What determines the direction of technological progress?

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What Determinesthe Direction of Technological Progress?*

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Abstract

Identifying the key factors determiningthe direction of technological progress is of central importance for macroeconomics.This paper develops a framework based on Acemoglu(2002,2003) in which profit-maximizing firms undertake both labor- and capital-augmenting technological improvements. It deviates from that framework by the introduction of nonlinear accumulation functions for the primary factors of production. It proves that, although in the short run the change of relative factor prices as suggested by Hicks(1932) and the relative market size as argued by Acemoglu(2002) indeed affect the direction of technological progress, in the long run that direction depends only on the relative supply elasticities of primary factors with respect to their respective prices. Moreover, it is biased towards enhancing the effectiveness of the factor with the relatively smaller elasticity.According to these results labor productivity has hardly increased during the pre-industrial era because labor supply was highly elastic during that time. In contrast, the industrial revolution and the concurrent demographic transition caused capital supply elasticity to increase and that of labor supply to decrease, inducing a labor-biased technological progress.

Key Words: direction of technological change, steady-state, Uzawa's theorem, non-linear accumulation processes, supply elasiticities.

JEL: E13; O33; O11; Q01

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1. Introduction

Technological change can equally increase the productivity of capital and labor, or it can be biased towards a specific factor.For example, according to Kaldor(1961), the stylized characteristics of economic growth in developed countries indicate that while per-capita output and physical capital have grown over time, the capital/output ratio and the income shares of labor and capital have remained basically constant since the industrial revolution.[3]These facts have been interpreted as indicating that technological progress has been purely labor-augmenting. In contrast, Ashraf and Galor(2011) show that during the preindustrial era,technological progress has generated population growth and higherdensity, butnot higher per-capita income, which may imply that during that period there were no labor-augmenting innovations.Why did technological progress hardly increase labor productivity during the preindustrial era but was focused on labor improvement afterwards? The current paper provides a very simple and clear answer to this question by identifying the factors affecting the long-run direction of technological progress.

Based on a standard growth model, we show that while in the short run changes of relative factor prices (as suggested by Hicks 1932) and the relative abundance of these factors (as argued by Acemoglu 2002) impact on that direction, in the long run it depends only on the relative supply elasticities of primary factors with respect to their prices, and is biased towards improving the exploitation of the factor with the relatively smaller elasticity.The intuition behind this result is the following. In the short run, a higher factor price encourages not only invention to economize its use but also its accumulation. If the supply elasticity of that factoris very large, it may not be optimal to invest in inventions that economize its use. Furthermore, to offset that factor's abundance, balanced growth requires an increased investment in technologies that augment the efficiency of the factor with the smaller supply elasticity.In the limit, when a factor's supply elasticity is infinite, it is not necessary to invest any resources economizing its use.

To fix ideas, consider the case of oil. During a long period oil was abundant, and hardly any effort was put into economizing its use, as evidenced by the MPG figures of U.S. produced cars before the 1973 oil crisis. That crisis has caused a sharp increase in oil prices, causing investment in energy-saving technologies (e.g., increasing MPG). However, the same price increase also induced search for new oil sources, such as shale oil. These new sources have again increased the supply of oil, contributing to sharp price decreases. Consequently, the incentives to further invest in energy-saving technologies have decreased.[4]

[3]These stylized characteristics are further supported by Jones(2015) using the latest available data.

[4] According to the PEW Environment Group, the model-year 1975 cars drove about 14 miles pergallon. This figure has doubled by 1985, and stayed roughly stagnant for the next two decades, rising to about 33 by 2005 (seehttp://www.pewtrusts.org/~/media/assets/2011/04/history-of-fuel-economy-clean-energy-factsheet.pdf).

With this intuition in mind, the paper suggests the following answers to the aforementioned questions.In the pre-industrial era technological progress did not increase labor productivity because labor supply was very elastic (as described by Malthus 1798). Approximately concurrent with the industrial revolution, the demographic transition reduced the supply elasticity of labor. Moreover, the industrial revolution has replaced land by reproducible physical capital. As the supply elasticity of capital increased, there were no incentives to economize on its use and improve its productivity. Consequently, technological progress was biased towards improving human capital,thereby increasing labor productivity.

The ideas in this paper are closely related to previous literature.As early as in 1932, Hicks (1932) wrote: "A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind-directed to economizing the use of a factor which has become relatively expensive" (pp. 124-125).However, as noted by Kennedy(1964), innovation faced not only the incentives created by relative factor prices but also the constraints of the “innovation possibility frontier”.Based on Kennedy, Samuelson(1965) and Drandakis and Phelps(1966) built growth models to formalize the contribution of the induced innovations idea, whereby firms choose their technologies to maximize the current rate of cost reduction.However, this literature was criticized for its lack of micro-foundations. Consequently, for almost thirty years there was little research on the direction of technological progress. Only the work of Acemoglu (1998, 2002, 2003, 2007, and 2009) which studied the issue using the framework of endogenous technological change(as developed by Romer 1990, and Aghion and Howitt 1992) has renewed interest in this question.In contrast to the papers of the 1960s, Acemoglu’s models start from microeconomic foundations of technological change, where innovations are carried out by profit-maximizing firms. Funk(2002) as well as Irmen and Tabakovic(2015) also study the determinants of technological progress within perfectly competitive environments. However, in their long-run equilibria these papers are all bound by the Uzawa(1961) theorem, whereby the steady-state direction of technological progress must be purely labor-augmenting. Moreover, they cannot answer why technological progress did not increase labor productivity in the pre-industrial era.

Many have noted that the Uzawa theorem lacks economic intuition(Aghion and Howitt,1998,p16;Acemoglu,2003,2009;Jones,2005;Jones and Scrimgeour,2008). Schlicht(2006) provides a very simple proof of the theorem, from which it becomes clear that the linear relationship between capital accumulation and investment is the key to the result. The introduction of adjustment costs may justify the inclusion of non-linear elements in that process.Subject to the corresponding nonlinearities, technological progress can include both labor- and capital-augmenting elements along a steady-state growth path (Sato and Ramachandran,2000; Li and Huang,2012,2015;

Irmen,2013). A different approach allowing both types of technological progress has been recently suggested by Grossman et al. (2016).[5]However, these papers are not cast in the framework of endogenous technological progress, and hence cannot expose the factors determining its long-run direction.

To overcome that shortcoming, the current paper combines several features.First, the framework in this paper is based on Acemoglu(2002,2003) and starts from microeconomic foundations of technological change, where profit-maximizing firms undertake both labor- and capital-augmenting technological improvements. Second, unlike Acemoglu's framework, the two primary factor accumulation processes are assumed to be concave functions of the respective investments, implicitly reflecting increasing marginal transactions or adjustment costs.Using this structure we show that *any* direction of technological change may be consistent with a steady-state growth path.Moreover, we provide a very clear characterization of the underlying factors determining that direction.

The rest of the paper is organized as follows. The second section describes the economic environment of the benchmark model. Building on Acemoglu (2002, 2003), it analyses the behavior of households and firms and characterizes the steady-state equilibrium path; The third section focuses on the determinants of the direction of technological progress and briefly compares the current results to those of the existing literature; The fourth section discusses some applications of the results and some alternative specifications; The fifth section concludes.

2. Benchmark model

The economic environment extends Acemoglu (2002,2003). The economy consists of two kinds of material factors, and three sectors of production; a final goods sector, an intermediate goods sector and a research and development (R&D) sector. The preference structure, production functions and the innovation possibilities frontier are identical to Acemoglu's. However, the current analysis differs from that of Acemoglu's in the factor accumulation functions.

2.1 The economy

This subsection provides the specification of the underlying structure.

2.1.1 The representative household

The representative household owns two kinds of material factors, denote by K and L.To facilitate the discussion, we refer to these factors as "capital" and "labor". In addition, the household is endowed by S "scientists" whose role is explained below.The household'sgoal is to maximize the discounted flow of utility, given by:

[5] One of the features distinguishing the Grossman et al. (2016) paper from the literature surveyed above is its inclusion of *embodied* technological change in the definition of capital-augmenting technological progress.

$$\mathrm{U} = \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \tag{1}$$

where $C(t)$ is consumption at time t, $\rho>0$ is the discount rate, and $\theta>0$ is a utility curvature coefficient of the household. The household's periodic budget constraint is given by:

$$C + I_K + I_L = Y = wL + rK + \mathrm{w}_S S \tag{2}$$

where the LHS stands for expenditures consisting of consumption and investments I_K and I_L into capital and labor, and the RHS is income, obtained from renting out labor at the rate w, capital at the rate r and scientists at the rate w_S.

2.1.2 Production

The final goods sector is competitive, using the production function

$$Y = \left[\gamma {Y_L}^{(\varepsilon-1)/\varepsilon} + (1-\gamma){Y_K}^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} \quad ,0 \le \varepsilon < \infty \tag{3}$$

where Y is output and Y_L and Y_K are the two inputs, with the factor-elasticity of substitution given by ε.

The factors of production are also produced competitively by constant elasticity of substitution (CES) production functions using a continuum of intermediate inputs, $X(i)$ and $Z(j)$:

$$Y_L = \left[\int_0^N X(i)^{\beta} di\right]^{1/\beta} \ and \ Y_K = \left[\int_0^M Z(j)^{\beta} dj\right]^{1/\beta} ,0 < \beta < 1 \tag{4}$$

where theelasticity of substitution is given by $v = 1/(1-\beta)$ and N and M represent the measure of different types of the respective intermediate inputs. For the ease of discussion, we associate the Y_L and Y_K inputs with respective "labor" or "capital" intensive production technologies, and accordingly interpret an increase in N or in M as labor- or capital-augmenting technological change.

Intermediate inputs are supplied by monopolists who hold the indefinite right to use the relevant patent, and are produced linearly from their respective primary factors:

$$X(i) = L(i) \text{ and } Z(j) = K(j) \tag{5}$$

2.1.3 The innovation possibilities frontier

The innovation possibilities frontier functions are given by[6]

$$\begin{cases} \dot{N} = d_N N S_N - \delta N \\ \dot{M} = d_M M S_M - \delta M \end{cases} \tag{6}$$

where S_N and S_M represent, respectively, the "number" of scientists who carry out

[6] In the extension section below we provide some alternative specifications for which the main results of the paper continue to hold.

R&D generating “patents” of the labor-and capital-intensive intermediate goods.[7] Once an R&D firm invents a new kind of an intermediate input, its patent rights are perfectly enforced and perpetual.

2.1.4 Material factors accumulation

While the above follows precisely the Acemoglu (2003) formulation, the factor accumulation processes are different. Specifically, we assume:

$$\begin{cases} \dot{K} = b_K I_K^{\alpha_K} - \delta_K K, \; b_K > 0, \; 0 \leq \alpha_K \leq 1, \delta_K > 0 \\ \dot{L} = b_L I_L^{\alpha_L} - \delta_L L, \; b_L > 0, \; 0 \leq \alpha_L \leq 1, \delta_L > 0 \end{cases} \tag{7}$$

where the factors *K* and *L* depreciates at the rates δ_K and δ_L.

The concave transformation of final output into material resources is key to the results derived below. It represents the idea that converting final output into useable production factors of either type is associated with increasing costs. For example, if *K* is to be interpreted as physical capital, the transformation may involve adjustment costs that are increasing as investment grows. Similarly, thinking of L as labor, increasing the size of the labor force is also likely to be associated with convex costs.

For the special case of $\alpha_K = 0$, in the long run *K* is fixed by $K^* = b_K/\delta_K$, which is equivalent to the assumption of Acemoglu(2002). Setting $\alpha_K = 1$ yields the usual case of the neoclassical growth model.Similarly, if $\alpha_L = 0$, in the long run L is fixed by $L^* = b_L/\delta_L$, which is again equivalent to the assumption of Acemoglu (2002). The case $\alpha_L = 1$ turns out to yield a Malthusian environment (Malthus,1798).

2.2 Profit maximization

This subsection describes the profit maximization problems of the various actors, replicating Acemoglu (2002,2003).

Letting the final good serve as numeraire, the representative competitive final good producer faces the input prices p_L and p_K and selects the respective Y_K and Y_L so as to maximize

$$\pi_Y = Y - p_L Y_L - p_K Y_K \tag{8}$$

subject to theproduction function (3), yielding the demand functions:

$$\begin{cases} p_K = (1-\gamma)[\gamma + (1-\gamma)(Y_K/Y_L)^{(\varepsilon-1)/\varepsilon}]^{1/(\varepsilon-1)}(Y_K/Y_L)^{-1/\varepsilon} \\ p_L = \gamma[\gamma + (1-\gamma)(Y_K/Y_L)^{(\varepsilon-1)/\varepsilon}]^{1/(\varepsilon-1)} \quad . \end{cases} \tag{9}$$

The reperesentative producers of Y_K and Y_L maximize their profits by choosing *Z(j)* and *X(i)* given the intermediate input prices $p_Z(j)$ and $p_X(i)$:

[7] Equation (6) is a simple case of the equation (8) in Acemoglu (2003).

$$\begin{cases} \pi_K = p_K Y_K - \int_0^M p_Z(j) Z(j) dj \\ \pi_L = p_L Y_L - \int_0^M p_X(i) X(i) di \end{cases} \tag{10}$$

subject to their respective production functions (4). This generates the demand functions

$$\begin{cases} Z(j) = Y_K (p_K / p_Z(j))^{1/(1-\beta)} \\ X(i) = Y_L (p_L / p_X(i))^{1/(1-\beta)} \end{cases} \tag{11}$$

The intermediate input producers which hold the exclusive right to produce their particular type of inputface the prices of the primary inputs and choose, respectively,$(p_Z(j), K(j))$ and $(p_X(i), L(i))$ to maximize

$$\begin{cases} \pi_M(j) = p_Z(j) Z(j) - rK(j) \\ \pi_N(i) = p_X(i) X(i) - wL(i) \end{cases} \tag{12}$$

subject to their technologies (5) and the demand functions (11).

From the maximization problem of the intermediate goods producers (12) we obtain:

$$\begin{cases} p_Z(j) = r/\beta \\ p_X(i) = w/\beta \end{cases} \tag{13}$$

which imply that all intermediate inputs have the same mark-up over marginal cost. Substituting equations (13) into (11), we find that all capital-intensive and all labor-intensive intermediate goods are produced in equal (respective) quantities.

$$\begin{cases} Z(j) = Z = Y_K (\beta p_K / r)^{1/(1-\beta)} \\ X(i) = X = Y_L (\beta p_L / w)^{1/(1-\beta)} \end{cases} \tag{14}$$

By the production functions of the intermediate inputs (3), all monopolists have the same respective demand for labor and capital.

Finally, the patent holders exctract the monopoly profits from the intermediate goods producers. However, due to the competition for the services of scientists, these profits are paid out as wages, yielding

$$\begin{cases} \mathrm{w}_S S_M = \int_0^M \pi_M(j) dj \\ \mathrm{w}_S S_N = \int_0^N \pi_N(i) di \end{cases} \tag{15}$$

2.3 Market equilibrium.

The material factor market clearing condition implies:

$$\begin{cases} Z(j) = K/M \\ X(i) = L/N \end{cases} \tag{16}$$

Substituting equations (16) into (4), we obtain the equilibrium quantities of labor-intensive and capital-intensive inputs:

$$\begin{cases} Y_L = \left[\int_0^N X(i)^\beta di\right]^{1/\beta} = N^{(1-\beta)/\beta} L \\ Y_K = \left[\int_0^M Z(j)^\beta dj\right]^{1/\beta} = M^{(1-\beta)/\beta} K \end{cases} \tag{17}$$

Finally, using equations (17) and (3), we obtain the amount of the final good:

$$Y = \left[\gamma (N^{(1-\beta)/\beta} L)^{(\varepsilon-1)/\varepsilon} + (1-\gamma)(M^{(1-\beta)/\beta} K)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} \tag{18}$$

In order to simplify notation, we follow Acemoglu (2003) by letting $A \equiv N^{(1-\beta)/\beta}$ and $B \equiv M^{(1-\beta)/\beta}$, to obtain:

$$Y = \left[\gamma (AL)^{(\varepsilon-1)/\varepsilon} + (1-\gamma)(BK)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} \quad . \tag{19}$$

Therefore, increasing the variety of capital-intensive or labor-intensive intermediate inputs, *M* and *N*, implies capital- or labor-augmentation.

Let $k \equiv BK/AL$ be the ratio of effective capital to effective labor, so that

$$k = (M^{(1-\beta)/\beta} K)/(N^{(1-\beta)/\beta} L). \tag{20}$$

Accordingly,equation (19) can be rewritten as:

$$f(k) \equiv Y/AL = \left[\gamma + (1-\gamma) k^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} \quad . \tag{21}$$

Usingequation (21), we transform the market prices of the capital-intensive and labor-intensive inputs (9) into the following forms:

$$\begin{cases} p_K = f'(k) \\ p_L = f(k) - kf'(k) \end{cases} \tag{22}$$

Using equations (16), (17), and (22) in (14), we obtain

$$\begin{cases} r = \beta M^{(1-\beta)/\beta} f'(k) = \beta B f'(k) \\ w = \beta N^{(1-\beta)/\beta} [f(k) - kf'(k)] = \beta A [f(k) - kf'(k)] \end{cases} \tag{23}$$

Equations (23) indicate that the returns to the primary factors are positively related to the respective “number” of the intermediate inputs.

The monopoly profits (10) become, by equations (13), (17) and (23):

$$\begin{cases} \pi_M = (p_Z - r) Z = (1-\beta) M^{(1-2\beta)/\beta} K f'(k) \\ \pi_N = (p_X - w) X = (1-\beta) N^{(1-2\beta)/\beta} L [f(k) - kf'(k)] \end{cases} \tag{24}$$

Substituting equations (23) intoequations (24), we obtain

$$\frac{\pi_M}{\pi_N} = \frac{r}{w} \frac{K}{L} / \frac{M}{N} \tag{25}$$

Equation (25) shows that for a given ratio of the technological levels (*M/N*), relative invention profits are positively related to the relative factor prices (r/w) and the relative factor supplies (*K/L*). Accordingly, a change of relative price encourages innovations directed at the scarce factor whose price has increased, as suggested by Hicks (1932).

The relative amount of the two factors, (*K/L*), has two countervailing effects on π_M/π_N. On the one hand, a higher *K/L* causes an increase in π_M/π_N, which in turn leads to a technological change favoring the abundant factor. This is what Acemoglu(2002) named "the market size effect". On the other hand, a higher *K/L* decreases r/w and π_M/π_N, which is the price effect of a change in *K/L*.[8]The total effect of a change in *K/L* is regulated by the elasticity of substitution ε between the two factors.If $\varepsilon > 1$, the market size effect dominates the price effect, and increasing *K/L* will encourage favoring improvements of the abundant factor.Otherwise, when $\varepsilon < 1$, improvements of the scarce factor will be favored (Acemoglu,2002).

However, holding *M/N* fixed implies that these effects are only the static or short-run ones. Specifically, when $\varepsilon > 1$, favoring innovation in the capital-intensive intermediate factor causes *M/N* to increase. Equation (25) shows that a higher *M/N* causes π_M/π_N to decrease, preventing further inverstment into innovations in the capital-intensive sector. Moreover, equation (25) represents only the demand side of technological change. To get the long effects, it is necessary to consider also factors affecting the supply of innovations and material factors, in particular that of r/w on *K/L* and of π_M/π_N on M/N, within a dynamic general equilibrium framework. As will be shown below, in such a context, even if there is a short-run "market size effect", *K/L* and *M/N* cannot be both continually increasing in the long-run.

Finally, we turn to the market for scientists which determines the supply of innovations. Owing to the free-entry into the R&D sector assumption, the marginal innovation value of scientists should be equal across technologies. Using the innovation possibilities frontier function (6), this implies

$$d_N N \pi_N = d_M M \pi_M \tag{26}$$

From equation (24) we obtain

$$\frac{d_N}{d_M} = \frac{kf'(k)}{f(k) - kf'(k)} \tag{27}$$

Applying equation (21) to (27) yields

$$k^* = \left[\frac{\gamma d_N}{(1-\gamma) d_M}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{28}$$

Equation (28) shows that market clearing implies that k^* is a constant determined solely by the parameters γ, d_M, d_N and ε. Equations (23), (24) and (28),also yield the following factor shares:

[8] It is worth noting that the price effect caused by a change of the relative factors supply *K/L* is different from the effect of an exogenous change of the relative price *r/w* when *K/L* is given.

$$
\begin{cases}
\varphi_L \equiv \dfrac{wL}{Y} = \beta \dfrac{d_M}{d_M + d_N} \\
\varphi_K \equiv \dfrac{rK}{Y} = \beta \dfrac{d_N}{d_M + d_N} \\
\varphi_N \equiv \dfrac{w_S S_N}{Y} = (1-\beta) \dfrac{d_M}{d_M + d_N} \\
\varphi_M \equiv \dfrac{w_S S_M}{Y} = (1-\beta) \dfrac{d_N}{d_M + d_N} \\
\varphi_{K/L} = \varphi_{M/N} = \dfrac{d_N}{d_M}
\end{cases} \tag{29}
$$

Equations (29) show that factor shares are determined solely by the market clearing conditions and depend only on the parameters β, d_M, d_N.[9]

2.4 Consumer behavior

Households maximize their objective (1) subject to the budget constraint (2), taking as given the technological change processes (6) and factor accumulation(7).

The corresponding Euler conditions are given by equations (30):[10]

$$
\begin{cases}
\dot{C}/C = \left[\alpha_K b_K I_K^{\alpha_K - 1} r - (\alpha_K - 1)\dot{I}_K/I_K - \rho - \delta_K\right]/\theta \\
\dot{C}/C = \left[\alpha_L b_L I_L^{\alpha_L - 1} w - (\alpha_L - 1)\dot{I}_L/I_L - \rho - \delta_L\right]/\theta
\end{cases} \tag{30}
$$

Equations (30) reflect the conditions of the optimal allocation of income among consumption and the two kinds of investment.The first equation in (30) is the necessary condition for the optimal allocation between physical capital investment and consumption. It is worth noting that when $\alpha_K = b_K = 1$, the equation simplifies to the familiar form $\dot{C}/C = (r - \rho - \delta_K)/\theta$.In that environment a constant value of $\dot{C}/C$ implies that r must be constant.However,if $0 < \alpha_K < 1$, when $\dot{C}/C$ and $\dot{I}_K/I_K$ are constant, the rate r cannot be constant unless $\dot{r}/r = (1 - \alpha_K)\dot{I}_K/I_K$. Thus steady-state growth does not necessarily imply a constant market rental price of capital.The second equation in (30) is the necessary condition for the optimal allocation between labor investment and consumption. The optimal allocation is achieved when the two equations hold simultaneously. As long as one equation of (30) is not satisfied, the household can obtain a higher level of utility by reallocating itsincome among consumption and investments.

Finally, the transversality condition is given by

$$
\lim_{t \to \infty} K(t) \, exp\left[-\int_0^t r(v)dv\right] = 0 \tag{31}
$$

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[9] Notice that the market equilibrium is identical to that of Acemoglu (2003). However, that paper does not exploit equation (29) and therefore fails to notice that ε has no impact on factor shares.

[10] See Appendix A.

2.5 Steady-state equilibrium

We summarize the section by stating the conditions implying a steady-state equilibrium for the above environment.

Definition 1: A *steady-state equilibrium path*(hereafter SSEP) is a dynamic path along which the endogenous variables (Y, C, I_K, I_L, K, L, M, N) are growing at constant rates, household utility and all producer profits are maximized and markets clear at each instant.

Using Definition 1, we obtain the following results:

Proposition 1: For the benchmark economy described above there exists a unique SSEP where equations (32) provide the growth rates of consumption (C), output(Y), investments(I_k, I_L), primary factors $(K)$and(L), and the measures of intermediate inputs(M, N), and the allocation of scientists and income are given by equations (33).

$$\begin{cases} \left(\frac{\dot{Y}}{Y}\right)^* = \left(\frac{\dot{I}_L}{I_L}\right)^* = \left(\frac{\dot{I}_K}{I_K}\right)^* = \left(\frac{\dot{C}}{C}\right)^* = g \\ \left(\frac{\dot{K}}{K}\right)^* = \alpha_K g \\ \left(\frac{\dot{L}}{L}\right)^* = \alpha_L g \\ \left(\frac{\dot{M}}{M}\right)^* = (1-\alpha_K)\frac{\beta}{1-\beta} g \\ \left(\frac{\dot{N}}{N}\right)^* = (1-\alpha_L)\frac{\beta}{1-\beta} g \\ g = \frac{1-\beta}{\beta}\frac{d_M d_N S - (d_M + d_N)\delta}{(1-\alpha_K)d_N + (1-\alpha_L)d_M} \end{cases} \tag{32}$$

$$\begin{cases} S_N{}^* = \frac{(1-\alpha_L)d_M S + (\alpha_L - \alpha_K)\delta}{(1-\alpha_K)d_N + (1-\alpha_L)d_M} \\ S_M{}^* = \frac{(1-\alpha_K)d_N S + (\alpha_K - \alpha_L)\delta}{(1-\alpha_K)d_N + (1-\alpha_L)d_M} \\ s_C{}^* = 1 - s_K{}^* - s_L{}^* \\ s_K{}^* = \frac{\beta\alpha_K d_N}{d_M + d_N}\frac{\delta_K + g\alpha_K}{\rho + \delta_K + g\alpha_K - g(1-\theta)} \\ s_L{}^* = \frac{\beta\alpha_L d_M}{d_M + d_N}\frac{\delta_L + g\alpha_L}{\rho + \delta_L + g\alpha_L - g(1-\theta)} \end{cases} \tag{33}$$

with $s_K \equiv I_K/Y$, $s_L \equiv I_L/Y$, $s_C \equiv C/Y$.

Proof: see appendix B.

From equations(32) and the definition of B and A,the rates of capital- and labor-augmenting technological progress aregiven by (34):

$$\begin{cases}(\dot{B}/B)^* = (1-\alpha_K)g \\ (\dot{A}/A)^* = (1-\alpha_L)g\end{cases} \tag{34}$$

Corollary 1: If $\alpha_K < 1$ and $\alpha_L < 1$, then an SSEP includes both types of technological progress.

The coexistence of both types of technological progress along an SSEP is due to the introduction of diminishing returns in the factor accumulation processes. Specifically, when α_K< 1, there is a gap between capital and output growth rates (see the second equation of (32)). That gap is closed by capital-augmenting technological progress (the first equation of (34), see also Irmen(2013)). Similarly, when $\alpha_L < 1$, labor is accumulated at a rate that falls short of output growth, and labor-augmentation makes up for the difference. Notice also that in the current framework the presence of both types of technological progress does not contradict the fact that factor shares remain constant (see equations (29) above).[11]

The stationary equilibrium solution enables us to infer what determines the direction of technological progress along an SSEP, which is the topic of the next section.

3. The determinants of the direction of technological progress

Identifying what determines the direction of technological progress is the main objective of this paper. Before stating the results we give a clear definition ofthatdirection.

Definition 2: The *direction of technological progress, DT,* is the ratio between the rates of capital-and labor-augmenting factors, i.e. $DT \equiv (\dot{B}/B)/(\dot{A}/A)$.

When $\dot{B}/B = 0$ and $\dot{A}/A > 0$ then *DT=0*, and technological progress is purely labor-augmenting (i.e. Harrod-neutral); when $\dot{B}/B > 0$and$\dot{A}/A = 0$ then $DT \to +\infty$, and technological progress is purely capital-augmenting (i.e. Solow-neutral); when

[11]This stands in contrast to Samuelson (1965), Drandakis and Phelps (1966), and Acemoglu (2003）who argued that labor-augmenting technological progress is one of the main assumptions needed to explain the stability of factor shares.

$\dot{B}/B = \dot{A}/A > 0$ then DT=*1*, and technological progress is Hicks-neutral.

Figure 1 shows different directions of technological progress:

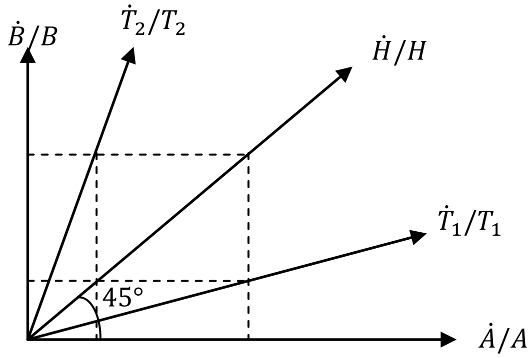

Figure 1: Direction of technological progress

Clearly, the axes represent Harrod-neutral (horizontal) and Solow-neutral (vertical) technological changes. The diagonal $\dot{H}/H$ represents the location of Hicks-neutral technological changes. The ray $\dot{T}_1/T_1$ indicates technological progress which tends to be more labor augmenting, while $\dot{T}_2/T_2$ is more capital augmenting.

3.1 Main results

Using definition 2, we can state the main results of this paper.

Proposition 2: Along an SSEP equations(34) immediately imply：

$$DT = \frac{1-\alpha_K}{1-\alpha_L} \tag{35}$$

Equation (35) shows that the direction of technological progress is determined by the exponents of the primary factor accumulation functions, namely α_K and α_L.It also shows that the parameters of the production function and the innovation possibilities frontier, such as the elasticity of substitution between labor and capital,ε, do not affect the direction of technical progress.

In order to provide an economic intuition for equation (35), we define next the primary factors' supply elasticities and then discuss the relationship between these elasticities and the direction of technological progress.

Definition 3: The *supply elasticity* of any primary factor *X* with respect to its price *p* is given by

$$\varepsilon_{X,p} \equiv \frac{\dot{X}/X}{\dot{p}_X/p_X} \tag{36}$$

With this definition in mind, we obtain the following relationships:

Corollary 2: Along an SSEP, the supply elasticities of capital and labor are given by:

$$\begin{cases} \varepsilon_{K,r} = \alpha_K/(1-\alpha_K) \\ \varepsilon_{L,w} = \alpha_L/(1-\alpha_L) \end{cases} \tag{37}$$

The result follows immediately from equations (32) alongside the time derivatives of (23) (using equation (34) and remembering that k^* is constant).Along an SSEP, equations (37) show that the factor supply elasticities are determined by the exponents of investment in the respective accumulation processes. This is because α_K and α_L regulate the degree to which returns to investment in factor accumulation are diminishing. Specifically, the higher α_K or α_L are, the higher are the returns to the respective investment. As a result, the quantitative response to a price change will increase, i.e. the supply elasticity will be higher.

Using equations (37) in (36) directly obtains:

$$DT = \frac{1+\varepsilon_{L,w}}{1+\varepsilon_{K,r}} \tag{38}$$

The interpretation of equation (38) is summarized as Proposition 3 which is the key result of the paper.

Proposition 3: Along an SSEP the direction of technological progress is determined solely by the relative primary factor supply elasticities and is biased towards the one with the relatively smaller elasticity.

Proposition 3 and equation (38) show thatthe direction of technological progress is *not* biased towards the relatively more or less abundant factor, but rather towards the harder to accumulate one.In other words, if one factor is relatively harder to accumulate, balanced growth requires that it must be augmented by technological change.

3.2 Comparison with Hicks (1932) and Acemoglu (2002).

Hicks (1932) argued that a change in the relative prices of the factors of production spurs invention and Acemoglu (2002) suggested that the relative market sizes is another factor that shapes the direction of technological progress. However, equations (35) and (38) show that when the economy is on an SSEP neither appears as a determinant of that direction, as stated by the following:

Proposition 4: Along an SSEP the direction of technological progress remains unchanged despite the continually changing relative factor prices and relative factor supplies.

Proof: Using equations (32) and applying equation (37), given initial values $A_0, B_0, K_0, L_0, r_0, w_0$ which are on an SSEP, the time evolutions of the relative primary factor supply (*K/L*), the relative price (*r/w*) and the relative technology level

(B/A) are given by:

$$\begin{cases} \left(\frac{K}{L}(t)\right)^* = \left(\frac{K_0}{L_0}\right) EXP\left(\frac{(\varepsilon_{K,r} - \varepsilon_{L,w})g}{(1+\varepsilon_{K,r})(1+\varepsilon_{L,w})} t\right) \\ \left(\frac{r}{w}(t)\right)^* = \left(\frac{r_0}{w_0}\right) EXP\left(-\frac{(\varepsilon_{K,r} - \varepsilon_{L,w})g}{(1+\varepsilon_{K,r})(1+\varepsilon_{L,w})} t\right) \\ \left(\frac{B}{A}(t)\right)^* = \left(\frac{B_0}{A_0}\right) EXP\left(-\frac{(\varepsilon_{K,r} - \varepsilon_{L,w})g}{(1+\varepsilon_{K,r})(1+\varepsilon_{L,w})} t\right) \end{cases} \quad (39)$$

where $\left(\frac{B_0}{A_0}\frac{K_0}{L_0}\right) = \left[\frac{\gamma d_N}{(1-\gamma)d_M}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \left(\frac{r_0}{w_0}\frac{K_0}{L_0}\right) = \frac{d_N}{d_M}$, are the respective initial values of these variables.

Equations (39) show that (K/L), (r/w) and (B/A) are evolving along the SSEP and that their growth (or decline) rates are impacted by the relative size of the primary factor price elasticities. Since the latter stay constant by equations (37), so does *DT* by equation (38). ■

It is important to notice the distinction between growth and level effects. Our definition of the direction of technological progress refers to the *relative change* in the capital- and labor-augmenting processes. However, even if this relative change is constant, the relative *levels* of the two technologies are changing (unless *DT*=1). Consider for example the case $\varepsilon_{K,r} > \varepsilon_{L,w}$. In that case, the second equation of (39) reveals that the price of labor, *w*,increases faster than that of capital, *r*. According to the Hicksian hypothesis this should induce more labor-augmenting technological progress thereby reducing *B/A*. This logic is fully consistent with the current model, as can be seen from the last equation of (39).

However, in this case, the first equation of (38) shows that *K/L* will keep increasingalong an SSEP, while the relative technology *B/A* will keep decreasing.This is because *k*=BK/AL* stays constant, so that a rising *K/L* is consistent only with a declining B/A. Accordingly, in the current framework Acemoglu's (2002) case, where an increase in *K/L* raises *B/A*, may happen only along a transition path, that is, only when the economy moves from one SSEP to another. Specifically, in the benchmark model $k*$ may be increasing from k_0^* to k_1^* due to some change in the exogenous parameters, such as d_N, d_M, ε or γ.[12]Once arriving at a new steady state, the direction of technological progress will again be determined by the equation (38).[13]

The above distinction also highlights the difference between a static and a dynamic concept of factor scarcity. In the static sense, a factor is relatively scarce if

[12]Acemoglu (2002, 2009) assumes that *K* and *L* are given and provides the determinants of the relative technology *levels* (B/A). In that environment, $k*$ may be changing due to exogenous changes in *K* or *L*. As argued above, fixing *K* and *L* amounts to setting $\alpha_K = \alpha_L = 0$.However, we have seen that with $\alpha_K \neq \alpha_L$, $\dot{B}/B \neq \dot{A}/A$ and B/A will be continually changing.

[13]In fact, in all cases considered by Acemoglu (2002) technological progress is Hicks-neutral (that is, *DT=1*) and not affected by changes in *K/L* along the steady-state path.

its quantity is smaller than that of the other. Acemoglu's (2002) aforementioned "market size effect" which implies that technological progress will favor the relatively more *abundant* factor, (see discussion below equation (25)), relates to that static scarcity sense. In the dynamic sense, a factor is relatively scarce if it is harder to accumulate, and by equation (37) has a smaller supply elasticity. In this sense it is the relatively *scarce* factor that enjoys the faster technological augmentation in the long-run.

4. Discussion and Extension

This section first provides a possible interpretation of the model's results, and then turns to some possible extensions.

4.1 The role of the supply elasticities

Equation (37) implies that when $\alpha_L \to 1$, $\varepsilon_{L,w} \to \infty$. Furthermore, from equation (34) we get $(\dot{A}/A)^* = 0$, i.e. there is no labor augmentation. In addition, equations (32) imply that Y and L grow at the same rate.

It is in this sense that this SSEP is Malthusian.[14]Labor supply is perfectly elastic, and while output may be growing due to capital (or land) augmenting technological change, labor grows just as fast, leaving no room for per-capita increases in income and consumption. In fact, many have argued that this feature characterizes, to a large extent, the growth path prior to the industrial revolution (see, e.g, Ashraf and Galor 2011).

In a similar vein, let $\alpha_K \to 1$. Clearly equations (7) imply that the capital accumulation process is linear in investment, which is the standard assumption in neoclassical growth models. From equations (32) we obtain that the capital/output ratio is constant, and equations (34) imply that there is no capital augmentation. These features are in line with the Kaldor (1961) stylized facts. Equation (37) implies in addition that $\varepsilon_{K,r} \to \infty$.[15]

To summarize, the current model is consistent with the "Kaldor facts" as long as $\varepsilon_{K,r} \to \infty$ and $\varepsilon_{L,w}$ is finite. A Malthusian path in which per-capita income remains constant is obtained if $\varepsilon_{L,w} \to \infty$. Additional conditions often found in the literature stating that a Malthusian path requires K (interpreted as "land") to be constant and technological progress to be nil do not apply in the current framework. Moreover, as shown above, the model finds that both types of technological progress may coexist with a bias towards labor (as long as $\varepsilon_{L,w} < \varepsilon_{K,r} < \infty$).This finding is consistent with empirical studies. Sato's (1970) analysis of 1909-1960 US national

[14] Li and Huang(2016) proves that in a Malthusian model technological change must be without labor-augmenting.

[15]Acemoglu (2003) seems to suggest that technological progress must be labor-augmenting because "capital, K, can be accumulated, while labor, L, cannot." The current model shows that technological progress is purely labor-augmenting not because labor cannot be accumulated but because the supply elasticity of capital is infinite.

income data pointed out that technological progress increased both capital and labor productivity, with labor efficiency increasing faster than capital's. More recently, Doraszelski and Jaumandreu(2015) used Spanish industrial panel data and found that at the firm level technological change was labor-biased.[16]

4.2 Possible extensions

The benchmark model is based on several assumptions concerning the underlying technologies. In particular, technological change is assumed to take the form of invention of new goods;R&D requires only the input of "scientists", i.e. that sector does not compete for investment goods; and the productivity in the R&D sector depends only on the number of existing goods in the same sector. It turns out that the key results (equations (35) and (38)) may be obtained under somewhat different specifications of the innovation technologies, albeit subject to some knife-edge conditions.

4.2.1 Knowledge spillover

The first extension takes the knowledge spillover model used in Acemoglu(2002,2009). According to that model, productivity in any of the R&D sectors depends on the number of existing varieties in *both* sectors. Specifically, the innovation possibilities frontier is defined by

$$\begin{cases}\dot{N} = d_N N^{(1+\eta)/2} M^{(1-\eta)/2} S_N - \delta N \\ \dot{M} = d_M M^{(1+\eta)/2} N^{(1-\eta)/2} S_M - \delta M\end{cases}, \quad 0 \le \eta \le 1, \text{where } S_N + S_M = S \qquad (40)$$

The above benchmark model is obtained when $\eta = 1$, which Acemoglu called "extreme state dependence".

Proposition 5: If the innovation possibilities frontier (6) is replaced by equations (40) with $\eta < 1$, while keeping the remaining assumptions of the benchmark model, an SSEP exists only under the knife-edge condition of $\alpha_L = \alpha_K$.

Proof: see appendix C.

Corollary 3: Under the conditions of Proposition 5, the direction of technological progress is determined by equation (38).

The intuition of the result follows directly from equations (40). To keep the growth rates of *M* and *N* constant, it must be the case that *M/N* is constant, so that they

[16] Sato (1970) reports yearly labor augmentation of about 2%, and capital augmentation of roughly 1.3%. Doraszelski and Jaumandreu (2015) decompose technological change into Harrod- and Hick-neutral components, finding that both have increased by an annual rate averaging 2% . In our terms this would translate into yearly rates of 4% labor- and 2% capital-augmentation.

both grow at the same rate. This implies that *B* and *A* also grow at the same rate. As a result, *K* and *L* grow at the same rate, which requires $\alpha_L = \alpha_K$.Clearly, by equation (37) we obtain $\varepsilon_{K,r} = \varepsilon_{L,w}$ and technological progress must be Hicks-neutral.

4.2.2 Lab equipment model

The lab equipment model was suggested by Rivera-Natiz and Romer (1991), and is used in Acemoglu (2002,2003,2009). In that model, the main input into the R&D sectors is final output. As a result, the accumulation processes and the R&D sectors compete for resources. To investigate the impact of this competition the current subsection generalizes the lab equipment model of Acemoglu(2002,2003,2009)and assumes the following innovation functions:

$$\begin{cases} \dot{M} = b_M I_M^{\alpha_M} - \delta_M M, \ b_M > 0, \ \ 0 \le \alpha_M \le \dfrac{\beta}{1-\beta}, \delta_M > 0 \\ \dot{N} = b_N I_N^{\alpha_N} - \delta_N N, \ b_N > 0, \ 0 \le \alpha_N \le \dfrac{\beta}{1-\beta}, \delta_N > 0 \end{cases} \tag{41}$$

where I_M and I_N are investments needed to develop new varieties *M* and *N* of the respective intermediate inputs, and δ_M and δ_N are respectively deprecation rates of blueprints of new varieties of capital- and labor-intensive intermediate inputs.[17]In this setting, the representative household's income can be used for either consumption or the corresponding four types of investments. In addition, the households are the direct owners of the patents and accordingly obtain the monopoly profits of the intermediate input producers. In this case too, for an SSEP path to exist,some knife-edge conditions must prevail among the parameters $\alpha_K, \alpha_L, \alpha_M, \alpha_N, \beta$ as summarize by:

Proposition 6: Subject to the proper modifications of the benchmark model, if the innovation possibility frontier takes the form of equations (41), an SSEP exists only under the following knife-edge conditions:[18]

$$\begin{cases} \alpha_K + [(1-\beta)/\beta]\alpha_M = 1 \\ \alpha_L + [(1-\beta)/\beta]\alpha_N = 1 \end{cases} \tag{42}$$

Proof: see appendix D.

Corollary 4: Subject to conditions (42), the direction of technological progress is determined by equation (38).[19]

[17]When $\alpha_M = 0$ and $\alpha_N = 0$, in the long run *M* and *N* will be fixed at $M^* = b_M/\delta_M$ and $N^* = b_N/\delta_N$, respectively. When $\alpha_M = \alpha_N = 1$, equations (40) are as same as the equation (19) (without depreciation) in Acemoglu (2002) or equation (34) in Acemoglu (2003). However, in the Acemoglu (2002, 2003) cases, there exists no SSEP unless $\alpha_K = \alpha_L = 0$ which is Acemoglu (2002)'s assumption, in contrast to $\alpha_K = 1$ and $\alpha_L = 0$ as assumed by Acemoglu (2003).

[18] Knife-edge conditions are often found in the growth literature. See, e.g., Jones (1995); Christiaans (2004); Growiec (2010); Grossman et al. (2016).

[19] Proof is available upon request.

4.3. Amending the input production function

The production functions of the inputs Y_L and Y_K of equation (4) may be replaced by identical constant elasticity of substitution (CES) production functions with corresponding intermediate inputs, *X(i)* and *Z(i)*:

$$\begin{cases} Y_L = \dfrac{1}{1-\beta}\left[\displaystyle\int_0^N X(i)^{1-\beta} di\right] L^\beta \\ Y_K = \dfrac{1}{1-\beta}\left[\displaystyle\int_0^M Z(i)^{1-\beta} di\right] K^\beta \end{cases} \tag{43}$$

Once invented, all kinds of intermediate inputs can be produced at a fixed marginal cost $\Psi = 1 - \beta$ in terms of the final output, as in Acemoglu(2009).

It can be shown that under the specification (43), the associated fixed marginal production cost of the intermediate inputs and the remaining assumptions of the benchmark model, the direction of technological progress is still determined by equations (35) and (38).[20]

6. Conclusions

What determines the direction of technological progress? This is one of the central issues of the theory of economic growth. To answer this question, this paper adopted a framework based on Acemoglu(2002,2003) in which profit-maximizing firms undertake both labor- and capital-augmenting technological improvements, amended by nonlinear accumulation functions of the two primary factors.It was shown that despite their short-run impact pointed out by Acemoglu(2002), the relative factor price and the market size effects disappear as long-run determinants of the direction of technological progress. Instead, along a stationary equilibrium path, it is the relative size of the supply elasticities of material factors with respect to their respective prices which determines that direction, biasing it towards the factor with the relatively smaller elasticity.

The paper provides new insights concerning the switch in the direction of technological progress between the preindustrial era and the period following the industrial revolution. Specifically, empirical studies by Kaldor(1961) and Ashraf and Galor(2011) show that technological progress before the industrial revolution was nearly completely devoid of labor-augmenting elements, while after the industrial revolution it was almost purely labor-augmenting. The paper argues that these facts may be due to the very high labor supply elasticity in a Malthusian world on the one hand, and very high renewable physical capital supply elasticity but much lower labor supply elasticity after the industrial revolution.

This paper also sheds some light on another important issue in current

[20]Proof is available upon request.

macroeconomics that has recently drawn growing attention, namely the global decline in labor shares and increased income inequality (e.g., Karabarbounis and Neiman, 2013; Piketty,2014). Some authors have argued that the bias of technological progress towards capital-augmentation is an important cause of these phenomena. However, the result of this paper implies that there is no necessary connection between capital-augmentation and declining labor shares even in transition path.

A key phenomenon not addressed by the current framework is the continued decline of the relative price of investment goods (Karabarbounis and Neiman, 2013; Grossman et al., 2016). This trend indicates that embodied technological progress has been playing a role for quite some time. The introduction of this option is likely to affect the resource allocation of profit-maximizing R&D firms between factor-augmentation and embodied technologies. This is an important problem for fruitful future research.

Appendix A: Deriving the Euler Equations (30)

Let the Hamilton associated with the optimization problem be:

$$H = U(C)e^{-\rho t} + \lambda_K\left(b_K I_K^{\alpha_K} - \delta_K K\right) + \lambda_L\left(b_L I_L^{\alpha_L} - \delta_L L\right)$$

$$+\mu[wL + rK + \pi_N N + \pi_M M + w_S S - C - (I_K + I_L) \qquad (A1)$$

The first-order conditions are:

$$\begin{cases} C^{-\theta}e^{-\rho t} = \lambda_K \alpha_K b_K I_K^{\alpha_K - 1} \\ C^{-\theta}e^{-\rho t} = \lambda_L \alpha_L b_L I_L^{\alpha_L - 1} \\ C^{-\theta}e^{-\rho t} = \mu \end{cases} \qquad (A2)$$

Taking log-derivatives of both sides of (A2) over time, we obtain

$$\begin{cases} -\theta\dfrac{\dot{C}}{C} - \rho = \dfrac{\dot{\lambda}_K}{\lambda_K} + (\alpha_K - 1)\dfrac{\dot{I}_K}{I_K} \\ -\theta\dfrac{\dot{C}}{C} - \rho = \dfrac{\dot{\lambda}_L}{\lambda_L} + (\alpha_L - 1)\dfrac{\dot{I}_L}{I_L} \\ -\theta\dfrac{\dot{C}}{C} - \rho = \dfrac{\dot{\mu}}{\mu} \end{cases} \qquad (A3)$$

The motion equations of λ are:

$$\begin{cases} \dot{\lambda}_K = -\partial H/\partial K = \lambda_K \delta_K - \mu r \\ \dot{\lambda}_L = -\partial H/\partial L = \lambda_L \delta_L - \mu w \end{cases} \qquad (A4)$$

Based on (A2) and (A4), we obtain

$$\begin{cases} \dot{\lambda}_K/\lambda_K = \delta_K - r\alpha_K b_K I_K^{\alpha_K - 1} \\ \dot{\lambda}_L/\lambda_L = \delta_L - w\alpha_L b_L I_L^{\alpha_L - 1} \end{cases} \qquad (A5)$$

Using (A5) in (A3), we obtain the Euler equations (30).

$$\begin{cases} \dfrac{\dot{C}}{C} = \dfrac{1}{\theta}\left\{r\alpha_K b_K I_K^{\alpha_K - 1} - (\alpha_K - 1)\dfrac{\dot{I}_K}{I_K} - \rho - \delta_K\right\} \\ \dfrac{\dot{C}}{C} = \dfrac{1}{\theta}\left\{w\alpha_L b_L I_L^{\alpha_L - 1} - (\alpha_L - 1)\dfrac{\dot{I}_L}{I_L} - \rho - \delta_L\right\} \end{cases}$$

Appendix B: Proof of Proposition 1.

We first conjecture there is a SSEP then verify it indeed exists by solving for it.

First, we prove that there is a SSEP given by equations (30). From the budget constraint (2) and Definition 1, we obtain

$$\frac{\dot{Y}}{Y} = \frac{\dot{I}}{I} = \frac{\dot{I}_L}{I_L} = \frac{\dot{I}_K}{I_K} = \frac{\dot{C}}{C} \qquad (B1)$$

Then, according to the primary factor accumulation functions and (7), the along an SSEP the following must hold:

$$\begin{cases} \dfrac{\dot{K}}{K} = \alpha_K \dfrac{\dot{I}_K}{I_K} = \alpha_K \dfrac{\dot{C}}{C} \\ \dfrac{\dot{L}}{L} = \alpha_L \dfrac{\dot{I}_L}{I_L} = \alpha_L \dfrac{\dot{C}}{C} \end{cases} \tag{B2}$$

From equation (21) we can obtain

$$Y = N^{(1-\beta)/\beta} L f(k) = M^{(1-\beta)/\beta} K f(k)/k \tag{B3}$$

Since k is constant on the SSEP, from (B3) we get

$$\frac{\dot{Y}}{Y} = \frac{1-\beta}{\beta}\frac{\dot{N}}{N} + \frac{\dot{L}}{L} = \frac{1-\beta}{\beta}\frac{\dot{M}}{M} + \frac{\dot{K}}{K} \tag{$B4$}$$

Equations (B1), (B2), (B4) together with the innovation possibilities frontier (6), yield:

$$\begin{cases} (1-\alpha_L)\dfrac{\dot{C}}{\mathrm{C}} = \dfrac{1-\beta}{\beta}\{\mathrm{d_N S_N} - \delta\} \\ (1-\alpha_K)\dfrac{\dot{C}}{\mathrm{C}} = \dfrac{1-\beta}{\beta}\{\mathrm{d_M(S - S_N)} - \delta\} \end{cases} \tag{B5}$$

From (B5) and $S_M + S_N = S$, we obtain the allocation of scientists between two kinds of intermediate R&D given by (B6).

$$\begin{cases} {S_N}^* = \dfrac{(1-\alpha_L)d_M S + (\alpha_L - \alpha_K)\delta}{(1-\alpha_K)d_N + (1-\alpha_L)d_M} \\ {S_M}^* = \dfrac{(1-\alpha_K)d_N S + (\alpha_K - \alpha_L)\delta}{(1-\alpha_K)d_N + (1-\alpha_L)d_M} \end{cases} \tag{B6}$$

Combining (B1) ,(B5) and (B6), we get the growth rates as given by (B7).

$$\left(\frac{\dot{Y}}{Y}\right)^* = \left(\frac{\dot{I}_L}{I_L}\right)^* = \left(\frac{\dot{I}_K}{I_K}\right)^* = \left(\frac{\dot{C}}{C}\right)^* = \frac{1-\beta}{\beta}\frac{d_M d_N S - (d_N + d_M)\delta}{(1-\alpha_K)d_N + (1-\alpha_L)d_M} \equiv g \tag{$B7$}$$

Substituting (B6) into the innovation possibilities frontier (6), and (B7) into (B2) we obtain

$$\begin{cases} (\dot{\mathrm{K}}/\mathrm{K})^* = \alpha_K g \\ (\dot{\mathrm{L}}/\mathrm{L})^* = \alpha_L g \\ (\dot{\mathrm{M}}/\mathrm{M})^* = (1-\alpha_K)\dfrac{\beta}{1-\beta}\mathrm{g} \\ (\dot{\mathrm{N}}/\mathrm{N})^* = (1-\alpha_L)\dfrac{\beta}{1-\beta}\mathrm{g} \end{cases} \tag{B8}$$

(B7) and (B8) confirm that the benchmark model indeed has a SSEP. While (B6) shows that there exist also an allocation of scientists which supports the SSEP, it still needs to be verified that there exists an appropriate allocation of income as given in (33).

Using equations (23), the Euler equations (30) can be written as:

$$\begin{cases}\dfrac{\dot{C}}{C}=\left[\alpha_K\dfrac{b_K I_K^{\alpha_K}}{K}\dfrac{Y}{I_K}\dfrac{M^{(1-\beta)/\beta}K}{Y}\beta f'(k)-(\alpha_K-1)\dfrac{\dot{I}_K}{I_K}-\rho-\delta_K\right]/\theta\\ \dfrac{\dot{C}}{C}=\left[\alpha_L\dfrac{b_L I_L^{\alpha_L}}{L}\dfrac{Y}{I_L}\dfrac{N^{(1-\beta)/\beta}L}{Y}\beta[f(k)-kf'(k)]-(\alpha_L-1)\dfrac{\dot{I}_L}{I_L}-\rho-\delta_L\right]/\theta\end{cases}\tag{B9}$$

Define $s_K\equiv I_K/Y$, $s_L\equiv I_L/Y$. Substituting (B1), s_K,s_L, the definitions (20) and (21), and rewriting (7) we get:

$$\begin{cases}\dfrac{\dot{C}}{C}=\left[\alpha_K\dfrac{1}{s_K}\dfrac{kf'(k)}{f(k)}\beta\left(\dfrac{\dot{K}}{K}+\delta_K\right)-(\alpha_K-1)\dfrac{\dot{I}_K}{I_K}-\rho-\delta_K\right]/\theta\\ \dfrac{\dot{C}}{C}=\left[\alpha_L\dfrac{1}{s_L}\dfrac{[f(k)-kf'(k)]}{f(k)}\beta\left(\dfrac{\dot{L}}{L}+\delta_L\right)-(\alpha_L-1)\dfrac{\dot{I}_L}{I_L}-\rho-\delta_L\right]/\theta\end{cases}\tag{B10}$$

Insert (B1), (B2) into (B10) to obtain

$$\begin{cases}\dfrac{\dot{C}}{C}=\left[\alpha_K\beta\dfrac{1}{s_K}\dfrac{kf'(k)}{f(k)}\left(\alpha_K\dfrac{\dot{C}}{C}+\delta_K\right)-(\alpha_K-1)\dfrac{\dot{C}}{C}-\rho-\delta_K\right]/\theta\\ \dfrac{\dot{C}}{C}=\left[\alpha_L\beta\dfrac{1}{s_L}\dfrac{f(k)-kf'(k)}{f(k)}\left(\alpha_L\dfrac{\dot{C}}{C}+\delta_L\right)-(\alpha_L-1)\dfrac{\dot{C}}{C}-\rho-\delta_L\right]/\theta\end{cases}\tag{B11}$$

Rearranging (B11) yields:

$$\begin{cases}\dfrac{\dot{C}}{\mathrm{C}}=\dfrac{\rho+\delta_\mathrm{K}\{1-\alpha_K\beta kf'(k)/[s_K f(k)]\}}{\beta\alpha_K{}^2kf'(k)/[s_K f(k)]+1-\alpha_K-\theta}\\ \dfrac{\dot{C}}{\mathrm{C}}=\dfrac{\rho+\delta_\mathrm{L}\{1-\beta\alpha_L[f(k)-kf'(k)]/[s_L f(k)]\}}{\beta\alpha_L{}^2[f(k)-kf'(k)]/[s_L f(k)]+1-\alpha_L-\theta}\end{cases}\tag{B12}$$

Using k^* in from (28), equation (B12) is rewritten as:

$$\begin{cases}\dfrac{\dot{C}}{C}=\dfrac{s_K(\rho+\delta_K)(d_M+d_N)-\beta d_N\alpha_K\delta_K}{s_K(1-\alpha_K-\theta)(d_M+d_N)+\beta d_N\alpha_K{}^2}\\ \dfrac{\dot{C}}{C}=\dfrac{s_L(\rho+\delta_L)(d_k+d_l)-\beta d_M\alpha_L\delta_L}{s_L(1-\alpha_L-\theta)(d_M+d_N)+\beta d_M\alpha_L{}^2}\end{cases}\tag{B13}$$

Inserting (B7) into (B13), we obtain

$$\begin{cases}s_K{}^*=\dfrac{\beta d_N\alpha_K(\delta_K+g\alpha_K)}{(d_M+d_N)[\rho+\delta_K+g(\alpha_K-1+\theta)]}\\ s_L{}^*=\dfrac{\beta d_M\alpha_L(\delta_L+g\alpha_L)}{(d_M+d_N)[\rho+\delta_L+g(\alpha_L-1+\theta)]}\end{cases}\tag{B14}$$

Define $s_C\equiv C/Y$ so that:

$$s_C+s_K+s_L=1\tag{B15}$$

Inserting (B14) in (B15) obtain that along a in SSEP, s_C given by

$$s_C{}^*=1-s_K{}^*-s_L{}^*\tag{B16}$$

Equations (B6), (B14) and (B16) given the allocation of scientists and income to reach the SSEP given by (32).

Finally, notice that the solution process implies that there exists only one allocation of scientists and income that is consistent with a SSEP. ■

Appendix C: Proof of Proposition 5.

From equation (40) we can obtain

$$\begin{cases} \frac{\dot{N}}{N} = d_N S_N \left(\frac{M}{N}\right)^{(1-\eta)/2} - \delta \\ \frac{\dot{M}}{M} = d_M S_M \left(\frac{M}{N}\right)^{-(1-\eta)/2} - \delta \end{cases} \tag{C1}$$

To keep the economy on a SSEP, $\dot{\mathrm{M}}/\mathrm{M} = \dot{\mathrm{N}}/\mathrm{N}$. This implies

$$(M/N)^* = \left(\frac{d_M(S - S_N)}{d_N S_{Nl}}\right)^{1/(1-\eta)} \tag{C2}$$

Furthermore,

$$\frac{\dot{M}}{M} = \frac{\dot{N}}{N} = [d_M d_N S_N (S - S_N)]^{1/2} - \delta \tag{C3}$$

Using equation (C3) in equation (B4) we get

$$\frac{\dot{L}}{L} = \frac{\dot{K}}{K} \tag{$C4$}$$

Applying (B2) to (C4), we obtain that in this case there exist a SSEP under the knife-edge condition

$$\alpha_L = \alpha_K \tag{C5}$$

■

Appendix D: Proof of Proposition 6

If $\alpha_i = 0$, then there may be $\dot{I}_i/I_i = 0$, where $i = K, L, M, N$; if $\alpha_i = 1, i = K, L$, then there may be $\alpha_j = 0$, $j = M, N$; if $\alpha_i = \beta/(1-\beta), i = M, N$,then there may be $\alpha_j = 0$, $j = K, L$. At these cases, equation (D1) cannot be consistent with the modified budget constraint and the definition of SSEP. So the proof of the necessary condition for the existence of a SSEP will include three steps. First, we prove it exists when $0 < \alpha_i < 1, i = K, L$, and if $0 < \alpha_j < \beta/(1-\beta), j = M, N$; Second we prove it exists when $\alpha_i = 0$, where $i = K, L, M, N$; third, we prove it exists when $\alpha_i = 1, i = K, L$, and $\alpha_i = \beta/(1-\beta), i = M, N$.

First, if $0 < \alpha_i < 1, i = K, L$, and if $0 < \alpha_j < \beta/(1-\beta), j = M, N$ then from the modified budget constraint and the definition of a SSEP, we obtain

$$\frac{\dot{\mathrm{Y}}}{Y} = \frac{\dot{I}_M}{I_M} = \frac{\dot{I}_N}{I_N} = \frac{\dot{I}_L}{I_L} = \frac{\dot{I}_K}{I_K} = \frac{\dot{\mathrm{C}}}{C} \tag{D1}$$

Then, according to the factor accumulation processes (7) and the innovation possibilities frontier (41), the following equations must hold along a SSEP:

$$\begin{cases}\dot{K}/K = \alpha_K \dot{I}_K/I_K \\ \dot{L}/L = \alpha_L \dot{I}_L/I_L\end{cases} \tag{D2}$$

$$\begin{cases}\dot{M}/M = \alpha_M \dot{I}_M/I_M \\ \dot{L}/L = \alpha_N \dot{I}_N/I_N\end{cases} \tag{D3}$$

Using the intensive form of the production function (21), we obtain

$$Y = N^{(1-\beta)/\beta} L f(k) = M^{(1-\beta)/\beta} K f(k)/\mathrm{k} \tag{D4}$$

In a SSEP, due to the fact that k is constant, we have:

$$\begin{cases}\dot{K}/K + (1-\beta)/\beta \dot{M}/M = \dot{Y}/Y \\ \dot{L}/L + (1-\beta)/\beta \dot{N}/N = \dot{Y}/Y\end{cases} \tag{D5}$$

Substituting (D1), (D2) and (D3) into (D5), if $\dot{Y}/Y > 0$ we obtain the necessary condition for the existence of a SSEP of equation (42)

$$\begin{cases}\alpha_K + [(1-\beta)/\beta]\alpha_M = 1 \\ \alpha_L + [(1-\beta)/\beta]\alpha_N = 1\end{cases}$$

Second, if $\alpha_K = 0$, then $\dot{K} = b_K - \delta_K K,$ and in the long run $\dot{K}/K = 0$. Then from (D5) we obtain

$$\dot{M}/M = \beta/(1-\beta)\dot{Y}/Y > 0 \tag{D6}$$

From (D3) we obtain

$$\dot{M}/M = \alpha_M \dot{I}_M/I_M = \beta/(1-\beta)\dot{Y}/Y > 0 \tag{D7}$$

From (D7) we get $\dot{I}_M/I_M > 0$. And when $\dot{I}_M/I_M > 0$, from the modified budget constraint we have $\dot{I}_M/I_M = \dot{Y}/Y$.Using this in (D7) we get

$$\alpha_M = \frac{\beta}{1-\beta} \tag{D8}$$

From (D8) we also have $\alpha_K + [(1-\beta)/\beta]\alpha_M = 1$.

Similarly, we can prove that if $\alpha_L = 0$ then α_N must be equal to $\beta/(1-\beta)$, and if $\alpha_i = 0$, where $i = M, N$, then it must be that $\alpha_j = 1$, where $j = K, L$. In all these cases, equation (42) must hold if a SSEP is to exist.

Third, if $\alpha_K = 1$ then $\dot{K}/K = \dot{I}_K/I_K > 0$, and the modified budget constraint implies:

$$\dot{I}_K/I_K = \dot{Y}/Y \tag{D9}$$

Using (D9) in (D5) we get $\dot{M}/M = 0$. From the innovation possibilities frontier (41) we know that only two possible cases can attain $\dot{M}/M = 0$. One is $\alpha_M = 0$.Then $M^* = \mathrm{b}_M/\delta_\mathrm{M}$, the other case is $\alpha_M > 0$ and $\mathrm{b}_M I_M^{\alpha_M} - \delta_\mathrm{M}\mathrm{M} = 0$. However, from the latter case we can get

$$\bar{I}_M = \left(\frac{\delta_M \bar{M}}{b_M}\right)^{1/\alpha_M} > 0 \tag{D10}$$

where $\bar{M}$ is a SSEP constant because of $\dot{M}/M = 0$. As a result, $\bar{I}_M$ will also be a constant. However, if $\bar{I}_M > 0$, then $\dot{I}_K/I_K$ cannot be a constant. So if $\alpha_K = 1$

then $\alpha_M = 0$ is the only possible way to get $\dot{M}/M = 0$ in the SSEP. As a result, we also obtain $\alpha_K + [(1-\beta)/\beta]\alpha_M = 1$.

Similarly, we can prove that if $\alpha_L = 1$ then it must be that $\alpha_N = 0$ .If $\alpha_i = \beta/(1-\beta)$, where $i = M, N$, then it must be that $\alpha_j = 0$, where where $j = K, L$. In all these cases equation (42) must hold.

To summarize, from the above three steps, we obtain when $0 \leq \alpha_K \leq 1$ and $0 \leq \alpha_L \leq 1$, equations (42) must hold if a SSEP is to exist.

■

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