Government Debt and Wealth Inequality: Theory and Insights from Altruism

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Abstract

This paper presents a "hybrid" Diamond model with altruistic households and studies the effects of public debt on wealth inequality. The results, as in some models based on incomplete financial markets, show that more government borrowing will suppress wealth inequality. This suggests that the heterogeneity in endowments might be the main driving force for that particular relationship. In terms of policy, it is likely then that debt adjustments might clash with demands to reduce wealth inequality.

Keywords: Public debt, wealth inequality, altruism, "Joy-of-giving" bequests, Diamond Model
JEL classification: H20, H30, H63, E62

1. Introduction

Public debt, conditioned in its prudent use, is among the policy instruments that can improve the social welfare. More recently, concerns over sustainability have provoked fiscal consolidation plans with unknown distributive outcomes. In addition, public debt is an asset people frequently invest in, and its manipulation by authorities can have a direct impact on the distribution of wealth. Hence, understanding the relationship between

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government debt and wealth inequality seems important. One main reason, as this paper will demonstrate, is that government debt might behave as a redistributive device, not only across generations (as the traditional perception is) but also within. In addition, the present analysis offers a theoretical benchmark case to address counter-factual questions such as: will the level of wealth inequality be lower (higher) if US spends less (more) on wars ? Or more generally, will the level of wealth inequality increase (decrease) with more "unproductive" government spending ? How debt adjustments might relate with wealth inequality ?

To answer those questions, I developed a "hybrid" version of Diamond (1965) model with altruistically linked families which differ in their endowments. However, parents instead of caring about the welfare of their offspring, enjoy utility directly from bequests. The novelty of this approach allows three distinct features to be combined. First, it compromises between the savings motive of a Ramsey and Diamond economy. Second, it maintains a minimum role for Government debt within a simple framework. And third, it permits a measure of wealth inequality which is analytically tractable.

In this paper, public debt has the interpretation of a differed tax. Therefore, I used three different tax instruments to approximate specific tax regimes. More specifically, I consider separately a "Regressive", a "Progressive" and an "Affine" tax instrument on labour. The different cases are meant to compare countries (or adjustments) that are identical, except for the choice of the tax instrument. Focusing on the stationary state, surprisingly enough, the qualitative results at the *macro*economic level are equivalent, although they have different *micro*economic mechanics.

At the macroeconomic level, a positive debt shock will crowd out capital and when the latter is sufficiently strong, *average* wealth will fall. Using as a measure of inequality the coefficient of variation, the fall in average wealth tends to amplify the initial disparity. In parallel, a permanent increase in public debt will also raise the tax burden and affect the interest rate. Savings responses then will determine the asset position among individuals and thus the change in variance. In the calibrated version of the model, this change is negative and inequality tends to decrease. In equilibrium, the drop in variance dominates the fall in the mean and therefore inequality is suppressed.

At the microeconomic level, as soon as debt is conceived as differed taxation, different tax instruments coupled with the bequest transfer, constitute a composite transfers scheme with diverse income and substitution effects (Polemarchakis (1983); Galor and Polemarchakis (1987)). Nonetheless, when the life-cycle considerations for savings (that is, the "egoistic" motive to finance future consumption, and the "altruistic" one to leave a positive bequest) are taken into account, the altruistic motive will play a critical role into agents savings behaviour. A consequence of heterogeneity is that "rich" and "poor" households will never have the same *qualitative* response over their own saving scopes. As a result, any change in debt policy will trigger *asymmetric* effects on asset holdings. In this context, the choice of the tax instrument is of relevance, solely to determine the nature of the change in the variance.

For instance, when the tax instrument is of "regressive" type, the drop in inequality is driven by the increase in savings of the "poor". The intuition for the behaviour of the "poor" is as follows; The "poorest" individuals (who had already low wages prior to the shock) would see their real wage to further decline and their tax liability (regressive in nature) to increase. Unless they start saving more, by exploiting the rise in the rate of interest, they will not be in a position to finance their future consumption *net of bequests*. This could never happen with the relatively "rich", since the (extended to altruism) income effect always dominates. In contrast, when the tax type is more "progressive", individuals will only differ in their quantitative responses. The drop in inequality in this case, is driven by the different in magnitude dissavings across individuals.

While this paper focuses primarily on the effects of debt on wealth inequality, its contribution can be extended onto another aspects. On a theoretical level, I show how the model replicates the steady state results of Diamond (1965), by utilizing the properties of the savings and bequests functions. Therefore, Diamond's original contribution (and in contrast to the general belief) is robust to a bequest motive, as soon as this is of the type mentioned earlier. This finding, to the best of my knowledge, was undiscovered in the literature. In an extension of Diamond's results, the findings of this paper suggest that when economies are dynamic inefficient, the "equity-efficiency" trade-off breaks down[1].

The paper is also related to the causal effects of debt on growth. As Bhandari et al. (2013) claim, this relationship cannot be empirically assessed

[1]See Rhee (1991) and King and Ferguson (1993) for a broader view on dynamic inefficiency.

unless,*inter alia*, the distributive costs of debt (within the population) are taken into account. Moreover, research shows wealth inequality may have adverse economic outcomes, either as a destabilizing factor (Ghiglino and Venditti (2011)) or as a variable affecting the risk premium (Gollier (2001)). For this reason, studying alternative explanations for the levels of wealth inequality is equally prominent.

The literature of public debt and economic inequalities, focused on the discussion mainly across generation (as in Romer (1988); Altig and Davis (1989)); whereas this paper examines the within cohort effects[2]. Notable exceptions include Floden (2001) and Rhrs and Winter (2013). Both papers are quantitatively orientated and rely on incomplete markets models (in the tradition of Aiyagari (1994)) to analyse the welfare properties of public debt.

The first pins down specific combinations of optimal transfers and debt levels and examines, among other things, the change in wealth inequality when at a particular combination. As this paper, Floden (2001) also emphasizes the negative relationship between public debt and wealth inequality. This might suggests that the heterogeneity in endowments rather than the incompleteness of financial markets could be the main driver for that particular relationship. With regard to the latter, its primary concern is to weight the relative (welfare) merits of debt adjustments at cases where the distribution of wealth is highly unequal[3].

In another study, although again different in scope, Heathcote (2005) examines the effects of fiscal policy (tax shocks) when agents face uninsurable idiosyncratic risk and compares his results against the first best[4]. Finally, Azzimonti et al. (2012) address the role of financial liberation to rationalize the joint evolution in the rise of public debt and income inequality observed in the data. However, their specific approach to endogenize public debt is hard to reconcile with episodes of debt adjustments[5]. Nonetheless, a common feature for the literature discussed so far, is the reliance on Ramsey

[2]On the interaction of public debt with bequests, prominent examples are Barro (1974), Laitner (1979), Drazen (1978) or Burbidge (1983)

[3]Nevertheless, the negative relationship between debt and wealth inequality is also implicit in their analysis.

[4]Within the general topic of Fiscal Policy, see also Garcia and Turnovsky (2007) or Alonso-Carrera et al. (2012) for the effects on income inequality.

[5]In their paper, public debt is endogenized through probabilistic voting within a framework of incomplete markets.

economies. In such environment, altruism is only implicit and it is assumed that any shock cannot affect consumers altruistic attitude. However, as I explain in this paper, this might be a seriously flawed approach.

The organization of the paper is as follows; In Section 2 the details of the model are set-up. In Section 3, the measure of wealth inequality is constructed and discussed. Section 4 focuses on the macroeconomic environment. Section 5 analyses the effects on wealth inequality, while Section 6 presents the main results.

2. The Model

The main environment modifies that of Diamond (1965) into two main dimensions. First, I introduce heterogeneity in labour or "ability" endowments and second I allow individuals to have a "joy-of-giving" bequest motive. Two factors motivate the particular choice for the bequest motive. The one factor is to allow some minimal role for debt (See Online Appendix) and the other one to gain analytical simplicity. Although Altonji et al. (1997) claim that the particular bequest motive might be more relevant in practice[6]. The present procedure also compromises, in a simple and tractable way, the two extremes between a Ramsey and a Diamond economy. In the first kind of economy, any life-cycle considerations are nullified while the latter abstracts from bequests. Finally, the method to assess wealth inequality borrows from Bossmann et al. (2007) with some variations.

On the demand side, consumer i, of generation t who lives in period t maximizes lifetime utility by choosing consumption c_{it}^{t} when young, consumption c_{it+1}^{t} when old and bequests x_{it+1} to transfer to his "son". In the notation, the superscript is the generation index and the subscript denotes calendar time. Total savings a_{it+1}^{t} of the young, determined in period t, are allocated between government bonds and capital. Bonds to be purchased have to pay the same interest rate as capital, making the portfolio composition indetermined. Moreover, agents differ in labour endowments l_i, which supply inelastically. This is the first source of heterogeneity in this model and it is assumed to be exogenous. The second one, implicit in this model, is the

[6]There is a consensus among the profession, over the presence of a bequest motive. For a survey see Laitner and Ohlsson (2001) or Light and McGarry (2003). For the "joy-of-giving" bequest motive as a reduced form specification of altruism see Abel and Warshawsky (1987).

bequest each "young" is endowed with[7]. If preferences are time separable, the individual maximization problem reads:

$$\max_{c_{it}^t, c_{it+1}^t, x_{it+1}} V_t^i = U(c_{it}^t) + \beta U(c_{it+1}^t) + \delta U(x_{it+1}) \tag{1}$$

$$\text{s.t}$$

$$c_{it}^t + a_{it+1}^t = D_t^i \tag{2}$$

$$c_{it+1}^t + (1+n)x_{it+1} = (1+r_{t+1})a_{it+1}^t \tag{3}$$

$$c_{it}^t > 0, \;\; c_{it+1}^t > 0, \;\; x_{it+1} > 0$$

The utility function is standard homothetic with the usual neoclassical assumptions. Equations (2) and (3) are the budgets constraint in the two periods of life. In the notation, D_t^i is the disposable income of the young and n is the population growth. The components of disposable income include the after tax wage and the transfer received by the "parent". The degree of altruism, δ is assumed to be lower than the discount factor β, with $0 < \delta < \beta < 1$. Thus, the "parent" values his own consumption greater. The first order conditions, assuming bequests are operative, are[8]:

$$(1+n)\beta U'(c_{it+1}^t) = \delta U'(x_{it+1}) \tag{4}$$

$$U'(c_{it}^t) = \beta(1+r_{t+1})U'(c_{it+1}^t) \tag{5}$$

For simplicity, I also assume the distribution of labour endowments to be constant over time with the mean normalized to 1 and some variance σ^2. Someone could justify this assumption based on recent research by Huggett et al. (2011). Their findings suggest that for life-time inequality in wealth and welfare, *ex-ante* heterogeneity is far more important than differences in luck (that is, *ex-post* heterogeneity). Alternatively, the assumption on the distribution of labour endowments, can also be thought as equivalent to an environment of a fixed intergenerational mobility, which it seems to be consistent with the US data (See Chetty et al. (2014)). Finally to characterize

[7]It is also assumed the "initial old" differ in their assets. The maximization problem then will only involve the consumption and bequests choices.

[8]The Inada assumptions for preference ensure that bequest will always be operative

the optimum plans, the assumptions on the distribution of "skills" and homothetic preferences, admit solutions in "Gorman Form" (that is,"linear in wealth"). Accordingly, the optimum plans for savings and bequests are:

$$a^t_{it+1} = S_D D^i_t = S_D\left[I^i_t + x^i_t\right] \equiv S_t(D^i_t, r_{t+1}) \tag{6}$$
$$x^i_{t+1} = X_S a^t_{it+1} \equiv X(r_{t+1}, a^t_{it+1}) \tag{7}$$

As usual, r_{t+1} is the rate of interest in period $t+1$, I^i_t is the after tax wage, while $S(D^i_t, r_{t+1})$ and $X(r_{t+1}, a^t_{it+1})$ are the optimal savings and bequests functions respectively. In their "Gorman form" counterparts shown in (6) and (7), S_D denotes the marginal propensity to save (MPS) out of disposable income, while $X_s > 0$ is the marginal propensity to bequeath out of savings (MPB). In Section 4, I will use the inequalities $0 < S_D, \overline{X_S} < 1$, where $\overline{X_S} \equiv \frac{(1+n)X_s}{(1+r_{t+1})}$ describes the *modified* or the *dynamic efficiency adjusted* MPB[9].

On the supply side, markets are competitive and capital depreciates fully within a period. I also assume that a exists a representative firm, which uses capital and labour to produce output. The production function is standard neoclassical with common assumptions. From profits maximization, factor prices in their intensive form equal to:

$$w_t = f(k_t) - f'(k_t)k_t \tag{8}$$
$$1 + r_t = f'(k_t) > 0 \qquad \text{with} \qquad f(k_t)'' < 0 \tag{9}$$

Moreover it is assumed that exist a government which in period t issues bonds, B_{t+1}, and collects taxes $\mathcal{T}_t$. The government uses its revenues to repay interest on the previously issued bonds, B_t. Thus, debt substitutes taxes to finance some given path of (unproductive) government expenses. In the text, I will analyse three different cases in terms of the available tax instrument; A flat (or "Regressive"), a proportional (or "Progressive") and an affine tax instrument. Concerning the latter as Bhandari et al. (2013)

[9]To ease the notation, I omit to make explicit the dependence of MPS and MPB on interest rates and hence on the time period. For the properties of the savings and bequest functions see On-line Appendix.

argue, it approximates better the tax system in the US. In that case, the analysis on "Flat" or "Proportional" tax systems will become instrumental to the "Affine tax" economy for reasons I will discuss later. Nevertheless, flat or more progressive tax systems alone are also quite common in practice (See Keen et al. (2012) for countries with "Flat tax" systems). Finally, it is assumed that all taxes fall on "labour" or the "young" generation[10]. For each case, total tax revenues equal:

$$\mathcal{T}_t = \left[\tau_t^f N_t \;\text{ or }\; \tau_t^p w_t \sum_{i=1}^{N_t} l^i \;\text{ or }\; \tau_t^A w_t \sum_{i=1}^{N_t} l^i - T_t^A N_t\right]$$

where N_t is the population size in period t, τ_t^F the lump-sum tax in "flat tax" economy, τ_t^p the marginal tax rate in the "Progressive tax" system and τ_t^A and T_t^A are the equivalent tax instruments for the "Affine tax" regime. In summary, government's budget constraint in absolute terms equals:

$$B_{t+1} + \mathcal{T}_t = (1 + r_t) B_t$$

Assuming constant debt per labour policy (and taking into consideration that for large economies averages converge to their means), the per capita cost for financing this policy in each case is:

$$\tau_t^F = (r_t - n)b \tag{10}$$

$$\tau_t^P = \frac{(r_t - n)b}{w_t} \tag{11}$$

$$T_t^A = (r_t - n)b - \tau_t^A w_t \tag{12}$$

where $b \equiv \frac{B_{t+1}}{N_{t+1}} = \frac{B_t}{N_t}$, is debt per labour. In all cases, the model assumes that debt generates positive savings. In the event of a debt shock and the subsequent permanent change in disposable income, I keep the terminology in the literature and refer to it as "wealth effects" (Baxter and King (1993)).

[10] Alternatively, we can assume that the tax on capital is set optimally to zero, following the traditional literature on optimal taxation (as in Chamley (1986)). In the case of bequests, these are taxed naturally with a "biological rate" n. The latter is equivalent to a redistributive policy from the "old" to the "young", with neutral government revenues. See also, Piketty and Saez (2013) for optimal bequest taxation and Bossmann et al. (2007) for the effect on wealth inequality.

For the affine tax economies there are two degrees of freedom: The government to maintain its policy can either adjust its flat tax component or the marginal tax rate. The specification in (12) implies the first to occur. If instead, the option was on the marginal tax rate, results would follow that of the proportional tax system[11]. Due to lack of space, I will restrict the attention to the one in the main text.

Another peculiarity that emerges in an "Affine" tax system is that individuals, for given endowments, are segregated between those who pay taxes and those who receive subsides. To see this, note that the after tax wages in this case are $w_t l^i - [(r_t - n)b + \tau_t^A w_t (l^i - 1)]$. Therefore, if the economy is dynamic inefficient, only those with endowment $l^i < 1$ are subsidised; Whether the rest will be paying taxes or not, depend on the degree of dynamic inefficiency and the marginal tax rate. On the other hand, if the economy is dynamic efficient, as we can see from equation (12), the T^A term can be either positive or negative. This depends, in turn, on whether debt repayments are "high" or not relative to the revenues collected from wages. As previously, someone can show that some "poor" ($l^i < 1 - \frac{(r_t - n)b}{\tau^A w_t}$) are subject to negative taxation (that is, "welfare benefits"). Nevertheless, none of those "discriminating" features will play any role for the qualitative pattern of the results.

3. Individual Wealth Accumulation and Wealth Inequality

Bequests facilitate the intergenerational link for the transmission of wealth among families. Substituting (7) in (6), wealth accumulation takes the following form:

$$a_{it+1}^t = S_D[I_t^i + X_s a_{it}^{t-1}] \Rightarrow a_{it+1}^t = S_D I_{it} + \underbrace{S_D X_s}_{=C_{4t}} a_{it}^{t-1} \tag{13}$$

For the three different tax structures, the after tax wage equals $I_t^i = \left[w_t l_i - \tau_t^F, \quad (1 - \tau_t^P) w_t l_i, \quad (1 - \tau_t^A) w_t l_i - T_t^A \right]$.

Equation (13) describes the transmission process of wealth accumulation. Its intuition is very simple: The wealth of a family (or "Dynasty") is the sum

[11] See footnote 21 in Section 5 and the discussion therein.

of the own wealth plus any financial wealth left by the predecessor in form of transfers. In the previous period, part $(1 - S_D)$ of transfers were consumed and the rest (S_D) were saved. From that financial stock, a certain amount was kept for consumption and the rest, X_S, passed on to the offspring. Thus, $S_D X_S$ reflects the fraction from total savings attributed to altruism. This component, along with own wealth, determines the available asset position in the current period.

It follows that in the absence of any willingness to bequeath, initial wealth levels persist forever, whereas some degree of altruism allows wealth to expand. Therefore, unless this degree is not too high (reflected in X_S), wealth accumulation might become explosive. In (13), a sufficient condition for mean-reversion is $C_{4t} \equiv S_D X_s < 1$. However, as soon as X_S depends on the interest rate, debt policies become critical for the existence of a stationary state in the *microeconomic* level. Nonetheless, as I will show in Section 4, a more stringent condition is needed for the *macroeconomic or financial stability*[12].

At present, is it useful to define the ratio $\frac{S_D}{1-S_D X_S}$. The numerator (S_D) is the fraction of savings attributed to the *gross* life-cycle considerations; that is, to finance future expenditures. The denominator $(1 - S_D X_S)$ is the fraction of own investments allocated to personal consumption. Then, the ratio of these two can define the *net* life-cycle component of savings; that is, the part of total wealth invested to finance future consumption *net of bequests*. I call this ratio as the *egoistic MPS*. In the subsequent analysis, this fraction will interact with the wage rate, giving a very natural meaning in the event of a shock. In fact, any perturbation on the $\frac{S_D w}{1-S_D X_S}$ will reflect a "battle" between the own life-cycle and the altruistic considerations.

3.1. Stationary wealth inequality

A tractable measure of inequality frequently used in the literature and the one also employed in Bossmann et al. (2007) is the coefficient of variation, $CV = \sqrt{\frac{Var(a_{it})}{E(a_{it})}}$. In this definition, $E(a_{it})$ is the mean wealth and $Var(a_{it})$ the variance. Restricting our analysis in the stationary state (assuming $C_4 < 1$, $\mathrm{E}(a_{it+1}) = \mathrm{E}(a_{it}) = \mathrm{E}(a_i)$, $\mathrm{Var}(a_{it+1}) = \mathrm{Var}(a_{it}) = \mathrm{Var}(a_i)$ and $k_{t+1} = k_t = k$), the first two long-run moments using (13) are respectively equal to:

[12]While S_D might not depend on interest rates (e.g with log utility), the MPB always does.

$$\mathrm{E}(a_i) = \frac{S_D[w-(r-n)b]}{1-S_D X_s} \tag{14}$$

$$\mathrm{Var}^{\mathrm{P}}(a_i) = \frac{\left[S_D[w-(r-n)b]\right]^2 \sigma^2}{1-(S_D X_s)^2} \tag{15}$$

$$\mathrm{Var}^{\mathrm{F}}(a_i) = \frac{(S_D w)^2 \sigma^2}{1-(S_D X_s)^2} \tag{16}$$

$$\mathrm{Var}^{\mathrm{A}}(a_i) = (1-\tau^A)^2 \mathrm{Var}^{\mathrm{F}}(a) \tag{17}$$

where in all cases above, I substituted for taxes using (10), (11) and (12) respectively. Note that in the "Progressive" tax system and in contrast to the rest, debt policies affect directly the dispersion of wealth (equation (15)). Second, an "Affine" tax instrument only rescales the variance of a flat tax economy (equation (17)). Therefore, the choice for the marginal tax rate only parametrizes the level of inequality and it is irrelevant for the qualitative effects[13]. Third, since the mean for all these economies is the same by construction, to compare the levels of wealth inequality between them is just sufficient to compare the variances. It turns out that if the economy is dynamic inefficient ($r < n$), the "Progressive" tax system will produce more wealth inequality. This seems intuitive, since the rich are more heavily subsidised. Note also that the wealth dispersion in affine tax economy will be higher relative to "Progressive" one, *iff* the marginal tax rate is not "too low" relative to the interest rate[14].

Finally, from (14), (17), (16) and (15) the stationary coefficient of variation for each case equals:

[13] If instead, the free parameter was set to be the T^A, the respective measure would had followed that of the "Progressive" tax system.

[14] Comparing the variances someone has to sign the term $S_D(\tau^A w) - (r-n)b$, if positive this implies that the "progressive" tax system has higher inequalities than the "Affine" one, otherwise is the opposite.

$$\mathrm{CV}^{\mathrm{P}} = \sigma\left(\sqrt{\frac{S_D[w-(r-n)b]}{(1+S_D X_s)}}\right) \tag{18}$$

$$\mathrm{CV}^{\mathrm{F}} = \sigma\left(\frac{S_D w}{\sqrt{(1+S_D X_s)}\sqrt{S_D[w-(r-n)b]}}\right) \tag{19}$$

$$\mathrm{CV}^{\mathrm{A}} = \sigma(1-\tau)\mathrm{CV}^{\mathrm{A}} \tag{20}$$

Note that the level of inequality is an implicit function of capital and debt. Therefore, macroeconomic shocks are of first-order importance in "shifting" the distribution of wealth (i.e. a change in inequality levels). It is necessary to put those into context, since at the macro level, the effect on inequality is essentially attributed to simultaneous changes in the mean and the variance.

4. The Macroeconomic environment

The equilibrium in financial markets will determine the macroeconomic state. The particular clearing condition, requires capital and bonds to compete for the average savings of the "young". For the large economies, average savings equal the mean assets, which in turn equal the savings of the "representative-mean" individual. Therefore, the financial markets clearing condition in per capita terms is:

$$\mathrm{E}(a^i_{t+1}) \equiv (1+n)(k_{t+1}+b) = S_t(D_t, r_{t+1}) \tag{21}$$

where $S(D_t, r_{t+1})$ is the optimal mean saving function (See (6)), $D_t = w_t - (r-n)b + X(r_t, S_{t-1})$ denotes the mean disposable income and $X(r_t, S_{t-1}))$ is the mean bequest function (See (7)), all in period t. For the macroeconomic state, only the mean taxes matter (i.e. the taxes the individual with the mean "skill" faces). As a consequence, for all of our three economies, the aggregate behaviour is governed by the same law of motion (21). For the specific law of motion, the stability condition is:

$$0 < \left.\frac{dk_{t+1}}{dk_t}\right|_{db=0} = \frac{S_D\Big[-(k+b)f'' + X_r f'' + (1+n)X_s\Big]}{[(1+n) - S_r f'']} < 1 \tag{22}$$

To gain some further insights from (22), this can alternatively be written as:

$$C_4 \equiv S_D X_s < 1 - \underbrace{\left(\frac{\left[S_r + S_D[(\frac{1}{Z+1} - 1)(k+b) \right]}{1+n} \right) f''}_{\Pi} \tag{23}$$

where $X_r \equiv \frac{\partial X(r_t, S_{t-1})}{\partial r_{t+1}} = \frac{\frac{S(D,r)}{1+n}}{Z+1} = \frac{(k+b)}{Z+1}$, and $Z = \frac{\delta U''(x_{t+1})}{(1+n)^2 \beta U''(c_{t+1})} > 0 \Rightarrow \frac{1}{Z+1} < 1$. In the "normal" case where $\frac{\partial S(D,r)}{\partial r} \equiv S_r < 0$, the sign of Π is positive for any (non-negative) debt level[15]. In consequence, for *macroeconomic or financial stability* a stronger restriction on $C_4 = S_D X_S$ is necessary. For example, in the special case of a logarithmic utility function $S_r = 0$. In that particular case, the stability condition implies a more definite upper bound $\overline{C(b)}$ (controlled by debt policy) such as the *average* intergenerational wealth persistence which should not only be less than one but also less than a specific threshold, i.e. $C_4 < \overline{C(b)} < 1$.

The main message of the paragraph above, deems financial stability to be *jointly* determined by the amount of debt and the level of inequality. More specifically, in the most plausible case ($S_r < 0$), the microeconomic condition for stationary distribution is only *necessary* and not *sufficient* for macroeconomic stability. Loosely put, "average *private* altruism" is detached from the "average *social* altruism" and debt policies can affect the distance between them, not necessarily directly but indirectly through its effect on marginal propensity to bequeath[16]. In what follows, I will assume that the

[15] $S_r < 0$ implies that any increase in the supply of assets will decrease interest rates for some given asset demand. In conventional models with CES (CRRA) utility function S_r is negative when the inter-temporal elasticity of substitution is less than one. Nonetheless, the case of $S_r < 0$ does not rule out "sunspot" behaviour (i.e. multiple steady states). See Galor and Ryder (1989). Therefore, in the numerical exercises, I will assume local analysis.

[16] In completely different settings and concerns, Farhi and Werning (2007) show the existence of long-run (consumption and welfare) distributions when the social discounting (altruism) is different from the private. However, in their paper, this separation was rather assumed on the basis of some welfare function. The existence of public debt, as the main text shows, might help to rationalize this separation.

steady state is stable and $S_r \leq 0$.

In the stationary state, the equilibrium in financial markets is:

$$\mathrm{E}(a) = \frac{S_D[w-(r-n)b]}{1-S_D X_s} \equiv (1+n)(k+b) = S(D,r) \tag{24}$$

Note from equation (24) that what matters for the supply of funds is the fraction of mean savings assigned to finance future consumption net of bequests (i.e. the egoistic MPS). When this feature is taken into consideration, then the model resembles that of Diamond (1965). To assess the effects of debt on capital, it is more convenient to totally differentiate the $(1+n)(k+b) = S(D,r)$ equality. This equals:

$$\frac{dk}{db} = \frac{S_D \overbrace{[\overline{X_S}-1]}^{(-)} f' + \overbrace{(S_D-1)}^{(-)}(1+n)}{\underbrace{(1+n-S_r f'')(1-\frac{dk_{t+1}}{dk_t}\big|_{db=0})}_{Q>0}} < 0 \tag{25}$$

where $Q = (1+n-S_r f'')(1-\frac{dk_{t+1}}{dk_t}\big|_{db=0}) > 0$ is positive from the stability condition and the numerator negative, since $0 < S_D, \overline{X_S} < 1$. Therefore, higher public debt always crowds out capital and increases the interest rates. Under the assumption of $S_r < 0$, the "income effect" dominates the "substitution effect" and in equilibrium, the rise in interest rate will depress *average* (mean) savings. The same occurs if $S_r = 0$ (i.e. when the utility is logarithmic)[17]. The result of the fall in average wealth, indicates that debt will crowd out capital by more than one. The implication of this statement is that average wealth falls. Thus, the direct macroeconomic effects of a positive debt shock tends to increase wealth inequality. However, the equilibrium effects on inequality also depend on the change in the variance. The next section takes this issue more closely.

[17]See Bertola et al. (2006, Chapter 5)

5. The Effects on Inequality

5.1. The "Mechanics" at the Macroeconomic level

To motivate the discussion on the mechanics of the model, the total change in inequality in the stationary state is decomposed as:

$$dCV(Var(a), E(a)) = W_1 dVar(a) + W_2(dE(a)) \Rightarrow \tag{26}$$

$$= W_1 dVar(a) + W_2(1+n)(dk + db) \Rightarrow \tag{27}$$

$$\frac{dCV}{db} = \underbrace{\underbrace{W_1}_{+} \frac{dVar(a)}{db}}_{\text{Variance effect}} + \underbrace{\underbrace{W_2(1+n)}_{-}(\frac{dk}{db} + 1)}_{\text{Mean Effect}} \tag{28}$$

The formulation above reveals the "mean" and "variance" effects akin to the one described by Bossmann et al. (2007), where $W_1 = \frac{\partial CV}{\partial Var}$ and $W_2 = \frac{\partial CV}{\partial E(a)}$ are the relevant contributions of each separate change in the mean and the variance. The mean is affected by the extent of crowding out, while the variance -as I will discuss later- by the equilibrium changes in savings behaviour.

In equation (28), the "mean effect" is positive since debt crowds out capital by more than one (i.e. $\frac{dk}{db} + 1 < 0$). In this instance, average wealth falls and therefore inequality tends to increase for unchanged variance. Nevertheless, individual savings responses will also affect the variance, due to the perturbation in factor prices. When for example, the variance drops, inequality tends to decrease. These two effects might compete with each other with an ambiguous total effect.

In our context, the debt shock will trigger factor price changes through the standard "OLG-Diamond model" mechanism. In short, when the *average* savings *net of government bonds* falls, the equilibrium interest rates will increase. In return, the change in factor prices will determine savings behaviour and thus the effect on the variance. The perturbation on the variance is crucially appended on factor price changes. In fact, it is the silent features of those *general equilibrium effects* that could possibly reverse the effects on wealth inequality. If those are not in operation, any conclusions will be overturned.

To gain some further insights for the mechanics at the macro level, I totally differentiate $C\hat{V}^P \equiv \frac{(CV^P)^2}{\sigma^2}$ in the case of the "Progressive Tax" economy, and $C\hat{V}^F \equiv \frac{(CV^F)^2}{\sigma^2}$ for the "Flat" tax economy[18]. For the latter economy, equation (19) can be restated in a functional form as $C\hat{V}^F = \Phi^F(C_3, C4, C_5)$, where $C_3 = S_D w,\ C_4 = S_D X_S,\ C_5 = S_D(r-n)b$ and $\Phi^F = \frac{C_3^2}{(1+C_4)(C_3-C_5)}$. By total differentiation, someone gets:

$$d\hat{CV}^F = \underbrace{\Phi_3^F}_{+} dC_3 + \underbrace{\Phi_4^F}_{-} dC_4 + \underbrace{\Phi_5^F}_{+} dC_5 \tag{29}$$

or equivalently,

$$\frac{d\hat{CV}^F}{db} = \underbrace{\left[\underbrace{\Phi_3^F}_{+}\underbrace{[w\Delta - S_D k]}_{-} + \underbrace{\Phi_4^F}_{-}\underbrace{\Gamma}_{?}\right]}_{\substack{\text{Average change}\\ \text{in Life-Cycle considerations}}}\frac{dr}{db} + \underbrace{\underbrace{\Phi_5^F}_{+}\overbrace{\left[\underbrace{E}_{+} b\frac{dr}{db} + \underbrace{\Phi_5^P}_{-} S_D(r-n)\right]}^{+}}_{\substack{\text{Average effect}\\ \text{of Taxation}}} \tag{30}$$

where the $phi's$ are $\Phi_j^F \equiv \frac{\partial C\hat{V}^F}{\partial C_j}$ for j=[3,4,5], and Γ, Δ and E are terms defined in Appendix B. In Table 1 below we can see the necessary and sufficient conditions to sign those. In the case of a logarithmic utility, the Γ and E terms are always positive and $\Delta = 0$ (See Appendix B for the proofs). For any other utility function, I will assume that $\Gamma < 0$ and $E > 0$. In other words, I implicitly assume that the level of debt is not "too high" and consumers have plausible marginal propensities to save [19].

From equation (29), the total change in wealth inequality is attributed to three main factors. First, it depends on how much more or less individuals will save out of their new wage (the dC_3 term). As soon as the level of debt is not too high, to avoid any "immiseration" from very low wages, the "rich" can save more than the "poor" and therefore inequality tends to increase (the $\Phi_3^F > 0$ term). Second, a change in debt will affect the fraction of "parent's"

[18] For the "Affine" tax economy, the equivalent expression is $\frac{(CV^A)^2}{((1-\tau^A)\sigma)^2}$. Since the qualitative effects are the same up to some constants, I will reserve the discussion for the "Flat tax" case only.

[19] See for example Jappelli and Pistaferri (2012) and the literature therein.

		Conditions
$\mathbf{\Gamma}$	(+)	$S_D > 0.5$
	(-)	$S_D < 0.5$
E	(+)	DI or $S_D \geq 0.5$ or $S^*_D < S_D < 0.5$
	(-)	$S_D < S^*_D < 0.5$

DI=Dynamic Inefficiency. The special cases are: $S_D = \frac{1}{2} \Rightarrow \Gamma = 0$, $b = \frac{w}{2(r-n)} \Rightarrow \Phi_3^F = 0$ and $S_D = S^* < 0.5 \Rightarrow E = 0$. S^*_D is a particular threshold which equals to $S^* = \frac{1}{2} - \frac{1}{2}\frac{1+n}{f'}$ and is assumed to be positive. In the case of logarithmic utility $\Gamma, E > 0$ always

Table 1: Main Conditions

savings that are "bequeathed" on top of "children's wealth (the dC_4 term). On average, leaving positive bequests tends to have an equalizing effect (the Φ_4^F term). Finally, a permanent increase in government borrowing will result to higher taxation (the dC_3 term), but as soon as taxes are of "regressive" nature they will tend to amplify the inequality levels (the Φ_5^F term).

In general, the effects on inequality are ambiguous. In particular, the source of ambiguity stems from the competing effects of the life-cycle and bequest motives (the $\Phi_3^F[w\Delta - S_D k] + \Phi_4^F\Gamma$ terms). Since these motives cannot be isolated at the individual level, as a result they also appear at the macro level. In the subsequent sections, where the individual behaviour is analysed, the "rich" and the "poor" will have opposite qualitative responses on these two saving scopes.

Similarly, equation (18) can be restated as $C\hat{V}^P = \Phi^P(C_3, C_4, C_5)$ and by total differentiation the total change is:

$$d\hat{CV}^F = \underbrace{\Phi_3^P}_{+} dC_3 + \underbrace{\Phi_4^P}_{-} dC_4 + \underbrace{\Phi_5^P}_{-} dC_5 \tag{31}$$

or equivalently,

$$\frac{d\hat{CV}^P}{db} = \underbrace{\left[\underbrace{\Phi_3^P}_{+}\underbrace{[w\Delta - S_D k]}_{-} + \underbrace{\Phi_4^P}_{-}\underbrace{\Gamma}_{?}\right]}_{\substack{\text{Average change}\\ \text{in Life-Cycle considerations}}}\frac{dr}{db} + \underbrace{\underbrace{\Phi_5^P}_{-}\overbrace{\left[\underbrace{E}_{+} b\frac{dr}{db} + \underbrace{\Phi_5^P}_{-} S_D(r-n)\right]}^{+}}_{\substack{\text{Average effect}\\ \text{of Taxation}}} \tag{32}$$

where the $phi's$ are $\Phi_j^P \equiv \frac{\partial \hat{CV}^P}{\partial C_j}$ for j=[3,4,5]. The analysis is similar to the one before. The only caveat is that while wages before-tax have an unequalising effect (the Φ_3^P term), the "progressivism" of the tax system tends to offset it (the Φ_5^P term).

Example: Consider the case of the proportional tax system. If preferences are logarithmic ($\Gamma > 0$, $\Delta = 0$), it follows from equation (32) that higher public debt will decrease inequality. In the case of a flat tax system, this is only true if the economy is dynamic inefficient. If not, it requires the average change in life-cycle consideration to dominate the "wealth effects" from taxation.

Notice also that under a "partial equilibrium analysis" (captured in $\Phi_5^j S_D(r-n)$ terms for j=[P,F]), the effects on inequality will follow what the tax instrument is supposed to do by design. However, the general equilibrium "feedback", due to price changes, might overturn this and in fact the effect on wealth inequality can go in either direction. It is proper therefore, not only to understand what will be observed at macro-level, but also what will trigger at micro-level a debt shock.

5.2. The "Mechanics" at the individual level

The change in the variance, while it seems of quantitative nature, we can still filter out some interesting qualitative properties. In general, to calculate the variance of wealth, it is sufficient to know the asset position of each individual. In our simple model, in order to understand the change in the variance, we should know the equilibrium savings responses. In this context, where altruism is taken into account, individuals will program their savings responses on the basis of their "egoistic needs" and their "altruistic liabilities". These two scopes can further decompose the change in wealth (and thus in the variance) into the "egoistic" and "altruistic" part respectively.

To begin the exposition, I will first consider a "Flat" tax system. In the stationary state $x^i_{t+1} = x^i_t = x^i$ (and at the macro-level $k_{t+1} = k_t = k$) substituting (7) in (6) and using (10) in $D^i = w_t l^i - \tau^F + x^i$, individual savings are equal to:

$$a^F_i = \frac{S_D}{1 - S_D X_S} w l^i - \frac{S_D}{1 - S_D X_S}(r - n)b \Rightarrow \tag{33}$$

$$a^F_i = E(a) + \frac{S_D}{1 - S_D X_S} w(l^i - 1) \Rightarrow \tag{34}$$

Equation (34) confirms that better endowed individuals are wealthier. Taxes in this case are only implicit to the individual decision making and what in effect matters is the fraction of wages someone can save for his own consumption; the $\frac{S_D}{1-S_D X_S} w(l^i - 1)$ term[20]. In fact, this term will determine the change in the variance. To see this, restate the *egoistic* MPS times the wage in functional form as $\Psi(C_3, C_4) = \frac{C_3}{1-C_4}$, where $C_3 = S_D w$ and $C_4 = S_D X_S$ as defined before. From equation (34), the total change in individual wealth becomes:

$$da^i = dE(a^i) + \left[\underbrace{\Psi_3}_{+} dC_3 + \underbrace{\Psi_4}_{+} dC_4 \right] (l^i - 1) \Rightarrow \tag{35}$$

or equivalently,

$$\frac{da^i}{db} = \underbrace{(1+n)(\frac{dk}{db} + 1)}_{\equiv \frac{dE(a)}{db}} + \left[\underbrace{\underbrace{\Psi_3}_{+} \underbrace{(w\Delta - S_D k)}_{-} (l^i - 1)}_{N_1} + \underbrace{\underbrace{\Psi_4}_{+} \underbrace{\Gamma}_{-} (l^i - 1)}_{N_2} \right] \underbrace{\frac{dr}{db}}_{+} \tag{36}$$

[20]Note, the model can also be interpreted as one where individuals differ in their MPS, i.e. the $S_D l^i$ term, but the distribution of l^i is such that the average MPS is S_D. For example, this is rationalized if agents differ in their discount factors and therefore the more impatient individuals end up having more wealth. In fact, the spirit of the model might be closer to this interpretation, however the convention in the literature considers the interpretation given in the main text as more appropriate.

where C_3, C_4, $\Delta < 0$ and Γ are terms discussed before and $\Psi_i = \frac{\partial \Psi(C_3, C_4)}{\partial C_i}$ for $i = [3, 4]$. In the most plausible case where the MPS is $S_D < 0.5$, it implies $\Gamma < 0$ (See also Table 1). From equation (35), the perturbation in individual wealth is decomposed into two parts; an aggregate (which essentially comes from the change in factor prices) and an idiosyncratic one (which essentially comes from the different endowments). On the one hand, a debt shock will affect everyone equally (the $\frac{d(E(a))}{db}$ term). On the other hand, individuals will adjust their savings (an intertemporal choice) and in parallel decide how much to bequeath from their investments (an inratemporal choice). In other words, consumers will save a *different* fraction of their *new wage* (the dC_3 term), and at the same time will alter the fraction gone to bequests (the dC_4 term). By looking at equation (36), the "rich" consumers will save and bequeath less (the $N_1 < 0$ and $N_2 < 0$ terms respectively) and the "poor" more. Thus, when a debt shock occurs, the "idiosyncratic" and "aggregate" components operate in the same direction for the "rich" but not for the "poor". In consequence, the wealth of the "rich" would fall but remain qualitatively uncertain for the other group.

Put simply, a rise in interest rates will induce the usual income and substitution effects. Nevertheless, those should be considered in terms of future expenditures (that is, future consumption and the bequest transfer). At the same time, the change in interest rate will affect the intratemporal choice. On the one hand, more bequests can be transferred due to the increase in X_S, whereas on the other hand, less can be bequeathed due to the effect in S_D. For people with different endowments, the intratemporal choice is not the same. Thus, the savings behaviour of the "rich" is aligned with the macroeconomic effect and their wealth falls. In this case, the extended to altruism "income effect" dominates the "substitution effect" and will reinforce the "wealth effect". For the "poor", the idiosyncratic component competes with the aggregate one, hence their equilibrium response is ambiguous.

However, if the utility is logarithmic ($\Delta = 0$ and $\Gamma > 0$), this implies that the change in savings will be ambiguous for either groups. In this case, the "rich" will now want to bequeath more and save less, whereas the "poor" will want to do the opposite. Nonetheless, as I show in Section 6, the relatively "rich" will still dis-save, which implies that the "egoistic" component (the N_1 term) will dominate the altruistic one (the N_2 term). Possibly, this is because future consumption is valued more. In this case though, the fraction of own investments that go to bequests will (unambiguously) increase on average ($dC_4 \uparrow$). This average increase, in turn, boils down to the special case analysed by Bossmann et al. (2007), where higher bequests induce the aforementioned "mean" and "variance" effects on the macroeconomic level.

For the "Affine tax", the analysis is analogous. To see this, note that the stationary individual wealth can be written as:

$$a_i^A = a_i^F - \tau^A \left[\frac{S_D}{1 - S_D X_S} w(l^i - 1) \right] \tag{37}$$

$$\frac{da_i^A}{db} = \frac{da_i^F}{db} - \tau^A \frac{\left[\frac{S_D}{1-S_D X_S} w(l^i - 1) \right]}{db} \tag{38}$$

Therefore, up to some constant determined by the marginal tax rate, the qualitative pattern remains identical[21].

For the "progressive" tax regime using equation (11), the disposable income is $D^i = (1 - \tau^p) w l^i + x^i$. As in the earlier procedure, the individual wealth equals:

$$a_i^P = \left(\frac{S_D(w - (r - n)b)}{1 - S_D X_S} \right) l^i = E(a) l^i \Rightarrow \tag{39}$$

$$a_i^P - E(a) = E(a)(l^i - 1) \tag{40}$$

And the effect of debt on individual wealth is:

$$\frac{da^i}{db} = \frac{dE(a)}{db} l^i = \left[(1 + n)(\frac{dk}{db} + 1) \right] l^i \tag{41}$$

From equation (39), each individual holds a fraction of total wealth which is in proportion of his endowment. In consequence, the effect of debt on

[21]If instead the free parameter for the "Affine tax" economy was T^A, stationery individual wealth would alter to $a_i^A = a_i^P - \left(\frac{S_d}{1-S_D X_S} \right) T^A$. The total change in savings would be $\frac{da_i^A}{db} = \frac{a_i^P}{db} - \frac{T^A}{1-S_D X_S} \left[\Delta + (\frac{S_D}{1-S_D X_S}) \Gamma \right] \frac{dr}{db}$. As soon as $0 < T^A < 1$, the qualitative pattern is similar to the one shown in Section 6 for the proportional tax.

individual savings will also be in the same proportion (See equation (41)). From the previous analysis, we know that higher public debt will decrease average wealth. Therefore, equation (41) also implies a negative impact on individual wealth. In particular, the effect will be stronger for the relatively "rich" and milder for the relatively "poor". However, the change in variance is still ambiguous, but of quantitative nature, since all agents are dis-saving. Nonetheless, some hidden qualitative differences still exist.

As earlier, any change in the dispersion of asset holdings can be decomposed into the part coming from own savings and the part stemming from the altruism of parents. This decomposition is interesting on its own. For the "Flat" or "Affine" tax economy, the behavioural elements of those were clear. But even in this case, a similar decomposition is still possible. By adding and subtracting the relevant terms, the asset holdings in the "proportional" economy can be written in terms of the flat tax economy as:

$$a_i^P = a_i^F - \frac{C_5}{1 - C_4}(l^i - 1) \tag{42}$$

where $C_5 = S_D(r - n)b$ and $C_4 = S_D X_S$. As expected, equation (42) confirms that the "rich" own less assets whereas the "poor" more, relative to a regressive tax system. Defining $\Psi^P = \frac{C_5}{1-C_4}$, using equation (36) and collecting terms, the effect of debt on asset holdings becomes:

$$\frac{da_i^P}{db} = \underbrace{\frac{dE(a^i)}{db}}_{-} + \Bigg[\underbrace{\underbrace{\Psi_3}_{+}\underbrace{(w\Delta - S_D k)}_{-}}_{N_1^P} + \underbrace{\underbrace{\overline{\Psi}_4}_{+}\Gamma}_{N_2^P}\Bigg]\underbrace{\frac{dr}{db}}_{+}(l^i - 1) - \underbrace{\Psi_5^P}_{+}\underbrace{\left[Eb\frac{dr}{db} + S_D(r-n)\right]}_{N_3^P > 0}(l^i - 1) \tag{43}$$

where $\overline{\Psi}_4 = \Psi_4 - \Psi_4^P = E(a^i)$, and $\Psi_i^P = \frac{\partial \Psi^P}{\partial C_i}$ for i=[4,5]. The N_3^P term is positive because of the change in taxes[22]. Equation (43) is a simple reformulation of (41), so all individual will have to dis-save in the end. Nonetheless, the different behavioural motives, as in the "Flat" tax economy, are still present (the second term in (43), with the $N_{3,4}^P$ replacing the $N_{1,2}$ terms). However, the "progressivism" of the tax system modifies the

[22] I assume that this is also true even in the dynamic inefficient case. Under a particular bound on debt, this is always the case.

personal "wealth effects" (the third term in (43)) and thus the incentive to save or dis-save. For instance, the "rich" expecting to bear higher taxes have an incentive to save more, while the "poor" less. In summary, heterogeneity itself becomes sufficient to generate different qualitative behaviour (in the presence of a debt shock) by interacting with the altruism of individuals. The tax instrument then, will only determine the nature of the change in inequality (i.e. whether this will be of quantitative or qualitative nature due to different savings behaviour).

6. Calibration and Main Results

As already mentioned in the text, the analysis is restricted to the stationary state, where $S_r \leq 0$. In this model debt is exogenous, therefore it will characterize the values of capital and the interest rates. The main concern for the numerical exercises is to satisfy the stability conditions and the stationarity for wealth accumulation (See equation (22)).

The results below rely on equation (25) to assess the effect of debt on capital, on equations (30) and (32) to assess the effects on wealth inequality, and on equations (41), (36) and (38) to assess the individual savings behaviour. To figure out the effects on the mean and the variance, I evaluate the $(1+n)(\frac{dk}{db}+1)$, $\frac{dVar^P}{db}$, $\frac{dVar^F}{db}$ and $\frac{dVar^A}{db}$ derivatives.

I also assume a CEIS ("CRRA") utility function of the form: $V = \frac{c_t^{1-\theta}-1}{1-\theta} + \beta\left(\frac{c_{t+1}^{1-\theta}-1}{1-\theta}\right) + \delta\left(\frac{x_{t+1}^{1-\theta}-1}{1-\theta}\right)$, where the choices for the inverse of the intertemporal elasticity of substitution θ are 1.5 and $\theta = 1$ (i.e. logarithmic utility). The production function in its intensive form is assumed to be Cobb-Douglas, $y = Ak^{\gamma}$, with $A = 9.37$, $\gamma = 0.3$. Population growth, n, is set to 1.81 to match the average post war growth rate in the US. The choices for the discount factor, β and the degree of altruism δ, are set to 0.3 and 0.10 respectively, when the value for debt is 0.05. This is the case of a dynamic efficient economy where it ensures that all conditions of the model are satisfied. Similarly, I set $\beta = 0.4$ and $\delta = 0.15$ with $b = 0.11$ for the dynamic inefficient case. The values for the discount factor are motivated by the RBC literature. There, $\beta = 0.99$ and each period represents a quarter. In my calibration, this is similar to consider the time period as 30 years in the first case and around 25 years in the second one. The choice for the degree of altruism is essentially arbitrary but closely follows that of Bossmann et al. (2007). In all cases, the MPS is restricted to be less than half. Finally, the choice for the marginal tax rate in the "Affine" tax economy is taken from Bhandari et al. (2013) and set to $\tau^A = 0.2$.

In Table 2 we can see the results obtained for our economy. At the macroeconomic level and in all cases, the mean and variance fall. This tends to move the inequality to opposite direction. Nevertheless, the "variance effect" dominates the "mean effect" and inequality is suppressed. But, the individual mechanics (between the tax systems) are different.

	Dynamic Efficiency			**Dynamic Inefficiency**		
Endowment	**Flat**	**Affine**	**Prop.**	**Flat**	**Affine**	**Prop.**
0.2	3.20	2.14	-0.42	2.97	2.01	-0.37
0.3	2.54	1.61	-0.63	2.37	1.53	-0.55
0.4	1.88	1.08	-0.84	1.77	1.05	-0.73
0.7	-0.11	-0.51	-1.47	-0.03	-0.39	-1.28
1	-2.10	-2.10	-2.10	-1.83	-1.83	-1.83
2	-8.73	-7.40	-4.20	-7.84	-6.64	-3.67
5	-28.62	-23.32	-10.50	-25.87	-21.06	-9.17
10	-61.77	-49.84	-21.00	-55.92	-45.10	-18.35
Mean effect	(+)	(+)	(+)	(+)	(+)	(+)
Change in Variance	(-)	(-)	(-)	(-)	(-)	(-)
Inequality	(-)	(-)	(-)	(-)	(-)	(-)

Mean effect $= W_2(1+n)(\frac{dk}{db}+1)$, with $W_2 < 0$, Variance effect $= W_1 \frac{dVar(a^i)}{db}$, with $W_1 > 0$. Each column describes the savings response for the different taxes.

Table 2: Main Results

In the "progressive" tax regime, everyone's wealth falls proportionally, as discussed in Section 5.2. However, in the rest of the cases, there is a clear qualitative difference between different groups of people. In fact, the "poorer" someone is the more is willing to save. Thus, the "poor" in this context seem to behave as "Ricardians"[23]. The intuition for the behaviour of the poor is the following: The "poorest" individuals (who had already

[23]See Laitner and Ohlsson (2001) for a clear exposition on the failure (at least on average) of the Ricardian equivalence under a "joy-of-giving" bequest motive. Moreover, the positive association between debt and the savings behaviour of the "poor" was also present in Heathcote (2005) .

low wages prior to the shock) would see their real wage to further decline and their tax liability (regressive in nature) to increase. Unless they start saving more, by exploiting the rise in the rate of interest, they will not be in a position to finance their future consumption *net of bequests*. In other words, the (extended to altruism) substitution effect is sufficiently strong to dominate both the income and the wealth effects. This is not the case for the "rich", where the income effect always dominates. In "Affine" tax system, as soon as the choice for the tax adjustment is on the common tax component, individual behaviour will follow that of a flat tax economy. However, since not everyone will pay the exact equal amounts in taxes, the quantitative response will be different. The same analysis goes through for the dynamic inefficient economies. Nevertheless, in this case and in contrast to the previous one, the "equity-efficiency" trade-off breaks down, which extends Diamond's original contribution.

In Table 3, we see the results obtained for the logarithmic utility. As someone can observe, the qualitative pattern is similar to the one described above and follows the discussion in Section 5.2

	Dynamic Efficiency			Dynamic Inefficiency		
Endowment	**Flat**	**Affine**	**Prop.**	**Flat**	**Affine**	**Prop.**
0.2	3.62	2.57	-0.33	2.40	1.77	-0.15
0.3	2.96	2.04	-0.49	2.00	1.45	-0.23
0.4	2.30	1.51	-0.65	1.61	1.14	-0.30
0.7	0.33	-0.06	-1.15	0.43	0.19	-0.53
0.8	-0.32	-0.59	-1.31	0.03	-0.13	-0.61
0.9	-0.98	-1.11	-1.47	-0.36	-0.44	-0.68
1	-1.64	-1.64	-1.64	-0.76	-0.76	-0.76
2	-8.20	-6.89	-3.27	-4.71	-3.92	-1.52
5	-27.90	-22.64	-8.18	-16.55	-13.39	-3.79
10	-60.72	-48.90	-16.36	-36.28	-29.18	-7.59
Mean effect	(+)	(+)	(+)	(+)	(+)	(+)
Change in Variance	(-)	(-)	(-)	(-)	(-)	(-)
Inequality	(-)	(-)	(-)	(-)	(-)	(-)

Mean effect $= W_2(1+n)(\frac{dk}{db}+1)$, with $W_2 < 0$, Variance effect $= W_1 \frac{dVar(a^i)}{db}$, with $W_1 > 0$. Each column describes the saving response for the different taxes.

Table 3: Logarithmic Utility

7. Conclusions

This paper offers a simple illustration on how debt policies affect wealth inequality when a reduced form of altruism is taken into account. In particular, I show a case where agents have different qualitative saving responses based on the sole features of heterogeneity in endowments and a "joy-of-giving" bequest motive. The different savings behaviour emanates from the life-cycle motives and more specifically from the different altruistic attitude between households. Consequently, it might be more prudent for the effects of debt on inequalities to be analysed within a framework where altruism is explicitly modelled.

Nevertheless, the findings here are in accord with Floden (2001) where the same negative long-run relationship between government debt and wealth inequality is also present. This might suggest that the role of heterogeneous endowments might be more important than the incompleteness of the financial markets for the effects of debt on asset accumulation, and hence on wealth inequality.

To highlight the last point, this paper documents the ambiguous savings behaviour of the "poor" (See also footnote 23). This feature, implies that the choice of the tax instrument influences solely the nature of the change in inequality. For instance, when the savings behaviour among individuals differs qualitatively (which occurs when taxes are regressive), public debt behaves as a redistributive device within the population. Debt adjustments, in this case, might not only clash with demands to reduce inequality but also redistribute wealth from some fraction of the population to the other. In our example, the redistribution of debt adjustments will occur from the "poor" to the "rich"[24].

Finally, the negative correlation between public debt and wealth inequality, seems to be also supported by some preliminary empirical work in the literature. More specifically, some evidence on the effects of pro-cyclical fiscal policies (i.e. proxy to an exogenous debt shock) on income inequality is provided by Vegh and Vuletin (2014). Their findings indicate that procyclical fiscal policies tend to exacerbate income inequality (a measure which is highly correlated with wealth inequality (OECD (2009)). Thus, the theoretical results here seems to have some validity in practise. However, those should only interpreted as indicative and certainly more work is needed in this direction.

To conclude, the results of the effects of debt on wealth inequality rely crucially on the general equilibrium effects, that is the indirect effects on the variance due to the change in the interest rate. Recent research by Laubach (2009) seems to confirm that budget deficits do affect the rate of interest. However, as I analysed in the main text, the scale of crowding out on capital also matters. In this respect, the assumption of a closed economy might be important.

[24] More generally, it is very easy then to note that the shape of distribution might matter as agents can be distinguished by their types, that is their endowments.

A. Proofs

A.1. Proof for $\Delta < 0$

Proof From $dC_3 = S_D dw + w dS_D$. Noting that S_D is a function of the interest rate and the MPB, then $dS_D = S_{Dr}dr + S_{Dx}dX_S$. Since X_S is a function of the interest rate, we also have $dX_S = X_{Sr}dr$. Therefore, $dS_D = S_{Dr}dr + S_{Dx}X_{Sr}dr$. Define, $\Delta = S_{Dr} + S_{Dx}X_{Sr}$, substituting the expressions for all terms and using the fact that $(1+n)X_{Sr} = \overline{X_s}$, someone gets $\Delta = \frac{2S_{Dx}}{1+n}[\overline{X_s} - 1] < 0$ which is unambiguously negative. If the utility is logarithmic then $dS_D = 0 \Rightarrow \Delta = 0$. For the definitions of S_{Dr}, S_{Dx} and X_{Sr} see Appendix B. For further details see On-line Appendix ■.

A.2. The conditions on Γ

Proof From $dC_4 = X_S dS_D + S_D dX_s$, using $dS_D = S_{Dr}dr + S_{Dx}dX_S$, then $dC_4 = [X_S\Delta + S_D X_{Sr}]dr$. Define $\Gamma = X_S\Delta + S_D X_{Sr}$. Using, $X_{Sr} = \frac{\overline{X_s}}{1+n}$ and the definition for $\overline{X_s}$, we have $\Gamma = X_S\left[\Delta + \frac{S_D}{f'}\right]$. So, if the utility is logarithmic $\Delta = 0 \Rightarrow \Gamma > 0$. Otherwise, Γ can be of either sign. In particular, $\Gamma > 0$ *iff*:

$$\Delta + \frac{S_D}{f'} > 0$$

$$\text{Using the definition for } \Delta = \frac{2S_{Dx}}{1+n}[\overline{X_s} - 1]$$

$$S_D > \frac{2S_{Dx}}{1+n}[1 - \overline{X_s}]f'$$

$$1 > 2S_D Z_1$$

Where in the last steps I used $S_{Dx} = (S_D)^2\beta\frac{U''(C_{t+1})}{U''(C_t)}(1+n)f'$ and $S_D = \frac{1}{Z_1+1}$ with $Z_1 = \beta\frac{U''(c_{t+1})}{U''(C_t)}[1 - \overline{X_s}](f')^2$. Therefore, using $S_D = \frac{1}{Z_1+1}$, $1 > 2S_D Z_1 \Rightarrow S_D > \frac{1}{2}$. The conditions in the main text follow ■.

A.3. The conditions on E

Proof From $dC_5 = S_D d(r-n)b + (r-n)b dS_D$ and using $\Delta = -\frac{2}{f'}S_D(1-S_D)$, $dC_5 = S_D d(r-n)b + [(\frac{S_D}{f'} + \Delta)f' - (1+n)\Delta]bdr$. Define $E = (\frac{S_D}{f'} +$

$\Delta)f' - (1+n)\Delta = S_D + (r-n)\Delta$. So, if the economy is dynamic inefficient this is always positive. Otherwise, some conditions must be put. With some algebra E is also equals to $E = 2S_D[S_D - \frac{1}{2} + \frac{1+n}{f'}(1-S_D)]$. Define, $P = S_D - \frac{1}{2} + \frac{1+n}{f'}(1-S_D)$, thus if $S_D > \frac{1}{2}$ then $P > 0 \Rightarrow E > 0$. Hence more investigation is required for a dynamic efficient economy and $S_D < \frac{1}{2}$. Consider the case $S_D < \frac{1}{2}$ and $f' > 1+n$. Then, $P < 0$ *iff*, $S_D < 1 - \frac{0.5}{1-\frac{1+n}{f'}}$. Thus, if the MPS is less than $S_D < 1 - \frac{0.5}{1-\frac{1+n}{f'}} < 0.5$ then $P < 0 \Rightarrow E < 0$, otherwise, if $1 - \frac{0.5}{1-\frac{1+n}{f'}} < S_D < \frac{1}{2} \Rightarrow P > 0 \Rightarrow E > 0$ $\blacksquare$.

B. Definitions and Notations

$$\mathbf{S}(\text{r,D}) = \text{saving function}$$
$$\mathbf{X}(r, \mathbf{S}) = \text{bequests function}$$
$$S_D = \frac{\partial \mathbf{S}(r, D)}{\partial D} < 1$$
$$= \frac{1}{Z_1 + 1}$$
$$\text{with } \; Z_1 = \beta(1 + r_{t+1})^2 \frac{U''(c_{t+1})}{U''(c_t)}(1 - \overline{X_s})$$
$$S_r = \frac{\partial \mathbf{S}(r, D)}{\partial r} < 1$$
$$X_S = \frac{\partial \mathbf{X}(r, \mathbf{S})}{\partial s}$$
$$\overline{X_S} \equiv \frac{(1+n)X_s}{f'} < 1$$
$$S_{Dr} = \frac{\partial S_D}{\partial r} < 0$$
$$= -(S_D)^2 \left[\beta \frac{U''(c_{t+1})}{U''(c_t)}(2 - \overline{X_s})(1 + r_{t+1})\right]$$
$$S_{Dx} = \frac{\partial S_D}{\partial X_S} > 0$$
$$= (S_D)^2 \left[\beta \frac{U''(c_{t+1})}{U''(c_t)}(1 + n)(1 + r_{t+1})\right]$$
$$X_{Sr} = \frac{\partial X_S}{\partial r} > 0$$
$$= \frac{\overline{X_s}}{1+n}$$

$$\Delta = S_{Dr} + S_{Dx}X_{Sr} =$$
$$= \frac{2S_{Dx}}{1+n}[\overline{X_s} - 1] < 0$$
$$= -\frac{2}{f'}S_D(1 - S_D) < 0$$
$$E = S_D + (r-n)\Delta$$
$$\Gamma = X_s[\Delta + \frac{S_D}{f'}]$$
$$C_3 = S_D w$$
$$C_4 = S_D X_S$$
$$C_5 = S_D(r-n)b$$
$$\Phi_3^F = C\hat{V}^F \left[\frac{2}{C_3} - \frac{1}{C_3 - C_5}\right] > 0$$
$$\textit{iff} \;\; b < \frac{w}{2(r-n)} \;\; \text{or} \;\; r < n$$
$$\Phi_4^F = -C\hat{V}^F(\frac{1}{1 + C_4}) < 0$$
$$\Phi_5^F = -C\hat{V}^F(\frac{1}{C_3 - C_5}) > 0$$
$$\Phi_3^P = \frac{1}{1 + C_4} > 0$$
$$\Phi_4^P = -\frac{C_3 - C_5}{1 + C_4}(\frac{1}{1 + C_4}) < 0$$
$$\Phi_5^P = -\Phi_3^P < 0$$
$$dC_3 = [w\Delta - S_D k]dr$$
$$dC_4 = \Gamma dr$$
$$dC_5 = S_D(r-n)db + Ebdr$$

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