Managing an Accumulative Inorganic Pollutant: An Optimal Tax Prescription for the Social Planner

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“MANAGING AN ACCUMULATIVE INORGANIC POLLUTANT”
AN OPTIMAL TAX PRESCRIPTION FOR THE SOCIAL PLANNER

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Abstract
The paper strives to postulate a possible optimal policy path for a social planner who is concerned with managing the stock of an accumulative pollutant within the society. Using Hamiltonian functions in a dynamic optimizing problem, the paper was able to show that the policy path that will minimize the damages of an accumulative pollutant depends on the steady state of the accumulative pollutant as compared to the level of the accumulative pollutant present within the society. This relationship between the steady state and level of accumulative pollutant determines both abatement levels and pollution tax: policy tool kits for the social planner. The paper concludes that the social planner should advocate for a tax regime that is below the steady state tax level which will imply lower optimal abatement levels initially. The tax can then be increased over time to ensure increased abatement of the stock pollutant.

Keywords: Accumulative Pollutant, Dynamic Optimization, Pollution, Social Welfare, Pollution Abatement, Environmental Abatement Tax

INTRODUCTION
The United Nations Conference on Environment and Development (UNCED) held in Rio de Janeiro in 1992 recognized the pressing environment and development problems of the world and, through adoption of Agenda 21, produced a global program of action for sustainable development into the 21st century (Sachs and Warner, 1995). After a decade known as the rhetoric decade, the World Summit for Sustainable Development (WSSD), held in Johannesburg in August 2002, made it clear that urgent formulation and elaboration of national strategies for sustainable development are necessary. The main result of the summit was that
one of the three pillars of sustainable development - the environment - is seriously damaged because of the distortions placed on it by the actions of human population. The collapse of the environmental pillar is a serious possibility if action is not taken as a matter of urgency to address human impacts, which have left: increased pollutants in the atmosphere, vast areas of land resources degraded, depleted and degraded forests, biodiversity under threat, reduction of the fresh water resources, depleted marine resources (Dasgupta and Maeler, 1994).

Sustainability does not mean that resources must remain untouched; rather it means the rates of use of their services must be chosen so as not to jeopardize future generations. Services are both in the sense of inputs for the production and consumption system and in the sense of dump for residuals (Faucheux et al, 1996). The Justification of this paper is premised on the need for providing a postulate for the social planner, that includes, a policy road map whereby sustainable progress can be achieved with minimum harm to the environment as well as ensuring societal welfare.

This paper is primarily concerned with the role of environmental economics in assessing how society can sustain its economy and environment. This paper will abstract from reality by using a case study to draw policy implications for managing a cumulative inorganic pollutant. In essence, the paper strives to examine the waste products or residuals from production and consumption and how to reduce or mitigate the flow of residuals so they have less damage on the natural environment and depletion of natural capital. According to Solow (1993) accumulative pollutants are a major source of pollution in the society because they stay in the environment in nearly the same amounts as they are emitted. Thus, their total stock thus builds up over time as these pollutants are released into the environment each year. The rest of the paper is arranged as follows: in (2) the paper provides some literature review on pollutant abatement and in (3) the paper defines and explains what accumulative pollutants are. In (4) the paper uses a Hamiltonian function to explain the evolution of the pollutant stock while, (5) and (6) derives the steady state and postulates policy implications of the Hamiltonian function respectively. In (7) the paper espouses management strategies for the social planner and in (8) conclusions are drawn.

LITERATURE REVIEW

Providing a feasible policy prescription for managing pollutants is the goal of environmental management. The literature on pollutant management is vast, with policy prescriptions ranging from the provision of an abatement tax regime to the advocates of a political economy solution. For example, Moslener and Requate (2009) investigated optimal abatement strategies for cumulative and interacting pollutants. They showed that different decay rates can cause non-
monotonic behaviour in the optimal paths of emissions, the aggregate level of pollution, and even the relative optimal price for emissions. Their findings contrast strikingly with the case of a single pollutant. Their results add to the scepticism existing about whether the concept of global warming potential is a useful indicator for the optimal relative abatement of different GHGs over time. In fact, they showed that a constant index suitable for comparing dynamically different pollutants with respect to their economic harmfulness does not exist.

Moslener and Requate (2007) also analysed a dynamic multi-pollutant problem where abatement costs of several pollutants are not separable. The pollutants they studied could be either technological substitutes or complements. Optimal emission paths are found to be qualitatively different for substitutes and complements. In particular they found out that optimal emission paths need not be monotonic, even for highly symmetric pollutants and they described a comparatively simple method to implement the optimal path without explicitly knowing its shape.

Requate (2005) surveyed and discussed recent developments on the incentives provided by environmental policy instruments for both adoption and development of advanced abatement technology. The main conclusion to be drawn from this work is that under competitive conditions market based instruments usually perform better than command and control. Moreover, taxes may provide stronger long term incentives than tradable permits if the regulator is myopic. If the government can anticipate new technology or is able to react on it optimally, regulatory policies by virtue of administered prices (taxes) and policies by setting quantities (issuing tradable permits) are (almost) equivalent.

Gonzalez (2007) argued that under certain conditions (including path dependence and lock-in), policies and measures leading to a cost-effective GHG emissions mitigation in the short term may not allow reaching long-term emissions targets at the lowest possible cost, that is, they might not be cost-effective in the long term. Simple models and a numerical simulation are provided to show this possible conflict between static and dynamic efficiency, which points out to the need to combine different instruments, some aiming at short-term cost-efficiency (such as incentive-based environmental policy) and others at encouraging dynamic cost reductions (such as technology/innovation policy).

Unold and Requate (2001) showed that for pollution control with imperfect information about aggregate abatement costs, a combination consisting of free permits and a menu of call options for additional permits with different striking prices. Accordingly, appropriately choosing permits and corresponding striking prices the regulator can approximate the marginal damage function arbitrarily well. Kuosmanen, Bijsterbosch and Dellink (2009) stressed the disadvantages of abatement policies. They argued that assessing the benefits of climate
policies is complicated due to ancillary benefits: abatement of greenhouse gases also reduces local air pollution. They conducted efficiency analysis of ten alternative timing strategies, taking into account the ancillary benefits. They concluded that if one is only interested in GHG abatement at the lowest economic cost, then equal reduction of GHGs over time is preferred. If society is willing to pay a premium for higher ancillary benefits, an early mid-intensive reduction strategy is optimal.

Leandri (2009) presented a model of optimal flow pollution control considering explicitly the dynamics of the corresponding assimilative capacity. His analysis shows that a minimum level of initial assimilative capacity is necessary to prevent its optimal extinction. He then allows for the restoration of assimilative capacity and characterizes the conditions under which this option frees the optimal policy from the dependency on the initial conditions. In both cases our results call for environmental standards based on the shadow price of assimilative capacity that are stricter than the static optimum commonly used in flow pollution control.

Also, Requate and Unold (2003) investigated incentives through environmental policy instruments to adopt advanced abatement technology. They study the case where the regulator makes long-term commitments to policy levels and does not anticipate arrival of new technology. The authors show that taxes provide stronger incentives than permits, auctioned and free permits offer identical incentives, and standards may give stronger incentives than permits.

Holland (2012) argued that the best emissions tax or emissions cap may be an inferior instrument under incomplete regulation (leakage). Without leakage, an intensity standard (regulating emissions per unit of output) is inferior due to an implicit output subsidy. This inefficiency can be eliminated by an additional consumption tax. With leakage, an intensity standard can dominate the optimal emissions tax, since the implicit output subsidy prevents leakage. The addition of a consumption tax improves an intensity standard's efficiency, may prevent leakage, and may be efficient. Comparing intensity standards to output-based updating showed that the latter dominates if updating is sufficiently flexible.

Using historical data, Baryshnikov (2010) studied pollution abatement and environmental equity in a dynamic panel model using data for 234 plants in the US pulp and paper industry observed over the period 1985–1997. He finds some environmental inequity with respect to the children (under 6 years) and adults with no high school diploma. Our findings show no evidence of environmental inequity against African–Americans, Hispanics, other minority races, or the poor.
ACCUMULATIVE POLLUTANTS: WHAT ARE THEY?
As already emphasized, studies in environmental economics examine the waste products or residuals from production and consumption and how to reduce or mitigate the flow of residuals so they have less damage on the natural environment and depletion of natural capital. Production and consumption create all types of materials residuals that may be emitted into the air or water or disposed of on land. The list is incredibly long: sulphur dioxide, volatile organic compounds, toxic solvents, animal manure, pesticides, particulate matter of all types, waste building materials, heavy metals, and so on. Waste energy, in the form of heat and noise, and radioactivity, which has characteristics of both material and energy, are also important production residuals. Consumers are also responsible for enormous quantities of residuals, chief among which are domestic sewage and automobile emissions. All materials in consumer goods must eventually end up as residuals, even though some may be recycled along the way. These are the source of large quantities of solid waste, as well as hazardous materials like toxic chemicals found in items such as pesticides, batteries, paint, and used oil.

One simple and important dimension of environmental pollutants is whether they accumulate over time or tend to dissipate soon after being emitted. The classic case of a non-accumulative pollutant is noise; as long as the source operates, noise is emitted into the surrounding air, but as soon as the source is shut down, the noise stops. At the other end of the spectrum we have accumulative pollutants that stay in the environment in nearly the same amounts as they are emitted. Their total stock thus builds up over time as these pollutants are released into the environment each year. Radioactive waste, for example, decays over time but at such a slow rate in relation to human lifespan that for all intents and purposes it will be with us permanently. Another accumulative pollutant is plastics. The search for a degradable plastic has been going on for decades and, while gains have been made, most plastics decay very slowly by human standards; thus, what we dispose of will be in the environment permanently (Asheim et al, 2001).

MANAGING A STOCK OF ACCUMULATIVE POLLUTANT
A stock pollutant is a pollutant that accumulates over time, and the damage it causes at a point in time is a function of how much has accumulated to that point (Dasgupta, 2001). For this paper, as already mentioned, the focus will be on inorganic pollutant / waste. The method of study employed is dynamic optimization (see. Cass and Shell, 1976; Fabbri and Gozzi, 2008; and Kamiean and Schwartz, 2012). The choice of dynamic optimization to espouse possible measures on managing an accumulative pollutant is premised on the characteristic of the social problem of:
1. Managing the stock of accumulative pollutants.
2. Decisions in one time period will affect opportunities and payoffs in the future.
3. All decisions are functions instead of single values. These decisions are time paths of actions over some time frame.

The fundamental problem for the social planner is to find a control which minimize or maximize a certain objective, subject to constraints on the evolution of the stock of accumulative pollutant. To do this, the paper employs a current – value Hamiltonian function.

**Notation**

- $S$ – Stock of accumulative pollutant
- $X$ – Potential aggregate effects: this is aggregate effect of the pollutant to the society without any control. The portion of production or consumption residuals that are placed in the environment, sometimes directly, sometimes after treatment
- $A$ – Abatement, i.e. Reduction or loss in welfare due to the presence of the pollutant (example: pollution tax)
- $(X-A)$ – Aggregate flow of pollutant
- $g$ – Percentage of the stock of the pollutant that decays in a time period, $g \in (0,1)$.

The state equation is

$$
\dot{S} = X - A - gS.
$$

Which showed the evolution of the accumulative pollutant over time

Let:

- $D(S)$ – aggregate damage function with $D'>0$ and $D''>0$. Damages are the negative impacts produced by environmental pollution—on people in the form of health effects, visual degradation, and so on, and on elements of the ecosystem through things like the disruption of ecological linkages or species extinctions
- $C(A)$ – aggregate abatement costs with $C'>0$ and $C''>0$.
- $i$ – discount rate held constant over [0,∞].

The social planner's objective is to minimize the present value of the flow of total costs—damage plus abatement costs:

$$
\text{Minimize } \int_{0}^{\bar{T}} [D(S) + C(A)e^{-it}] dt
$$

Subject to

$$
\dot{S} = X - A - gS
$$

$S(0)=S_0$ (the initial stock of $S$).

$T = \bar{T}$, $S(T) = \bar{S}$
The current-value Hamiltonian is
\[
H^c = D(S) + C(A) + \mu(X - A - gS)
\]  
Since \(D\) and \(C\) are strictly convex, the Hamiltonian is strictly convex in \((S, A)\). Therefore, the following necessary conditions are also sufficient.
\[
H_A^c = C'(A) - \mu = 0
\]
\[
\frac{d(\mu e^{-it})}{dt} = -H_S e^{-it} \Rightarrow \mu e^{-it} - i\mu e^{-it} = -(D'(S) - \mu g)e^{-it}
\]
\[
\dot{\mu} = -D'(S) + \mu(i + g)
\]
\[
\dot{S} = X - A - gS
\]

The current-value costate variable \((\mu)\) is the marginal reduction in future damages and abatement costs from abating one unit now. Thus, \(\mu = C'(A)\) balances the marginal benefit of current abatement against marginal costs. Since \(\mu\) is the shadow value of abating a unit of emissions, we can interpret it as the optimal tax on emissions. Graphically:

Optimal Tx on Emissions form of tax is not in the strict sense. It could also include all effort (assume the effort can be measured) directed towards abatement i.e government policy favouring recycling

The Steady State
To identify the steady state we use the first three first order conditions and set \(S = \mu = 0\). Take the costate equation first and solve,
\[
\dot{\mu} = -D'(S) + \mu(i + g) = 0
\]
Which yields
\[
\mu = \frac{D'(S)}{(i + g)}
\]
Now consider the state equation:
\[ \dot{S} = X - A - gS \]

Then, with \( \mu = C'(A) \),

We have a system of three equations, with three unknowns \((S, A, \mu)\). The solution to these is the steady state \((S^s, A^s, \mu^s)\).

To illustrate the solutions in two dimension, focus on \( \mu \) and \( S \) for which \( \mu = 0 \). Note that;

\[ \frac{\partial \mu}{\partial S} = D''(S)/(i + g) > 0 \]

Graphically,

Figure 2: Relationship between \( \mu \) And \( S \) For Which \( \dot{\mu} = 0 \)

Given \( \dot{S} = X - A - gS = 0 \),

To get \( \mu \), note first that \( \dot{S} = X - A - gS = 0 \) implies \( A = X - gS \)

In the steady state, \( \mu = C'(A) \) and \( A = X - gS \)

\[ \mu = C'(X - gS) \]

And this equation collects all combinations of \( \mu \) and \( S \) for which \( \dot{S} = 0 \)

\[ \frac{\partial \mu}{\partial S} = -C''(X - gS) < 0 \]

Graphically:
The steady-state value of \( \mu \) and \( S \) are determined as the simultaneous solution to
\[
\mu = D'(S) + \mu(i + g) = 0, \quad [\mu = 0], \quad 14
\]
\[
\mu = C'(X - gS), \quad [S = 0]. \quad 15
\]
The steady-state level of abatement is determined easily with the state equation
\[
\dot{S} = X - A - gS = 0; \text{ that is, } A^S = X - gS^S
\]

**POLICY IMPLICATIONS—A PHASE DIAGRAM**

One of the most interesting aspects about steady-state analysis is the analysis of how \( \mu \), \( S \), and \( A \) move toward the steady state. To do this we construct a phase diagram by examining how \( \mu \) moves when the system is away from \( \dot{\mu} = 0 \) and doing the same with \( S \).

Begin with \( \mu \):
\[
\dot{\mu} = \mu(S, \mu) = -D'(S) + \mu(i + g). \quad 16
\]

Note that
\[
\frac{\partial \dot{\mu}}{\partial S} = -D''(S) < 0
\]

Consider a pair \((\bar{S}, \bar{\mu})\) such that \( \dot{\mu} = 0 \). That is,
\[
\dot{\mu} = \bar{\mu} = (\bar{S}, \bar{\mu}) = -D'(\bar{S}) + \bar{\mu}i + g = 0 \quad 17
\]

Leave \( \mu = \bar{\mu} \), but consider \( S^0 < \bar{S} \). Since \( \frac{\partial \dot{\mu}}{\partial S} < 0 \) and \( \dot{\mu}(\bar{S}, \bar{\mu}) = 0 \), then \( \dot{\mu}(S^0, \bar{\mu}) > 0 \) For \( S^1 > \bar{S}, \dot{\mu}(S^1, \bar{\mu}) < 0 \). Graphically:

![Figure 3: Movements of \( \dot{\mu} \) with Different Values of \( S \)](image)

Thus, for pairs \((\mu, S)\) above \( \dot{\mu} = 0, \dot{\mu} > 0 \). For every \((\mu, S)\) below \( \dot{\mu} = 0, \dot{\mu} < 0 \).

Now consider \( \dot{S} \) away from \( \dot{S} = 0 \). Take the state equation \( \dot{S} = X - A - gS \). We need this in terms of \( \mu \), so consider \( \mu = C'(A) \). Since \( C' \) is strictly increasing \((C'' > 0)\), it has an inverse that is also strictly increasing. Let \((C') - 1 = h\).
Then
\[ \dot{S} = S(S, \mu) = X - h(\mu) - gS, \]
with \( \partial \dot{S} / \partial \mu = h'(\mu) < 0 \)

Take a pair \((S', \mu')\) such that
\[ \dot{S}(S', \mu') = X - h(\mu') - gS' = 0 \]

Fix \( S = S' \) and consider \( \mu^0 < \mu' \). Since \( \partial \dot{S} / \partial \mu < 0 \) and \( \dot{S}(S', \mu') = 0 \), then \( \dot{S}(S', \mu') > 0 \). For \( \mu^1 > \mu' \), \( \dot{S}(S', \mu^1) < 0 \).

Graphically:

Figure 4: Relationship between \( \mu \) and \( S \), Showing Movements in \( \dot{S} \) with different Values of \( \mu \)

Therefore, for any \((\mu, S)\) above \( \dot{S} = 0 \), \( \dot{S} < 0 \). For any \((\mu, S)\) below \( \dot{S} = 0 \), \( \dot{S} > 0 \). Combining our findings about \( \dot{\mu} \) and \( \dot{S} \) away from \( \dot{\mu} = \dot{S} = 0 \) yields the following graph.

The directional arrows (like \( \rightarrow \)) tell us the qualitative movement of \((\mu, S)\) away from or toward the steady state in each of the four regions of the diagram. Given some initial stock of accumulative pollution, \( S^0 \neq S^s \), we can draw qualitative conclusions about the path of \((\mu, S)\) toward the steady state. This will be particularly revealing for the \( \mu^- \) path, because this will be the path of the optimal tax on emissions (management strategy).
Figure 5: Interaction between $\dot{\mu}$ and $\dot{S}$ to determine steady state $\mu^s$ and $S^s$

MANAGEMENT STRATEGY

Following graph, the paper assume that $S^0 < S^s$, drawn the paths (a), (b) and (c), but only path (b) reaches the steady state. Note that a tax policy that originates above $\mu^s - \mu(0) > \mu^s$ as for path (a) cannot reach the steady state. Therefore the optimal initial tax must be below the steady state tax. However, it cannot be too low like $\mu(0)$ for path (c).

Path (b) converges to the steady state. Note the policy prescription: set the initial emissions tax lower than the steady state tax and increase it toward $\mu^s$ as time goes by. This also implies that optimal abatement starts out relatively low and increases as the tax is increased [use $\mu = C(A)$]. Abatement is higher in later periods, because in present value terms it is cheaper to push it off into the future. Of course, doing so has to be balanced against increased damage in earlier periods.
Figure 6: Using movements towards the steady state, a, b, and c provide different evolution paths for different tax regimes. In a, the tax is too high, and in c the tax is too low. Scenario b provides the optimal tax.

Using the same phase-diagram we can draw the opposite conclusions if the initial stock is greater than the steady-state stock. In this case the initial tax is set higher than the steady state tax and the tax is decreased toward the steady state as time goes by. Consequently, abatement starts out relatively high and is gradually decreased as the system moves toward the steady state.

The success of the social planner in managing any accumulative pollutant depends on the measures taken. The social planner has to first consider the stock of pollutant – its evolution towards the steady state, the policy measure he deems appropriate for managing the accumulative pollutant and the policy prescription that will guide the attainment of the desired social outcome. Depending on the initial stock of the accumulative pollutant and its movement towards a steady state, the social planner should, by the analysis of the paper;
• Make sure that the initial pollution tax measure should not be above the steady state tax.
• The initial optimal pollution tax measure for abatement should be below the steady state level of tax but not too low. This optimal Tax should be increased over time till it gets to $\mu$. 
• This means that abatement will be relatively lower in earlier periods but will increase as the optimal tax is increased.
• If the optimal tax measure for abatement is too low, the policy of having an initial optimal pollution tax below the steady state tax will not be effective.

CONCLUSION

From the above analysis, it becomes imperative that the social planner is laden with the task of providing an optimal control for managing accumulative pollutants in the society. The Hamiltonian model used to espouse possible policy paths in the paper can also be used in determining other social problems not restricted to accumulated pollutants. The paper has been able to derive a valid (both in the short and long time horizons) solution that the social planner can use to stymie growth in accumulative pollutants as well as inherent damages in form of loss in welfare to the society.

The social planner’s strategy depends on the steady state stock of accumulative pollutants in the society. Once this is known, the social planner has the responsibility of postulating optimal tax on emissions of these pollutants and invariably the level of abatement. As seen in the model, the optimal tax policy is one which must be lower than the steady state of the stock pollutant, if and only if the present stock of accumulated pollutant is lower than the already established steady state. This implies that abatement will be lower in earlier periods and higher in later periods as pollutant tax increase. Conversely, if the present stock of accumulated pollutant is higher than the established steady state, abatement will be high in earlier periods (with an attendant high tax) and lower in later periods(lowering taxes).

The limitation of this management strategy is the possibility of the tax levied of producers of accumulated stock pollutants being transferred to the consumers, especially low income consumers. Thus, revenues generated from the optimal abatement tax, can be used to reduce the incidence of the abatement tax to low income earners, if the generated revenues are prioritized for low income earners. Also, this optimal abatement tax strategy may be unable to differentiate, sources of the accumulated pollutants. This will lead to inefficiency, because a uniform abatement tax will be levied against all polluters, irrespective of their contributions towards the stock of pollutant.
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