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Abstract. Structural change is associated with high costs for the economy and the society ranging from environmental pollution to unemployment. We focus on the three-sector framework (related to agriculture, manufacturing and services) and assume that the structural change costs increase with the strength of structural change. We show that monotonous structural change paths are minimizing the structural change costs in this framework. By using this result and the (qualitative) stylized facts of structural change based on the theoretical and empirical literature consensus, we derive the cost-minimizing strategy for a developing country. We use these results to discuss some well-known structural/trade strategies.

Keywords: employment, structure, transformation, industrial, structural, policy, costs, long run, geometry, vector, simplex, dynamic optimization, uncertainty, calculus of variations.

JEL: O14, O41, O21, O24, C61

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1. Introduction

1.1 Motivation of the Paper

One of the key characteristics of the long-run development process is structural change as measured by the long-run changes in the sectoral GDP and employment shares. We focus on the three-sector framework dividing the economy into the agricultural, manufacturing and services sector, which has been studied in numerous empirical and theoretical studies.[1]

Structural policy within the three-sector framework means fostering policies (e.g., choosing taxes, tariffs, subsidies, education system structure, infrastructure, research funding schemes and legal entry barriers) that favor one sector over the others. The development literature provides different arguments for such structural policy, as discussed in Section 2. Some of these arguments are favoring agriculture, while others are favoring manufacturing or services. Moreover, as shown in Section 2, most of the arguments (a) refer to an underdeveloped (i.e. not fully industrialized country) that seeks for an optimal structural policy (in the three-sector framework) over the initial phase of its development and (b) do not address the myopic development planer or policy maker (who seeks to maximize initial growth, while neglecting the long-run effects of its policy), but the planer who seeks to maximize and sustain the welfare and growth in the long run; i.e. the arguments refer to the effects of the present-day's policy in a more or less distant future.

Our paper is a contribution to this discussion of optimal structural policy in the three-sector framework. We focus on the costs of structural change; in particular, we assume that the economic and social costs of structural change increase (monotonously) with the magnitude of structural change (as measured by the magnitude of the changes in the sectoral employment shares or sectoral GDP shares). The historical experiences of present-day's developed and developing countries reveal severe costs of structural change, among others, increasing environmental pollution and global warming (over the industrialization phase), costs associated with unemployment (over the de-industrialization phase) and geographical re-location of labor (e.g. negative aspects of hasted urbanization over the industrialization phase) and abandoned/unused/sunk capital, e.g. ghost cities/facilities (over the de-industrialization phase). These costs are still being discussed in highly developed economies (e.g. in election campaigns), which reveals their lasting impact on the society.

[1] For an overview of the structural change literature, see, e.g., Schettkat and Yocarini (2006), Krüger (2008), Silva and Teixeira (2008), Stijepic (2011, Chapter IV), and Herrendorf et al. (2014). Recent contributions to the three-sector modeling literature include, e.g., Kongsamut et al. (2001), Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), Uy et al. (2013) and Stijepic (2015).

1.2 Aims of the Paper

Considering the magnitude of the structural change costs, it seems to make sense to discuss the structural policy alternatives based on the structural change costs they cause. In particular, following the discussion from above (cf. points (a) and (b)), we search for an answer to the following (theoretical) problem: assume that the non-myopic (cf. point (a)) social planer in an underdeveloped (i.e. non-industrialized) country seeks to choose a structural policy over the initial development phase of its country that minimizes the future structural change costs (over the planning horizon); which structural change path (among the many feasible structural change paths) should the social planer choose? We provide a solution to this calculus-of-variations problem and demonstrate that it can be used to (i) design a structural policy that minimizes the structural change costs in a developing country, (ii) evaluate the prominent structural policy alternatives discussed in the literature based on the structural change costs they cause and (iii) easily estimate the aggregate magnitude of the past structural change costs beared by the present-day's developed economies on the basis of macroeconomic historical data (cross-country comparison of cost-efficient structural change).

1.3 Method/Approach

We model structural change as a trajectory/path on a standard 2-simplex (cf. Stijepic (2015)) and assume that the structural change costs are monotonously increasing in the structural change magnitude (as measured by the magnitude of the changes in the sectoral employment shares or the sectoral GDP shares). As we will see, it is not difficult to determine the cost-minimizing structural change path if we know the (optimal)[2] sector structure that will be realized at the end of the planning horizon of the social planer. Figuratively speaking, it is relatively easy to find a cost-minimizing path if we know the destination of the economy/path. We show that such a path must be monotonous on the 2-simplex (Result 1). Unfortunately, we do neither know the planning horizon of the social planer nor the destination of a developing economy; in particular, we do not know what the (optimal) sector structure of a developed economy will be in, e.g., 20 years given all the thinkable and unthinkable exogenous determinants of the sector structure (in 20 years). Therefore, we study the historical evidence on the structural change patterns in present-day's developing and developed countries and the (normative and positive) structural change models' predictions of the (optimal) sector

[2] 'optimal' refers here to the normative multi-sector growth models' predictions of the structural change path choice by the utility-maximizing representative household.

structures. As we discuss in Section 3, the evidence and the models generate very different predictions. (This problem is exacerbated by the fact that we do not know the planning horizon of the social planner.) The only consensus forecast that we can derive from the previous literature is that (probably) the (distant) future agricultural/services share of a present-day's developing economy will be lower/higher than it is today (Result 2). Finally, we combine Results 1 and 2 to derive the cost-minimizing policy in an underdeveloped economy. Since Results 1 and 2 are qualitative statements, our analysis relies on geometrical methods studying the geometrical properties of trajectories and tangential vectors.

1.4 Results

We show that a social planer in an underdeveloped country seeking to minimize the future structural change costs and facing the global uncertainties regarding the optimal future sector structure should choose a structural policy that is consistent with: a decreasing agricultural share, a constant manufacturing share and an increasing services share (in GDP or in employment) over the initial phase of development.

This result implies that structural policies, e.g., the Washington Consensus strategy and the Kaldorian strategies (cf. Section 2), that emphasize the agricultural and manufacturing sector at the initial phases of development are associated with relatively high structural change costs (in future). Thus, our results predict that the countries that emphasized the agricultural sector (e.g. many developing countries) or the manufacturing sector (e.g. UK, China and Germany) faced or will face relatively high structural change costs, e.g. costs of environmental pollution over the industrialization phase and (future) costs of de-industrialization (e.g. unemployment related costs). Moreover, many present-day's highly developed economies (e.g. UK) that are characterized by a heavily 'hump-shaped' manufacturing sector development (i.e. overshooting industrialization followed by strong de-industrialization) are characterized by relatively high structural change costs according to our results. In contrast, India's recent development strategy of emphasizing the role of the service sector seems to minimize the structural change costs.

Overall, our paper implies that the strategy of manufacturing sector restructuring (towards more modern industries/branches) is preferable to the strategy of increasing the manufacturing's share in GDP and employment over the initial phases of development. Of course, these results refer only to the structural change costs. There are many other aspects (discussed in Section 2) that should be considered when choosing a structural strategy.

1.5 Structure of the Paper

The rest of the paper is set up as follows. In Section 2, we discuss the literature providing arguments on structural policy in the three-sector framework. In Section 3, we discuss the empirical evidence and the theoretical literature results regarding the destination of the structural change process. Sections 4 and 5 derive the mathematical lemmas regarding the minimal structural change costs. We interpret and discuss these results in Section 6. Concluding remarks are provided in Section 7.

2. Arguments from the Development Literature related to Structural Policy in the Three-Sector Framework

The development literature provides different arguments for structural policy favoring one sector over the others. For an overview of such arguments see the manifold contributions (e.g. Harrison and Rodríguez-Clare (2010)) collected by Rodrik and Rosenzweig (2010) as well as Robinson (2009). We start with the arguments for agriculture.

The policy implications of the neoclassical growth and development literature, which are often summarized under the term 'Washington Consensus', favor a trade liberalization (see, e.g., Rodrik (2006)). In the context of north-south trade, where a (highly) underdeveloped country trades with more developed countries, trade liberalization implies that the underdeveloped country specializes in agricultural goods production and export while importing manufactured goods because of comparative advantage (Ricardian argument) and resource constraints regarding, e.g., education required for manufacturing (Heckscher-Ohlin argument). Thus, according to these arguments (and the evidence on the trade structures of underdeveloped economies), an uncontrolled trade liberalization is de facto a structural policy favoring the *agricultural sector*.

This fact has been a basis for a critique of the trade liberalization policy (and the 'Washington Consensus') on behalf of the literature branch favoring the *manufacturing sector*. This critique is based on terms-of-trade arguments ('Prebisch-Singer thesis') stating that the long-run terms-of-trade development is such that the agricultural goods exporting countries (the South) have disadvantages in comparison to the manufacturing goods exporting countries (the North) (see, e.g., Hadass and Williamson (2003)). Moreover, Kaldorian arguments have been elaborated stating that subsidizing/protection of the manufacturing sector is decisive for the long-run growth of a country, since the manufacturing sector is a source of technological progress (see, e.g., Greenwald and Stiglitz (2006) and Stiglitz et al. (2013)). These arguments for an industrialization are contrasted by some well-known counterarguments related to the negative effects of strong (and quick) manufacturing sector development, e.g., environmental pollution (as in the case of modern China) and problems associated with hasted urbanization as is documented in the case of the USA in the 19^{th} century and later.

The literature provides arguments regarding the *services sector* as well. Some arguments imply that in less developed countries that have some structural characteristics, e.g., a great share of English-speaking population, a policy favoring the (modern) services sector may enhance growth (while omitting the negative effects of industrialization). The major example for this argument is India, which is characterized by a relatively high share of highly educated English-speaking population that can be employed in IT branches (exporting IT services to the USA and UK). Moreover, there is literature that emphasizes the importance of the development of the financial (services) sector for generating economic growth (see, e.g., Demirgüç-Kunt and Levine (2004)) and the fact that the services sector seems to be less volatile in comparison to the manufacturing sector (thus, a greater services share implies lower volatility of the economy; see, e.g., Moro (2012)). One of the major arguments against the services sector is pioneered by Baumol (1967) and Baumol et al. (1985) stating that it is relatively difficult to generate innovation and productivity growth in the (personal) services sector (due to the personal nature of services, among others); thus, an economy characterized by a relatively great services share will have problems in generating high growth rates (in the long run).

As we can see, there are advantages and disadvantages associated with each of the sectors. Most of the arguments (a) refer to an underdeveloped (i.e. not fully industrialized country) that seeks for an optimal structural policy (in the three-sector framework) over the initial phase of its development and (b) do not address the myopic development planer or policy maker (who seeks to maximize initial growth, while neglecting the long-run effects of its policy, e.g. pollution or a bad positioning on the world market due to specialization on agriculture) but the planer who seeks to maximize/sustain the welfare and growth in the long run; i.e. the arguments refer to the effects of the present-day's policy in a more or less distant future.

3. Implications of the Empirical Evidence and the Theoretical Models Regarding the Destination of the Structural Change Path

In this section, we focus on the discussion of the sectoral employment shares. (The term 'employment share of sector i' refers to the share of aggregate employment devoted to sector i.) We omit the discussion of the sectoral GDP shares, because it is very similar to the discussion of the sectoral employment shares. Since we do not know the planning horizon of the social planner in our cost-minimization problem, not only the limit structure of the economy (i.e. the structure to which the economy converges as time goes to infinity) but also the transitional structures (i.e. the shape of the structural trajectory) is/are relevant for the discussion of the destination of the structural change trajectory (i.e. the structure that materializes at the end of the social planer's horizon), as explained in Section 3.2.

3.1 Implications of Structural Change Models

In this section, primarily, we refer to the following models of structural change: Kongsamut et al. (1997), Kongsamut et al. (2001), Ngai and Pissarides (2007), Foellmi and Zweimuller (2009), Uy et al. (2013) and Stijepic (2015). We restrict our discussion to these models, since the inclusion of a greater number of models into the following discussion does not change the main result of this section, namely, the fact that the theoretical literature makes very heterogeneous predictions regarding the future structure of a today's developing country.

In general, the papers listed above make very different predictions of structural change. The shape of the structural change trajectory and the limit structure (where the latter term refers to the sector structure to which the economy converges as time goes to infinity) depend on the model assumptions. For example, the trajectory shapes of the Kongsamut et al. (2001) model and the Ngai and Pissarides (2007) model differ significantly, where the latter predicts a curved trajectory (cf. Stijepic (2015), p.80) and the former a linear trajectory (cf. Stijepic (2016a)); the same is true for the limit structure, where the Kongsamut et al. (2001) model predicts that in the limit, the manufacturing share is the same as in the initial state, while the Ngai and Pissarides (2007) model predicts a set of different limit manufacturing shares depending on the parameterization of the model. In general, the shapes and the limit properties of the structural change trajectories generated by these models depend on the parameter settings; we have no clear evidence/theory regarding these model's parameter values; moreover, the sets of parameters determining the shape and the limit properties of the model's trajectories differ strongly across models.

Our study of the models listed above implies the following consensus statements (i.e. statements that are consistent with the predictions of all these models):

Meta-theorem 1. *In a developing economy, the services employment share grows and the agricultural employment share declines over the very long run. In other words, the models imply that in a more or less distant future ('long run perspective'), a developing country's services/agricultural employment share will be greater/smaller than it is today.*

Meta-theorem 2. *A developing country's manufacturing employment share may be growing, decreasing or constant. Moreover, it may follow a non-monotonous pattern ('hump-shaped development') over the long run (as predicted by, e.g., Ngai and Pissarides (2007) and Uy et al. (2013)).*

3.2 Empirical Evidence on Shapes and Destinations of the Structural Change Trajectories

For a discussion of the empirically observable shapes and the limit properties of structural change trajectories, we refer to Stijepic (2016b), who collected structural change data from different sources covering a large set of countries and depicted this data on standard 2-simplexes. The following facts becomes immediately apparent when studying the figures (and, in particular, the Figures 10-17) presented by Stijepic (2016b):

(1.) the shapes and the endpoints of the trajectories differ significantly across countries;

(2.) many trajectories are strongly curved; thus, depending on the planning horizon (i.e. the point of time that we define to be the end of the planning horizon), the sector structure at the end of the planning horizon (which is simply a point on the trajectory corresponding to the time point representing the end of the planning horizon) varies strongly even when considering the trajectory of only one country;

(3.) the empirical evidence depicted by Stijepic (2016b) supports the Metha-theorems 1 and 2 (see also Stijepic (2016b), pp.16-21).

4. Monotonous Paths as Structural Change Costs-Minimizing Paths when the Path-Destination is Known

In this section, we show that if the destination of the development path is given, the structural change costs-minimizing path is monotonous. We require this result as a basis for our main results. Again, we focus our discussion on the sectoral employment shares. Analogous results can be obtained for the sectoral GDP shares. In the rest of the paper, the mathematical notation is as follows: small letters denote scalars, capital letters denote vectors, bold capital letters denote sets, and Greek small letters denote angles.

Definition 1. *The* ***sector structure*** *(indicated by the labor allocation) at time* $t \in [0,\infty)$ *is given by the vector* $X(t) = (x_1(t), x_2(t), ... x_n(t)) \in \mathbf{R}^n$*, where* $x_i(t)$ *denotes the* ***share of employment*** *devoted to sector i, i = 1,…n, and* $\mathbf{R}^n$ *is the n-dimensional Real space.*

Thus, for example, if $l(t)$ is the aggregate employment (e.g., the number of employees in the economy) at time *t* and $l_i(t)$ is the employment in sector i (e.g., the number of employees in sector *i*) at time *t*, then $x_i(t) = l_i(t)/l(t)$.

Assumption 1. *The **sector structure** $X(t)$ (cf. Definition 1) satisfies the following conditions:*

(1) $\forall t \in [0,\infty) \forall i \in \{1,2,...n\} \ \ 0 \le x_i(t) \le 1$

(2) $\forall t \in [0,\infty) \ \ x_1(t) + x_2(t) + ... x_n(t) = 1$.

Equation (2) and Definition 1 imply that the aggregate employment is the sum of sector employment. This is a standard assumption in structural change modelling. It can be always satisfied by defining a residual sector; cf. Stijepic (2015). Equation (1) is obviously meaningful, since employment cannot be negative (and, thus, (2) implies that the employment share cannot be greater than one).

Assumption 2. (a) *The **initial sector structure** (of the economy) is given, i.e.* $X(0) = X^0 \equiv (x_1^0, x_2^0, ... x_n^0) \in \mathbf{R}^n$. ***(b)*** *The economy moves along a **continuous path**, i.e.* $\forall t \forall i \ \ x_i(t)$ *is continuous in t.*

It is obvious that the today's labor allocation (X^0) is given. The assumption of a continuous path is due to the long-run modelling horizon, i.e. we consider only the long-run dynamics and neglect shorter-run jumps and fluctuations. Again, this is a standard assumption in long-run growth modelling. For example, all the models listed in Section 3.1 choose a continuous modelling framework.

Definition 2. *The **development path** over the time-interval* $[0,\bar{t}]$ *is given by the curve* $X(t)$, $0 \le t \le \bar{t}$ *(cf. Definition 1), and the set* $\mathbf{P} := \{X(t) \in \mathbf{R}^n : t \in [0,\bar{t}]\}$.

Thus, we can imagine a development path as a curve/path connecting the points $X(0)$ and $X(\bar{t})$ in the n-dimensional Euclidean space.

Definition 3. *A development path (cf. Definition 2) is* ***monotonous*** *on the time-interval* $[0,\bar{t}]$ *if* $\nexists\, i \in \{1,2,...n\}: (\exists t_a \in [0,\bar{t}] \wedge \exists t_b \in [0,\bar{t}]: t_a \neq t_b \wedge x_i'(t_a) < 0 \wedge x_i'(t_b) > 0)$.

Remark 1. Definition 3 implies the following properties of a monotonous path. (1.) All x_i are behaving monotonously. Thus, for any $i \in \{1,2,...n\}$ the following is true: either $\forall t \in [0,\bar{t}]\ x_i'(t) \geq 0$ or $\forall t \in [0,\bar{t}]\ x_i'(t) \leq 0$. (2.) Some x_i may be monotonously decreasing, while at the same time some x_i may be monotonously increasing and at the same time some x_i may be constant. That is, if the economy moves along a monotonous development path, the following scenario is possible, for example: at the time $t_a \in [0,\bar{t}]$, $x_1'(t_a) > 0$, $x_2'(t_a) < 0$ and $x_3'(t_a) = 0$.

Assumption 3. *The (cumulative) costs ($c^{0\bar{t}}$) of structural change associated with the development path* $X(t)$*,* $0 \leq t \leq \bar{t}$*, are given by*

$$\text{(3)} \qquad c^{0\bar{t}} := f\left(r^{0\bar{t}}\right),\ r^{0\bar{t}} := \int_0^{\bar{t}} \sum_{i=1}^{n} \left|x_i'(t)\right| dt,\ x_i'(t) = \frac{dx_i}{dt},\ f: \mathbf{R} \to \mathbf{R},\ f'(.) > 0$$

The structural change costs index (3) requires some explanation. Assume that *l* is the aggregate labor force. Furthermore, assume that *l* is constant. In this case, $r_i(t) := x_i'(t) l$ is the change in employment in sector *i* at time *t*. If $r_i(t) > 0$, then $r_i(t)$ is the (net) number of workers reallocated to sector *i* at time *t*. If $r_i(t) < 0$, then $r_i(t)$ is the (net) number of workers reallocated (or: withdrawn) from sector *i* at time *t*. Thus, $r(t) := |r_1(t)| + |r_2(t)| + ...|r_n(t)|$ is an index of the number of re-allocated workers at time *t*. Note that we must take the absolute values of $r_i(t)$, since $r_1(t) + r_2(t) + ...r_n(t)$ is always equal to zero (cf. (2)). Furthermore, we should multiply $r(t)$ with 0.5, since 're-allocation of workers across sectors' means that a withdrawal of the workers from one sector is always associated with the hiring of these workers in another sector (in long-run modelling). Since multiplying $r(t)$ with 0.5 does not change any of our results, we omit it here. Overall, $r(t)$ is the index of re-allocation at time *t*. To obtain an index of re-allocation over the time period $[0,\bar{t}]$, we must sum up all $r(t)$ over this period, which in continuous time, corresponds to taking the integral over *t*. This integral is equal to $r^{0\bar{t}}$. In fact, $r^{0\bar{t}}$ is an index of the magnitude of re-allocation (or: an index of the number of re-allocated workers). As noted in the introduction, we assume that the structural change costs ($c^{0\bar{t}}$) are a (strictly) monotonously increasing function (f) of this magnitude of re-allocation ($r^{0\bar{t}}$). Analogous, results could be obtained if we used a measure of magnitude of the changes in the sectoral GDP shares.

Now, we study the following (calculus-of-variations) problem. Assume that Assumptions 1-3 are satisfied (and, thus, $X(0) = X^0$ is given) and that the path-destination (at time $\bar{t}$) is determined, i.e. $X(\bar{t}) = X^{\bar{t}}$ is given. There exist different paths that connect X^0 and $X^{\bar{t}}$ in Euclidean space (cf. Figure 1). A path is "admissible" if it is continuous (cf. Assumption 2b) and if it connects X^0 and $X^{\bar{t}}$. The functional (3) associates each of these admissible paths with a certain magnitude of structural change costs $c^{0\bar{t}}$. We search for an answer to the following question: 'Which of the admissible paths is associated with minimal structural change costs $(c^{0\bar{t}})$?' That is, we want to find the (admissible) path that minimizes the structural change costs $c^{0\bar{t}}$. Lemma 1 provides the solution of this problem.

Figure 1. *The calculus-of-variations problem solved by Lemma 1.*

- insert Figure 1 here -

Lemma 1. *Assume that Assumptions 1 to 3 are satisfied and that the path-destination at time* $\bar{t} > 0$ *is given, i.e.* $X(\bar{t}) = X^{\bar{t}} \equiv (x_1^{\bar{t}}, x_2^{\bar{t}}, ... x_n^{\bar{t}}) \in \mathbf{R}^n$. *Under these conditions, any* ***monotonous*** *(and continuous) development path (cf. Definition 2) that connects* X^0 *and* $X^{\bar{t}}$ *(in Euclidean space) is associated with minimal structural change costs* $c^{0\bar{t}}$ *(cf. Definition 3).*

For a **proof of Lemma 1** you could apply the theorems of the calculus of variations (see, e.g., Gelfand and Fomin (1963), Chapter 15). In the APPENDIX, we provide a more detailed (geometrical) proof, which uses the techniques familiar to calculus of variations. This detailed proof provides us with lemmas and interpretations that are helpful for proving and understanding the properties of the minimal-costs paths that will be discussed later.

Simply speaking, Lemma 1 states that if we want minimal structural change costs, it does not matter which path we take from X^0 to $X^{\bar{t}}$ as long as it is monotonous (and per assumption continuous).

Note that since Lemma 1 is valid for any $X^{\bar{t}} \in \mathbf{R}^n$, we could formulate it more generally, i.e. we can omit the reference to X^0 and $X^{\bar{t}}$, as follows: any monotonous path (in Euclidean space) is associated with minimal structural change costs.

Note that we assume throughout the paper that $X(t)$ is C[1] (see, e.g., Definition 3 and Assumption 3), i.e. the development path has a certain degree of smoothness. This argument is valid, since we study here only long-run trend paths, i.e. the smoothness of $X(t)$ is per definition (of the term 'long run trend'.

Obviously, if $X^{\bar{t}} = X^0$, the structural change costs-minimizing strategy (for 'moving' from X^0 to $X^{\bar{t}}$) is: stay in X^0 for all $t \in [0, \bar{t}]$, i.e. no structural change at all! Such a 'path' is per Definition 3 monotonous.

5. Monotonous Paths in the Three-Sector Framework when the Path-Destination is Determined by Meta-Theorems 1 and 2

In this section, we prove the following lemma. As we will see later, this lemma and Lemma 1 imply jointly the existence of a structural change costs-minimizing path given Meta-theorems 1 and 2.

Assumption Set 1. *We consider the three-sector economy (n = 3) over the period* $[0, \infty)$, *where t = 0 denotes the present. Assume that the initial structure of the economy (at t = 0) is given by the vector*

(4) $\quad X^0 \equiv (x_1^0, x_2^0, .x_3^0) \in \mathbf{R}^3$.

Let $\bar{t}$ *denote a future time point, i.e.*

(5) $\quad \bar{t} \in (0, \infty)$

and $X^{\bar{t}}$ *denote the structure of the economy at* $\bar{t}$, *where*

(6) $\quad X^{\bar{t}} \equiv (x_1^{\bar{t}}, x_2^{\bar{t}}, x_3^{\bar{t}}) \in \mathbf{R}^3$

Let Meta-theorems 1 and 2 be valid, i.e. assume that

(7) $\quad x_1^{\bar{t}} < x_1^0 \wedge x_3^{\bar{t}} > x_3^0$

Moreover, let the vectors X^0 *and* $X^{\bar{t}}$ *satisfy the following conditions*

(8) $\quad \forall t \in \{0, \bar{t}\} \; \forall i \in \{1,2,3\} \; 0 \le x_i^t \le 1 \wedge x_1^t + x_2^t + x_3^t = 1$.

Lemma 2. a) *Let the Assumption Set 1 be valid. Then, there exists a path* $X^*(t) \equiv (x_1^*(t), x_2^*(t), x_3^*(t))$, $t \in [0, \bar{t}]$, *that has the following characteristics*

(I) $\forall t \in [0, \bar{t}]\ \forall i \in \{1,2,3\}\ \ 0 \le x_i^*(t) \le 1 \wedge x_1^*(t) + x_2^*(t) + x_3^*(t) = 1$

(II) $\forall t \in [0, \bar{t}]\ \ X^*(t)$ *is continuous in t*

(III) $\forall t \in [0, \bar{t}]\ \ X^*(t)$ *is monotonous in t*

(IV) $X^*(0) = X^0$

(V) $X^*(\bar{t}) = X^{\bar{t}}$

(VI) $\exists t' \in (0, \bar{t}) : \forall t \in [0, t')\ x_2^*(t) = x_2^0$

b) *Let the Assumption Set 1 be valid. Then, for some* $X^{\bar{t}} \in \mathbf{R}^3$ *(satisfying (8)) there does not exist a path* $X^*(t) \equiv (x_1^*(t), x_2^*(t), x_3^*(t))$, $t \in [0, \bar{t}]$, *satisfying the conditions (I), (II), (III), (IV), (V) and (VI'), where*

(VI') $\exists t' \in (0, \bar{t}) : \forall t \in [0, t')\ dx_2^*(t)/dt < 0$.

c) *Let the Assumption Set 1 be valid. Then, for some* $X^{\bar{t}} \in \mathbf{R}^3$ *(satisfying (8)) there does not exist a path* $X^*(t) \equiv (x_1^*(t), x_2^*(t), x_3^*(t))$, $t \in [0, \bar{t}]$, *satisfying the conditions (I), (II), (III), (IV), (V) and (VI''), where*

(VI') $\exists t' \in (0, \bar{t}) : \forall t \in [0, t')\ dx_2^*(t)/dt > 0$.

We choose here a rather 'informal' way of proving Lemma 2 allowing us to discuss the aspects being proven and derive some corollaries that will be of interest in Section 6. The **proof** is structured as follows: first, we show that the path characterized by Lemma 2 is located in a subset (**D**) of a plane in $\mathbf{R}^3$ and that the path-destination (which is determined by Meta-theorems 1 and 2) is located in a subset ($\mathbf{D}^{\bar{t}}$) of **D**; then, we partition the subset $\mathbf{D}^{\bar{t}}$ and show that (a) a path characterized by (VI) can be constructed to any location in any partition while satisfying requirements (I)-(V) and (b) a path characterized by (VI') or (VI'') cannot lead to some of the partitions if (I)-(V) are satisfied.

We start the proof by defining the path $\mathbf{P}^*$ as follows:

(9) $\mathbf{P}^* := \{X^*(t) \in \mathbf{R}^3 : t \in [0, \bar{t}]\}$

Lemma 2 states that $\mathbf{P}^*$ satisfies the condition (I) among others. Condition (I) states that the path $\mathbf{P}^*$ is located in the set

(10) $\mathbf{D} := \{(x_1, x_2, x_3) \in \mathbf{R}^3 : \forall i \in \{1,2,3\}\ 0 \le x_i(t) \le 1 \wedge x_1(t) + x_2(t) + x_3(t) = 1\}$

In other words,

(11) $\mathbf{P}^* \subset \mathbf{D}$

Thus, when searching for $\mathbf{P}^*$ satisfying the characteristics (I)-(VI), we do not need to analyze the whole $\mathbf{R}^3$, but can restrict our attention to **D**.

As discussed by Stijepic (2015), (10) states that **D** is a standard 2-simplex, which is a subset of a plane in $\mathbf{R}^3$; in particular, **D** is a triangle with the vertices V_1:=(1,0,0), V_2:=(0,1,0) and V_3:=(0,0,1) in the Cartesian coordinate system (see Figure 2).

Figure 2. *The standard 2-simplex (***D***) in the Cartesian coordinate system.*

- insert Figure 2 here -

Henceforth, we depict **D** without the coordinate system, as depicted in Figure 3.

Figure 3. *The standard 2-simplex (***D***) depicted without the coordinate system.*

- insert Figure 3 here -

(4), (6), (8) and (10) imply

(12) $X^0 \in \mathbf{D} \ \wedge \ X^{\bar{t}} \in \mathbf{D}$

(11), (12), (IV) and (V) imply that the path $\mathbf{P}^*$ connects X^0 and $X^{\bar{t}}$ on **D** (cf. Figure 4).

Figure 4. *An example of the path* $\mathbf{P}^*$.

- insert Figure 4 here -

Given an initial state $X^0 \in \mathbf{D}$, we define the set $\mathbf{D}^{\bar{t}}$ and its partitioning ($\mathbf{D}^{\bar{t}}_a$, $\mathbf{D}^{\bar{t}}_b$, $\mathbf{D}^{\bar{t}}_c$) as follows :

(13) $\mathbf{D}^{\bar{t}} := \{(x_1,x_2,x_3) \in \mathbf{D} : x_1 < x_1^0 \wedge x_3 > x_3^0\}$

(14) $\mathbf{D}^{\bar{t}}_a := \{(x_1,x_2,x_3) \in \mathbf{D} : x_1 < x_1^0 \wedge x_3 > x_3^0 \wedge x_2 < x_2^0\} = \{(x_1,x_2,x_3) \in \mathbf{D}^{\bar{t}} : x_2 < x_2^0\}$

(15) $\mathbf{D}^{\bar{t}}_b := \{(x_1,x_2,x_3) \in \mathbf{D} : x_1 < x_1^0 \wedge x_3 > x_3^0 \wedge x_2 = x_2^0\} = \{(x_1,x_2,x_3) \in \mathbf{D}^{\bar{t}} : x_2 = x_2^0\}$

(16) $\mathbf{D}^{\bar{t}}_c := \{(x_1,x_2,x_3) \in \mathbf{D} : x_1 < x_1^0 \wedge x_3 > x_3^0 \wedge x_2 > x_2^0\} = \{(x_1,x_2,x_3) \in \mathbf{D}^{\bar{t}} : x_2 > x_2^0\}$

As we can see, $\mathbf{D}^{\bar{t}}$ is the set of all points (on **D**) satisfying Meta-theorems 1 and 2 (cf. (7) and (13)). $\mathbf{D}^{\bar{t}}$ and its partitioning ($\mathbf{D}^{\bar{t}}_a$, $\mathbf{D}^{\bar{t}}_b$, $\mathbf{D}^{\bar{t}}_c$) are illustrated in Figure 5.

Figure 5. *The set* $\mathbf{D}^{\bar{t}}$ *and its partitioning.*

- insert Figure 5 here -

Note. **A** and **C** are open sets. The sets associated with line-segments do not contain the end-points of the line-segments, e.g. the set $\overline{XY}$ associated with the line-segment connecting the points X and Y does not contain the points X and Y.

Note that (10) and (14)-(16) imply that $\mathbf{D}_a^{\bar{t}}$, $\mathbf{D}_b^{\bar{t}}$ and $\mathbf{D}_c^{\bar{t}}$ are pairwise disjoint and their union is equal to $\mathbf{D}^{\bar{t}}$. Thus, ($\mathbf{D}_a^{\bar{t}}, \mathbf{D}_b^{\bar{t}}, \mathbf{D}_c^{\bar{t}}$) is a partitioning of $\mathbf{D}^{\bar{t}}$, i.e.

(17) $\mathbf{D}_a^{\bar{t}} \cup \mathbf{D}_b^{\bar{t}} \cup \mathbf{D}_c^{\bar{t}} = \mathbf{D}^{\bar{t}}$

(18) $\forall i \in \{a,b,c\}\, \forall j \in \{a,b,c\} \setminus i \quad \mathbf{D}_i^{\bar{t}} \cap \mathbf{D}_j^{\bar{t}} = \emptyset$

(6), (7), (12) and (13) imply that $X^{\bar{t}}$ is located in $\mathbf{D}^{\bar{t}}$, i.e.

(19) $X^{\bar{t}} \in \mathbf{D}^{\bar{t}}$

Overall, (17)-(19) imply that $X^{\bar{t}}$ is located in one and only one of the sets $\mathbf{D}_a^{\bar{t}}$, $\mathbf{D}_b^{\bar{t}}$ and $\mathbf{D}_c^{\bar{t}}$. Thus, we can distinguish between three cases: (1.) $X^{\bar{t}} \in \mathbf{D}_a^{\bar{t}}$, (2.) $X^{\bar{t}} \in \mathbf{D}_b^{\bar{t}}$, and (3.) $X^{\bar{t}} \in \mathbf{D}_c^{\bar{t}}$.

Before analyzing these cases, we introduce the following vector angle definition, which allows us to analyze the dynamics on **D** by referring to vector angles.

Definition 4. *Let X be a point on* **D** *and D(X) be a vector indicating the direction of movement associated with point X. (For example, X may be a point on a curve/trajectory on* **D** *and D(X) a tangential/directional vector associated with point X.) The vector angle δ(D(X)) is the angle between D(X) and the simplex-edge* V_1V_2, *i.e.* $\delta(D(X)) := \angle(D(X), \overline{V_1V_2})$.

This definition and the definition of **D** imply the following properties of a directional vector D on the simplex **D**.

Property 1. *a) If* $\delta(D(X)) = 0°$, *the movement indicated by vector D(X) is characterized by a decrease in* x_1, *an increase in* x_2 *and a constant* x_3.

b) If $0 < \delta(D(X)) < 60°$, *the movement indicated by vector D(X) is characterized by a decrease in* x_1, *an increase in* x_2 *and an increase in* x_3.

c) If $\delta(D(X)) = 60°$, *the movement indicated by vector D(X) is characterized by a decrease in* x_1, *a constant* x_2 *and an increase in* x_3.

d) If 60° < δ(D(X)) < 120°, the movement indicated by vector D(X) is characterized by a decrease in x_1, a decrease in x_2 and an increase in x_3.
e) If δ(D(X)) = 120°, the movement indicated by vector D(X) is characterized by a constant x_1, a decrease in x_2 and an increase in x_3.
f) If δ(D(X)) > 120°, the movement indicated by vector D(X) is characterized by an increase in x_1 or a decrease in x_3.

Figure 6 illustrates Property 1.

Figure 6. *Examples of vectors characterized by Property 1.*

- insert Figure 6 here -

Henceforth, we use Definition 4 and Property 1 to characterize the path $\mathbf{P}^*$ as follows. The path $\mathbf{P}^*$ assigns to each $t \in [0,\bar{t}]$ an $X^*(t)$ (cf. (9)). We can assign to each $X^*(t)$ a directional vector $D(X^*(t))$ indicating the direction of movement along the path $\mathbf{P}^*$ at the point $X^*(t)$ (cf. Definition 4). (In case of differentiable functions, i.e. if $X^*(t)$ is differentiable with respect to t, $D(X^*(t))$ can be interpreted as the tangential (or directional) vector at point $X^*(t)$ of the curve $X^*(t)$, $t \in [0,\bar{t}]$, associated with the path $\mathbf{P}^*$.) Moreover, via Definition 4, we can measure the vector angle $\delta(D(X^*(t)))$ and identify the changes in (x_1,x_2,x_3) at the point $X^*(t)$, i.e. we can identify the signs of $dx_1^*(t)/dt$, $dx_2^*(t)/dt$ and $dx_3^*(t)/dt$ at each point of $\mathbf{P}^*$.

Now, we return to the three cases. First, we analyze case 1, i.e.

(20) $X^{\bar{t}} \in \mathbf{D}_a^{\bar{t}}$

(6), (14) and (20) imply

(21) $x_1^{\bar{t}} < x_1^0 \wedge x_2^{\bar{t}} < x_2^0 \wedge x_3^{\bar{t}} > x_3^0$

(21) states that at the destination $X^{\bar{t}}$ of the path $\mathbf{P}^*$, x_3 (x_1 and x_2) is (are) greater (smaller) than in the initial state X^0. (III) and Definition 3 imply that, thus, x_3 (x_1 and x_2) must grow (decrease) monotonously along the path $\mathbf{P}^*$ (cf. Remark 1), i.e.

(22) $\forall t \in [0,\bar{t})\ dx_1^*(t)/dt \le 0 \wedge dx_2^*(t)/dt \le 0 \wedge dx_3^*(t)/dt \ge 0$

(23) $(\exists t_1 \in [0,\bar{t})\ dx_1^*(t_1)/dt < 0) \wedge (\exists t_2 \in [0,\bar{t})\ dx_2^*(t_2)/dt < 0) \wedge (\exists t_3 \in [0,\bar{t})\ dx_3^*(t_3)/dt > 0)$

where (23) states that x_1, x_2 and x_3 must change over time (according to (21)), since otherwise (21) cannot be satisfied.

By using Property 1, we can translate (22) and (23) as follows:

(24) $\forall t \in [0,\bar{t})\ 60° \le \delta(D(X^*(t))) \le 120°$

(25) $\exists t \in [0,\bar{t})\ 60° < \delta(D((X^*(t))) < 120°$

By now, we have shown that if (20) is true, $\mathbf{P}^*$ must satisfy (24) and (25) due to the monotonicity requirement (III) among others. Moreover, (25) does not prohibit $\delta(D((X^*(0)))$ = 60° or for some t, $\delta(D((X(t))) = 120°$. That is, we can construct a path $\mathbf{P}^{**} := \{X^{**}(t) \in \mathbf{D}:$ $t \in [0,\bar{t}]\}$ that can be partitioned into two linear segments

(26) $\mathbf{P}_I^{**} := \{X^{**}(t) \in \mathbf{P}^{**} : t \in [0,t')\}$

(27) $\mathbf{P}_F^{**} := \{X^{**}(t) \in \mathbf{P}^{**} : t \in [t',\bar{t}]\}$

where the initial path-segment ($\mathbf{P}_I^{**}$) is characterized by a tangential vector angle of 60°, i.e. $\forall t \in [0,t')\ \delta(D((X^{**}(t))) = 60°$, and the final path-segment ($\mathbf{P}_F^{**}$) is characterized by a tangential vector angle of 120°, i.e. $\forall t \in [t',\bar{t})\ \delta(D(X^{**}(t))) = 120°$, while being consistent with (24) and (25) and all the other requirements (e.g. (IV) and (V)) listed in Lemma 2. That is:

(28) $\mathbf{P}^{**} := \{X^{**}(t) \in \mathbf{D} : t \in [0,\bar{t}]\} \wedge X^{**}(0) = X^0 \in \mathbf{D} \wedge X^{**}(\bar{t}) = X^{\bar{t}} \in \mathbf{D}_a^{\bar{t}} \wedge (\forall t \in [0,t')$ $\delta(D(X^{**}(t))) = 60°) \wedge (\forall t \in [t',\bar{t})\ \delta(D(X^{**}(t))) = 120°)$

An example of the path $\mathbf{P}^{**}$ is depicted in Figure 7.

Figure 7. *An example of* $\mathbf{P}^{**}$.

- insert Figure 7 here -

This discussion states that it is possible to construct a path $\mathbf{P}^{**}$ that has (a) the characteristics (I)-(VI) and (b) an initial segment ($\mathbf{P}_I^{**}$) that is characterized by a vector angle $\delta(D(X^{**}(t))) = 60°$ (over the initial phase $[0,t')$). However, this discussion does not tell us how long the initial segment $\mathbf{P}_I^{**}$ is (given a X^0 and a $X^{\bar{t}}$); in other words, we have not determined t' in (26)-(28). The magnitude of t' will be later of importance (when determining the length of the optimal policy).

We use Figure 7 to illustrate the geometrical derivation of the length of $\mathbf{P}_I^{**}$ for any $X^0 \in \mathbf{D}$ and any $X^{\bar{t}} \in \mathbf{D}_a^{\bar{t}}$. Given a $X^0 \in \mathbf{D}$, we construct a line-segment going through X^0 and being parallel to the simplex-edge V_1V_3. Moreover, given a $X^{\bar{t}} \in \mathbf{D}_a^{\bar{t}}$, we construct a line-segment going through $X^{\bar{t}}$ and being parallel to the simplex-edge V_2V_3. Let T be the point of intersection between the two line-segments. $\mathbf{P}_I^{**}$ is the linear path from X^0 to T; $\mathbf{P}_F^{**}$ is the linear path from T to $X^{\bar{t}}$; $\mathbf{P}^{**}$ is the union of $\mathbf{P}_I^{**}$ and $\mathbf{P}_F^{**}$. The length of $\mathbf{P}_I^{**}$ is equal to the distance between X^0 and T. As we can see in Figure 7, the length of $\mathbf{P}_I^{**}$ is equal to the distance between $X^{\bar{t}}$ and $\overline{X^0 A}$ and is non-trivial except in the limiting case of $X^{\bar{t}} \to \overline{X^0 A}$. (As implied by Figure 2, the distance between $X^{\bar{t}}$ and $\overline{X^0 A}$ depends on the difference $x_1^{\bar{t}} - x_1^0$, where $X^{\bar{t}} \to \overline{X^0 A}$ for $x_1^{\bar{t}} \to x_1^0$.) The limiting case $x_1^{\bar{t}} \to x_1^0$ is not of interest (cf. Meta-theorem 1). If the length of $\mathbf{P}_I^{**}$ is non-trivial and if the velocity of structural change (i.e. the velocity of movement along $\mathbf{P}_I^{**}$) is not infinitely large, the fact that the length of $\mathbf{P}_I^{**}$ is non-trivial implies that *t'* is non-trivial, i.e. the duration of movement along $\mathbf{P}_I^{**}$ is non-trivial.

Finally, note that (24) states that $\forall t \in [0, \bar{t})$, $\mathbf{P}^*$ must not be characterized by $\delta(D(X^*(t))) < 60°$ or $\delta(D(X^*(t))) > 120°$ (in case 1, i.e. if $X^{\bar{t}} \in \mathbf{D}_a^{\bar{t}}$). Moreover, the movement along path-segment $\mathbf{P}_F^{**}$ is characterized by a decreasing manufacturing share x_2 and a growing services share x_3 (cf. (28), Figures 2 and 7 and Property 1e).

Overall, by now, we have considered case 1, i.e. we assumed that $X^{\bar{t}} \in \mathbf{D}_a^{\bar{t}}$. We have shown that in this case:

(A) a monotonous and continuous path $(X^{**}(t),\, t \in [0, \bar{t}])$ can be constructed that

(i) connects $X^0 \in \mathbf{D}$ and $X^{\bar{t}} \in \mathbf{D}_a^{\bar{t}}$ and

(ii) is characterized by $\delta(D(X^{**}(t))) = 60°$ over some initial period $[0, t')$ of non-trivial length;

(B) there does not exist a continuous and monotonous path $(X^{**}(t),\, t \in [0, \bar{t}])$ that

(i) connects $X^0 \in \mathbf{D}$ and $X^{\bar{t}} \in \mathbf{D}_a^{\bar{t}}$ and

(ii) is characterized by $\delta(D(X^{**}(t))) < 60°$ or $\delta(D(X^{**}(t))) > 120°$ over some initial period $[0, t')$ of non-trivial length.

Analogously, it can be shown that

(C) in case 2, i.e. if $X^{\bar{t}} \in \mathbf{D}_b^{\bar{t}}$, a continuous and monotonous path $(X^{**}(t), t \in [0,\bar{t}])$ that connects $X^0 \in \mathbf{D}$ and $X^{\bar{t}} \in \mathbf{D}_b^{\bar{t}}$ must be characterized by $\delta(D(X^{**}(t))) = 60°$ $\forall t \in [0,\bar{t})$;

(D) in case 3, i.e. if $X^{\bar{t}} \in \mathbf{D}_c^{\bar{t}}$:

(a) a monotonous and continuous path $(X^{**}(t), t \in [0,\bar{t}])$ can be constructed that

(i) connects $X^0 \in \mathbf{D}$ and $X^{\bar{t}} \in \mathbf{D}_c^{\bar{t}}$ and

(ii) is characterized by $\delta(D(X^{**}(t))) = 60°$ over some initial period $[0,t')$ of non-trivial length;

(b) there does not exist a continuous and monotonous path $(X^{**}(t), t \in [0,\bar{t}])$ that

(i) connects $X^0 \in \mathbf{D}$ and $X^{\bar{t}} \in \mathbf{D}_c^{\bar{t}}$ and

(ii) is characterized by $\delta(D(X^{**}(t))) > 60°$ over some initial period $[0,t')$ of non-trivial length.

These facts (i.e. points (A)-(D)) imply that in all three cases, only an initial angle of 60°, i.e. $\delta(D(X^{**}(0))) = 60°$, ensures that we can reach our destination along a monotonous (and continuous) path. (Moreover, the vector angle $\delta(D(X^{**}(t))) = 60°$ can be sustained over some initial period $[0,t')$ of non-trivial length while ensuring that the path is monotonous and continuous and the destination is reached.) Any other initial vector angle cannot ensure in all cases that we can reach the destination along a monotonous and continuous path. For example, if the initial vector angle is equal to 80°, i.e. $\delta(D(X^{**}(0))) = 80°$, a monotonous and continuous path can be constructed to a destination in $\mathbf{D}_a^{\bar{t}}$ but not to a destination in $\mathbf{D}_c^{\bar{t}}$. Finally, note that Property 1 states that $\delta(D(X^{**}(t))) = 60°$ for $[0,t')$ means that the employment share of manufacturing is constant over the period $[0,t')$. Moreover, $\delta(D(X^{**}(t))) \neq 60°$ for $[0,t')$ means that the employment share of manufacturing is not constant over the period $[0,t')$. These facts prove Lemma 2.

Note that the proofs of the following facts are analogous to the corresponding proofs discussed in this section: (a) the length of $\mathbf{P}_I^{**}$ and, thus, the magnitude of t' depends on the difference $x_3^{\bar{t}} - x_3^0$ if $X^{\bar{t}} \in \mathbf{D}_c^{\bar{t}}$; (b) $t' = \bar{t}$ if $X^{\bar{t}} \in \mathbf{D}_b^{\bar{t}}$; (c) if $X^{\bar{t}} \in \mathbf{D}_c^{\bar{t}}$, the path-segment $\mathbf{P}_F^{**}$ is characterized by a growing manufacturing share x_2 and a decreasing agricultural share x_1.

We provide now an interpretation of Lemma 2.

6. Discussion

6.1. Implications of Lemmas 1 and 2: Cost-Minimizing Development Strategy

We can use Lemmas 1 and 2 to derive the optimal structural change policy as follows. *Lemma 1* states that monotonous development paths minimize the structural change costs. *Lemma 2a* states that for any path destination $X^{\bar{t}}$ (cf. (V)) satisfying Meta-theorems 1 and 2 (cf. (7)), there exists a monotonous path (cf. (III)) that is characterized by a constant manufacturing employment share over some initial phase [0,*t*') (cf. (VI)); moreover, Lemma 2a implies that this path is characterized by a monotonously growing (decreasing) services (agricultural) share (cf. (7), (III) and Definition 3). *Lemmas 2b and 2c* state that if the social planer does not choose a policy that ensures a constant manufacturing share over the initial development phase (cf. (VI') and (VI'')), then the economy may not be able to reach its destination along a monotonous path (cf. (III)).

Jointly, Lemmas 1 and 2 imply that an underdeveloped country not knowing the exact destination of its structural change path should choose the following policy:

(a) decreasing agricultural share,

(b) constant manufacturing share and

(c) increasing services share.

This policy is consistent with the theoretical and empirical literature consensus on the path-destination of a developing economy (cf. Meta-theorems 1 and 2) and minimizes the country's future structural change costs.

6.2 On the Optimal Duration of Policy (a)-(c)

Lemma 2 states that the structural policy (a)-(c) is only optimal over the initial phase of development, which is in our model denoted by the time-interval [0,*t*'). As implied by the discussion (cf. Section 5), the length of this phase (which can be derived from the length of the initial path-segment $\mathbf{P}_I^{**}$) depends on the differences between the initial and the destined agricultural and services employment shares ($x_1^{\bar{t}} - x_1^0$ and $x_3^{\bar{t}} - x_3^0$). Since, in general, these differences are relatively large in an underdeveloped yet developing country, it seems that policy (a)-(c) is optimal over a relatively long phase, as demonstrated by the following example referring to the USA.

The USA accomplished their structural transformation from an agricultural to a services economy over a period of ca. 170 years, as illustrated by Figure 8, which depicts among others the US structural change over the period 1820-1992. Figure 8 implies that it is possible to construct a linear line-segment that (a) is approximately parallel to the V_1V_3 edge of the simplex and (b) connects the initial point (representing 1820) and the last point (representing 1992) of the US trajectory.[3] In our modeling framework, this line-segment is denoted by $\mathbf{P}_I^{**}$ (cf. Figure 7) and represents policy (a)-(c), i.e. a structural change path that is characterized by a constant manufacturing share (over the period 1820-1992). Thus, our results imply that in the case of the USA, policy (a)-(c) would have been optimal over a period of ca. 170 years and would have avoided the costs of industrialization (e.g. the declining health of the population and the problems with urbanization) and the costs of de-industrialization (e.g. urban decline and unemployment). Of course, these arguments only refer to the structural change cost-minimization problem and neglect other aspects of optimal structural policy discussed in Section 2.

Figure 8. *Labor allocation trajectories for the USA, France, Germany, Netherlands, UK, Japan, China, and Russia.*

- insert Figure 8 here -

Notes. *Data source: Maddison (1995). The black dot represents the barycenter of the simplex. Abbreviations: C – China, F – France, G – Germany, J – Japan, N – Netherlands, R – Russia, US – United States, UK – United Kingdom. Data points (years in parentheses): USA (1820, 1870, 1913, 1950, 1992), France (1870, 1913, 1950, 1992), Germany (1870, 1913, 1950, 1992), Netherlands (1870, 1913, 1950, 1992), UK (1820, 1870, 1913, 1950, 1992), Japan (1913, 1950, 1992), China (1950, 1992), Russia (1950, 1992).*

6.3 Optimal Policies Following Policy (a)-(c)

As discussed in Sections 5 and 6.2 (in the case of the USA), policy (a)-(c), which is represented by path-segment $\mathbf{P}_I^{**}$, may be optimal over a relatively long period. However, the discussion in Section 5 has shown that this is a special case and in general, policy (a)-(c) must be followed by a de-industrialization accompanied by a tertiarization or an industrialization accompanied by an agricultural decline (cf. the discussion of path-segment $\mathbf{P}_F^{**}$) if we seek to minimize the

[3] Note that Figure 8 depicts the development of the USA until 1992. Since 1992, the USA have come even closer to the simplex-edge V_1V_3 such that the line-segment connecting their present-day's location and their initial (i.e. 1820) location on the simplex is approximately parallel to the simplex-edge V_1V_3.

structural change costs. Thus, policy (a)-(c) does not only minimize the structural change costs but also allows for a postponing of the industrialization/de-industrialization decision to a later phase of development, where additional information on the global environment may be available.

6.4 Comparison of Policy (a)-(c) to the Standard Structural Policies

As discussed in Section 2, the previous literature implies different structural change strategies. We compare now these strategies with policy (a)-(c).

Our results imply that the 'Washington Consensus strategy' (in particular, trade liberalization) emphasizing the agricultural sector in the early stages of development is associated with high structural change costs. It contradicts the policy aspect (a) ('decreasing agricultural share'). In general, nearly all highly developed countries are characterized by relatively low agricultural shares (cf. Figure 8). Thus, the increases in the agricultural share (induced by the Washington Consensus strategy) must be reversed at some later stages of development, which causes unnecessary structural change costs.

Moreover, the Kaldorian strategy of emphasizing the manufacturing sector, which has been pursued by many socialist countries (e.g. China) contradicts the policy aspect (b) ('constant manufacturing share'). Examples of the negative effects of a manufacturing sector emphasis are well known from the history (e.g. the food shortages in USSR and China) and the present experiences (e.g. the environmental pollution in China) of socialist countries. Many highly-developed countries (e.g. UK) went through severe phases of de-industrialization, which were characterized by unemployment, urban decline and political/social instabilities. These crises can be avoided if an overshooting of the manufacturing sector is avoided and, in particular, the manufacturing share (in GDP or employment) is kept approximately constant as suggested by policy (a)-(c). However, our results do not prohibit a restructuring of the manufacturing sector towards more modern products and technologies, while keeping the employment share of the manufacturing sector constant. Thus, policy (a)-(c) is rather a policy of restructuring the manufacturing sector than a policy of increasing its share/size disproportionately.

Finally, it seems that the 'recent Indish' strategy, which refers to a transformation from an agricultural to a services economy, is consistent with policy (a)-(c).

6.5 A Comparison of Empirically Observed Structural Change Paths and Policy (a)-(c)

Discussing and comparing the structural change paths and their costs across countries is a relatively extensive task and an interesting topic for further research. To demonstrate the direct and simple applicability of the concepts developed in our paper, we briefly discuss here the long-run data on structural changes in present-day's most developed and emerging countries. By using Property 1, we can analyze the monotonicity features of this long run data. This property and Figure 8 imply that the countries' agricultural (services) shares decreased (increased) in the long run, thus being consistent with the aspects (a) and (c) of the policy derived in our paper. Moreover, Figure 8 shows that Germany and UK had developed the highest manufacturing shares over time (as implied by Property 1).[4] UK has reduced the employment share again, resulting in a very curved[5] structural change path. This contradicts policy (a)-(c), and our measure $c^{0\bar{t}}$ implies that the structural change costs associated with this path are relatively high. Whether Germany will face high overall structural change costs depends on its future development (i.e. the future degree of de-industrialization). Moreover, Figure 8 reveals that China has developed against policy (a)-(c) by pursuing a strong industrialization program.

7. Concluding Remarks

The growth and development process is characterized by massive structural change, which generates high costs for the society and the economy ranging from pollution to unemployment. In this paper, we have derived the properties of the development path that minimizes the structural change costs in the three-sector framework depending on the destination of the path, where we assumed that the structural change costs increase with the strength of structural change. Moreover, we have discussed the structural change theories and the empirical evidence and derived the literature consensus/prediction regarding the destination of the structural change path of a today's underdeveloped economy. The consensus statements are crude and qualitative such that the set ($\mathbf{D}^{\bar{t}}$) of potential destinations implied by the consensus is relatively great. For this reason, among others, we had to apply qualitative/geometrical modeling techniques for deriving the structural change costs-minimizing policy in a today's underdeveloped country when assuming that the country's destination is located in the set $\mathbf{D}^{\bar{t}}$. We have shown that the cost-minimizing policy is characterized by a decreasing agricultural employment share, a constant manufacturing employment share and a growing services

[4] The magnitude of the manufacturing employment share in Figure 8 is indicated by the closeness to vertex V_2 (see also Stijepic (2015)). As we can see, the trajectories of Germany and UK come very close to vertex V_2.
[5] In particular, the fact that the path is curved with respect to the V_1V_3-edge of the simplex is relevant. It implies that the manufacturing share increased strongly (as the economy moved away from the V_1V_3-edge) and, then, decreased strongly (as the economy moved towards the V_1V_3-edge), as discussed by Stijepic (2015).

employment share. Finally, we applied this theoretical result for evaluating (a) the standard development strategies and (b) some historically observed structural change paths in developed economies regarding the structural change costs they generate (cf. Section 6). As we have shown, our results imply among others that the standard development strategies generate relatively high structural change costs and that, e.g., UK, Germany and China have chosen structural change paths that are (potentially) associated with high structural change costs.

While these applications are only brief demonstrations of the applicability of our results, future research could focus on more elaborate (empirical) studies of these aspects. For example, countries could be grouped into groups with relatively high and relatively low structural change costs and the properties of these groups (e.g. prevalence of crises, political regime, etc.) could be analyzed. Moreover, the importance of the structural change costs in relation to the other effects of structural policies discussed in Section 2 for welfare and growth could be estimated.

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APPENDIX (Proof of Lemma 1)

Lemma 1 refers to the solution of the following problem*: $\min c^{0\bar{t}}$, where $c^{0\bar{t}}$ is given by (3) and $X(0) = X^0$ and $X(\bar{t}) = X^{\bar{t}}$!* Since, among others, $c^{0\bar{t}}$ is monotonous in $r^{0\bar{t}}$, we can rewrite this problem as follows:

(A1) $\quad$ *$\mathrm{Min}\, r^{0\bar{t}}$, where $r^{0\bar{t}} := \sum_{i=1}^{n} r_i^{0\bar{t}}$, $r_i^{0\bar{t}} := \int_0^{\bar{t}} |x_i'(t)| dt$ and $X(0) = X^0$ and $X(\bar{t}) = X^{\bar{t}}$!*

First, we solve the following problem, which is simpler:

(A2) $\mathrm{Min}\, r_i^{0\bar{t}}$, where $r_i^{0\bar{t}} := \int_0^{\bar{t}} |x_i'(t)| dt$ and $x_i(0) = x_i^0$ and $x_i(\bar{t}) = x_i^{\bar{t}}$ are given.

Note that $\forall t \forall i$, $x_i(t)$ must be continuous in *t* (see Assumption 2 and Lemma 1). First, assume that $x_i^{\bar{t}} > x_i^0$. Problem (A2) is about finding the path $x_i^*(t)$, $0 \le t \le \bar{t}$, that minimizes $r_i^{0\bar{t}}$, where we must search among all the (continuous) paths that connect x_i^0 and $x_i^{\bar{t}}$ on the Real line (one-dimensional Euclidean space, **R**). Obviously, if $x_i^{\bar{t}} > x_i^0$, a monotonously decreasing path ($\forall t\ x_i'(t) \le 0$) cannot connect x_i^0 and $x_i^{\bar{t}}$ (see Figure A1). Thus:

Property A1. *If $x_i^{\bar{t}} > x_i^0$, only two classes of paths are admissible in the solution of problem (A2): (A) monotonously increasing paths ($\forall t\ x_i'(t) \ge 0$) and (B) non-monotonous paths (see Figure A2).*

Figure A1.

- insert Figure A1 here -

Figure A2.

- insert Figure A2 here -

First, consider class A. The geometrical interpretation of a monotonously increasing path (connecting x_i^0 and $x_i^{\bar{t}}$) on **R** is relatively straight forward: it is a path on the real line along which the economy moves from x_i^0 to $x_i^{\bar{t}}$ monotonously, i.e. the movement (from x_i^0 to $x_i^{\bar{t}}$)

is *unidirectional* (see Figure A3). The length of this path is equal to the length of the real line-segment x_i^0 - $x_i^{\bar{t}}$, i.e. $\left|x_i^{\bar{t}} - x_i^0\right|$.[6]

Figure A3.

- insert Figure A3 here -

In contrast, a non-monotonous path (class B) is characterized by at least one *change in direction*. A non-monotonous path (on **R**) is associated with at least one point in time $t_1 \in [0, \bar{t}]$ at which the economy does not move towards $x_i^{\bar{t}}$ but away from $x_i^{\bar{t}}$, i.e. there is a "backward step" or an "overshooting step" (see Figures A4 and A5 for illustrative examples). Furthermore, we know that the economy must turn towards $x_i^{\bar{t}}$ again at some later point in time $t_2 \in (t_1, \bar{t})$, since the economy must arrive at $x_i^{\bar{t}}$ at time $\bar{t}$. Obviously, such a path (i.e. a path with at least one change in direction) is longer than a monotonous path: the length of the path with a "backward/overshooting step" is equal to the length of the monotonous path ($\left|x_i^{\bar{t}} - x_i^0\right|$) plus two times the length of the "backward/overshooting step"; cf. Figures A4 and A5. Overall, these facts imply the following statement:

Property A2. *If $x_i^{\bar{t}} > x_i^0$, the length of a non-monotonous path connecting x_i^0 and $x_i^{\bar{t}}$ on the Real line is greater than the length of a monotonous path connecting x_i^0 and $x_i^{\bar{t}}$ on the Real line.*

Figure A4.

- insert Figure A4 here -

Figure A5.

- insert Figure A5 here -

[6] Recall that the (Euclidean) length of an interval (or line-segment) on the real line is given by the absolute value of the difference between its endpoints. Most introductory books on analysis discuss this fact. For a discussion of the length of paths in two-dimensional space, where the (Euclidean) length of the path is measured by a quadratic formula, see, e.g., Gelfand and Fomin (1963). In one-dimensional space this quadratic formula becomes the absolute value function that we use.

The proof of the following two properties is analogous to the proof of Properties A1 and A2.

Property A3. *If* $x_i^{\bar{t}} < x_i^0$, *only two classes of paths are admissible in the solution of problem (A2): (I) monotonously decreasing paths (*$\forall t \;\; x_i'(t) \leq 0$*) and (II) non-monotonous paths.*

Property A4. *If* $x_i^{\bar{t}} < x_i^0$, *the length of a non-monotonous path connecting* x_i^0 *and* $x_i^{\bar{t}}$ *on the Real line is greater than the length of a monotonous path connecting* x_i^0 *and* $x_i^{\bar{t}}$ *on the Real line.*

Now, we show that the length of a path is equal to the $r_i^{0\bar{t}}$ associated with this path. Let t_1, t_2, $\ldots t_Z$ denote the points of time at which the economy changes its direction. See Figure A6. A direction change at time $t_B \in (0,\bar{t})$ is given if there exists a $t_A \in (0,\bar{t})$ and $t_C \in (0,\bar{t})$ such that **either** $(\forall t \in (t_A, t_B]\, x_i'(t) \geq 0) \wedge (\exists t \in (t_A, t_B]\, x_i'(t) > 0) \wedge (\forall t \in (t_B, t_C)\, x_i'(t) < 0)$ **or** $(\forall t \in (t_A, t_B]\; x_i'(t) \leq 0) \wedge (\exists t \in (t_A, t_B]\, x_i'(t) < 0) \wedge (\forall t \in (t_B, t_C)\, x_i'(t) > 0)$. In this case:

(A3) $$r_i^{0\bar{t}} := \int_0^{\bar{t}} |x_i'(t)| dt = \int_0^{t_1} |x_i'(t)| dt + \int_{t_1}^{t_2} |x_i'(t)| dt + \ldots \int_{t_Z}^{\bar{t}} |x_i'(t)| dt$$

Since there are no changes in direction within the intervals $(t_1, t_2]$, $(t_2, t_3]$,… $(t_Z, \bar{t}]$ per definition of the t_1, t_2 ,… t_Z, $x_i(t)$ is monotonous within these intervals and we can rewrite (A3) as follows:

(A4) $$\begin{aligned} r_i^{0\bar{t}} &= \left| \int_0^{t_1} x_i'(t) dt \right| + \left| \int_{t_1}^{t_2} x_i'(t) dt \right| + \ldots \left| \int_{t_Z}^{\bar{t}} x_i'(t) dt \right| \\ &= |x_i(t_1) - x_i(0)| + |x_i(t_2) - x_i(t_1)| + \ldots |x_i(\bar{t}) - x_i(t_Z)| \end{aligned}$$

In fact, our definition of the points t_1, t_2 ,… t_Z implies a partitioning of the path (connecting x_i^0 and $x_i^{\bar{t}}$) into sections/partitions of monotonous dynamics (see Figure A6). (A4) implies that $r_i^{0\bar{t}}$ is equal to the sum of the lengths of the partitions of monotonous dynamics (see Figure A6 for an example). This is consistent with the natural/standard definition of path length used in Properties A2 and A4.[7] Thus, we can state the following property:

[7] That is, the length of a path on **R** is equal to the sum of the lengths of its partitions of monotonous dynamics. This result is consistent with the standard definition of path length in two-dimensional Euclidean space (see the previous footnote).

Property A5. *$r_i^{0\bar{t}}$ is equal to the length of the path connecting x_i^0 and $x_i^{\bar{t}}$ on the Real line.*

Figure A6.

- insert Figure A6 here -

Obviously, if $x_i^{\bar{t}} = x_i^0$, $r_i^{0\bar{t}}$ is minimized if the economy stays in x_i^0 for all *t*, i.e. $\forall t\, x_i'(t) = 0$, which corresponds per Definition 3 to a monotonous path. Thus:

Property A6. *If $x_i^{\bar{t}} = x_i^0$, the solution of the problem (A2) is given by a monotonous path ($\forall t\, x_i'(t) = 0$). In this case, the minimal $r_i^{0\bar{t}}$ is equal to 0.*

Furthermore, if $x_i^{\bar{t}} \neq x_i^0$, all monotonous paths connecting x_i^0 and $x_i^{\bar{t}}$ on **R** have the same length and, thus, the same value of $r_i^{0\bar{t}}$, since if the path is monotonous we can write:

(A5) $$r_i^{0\bar{t}} := \int_0^{\bar{t}} |x_i'(t)| dt = \left| \int_0^{\bar{t}} x_i'(t) dt \right| = \left| x_i^{\bar{t}} - x_i^0 \right| > 0 .$$

Property A7. *Any monotonous path connecting x_i^0 and $x_i^{\bar{t}}$ on **R** is characterized by $r_i^{0\bar{t}}$ = $\left| x_i^{\bar{t}} - x_i^0 \right|$.*

Overall, Properties A1-A7 imply the following lemma:

Lemma A1. *The solution of problem (A2) is given by a monotonous path. In particular, any monotonous path connecting x_i^0 and $x_i^{\bar{t}}$ on* **R** *is associated with minimal $r_i^{0\bar{t}}$. If $x_i^{\bar{t}} \neq x_i^0$, the minimal $r_i^{0\bar{t}}$ is equal to $\left| x_i^{\bar{t}} - x_i^0 \right| > 0$. If $x_i^{\bar{t}} = x_i^0$, the minimal $r_i^{0\bar{t}}$ is equal to 0. Here, the path connecting x_i^0 and $x_i^{\bar{t}}$ on* **R** *is monotonous if either $\forall t \in [0, \bar{t}]\; x_i'(t) \geq 0$ or $\forall t \in [0, \bar{t}]$ $x_i'(t) \leq 0$.*

Now, we can turn to the solution of the problem (A1). Since x_i are independent of each other (cf. Definition 1), $r_i^{0\bar{t}}$ are independent of each other (cf. (A1)). Furthermore, as implied by (A1), $\forall i\;\; r_i^{0\bar{t}} \geq 0$. Thus, the cost-index $r^{0\bar{t}} = r_1^{0\bar{t}} + r_2^{0\bar{t}} + ... r_n^{0\bar{t}}$ is separable. That is, minimizing

$r^{0\bar{t}}$ is equivalent to minimizing $r_i^{0\bar{t}}$ $\forall i$. The minimum of $r^{0\bar{t}}$ is attained if and only if all $r_i^{0\bar{t}}$ are minimal. This fact and Lemma A1 imply that $r^{0\bar{t}}$ is minimal if and only if all x_i behave monotonously. In other words, $r^{0\bar{t}}$ is minimal if and only if there does not exist any x_i that behaves non-monotonously. That is, $r^{0\bar{t}}$ is minimal if and only if:

(A6) $\quad \nexists\, i \in \{1,2,\ldots n\} : (\exists t_a \in [0,\bar{t}] \wedge \exists t_b \in [0,\bar{t}] : t_a \neq t_b \wedge x_i'(t_a) < 0 \wedge x_i'(t_b) > 0)$.

(A6) corresponds to the definition of a monotonous development path (see Definition 3). Finally note that $c^{0\bar{t}}$ is monotonously increasing in $r^{0\bar{t}}$. These facts prove Lemma 1.

FIGURES

Figure 1

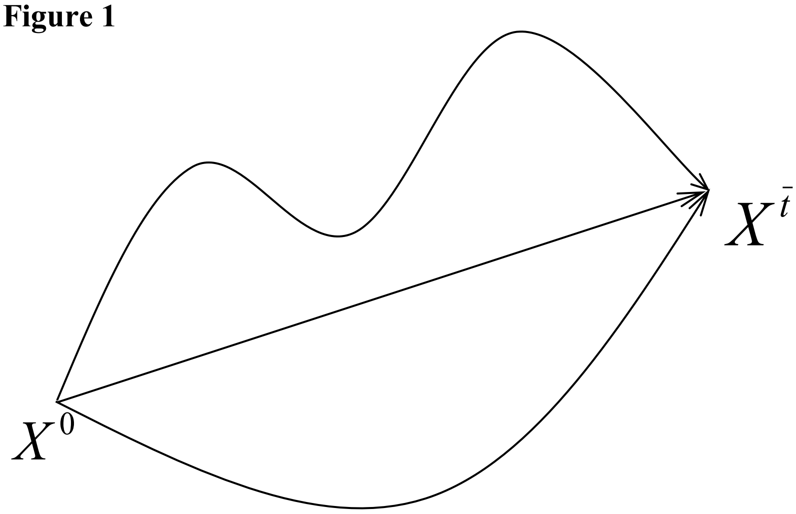


Figure 2

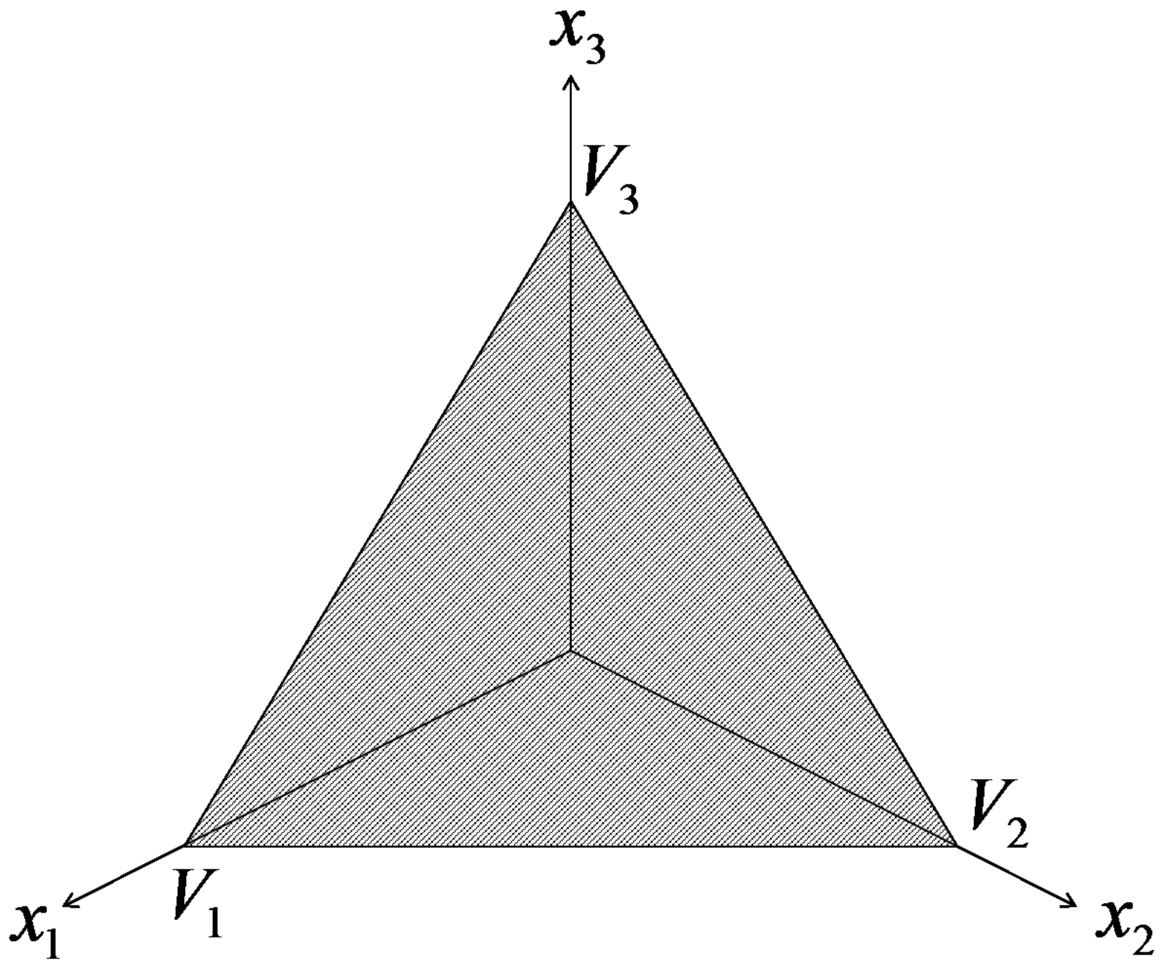


Figure 3

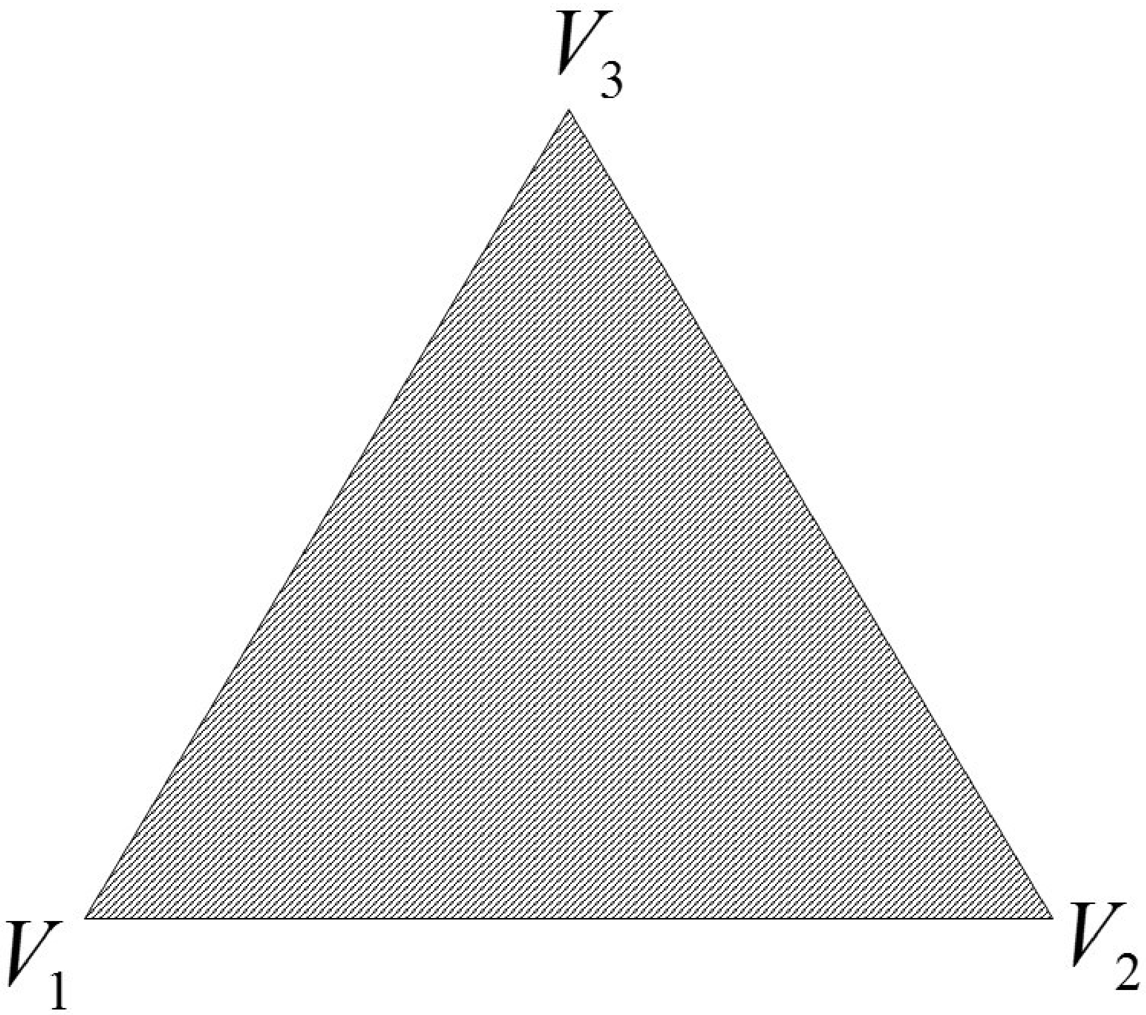


Figure 4

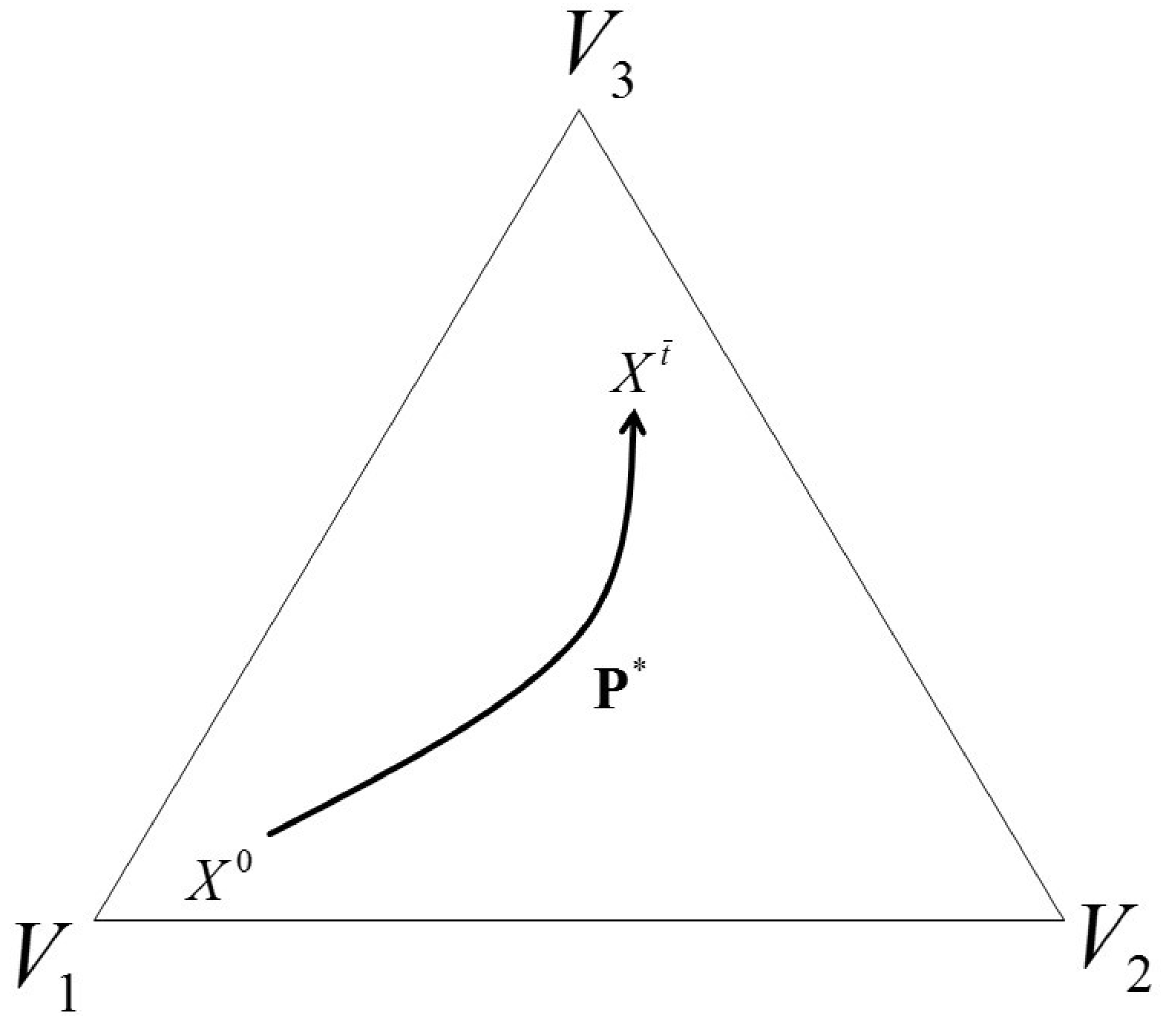
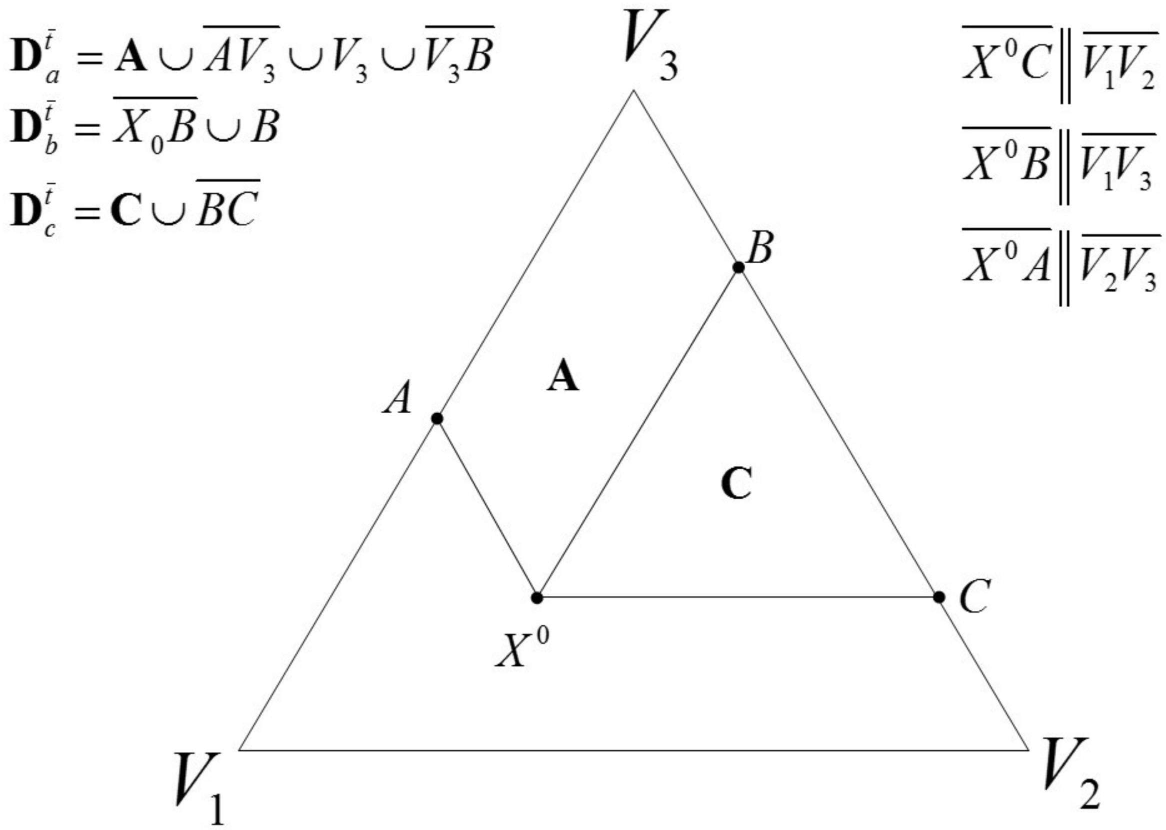


Figure 5



$\mathbf{D}_a^{\bar{t}} = \mathbf{A} \cup \overline{AV_3} \cup V_3 \cup \overline{V_3 B}$
$\mathbf{D}_b^{\bar{t}} = \overline{X_0 B} \cup B$
$\mathbf{D}_c^{\bar{t}} = \mathbf{C} \cup \overline{BC}$
$\overline{X^0 C} \parallel \overline{V_1 V_2}$
$\overline{X^0 B} \parallel \overline{V_1 V_3}$
$\overline{X^0 A} \parallel \overline{V_2 V_3}$
V_3
B
A
A
C
C
X^0
V_1
V_2

Figure 6

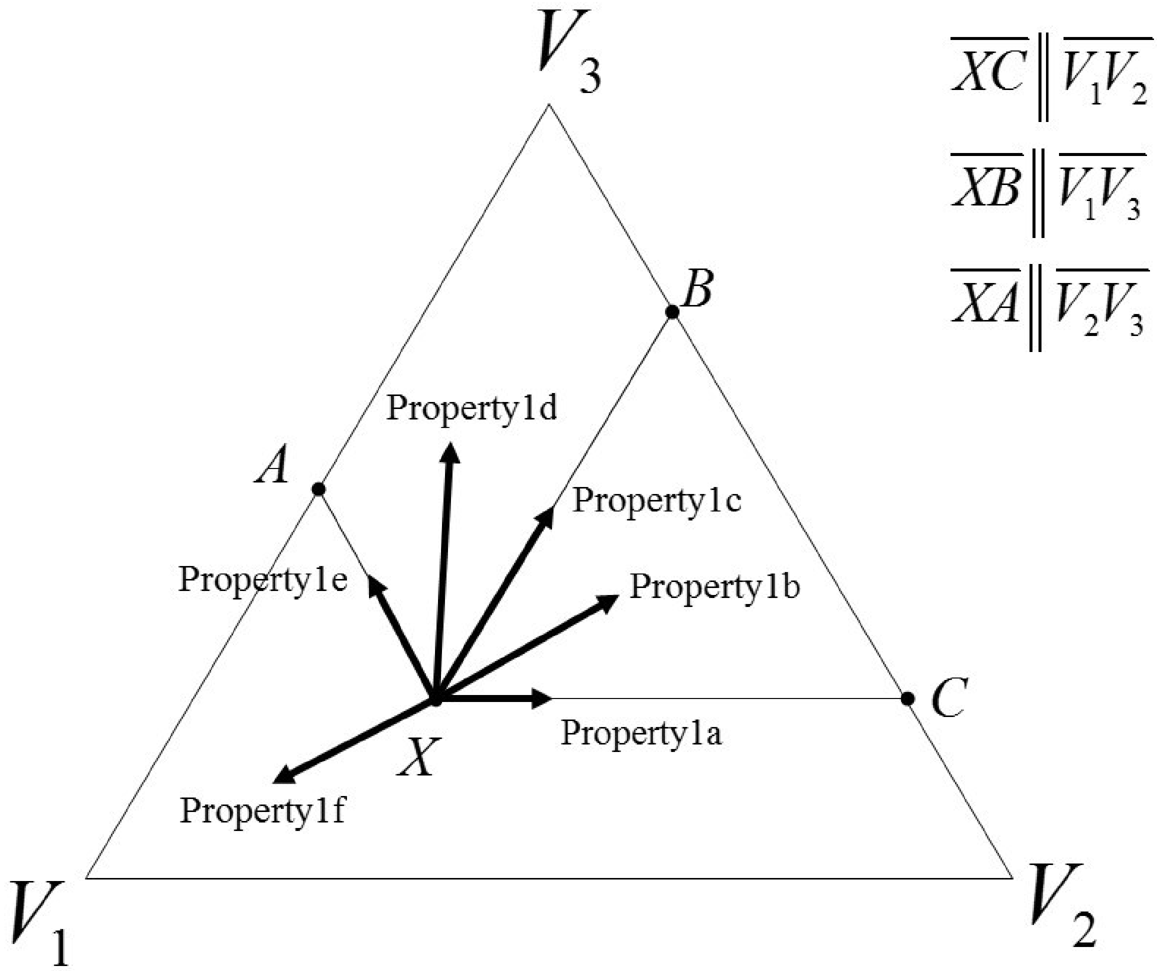
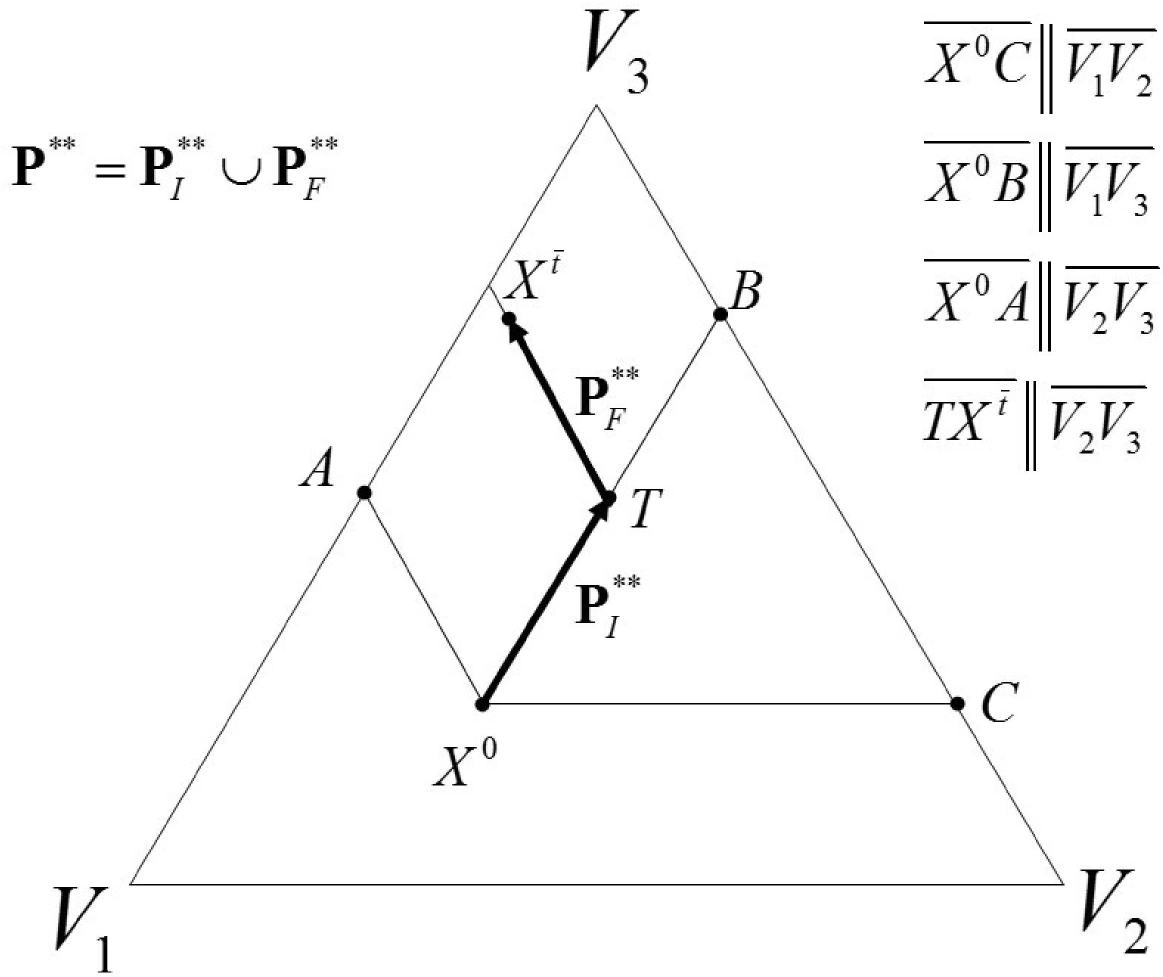

V_3
$\overline{XC} \parallel \overline{V_1V_2}$
$\overline{XB} \parallel \overline{V_1V_3}$
$\overline{XA} \parallel \overline{V_2V_3}$
B
Property1d
A
Property1c
Property1e
Property1b
C
Property1a
X
Property1f
V_1
V_2

Figure 7



V_3
$\overline{X^0C} \parallel \overline{V_1V_2}$
$\mathbf{P}^{**} = \mathbf{P}_I^{**} \cup \mathbf{P}_F^{**}$
$\overline{X^0B} \parallel \overline{V_1V_3}$
$X^{\bar{t}}$
B
$\overline{X^0A} \parallel \overline{V_2V_3}$
$\mathbf{P}_F^{**}$
$\overline{TX^{\bar{t}}} \parallel \overline{V_2V_3}$
A
T
$\mathbf{P}_I^{**}$
C
X^0
V_1
V_2

Figure 8

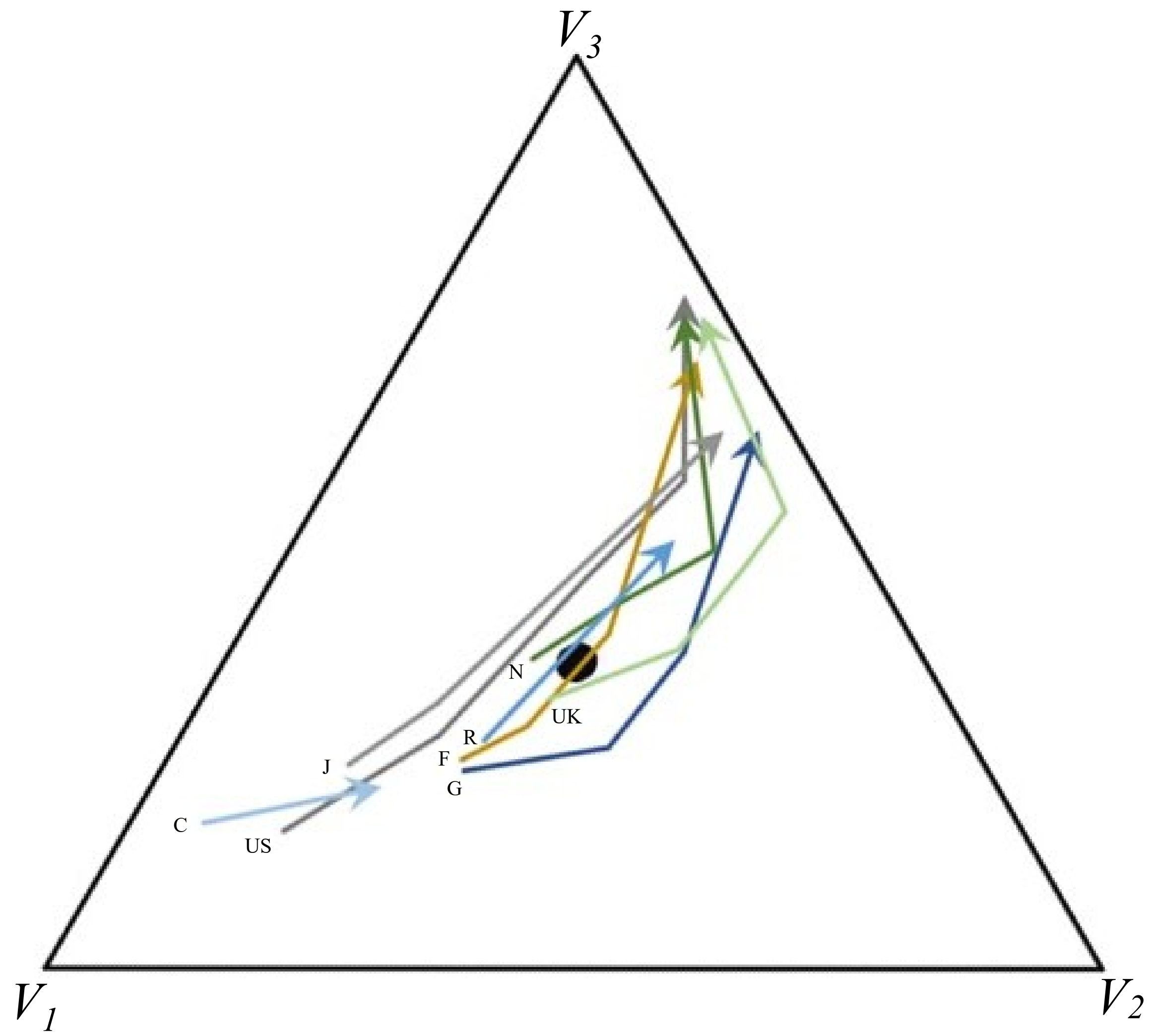


Figure A1

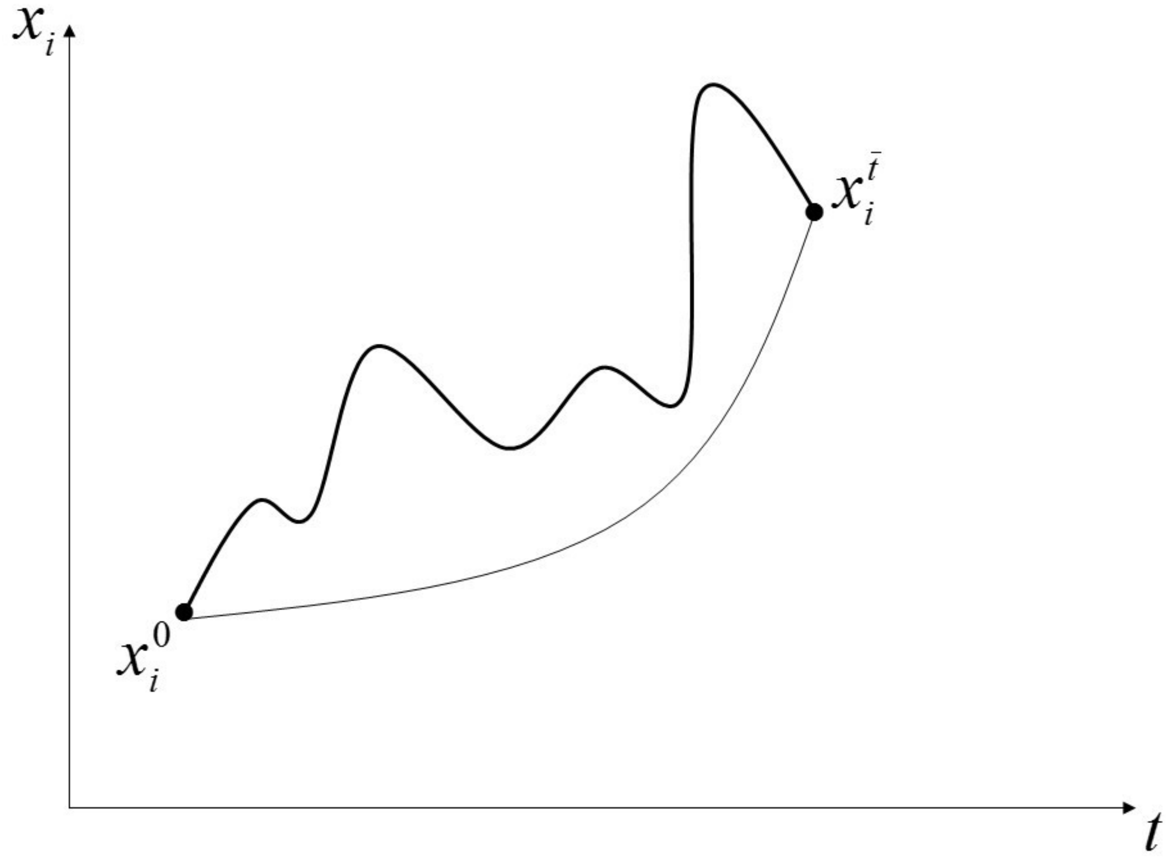


Figure A2

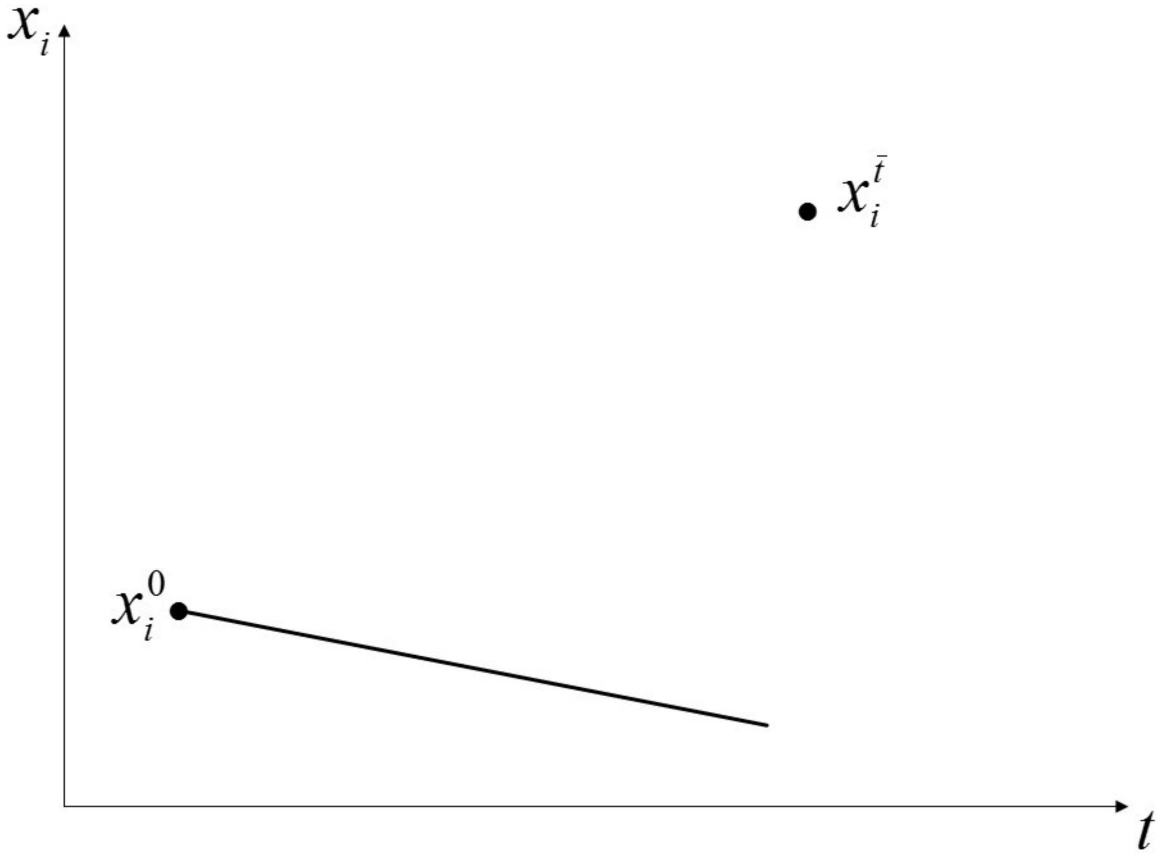


Figure A3

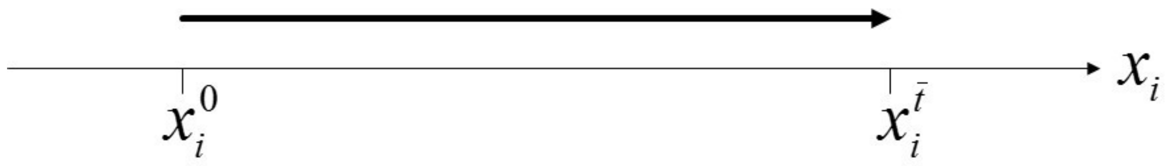


Figure A4

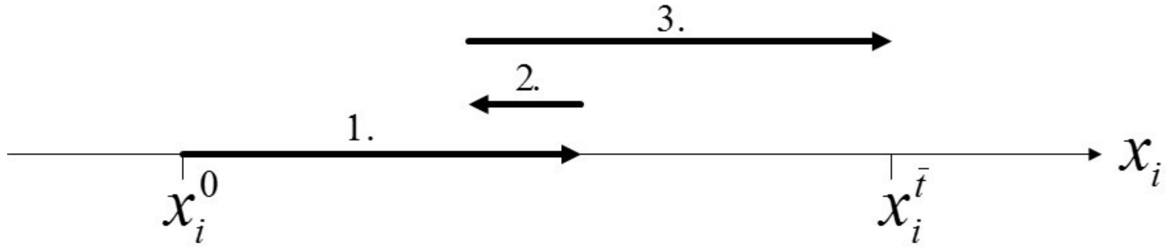


Figure A5

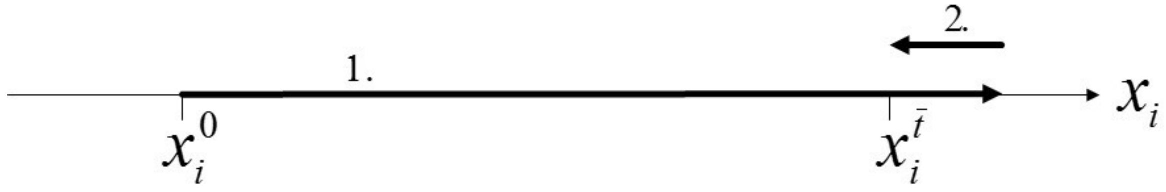


Figure A6