Spatial Price Discrimination and Privatization on Vertically Related Markets

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Abstract

We consider a vertically structured market with two retail firms of mixed ownership competing against each other exercising spatial price discrimination. We examine the strategic behavior of downstream rivals as well as the effect of privatization on the intensity of competition and welfare in two cases; when location decisions are taken sequentially and when location decisions are taken simultaneously. We show that production cost differentials are crucial in determining the Nash equilibrium locations (hence market shares) and the impact of the degree of privatization on the level of downstream competition. Privatization leads to stiffer competition when the mixed ownership firm has the cost advantage. However, it can be welfare enhancing only when decisions are taken sequentially with the follower being the semi-public firm having a moderate production cost advantage over the market leader. The results of our model generalize to capture the case of vertical mergers.

JEL classification: L13, L33, L42, R32
Keywords: mergers; mixed oligopoly; privatization; spatial competition

1 Introduction

De Fraja and Delbono (1989) initiated a large literature on mixed oligopoly, where public and private firms coexist in the same market.[1] The existing studies can be classified as falling into two groups; one adopting a restricted 'binary' approach where firms are either private or public (e.g., Cremer et al., 1991; Matsumura and Matsushima, 2003; Lu, 2006; Heywood and Ye, 2009a, b) and the other allowing for partially privatized firms (e.g., Matsumura, 1998; Fershtman, 1990; Bennett and Maw, 2003; Kumar and Saha, 2008; Beladi et al., 2014).

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[1]The industries where public and private firms coexist are numerous, and include, amongst others, auto, steel, health, education, telecommunications etc. For a comprehensive review about mixed oligopoly literature, see De Fraja (2009).

Mixed oligopoly, however, has rarely been examined within the context of spatial price discrimination introduced by Hoover (1937) and Lerner and Singer (1937). This type of spatial competition differs from the one introduced by Hotelling (1929) in the fact that the firms do not compete in mill prices but instead bear transportation costs and set delivered price schedules.[2] Few papers in mixed oligopoly theory account for spatial price discrimination. Two such examples are the papers by Heywood and Ye (2009a, b) and Beladi et al. (2014). These works, however, do not account for a vertical structure in the market[3] and assume that competing firms are homogeneous regarding their marginal production costs. A notable exception is Beladi et al. (2016) but unfortunately this paper is plagued with conceptual errors.

Our study is driven by the need to better understand the impact of the recent privatization and merger waves[4] on the strategic behavior of geographically differentiated firms and the ensuing welfare implications; a topic of profound interest for both academics and policy makers. Regarding mergers, we focus our attention on vertical[5] and upstream horizontal mergers. The latter case is examined within the context of cross-border mergers[6] driven by trade liberalization.

The present work contributes to the existing literature in manifold ways. Firstly, it adds to the limited number of studies on mixed oligopoly where retailers compete in a vertically structured market exercising spatial price discrimination. Within this context, our work complements a rather inconclusive literature on the welfare effects of privatization.[7] Secondly, it extends recent results to include and compare retailer location decisions under simultaneous and sequential competition in a market exhibiting vertical structure. Thirdly,

[2] Applications of spatial price discrimination can be found in Lederer and Hurter (1986), Hamilton et al. (1989), Hamilton et al. (1991), McLeod et al. (1992), Braid (2008), and Vogel (2011). Anderson et al. (1992) present an overview of the related literature.

[3] The assumption of a vertically linked market enhances the validity of any policy implications since the majority of the products are processed through the various stages of the vertical production chain before reaching the final consumer.

[4] For more information about the arguments for the existence of a seventh merger wave, see https://meritocracycapital.com/another-merger-wave-unwinds/. Information about privatization trends can be retrieved from the Privatization Barometer Report 2014/2015 (http://www.feem.it/userfiles/attach/2015112392244PB_Annual_Report_2014-2015.pdf).

[5] For a review on vertical mergers, see Lafontaine and Slade (2007).

[6] For a discussion about the significance of cross-border mergers, see Chapman (2003).

[7] See Hamada (2017) for a brief review of the literature on welfare effects of privatization.

it establishes a general setting for the study of related problems enabling us to provide concise and thorough explanations for the mistakes appeared in Beladi et al. (2010a) and Beladi et al. (2010b) already announced in Eleftheriou et al. (2016a). Fourthly, it corrects further mistakes pertaining in recent literature as in Beladi et al. (2016) setting the record straight.

Our main findings can be summarized as follows. When downstream firms take their location decision simultaneously, their Nash equilibrium locations are determined by the difference in the marginal production costs (as expressed by the wholesale prices); the firm having the cost advantage increases its market share. When location decisions are taken sequentially, two effects influence the equilibrium outcome; the first-mover effect and the cost-advantage effect. A high cost producing firm can still increase its market share provided it chooses its location first and the cost advantage of its competitor is relatively small. In other words, a downstream follower can still increase its market share provided it has a sizeable cost advantage. An important policy-related implication of our analysis is that the adaptability of a partially privatized public firm to changing market conditions (such as changes in wholesale prices) increases with the degree of privatization. It is important to observe that the results regarding the social optimality of the market outcome of a market exhibiting a vertical structure are in sharp contrast with the corresponding results concerning a market absent of vertical structure.

Our study further generates interesting results regarding the welfare implications of privatization. Specifically, the welfare effect of privatization depends on how the latter affects the balance between the first-mover and the cost-advantage effect. When no downstream rival has a leading position (simultaneous move game) the cost-advantage effect is enhanced by privatization resulting to a lower welfare (despite the fact that privatization leads to stiffer competition when the mixed ownership firm has a cost advantage). In a sequential move game, the balance between the two effects depends on the nature of the leader. Privatization is not efficient when the semi-public firm is the leader. On the other hand, a public follower exhibiting a cost advantage is a prime privatization target. Therefore, a vertical merger between a public follower and a private upstream monopolist, which increases the degree of privatization can be welfare enhancing.

The rest of the paper is structured as follows. Section 2 presents the benchmark model where downstream firms decide on their locations simultaneously. The sequential move game is analyzed in Section 3. Section 4 concludes.

2 Simultaneous choice of location: The baseline model

The setting of our baseline model follows that of Beladi et al. (2016). Two downstream firms, R_i, $i = 1, 2$, compete in a vertically related industry where the only input (intermediate good) required for downstream production is provided by an upstream supplier, M. The intermediate good is transformed (on a one-to-one basis) by R_1 and R_2 into differentiated final goods they sell to uniformly distributed consumers on a uni-dimensional (linear) market interval with support on $[0, 1]$. The locations of R_1 and R_2 are denoted by x and y, respectively, with $x < y$ in $[0, 1]$. Three varieties of a differentiated product are offered: U and W from firm R_1 and V and W from firm R_2. We assume that the fraction of consumers buying only good U is equal to the fraction of consumers buying good V, both set equal to c while a fraction b of consumers buys the common product W.[8] Our assumptions about the provision of downstream goods imply that R_1 and R_2 enjoy monopoly power over the goods (or varieties) U and V while they compete for market share regarding the common good (or variety) W. R_1 is privately owned whereas R_2 is partly privately owned and partly publicly owned (mixed) in proportions a and $1-a$, respectively with $a \in [0, 1]$. The profit function of R_2 is equal to the weighted average of its own profits and social welfare with weights a and $1 - a$, respectively. Social welfare is equal to the sum of the aggregate profits (the profits of both firms when they are both under private ownership) and consumers' surplus. Transportation costs are equal to td, where t is a positive scalar and d is the distance shipped. The maximum reservation price a consumer is willing to pay for any product is denoted by k. This price is sufficiently large and becomes relevant only for products (or varieties) the retailers enjoy monopoly power over. Downstream firms bear transportation costs. Their marginal delivered cost for selling at location z is equal to the marginal production cost w_i,

[8] If q denotes the fraction of consumers deciding to buy no good, then it is clear that $b + 2c + q = 1$. Fractions b, c and q are constant throughout.

$i = 1, 2$, plus the transportation cost for shipping the good to the consumer's location z. The pricing of downstream goods is as follows. For goods U and V, the downstream firms taking advantage of their monopoly power charge all consumers infinitesimally less than k. Good W is the object of Bertrand competition à la Hoover (1937) and Lerner and Singer (1937). Specifically, the price charged for W by the firm that is closer to the consumer is equal to (or infinitesimally less than) the delivered cost of the firm that is further away.

Upstream supplier, downstream retailers and consumers play a four-stage game of complete information. In the first stage, M makes a take-it-or-leave-it two-part tariff offer (w_i, F_i), $i = 1, 2$, to firm R_i where w_i is the wholesale price and F_i is the fixed fee extracted by the upstream supplier.[9] At this stage, R_1 and R_2 simultaneously choose their locations in the market. In stage two of the game, the downstream competitors, R_1 and R_2, simultaneously decide whether to accept or decline the two-part tariff contract offered by the upstream monopolist. Once an offer is accepted by the downstream firms, the fixed fee is collected by the monopolist. In the third stage, R_1 and R_2, having observed each other's location, choose delivered price schedules. In the final stage of the game consumers make their purchasing choices to clear the market. The solution of the game is given by backward induction.

In order to write down the profit functions of the downstream firms, we first need to determine the location of the indifferent consumer, s, for the common good W. To this end we equate the respective delivered schedules to get $t\,(y - s) + w_2 = t\,(s - x) + w_1 \Longrightarrow s = \frac{x+y}{2} + \frac{w_2 - w_1}{2t}$.

It should also be observed that if $\frac{|w_2 - w_1|}{2t} > \frac{y-x}{2}$, both firms are reduced to spatial-price discriminating monopolists where the common good W is now provided only by either R_1 or R_2. We consider this case trivial and focus only on the case

$$\frac{|w_2 - w_1|}{2t} \leq \frac{y - x}{2}\,. \tag{1}$$

Thus, the profit functions of R_1 and R_2, are respectively:

[9] Vertically related firms usually trade through non-linear two-part tariff contracts (see Bonnet and Dubois, 2010).

$$\begin{aligned}\Pi_{R_1}(x,y) &= c(k-w_1) - \frac{ct}{2}[x^2+(1-x^2)] + \int_0^x b[t(y-x)+w_2-w_1]dz \\ &+ \int_x^{\left(\frac{x+y}{2}+\frac{w_2-w_1}{2t}\right)} b[t(x+y-2z)+w_2-w_1]dz - F_1 \end{aligned} \tag{2}$$

$$\begin{aligned}\Pi_{R_2}(x,y) &= c(k-w_2) - \frac{ct}{2}\left[y^2+(1-y)^2\right] + \int_{\left(\frac{x+y}{2}+\frac{w_2-w_1}{2t}\right)}^y b[t(2z-x-y)+w_1-w_2]dz \\ &+ \int_y^1 b[t(y-x)+w_1-w_2]dz - F_2 + (1-a)g(x,y) \end{aligned} \tag{3}$$

with

$$\begin{aligned} g(x,y) &= \Pi_{R_1}(x,y) \\ &+ \left(\int_0^{\left(\frac{x+y}{2}+\frac{w_2-w_1}{2t}\right)} b[k-t(y-z)-w_2]dz + \int_{\left(\frac{x+y}{2}+\frac{w_2-w_1}{2t}\right)}^1 b[k-t(z-x)-w_1]dz \right) \\ &+ \left(\begin{array}{c} c\int_0^1 w_1 dz + c\int_0^1 w_2 dz + b\int_0^{\left(\frac{x+y}{2}+\frac{w_2-w_1}{2t}\right)} w_1 dz + b\int_{\left(\frac{x+y}{2}+\frac{w_2-w_1}{2t}\right)}^1 w_2 dz \\ +F_1+F_2 \end{array} \right) \end{aligned} \tag{4}$$

The term inside the first set of parentheses in (4) corresponds to the total consumer surplus (CS), while the term inside the second set of parentheses denotes the profits of the upstream monopolist (Π_M). The objective of the mixed firm (R_2) is to maximize the weighted average of its own profits and social welfare, where the weights are determined by the degree of privatization, a, i.e. $\Pi_{R_2}(x,y) = a\Pi_{R_2}|_{a=1} + (1-a)[\Pi_{R_1} + \Pi_{R_2}|_{a=1} + CS + \Pi_M]$.[10] Equations (2) and (3) differ from the corresponding ones in Beladi et al. (2016) in three points. Firstly, Beladi et al. (2016) (incorrectly) do not account for the fact that the delivered cost for good

[10] Where $\Pi_{R_2}|_{a=1} = c(k-w_2) - \frac{ct}{2}\left[y^2+(1-y)^2\right] + \int_{\left(\frac{x+y}{2}+\frac{w_2-w_1}{2t}\right)}^y b[t(2z-x-y)+w_1-w_2]dz + \int_y^1 b[t(y-x)+w_1-w_2]dz - F_2$.

W of the firm that is located further away, which is equal to the price charged to consumers by its rival firm, is equal to the sum of its transportation cost and its marginal cost (see Braid, 2008, p. 345). Since the marginal cost of R_2 is equal to the wholesale price, w_2, the profits of R_1 realizing a sale of good W to a customer located at place z, will be equal to $w_2 + t(y-z) - t|z-x| - w_1 = t(y-z) - t|z-x| + w_2 - w_1$. Secondly, Beladi et al. (2016) fail to take into account the difference of the two wholesale prices in determining the location of the indifferent consumer which is correctly evaluated to $\frac{x+y}{2} + \frac{w_2 - w_1}{2t}$ instead of $\frac{x+y}{2}$ as they have. Thirdly, Beladi et al. (2016) do not include the profits of the upstream monopolist (term inside the second set of parentheses in (4)) in the calculation of social welfare. Having evaluated the integrals, (2) and (3) become

$$\begin{aligned}\Pi_{R_1}(x,y) &= c(k-w_1) - \frac{ct}{2}\left[x^2 + (1-x)^2\right] \\ &\quad + bx\left[t(y-x) + w_2 - w_1\right] + \frac{b}{4t}\left[t(y-x) + w_2 - w_1\right]^2 - F_1 \end{aligned} \tag{2b}$$

$$\begin{aligned}\Pi_{R_2}(x,y) &= c(k-w_2) - \frac{ct}{2}\left[y^2 + (1-y)^2\right] + b(1-y)\left[t(y-x) + w_1 - w_2\right] \\ &\quad + \frac{b}{4t}\left[t(y-x) + w_1 - w_2\right]^2 - F_2 + (1-a)g(x,y) \end{aligned} \tag{3b}$$

R_1 chooses x to maximize (2b), and R_2 chooses y to maximize (3b), leading to the following Nash equilibrium locations

$$(x,y) = \left(\frac{1}{2} - r - [b(3+a) + 4c]\Delta,\ \frac{1}{2} + r - [b(1+3a) + 4ca]\Delta\right) \tag{5}$$

where $\Delta = \frac{b(w_1 - w_2)}{8t(b+2c)(c+b)}$ and $r = \frac{b}{4(b+c)}$.

We deduce,

Proposition 1. *In a vertically related mixed downstream duopoly with simultaneous decision taking, the Nash equilibrium locations of the two downstream rivals are* $\left(\frac{1}{2} - r - [b(3+a) + 4c]\Delta\right)$ *for the privately owned firm and* $\left(\frac{1}{2} + r - [b(1+3a) + 4ca]\Delta\right)$ *for the mixed ownership firm.*

It can be shown that the socially optimal locations are equal to the ones determined by Braid (2008), namely

$$(x^*, y^*) = \left(\frac{1}{2} - r,\ \frac{1}{2} + r\right) \tag{6}$$

From Proposition 1 and (6), we get

Corollary 1. *In a vertically structured market, where the downstream rivals decide on their locations simultaneously*

1. *their Nash equilibrium locations are not socially optimal, unless both firms are privately owned*[11]

2. *both firms move to the left (resp. right) of the socially optimal location if $w_1 > w_2$ (resp. $w_1 < w_2$)*

3. *an increase in the degree of privatization will induce both firms to move away from their socially optimal locations, increasing (resp. decreasing) their in-between distance, $\frac{b}{2(b+c)} + \frac{(1-a)b(w_1-w_2)}{4t(c+b)}$, if $w_1 < w_2$ (resp. $w_1 > w_2$) and*

4. *total welfare decreases in the degree of privatization.*

Proof. See Appendix.

Beladi et al. (2016) (in their Proposition I) arrive, indeed, at the same conclusion as in 1 of Corollary 1. However, they do so by a fluke because the Nash equilibrium locations they find are independent from the difference of wholesale prices ($w_1 - w_2$) which, in general, is not zero.

Eleftheriou and Michelacakis (2016b) have shown that in the absence of an upstream supplier, the social optimality of Nash equilibrium locations is restored. When the market exhibits a vertical structure it is the participation of the profits of the upstream supplier that affects the socially optimal location of the retail competitors.

[11] Eleftheriou and Michelacakis (2016a) showed that Nash equilibrium locations can be socially optimal when both downstream firms are privately owned (i.e., $a = 1$).

Part 2 of Corollary 1 implies that downstream competitors move towards the direction of the firm producing with the higher wholesale price. The intuition behind this finding is that the firm facing the higher wholesale price, loses its competitive edge and is forced to give away part of its market share.

Part 3 of Corollary 1 is different than (and serves to correct) Propositions II and III in Beladi et al. (2016). The intuition behind this statement appears counterintuitive albeit with strong policy implications; when the mixed ownership firm has a production cost disadvantage (i.e., $w_1 < w_2$) then the intensity of competition decreases as the degree of privatization increases. In other words, the nationalization of a costly partly public-partly private firm is a desirable policy. While the adverse welfare effect of privatization is reported in the literature (see for example Sanjo, 2009; Heywood and Ye, 2009c; Martinez-Sanchez, 2011), it is, to the best of our knowledge, the first time that this result manifests itself in a model with heterogeneous costs without R&D, where the mixed ownership firm has the cost disadvantage.

Through normalization if we let $w_1 = 0$ (resp. $w_2 = 0$) the model captures the case of a vertical merger between the upstream monopolist and the private (resp. mixed ownership) firm. We get the following corollary.[12]

Corollary 2. *When the downstream private firm (R_1) merges upstream, the Nash equilibrium locations of the two downstream firms, are* $\left(\frac{1}{2} - r + [b(3+a) + 4c]\frac{bw_2}{8t(b+2c)(c+b)}\right)$ *for the integrated firm and* $\left(\frac{1}{2} + r + [b(1+3a) + 4ca]\frac{bw_2}{8t(b+2c)(c+b)}\right)$ *for the un-integrated firm. Moreover, both downstream firms move to the right of their socially optimal locations. Their in-between distance,* $\frac{b}{2(b+c)} - \frac{(1-a)bw_2}{4t(c+b)}$*, is an increasing function of the degree of privatization and it is bounded above by the distance separating their socially optimal locations.*

Assuming that a merger between R_2 -with pre-merger private share $a \in [0, 1]$- and the upstream supplier leads to an integrated firm with private ownership share a_m with $0 \leq a \leq a_m \leq 1$, the following corollary is in line:

Corollary 3. *When the downstream mixed ownership firm, R_2, merges upstream, the Nash equilibrium locations of the two downstream firms, are* $\left(\frac{1}{2} - r - [b(3+a_m) + 4c]\frac{bw_1}{8t(b+2c)(c+b)}\right)$

[12] Setting $a = 1$ and $w_1 = 0$, we get the results for the post-merger case in Eleftheriou and Michelacakis (2016a).

for the the un-integrated firm and $\left(\frac{1}{2} + r - [b(1+3a_m) + 4ca_m]\frac{bw_1}{8t(b+2c)(c+b)}\right)$ *for the integrated firm. Both downstream firms move to the left of their socially optimal locations. Their in-between distance,* $\frac{b}{2(b+c)} + \frac{(1-a_m)bw_1}{4t(c+b)}$*, is a decreasing function of the degree of privatization and it is bounded below by the distance separating their socially optimal locations.*

The rationale behind the location choice of the downstream rivals in Corollaries 2 and 3 is the same as in Corollary 1; higher wholesale price goes 'hand in glove' with smaller market share. A vertical merger in this case is equivalent to the maximization of the cost-advantage for the integrated firm. The deviation of the in-between equilibrium distance from its socially optimal level can be attributed to the fact that the welfare-oriented objective of R_2 prevents it from giving up (gaining) market share in favor of (against) R_1, when the latter (former) enjoys the competitive advantage of a lower wholesale price due to its merger with the input supplier. The above result has an important policy implication; the degree of privatization determines the responsiveness of the firm under mixed ownership to changes in production cost conditions. The lower the a, the more sluggish the reaction of the mixed firm.

Both Proposition 1 and Corollary 1 can be extended to include the case of cross-border horizontal mergers. For $a = 1$ (and $w_1 = w_2$), we get the results for the pre-merger autarkic (pre-merger free trade and cross-border merger) case(s) announced in Eleftheriou et al. (2016a) correcting the corresponding ones in Beladi et al. (2010b). An alternative explanation following a case-specific treatment can be found in Eleftheriou et al. (2016b).

3 The sequential move game

In this section, we examine the case where the downstream firms decide on their locations sequentially instead of simultaneously. The game is now played in six stages and at the end of each stage all players have complete knowledge of the moves played in earlier stages. In the first stage, M makes a take-it-or-leave-it two-part tariff offer, (w_i, F_i) to firm R_i in a fashion similar to the simultaneous move game. In the second stage, R_i chooses its location in the market. Having observed the location decision of R_i, R_j, $j \neq i$, chooses its location in the third stage. In the fourth stage, R_1 and R_2 simultaneously decide whether or not to accept or decline the two-part tariff contract offered by the upstream monopolist, M. In stage five of the game downstream firms engage in spatial price discrimination by choosing delivered price schedules. In the final stage, consumers make their purchasing choices to clear the market.

The game is solved by backward induction. We first examine the case where the privately owned firm is the leader and the mixed ownership firm the follower. Maximizing (3) with respect to y, we obtain R_2's reaction function:

$$y(x) = \frac{4t(b+c) + 2tbx + 2ba(w_2 - w_1)}{2t(4c+3b)} \tag{7}$$

The profit function of R_1 is

$$\begin{aligned} \Pi_{R_1}(x, y(x)) &= c(k - w_1) - \frac{ct}{2}[x^2 + (1 - x^2)] + \int_0^x b[t(y(x) - x) + w_2 - w_1]dz \\ &+ \int_x^{\left(\frac{x+y(x)}{2} + \frac{w_2 - w_1}{2t}\right)} b[t(x + y(x) - 2z) + w_2 - w_1]dz - F_1 \end{aligned} \tag{8}$$

Solving the first order condition for profit maximization of R_1 and substituting the solution into (7), we get the following Nash equilibrium locations.

$$x = \left(\frac{1}{2} - r\right) + \frac{b\{bt(3b+4c) + [12b^2 + 16c^2 + 28cb + 4ba(b+c)](w_2 - w_1)\}}{4t(b+c)\Lambda} \tag{9a}$$

$$y = \left(\frac{1}{2} + r\right) + \frac{b[tb^2 + (8b^2a + 24cba + 4b^2 + 4cb + 16ac^2)(w_2 - w_1)]}{4t(b+c)\Lambda} \tag{9b}$$

where $\Lambda = 16c^2 + 20cb + 5b^2$. This leads to the following proposition.

Proposition 2. *In a vertically related mixed downstream duopoly, the Nash equilibrium locations of the two downstream firms deciding on their locations sequentially with the privately owned firm being the leader, are* $\left(\left(\frac{1}{2} - r\right) + \frac{b\{bt(3b+4c)+[12b^2+16c^2+28cb+4ba(b+c)](w_2-w_1)\}}{4t(b+c)\Lambda}\right)$ *for the leader and* $\left(\left(\frac{1}{2} + r\right) + \frac{b[tb^2+(8b^2a+24cba+4b^2+4cb+16ac^2)(w_2-w_1)]}{4t(b+c)\Lambda}\right)$ *for the follower.*

Proposition 2 together with equation (6) lead to

Corollary 4. *When, in a vertically related mixed downstream duopoly, the downstream firms decide on their locations sequentially with the privately owned firm being the leader then,*

(i) if $w_1 > w_2$ *and* $\min\{\frac{t(16c^3+28c^2b+15cb^2+2b^3)}{2b(c+b)(b+2ba+4ca)}, \frac{bt(8c^2+9cb+2b^2)}{(c+b)(b+4c)(ba+3b+4c)}\} > w_1-w_2 > \frac{bt(4c+3b)}{4(c+b)(ba+3b+4c)}$ *both downstream firms move away from their socially optimal locations,* $(x^*, y^*) = \left(\frac{1}{2} - r, \frac{1}{2} + r\right)$*, to the direction of the downstream leader*

(ii) if $w_1 > w_2$ *and* $\frac{b^2t}{4(c+b)(b+2ba+4ca)} < w_1 - w_2 < \frac{bt(4c+3b)}{4(c+b)(ba+3b+4c)}$*,* R_1 *(resp.* R_2*) moves to the direction of the downstream follower (resp. leader)*

(iii) if $w_1 > w_2$ *and* $w_1 - w_2 < \frac{b^2t}{4(c+b)(b+2ba+4ca)}$ *or if* $w_1 < w_2$ *both downstream firms move to the direction of the downstream follower.*

The interpretation of Corollary 4 is quite straightforward. A downstream firm can have two competitive advantages; one from the first-mover (leader) effect and the other from the cost effect due to the lower wholesale price charged by the upstream supplier. When $w_1 < w_2$, both effects work concurrently in favor of R_1. If, however, $w_1 > w_2$, the private firm loses market share against the mixed ownership firm provided the difference between the wholesale prices is high enough (adverse cost effect) to offset the first-mover effect. Finally, if $w_1 - w_2 > 0$ is within a certain interval $(\frac{b^2t}{4(c+b)(b+2ba+4ca)}, \frac{bt(4c+3b)}{4(c+b)(ba+3b+4c)})$, then the adverse cost effect completely offsets the first-mover effect, intensifying competition and leading both downstream rivals to move towards each other. A graphic illustration of Corollary 4 for the case $w_1 > w_2$ is presented in Figure 1.

[Figure 1 about here]

Further calculations using Proposition 2 lead to

Corollary 5. *The in-between distance of the two downstream firms is* $\frac{b(8c^2+9cb+2b^2)}{(c+b)(16c^2+20cb+5b^2)} + (w_1 - w_2)\frac{b[b(2-a)+4c(1-a)]}{(16c^2+20cb+5b^2)t}$*. If both downstream firms operate on equal marginal costs privatization does not affect competition.*

Corollary 5 confirms the crucial role of the cost advantage effect in determining the impact of privatization on the intensity of competition and consequently on the welfare (see Appendix).

We observe that privatization affects the intensity of competition and welfare only under the existence of production cost differences.

From Corollary 5 one deduces

Corollary 6. *If the follower (R_2) has a production cost advantage over the leader (R_1), i.e. if $w_1 > w_2$ then, the higher the degree of privatization the stiffer the competition is because their in-between distance is a decreasing function of the degree of privatization. If the leader has a production cost advantage, i.e. if $w_1 < w_2$ then, the higher the degree of privatization the weaker the competition is because their in-between distance is an increasing function of the degree of privatization.*

We arrive finally at the following corollary whose proof is explained in the Appendix.

Corollary 7. *If $w_1 < w_2$ (resp. $w_1 > w_2$) total welfare decreases (resp. increases) as the degree of privatization increases (resp. if $\frac{b^2 t}{4(c+b)(b+2ba+4ca)} < w_1 - w_2 < \frac{bt(4c+3b)}{4(c+b)(ba+3b+4c)}$ or $w_1 - w_2 < \frac{b^2 t}{4(c+b)(b+2ba+4ca)}$).*

According to Corollary 6, a private leader with a cost handicap will suffer more, in terms of competition, from a private rather than a public follower. See, relatively, the discussion about the responsiveness of the public firm to changes of the production cost in the previous section. Corollary 7 implies that privatization will have a positive impact on total welfare as long as the cost advantage of the mixed ownership firm is not very strong (i.e., cases (ii) and (iii) as opposed to case (i) in Corollary 4). If $w_1 < w_2$, then R_1 has both advantages and an increase in the degree of privatization will decrease the competitive resistance of R_2 (follower) leading to a lower level of total welfare and weaker competition.

Maximizing (2) with respect to x, we obtain R_1's reaction function when the mixed firm is the leader.

$$x(y) = \frac{2t(by + 2c) + 2b(w_2 - w_1)}{2t(4c + 3b)} \tag{10}$$

The profit function of R_2 in this case will become

$$\begin{aligned}\Pi_{R_2}(x(y),y) &= c(k-w_2)-\frac{ct}{2}\left[y^2+(1-y)^2\right] \\ &+\int_{\left(\frac{x(y)+y}{2}+\frac{w_2-w_1}{2t}\right)}^{y} b[t(2z-x(y)-y)+w_1-w_2]dz \\ &+\int_{y}^{1} b[t(y-x(y))+w_1-w_2]dz - F_2+(1-a)g(x(y),y) \qquad (11)\end{aligned}$$

Solving the first order condition for profit maximization of R_2 and plugging the solution into (10), we get the following Nash equilibrium locations.

$$x=\left(\frac{1}{2}-r\right)+\frac{b[(16c^2+24cb+4cba+4b^2a+8b^2)(w_1-w_2)+b^2at]}{4t(b+c)(-6b^2+b^2a-20cb-16c^2)} \qquad (12a)$$

$$y=\left(\frac{1}{2}+r\right)+\frac{ba[(2b+2c)^2(w_1-w_2)+(\frac{3}{4}b^2t+ctb)]}{t(-6b^3+cb^2a-16c^3-26cb^2-36bc^2+b^3a)} \qquad (12b)$$

We are led to

Proposition 3. *In a vertically related mixed downstream duopoly, the Nash equilibrium locations of the two downstream firms deciding on their locations sequentially with the mixed ownership firm being the leader, are* $\left(\left(\frac{1}{2}+r\right)+\frac{ba[(2b+2c)^2(w_1-w_2)+(\frac{3}{4}b^2t+ctb)]}{t(-6b^3+cb^2a-16c^3-26cb^2-36bc^2+b^3a)}\right)$ *for the leader and* $\left(\left(\frac{1}{2}-r\right)+\frac{b[(16c^2+24cb+4cba+4b^2a+8b^2)(w_1-w_2)+b^2at]}{4t(b+c)(-6b^2+b^2a-20cb-16c^2)}\right)$ *for the follower.*

Straightforward calculations using Proposition 3 and (6) lead to[13]

Corollary 8. *In a vertically related market when the downstream competitors decide on their locations sequentially with the mixed ownership firm being the leader, the following hold true:*

(i) when $w_1>w_2$, *if* $a<\frac{4c+2b}{4c+3b}$ *and* $w_1-w_2\leq\frac{t(b^2a-8c^2+cba-10cb-3b^2)}{(c+b)(2b-3ba+4c-4ca)}$ *or if* $a>\frac{4c+2b}{4c+3b}$ *and* $w_1-w_2\leq\frac{-bt(-10cb-8c^2+b^2a+cba-3b^2)}{2(b+c)(b+2c)(2b+ba+4c)}$, *both downstream firms move away from their socially optimal locations,* $(x^*,y^*)=\left(\frac{1}{2}-r,\frac{1}{2}+r\right)$, *to the direction of the downstream follower*

[13] In our analysis, the public firm remains the leader after privatization. This assumption is not unrealistic (see Fjell and Heywood, 2002).

(ii) when $w_1 < w_2$*, if* $a > \frac{4c+2b}{4c+3b}$ *and* $w_2 - w_1 \leq \frac{bt(-10cb-8c^2+b^2a+cba-3b^2)}{4(b+c)^2(ba-2b-4c)}$ *or if* $a < \frac{4c+2b}{4c+3b}$ *and* $w_2 - w_1 \leq -\frac{t(b^2a-8c^2+cba-10cb-3b^2)}{(c+b)(2b-3ba+4c-4ca)}$ *then*

(iia) if $w_2 - w_1 > \frac{bt(3b+4c)}{16(c+b)^2}$ *both firms move to the direction of the downstream leader*

(iib) if $\frac{b^2at}{4(c+b)(ba+2b+4c)} < w_2 - w_1 < \frac{bt(3b+4c)}{16(c+b)^2}$*,* R_1 *(resp.* R_2*) moves to the direction of the downstream leader (resp. follower)*

(iic) if $w_2 - w_1 < \frac{b^2at}{4(c+b)(ba+2b+4c)}$ *both firms move to the direction of the downstream follower.*[14]

The rationale behind Corollary 8 is the same as in Corollary 4; the interaction between the first-mover advantage effect with the wholesale advantage effect. The majority of the technical conditions appearing in Corollary 8 aim at ensuring that $0 \leq x < y \leq 1$ and (1) hold (see proof in the Appendix).[15] A graphic illustration of point (ii) in Corollary 8 is presented in Figure 2.

[Figure 2 about here]

Proposition 3 allows us to calculate the distance between the Nash equilibrium locations of the two rivals.

Corollary 9. *The in-between distance of the downstream competitors is equal to* $\frac{b(b^2at+actb-8tc^2-3b^2t-10ctb)}{t(c+b)(-6b^2+b^2a-20cb-16c^2)} + (w_1 - w_2)\frac{b[3b^2a+7acb+4c^2a-6cb-2b^2-4c^2]}{t(c+b)(-6b^2+b^2a-20cb-16c^2)}$*. Privatization affects competition even if both downstream firms operate on equal marginal costs (i.e.,* $w_1 = w_2$*).*

If the leader (R_2) has a production cost advantage over the follower (R_1), i.e. if $w_1 > w_2$ then, the higher the degree of privatization the stiffer the competition is because their in-between distance is a decreasing function of the degree of privatization. If the follower has a production cost advantage, i.e. if $w_1 < w_2$ and if $w_2 - w_1 < \frac{bt(3b+4c)}{16(b+c)^2}$ (resp. $w_2 - w_1 > \frac{bt(3b+4c)}{16(b+c)^2}$) then, the higher the degree of privatization the stiffer (resp. weaker) the competition is

[14] It can be shown that $\frac{bt(3b+4c)}{16(c+b)^2}$ is always less than $\frac{bt(-10cb-8c^2+b^2a+cba-3b^2)}{4(b+c)^2(ba-2b-4c)}$ and always less than $-\frac{t(b^2a-8c^2+cba-10cb-3b^2)}{(c+b)(2b-3ba+4c-4ca)}$ for $a < \frac{4c+2b}{4c+3b}$.

[15] Further details can be found in the Appendix under the title "Conditions for the validity of equilibrium locations when the mixed ownership firm is the leader".

because their in-between distance is an decreasing (resp. increasing) function of the degree of privatization.[16]

Equation (A.3) in the Appendix has the following implication. When the semi-public firm enjoys both advantages (first-mover and cost), then an increase in a will make it more aggressive (competitive), resulting to a decrease in the in-between distance of the downstream rivals. However, if the leading position is combined with a cost handicap, the final result depends on the size of difference between the production costs. If this is small, the first-mover advantage prevails increasing the aggressiveness of R_2 as the degree of privatization increases (i.e., the in-between distance will decrease in a).

Corollary 10. *Total welfare decreases in the degree of privatization even if both downstream firms operate on equal marginal costs (i.e., $w_1 = w_2$).*

Proof. See Appendix.

According to Corollary 10, when the semi-public firm is the leader, an increase in privatization will always have a negative impact on total welfare, regardless of differences in production costs.

On equal production costs, an increase in a will increase competition since the first-mover advantage will be better exploited by the mixed ownership firm (i.e., the in-between distance depends on a even if $w_1 = w_2$). Corollary 10 stresses the adage: "*Never privatize a public leader*". For a similar result, at the presence of government subsidies, when firms compete in quantities see Fjell and Heywood (2004).

Propositions 2 and 3 and Corollaries 4-10 may be extended to include the case of a vertical merger between the upstream supplier and the leader or the upstream supplier and the follower of the market respectively. Specifically, the corresponding results about the Nash equilibrium locations as well as their in-between distance can be derived by setting $w_i = 0$, $i = 1, 2$, with i representing the integrated downstream firm and a_m with $0 \leq a \leq a_m \leq 1$ as the post-merger private share of the integrated firm. For $a = 1$, we get the results announced in Eleftheriou et al. (2016a) correcting the corresponding ones in Beladi et al. (2010a). An

[16] Further details can be found in the Appendix under the title "Proof of the relationship between competition intensity and privatization under R_2 leadership".

explanation following a case-specific treatment can be found in Eleftheriou and Michelacakis (2016c).

To the end of shedding more light on the effect of a on the competition in the case where R_2 is the leader, we consider the following example.

Example 1. *Let $c = 0$, (i.e., no downstream firm has monopoly power and only the common good W is provided). If R_2 is completely public (i.e., $a = 0$) then the in-between distance is $\frac{1}{2} + (w_1 - w_2)\frac{1}{3t}$. If $a = 1$, i.e. R_2 is completely private then the in-between distance is $\frac{2}{5} - (w_1 - w_2)\frac{1}{5t}$. Hence, if $w_1 = w_2$ or $w_1 > w_2$ or $w_2 > w_1$ with $w_2 - w_1 < \frac{3t}{16}$, then competition is more intense when R_2 is fully privatized. Furthermore, $TW|_{a=0} - TW|_{a=1} = \frac{b[16(w_1-w_2)+3t]^2}{600t} > 0$. If we further set $w_1 = 0$ we capture the case of a vertical merger of R_1 with the upstream monopolist. Furthermore, the in-between distance following a vertical merger between a fully public leader and the upstream monopolist leading to a semi-public integrated firm with private share a_m is $\frac{(3-a_m)t-(3a_m-2)w_1}{(6-a_m)t}$. It is observed that $\frac{(3-a_m)t-(3a_m-2)w_1}{(6-a_m)t}$ decreases in a_m.*

4 Conclusion

In the present paper, we examine the location behavior of two downstream firms in a vertically linked market competing for market share within a frame of spatial price discrimination using a single input supplied by an upstream monopolist. In the downstream market, we allow for the coexistence of a private firm and a mixed ownership, private-public, firm of variable degree of privatization. We assume that each retailer can provide only a fraction of the product varieties demanded by consumers. Decisions are taken either simultaneously or sequentially. When decisions are taken simultaneously the production cost difference between retailers (due to different wholesale prices charged by the upstream supplier) is identified as the driving force of our findings. Results along this line of research have been obtained by Beladi et al. (2010b) and Beladi et al. (2016). We set the record straight as these papers are all plagued by similar mistakes and misconceptions.

We further contribute to the literature by considering a sequential decision game. It turns out that a high enough marginal production cost advantage for the downstream follower can more than offset the first-mover advantage of the leader leading the latter to lose part of her market share. We show that privatization of public firms increases their adaptability to changes in their marginal production cost. Nevertheless, when the mixed ownership firm is the market follower enjoying a marginal production cost advantage privatization leads to more intense competition and increased total welfare only when this advantage is of moderate size. Contrary to common belief, we prove that privatizing a public leader is not a beneficial move.

Our findings have implications for mixed oligopolistic markets and markets where vertical mergers are taking place. In addition, they provide theoretical answers to pertinent economic policy questions regarding the feasibility of privatization of public firms.

Appendix

Proof of Corollary 1

Parts 1, 2 and 3 are derived from Proposition 1 and (6). Substituting (5) into total welfare $TW = \Pi_{R_1} + \Pi_{R_2}|_{a=1} + CS + \Pi_M$ and differentiating with respect to a, we get

$$\frac{\partial TW}{\partial a} = \frac{-b^2(w_1 - w_2)^2(3ba + b + 4ca)}{16t(b+c)(b+2c)} < 0 \tag{A.1}$$

which proves part 4. Finally, equilibrium locations satisfy $0 \leq x < y \leq 1$ and (1) if (i) $w_1 - w_2 \leq \frac{2bt}{3b+4c+ba}$ when $w_1 > w_2$ and (ii) $w_2 - w_1 \leq \frac{2bt}{5b+4c-ba}$ when $w_1 < w_2$.

Proof of Corollary 7

Substituting (9a) and (9b) into the total welfare (TW) and differentiating with respect to a, we get

$$\frac{\partial TW}{\partial a} = -(w_1 - w_2)[(w_1 - w_2)L_1 - L_2] \tag{A.2}$$

with $L_1 = \frac{(4b^3+32ac^3+12cb^2+\frac{11}{2}b^3a+56bac^2+8c^2b+30cb^2a)b^2}{(16c^2+20cb+5b^2)^2t}$ and $L_2 = \frac{(tb^3+2tcb^2)b^2}{(16c^2+20cb+5b^2)^2t}$.

If $w_1 > w_2$, then $\frac{\partial TW}{\partial a} > 0$ if $w_1 - w_2 < \frac{L_2}{L_1} = \frac{2b^2t(b+2c)}{8b^3+64ac^3+24cb^2+11b^3a+112bac^2+16c^2b+60cb^2a}$. However, it can be shown that this always holds for cases (ii) and (iii) in Corollary 4 (i.e., the difference between the wholesale prices should not be very high). For case (i) in Corollary 4, $\frac{\partial TW}{\partial a} < 0$. Moreover, if $w_1 < w_2$, then $\frac{\partial TW}{\partial a} < 0$.

Conditions for the validity of equilibrium locations when the private firm is the leader

Equilibrium locations satisfy $0 \leq x < y \leq 1$ and (1) if

(i) $w_1 - w_2 \leq \min\{\frac{t(16c^3+28c^2b+15cb^2+2b^3)}{2b(c+b)(b+2ba+4ca)}, \frac{bt(8c^2+9cb+2b^2)}{(c+b)(b+4c)(ba+3b+4c)}\}$ when $w_1 > w_2$ and (ii) $w_2 - w_1 \leq \min\{\frac{t(8c^2+9cb+2b^2)}{(c+b)[b(2-a)+4c(1-a)]}, \frac{t(16c^3+28c^2b+15cb^2+2b^3)}{2b(c+b)(b+2ba+4ca)}, \frac{bt(8c^2+9cb+2b^2)}{(b+c)(7b^2+24cb+16c^2-4cba-ba^2)}\}$ when $w_1 < w_2$.

Proof of the relationship between competition intensity and privatization under R_2 leadership

We differentiate the in-between distance provided in Corollary 9 with respect to a to get

$$-(w_1 - w_2)\left[\frac{b(b+2c)^2(16b^2+32cb+16c^2)}{t(b+c)(-6b^2+b^2a-20cb-16c^2)^2}\right] - \frac{b(b+2c)^2(3b^2t+4ctb)}{t(b+c)(-6b^2+b^2a-20cb-16c^2)^2} \tag{A.3}$$

It can be easily shown that A.3 is always negative if $w_1 > w_2$ or if $w_1 < w_2$ with $w_2 - w_1 < \frac{bt(3b+4c)}{16(b+c)^2}$ (cases (iib) and (iic) in Corollary 8). If $w_1 < w_2$ with $w_2 - w_1 > \frac{bt(3b+4c)}{16(b+c)^2}$ then A.3 is always positive (case (iia) in Corollary 8).

Proof of Corollary 10

The derivative of the total welfare with respect to the degree of privatization is

$$\frac{\partial TW}{\partial a} = \frac{b^2a(b+2c)^2[(w_1-w_2)(32cb+16b^2+16c^2)+3b^2t+4ctb]^2}{2t(b+c)(-6b^2+b^2a-20cb-16c^2)^3} < 0 \tag{A.4}$$

Conditions for the validity of equilibrium locations when the mixed ownership firm is the leader

Equilibrium locations satisfy $0 \leq x < y \leq 1$ and (1) if (i) $w_1 - w_2 \leq \frac{t(b^2a-8c^2+cba-10cb-3b^2)}{(c+b)(2b-3ba+4c-4ca)}$ for $a < \frac{4c+2b}{4c+3b}$ or $w_1 - w_2 \leq \frac{-bt(-10cb-8c^2+b^2a+cba-3b^2)}{2(b+c)(b+2c)(2b+ba+4c)}$ for $a > \frac{4c+2b}{4c+3b}$, when $w_1 > w_2$ and

(ii) $w_2 - w_1 \leq \frac{bt(-10cb-8c^2+b^2a+cba-3b^2)}{4(b+c)^2(ba-2b-4c)}$ for $a > \frac{4c+2b}{4c+3b}$ or $w_2 - w_1 \leq -\frac{t(b^2a-8c^2+cba-10cb-3b^2)}{(c+b)(2b-3ba+4c-4ca)}$ for $a < \frac{4c+2b}{4c+3b}$, when $w_1 < w_2$.

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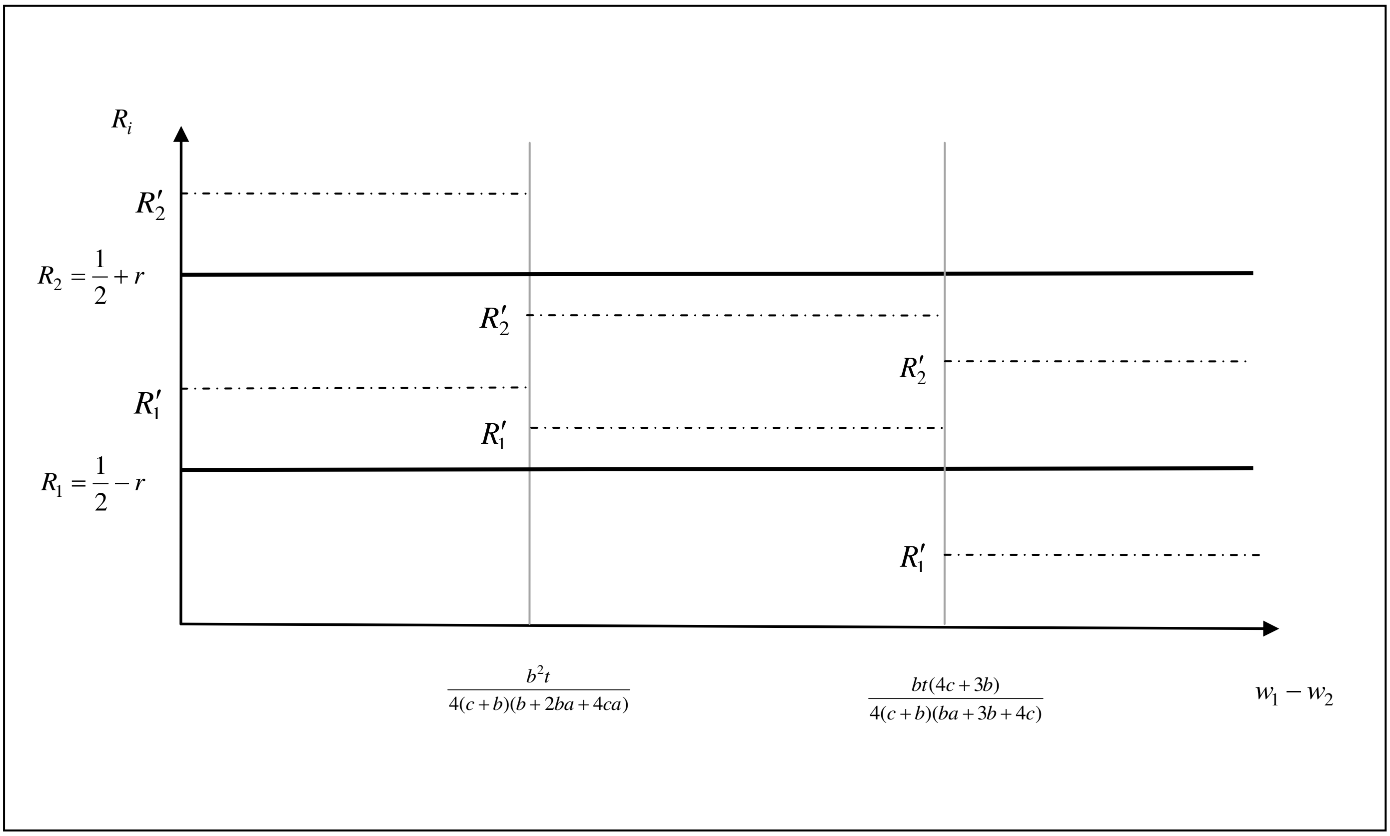
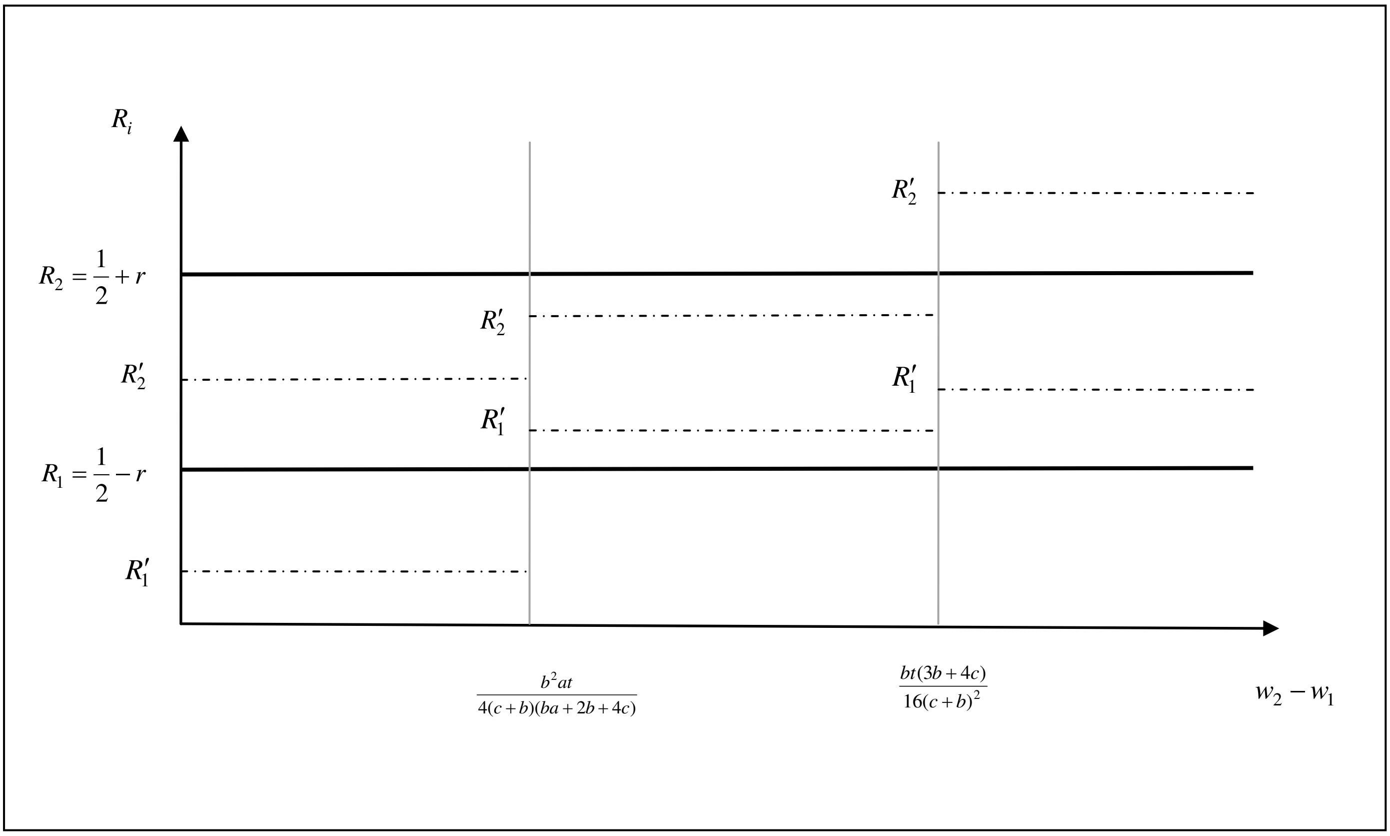
Figure 1: Graphic illustration of Corollary 4 for the case $w_1 > w_2$



Note: R_i and $R_i^{'}$ denote the socially optimal and the Nash equilibrium location of firm i, respectively.

Figure 2: Graphic illustration of point (ii) in Corollary 8



Note: R_i and $R_i^{'}$ denote the socially optimal and the Nash equilibrium location of firm *i*, respectively.