τ is the profit share it must lie on the unit space. The output-capital ratio has also historically moved in a narrow band. Although history gives initial conditions in the interior, the dynamics of the system would tend to keep u and τ within the boundary values.

The system is non-linear. Therefore the local flow $\Phi_t: R^2 \rightarrow R^2$ is obtained by linearizing (22) at equilibrium x^* given by $x' = 0$, to obtain the Jacobian matrix of first partial derivatives. Under certain conditions[5], the dynamic flow can be analyzed by examining the Jacobian of the linearization.

[5] The Hartman-Grobman and Stable Manifold Theorem ensure that if the real parts of the eigenvalues or the trace of the Jacobian is not equal to zero, then the local behaviour of the linearized system is a valid approximation (Hirsch and Smale, 1974, Pg. 242).

The firm can foresee the dynamic flow but there is uncertainty with regard to the timing and size of possible bifurcations. After a shock the firm can foresee the new equilibrium. With reasonable parameter values adjustment to an equilibrium $u^*\tau^*$ can take years; long enough to make it necessary for the firm to take account of profits made on the adjustment trajectories.

Figures 1 and 2, graph the dynamic flow Φ for the case of $g_u < 0$ and $g_u > 0$ respectively. Dashed (solid) lines refer to the trajectories when f_u and $f_\tau > 0$ (< 0). The slopes of the isoclines $u' = 0$, and $\tau' = 0$ derive from the restrictions on the partial derivatives. The dashed $u' = 0$ curve refers to the unstable case[6].

If the mark-up rises with u (or $g_u > 0$) and $\tau' = 0$ as in standard competitive theories of firm pricing, as f_u changes from less to greater than zero, a bifurcation or sharp change in the dynamic flow occurs at $f_u = 0$. Aggregate demand becomes unstable.

Proposition 1: If $g_u > 0$ and $\tau' = 0$, as f_u changes from negative to positive, the equilibrium changes from a sink to a source. For values of f_u greater than or equal to zero, unstable trajectories exist.

An intuitive explanation of the propositions is given here; the proofs are in Appendix B. With a very high quantity response undamped by a change in mark-ups, the equilibrium would be unstable (a source). Trajectories would recede from it. There would be disorderly fluctuations in prices and quantities. Proposition 2 lists restrictions that damp price adjustment. Smooth adjustment paths then exist.

Proposition 2: If $g_\tau < 0$, and $|g_\tau|$ is large, dynamic instability would be avoided. The mark-up would show small movements.

[6] The $u' = 0$ curve would shift with changes in e. As $f_u > f_\tau$, the slope of $u' = 0$ when f_u and $f_\tau > 0$ is less when f_u and $f_\tau < 0$, and for the same absolute value of e the dashed $u' = 0$ would lie below the solid one. This can be seen, for example, by working out the slopes for the functional forms given in Appendix A. Moreover, as the deterministic part of investment rises when $f_u > 0$, e_i and therefore e will be low. Indeed, for the unstable IS curve to lie in the positive quadrant e has to be less than zero.

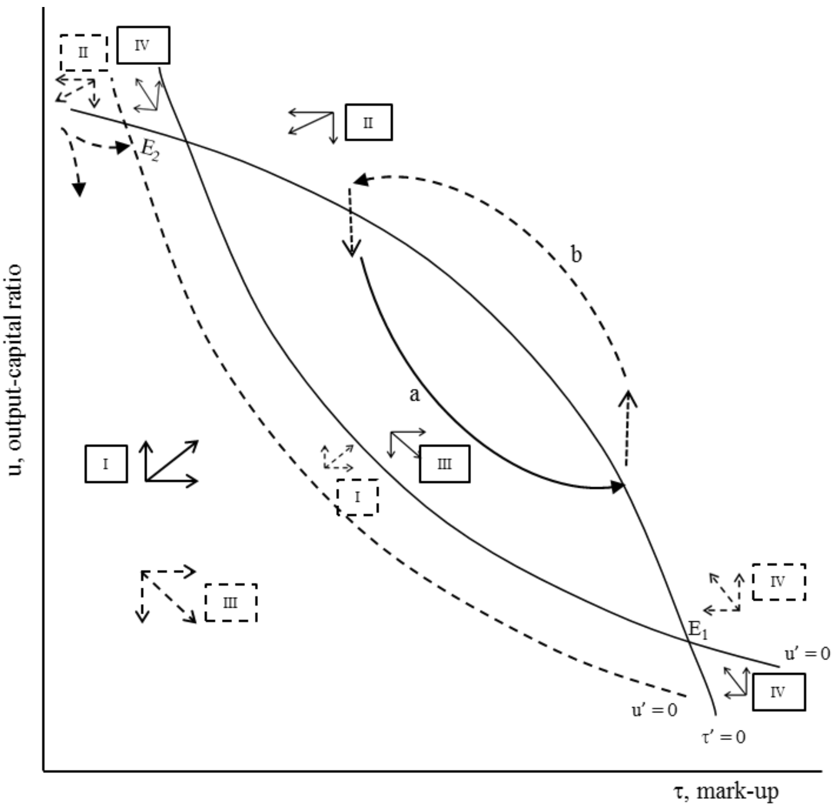

Figure 1: The dynamic flow when mark-up varies inversely with u

Note: Solid lines refer to the case of multiplier stability and dashed lines to the reverse.

So far we have the restriction on the g function that $g_\tau < 0$. The assumption of a gradual approach of the mark-up to its boundary values gives the restriction $g_\pi < 0$.

A stable bifurcation of the dynamic flow occurs at $f_u = 0$, as f_u changes from negative to positive, if $g_\tau < 0$. Proposition 3 asserts that $g_u < 0$ maximizes profits; the mark-up varies inversely with demand. If $f_u > 0$ and, at the same time $f_\tau < 0$ the $u' = 0$ curve would slope upwards. But this is possible only for a very narrow range of parameter values, and even in this range the slopes of the dominant trajectories remain the same. Therefore the proof of Proposition 3 continues to be valid[7].

[7] In the case of $f_u > 0$ and $f_\tau < 0$, the $u' = 0$ curve is upward sloping. The resulting equilibrium is a saddle point both with $g_u < 0$ (case 1) and $g_u > 0$ (case 2). It can be shown that the dominant trajectories remain the same as in Figures 1 and 2, in each of the two cases. In Case 1 $g_u < 0$; u' and $\tau' > 0$ or $u' > 0$ and $\tau' < 0$. In Case 2 $g_u > 0$; $\tau' > 0$ and $u' > 0$ or $\tau' < 0$. In the first case, however, the trajectories will recede from equilibrium approaching the $\tau' = 0$ isocline, and in the latter the stable arm will dominate.

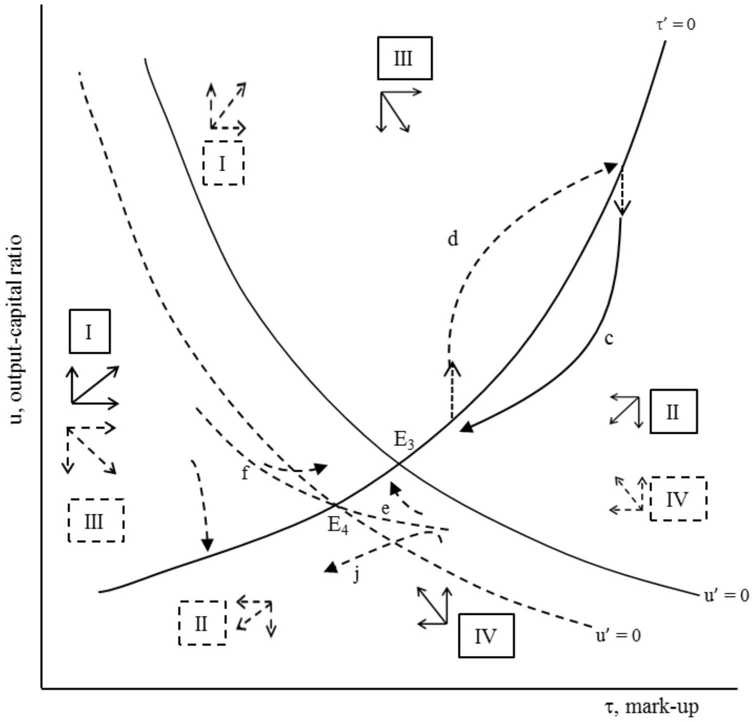

Figure 2: The dynamic flow when mark-up varies positively with u

Note: Interpretations as in Figure 1.

Proposition 3: A risk-averse firm will be maximizing profits over the set of adjustment trajectories, if $g_u < 0$, given the possibility of a bifurcation of the dynamic flow.

The proof uses three Lemmas:

Lemma 1: In Figure 1 as f_u and f_τ change from < to > 0, unique trajectories change from those approaching the saddle-point E_1 to those approaching the saddle-point E_2. In Figure 2 as f_u and f_τ, change from < to > 0, the trajectories change from those approaching the sink E_3 to those receding from the saddle-point E_4.

The Lemma is proved by examining the trace and determinant of the Jacobian. E_1 will be a saddle if prices are stickier downwards than upwards.

Lemma 2: (i) When $g_u < 0$, unique trajectory *a* approaching E_1 would dominate with f_u and $f_\tau < 0$, and unique trajectory *b* approaching E_2 with f_u and $f_\tau > 0$. Both are downward-sloping. (ii) When $g_u > 0$, trajectory *c* approaching E_3 would dominate with f_u and $f_\tau < 0$, and trajectory *d* receding from E_4 would dominate, with f_u and $f_\tau > 0$. Both are upward-sloping.

The structure of the dynamic flow is such that in Figure 1 downward sloping trajectories would dominate, and upward sloping in Figure 2. All others would lead to them. Only *b* and *a* are unique, as they are saddle paths, approaching saddle-point equilibria.

Lemma 3: Expected profits will be higher, for a risk-averse firm, on the dominant downward, as compared to the upward sloping trajectories.

The variability of profits is less along *ab* than on *cd*. A risk-averse firm prefers average profits. On *ab*, u and τ move in opposite directions, so that in periods of falling u profits fall less. As the variability of output is greater than that of τ, however, profits would be higher on *b* compared to *a*.

It is clear that a risk-averse firm will maximize expected profits when $g_u < 0$, as only in this case will the downward sloping trajectories dominate.

If the perfect foresight, or rational expectation with uncertainty, assumption is relaxed to allow for learning, it can be shown that expectations will converge only if $g_u < 0$.

Finally we prove that the dominant trajectories constitute a medium-run growth cycle.

Proposition 4: (4.1) *ba* and the trajectories joining them constitute a growth cycle in which rising (falling) capacity utilization is accompanied by falling (rising) mark-ups. (4.2) On *b* growth would be higher than on *a*.

The proof uses a line integral to show that *ba* and the trajectories joining them are a closed orbit. As profits and sales would be higher on *b* than on *a*, so would investment, induced savings and therefore growth. Excess capacity and excess labour in our model allows growth to be demand driven. On a cycle such as *ba*, variability in profits would be less than that in output. This could be used as an empirical test of the model.

Therefore the restrictions on the partial derivatives of the $\tau = 0$ function (27) and intuitive explanations for them are:

$$g_u < 0,\ g_\tau < 0,\ g_{\tau\tau} < 0 \text{ and large } |g_\tau|$$

a) $g_\tau < 0$ and large $|g_\tau|$ prevent chaotic fluctuations given the possibility of instability in quantity adjustments, and allow smooth adjustment paths and stable equilibria to exist. A large absolute value of g_τ means that the mark-up will have only small variations.
b) $g_u < 0$ avoids recessionary trajectories of falling τ and u. In this case the firm maximizes expected profits over unique classes of adjustment paths.
c) $g_{\tau\tau} < 0$ allows non-linear adjustment to upper and lower bounds on τ.
d) Higher i and k in periods of rising u would imply greater variability of y or a higher ε_τ, so that $g_u < 0$ is consistent with the firm's short-run maximization by which τ should vary inversely with the elasticity of demand (see Goyal 1994).

A model is credible if it maps reality. In Goyal (1994a, b), empirical simulations based on the dynamic flow Φ were able to reproduce the Indian aggregate price and output series for the period 1960/61 to 1984/85. The calibrated parameter set obtained is given in Appendix A, and the simulated trajectories in state space can be seen in Figure 3.

4. Foreign Inflows and Parameter Changes

A rise in investment propensities causes a bifurcation in the dynamic flow, or a switch from f_u and $f_\tau < 0$ to f_u and $f_\tau > 0$. This occurs if the present value of expected future output rises, in response to an exogenous shock, as the economy shifts to a growth path approaching a better equilibria. If the response of investment to u now exceeds that of savings[8] the exogenous shock is amplified. Such a bifurcation, with a switch from trajectory a to b, makes precise the idea of a rise in animal spirits. One such exogenous shock could be a change in the regime of foreign inflows, or in their cost.

Recall the investment function (7). It is written in terms of average q, a rise in the present discounted value of future marginal products will raise marginal q and be reflected in the coefficients of average q. These coefficients will converge to a constant value on anyone growth or adjustment path. A major cause of the difference between average and marginal q

[8] For our arguments to go through it is enough that any change in savings propensities is less than that in investment propensities. Constant savings propensities simplify the argument, and are not inappropriate, especially as we consider savings out of profit income.

or of a change in the coefficients of equation (7) is a bifurcation in response to an exogenous shock. Although in equilibrium:

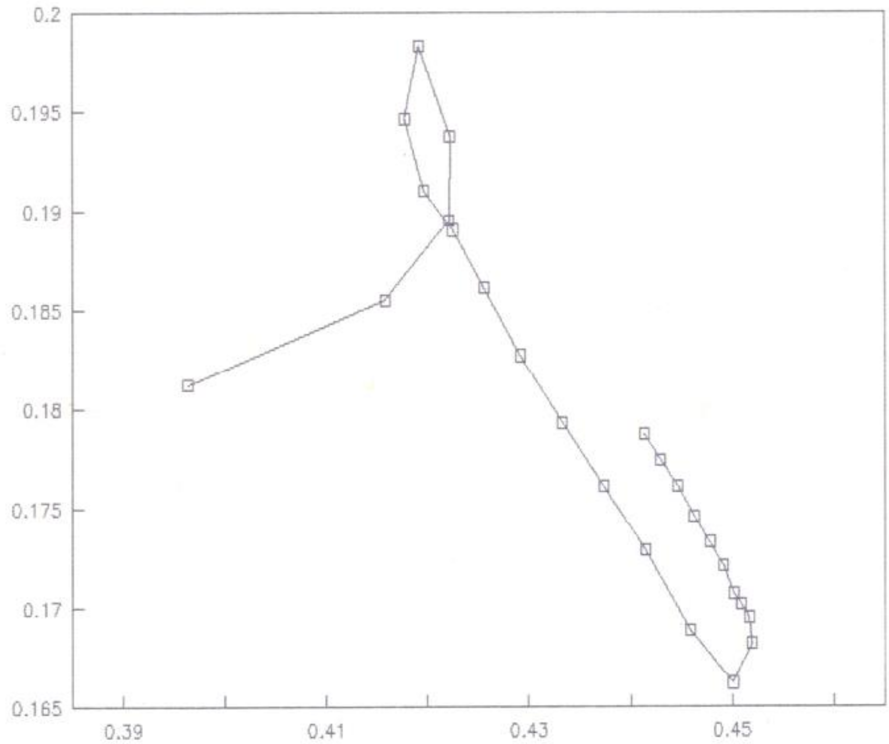

Figure 3: Simulated trajectories

$$Eu_t = u_t = Eu_{t+1}$$

On an adjustment trajectory:

$$Eu_t \neq Eu_{t+1}$$

In that case, if (24) were written in terms of current variables, it would be:

$$\frac{i_t}{K_t} = e_{it} + i_1\, E\tau_{t+1}\, u_{t+1} + i_2\, Eu_{t+1}$$

$$= e_{it} + i_1\,(1 + z_{1t})\tau_t\, u_t + i_2(1 + z_{2t})u_t \qquad (24)$$

Where z_{1t} and z_{2t} are the expected and actual rates of growth of τu and u respectively in period t.

Therefore, a rise in expected rates of growth will lead to a rise in investment propensities and intensify the bifurcation process. Any permanent shock can alter expectations and multiplicative parameters of aggregate demand, thus amplifying growth.

The conditions for stability of equilibria E_1 and E_2 defined earlier will be sufficient to ensure that $\sum_{t=10}^{\infty}(1+z_{it}), \quad i=1,2$ defines a contraction mapping, so that the coefficients would converge in an equilibrium where $\tau_t\, u_t = \tau_{t+1}\, u_{t+1}$. This is obvious along trajectory *a* where $Z_{it} < 0 \ \forall t$, so that $|1 + z_{it}| < 1$ and along trajectory *b* would imply that Z_{it} is first increasing and then decreasing. Expectations jump to a well-defined equilibrium, and determine the change in the coefficients of the investment function written in terms of average *q*. The dynamic system generates switching equilibria and cycles driven by expectations of future output; the Harrod-Domar mechanism is made stable.

In India a major cause of such a switch has been changes in the regime of foreign inflows. In the mid-sixties there was a fall in growth as aid and public investment was cut. Growth rates improved when foreign remittances rose sharply in the seventies and public investment was raised to bring down accumulating foreign exchange reserves (Goyal 1993). In the nineties improvement in structural efficiency after reform was a positive shock and as private inflows boomed with liberalization, private investment also rose. But unfortunately the reforms were prejudiced against any rise in public investment, and severe stabilization measures raised domestic interest rates and expected depreciation of the rupee, so that growth faltered. A rise in domestic interest rates lowers the present discounted value of future output and the coefficient of *u* in the investment function.

Traditionally foreign exchange resources have been regarded as an exogenous constraint in development models. But the liberalizing regime change together with technological advances have made global capital more mobile. In any regime where foreign reserves are accumulating, it means that foreign resources become endogenous, and are available to fill any gap between investment and saving. A positive shock to exogenous foreign inflows may stimulate investment, but domestic savings would also rise with output. But encouraging FDI and preventing an accumulation of mobile short-term debt-capital, requires monetary policies that keep domestic interest rates aligned to foreign interest rates. These interest rates and the inflows that occur at such rates, in response to investment demand, define the sustainable high growth equilibrium for a small open economy (see Goyal1997, 1998).

5. Conclusion

In a simple aggregative macro model, when the representative firm maximizes profits over disequilibrium adjustment paths, it is shown that:

a) An evolutionarily optimal pricing rule gives rise to non-market clearing multiple equilibria, with excess capacity and positive mark-ups. Therefore quantity adjustments dominate price adjustments.
b) Unique classes of adjustment paths exist, and cause difference stationary growth cycles.
c) The latter arise from jumps in expectations which cause an endogenous amplification of exogenous shocks.
d) A change in the regime of foreign inflows facing a country is a major source of such shocks. But for it to lead to a switch to a higher growth path requites complementary government policies such as a rise in public investment and keeping domestic interest rates aligned to world interest rates, to stimulate private investment and prevent the over-accumulation of short-term debt capital.

The causality runs as follows: investment is determined by intertemporal optimization given the pricing rule and long-term output expectations; investment then determines expected demand, output and capital stock in anyone period. The pricing rule turns out to maximize expected profits. Surplus labour and excess capacity make higher growth paths possible in response to shocks, but excess capacity is sustained by the dynamics. Since growth occurs in the form of fluctuations, even though it is not driven purely by supply shocks, output is difference rather than trend stationary, and shows a unit root. A parsimonious explanation of the structure of macroeconomic time series is provided without making *ad hoc* assumptions about serial correlations in error terms.

Appendix A

The dynamic system was given the specific functional form:

$$u' = e + \Big(\big(i_1 + g_1\, s_p(1-f) - s_p\big)\tau + j\Big)u \qquad (A.1)$$

$$\tau' = w_1 u - w_2 \tau^2 + w_3 \qquad (A.2)$$

They were selected to be consistent with the restrictions on the partial derivatives obtained from the theoretical arguments in the text. If $w_1 < 0\ (>0)$ then $g_u < 0\ (>0)$. w_3 and e capture supply and demand shocks respectively. The dummy variable f, stands for the effect of exogenous changes on public sector propensity to invest out of private savings $g_1 s_p$,i_1 and s_p, the propensity to invest and save respectively, out of profit income, and j captures the additional response of investment to a rise in capacity utilization.

The model was calibrated using data for the Indian economy over the period 1960/61 to 1984/85. This yielded the following values of the parameters.

From 1960/61 to 1974/75:
$i_1 = 0.258$, $j = 0.002$, $s_p = 0.429$, $g_1 = 0.5$, $w_2 = 0.9$
with $f = 0$ for 1960/61 to 1964/65 and $f = 0.5$ thereafter.

From 1975/76 to 1984/85:
$i_1 = 0.32$, $j = 0.002$, $s_p = 0.479$, $g_1 = 0.7$, $w_2 = 0.9$, $f = 0.5$.

These, with $w = -1$ and $w_3 = 0.3524$, generate series that closely track the historical or actual u and τ, and other macroeconomic series of the structural model. The simulated trajectories switch from b for the period 1960/61 to 1964/65, to a for 1964/65 to 1974/75, and b again subsequently. They are shown in Figure 3. Simulated normalized profits τu for $g_u < 0$ are in general higher than for $g_u > 0$. Keeping $g_u < 0$ yields higher profits for the firm.

Appendix B: Proofs

Proposition 1: We have $f_u > f_\tau$ when induced expenditure exceeds induced leakage. Therefore, at a point of bifurcation where $f_u = 0$, the Jacobian of the dynamic system would be:

$$Dh = \begin{bmatrix} f_u & f_\tau \\ 0 & < 0 \\ g_u & g_\tau \\ > 0 & 0 \end{bmatrix}_{x=x^*}$$

Let tr stand for trace and | | denote a determinant. When $f_u < 0$, then $f_\tau < 0$, $tr\ Dh < 0$, and $|Dh| > 0$. The equilibrium is a sink. $Tr\ Dh = f_u$ and $|Dh| = -g_u f_\tau$.

As f_u changes from zero to a small positive number, f_τ is still < 0, because the additional term in u, denoting a higher response of investment to capacity utilization, enters f_u but not f_τ. This can be easily understood by looking at the specific functional forms in Appendix A. The equilibrium is then a source as $tr\ Dh > 0$ and $|Dh| > 0$.

As f_u, f_τ are small near a bifurcation point, with $|f_u| < |f_\tau|$, $tr^2 - 4\,|Dh| < 0$. The positive first term f_u^2 is exceeded by the negative second term. The equilibrium would be a periodic unstable focus. Under such conditions chaotic trajectories could exist, with large variations in u and τ.

Proposition 2: If $g_\tau < 0$ and $|g_\tau|$ is large so that $|g_\tau| > |f_u|$, the trace would be negative even if $f_u > 0$. Also $(tr^2 - 4\,|Dh|) > 0$, so that imaginary eigenvalues are ruled out, and the equilibrium would not be periodic.

Lemma 1: In Figure 1 as f_u and f_τ change from < to > 0, trajectories change from those approaching the sink or saddle E_1 to those approaching the saddle E_2. The signs of the Jacobian are as follows:

$$Dh\ (E_1) = \begin{vmatrix} < 0 & < 0 \\ < 0 & < 0 \end{vmatrix}, Dh\ (E_2) = \begin{vmatrix} > 0 & > 0 \\ < 0 & < 0 \end{vmatrix}$$

$Tr\ Dh\ (E_1) < 0$, $|Dh\ (E_1)| > (<)\ 0$ or E_1 is a sink (saddle), if $|f_u\ g_\tau| > (<)\ |g_u\ f_\tau|$.

$Tr\ Dh\ ((E_2) < 0$, $|Dh\ (E_2)| < 0$ or E_2 is a saddle-point, if $|f_u\ g_\tau| > |g_u\ f_\tau|$.

E_2 can be a source if $|g_\tau|$ is small enough so that $tr\ Dh\ (E_2) > 0$ and $|g_u f_\tau| > |g_\tau f_u|$, because in that case $|Dh\ (E_2)| > 0$. It is unlikely to be a sink for that would require $|g_\tau| > |f_u|$ and yet $|g_u f_\tau| > |g_\tau f_u|$. Both E_2 and E_1 will be saddle-points, if as f_u changes from > to < 0, $|g_u f_\tau| < |g_\tau f_u|$ changes to $|g_u f_\tau| > |g_\tau f_u|$. This can occur if $|g_\tau|$ is larger when $f_u > 0$ as compared to when $f_u < 0$.

That is, firms would increase mark ups by more than they would decrease them. This accords well with evidence that prices are stickier downwards than upwards.

In Figure 2 as f_u changes from < to > 0, trajectories change from those approaching the sink E_3 to those receding from the saddle E_4. The signs of the partial derivatives of the Jacobian are:

$$Dh\ (E_3) = \begin{vmatrix} < 0 & < 0 \\ > 0 & < 0 \end{vmatrix}, Dh\ (E_4) = \begin{vmatrix} > 0 & > 0 \\ > 0 & < 0 \end{vmatrix}$$

$Tr\, Dh\, (E_3) < 0$, $|Dh\, (E_3)| > 0$, or E_3 is a sink.
$Tr\, Dh\, (E_4) < 0$, $|Dh\, (E_4)| < 0$ or E_4 is a saddle.

Lemma 2: The isoclines $u' = 0$ and $\tau' = 0$, divide the state space, u, τ, into basic regions of the following four types:

$$\begin{array}{rl} I & u' > 0, \tau' > 0 \\ II & u' < 0, \tau' < 0 \\ III & u' < 0, \tau' > 0 \\ IV & u' > 0, \tau' < 0 \end{array}$$

Trajectories of Types I and II dominate in Figure 2, in the sense that all others lead to some other basic region. Similarly in Figure 1, those of Type III and IV dominate (the analysis follows Hirsch and Smale, pp. 267). The numerals are marked in the figures. Dashed lines refer to the flow with $f_u > 0$, solid lines to $f_u < 0$.

With (i) initial conditions given by history, so that we may take an interior rather than an extreme value in the state space, (ii) the pattern of trajectories, and (iii) the results on the local stability of the fixed points, we can obtain some information on the structure of the global dynamics.

(a) If $g_u < 0$ (Figure 1), trajectory *a* approaching E_1 would dominate when f_u and $f_\tau < 0$, and *b* approaching E_2 when f_u and $f_\tau > 0$.
(b) If $g_u > 0$ (Figure 2), trajectory *c* approaching E_3 would dominate when f_u and $f_\tau < 0$, and *d* receding from E_4, when f_u and $f_\tau > 0$.

We note that while *b*, and if E_1 is a saddle (see Lemma 1), *a* are unique saddle-paths, the other paths are defined with respect to a class of trajectories with the same slope. Only on *b* and on *a* is the path unique.

Since $u' = 0$ with f_u and $f_\tau > 0$ will lie below $u' = 0$ with f_u and $f_\tau < 0$, when f_u changes from < to > 0, the economy will not be able to reach the stable arms *e* and *f* of the saddle-equilibrium E_4 after a switch (see footnotes 6 and 7). The dynamic flow is such that, *b* in Figure 1 will always be reached.

Lemma 3: We take E_1 and E_2 and the trajectories joining them as approximately lying on a rectangular hyperbola, so that $\tau u = K$, where K is nearly constant. Let the values of τu at the extreme points of a potential cycle *dc* in Figure 2, corresponding to *kh* in Figure 4, be such that: $h(\tau u) < K < k(\tau u)$.

Then, for a risk-averse firm with a normalized concave expected profit function $E\pi(.)$, it follows:

$$E\,\pi\,(ph + (1-p)k) < E\,\pi\,(K)$$

Where p is a measure of the probability of state h. This is shown graphically in Figure 4. The above inequality would be satisfied for the range of p values, such that the expected profits from the combination of h and k is less than or equals that from q in Figure 4. A risk-averse firm would certainly impute a probability exceeding zero and approaching 1/2 to the worst state. In addition, the firm would wish to avoid the possibility of being trapped on a trajectory such as *j* in Figure 2. Therefore it would never adopt $g_u > 0$ as a behavioural rule and thus avoid the potential cycle *dc* and trajectory *j*. The risk averse firm will maximize expected profits when $g_u < 0$.

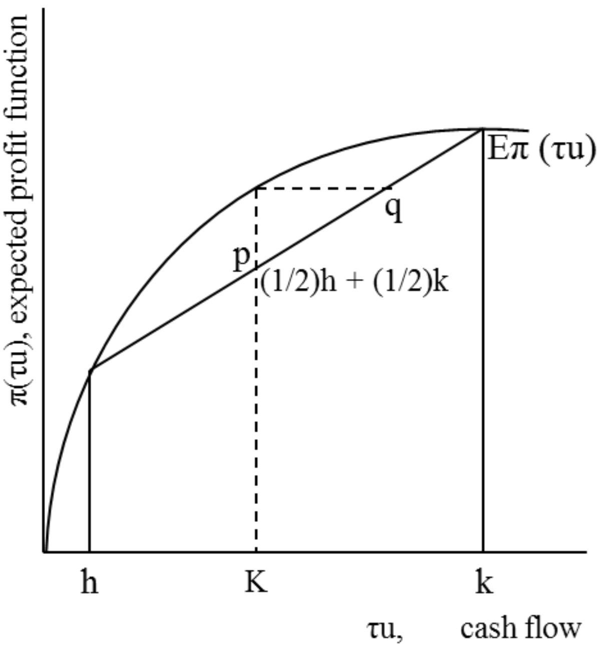

Figure 4: A concave expected profit function implies that expected profits are higher when cash flow is average

Proposition 4

4.1: The equation of a trajectory is given by:

$$\frac{du}{d\tau} = \frac{f(.)}{g(.)} \qquad (B.1)$$

Along any trajectory $g(.)du - f(.)d\tau = 0$. In Figure 1, with $g_u < 0$:

(i) $\qquad If\ |f_u| > |g_\tau|$

Bendixson's Criterion, a corollary of the Poincare-Bendixson Theorem (see Guckenheimer and Holmes pp.44), states that if on a simply connected region $D \subseteq R^2$, the expression $f_u + g_\tau$ is identically zero or changes sign then equation. B.l has a dosed orbit lying entirely in D.

On any solution curve of f we have from B.l:

$$\int_\gamma (f(.)d\tau - g\,(.)du) = 0 \qquad (B.2)$$

On any closed orbit γ. This implies via Green's Theorem that:

$$\iint_s (f_u + g_\tau)du\ d\tau = 0 \qquad (B.3)$$

Where s is the interior of γ. Given two successive bifurcations such that along *ba* in Figure 1, $tr = f_u + g_\tau$ changes from positive to negative as f_u changes from > to < zero and back again B.3 can be satisfied and a closed orbit, *ab* can exist.

(ii) $\qquad If\ |g_\tau| > |f_u|$ or trace < 0

Take the line integral:

$$\int_\gamma \frac{f(.)}{g(.)} u'(t)dt \qquad (B.4)$$

over $t_0 < t < t_1$ assuming the cycle *ba* is contained in t_0 to t_1. As u' changes from positive to negative and back again the line integral could sum to zero, and a closed orbit can exist.

4.2: We know that:

$$\Gamma_s = \left(s_p\,\tau + \frac{f^i}{y}\right)u \qquad \Gamma_i = \frac{i}{K} \qquad (B.5)$$

where $\Gamma_i\ (\Gamma_s)$ is the rate of growth of capital made possible by investment (savings). Since $f_u > f_\tau$, the effect of u on Γ_i would exceed that of τ. In the model growth is investment led with flows of f^i ensuring that $\Gamma = \Gamma_i = \Gamma_s$. On trajectory b, u is rising and τ is falling. The rate of growth is therefore higher than on trajectory a where the opposite holds. B.5 also implies that the steady state rate of growth at E_2 is greater than at E_1.

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