Dividend Taxes, Household Heterogeneity, and the US Great Depression

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Abstract: As shown by McGrattan (2012), an anticipated increase in dividend taxes plays an important role in explaining the dramatic investment decline during the U.S. Great Depression. This paper attempts to investigate whether this is still robust to a model with household heterogeneity and precautionary saving motives. I build an Aiyagari model with dynamic firms, dividend taxes, and labor productivity shocks, which are calibrated to account for the U.S. earnings and wealth inequality during the Great Depression using 1930s data. The conclusion is that the impact of anticipated increases in dividend taxes is very sensitive to the presence of household heterogeneity. The quantitative results show that, in the presence of household heterogeneity, the predicted investment is 50% smaller than in the standard business cycle model proposed by McGrattan (2012). The decline in output and working hours also become much less significant. Intuitively, although the anticipated hike in dividend taxes diminishes the expected return to the investment, it also shrinks the total assets that households use for self-insurance against the highly persistent idiosyncratic shocks. In order to retain the desired asset level, precautionous households keep their savings at a lower capital return and the model ultimately generates a lower aggregate investment decline.

Keywords: Dividend Taxes, Household Heterogeneity, Investment, and the U.S. Great Depression

JEL Classification Numbers: E23, E44, E62, D52

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1 Introduction

Recent studies question the traditional opinion that fiscal policies contribute little to the dramatic economic downturn and subsequent slow recovery during the US Great Depression. Their neoclassical growth model with disaggregated taxes and a specific anticipation predicts patterns of output, investment and working hours that are more comparable to data. The anticipated increases in capital taxes, dividend tax and undistributed profit tax, are claimed as the most important reason for this conclusion. This paper explores the importance of fiscal policies during the Great Depression again but in an incomplete market framework. I establish a heterogeneous-agent model with disaggregated taxes and government spending and assume that all the households have a perfect foresight anticipation of the future fiscal changes. With all parameters calibrated to the US economy in 1929, including aggregates and inequalities in income and wealth, I solve for the transition of economic aggregates from 1929 to 1939, in which the actual changes of fiscal policy are imposed annually. My main finding is that the impacts of anticipated increases in dividend tax and undistributed profit tax are very sensitive to the presence of household heterogeneity. The anticipated increase of dividend tax and undistributed profit tax fails to take down the economy as they do in the representative-agent framework. The predicted decrease in investment is 50% smaller. The decreases in output and working hours are much less significant. My explanation to these differences is that households are forced to require a lower return to their savings when the increase of dividend tax or undistributed profit tax shrinks the total asset value in an incomplete market because the extremely high persistency of idiosyncratic labor productivity shocks gives households a stronger motive to acquire assets. The low return to assets furthermore leads to a smaller decline in investment if the no arbitrage condition holds.

There is no consensus yet about the cause of the US Great Depression, although enormous amount of literatures attempt to explain the dramatic economic contraction and subsequent slow recovery in the 1930s from different perspectives. However, it is surprising that only a few of them pay attention to fiscal policy regardless of the large and frequent fiscal changes at that time: The government spending increases a lot as a fraction of GDP. The annual data from National Income and Product Accounts (called NIPA later on) exhibits that the proportion of
government spending in GDP got doubled from 1929 to 1939; The absolute level and progressivity
of personal income tax and corporate income tax are both skyrocketing\(^1\), as many literatures
also confirm such polytropical situations: Joines (1981) estimates the average marginal tax rates
on corporate profit and labor income respectively; McGrattan (2010) constructs dividend and
undistributed profit tax rates from the *Statistic of Income* and sale and property tax rates from
NIPA, and also modifies corporate profit tax rate in Joines (1981) by adding the impact of
undistributed profit tax. Figure 1 summarizes the trends of effective rates of different tax and
government spending during the US Great Depression according to the above literatures. The
efforts to retrieve tax rates and government spending make the recent studies on the impacts of
fiscal policies during Great Depression feasible. Cole and Ohanian (1999) constructs a growth
model with labor income tax, capital income tax and government spending. They calibrate the
steady state of their model to US economy under fiscal policy of 1929 first and then solve for
another steady state under fiscal policy of 1939. The result shows that the change in fiscal factors
only explains 4% of the decline in outputs so they conclude that fiscal policies play little role.
McGrattan (2010) challenges this conclusion by adding more disaggregated taxes and a specific
anticipation of tax changes to the growth model above. The additional taxes include sale tax,
property tax, dividend tax and undistributed profit tax. The anticipation, which is built on the
basis of news reports then, helps match the data series well along with more delicate fiscal setup.
The investment simulation in the model almost accounts for all the decline of its US counterpart
in 1930s; The simulated output and working hours do a much better job than Cole and Ohanian
(1999) in matching actual data series. In addition, the similar methodology to study short-run
tax change under forward anticipation is first discussed by Auerbach and Hines (1988), in which
they analyze the impact of anticipated corporate tax reform on the annual investment decision
of firms and also evaluate three possible reform treatments with the simulation of a neoclassic model.

\[\text{Figure 1 about here.}\]

In my model the households have the same preference but differ from each other in asset
holdings and idiosyncratic labor productivity shocks. Households can only access to a unique

\(^{1}\) *Tax Foundation* provides historical series in personal income tax and corporate income tax. See
aggregate asset market where short selling is not allowed. The features of such incomplete market setup have been studied by Aiyagari (1994) that households do precautionary savings to partially insure against the idiosyncratic labor productivity risk and therefore possess an upward-sloping asset demand curve. Recently heterogeneity and incomplete market have become an key perspective of the study of taxation. Anagnostopoulos et al. (2011) investigates the impacts of Jobs and Growth Tax Relief Reconciliation Act (JGTRRA) of 2003 with household heterogeneity. They build a heterogeneous household model with dividend tax and capital tax and find out that dividend tax cuts, contrary to capital gains tax cuts, lead to a decrease in investment and capital. This surprising conclusion is because the dividend tax cuts increase the market value of existing capital and households require a higher return to hold this additional wealth. Such mechanism is similar to the wealth effect in the classic microeconomic analysis. Gourio and Miao (2010) studies the long-run effect of dividend taxation on aggregate capital accumulation, they build a dynamic general equilibrium model in which there is a continuum of firms subject to idiosyncratic productivity shocks and find that a dividend tax cut raises aggregate output through reducing the frictions in the reallocation of capital across firms. Furthermore, Gourio and Miao (2011) shows how to solve for a transition between two steady states with different dividend tax and capital gain tax regimes in the heterogeneous firm framework, and that dividend payments, equity issuance, and aggregate investment rise immediately when the dividend and capital gains tax cuts are unexpected and permanent. By contrast, when these tax cuts are unexpected and temporary, aggregate investment falls in the short run. This fall allows firms to distribute large dividends initially in response to the temporary dividend tax cut. The effects of a temporary dividend tax cut are very different from those of a temporary capital gains tax cut.

Although heterogeneous-agent is more realistic and improves the comparability between models and realities, there is a considerable price of such advantages in practice. For instance, it is difficult to obtain a satisfactory earning process which can generate income and wealth inequalities consistent with their counterparts in data. Because, first, it is not always feasible to calibrate an earning process computationally. Castaeda et al. (2003) suggests an accomplishable method, in which households are assumed to face three risks from labor productivity, retirement and mortality. But all these three shocks are assumed to be governed by the same Markov process for convenience.
They succeed in finding parameters to account for the earning and income Lorenz curve estimated from *Survey of Consumer Finance 1986* and also the corresponding economic aggregates. Pijoan-Mas et al. (2001) simplifies this method to fit the circumstance where households face only an idiosyncratic productivity shock. This approach can be briefly described as to solve a nonlinear equation system consisting of equal numbers of unknown parameters and target conditions. Second, qualified income and wealth inequalities data are not always available, especially during the interested inter-war era. The qualification problem arises from the limitation of parsimonious model setup. Usually more state dimensions in the labor productivity shock can generate more delicate wealth and income Lorenz curves. However, there exists a tradeoff between the accuracy and efficiency in computation. So the dimension in productivity shocks can not be very large\(^2\). Moreover, the productivity shocks with limited dimension are just able to identify a rough shape of Lorenz curve characterized by a handful of sparse grids. It in turn requires that the distribution grids estimated from data should be in the same number and also sparse enough along the Lorenz curve that they can contain as much information of inequalities as possible. Unfortunately the surveys that meets the above standard merely exists in the first half of 20th century and even fewer before World War II. Fortunately some individual studies try to retrieve data before 1930s: *World Top Income Database*\(^3\) collects the longitudinal record of US top income shares from Piketty and Saez (2006); *Handbook of Income distribution* presents some statistics of income distribution from Kuznets (1955) and Goldsmith (1967), and also some statistics of the wealth distribution from Edward N. Wolff (1989); Williamson and Lindert (1980) incorporate "king's and Williamson-Lindbert's estimates" of wealth distribution from a Federal Trade Committee special survey of 43,512 probate estate valuations from 23 counties in 13 states plus the District of Columbia in 1926.

The paper is organized as follows. In section 2, I construct a benchmark model without household heterogeneity to duplicate the results of McGrattan (2010) for later comparison, and also decompose the impacts of different fiscal factors to confirm the mechanism. In section 3, I introduce the setup of the heterogeneous agent model and define the recursive competitive equilibrium. In section 4, I summarize the data source and calibration strategies. The section 5, 6 and 7 explain

\(^2\)For instance, Castaeda et al. (2003) uses 4-dimension productivity shocks and 4 by 4 transition matrix to target the 10 quintiles of income and wealth distribution.

\(^3\)See the website http://g-mond.parisschoolofeconomics.eu/topincomes/
the solution method, results and intuition respectively. The final section concludes.

2 Benchmark Model

This section is to build a calibrated homogeneous-agent model with disaggregated taxes and a forward anticipation pattern as a benchmark for the future study on heterogeneous-agent cases. All the tax rates, government spending and anticipation pattern are consistent with McGrattan (2010). Therefore, it duplicates the results in McGrattan (2010). Furthermore, many additional experiments are taken to examine the impact of different fiscal factors and anticipations.

2.1 Setup

Households live in an infinite horizon and gain utility streams from consumption $c_t$ and leisure $l_t$. The discount factor is $\beta$. Their total labor endowment is normalized into unity so the leisure can be considered as $1 - h_t^h$ where $h_t^h$ represents working hours, that’s the fraction of 24 hours devoted to working. Households own mature dynamic firms through holding shares $s_t$ and hence receive the corresponding dividends $d_t$. They are able to sell or buy the shares next period $s_{t+1}$ but no short selling is allowed, namely $s_{t+1} \geq 0$. The government collects consumption (sale) tax at $\tau_c$, labor income tax at $\tau_h$ and dividend tax at $\tau_d$ from households and gives the budget surplus or deficit back to households as a lump-sum transfer $\kappa_t$. The total tax payment of households is denoted by $\Gamma^h$. All the fiscal factors are governed by the fiscal state $z_t$. $z_t$ is exogenous and perceived as a Markov process by households. The households’ perception of $z_t$ will be discussed later in section 2.2. The optimization problem of households can be put as follows,

$$\max_{\{c_t, s_{t+1}, h_t^h\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(c_t) + \psi \log(l_t) \right] \quad (1)$$

s.t. $c_t + p_t s_{t+1} \leq (p_t + d_t) s_t + w_t h_t^h - \Gamma^h(z_t) + \kappa_t$

$l_t = 1 - h_t^h$

$s_{t+1} \geq 0$

$$\Gamma^h(z_t) = \tau_d(z_t) d_t s_t + \tau_c(z_t) c_t + \tau_h(z_t) w_t h_t^h$$
Thus, the first order conditions to maximize their utility stream subject to $h^t_t$ and $s_{t+1}$ are respectively:

$$\psi = [1 - \tau_h(z_t)]w_t$$
$$E_t[\beta p_{t+1} + [1 - \tau_d(z_{t+1})]d_{t+1}] = 1$$

The dynamic firms in this economy are assumed to possess identical Cobb-Douglas technology. They input their own capital $k_t$ and labor $h^F_t$ rented from households at wage rate $w_t$ into production each period. They pay out the dividends $d_t$ after investment $x_t$ and tax payment $\Gamma^f$, which includes profit tax with rate $\tau_p$, property tax with rate $\tau_k$ and undistributed profit tax with rate $\tau_u$. The discount factor of firms $\Lambda_t$ is consistent with the time preference of households, namely $\Lambda_t = E_t[\beta^{1 + \tau_d(z_{t+1})}]$. Then the problem of dynamic firms is as follows:

$$\max \sum_{t=0}^{\infty} \Lambda_t (1 - \tau_{dt})d_t$$

s.t. $d_t = f(k_t, h^F_t) - w_t h^F_t - x_t - \Gamma^f(z_t)$

$$f(k_t, h^F_t) = k_t^\theta (Ah^F_t)^{1-\theta}$$

$$x_t = k_{t+1} - (1 - \delta)k_t$$

First order conditions to maximize firm value subject to $k_{t+1}$ and $h^F_t$ are listed below,

$$1 = \frac{\Lambda_{t+1}[1 - \tau_d(z_{t+1})][f_{k_{t+1}}(k_{t+1}, h^F_{t+1}) - \delta - \tau_k(z_{t+1})]}{\Lambda_t[1 + \tau_u(z_t)][1 - \tau_d(z_t)]}$$

$$w_t = f_{h^F_t}(k_t, h^F_t)$$

The government collects the taxation revenue $\Gamma^h(z_t)$ and $\Gamma^f(z_t)$ from households and firms, spends $g(z_t)$ and transfers the surplus or deficit $\kappa_t$ to households for a balanced budget each period.

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4It implicitly requires a no-arbitrage condition holds. See the proof in Appendix.
The budget constraint for the government is:

\[ \Gamma^h(z_t) + \Gamma^f(z_t) = g(z_t) + \kappa_t \]  

(7)

There are totally three markets in this economy: common goods, labor and stock shares. Their market clearing conditions are respectively,

\[ c_t + x_t + g(z_t) = f(k_t, h_t) \]  

(8)

\[ s_{t+1} = 1 \]  

(9)

\[ h_t^h = h_t^f \]  

(10)

### 2.2 Anticipation

\( z_t \) determines tax rates and government spending \( \{\tau_p, \tau_d, \tau_k, \tau_c, \tau_u, \tau_h, g\} \). I put the same assumption on the anticipation of fiscal regimes \( z_t \) as McGrattan (2010) does. According to the perception of households, \( z_t \) can only take on 11 possible states, which correspond to 11 annual fiscal policy states in US from 1929 to 1939. However, households don’t always have an accurate knowledge of the incoming fiscal state next period and only infer it with certain anticipations, which could be abstracted by an 11-by-11 transition matrix \( \Pi(z_{t+1} | z_t) \). In this way \( z_t \) is considered to follow a Markov process by households when they make consumption or saving or working decisions. The specific transition matrix \( \Pi(z_{t+1} | z_t) \) is taken from McGrattan (2010) and shown in table 1. Actually this kind of setup makes the roles of fiscal policies uncertainty similar to aggregate technology risk in the standard Real Business Cycle model and brings many advantages in computation. To identify the role of this specific anticipation pattern, I also introduce another two anticipation patterns for comparisons: myopic foresight and perfect foresight. The former one supposes that households have no access to know the future fiscal regimes change and always consider that the current regime lasts for ever, whereas the alternative implies the totally opposite situation that households know exactly the tax rates and government spending next period in

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5See relevant sections in McGrattan (2010). The political process to generate the fiscal policies are usually discussed by political science or political economics.

6See the discussion in section 2.3
advance.

Table 1 summarizes the anticipation of households on future fiscal regimes. The foundation of this format is the news report in the 1930s. The row represents the fiscal state current period and the column shows the possible fiscal states next period. The number in each cell indicates the probability the column state comes next period given the row fiscal state current period. The $z$ subscripted by numbers denotes 11 possible states of fiscal regimes while the Year denotes the year when the corresponding fiscal policy is actually carried out in US. The context in the table can be generally divided into three cases in general: First, taking the first row as an example, it’s a myopic foresight: if households are in the fiscal regime of year 1929 current period, they believe that they will stay with this regime next period; Second, taking the forth row as an example, taking the second row as an example, if households are in the fiscal regime of year 1930, they think that the fiscal regimes of 1929, 1930 and 1931 are all possible to come with equal probabilities. This is an ambiguous inference about the fiscal regime next year; Third, it’s a perfect foresight: if households are in the fiscal regime of year 1932, they believe that the incoming fiscal regime would be the one of year 1933, that is they make the correct inference of incoming fiscal policy next period. The anticipation pattern in table 1 is used as a benchmark for the following numerical experiments.

2.3 Computation and Results

2.3.1 Methodology

As mentioned in previous sections, the fiscal uncertainty in this paper is more or less equivalent to the aggregate risk in the RBC model. As a result, I apply the similar strategy usually to solve stochastic growth model. $z_t$ is considered as an aggregate "technology" shock with the transition matrix in table 1. Given share holdings $s_t$ and fiscal state $z_t$ are state variables, Solve for decision functions of households mainly through iterating the household first order conditions (2) and (3). Besides solving and simulating the benchmark model, two more experiments are made to study the impacts of different factors in this economy. Experiment I: solve and simulate the model
under perfect and myopic anticipation patterns respectively, which obviously requires to solve the model three times in total; Experiment II: solve and simulate the model with keeping only one tax constant under each anticipation pattern in turn, which requires to solve the model eighteen times in total. Experiment I is to explore the roles of anticipations in this model while Experiment II is to identify the impact of each tax under different anticipation.

2.3.2 Results

Figure 1 above illustrates the simulation results of benchmark model along with simulation results of extended model in McGrattan (2010) and data series. It tells that the benchmark model with the disaggregated taxes and same anticipation pattern can predict a similar aggregates trend to the one in McGrattan (2010) although it doesn’t include intangible capital. It implies that the intangible capital contributes little to match the data series. The simulations show a large decline at the beginning of 1930s followed by a quick recovery and then another small decline around 1937. The predicted investment and the actual investment almost overlap each other before the lowest point in 1932. Afterwards the predicted one recovers immediately and faster than its counterpart in reality. The absence of investment recovery delay has an enduring impact on the whole simulation results and makes the investment after 1932 always above the data series. The predicted output and working hours also have a significant decline but not as much as in the data. However, the model fails to capture the decline of consumption in the early 1930s too.

Figure 2 gives the results of experiment I. The myopic anticipation can barely produce any decline or recovery but a flat economic trend instead, although all fiscal policy changes are feed into the simulation in the same way. The intuition behind this is very straightforward. If the households have the belief that the current fiscal regime lasts will not change next period, it’s reasonable for them to not change their current investment decision dramatically; On the other hand, the perfect foresight anticipation provides households an accurate knowledge of tax change next period, including the dividend tax or undistributed profit tax increase, then they are able to
take actions in advance to accommodate these changes. A proper response to the capital tax hike next period should be to cut the investment today.

Figure 3, 4 and 5 show the results of experiment II under myopic foresight, perfect foresight and benchmark anticipation in table 1 in order. The experiment is made in the following way: First, given a specific anticipation pattern, solve and simulate the model with only one of six taxes constant. Then, change the anticipation pattern and repeat the same procedure in the first step. Figure 3 illustrates the impact of each tax under the myopic foresight anticipation. It implies that no tax can solely generate a trend similar to the data. All economic variables decrease along the time axis because of an increasing government spending. Such result also confirms the conclusion of experiment I implicitly. Figure 4 illustrates the impact of each tax under perfect foresight anticipation. The dividend tax and undistributed profit tax seem to be very interesting: Once the dividend tax is fixed, the decline of investment at the beginning of 1930s disappears. Moreover, the decline of investment in the late 1930s will be missing if the undistributed profit tax is constant. Similarly, output and working hours have declines when those two taxes are eliminated respectively. All the other taxes have no significant effects on any economic aggregates. However, it’s not so satisfactory that the declines occur earlier and recover faster than the actual series and also that the simulated consumption increase at the beginning. The results from the benchmark anticipation provide the best match with data in the sense that the timing of investment is much improved. Figure 5 shows that the investment, output and working hours share similar trends to data series although there are still large differences in absolute levels, especially for output and investment.

In sum, capital tax (dividend tax and undistributed profit tax) increase and forward anticipation (perfect foresight and benchmark anticipation) together result in investment decline and therefore the decrease in output and working hours. The dividend tax and undistributed profit tax seem to contribute the most to the dramatic economic downturn and slow recovery during

7Actually the opposite implementation is also feasible, that’s to eliminate all taxes but the interested one. For computation convenience I don’t apply the alternative experiment. The absence of so many taxes may lead the policy functions far from the ones of the benchmark model.
Great Depression. Only when households expect the increase of dividend or undistributed profit tax increase, they can cut down the investment in time. This is also the main mechanism implied by the benchmark model.

3 Heterogeneous Agent Model

3.1 Model

3.1.1 Setup

Now I start to discuss about the heterogeneous agent model. Assume that households are infinitely lived and in a continuum mass equal to one. They differ with regard to their share holdings $s_{it}$ and labor productivity shocks $\varepsilon_{it}$. $i$ is the index for different types of households. They make decision on their consumption $c_{it}$, share holding next period $s_{it+1}$ and working hours $h_{it}$ each period to maximize their discounted utility flow into the infinite horizon. Their discount factor is $\beta$ and $\beta < 1$. As the total labor endowment of households is normalized to unity, the leisure of each period is defined as $1 - h_{it}$. All the households pay consumption tax at $\tau_{ct}$, labor income tax at $\tau_{ht}$ and dividends tax at $\tau_{dt}$, and also receive a lump-sum transfer $\kappa_t$. Those four fiscal factors are uniform across all households. Note, households are assumed to have a perfect knowledge of the fiscal state next period. Therefore, their expectation is only on the labor productivity shock they might experience next period. $\Pi(\varepsilon_{t+1}|\varepsilon_t)$ is the conditional probability of the labor productivity shock next period to be $\varepsilon_{t+1}$ given the current one is $\varepsilon_t$. Then, the problem of the heterogeneous household $i$ is summarized as below:

$$\max_{\{c_{it}, s_{it+1}, h_{it}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(c_{it}) + \psi \log(l_{it}) \right]$$

(11)
\[
\begin{align*}
\text{s.t.} \quad & c_{it} + p_t s_{it+1} \leq (p_t + d_t) s_{it} + w_t h_{it} \varepsilon_{it} + \kappa_t - \Gamma^h_{it} \\
& s_{it+1} \geq 0 \\
& l_{it} = 1 - h_{it} \\
& \Gamma^h_{it} = \tau_{ct} c_{it} + \tau_{dt} d_{it} + \tau_{ht} w_{it} h_{it} \varepsilon_{it}
\end{align*}
\]

Then the first order conditions subject to \( h_{it} \) and \( s_{it+1} \) to maximize household utility are respectively:

\[
\begin{align*}
\psi & = \frac{(1 - \tau_{ht}) w_{it} \varepsilon_{it}}{1 - h_{it}} \\
E_t \beta & \frac{p_{t+1}}{p_t} + \frac{(1 - \tau_{dt+1}) d_{t+1}}{1 + \tau_{ct+1}} c_{it+1} \leq \frac{1}{1 + \tau_{ct+1}} c_{it} 
\end{align*}
\]

The equal sign of (13) holds only if \( s_{it} > 0 \). The setup of firms in this economy is similar to the one in the benchmark model except that its discount factor change according to the existence of household heterogeneity. The dynamic firms produce with their own capital \( k_t \) and the labor \( h^f_t \) rent from households. To be consistent with the heterogeneous shareholders, the discount factor \( \tilde{\Lambda}_t \) is defined as,

\[
\tilde{\Lambda}_t = \begin{cases} 
E\beta^t \frac{c_{it}}{c_{it}} & t = 1 \ldots \infty \\
1 & t = 0
\end{cases}
\]

First order conditions for the dynamic firms to maximize their value subject to \( k_{t+1} \) and \( h_t \) are shown as below:

\[
1 = \tilde{\Lambda}_{t+1}(1 - \tau_{dt+1}) \{ (1 - \tau_{pt+1}) \left[ f_{k_{t+1}}(k_{t+1}, h^f_{t+1}) - \delta - \tau_{kt+1} \right] + 1 + \tau_{ut+1} \} \frac{\tilde{\Lambda}_t(1 + \tau_{ut})(1 - \tau_{dt})}{\tilde{\Lambda}_t(1 - \tau_{dt})} \\
w_t = f_{h^f_t}(k_t, h^f_t)
\]

These two first order conditions are very important. By forward iteration, a P-K mapping, which is very important for the computation, can be proved.

\[
p_t = (1 - \tau_{dt})(1 + \tau_{ut}) k_{t+1}
\]

That is, the share price is an indicator of capital stocks next period and influenced by two
capital taxes, dividend tax and undistributed tax. The P-K mapping also implies that there is a wedge between the inside-firm capital and outside-firm capital. Given the same inside-firm capital level in this economy, the increase of dividend tax rate leads to an increase in the value of outside-firm capital, while the increase of undistributed tax rate leads to the opposite change on the outside-firm capital. Furthermore, the outside-firm capital is the asset actually traded in the economy so the change of capital taxation can cause the total volume of wealth in this economy. The detailed discussion on the P-K mapping will continue in the computation and intuition sections.

The government budget also changes as the households become heterogeneous. The total taxation revenue from households are also determined by the joint distribution of share holdings and productivity shocks among households $\Phi(s_{it}, \varepsilon_{it})$.

$$\int \Gamma_{it}^{h} \Phi(s_{it}, \varepsilon_{it}) + \Gamma_{t}^{f} = g_{t} + \kappa_{t} \quad (18)$$

Then the market clearing conditions are,

$$\int c_{it} d\Phi(s_{it}, \varepsilon_{it}) + x_{t} + g_{t} = f(k_{t}, h_{t}) \quad (19)$$

$$\int s_{it+1} d\Phi(s_{i,t}, \varepsilon_{it}) = 1 \quad (20)$$

$$\int h_{it} d\Phi(s_{i,t}, \varepsilon_{it}) = h_{t}^{f} \quad (21)$$
3.1.2 Recursive Competitive Equilibrium

Finally, the recursive competitive equilibrium in this economy can be defined as follows: Given the initial capital level $k_0$, the initial joint distribution of share holdings and productivity shock $\Phi_0(s, \varepsilon)$, a recursive competitive equilibrium subject to the fiscal policy $\{\tau_p, \tau_d, \tau_c, \tau_u, \tau_h, g\}$, consists of a set of laws of motion $\{k' = \Omega(k, h, \Phi), h' = \Xi(k, h, \Phi), \Phi' = \Upsilon(k, h, \Phi)\}$, a price set $\{w, p\}$, firm choices $\{d, h^f, k'\}$, and the individual household decision functions and value function $\{c(s, \varepsilon), s'(s, \varepsilon), h(s, \varepsilon), V(s, \varepsilon)\}$ such that:

- Given the price $\{w, p\}$ and laws of motion $\{k' = \Omega(k, h, \Phi), h' = \Xi(k, h, \Phi), \Phi' = \Upsilon(k, h, \Phi)\}$, the individual household decision functions and value function $\{c(s, \varepsilon), s'(s, \varepsilon), h(s, \varepsilon), V(s, \varepsilon)\}$ solve the household optimization problem:

$$V(s, \varepsilon) = \max_{\{c, s', h\}} \{U(s, \varepsilon) + E[V(s', \varepsilon')|\varepsilon]\}$$

subject to

$$c + ps' \leq (p + d)s + wh \varepsilon + \kappa - \Gamma^h$$

where

$$U(s, \varepsilon) = \log(c) + \psi \log(l)$$

and

$$s' \geq 0$$

$$l = 1 - h$$

$$\Gamma^h = \tau_c c + \tau_d ds + \tau_h wh \varepsilon$$

- The dynamic firms satisfy the profit maximization conditions below:

$$p = (1 - \tau_d)(1 + \tau_u)k'$$

$$w = f_{h^f}(k, h^f)$$

$$d = f(k, h^f) - wh^f - x - \Gamma^f$$

- The government operates on a balanced budget:

$$\int \Gamma^h(s, \varepsilon)\Phi(s, \varepsilon) + \Gamma^f = g + \kappa$$
\[ \Gamma^f = \tau_p [f(k, h^f) - wh^f - \delta k - \tau_k k] + \tau_k k + \tau_u (k' - k) \]

- All the market clear each period;

\[ \int c(s, \varepsilon) d\Phi(s, \varepsilon) + x + g = f(k, h) \]

\[ \int s'(s, \varepsilon) d\Phi(s, \varepsilon) = 1 \]

\[ \int h(s, \varepsilon) d\Phi(s, \varepsilon) = h^f \]

- The laws of motion are consistent with the individual household behavior;

4 Calibration

The objective of this paper is to exam the impacts of the fiscal changes during Great Depression. It is straightforward to take the US economy in 1929, the year when Great Depression began, as the benchmark for calibration and also as the starting point of the computational experiment. As the fiscal policy in the experiment evolutes in the way with its counterpart in reality, the model shows the simulated response of aggregate economy to those fiscal changes. However, why is it reasonable to consider the US economy in 1929 as a steady state subject to the fiscal policy at that time? According to historic data in fiscal policies, the taxation and government spending was both very stable during 1920s, especially after 1924 Post War Reduction. In addition, the latest expectation on fiscal changes then didn’t come into being until 1930 when abrupt breakout of Great Depression forced President Roosevelt to take new economic measures. Therefore, it’s not unreasonable to assume that the US economy has arrived at steady state through almost a decade with a stable fiscal environment.

\[ \text{For taxation, see } \text{tax foundation}; \text{ For government spending, see } \text{national income and product accounts.} \]
4.1 Economic Aggregates

To make the one-sector model comparable to the real data, the capital level in the model is calibrated to the reproducible capital stocks from all production sectors in US economy. Namely it includes private fixed assets, durable consumption goods, government fixed assets, corporate inventory, and value of lands. The data source of the first three categories is table 1 in Katz and Herman (May 1997), which is an adjusted summary of NIPA tables from Bureau of Economic Analysis. The *Statistics of Income for 1929* provides the information of inventory under the assets category of of US corporate balance sheet.\(^9\) The land value is from the nonresidential land value in table \(W – 30\) of Goldsmith et al. (1956).

The consistency in calibration also requires that the outputs of capital measured by the above strategy should be all considered as part of total products. So it’s necessary to include the service flow from durable goods and government fixed assets, which is not imputed in the GDP of NIPA tables. The return to capital \(r\) is essential to infer these series. The procedure to obtain the return rate \(r\) will be discussed in later part of this section. Given \(r\) is available, I can add the term \(r\) multiplying the sum of durable goods and government fixed assets to GDP and consumption value from NIPA table respectively to impute the adjusted GDP and consumption. Moreover, as McGrattan (2010) suggests, the adjustments relevant to sale tax expenditures are also made to the consumption and investment: The sale tax on durable consumption and nondurable consumption are respectively less from the durable consumption and nondurable consumption, because the consumption expenditure in NIPA doesn’t differentiate the price and tax. Investment is the gross investment plus consumption of durable goods in the GDP components table of NIPA. Government spending directly obtained from the government consumption in NIPA. All the series above are divided by GDP deflator and mid-year population in NIPA. They are also detrended by technology growth rate set to be 1.9%.

\(^9\)see website http://www.irs.gov/taxstats/productsandpubs/article/0,,id=125133,00.html, and download the SOI report from SOI Publications Archive session.

Cooley and Prescott (1995) has provided the methodology to calibrate the capital return \(r\),
capital income share $\theta$ and capital depreciation rate $\delta$. I follow the standard procedures in that paper. Extract the labor income, unambiguous capital income and ambiguous capital income respectively from NIPA. With consumption of fixed assets, that’s the capital depreciation if steady state assumption holds, solve for the private capital income share $\theta_p$ first by:

$$\theta_p = \frac{\text{unambiguous capital income} + \text{capital depreciation}}{\text{GNI} - \text{ambiguous capital income}}$$ (22)

Then, calculate the private capital income $Y_{KP}$ by $\theta_p$ multiplying GNI. The return rate to capital $r$ is determined by:

$$r = \frac{Y_{KP} - \text{capital depreciation}}{K_p}$$ (23)

Here, $K_p$ includes private fixed assets, corporate inventory and land value. With the assumption that $r$ is unique in the economy, I can impute the gross service flow from the durable goods $Y_d$ and government fixed assets $Y_g$.\(^{10}\) Then capital income share $\theta$ is finally determined by the following formula:

$$\theta = \frac{Y_{KP} + Y_d + Y_g}{\text{GNP} + Y_d + Y_g}$$ (24)

### 4.2 Income and Wealth Inequality

In this model, the earning process is responsible for the endogenous income and wealth heterogeneity. The classic methodology to calibrate the earning process has been discussed by Castaeda et al. (2003). However, it requires a delicate micro-data source to detail the earning and wealth Lorenz curves.\(^{11}\) Unfortunately there exists quite few micro-level data sources in early 20th century. Only a handful of literatures have disclosed very limited information in income and wealth Gini coefficients and their top group shares. Earning inequality at that time is barely exposed. Lindert

---

\(^{10}\)Estimation over the depreciation rates $\delta_d$ and $\delta_g$ are also required

\(^{11}\)For instance, *Survey of Consumer Finance* used by Castaeda et al. (2003) is eligible to calculate the quintile of earning and wealth distribution.
(2000) does a systematical survey on income and wealth inequality of the early 20th century and introduces potential income and wealth inequality data sources for heterogeneous household study. In this paper, the household income and wealth inequalities are respectively taken from Goldsmith (1967) and Williamson and Lindert (1980). The former one estimates the top 20% income share and income Gini coefficient in 1929, whereas the later one displays the top 10% wealth share and wealth Gini coefficient in 1913 and 1925 from King (1915) and P. H. and Williamson (1976). We infer the wealth inequalities through those two sets of statistics, because Williamson and Lindert (1980) mentions "The period from 1860 to 1929 is thus best described as a high uneven plateau of wealth inequality. When did wealth inequality hit its historic peak? We do not yet know. We do know that there was a leveling across the 1860s. We also know that there was a leveling across the World War I decade (1912-22), which was reversed largely or entirely by 1929.", which implies that the wealth Gini coefficients and top wealth shares in 1913 and 1925 should be very close to the ones in 1929. In practice, I just take the average of these data as the proxies for wealth inequalities in 1929.

4.3 Parametrization

For convenience, the earning process is simply considered to have a symmetric transition matrix, which help reduce the total number of unknown parameters in the transition matrix to 313. The labor productivity shock values \{\epsilon_i\}_{i=1}^3 add 3 more unknowns. Besides, we have another 5 aggregate parameters \{\beta, \psi, \theta, \delta, z\} to settle down. Nonetheless, \theta, \delta and \epsilon are able to be determined by capital income share, investment-capital ratio and capital-working hours ratio respectively with no computational experiment. As a result there are totally 8 parameters to be calibrated systematically. The solvability consequently requires at least 8 conditions as the calibration targets. In addition to the clearing condition in labor and stock market, the remaining can be found in the normalization of labor productivity shock, the unit expected labor productivity shock in ergodic

\footnote{Most of Wealth inequality literature only measure the top 0.01% top 0.5% top 1% or top 5% wealth shares. Although they are very useful information, it is very difficult to capture them by a parsimonious model with limited labor shock states.}

\footnote{See the following example for the symmetric transition matrix used for calibration computation.}

\[ \Pi_{ij} = \begin{pmatrix}
    P_{11} & 1 - P_{11} & 0 \\
    1 - P_{22} & P_{22} & 1 - P_{22} \\
    0 & 1 - P_{33} & P_{33}
\end{pmatrix} \] (25)
space, income Gini coefficient, wealth Gini coefficient, top 20% income share and top 10% wealth share. The benchmark targets and corresponding predicted value are posted in table 3. The table 4 and 5 show the parameters from calibration.

5 Solution and Results

5.1 Solution Methods

The objective of the numerical experiments in this paper is to find predicted economic trends and contrast them with the corresponding actual data series. It requires to solve out the transition from 1929 to 1939 under the changing fiscal policies then. Therefore, I use a shooting method to solve the transition between steady states in brief. To make this method feasible, the following conditions on the model must hold: First, the economy is at steady state in 1929; Second, households have a perfect foresight of the fiscal policy path from 1929 to 1939; Third, households have no knowledge of the fiscal states after 1939 and believe that the fiscal policy stays the same afterwards. Thanks to the above assumptions, the interested solution can be considered as an economic transition from the steady state under the fiscal policy of 1929 to another one under fiscal policy of 1939. The feature different from the standard shooting method is that the fiscal policies change in the first 11 periods and constant in the rest. Thus, the brief algorithm is listed as below:

- **Step 1:** Solve for the stationary equilibria under the fiscal regime of 1929 and 1939 respectively and store the invariant distributions of households, value functions, aggregate capital stocks, aggregate working hours and government transfers;

- **Step 2:** Choose the total periods of the transition between the above two steady state and make sure the first 11 periods with the fiscal policies changing each period;

- **Step 3:** Guess sequences of household distributions, aggregate capital stocks, aggregate working hours and value functions along the whole transition excluding the end period. Note,
the share price, wage rate and government transfer can be imputed each period once the guess is made;

• **Step 4:** Given projected paths of share price, wage rate, government transfer and value function, do the value function iteration to update value function and decision functions each period;

• **Step 5:** Aggregate the capital stocks and working hours by updated decision functions and projected distribution to update the guess of capital stocks, working hours and distribution of households each period;

• **Step 6:** Check the convergence, otherwise go back to step 4 with the updated aggregates, value functions and distribution of households.

### 5.2 Results

The benchmark model solved under the anticipation in table 1 fails to provide a good contrast to the heterogeneous agent model in this paper as a result of different anticipation patterns. A reasonable practice is to compare the solutions of the benchmark model and heterogeneous agent model both under the perfect foresight anticipation. Figure 7 shows the trends of different economic aggregates projected by benchmark model, heterogeneous agent model and actual data series respectively. The graphics of investment shows that the impacts of the dividend tax and undistributed profit tax increase are very sensitive to the presence of household heterogeneity. The investment decline caused by the dividend tax increase is almost 50% smaller in the heterogeneous model than in the homogeneous model. Consequently, the simulated output and working hours don’t show a significant decline in the heterogeneous model as they do in the homogeneous model either. However, the counterfactual consumption increase in the early 1930s gets a little improved. My conjecture is that the smaller consumption jump mainly comes from the effect of a smaller investment drop\(^{14}\). Although the simulation under perfect foresight anticipation cannot provide a satisfactory match for the actual data series in neither benchmark model and heterogeneous model, it provides a straightforward perspective to understand the mechanism how the capital

\(^{14}\)If the capital stock and working hours don’t fall much, then the output will more or less remain the same level as before. In this sense, a smaller decline in investment implies a larger fraction of output goes to investment and then a smaller consumption increase.
taxation affect the investment and further more economic aggregates. The absence of dramatic investment decline in the heterogeneous model with perfect foresight anticipation implies that the mechanism suggested by the benchmark model is not significant when the household heterogeneity exists. Figure 8 shows the full computational solution of capital stocks and working hours. As discussed before, the full solution is just a transition from one steady state to another with the fiscal policies changing only in the first 11 periods. Capital stocks and working hours are both consistent with such conditions: On one hand, the capital stock decreases at the beginning as a response to the dividend tax increase, and then decreases again as a response to the undistributed profit tax increase after a small and short recovery, and finally increases to the new steady state smoothly once the fiscal policy is constant; On the other hand, the working hours show the same trend with the capital stocks but within a smaller fluctuation range.

6 Explanation

The disappearance of significant investment decline in the heterogeneous household model can be explained by the wealth effect discovered by Anagnostopoulos et al. (2011). When the dividend tax increases, the capital demand decrease is offset by households’ strong desire to insure against bad productivity shocks with less available assets. The increase of dividend tax in heterogeneous household economy generally causes two effects: anticipation effect and wealth effect. On one hand, the anticipation on the dividend tax increase plays the same role as it dose in homogeneous household economy. Firms find that the future capital return decrease and cut down their capital investment. That’s to low the inside-firm capital demand in this economy, which tends to decrease the equilibrium capital stock; On the other hand, the outside-firm capital or assets supply in the economy shrinks when the dividend tax increases according to the P-K mapping. The precautious saving motive drives households to compete for scarcer assets and then to require a lower return rate. If the decreasing marginal productivity and no arbitrage are presumed, the fall of return
to capital leads to a higher equilibrium capital stock. In sum, the two effects above move the
equilibrium capital stock in different directions. Therefore, the total impact of dividend tax
increase is a quantitative issue.

In the following section these two effects will be put into concrete exposition. First, the original
model setup need be transferred to a classic heterogeneous agent model in Aiyagari (1994) by
replacing $p_t s_{it+1}$ and $p_{t+1} \frac{(1-\tau_{dt+1})d_{t+1}}{pt}$ with $a_{it+1}$ and $r_{t+1}$ respectively. Then the assets demand
curve $A_h^t$ in this economy is in the similar shape to the aiyagari model, upward sloping, concave
and converge to $\frac{1}{\beta} - 1$. Second, capital demand curve $K_f^t$ is obtained from conditions (13) (14) and (15)
in the firm value maximization problem. Third, construct the assets supply curve $A_f^t$ on the basis of
capital demand curve $K_f^t$ and the P-K mapping. The total share in the economy is normalized into
unity so the mapping between share price and interest rate is just the mapping between the total
assets and interest rate. The typical household asset demand curve can be solved from the problem
(26); The equation (27) is an implicit function of capital demand of firms; The equation (28) re-
fects the relation between assets supply (outside-firm capital demand) and (inside) capital demand.

Assets supply curve:

\[
\max \begin{align*}
\{c_{it}, a_{it+1}, h_{it}\} & E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(c_{it}) + \psi \log(1 - h_{it}) \right] \\
\text{s.t.} & c_{it} + a_{it+1} \leq (1 + r_t)a_{it} + w_t h_{it} \varepsilon_{it} + \kappa_t \\
& a_{it+1} \geq 0
\end{align*}
\] (26)

Capital demand curve:

\[
(1 + r_{t+1}) = \frac{(1 - \tau_{dt+1}) \left\{ (1 - \tau_{pt+1}) \left[ f_{kt+1}(k_{t+1}, h_{t+1}) - \delta - \tau_{kt+1} \right] + 1 + \tau_{ut+1} \right\}}{(1 + \tau_{ut})(1 - \tau_{dt})} 
\] (27)

Assets demand curve:

\[
A_h^t = p_t * 1 = (1 - \tau_{dt})(1 + \tau_{ut})k_{t+1}
\] (28)
6.1 Anticipation Effect

According to the decomposition results in benchmark model, only the increase of dividend tax and undistributed profit tax leads to investment decrease while all the other fiscal changes have no significant impacts. Hence, it’s reasonable to focus on the capital taxes and eliminate all the other fiscal parameters for a while. Then the capital demand curve $K_f^t$ can be simplified as:

$$r_{t+1} = \frac{(1 - \tau_{dt+1})(f_{k_{t+1}}(k_{t+1}, h_{t+1}) - \delta + 1)}{1 - \tau_{dt}} - 1$$  \hspace{1cm} (29)$$

Then it’s more convenient to observe the impact of anticipated increase of $\tau_{dt+1}$. It pushes the capital demand curve $K_f^t$ to left. So does the assets supply curve $A_f^t$ at the same time. The assets market equilibrium change from $A_{old}$ to $A_{new}$ and arrive at a lower interest rate. Accordingly the lower interest rate drive the capital down to a new equilibrium $K_{new}^t$ from $K_{old}^t$. Note, if the increase of $\tau_{dt+1}$ is not expected, then $\tau_{dt+1}$ equals to $\tau_{dt}$ in the perspective of firms, which implies the capital demand $K_f^t$ doesn’t receive any influence from the dividend tax change next period. So anticipation is very important for the dividend tax change to influence the demand for capital stock next period. The anticipation effect is marked by A.E. in figure 9.

[Figure 9 about here.]

6.2 Wealth Effect

The assets in this model can be considered as the outside-firm capital and also equal to the share price. When $\tau_{dt}$ increases, the wedge increase. Namely, the value of outside-firm capital decreases even given the same inside-firm capital stock. In figure 10, the assets supply curve $A_f^t$ is pushed to left with the capital demand curve $K_f^t$ untouched. The new equilibrium interest rate becomes lower than the original one. It requires a higher equilibrium capital stock $K_{new}^t$. Note, this effect is absent in the complete market as the assets demand there is absolutely elastic. The W.E. marks the wealth effect in figure 10.

\[\text{Consider the total share volume in this economy always equal to unity.}\]
6.3 Total Effect

When the expected dividend tax rate increases continuously for many periods, the anticipation effect and wealth effect occur at the same time each period. For instance, the anticipated dividend tax increase in period $t+1$ leads to the drop of capital demand in period $t$. Nevertheless, the current dividend tax increase introduce the wealth effect and increases the equilibrium capital stock. In total, these two effect offset each other. However, which effect ultimately dominates is a quantitative issue. In the quantitative analysis in this paper, the anticipation effect is larger than the wealth effect but not very much. The transition results show that the investment only fall a little from the steady state level after the dividend tax hikes start. Figure 11 illustrates the total effect in this paper. Without the wealth effect, the anticipation effect should move the assets supply curve $A_f$ to the dot line position rather than $A'_f$. Then the decrease of capital stock in that case will be much larger than what the model actually produce. Note, the figure 11 just illustrates a specific situation in this paper. The numerical results could be different in other experiments. Suppose that the wealth effect is much larger than figure 11 shows, the result could be that the wealth effect dominates and that the capital stock increases finally.

7 Conclusion

In this paper, I first confirm the results of McGrattan (2010) from a benchmark model: The homogeneous agent model with disaggregated taxes and anticipation over the future fiscal policy can explain the dramatic drop of investment during the Great Depression. Then, the decomposition of taxes under different anticipation patterns identifies the impacts of different factors. As expected, the dividend tax increase, undistributed profit tax increase and forward expectation are together responsible for the investment decline. The benchmark anticipation in table 1 does a better job than the perfect foresight in matching the timing of the economic downturns. Subsequently, I
extend the benchmark model to a heterogeneous agent model whose endogenous income and wealth inequalities are consistent with the ones in US 1929. The solution implies that the impact of capital taxes increase and forward anticipations is very sensitive to the presence of household heterogeneity. Given the same capital stock level, the increase of dividend tax rate decreases the value of total assets in the economy and forces households to require a lower return for much fewer accesses to assets. Under no arbitrage condition, the lower return rate lures the firms to cut capital stock demand and then leads to an investment decline. The quantitative experiment illustrates that the heterogeneous agent model can only project half of investment decrease suggested by the benchmark model. The downturn in the working hours and output also become less significant. It seems that the role of fiscal policy during Great Depression is still in question.

References


A Proof of P-K Mapping

By the first order condition subject to $k_{t+1}$ to optimize the value of firms, we can prove that the share price is actually a function of capital in this economy.

$$(1 + \tau_{ut})(1 - \tau_{dt}) = \frac{\Lambda_{t+1}}{\Lambda_t}(1 - \tau_{dt+1})\{(1 - \tau_{pt+1})[f_{k_t+1}(k_{t+1}, h_{t+1}) - \delta - \tau_{kt+1}] + 1 + \tau_{ut+1}\}$$

$$(1 + \tau_{ut})(1 - \tau_{dt})k_{t+1} = \frac{\Lambda_{t+1}}{\Lambda_t}(1 - \tau_{dt+1})\{(1 - \tau_{pt+1})[f_{k_t+1}(k_{t+1}, h_{t+1}) - \delta - \tau_{kt+1}] + 1 + \tau_{ut+1}\}k_{t+1}$$
\[
\begin{aligned}
\frac{\Lambda_{t+1}}{\Lambda_t} (1 - \tau_{dt+1}) & \left\{ \left[ f(k_{t+1}, k_{t+1} - \delta k_{t+1} - \tau_{kt+1} k_{t+1}) + k_{t+1} + \tau_{ut+1} k_{t+1} \right] \\
& + \frac{\Lambda_{t+1}}{\Lambda_t} (1 - \tau_{dt+1}) \left[ f(k_{t+1}, h_{t+1}^f) - \delta k_{t+1} - \tau_{kt+1} k_{t+1} \right] + k_{t+1} - k_{t+2} + \tau_{ut+1} (k_{t+1} - k_{t+2}) \right\} \\
& + \frac{\Lambda_{t+1}}{\Lambda_t} (1 - \tau_{dt+1}) (1 + \tau_{ut+1}) k_{t+2} \\
& = \frac{\Lambda_{t+1}}{\Lambda_t} (1 - \tau_{dt+1}) d_{t+1} + \frac{\Lambda_{t+1}}{\Lambda_t} (1 - \tau_{dt+1}) (1 + \tau_{ut+1}) k_{t+2} \\
& = \frac{\Lambda_{t+1}}{\Lambda_t} (1 - \tau_{dt+1}) d_{t+1} + \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1 - \tau_{dt+2}) d_{t+2} + \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\Lambda_{t+2}}{\Lambda_{t+1}} \frac{\Lambda_{t+3}}{\Lambda_{t+2}} (1 - \tau_{dt+3}) d_{t+3} + \ldots \\
& = \sum_{j=1}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} (1 - \tau_{dt+j}) d_{t+j} \\
& = p_t
\end{aligned}
\]

This conclusion obviously leads to the price-capital mapping:

\[p_t = (1 + \tau_{ut}) (1 - \tau_{dt}) k_{t+1}\]

**B Detailed Computation Algorithm**

**B.1 Algorithm to Solve the Stationary Equilibrium**

- **Step 1:** Guess the aggregate capital stock and working hours at stationary equilibrium as \( k_{ss}^0 \) and \( h_{ss}^0 \)

- **Step 2:** According to the production function and government budget constraint, we can solve out \( \kappa_{ss}^0 \) in the following way:

\[
\Gamma_{ss}^0 = \tau_p \left[ f(k_{ss}^0, h_{ss}^0) - w_{ss}^0 h_{ss}^0 - \delta k_{ss}^0 \right]
\]

\[
\kappa_{ss}^0 = w_{ss}^0 h_{ss}^0 \tau_h + \Gamma_{ss}^0 - g
\]

- **Step 3:** For \( l = 0 \ldots L \), where \( l \) is the inner-loop iteration and \( L \) is the maximum iteration
number allowed. Do the policy iteration with the following equations for all the grid points \( \{s, \varepsilon\} \) with initial guess \( c^0(s, \varepsilon) \):

\[
\frac{1}{c^{l+1}(s, \varepsilon)} = \sum_{\varepsilon'} \pi(\varepsilon' | \varepsilon) \beta[1 + (1 - \tau_p)(f^0_1 - \delta)] c^l(s', \varepsilon')
\]

\[
\psi \frac{1}{1 - h^l(s, \varepsilon)} = \frac{(1 - \tau_h)w^0_{ss} \varepsilon_{i,t}}{c^l(s, \varepsilon)}
\]

\[
c^l(s, \varepsilon) + p^0_{ss} s' = (p^0_{ss} + d^0_{ss})s + (1 - \tau_h)w^0_{ss} h^l(s, \varepsilon)\varepsilon + \kappa^0_{ss}
\]

\( p^0_{ss}, d^0_{ss} \) and \( w^0_{ss} \) can be derived from the p-k mapping, dividend definition and production respectively. The linear interpolation will be applied when we look for the grids of \( c^l(s', \varepsilon') \).

Calculate the policy function \( s^{l'}(s, \varepsilon) \) with \( c^l(s, \varepsilon) \) and \( h^l(s, \varepsilon) \). If \( s^{l'}(s, \varepsilon) \leq 0 \) or \( s^{l'}(s, \varepsilon) \geq 1 \), set \( s^{l'}(s, \varepsilon) = 0 \) or \( s^{l'}(s, \varepsilon) = 1 \). Take out the intertemporal condition and re-solve out \( c^l(s, \varepsilon) \) and \( h^l(s, \varepsilon) \). Continue until the consumption and labor supply policy functions both converge. Store the converged \( c^l(s, \varepsilon), s^{l'}(s, \varepsilon) \) and \( h^l(s, \varepsilon) \) as \( c^0(s, \varepsilon), s^0(\varepsilon) \) and \( h^0(s, \varepsilon) \) for the future steps.

- **Step 4:** Then derive the invariant distribution \( \Phi^0(s, \varepsilon) \) by constructing the transition matrix of \( (s, \varepsilon) \) with \( s^0(\varepsilon) \) and \( \pi(\varepsilon' | \varepsilon) \).

- **Step 5:** With \( \Phi^0(s, \varepsilon) \) we can solve out the total demand for capital \( k^1_{ss} \) and the total labor supply \( h^1_{ss} \) as below:

\[
h^1_{ss} = \int h^0(s, \varepsilon) d\Phi^0(s, \varepsilon)
\]

\[
k^1_{ss} = \frac{f(k^0_{ss}, h^0_{ss}) + (1 - \delta)k^0_{ss} - \int c^0(s, \varepsilon) d\Phi^0(s, \varepsilon) - g}{(1 + \gamma)(1 + \eta)}
\]

- **Step 6:** Compare \( \{h^1_{ss}, k^1_{ss}\} \) with \( \{h^0_{ss}, k^0_{ss}\} \). If convergence occurs, stop; Otherwise, update
the guess as follows and go back to step 2:

\[ k_{ss}^0 = k_{ss}^0 + \lambda_k (k_{ss}^1 - k_{ss}^0) \]  

(37)  

\[ h_{ss}^0 = h_{ss}^0 + \lambda_h (h_{ss}^1 - h_{ss}^0) \]  

(38)

B.2 Algorithm to Solve the Transition with Perfect Foresight

- **Step 1:** Choose the total number of the transition periods \( T \). The first 11 periods represent the actual economic period 1929 – 1939 with correspondent changing taxation regime of each year, and the late (\( T - 11 \)) periods represent the transition path the economy takes from the state at the end of 11th period to the steady state under the fixed taxation regime of 1939;

- **Step 2:** As we discussed in the last section, solve the stationary equilibria under the taxation regime of 1929 and 1939 respectively and store the invariant distribution of individual states, policy functions, aggregate capital, aggregate working hours and government transfer as \( \{\Phi_{\text{initial}}(s, \varepsilon), c_{\text{initial}}(s, \varepsilon), k_{\text{initial}}, h_{\text{initial}}, \kappa_{\text{initial}}\} \) and \( \{\Phi_{\text{end}}(s, \varepsilon), c_{\text{end}}(s, \varepsilon), k_{\text{end}}, h_{\text{end}}, \kappa_{\text{end}}\} \);

- **Step 3:** Guess the distribution of individual states, aggregate capital stock, and aggregate working hours sequence \( \{\Phi^0_{\tau}(s, \varepsilon), k^0_{\tau}, h^0_{\tau}\}_{\tau=1}^{T-1} \), where \( k^0_{\tau} = k_{\text{initial}} \) and \( \Phi^0_{\tau}(s, \varepsilon) = \Phi_{\text{initial}} \); Also guess a sequence of policy functions for households \( \{c^0_{\tau}, s^0_{\tau}, h^0_{\tau}\}_{\tau=1}^{T-1}; \)

- **Step 3:** Use the previous guess \( \{\Phi^0_{\tau}(s, \varepsilon), k^0_{\tau}, h^0_{\tau}\}_{\tau=1}^{T-1} \), government budget constraint and determination equation of government’s profit tax income to solve out \( \{\kappa^0_{\tau}\}_{\tau=1}^{T-1} \):

\[ \Gamma^0_{\tau} = \tau_{pt} [f(k^0_{\tau}, h^0_{\tau}) + (1 - \delta)k^0_{\tau} - w^0_{\tau}h^0_{\tau} - (1 + \eta)(1 + \gamma)k^0_{\tau+1}] \]  

(39)  

\[ \kappa^0_{\tau} = w^0_{\tau}h^0_{\tau}\tau_{ht} + \Gamma^0_{\tau} - g_{\tau} \]  

(40)

- **Step 4:** For all the periods \( t = T - 1...1 \), do the following period by period backwards:

  - **Step 4.1:** Given \( \{k^0_{\tau}, h^0_{\tau}, \kappa^0_{\tau}\} \) and \( c^0_{\tau+1}(s, \varepsilon) \), we can update \( \{c^0_{\tau}(s, \varepsilon), h^0_{\tau}(s, \varepsilon), s^0_{\tau}(s, \varepsilon)\} \) into \( \{c^1_{\tau}(s, \varepsilon), h^1_{\tau}(s, \varepsilon), s^1_{\tau}(s, \varepsilon)\} \) using the following conditions:
\[
\frac{1}{c_t^1(s, \varepsilon)(1 + \tau_{ct})} = \sum_{\varepsilon'} \Pi(\varepsilon' | \varepsilon) \beta [1 + (1 - \tau_{pt+1})(r^0_{t+1} - \delta)]
\]

\[\psi = \frac{(1 - \tau_{ht})w^0_t \varepsilon}{c_t^1(s, \varepsilon)(1 + \tau_{ct})} \]

\[
(1 + \tau_{ct})c^1_t(s, \varepsilon) + p^0_t s' = (p^0_t + d^0_t) s + (1 - \tau_{ht})w^0_t h^1_t(s, \varepsilon) \varepsilon + \kappa^0_t
\]

\[p^0_t = (1 + \gamma)(1 + \eta)k^0_{t+1}\]

\[d^0_t = (k^0_t)^{\theta}(Ah^0_t)^{1-\theta} + (1-\delta)k^0_t - (1+\eta)(1+\gamma)k^0_{t+1} - w^0_t h^1_t - \tau_{pt}[f(k^0_t, h^0_t) - w^0_t h^1_t - \delta h^0_t]\]

\[w^0_t = (1 - \theta)A(k^0_t)^{\theta}(Ah^0_t)^{-\theta}\]

\[r^0_t = \theta(k^0_t)^{\theta-1}(Ah^0_t)^{1-\theta}\]

The linear interpolation will be applied when we look for the grid of \(c^0_{t+1}(s', \varepsilon')\). In addition, the solution of \(s'\) has to be checked for each period to guarantee \(0 \leq s_t \leq 1\). If binding solutions \(s' < 0\) or \(s' > 1\) are found, replace the intertemporal condition with \(s = 0\) or \(s = 1\), solve the system again;

- Step 4.2: With policy functions \(\{c^1_t(s, \varepsilon), h^1_t(s, \varepsilon), s_{t+1}^1(s, \varepsilon)\}\) obtained from last step and transition matrix \(\pi(\varepsilon' | \varepsilon)\), we can calculate the transition matrix of individual state from period \(t\) to period \(t + 1\), \(\Omega_t\). Then we can update the distribution of households over shares and labor shock at period \(t\) into \(\Phi^1_t\).

\[\Phi^1_t = \Omega^T_{t+1} \Phi^0_{t+1}\]

Here you might notice that \(\Phi^1_T = \Phi^0_T = \Phi_{end}\). Finally, use the \(\Phi^1_t\) to solve out the aggregate capital of period \(t + 1\) and labor supply of period \(t\) as \(\{k^1_{t+1}, h^1_t\}\) in the following way:

\[h^1_t = \int h^1_t(s, \varepsilon) d\Phi^1(s, \varepsilon)\]
\( k_{t+1}^1 = \frac{f(k_t^0, h_t^0) + (1 - \delta)k_t^0 - \int c^1(s, \varepsilon) d\Phi_t^1(s, \varepsilon) - g_t}{(1 + \eta)(1 + \gamma)} \) (50)

- **Step 5:** Compare \( \{\Phi_t^1, k_{t,sim}, h_t^1\}_{t=1}^{T-1} \) with \( \{\Phi_t^0, k_{t}^0, h_t^0\}_{t=1}^{T-1} \). If convergence occurs, stop; otherwise, update the guess in the following way:

\[ k_t^0 = k_t^0 + \lambda_k(k_t^1 - k_t^0) \text{ if } t > 1 \] (51)

\[ k_1^0 = k_1^1 = k_{initial} \] (52)

\[ h_t^0 = h_t^0 + \lambda_h(h_t^1 - h_t^0) \] (53)
Table 1: Anticipation on Fiscal Policies $\Pi(z_{t+1}|z_t)$

<table>
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<tr>
<th>Year</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
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<th>$Z_6$</th>
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<th>$Z_8$</th>
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Table 2: Calibration targets

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<th>predicted value</th>
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<td>3.6681</td>
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<tr>
<td>working hours (fraction of 24 hours)</td>
<td>0.2892</td>
<td>0.2892</td>
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<tr>
<td>Wealth Gini coefficients</td>
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<td>0.8990</td>
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<tr>
<td>Income Gini coefficients</td>
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<td>0.5009</td>
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<tr>
<td>Top 10% wealth share</td>
<td>90.0%</td>
<td>87.0%</td>
</tr>
<tr>
<td>Top 20% income share</td>
<td>54.0%</td>
<td>52.5%</td>
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<tr>
<td>Value</td>
<td>$\pi(\epsilon_1</td>
<td>\epsilon_i)$</td>
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<tr>
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Table 4: parameters

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Figure 1: disaggregated tax rates and government spending from 1929 to 1939
Figure 2: Benchmark Model vs Extended Model in McGrattan 2010 and Data
Figure 3: Basic Model with Different Expectation
Figure 4: Single Tax Experiment under Myopic Expectation
Figure 5: Single Tax Experiment under Perfect Expectation
Figure 6: Single Tax Experiment under Benchmark Expectation
Figure 7: Heterogeneous Model vs Homogeneous Model under Perfect Expectation
Figure 8: Capital and Labor Transition Path
Figure 9: Anticipation Effect
Figure 10: Wealth Effect
Figure 11: Total Effect