The E-Monetary Theory

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Abstract

Using the sparse grid, we solve a DSGE model where there are two types of electronic money: reserves (e-money that is issued by the central bank for banks) and zero maturity deposits (e-money that is issued by banks). Transactions between bankers are settled by reserves, while transactions in the non-bank private sector are settled by zero maturity deposits. We use our model to discuss about unconventional monetary policy tools during the Great Recession. Due to the maturity mismatch between deposits and loans, we find that keeping the federal funds rate at the lower bound for a long but finite time stimulates the economy in the short run but creates deflation and lower outputs in the long run. To get out of the zero lower bound, the central bank can conduct helicopter money and increase the interest rate paid on reserves simultaneously, which is impossible in the Keynesian theory, but possible with the current electronic money system.

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1 Introduction

Nowadays, money mostly exists in the electronic form. According to the data from the Federal Reserve Bank, the total stock of M1 in Jun 2016 is around USD 3274 billion, consisting of USD 1381 billion in currency and USD 1850 billion in checkable deposits. However, as the world currency, most US dollar bills are held outside US. Judson (2012) estimates that 60 percent of US dollar bills are in foreign countries. If we exclude that number from M1 and M2, currency only accounts for 15 percent of the M1 stock and 5 percent of the M2 stock. Moreover, the value of transactions conducted by currency is far less than the ones with electronic money. Schreft and Smith (2000), based on the *"Nilson Report 1997"*, shows that cash accounts for 20 percent of the dollar volume of U.S. payments made by consumers in 1990 and 18 percent in 1996, and is projected to account for only 16 percent in 2000 and 12 percent in 2005. In this paper, we focus on a particular group of e-money issued by commercial banks, including checkable deposits, money market deposit account and saving account. For convenience, we call this group as zero-maturity deposits (ZMDs) thereafter.

ZMDs are different from currency in two salient features. First, in nature, currency is an IOU issued by the central bank while ZMDs are IOUs issued by commercial banks. In economic terms, currency is outside money while ZMDs are inside money. Second, unlike currency, ZMDs pay the (positive) nominal interest rate. Banks pay the interest rate for saving account and money market deposits account for a long time, but the tricky part is the checking account. In a frictionless perfectly competitive banking market, the interest rate on checkable deposits should be positive and follow the federal funds rate. However, before 2012, under the Regulation Q, banks in US were prohibited from paying interest on checking account. During this period, banks still implicitly paid the demand deposit rate under the form of giving gifts or reducing the cost of additional services to their customers, see Mitchell (1979), Startz (1979), Dotsey (1983). Becker (1975) estimates that the implicit demand deposit rate in US during 1960-1968 was around 2.64 percent-3.74 percent.

Since 2012, the Regulation Q has been no longer valid, and most banks are now paying interest rate on the checkable deposits (see Table 8 in Appendix A). According to the data in September 2016 of Federal Deposit Insurance Corporation (FDIC), the national average interest on checkable account is 0.04 percent, on saving account is 0.06 percent. These rates are low as the federal funds rate is near zero. If the federal funds rate is around 4 percent, these rates are likely from 1 percent to 2 percent. As a result of that, in the era of electronic money, it is more natural to model money as an interest-earning asset. Even during the period of 2000-2012, as the popularity of credit cards and Internet banking helped people to conduct transaction even with only saving account, it is more reasonable to model that households earn interest rate from holding ZMDs.

This paper builds a dynamic general equilibrium model where currency does not exist. There are two forms of money in our model: reserves and ZMDs. In our model, ZMDs are inside money issued by the commercial banks. They are used for settling the transactions between every pair of agents in the economy, except between bankers-bankers, bankers-government, and bankers-central bank. In these types of transactions, bankers have to use reserves- another type of e-money that commercial banks deposit at the central bank. Only government and commercial banks have an account at the central bank. The amount of ZMDs banks can issue is restricted by two constraints: the reserves requirement and the capital requirement. In our model, the central bank only controls the level of reserves while the level of money supply (total amount of ZMDs) depends on the interaction between the central bank, the commercial banks and the public. In the normal time when the reserves requirement is binding, by adjusting the level of reserves, the central bank can target the interbank lending rate; and therefore the prime rate that banks lend out to households. In the situation when the reserves requirement is not binding, the central bank can control the interbank lending rate by adjusting the interest rate paid on reserves.

We use our theory to set lights on what happened in the Great Recession. Banks, in our model, hold an asset related to the construction sector. When a big negative housing demand shock is realized, the price of this asset drops. Bankers fire-sell this asset to satisfy the capital constraint, which further pushes down its price. Consequently, bankers' net worth decline to a new level that they need to cut loans to satisfy the capital requirement constraint. The level of ZMDs declines even though the level of reserves go up. As the price of loan is higher and the deflation episode is realized, the private investment declines strongly. If the central bank only follows the conventional monetary policy where they purchase the small amount of government bond from bankers to inject reserves, the recession will be severe. It takes a long time for the loan market to be back into the normal mode.

Even though the only shock in our model comes from the households' demand side, it does not affect the household directly, but rather indirectly through the banking sector. When banks cut loans, the total amount of money in the economy declines. As households need money in advance to purchase the final good and make investment, the tighter of liquidity constraint forces them to cut down the level of investment sharply.

Moreover, unlike the story in the collateral constraint[1] households want to sell house more quickly to relax their liquidity constraint, which in turn, pushes down the housing price further and amplified the initial housing demand shock.

We discuss in detail the unconventional monetary policy[2] when the central bank changes the component of its balance sheet by purchasing the distressed assets rather than the government bonds. This program, through the asset price channel, immediately pushes the commercial banks out of the capital constraint and recover the credit flows between households and bankers. In the short run, this unconventional monetary policy can prevent the prolonged deflation episode. However, in the medium run, without adjusting the interest rate paid on reserves, deflation might happen as the result of keeping the interbank rate at the lower bound for too long.

The intuition for the mechanism why deflation might happen in the medium run after the quantitative easing (QE) can be explained by the mismatch between the maturity of money and loans in our model. Bankers know that in the long run, the federal funds rate will be back to the steady state level, so when making a long term loan, they have to take into account this effect into the current loan rate. If the central bank keep the federal funds rate at the lower bound, the nominal deposit rate is also at the lower bound, while the real loan rate might start rising if bankers know the rate will rise in the future. In equilibrium, the real return on deposits will be equal to the real loan rate minus the liquidity premium. If the liquidity premium does not change much, the deflation must realize to increase the real return on deposits to match it up with the real loan rate.

The recommended policy for the central bank is to raise the interest rate paid on reserves on the exit date from the zero lower bound, even if the central bank does not see the inflation signal. When the central bank raises the federal funds rate, the deflation goes down further but it will pick up quickly as the nominal interest rate on deposits start matching up with the real loan rate. To reduce the negative effect of temporary rise in the federal funds rate, the Fed can use the helicopter money simultaneously. This policy is impossible to conduct in the Keynesian theory, but is totally possible with the electronic money.

Our model can generate some key facts in the Great Recession: (i) the level of reserves skyrockets but the money multiplier plummets, (ii) the interbank rate is equal to the interest rate paid on reserves for a

[1] If housing is in the collateral constraint, in the model without default, the price of housing will recover quickly as the shadow price of the collateral constraint pushes up the housing price

[2] In many contexts, it is called "quantitative easing" (QE) in our paper, even though our model does not examine the situation when the central bank purchases the massive amount of government bonds.

long time after the quantitative easing, (iii) the deleveraging process when banks actively cut loans, (iv) the sharp cut of investment and outputs at the beginning of the housing crisis, (v) the sharp decline then suddenly increase again of the inflation in only two quarters. Indeed, the last fact is the challenging to any New Keynesian model to generate.

The are two main challenges in the computation method for our model. First, our model features the three occasionally binding constraints, where the switch between reserves requirement and capital requirement play the key role. Second, our model has six state variables, one of which is the aggregate reserves level. The level of reserves can be nearly five times as high as its steady state level in the simulation, creating the difficulty for building a good grid. To conduct the quantitative exercise, we build a Smolyak grid in Brumm and Scheidegger (2015) and use the nonlinear certainty equivalent (NLCEQ) approximation method in Cai, Judd and Steinbuks (2015) to estimate the decision rule. This global solution method is stable and can handle three occasionally binding constraints in our model. For each point in the grid, NLCEQ uses it as the initial state and transforms our stochastic problem into a deterministic problem. For large shock, to solve the deterministic system, we also combine NLCEQ with the continuation method.

Related Literature

On the money supply side, our approach is similar to Bianchi and Bigio (2014) and Afonso and Lagos (2015) when the central bank can increase the level of money supply and cut down the federal funds rate by injecting more reserves in the banking system. These papers explicitly model the search and matching process of heterogeneous agents in the interbank market while the one in our model is frictionless with identical bankers. In exchange of that, our model can connect the central bank policy to not only banks' balance sheet but also the production sector, which is missing in both Bianchi and Bigio (2014) and Afonso and Lagos (2015).

On the money demand side, our model follows the cash-in-advance literature. As our model does not have currency, "cash" here should be interpreted as ZMDs. In Belongia and Ireland (2006, 2014), currency and deposits are bundled together and provide the liquidity service to households. Our deposits, however, are zero maturity deposits. Like Lucas and Stokey (1987), households need money in advance to purchase the consumption good. Even though money in our model earns the positive interest rate, its rate of return is still less than the rate they borrow from bankers. We also extend the Clower constraint to the investment and housing purchases (Stockman (1981), Abel (1985), Fuerst (1992), Wang and Wen (2006)). Indeed, most empirical research, for example Friedman (1959) and Mulligan and Sala-I-Martin (1997), usually uses the income, rather than the consumption alone, to estimate the money demand function. The appearance investment in the Clower constraint is important to generate the persistence of output and deflation.

We also share the sticky price feature with the New Keynesian framework. The important role of financial frictions in the New Keynesian has been emphasized for a long time (see Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2004)). Recently, many New Keynesian research (Gertler and Kiyotaki (2010), Curdia and Woodford (2011), Gertler and Karadi (2011)) incorporates the banking sector to their models, aiming to answer what happened in the Great Recession and the role of the unconventional monetary policy, Our paper differs mainly from these line of research in the money supply process. We can characterize the micro-foundation link between reserves, banks' balance sheets, money supply and the federal funds rate; hence, our model can exhibit the long duration of federal funds rate at the lower bound after quantitative easing while this link is often missing in the New Keynesian literature. Moreover, the New Keynesian often focuses on the interest rate channel, while the unconventional monetary policy in the Great Recession inclines to the asset price channel and bank lending channel. Our model, somehow, can exhibit all of those channels in a succinct framework.

Our approach also relates to a large literature in macro-finance where macro shocks can affect strongly to the balance sheets of intermediaries. Both Brunnermeier and Sannikov (2016) and our model emphasize the importance of inside money in the deflation episode. Both show the intermediaries cut the amount of inside money they issue during the time their net worth decline, leading to the sharp decline in the money multiplier during the crisis. However, two papers differ mainly in the role of money and the money supply process. There is no reserves and two-tier money in Brunnermeier and Sannikov (2016). Moreover they emphasize the role of storing value in money while our paper focus on the function as the medium of exchange in money. Gertler and Karadi (2011), He and Krishnamurthy (2013) and Caballero and Simsek (2013) also show the circumstances when intermediaries fire sell their risky assets under the capital constraint, which is also found in our paper.

2 The Environment

2.1 Notation:

Let P_t be the price level (price of the final consumption good), we use the lower letter to exhibit the real balance of a variable or its relative price. For example, the real reserves balance $n_t = N_t/P_t$, or the real price of $p^m_t = P^m_t/P_t$. The timing notation follows this rule: If a variable is determined or chosen at time t, it will have the subscript t. All of the interest rates in the model are gross nominal rates, except when explicitly stated differently. The gross inflation rate is $\pi_t = P_t/P_{t-1}$.

2.2 Time, Demographics and Preferences

Time is discrete, indexed by t and continues forever. The economy has seven types of agents: bankers, households, wholesale firms, retail firms, construction firms, the government and the central bank.

There is a measure one of identical infinitely lived bankers in the economy. Bankers discount the future with the rate β. Each period, they gain utility from consuming the final consumption good c_t. Their expected utility at the period t can be written as:

$$E_t\left[\sum_{i=0}^{\infty}\beta^i\log(c_{t+i})\right]$$

There is also a measure one of identical infinitely lived households. Households discount the future with the rate $\widetilde{\beta}<\beta$, so they will borrow from the bankers. Each period, households gain utility from consuming the final consumption good c_t and enjoying the flow of housing service from the housing stock h_t. However, they lose utility when providing labor $\widetilde{l}_t$ to their own production. Household's expected utility at the period t can be written as:

$$E_t\left[\sum_{i=0}^{\infty}\widetilde{\beta}^i\left(\log(\tilde{c}_{t+i})+\xi_t\log(\tilde{h}_{t+i})-\chi\frac{\tilde{l}_{t+i}^{1+\eta}}{1+\eta}\right)\right]$$

where ξ_t is time-varying housing demand shock and η is the inverse Frisch elasticity of labor supply.

Wholesale firms, retail firms are infinitely lived, owned by households. Construction firms, like in Gertler and Karadi (2011), finance their capital purchase by issuing a perfectly state-contingent securities to bankers, so their profits are zero in every period.

The government is independent from the central bank in our model. Each period, the government collects tax from households to pay for the coupon for the outstanding government bonds (perpetuities). The government is assumed to not issue new government bonds in our model.

The central bank uses the payoffs they get from holding the government bonds (or other assets) to pay for the interest paid on reserves to bankers. The remaining payoffs are transferred back to the government to transfer to households. The central bank also purchases or sells the government bonds to bankers every period to adjust the level of reserves and the federal funds rate. In the unconventional monetary policy, the central bank might purchase the financial claims on the construction firms from bankers.

2.3 Goods and Production Technology

There are four types of goods in the economy: final consumption good y_t produced by retailers, wholesale goods $y_t(j)$ produced by wholesale firm j, intermediate good y_t^m produced by households, and housing h_t produced by construction firms. Besides that, there are two types of capital in the economy: K_t and H_t.

Each period households self-employ their labor $\widetilde{l_t}$ and use the capital K_{t-1} to produce the homogeneous intermediate good y_t^m under the Cobb-Douglas production function:

$$y_t^m = K_{t-1}^{\alpha} \widetilde{l_t}^{1-\alpha}$$

where α is the share of capital in the production function. Capital K depreciates with the rate δ. Households also own a technology to convert one unit of final good y_t to one unit of capital type K and vice versa. So each period they also make an investment I_t. Households sell y_t^m to wholesale firms.

There is a continuum of wholesale firms indexed by $j \in [0,1]$. Each wholesale firm purchases the homogeneous intermediate good y_t^m from households and differentiates it into a distinctive wholesale goods $y_t(j)$ under the following technology:

$$y_t(j) = y_t^m$$

Then retail firms produce the final good y_t by aggregating a variety of differentiated wholesale goods $y_t(j)$:

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$$

Construction firms use the capital H_{t-1} they purchase from other construction firms at the end of the period $t-1$ to build new housing. Let i_t^h be the amount of new housing which is built in the period t. The production function of a construction firm is:

$$i_t^h = H_{t-1}$$

We assume that capital H is not depreciated and there is a total fixed amount $\overline{H}$ of this capital in the economy. In equilibrium, $H_t = \overline{H}$. The stock of housing in the economy is h_t. Housing is depreciated with the rate δ^h, so:

$$h_t = (1-\delta^h)h_{t-1} + i_t^h$$

2.4 Asset Technology

There are four types of financial assets: government bonds b_t^g, bank loans b_t^h, financial claims on the construction firms x_t, and interbank loans b_t^f. Let P_t be the price of the final good y_t, some of our financial instruments are indexed to the price level P_t.

(a) **Government bonds (b_t^g)**: are issued by the government. They are traded between bankers and the central bank. We assume that households do not hold government bonds. We model government bonds b_t^g as consols, which are indexed to the price level P_t. The owner of 1 unit of government bonds at time t will receive P_j units of dollars for every period j where $j \geq t+1$. The price of one unit of government bond is q_t. The total number of government bonds issued by the government is fixed.

(b) **Bank loans to households (b_t^h)** : We follow Leland and Toft (1996) and Bianchi and Bigio (2014) to model the loan structure between bankers and households. We assume that the loan contract between bankers and households is in a particular form that each period households only need to pay a fraction δ_b of the total real debt stock in the previous period b_{t-1}^h. Let b_t^h be the real loan stock in the period t, let s_t be the real flow of new loan issuance, we have:

$$b_t^h = (1-\delta_b)b_{t-1}^h + s_t$$

In detail, the real flow new loans issuance s_t constitutes a promise from households to pay for the bankers the nominal amount $s_t(1-\delta_b)^n\delta_b P_{t+1+n}$ in period $t+n+1$, for all $n \geq 0$. So banks' loans are also indexed to the price level. The price of loan r_t is determined by a perfectly competitive market. When issuing loans s_t, bank will open an account for households and only deposit $r_t s_t$ into their account.

So if $r_t < 1$, bankers earn an immediate accounting profit $(1 - r_t)s_t$. Loans are illiquid and bankers cannot sell loans.

(c) **Financial claim on the construction firms (x_t)** : are issued by the construction firms. We assume that only bankers can purchase these claims as they have technology to verify the revenue of the construction firms. It is also assumed that these claims can be traded between bankers. Each financial claim has a price v_t and pays a stochastic real payoff r_t^x.

(d) **Interbank loan (b_t^f)**: Bankers can borrow reserves from other bankers in the federal funds market. The nominal gross interest rate in the federal funds market is the federal funds rate R_t^f.

2.5 Money

There are two types of electronic money in our economy: reserves n_t and zero maturity deposits m_t.

(a) **Reserves (n_t)**: is a type of e-money issued by the central bank. Only government and bankers have an account at the central bank, so only government and bankers have reserves[3]. Each period, the central bank pays a gross interest rate R^n on these reserves[4]. Reserves are used for settling the transactions between bankers and bankers, bankers and central bank, bankers and government.

(b) **Zero maturity deposits (m_t)**: is a type of e-money issued by the bankers. Each period, banks pay the interest rate R_t^m for these ZMDs which is determined by the perfectly competitive market. Money m_t is used for settling the transaction in the private sector. These ZMDs are insured by the central bank, so they are totally safe. ZMDs and reserves have the same unit of account.

In the electronic payment system, there is a connection between the flows of reserves and deposits. For example, we assume that wholesale firm A (whose account at bank A) pays 1 dollar for household B (whose account at bank B). Then the flow of payment will follow Table (1). For a transaction between government and households, or between the central bank and households, money still flows through banks, so we can think this contains two sub-transactions. One is between government and banks, which is settled by reserves. One is between banks and households, which is settled by ZMDs.

[3]The amount of US Treasury deposits at the Fed is not considered as reserves in reality. However, for convenience, we also call that money as reserves in our model. In equilibrium, the balance of the government account at the central bank is zero, so it does not matter.

[4]Throughout our paper, it is assumed that R^n is a constant. Only in the section 6.8, we consider R_t^n is a time-varying variable decided by the central bank.

Wholesale firm A	
Deposit: -1	Payable:-1

Household B	
Deposit: +1	
Receivable:-1	

Bank A	
Reserve: -1	Deposit: -1

Bank B	
Reserve: +1	Deposit: +1

The Fed	
	Reserve (bank A): -1
	Reserve (bank B):+1

Table 1: Electronic Payment System

2.6 Regulations

There are two central bank's regulations that bankers have to face each period: reserves requirement and capital requirement

(a) **Reserves Requirement**: At the end of each period, bankers have to hold enough reserves that is greater or equal to a fraction φ of total deposits.

(b) **Capital Requirement**: At the end of each period, bankers' net worth must be greater or equal to a fraction κ of the sum of their loan issuance and the total value of financial claims on the construction firms.

2.7 Timing within one period

1. The housing demand shock ξ_t realizes.

2. Production take places. Households produce the homogeneous intermediate goods y_t^m and sell them to wholesale producers . Wholesale producers differentiate them and sell to retailers. Retailers then produce the final consumption good. The construction firms also build new houses. All of the payments of these transaction are delayed until the step 5.

3. Bankers pays the interest rate for ZMDs that households deposit at time $t-1$. The loan market between bankers and households open.

4. The final good market and housing market open. Households need ZMDs in advance to purchase both housing and final goods.

5. ZMD payments are settled: The retailers pay the wholesale firms. The wholesale firms pay households. The profits of wholesale firms are also distributed to households. Tax/Transfer is sent from the government to households. Payoffs from the financial claims on the construction firms go to bankers.

6. The banking market opens. Banker can adjust the level of reserves by borrowing in the interbank market, receiving new deposits, trading x_t or b_t^g with other bankers and central bank. The central bank also pays the interest rate on reserves for bankers.

3 Agents' Problems

3.1 Bankers

There is a measure one of identical bankers in the economy. These bankers have to maintain a good balance sheet under the regulation of the central bank. There are five types of assets on the bankers' balance sheet: reserves (n_t), government bonds (b_t^g), loans to households (b_t^h), loan to other bankers (b_t^f) and financial claims on the construction firms (x_t). The bankers' liability side contains the zero-maturity deposits that households deposit here (m_t).

Banker	
Reserves: n_t	Zero Maturity Deposits: $\frac{m_t}{R_t^m}$
Govt bonds: $b_t^g \times q_t$	Net worth
Loans to households: b_t^h	
Claims on construction firms: $x_t \times v_t$	
Loans to other bankers: $\frac{b_t^f}{R_t^f}$	

Cost: We assume that banks face two kinds of cost when issuing loan. First, it is the operating cost when bankers have to manage to collect the payment from households on time. The operating cost, measured by the unit of final goods, is proportional to the loan size νb_t^h, where ν is a parameter.

The second cost is an asymmetric adjustment cost when bankers change the size of loan stock:

$$f\left(\frac{s_t}{b^h_{t-1}}\right) = \frac{\upsilon}{4}\max\left\{\frac{s_t}{b^h_{t-1}} - \delta_b, 0\right\}^4$$

When households want to reduces their debt stock by borrowing less, banker incurs no cost. However, when bankers want to increase the loan stock, they have to pay the marketing expense. These costs are in terms of final goods. The parameter υ governs the size of this cost.

On the timing of the market, it is worth noting that bankers can adjust the level of their deposits and reserves after households and firms pay for each other. When a wholesale firm, who has an account a bank B, pays for a household, who has an account at bank A, the deposits and reserves of bank B will reduce by the same amount, while the deposits and reserves of bank A will increase by the same amount. When the different parties in the economy pay each other, a banker can witness the deposits and reserves outflow from or inflow to his bank. Let NIF_t be the net inflow of deposits and reserves go into their bank, bankers will take NIF_t as given. When the banking market opens, as the deposit market is perfectly competitive, banker can choose any amount $\hat{d}_t$ of deposit inflows or outflows to his bank. Hence, if we let $d_t = NIF_t + \hat{d}_t$, the variable d_t will be a choice variable in bankers' problem.

In each period, bankers treat all the prices as exogenously and choose $\{c_t, n_t, b^h_t, s_t, m_t, b^f_t, x_t, d_t, b^g_t\}$ to maximize their expected utility over a stream of consumptions:

$$\max E_t\left[\sum_{i=0}^{\infty}\beta^i \log(c_{t+i})\right]$$

subject to

$$\frac{R^n n_{t-1}}{\pi_t} + b^g_{t-1} + \frac{b^f_{t-1}}{\pi_t} + d_t - T_t = n_t + q_t(b^g_t - b^g_{t-1}) + v_t(x_t - x_{t-1}) + \frac{b^f_t}{R^f_t} \quad \text{(Reserve Flows)} \tag{1}$$

$$\frac{m_t}{R^m_t} = \frac{m_{t-1}}{\pi_t} + f\left(\frac{s_t}{b^h_{t-1}}\right) + \nu b^h_t + r_t s_t - \delta_b b^h_{t-1} + c_t - x_{t-1} r^x_t + d_t - T_t \quad \text{(Deposit Flows)} \tag{2}$$

$$b^h_t = (1-\delta_b) b^h_{t-1} + s_t \quad \text{(Loan Stocks)} \tag{3}$$

$$n_t \geq \varphi \frac{m_t}{R^m_t} \quad \text{(Reserves Requirement)} \tag{4}$$

$$n_t + b^g_t q_t + \frac{b^f_t}{R^f_t} + b^h_t + x_t v_t - \frac{m_t}{R^m_t} \geq \kappa\left(b^h_t + x_t v_t\right) \quad \text{(Capital Requirement)} \tag{5}$$

Banker	
Reserves: $+d_t$	Deposits: $+d_t$

Banker	
Reserves: - $(b^g_t - b^g_{t-1})q_t$	
Bonds: + $(b^g_t - b^g_{t-1})q_t$	

Table 2: Bankers choose more deposits (left) and purchase bonds (right)

The Fed	
	Bank Reserves: -T
	Treasury Deposit: +T

Banker	
Reserves: -T	Deposits: -T

Households	
Deposits: -T	Tax Due: -T

Table 3: Government collects tax from households

Banker	
Loans: $+s_t$	Deposits: $+r_t s_t$
	Net worth: + $(1-r_t)s_t$

Banker	
Loans: -$\delta_b b^h_{t-1}$	Deposits: -$\delta_b b^h_{t-1}$

Table 4: Banker issues loans (left) and collects loans (right)

Banker	
	Deposits: $+c_t$
	Net worth: - c_t

Banker	
	Deposits: $+(f(.)+\nu b^h_t)$
	Net worth: - $(f(.)+\nu b^h_t)$

Table 5: Banker consumes (left) and pays for cost (right)

The equation (1) shows the evolution of reserves in the banker's balance sheet. After receiving the interest rate paid on reserves, the previous reserves balance becomes $R^n n_{t-1}/\pi_t$. He also receives the payoffs from holding government bonds b^g_{t-1}; and collects the payment from the loan he lends out to other bankers in the previous period b^f_{t-1}/π_t. He can also increase his reserves by taking more deposits d_t. When the banker receives more deposit inflows, his reserves and his liability increase by the same amount (Table 2). That is the reason we see d_t appear on both the equation (1) and (2). The similar effect can be found on T_t- the net tax that the government imposes on households. In this case, bankers debit the households' deposit accounts then transfer reserves from their accounts to the account of the government at the central bank (Table 3). Banker can then leave reserves n_t at the central bank's account to earn the interest rate, or purchase the government bonds $q_t(b^g_t - b^g_{t-1})$, or purchase the financial claims $v_t(x_t - x_{t-1})$, or lend reserves to another bankers b^f_t/R^f_t.

The equation (2) shows the evolution of deposits on the liability side. Bankers make loans by issuing deposits (Table 4). So when they make a loan ($r_t s_t$), the balance sheet expands. When they collect the payoffs from the loan ($\delta_b b^h_{t-1}$), the balance sheet shrinks. The banker also issues their own deposits to purchase the consumption good from firms (c_t) and to pay the cost (in terms of final goods) related to lending activities ($f(s_t/b^h_t) + \nu b^h_t$) (Table 4). The total amount of deposits also decline when the construction firms pay for the bankers ($x_{t-1} r^x_t$).

Bankers face two constraints in every period. At the end of each period, banks have to hold enough reserves as a fraction of total deposits (4). This constraint should be interpreted more broadly than the the real life reserves requirement in US because the total ZMDs here include not only the checkable deposits but also the saving account and money market deposit account.

The second constraint plays the key role in our model - the capital requirement constraint. The left hand side of (5) is the banker's net worth (capital), which is equal to total assets minus total liabilities. The constraint requires bank to hold capital greater than a fraction of total loans they issue, where financial claim is also considered as a type of commercial loans.

Let λ_t, μ_t be the Lagrangian multipliers attached to the reserves constraint and the capital constraint. Let R^x_t be gross nominal return on the financial claim x_t, which is defined as:

$$R^x_{t+1} = \frac{(v_{t+1} + r^x_{t+1})\pi_{t+1}}{v_t}$$

The first order conditions of the banker's problem can be written as:

$$\frac{1}{c_t} = \beta R^f_t E_t \left[\frac{1}{c_{t+1}\pi_{t+1}} \right] + \mu_t \tag{6}$$

$$\frac{1}{c_t} = \beta R^m_t E_t \left[\frac{1}{c_{t+1}\pi_{t+1}} \right] + \mu_t + \varphi\lambda_t \tag{7}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{R^x_{t+1}}{c_{t+1}\pi_{t+1}} \right] + (1-\kappa)\mu_t \tag{8}$$

$$\frac{1}{c_t} = \beta R^n E_t \left[\frac{1}{c_{t+1}\pi_{t+1}} \right] + \lambda_t + \mu_t \tag{9}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{1 + q_{t+1}}{q_t c_{t+1}} \right] + \mu_t \tag{10}$$

In all cases, we have $\max\{R^n, R^m_t\} \leq R^f_t$. The natural lower bound for the federal funds rate R^f_t will be the interest rate paid on reserves R^n. Holding reserves brings two benefits to the bankers: (i) it earns the interest rate R^n; (ii) it helps bank to satisfy the reserve requirement, showed in λ_t. The opportunity cost of holding reserves is the federal funds rate that bankers can earn when lending out reserves to other bankers. From the equation (6) and (9), when the reserves requirement is no longer bound, $R_t^f = R^n$.

We denote $f'_t = f'(s_t/b^h_{t-1})$ and $f'_{t+1} = f'(s_{t+1}/b^h_t)$. The first order condition for choosing the loan stock can be written as:

$$\frac{1}{c_t}\left(\nu + r_t + \frac{f'_t}{b^h_{t-1}}\right) = E_t\left[\frac{\beta\delta_b}{c_{t+1}}\right] + E_t\left[\frac{\beta s_{t+1} f'_{t+1}}{c_{t+1}(b^h_t)^2}\right] + E_t\left[\frac{\beta(1-\delta_b)(r_{t+1}b^h_t + f'_{t+1})}{c_{t+1}b^h_t}\right] + (1-\kappa)\mu_t \qquad (11)$$

The price of loans that households borrow from bankers r_t depends on μ_t- the shadow price of the capital constraint, ν- the monitoring cost and the adjustment cost. When the capital constraint is binding, r_t is lower than before, implying that the real cost of borrowing for households increase.

3.2 Households/Intermediate Good Producers

There is a measure one of identical households. These self-employed households produce the homogeneous intermediate good y^m to sell to the wholesale firms at P_t^m, or the real relative price p_t^m. In each period, household purchases the final consumption good ($\widetilde{c}_t$) from the retail firms and housing ($\widetilde{h}_t$) from the construction firms. They also have a technology to convert one unit of consumption goods to one unit of capital type K and vice versa . We assume the discount factor of the households is smaller than the discount factor of the bankers, $\widetilde{\beta}(1+\nu) < \beta$, therefore, households will borrow from the bankers. Let $\widetilde{b}_t^h$ be the debt stock that households borrow from bankers. Recalling from the section 2.4, each period households only pay a fraction δ_b of the old debts. We impose an exogenous borrowing constraint for households with the debt limit $\overline{b^h}$.

Households face the "ZMD-in-advance" when the good market opens. Let a_t be the amount of total ZMDs that they hold when entering the goods market and housing market. Their total purchase of final goods and new housing has to be less than or equal to a_t.

In each period, households choose $\{\widetilde{c}_t, \widetilde{m}_t, a_t, \widetilde{h}_t, \widetilde{l}_t, \widetilde{b}_t^h, \widetilde{s}_t, I_t, K_t, \widetilde{i}_t^h\}$ to maximize their expected utility:

$$\max E_t\left[\sum_{i=0}^{\infty}\widetilde{\beta}^i\left(\log(\tilde{c}_{t+i}) + \xi_t\log(\tilde{h}_{t+i}) - \chi\frac{\tilde{l}_{t+i}^{1+\eta}}{1+\eta}\right)\right]$$

subject to

$$a_t + \delta_b \widetilde{b}^h_{t-1} = \frac{\widetilde{m}_{t-1}}{\pi_t} + r_t \widetilde{s}_t \qquad \text{(Loans Market Open)} \tag{12}$$

$$\widetilde{c}_t + I_t + q^h_t \widetilde{i}^h_t \leq a_t \qquad \text{(ZMDs-in-advance)} \tag{13}$$

$$\frac{\widetilde{m}_t}{R^m_t} + \widetilde{c}_t + \delta_b \widetilde{b}^h_{t-1} + q^h_t \widetilde{i}^h_t + I_t + T_t = \frac{\tilde{m}_{t-1}}{\pi_t} + r_t \widetilde{s}_t + p^m_t y^m_t + \frac{\Pi_t}{P_t} \qquad \text{(Budget Constraint)} \tag{14}$$

$$\widetilde{i}^h_t = \widetilde{h}_t - (1-\delta^h)\tilde{h}_{t-1} \qquad \text{(Residential Investment)} \tag{15}$$

$$I_t = K_t - (1-\delta)K_{t-1} \qquad \text{(Investment)} \tag{16}$$

$$\widetilde{b}^h_t = (1-\delta_b)\widetilde{b}^h_{t-1} + \widetilde{s}_t \qquad \text{(Loan Stock)} \tag{17}$$

$$y^m_t = K^{\alpha}_{t-1} \widetilde{l}^{1-\alpha}_t \qquad \text{(Production Function)} \tag{18}$$

$$\widetilde{b}^h_t \leq \overline{b^h} \qquad \text{(Borrowing Constraint)} \tag{19}$$

The equation (12) shows the constraint households face when the loan market opens but the final good market has not opened yet. Households bring $\widetilde{m}_t/\pi_t$ amount of money from the previous period, obtaining new loan $r_t\widetilde{s}_t$, paying back a fraction of the previous loan $\delta_b\widetilde{b}^h_{t-1}$, and bring a_t amount of money into the final good market.

The constraint (13) is the "cash-in-advance" constraint in Lucas and Stokey (1987). The only difference here is that currency does not exist in our model, so "cash" should be interpreted as ZMDs. Households need ZMDs in advance to purchase the consumption goods, housing and making investment. Wang and Wen (2006) shows that putting investment in the cash-in-advance constraint can generate the persistence of output and inflation in the data.

The equation (14) shows the typical budget constraint households face in each period. After paying the debts ($\delta_b\widetilde{b}^h_{t-1}$) in the previous period to the banker, and obtaining new loans ($r_t\widetilde{s}_t$), in addition to their previous balance in the deposit account ($\tilde{m}_{t-1}/\pi_t$), households will receive income from selling intermediate goods ($p^m_t y^m_t$) and the profits from the wholesale firms (Π_t). They spend on consumption goods ($\widetilde{c}_t$), investment I_t, new housing $\widetilde{i}^h_t$, leave some money as the zero-maturity deposits at the banks $\widetilde{m}_t/R^m_t$, and pay (receive) the tax (transfer) T_t to the government.

The equation (18) shows the production function of households. In each period, households combine their own labor and capital in the Cobb-Douglas form to produce the intermediate goods. Here we assume the household self-employs their workers, but the analysis will not change if households supply labor and hire labor in the competitive market.

We assume that households face an exogenous borrowing constraint, rather than a collateral borrowing constraint like Kiyotaki and Moore (1997) and Iacoviello (2005). Our purpose is to emphasize that the mechanism of the shock transmission in our model is not related to the collateral constraint literature.

We model housing as a durable good. Housing is depreciated at the rate δ^h. There is a housing demand shock in our model, captured by the time-varying parameter ξ_t. We assume the motion of ξ_t follows the equation:

$$\log(\xi_t) = (1-\rho)\log(\bar{\xi}) + \rho\log(\xi_{t-1}) + \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma^2) \tag{20}$$

Let $\gamma_t, \Lambda_t, \omega_t$ be respectively as the Lagrangian multiplier for the constraint (13), (14) and (19). We have:

$$\frac{1}{\widetilde{c}_t} - \gamma_t - \Lambda_t = 0 \tag{21}$$

$$\Lambda_t = \widetilde{\beta} R_t^m E_t\left[\frac{1}{\widetilde{c}_{t+1}\pi_{t+1}}\right] \tag{22}$$

$$\frac{q_t^h}{\widetilde{c}_t} = \frac{\xi_t}{\widetilde{h}_t} + \widetilde{\beta}(1-\delta^h)E_t\left[\frac{q_{t+1}^h}{\widetilde{c}_{t+1}}\right] \tag{23}$$

$$\chi \widetilde{l}_t^{\eta+1} = (1-\alpha)\Lambda_t p_t^m y_t^m \tag{24}$$

$$\frac{r_t}{\widetilde{c}_t} = \widetilde{\beta}\delta_b E_t\left[\frac{1}{\widetilde{c}_{t+1}}\right] + \widetilde{\beta}(1-\delta_b)E_t\left[\frac{r_{t+1}}{\widetilde{c}_{t+1}}\right] + \omega_t \tag{25}$$

$$\frac{1}{\widetilde{c}_t} = \widetilde{\beta}(1-\delta)E_t\left[\frac{1}{\widetilde{c}_{t+1}}\right] + \widetilde{\beta}E_t\left[\frac{\alpha\Lambda_{t+1}p_{t+1}^m y_{t+1}^m}{K_t}\right] \tag{26}$$

From the equations (21) and (22), we can rewrite:

$$\frac{1}{\widetilde{c}_t} = \widetilde{\beta} R_t^m E_t\left[\frac{1}{\widetilde{c}_{t+1}\pi_{t+1}}\right] + \underbrace{\gamma_t}_{\text{Liquidity Premium}}$$

As money plays the role of medium of exchange in our model, it's value contains the liquidity part. In the steady state, the rate of return on money has to be less than $1/\widetilde{\beta}$- the risk-free rate that households lend out to each other (if possible). This equation plays the important role when we analyze the implication when the central bank keeps the interest rate at the lower bound R^n for a long time.

The equation (25) gives us the marginal cost and the marginal benefit when households borrow one more unit of loans from bankers. As the loan market opens before the goods market, one more unit of loan can relax both cash-in-advance constraint and the general budget constraint, allowing the households to consume more. The left hand side of (25) is the marginal benefit when household borrow one more unit of s_t with the price r_t. The marginal cost is more challenging to understand as the maturity of loan is not finite. When households borrow one more unit of s_t, in the next period, they have to pay $\widetilde{\beta}\delta_b s_t$, so it explains the first part on the right hand side of the equation (25). The second part is the present value of the remaining debts carrying on to the following period after households borrow one more unit of s_t. The final component is the shadow price ω_t of the borrowing constraint. In our model, the borrowing constraint will be bound at the steady state. However, the interesting part of our paper happens when bankers actively cut the level of loans and the constraint (19) is no longer binding, ω_t=0.

Similarly, the marginal cost and the marginal benefit when household invest one more unit of capital are showed in the equation (26). When households decide to transform 1 unit of consumption good to 1 unit of capital, the marginal cost (left hand side) is the marginal utility lost from not consuming this 1 unit. However, if investing into capital, households receive two benefits in the next period. The first thing is capital, after depreciation, can transform back to the consumption good for consuming in the future. The second thing is the increase in the level of output in the next period from investing 1 more unit of capital in this period.

3.3 Construction Firms

Competitive construction firms build houses that are eventually sold to households. At the end of the period t, a construction firm acquires the capital H_t for use in the production in the subsequent period. It is noted that the capital type H is totally different from the capital type K, and they are not substitutable. We assume capital H is the only factor in the production function of construction firms. After production in the period $t+1$, capital is not depreciated and the firm can sell the capital in the open market to other construction firms. There are no adjustment costs at the firm level, so the firm's capital choice problem is always static.

The construction firms finance their capital acquisition in each period by obtaining funds from bankers. Follow Gertler and Karadi (2011), to acquire the funds to purchase capital at the end of t, the firm issues x_t claims equal to the number of units of capital they acquire H_t. Under the no arbitrage condition, the price of each claim v_t will be equal to the price of a unit of capital type H.

It is assumed that the bankers has perfect information about the firm and has no problem enforcing payoffs. The firms is thus able to offer the banker a perfectly state-contingent securities. At the end, the construction firms will always get zero profits, while the bankers might earn positive or negative profits depended on the realization of the housing demand shock.

The production function of the construction firms is:

$$i_t^h = H_{t-1} \tag{27}$$

Let q_t^h be the price of houses in the period t. Given that the construction firm earns the zero profit state-by-state, r_t^x- the real payoffs (excluding the capital gain) paid to the owner of one financial claim-and R_t^x - the gross nominal return for holding this claim- will be:

$$r_t^x = q_t^h \tag{28}$$

$$R_t^x = \frac{(q_t^h x_t + v_t x_t)\pi_t}{v_{t-1} x_{t-1}} \tag{29}$$

3.4 Retail Firms

Retail firms competitively produce the final consumption goods. The retail firm buy and aggregate a variety of differentiated wholesale goods indexed by $j \in [0,1]$ using a CES technology:

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution among the wholesale goods. Profit maximization and the zero profit condition give the demand for the wholesale good j

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} y_t \tag{30}$$

3.5 Wholesale Firms

There is a unit mass of wholesale firms on $[0,1]$ that are monopolistic competitors. Wholesale firms buy the intermediate good y_t^m at P_t^m from the households in a competitive market, differentiate the good at no cost into $y_t(j)$ and sell it with the price $P_t(j)$ to the retailer.

$$y_t(j) = y_t^m \tag{31}$$

Follow Rotemberg pricing, we assume that each wholesale goods firm j faces costs of adjusting prices, which are measured in terms of final good and given by:

$$\frac{\iota}{2}\left(\frac{P_t(j)}{P_{t-1}(j)}-1\right)^2 y_t$$

where ι is the adjustment cost parameter which determines the degree of nominal price rigidity. The wholesale firm j discounts the profit in the future with rate $\widetilde{\beta}^i \Lambda_{t+i}/\Lambda_t$, where Λ_t is the shadow price attached to the households' budget constraint. The problem of firm j is given by:

$$\max_{\{P_t(j)\}} E_t \sum_{i=0}^{\infty}\left\{\widetilde{\beta}^i \frac{\Lambda_{t+i}}{\Lambda_t}\left[\left(\frac{P_{t+i}(j)}{P_{t+i}}-p^m_{t+i}\right)y_{t+i}(j)-\frac{\iota}{2}\left(\frac{P_t(j)}{P_{t-1}(j)}-1\right)^2 y_{t+j}\right]\right\}$$

subject to its demand in equation (30).

In a symmetric equilibrium, all firms will choose the same price and produce the same quantity $P_t(j) = P_t$ and $y_t(j) = y_t = y^m_t$. The optimal pricing rule then implies that:

$$1-\iota\left(\pi_t-1\right)+\iota\widetilde{\beta}E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\pi_{t+1}-1\right)\pi_{t+1}\frac{y_{t+1}}{y_t}\right]=(1-p^m_t)\theta \tag{32}$$

3.6 The Central Bank

The central bank conducts open market operation, by purchasing and selling government bonds with bankers every period, to target the the interest rate R^f_t that bankers borrow from each other. By purchasing the government bonds held by bankers, it increases the level of reserves, relaxing the reserves requirement constraint, and therefore lowering the federal funds rate and the prime rate.

When we model explicitly the process of how the central bank controls the federal funds rate, the "traditional" Taylor rule is not enough for the determinacy. There are infinite levels of reserves that satisfy $R^f_t = R^n$. Hence, when the federal funds rate is at its lower bound, we need a rule governing the motion of the level of reserves (reserves targeting) (Figure 1).

We assume that the central bank follows the hybrid rule which is resulted from the combination of federal funds rate targeting and reserves targeting:

$$R^f_t = \overline{R^f} + \phi_n\left(\frac{n_t-(1-\rho_n)\bar{n}-\rho_n n_{t-1}}{\bar{n}}\right)+\phi_\pi(\pi_t-\bar{\pi})+\phi_y(\log(y_t)-\log(\bar{y})) \tag{33}$$

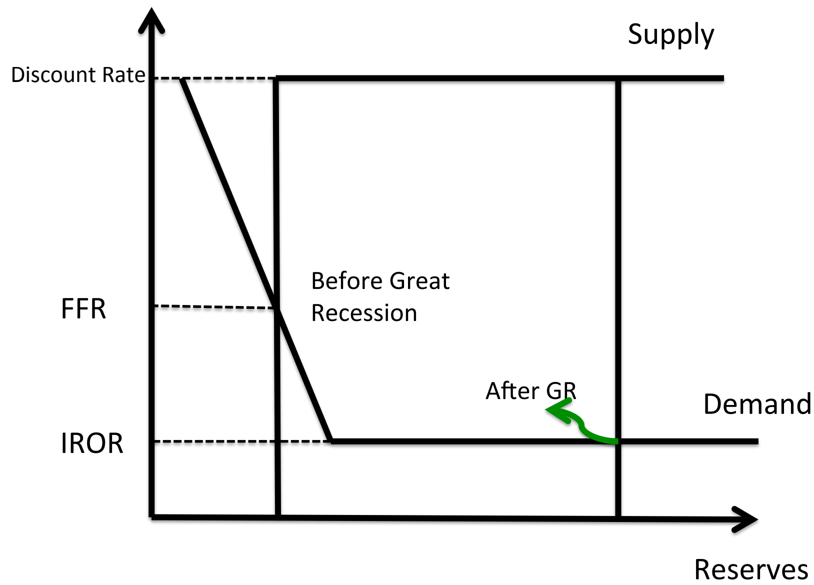


Figure 1: The Federal Funds Market

where ρ_n is the persistence of the level of reserves; ϕ_n, ϕ_π and ϕ_y is the coefficient responding to the reserve gap, inflation gap and output gap; $\overline{R^f}$, $\bar{n}$, $\bar{\pi}$ and $\bar{y}$ are the federal funds rate, reserves, inflation and final good outputs at the deterministic steady state.

Like the Taylor rule in the New Keynesian literature, the interbank rate responds to the output gap and the inflation rate. However, this rule departs from the traditional Taylor rule in two important points. First, it also includes the gap between the current level of reserves and its steady state value. The return of the federal funds rate to the steady state value is not enough to guarantee that the level of reserves and government bonds held by bankers also back to the steady state. Therefore, the federal funds rate should respond to the reserves gap. Does the appearance of reserves gap change significantly the Taylor rule? The answer is no, ϕ_n is positive but very small so it nearly does not have any effects when the reserves requirement is still binding.

Second, unlike the conventional Taylor rule, we do not see the zero lower bound here. This is one of the most interesting points in our model. The natural lower bound for the federal funds rate in our model is the interest rate paid on reserves. When the central bank injects the amount of reserves over the critical point where the reserves requirement is no longer binding, R_t^f will be automatically equal to R^n from (6) and (9). So what does the equation (33) mean in this case ? When $R_t^f = R^n$, it implies the central bank will switch from targeting the federal funds rate to targeting the level of reserves to responds to the output gap and the inflation rate. It also implies that when the banking system is awash of reserves, the central bank can still control the federal funds rate by changing its policy on interest rate paid on reserves. We will discuss this policy later in the paper. For now, we assume R^n is constant and the central bank will only follow the rule in (33).

3.7 The Government

We assume the government is totally independent from the central bank. It does not change its fiscal policy in our model. The total outstanding government bonds is fixed. Each period, the government collects tax from households to pay coupons for the government bond holders, which are central bank and bankers.

When the central bank receives the payoffs from government bonds, they use them to pay the interest rate paid on reserves for bankers $(R^n - 1)n_{t-1}/\pi$. We assume that central bank holds a big number of government bonds such that the payoffs are greater than $(R^n - 1)n_{t-1}/\pi$. After that, for the remaining payoffs, the central bank will transfer them back to the government, who in turn will transfer it back to households. So let T_t be the net tax households (in real term) have to pay:

$$T_t = b^g_{t-1} + \frac{(R^n - 1)n_{t-1}}{\pi_t} \tag{34}$$

It is worth noting that, as government bonds are indexed, we do not have to keep track the balance sheet of the Fed explicitly. It is no longer true if we assume the central bank holds nominal long term government bond.

4 Equilibrium

Definition: *A competitive equilibrium is a stochastic sequence of bankers' decision choice* $\{c_t, n_t, b^h_t, s_t, m_t, b^f_t, x_t, d_t, b^g_t\}$, *household's choice* $\{\widetilde{c}_t, \widetilde{b}^h_t, \widetilde{s}_t, \widetilde{m}_t, \widetilde{l}_t, K_t, I_t, \widetilde{y}^m_t\}$, *wholesale firm's choice* $\{y^m_t\}$, *retail firm's choice* $\{y_t\}$, *the government tax* $\{T_t\}$, *and the market price* $\{r_t, R^f_t, q_t, \pi_t, p^m_t\}$ *such that given the market price, banker's choices solve the banker's problem, household's choices solve the household's problem, wholesale firm's choice solves the equation (32), the government's budget is balance, the stochastic sequence* $\{R^f_t, n_t\}$ *satisfy the central bank rule and all the markets clear.*

The market clearing condition implies that:

Net flows of reserves between bankers:	$d_t = 0$	(35)
The interbank market:	$b_t^f = 0$	(36)
Total ZMDs:	$m_t = \widetilde{m}_t$	(37)
Loan Market:	$b_t^h = \widetilde{b}_t^h$	(38)
Capital H and Financial claim:	$x_t = H_t = \overline{H}$	(39)
Housing Market:	$\widetilde{i^h}_t = i_t^h = \overline{H}$	(40)
Consumption Good Market:	$y_t = c_t + \widetilde{c}_t + I_t + \nu b_t^h + f\left(\frac{s_t}{b_{t-1}^h}\right) + \frac{\iota}{2}(\pi_t - 1)^2 y_t$	(41)

It is worth noting that, as the reserves only flow from one bank to another bank, the total net flows of reserves in equilibrium will be zero. If we only consider the symmetric competitive equilibrium where all banks are identical, then d_t must be zero. This condition, (34), (36) and (39) imply that the evolution of reserves (1) in the banking system, in equilibrium, can be written as:

$$\frac{n_{t-1}}{\pi} = n_t + q_t(b_t^g - b_{t-1}^g)$$

In the conventional monetary policy, we assume that the central bank only purchase the government bonds. Under that condition, the level of reserves in the banking system will increase when the central bank conducts the open market purchase and reduce when they conducts the open market sales. It is emphasized here that the level of reserves in the banking system is decided solely by the central bank. Each individual commercial bank can change its own level of reserves, but the sum amount of reserves are not decided by commercial banks. Under the normal condition, by manipulating the level of reserves, the central bank can adjust the federal funds rate and the level of money supply.

The right hand side of the equation (41) shows the aggregate demand of the economy. We can decompose it into four components: (i) the aggregate consumption as the sum of bankers' consumption (c_t) and households' consumption ($\widetilde{c}_t$), (ii) the private investment (I_t), (iii) the cost of providing services in the banking sector $\nu b_t^h + f(.)$ and (iv) the inflation cost. When the housing demand shock ξ_t is realized, later we will see that the decline in I_t is the biggest factor for the collapse of the aggregate demand.

The system of equilibrium conditions (C.1)-(C.28) is written in a compact form in the Appendix.

5 The Deterministic Steady State

To ensure that in the steady state, households will borrow from banker and the borrowing constraint is binding, we assume that the discount rate of households is smaller than the one of bankers after adjusting for the cost of loan.

Assumption 1:

$$\widetilde{\beta}(\nu+1) < \beta$$

Assumption 2:

The long-run interbank rate that the central bank target is $1/\beta$.

$$\overline{R^f} = 1/\beta > R^n$$

Theorem 1: *Under the Assumption (1) and (2), the deterministic steady state where* $\overline{R^f} = 1/\beta$ *will lie in the region where the capital constraint is slack (*$\overline{\mu} = 0$*) and the reserves constraint is binding (*$\bar{\lambda} > 0$*).*

From the Assumption 2 and (6), if $\mu > 0$ then $\overline{R^f} < 1/\beta$, lower than the central bank long run target. If $\lambda = 0$, then $\overline{R^f} = \overline{R^n}$, which is contradict with the Assumption 2. We want to restrict ourself to the steady state similar to the banking state before the Great Recession to illustrate the effect of housing crisis on the economy.

6 Quantitative Analysis

6.1 Calibration

We calibrate the banker's discount factor $\beta = 0.99$, implying the federal funds rate at the steady state is around 4.04 percent annually, which is roughly matched with the average of the effective federal funds rate before the Great Recession. The reserves requirement φ and $\bar{n}$ are set to match the ratio between the amount of zero maturity deposits and the level of reserves in the banking system. ZMDs are calculated as the difference between the total MZM (money zero maturity) and the currency. We average the ratio data between ZMDs and reserves during the 2008Q1, which is around 527 times, or $\varphi = 0.199$. For the capital requirement, we calibrate $\kappa = 0.2$ so the banker's net worth is 1 percent higher than the constraint. The parameter ν is set to make the bank's loan rate to household is around 0.5 percent higher than the federal funds rate in the steady state. The fraction of paid loan δ_b is set to 0.2, which is 20 percent of the total loan stock. It is important to remark here that our banker's balance sheet does not contain the time deposit (both small-denomination and large denomination), which accounts for approximately 25 percent of total bankers' asset.

For the households, the parameters $\{\ \widetilde{\beta},\ \chi,\ \eta,\ \delta,\ \alpha\ \}$ are calibrated in the standard way. The two nonstandard parameters are $\bar{\xi}$, the relative utility weight of housing, and $\overline{b^h}$, the exogenous borrowing constraints. We calibrate $\bar{\xi}$ so the total value of financial claims on the construction firms accounts for 35 percent of total banker's asset. We should interpret that these financial claims also account partial to the level of mortgage loans in the commercial bank's balance sheet. So more than one third of commercial banks' balance sheet in our model is exposed to the housing demand shock. The other nonstandard parameter is $\overline{b^h}$, which is set so the total household debt is around 100 percent of household's consumption.

On the supply side, we calibrate the elasticity of substitution between wholesale goods is 4, which is in range from 3 to 5, that is common in the literature. The cost of changing price in the Rotemberg model is set to 50.

About the central bank's rule, the reaction of the federal funds rate to the output gap is set to 0.25/4, to the inflation gap is set to 1.5. A new parameter appears in our model is ρ_n, the persistence in the level of reserves. We set it at the level of 0.8, means that once the Fed injects a lot of reserves in the banking system, the central bank will not drain it quickly in the following period. We can say that the flow of reserves is smooth.

6.2 Solution Method

We focus on the recursive competitive equilibrium when the prices and decision rules are the functions of state variables. Our problem has 6 state variables $\{b^g_{t-1}, n_{t-1}, m_{t-1}, b^h_{t-1}, K_{t-1}, \xi_t\}$ when we assume the central bank follows the conventional monetary policy by purchasing only government bonds. The problem has 3 occasionally binding constraints, which are the reserves requirement constraint, the capital constraint and the borrowing constraint. These constraints require us to use a global solution to solve the model. Moreover, some variables, like reserves and the amount of government bonds held by bankers, can jump to points very far from the steady state. Reserves can be five times as high as the level at the steady state. As the result of that, the solution is very nonlinear and non-smooth in nature. We use the nonlinear certainty equivalent

Table 6: Parameter values

Param.	Definition	Value
Bankers		
β	Banker's discount factor	0.99
φ	The reserves requirement	0.0019
κ	The risk weight	0.199
ν	The monitoring cost	0.0005
δ_b	The fraction of loan is paid	0.2
υ	The adjustment cost for loan portfolio	1000
Households		
$\widetilde{\beta}$	Household's discount factor	0.985
$\bar{\xi}$	Relative Utility Weight of Housing	0.0075
χ	Relative Utility Weight of Labor	0.92
η	Inverse Frisch Elasticity of Labor Supply	0.35
$\overline{b^h}$	The borrowing limit	1.485
δ^h	House's depreciation rate	0.025
δ	Capital's depreciation rate	0.025
α	Capital share in production function	0.34
Firms		
θ	Elasticity of substitution of wholesale goods	4
ι	Cost of changing price	50
$\overline{H}$	The total capital H	0.04
Central bank		
$\bar{n}$	Reserves at steady state	0.0035
ϕ_n	Policy respond to reserves gap	0.0035
ρ_n	The persistence of reserves	0.8
ϕ_π	Policy respond to inflation	1.5
ϕ_y	Policy respond to output gap	0.25/4
Shock Process		
ρ	Persistence of shock	0.99
σ	The standard deviation of shock	0.047

approximation method in Cai, Judd and Steinbuks (2015) to solve our model. This method contains three main steps: (i) build a grid and transform a stochastic problem into a deterministic problem, (ii) solve the deterministic problem in each grid point to find the next period decision's rule, (iii) finally approximate the decision rule in whole grid from the results in step 2. The method is global and deals well with the inequality constraint. However, we will lose some accuracy from ignoring the uncertainty when agents make decision. In this paper, we use the software Ipopt (Interior Point Method) written by Wachter and Biegler (2006) to solve the large system of equation in each grid point.

Follow Brumm and Scheidegger (2015), we build a Clenshaw-Curtis sparse grid (level 4), then add points to refine the grid adaptively. The description of how to build this grid is presented in the Appendix (B). Smolyak grid can deal with the "curse of dimensionality" problem in macroeconomics. It detail description and applications in economics can be found in Malin, Krueger and Kubler (2011), Judd et al. (2014), Brumm and Scheidegger (2015). In our problem, when building grid, we also use the adaptive domain technique in Judd et al. (2014) for the two pairs of state variables b^g -n, and b^h-m. When the central bank conducts the open market purchase, the increase of reserves n_t is associated with the decrease the government bond held by banker. It is likely that we are never in a state that both reserves and government bonds held by banker are at the high level, so for these two states, we put them in a parallelogram rather than a rectangular. The same principle is applied for b^h and m as they tend to move together in the same direction. Judd et al. (2014) use the stochastic simulation from the solution of linear model to identify the adaptive domain region while we only use the ad-hoc way to identify the adaptive domain.

For each grid point (each initial state), we solve the deterministic problem, and assume that after T=500, the economy will be back to the steady state. To deal with the inequality constraints, we use the penalty method found in McGrattan (1996) to transform the hard constraint into soft constraint. We assume the utility in one period of bankers has the form:

$$
\begin{aligned}
u(c_t, \lambda_t^*, \mu_t^*) &= \log(c_t) - \frac{1}{4} e \max\{\lambda_t^*, 0\}^4 - \frac{1}{4} e \max\{\mu_t^*, 0\}^4 \\
\lambda_t^* &= \frac{m_t}{R_t^m} - \frac{n_t}{\varphi} \\
\mu_t^* &= \frac{m_t}{R_t^m} - \left[n_t + q_t b_t^g + (1-\kappa)(b_t^h + x_t v_t) \right]
\end{aligned}
$$

where $e = 1e8$ is the penalty coefficient. It means that any time, bankers violate the reserves requirement constraint or the capital constraint, they suffer the reduction in their utility. However, if they hold more reserves or capital than the required amount, they are not rewarded for that. This penalty method is also applied for the households' borrowing constraint. Cash-in-advance constraint is always binding in the deterministic model, so we do not need to transform it.

This setup still ensures that all of our first order conditions are twice differentiable. However, even with this transformation, our system of equations is still ill-conditioning. Most of the time, we have to use the homotopy method in combination with nonlinear large scale optimization tool Ipopt to solve the big deterministic system.

6.3 Housing Crisis and Conventional Monetary Policy

6.3.1 Shock Transmission in Interbank Market

When a big negative shock on housing demand ξ_t is realized, price of housing q_t^h drops. Consequently, the price v_t of the financial claims x_t on the construction firms and bankers' net worth must go down. In the conventional monetary policy, the central bank will conduct the open market purchase to push down the interbank rate R_t^f. As the price is sticky, the real rate will go down, pushing up the price of government bonds and other assets. This can mitigate the initial decline in v_t and bring the economy back to the normal state.

However, when the shock is large and persistent, the conventional monetary policy is not enough for controlling the decline in v_t, especially when the capital constraint is bound. The decline of v_t pushes down the banker's capital. If bankers' net worth goes down to the level where the capital constraint is binding, bankers have to relax this constraint by reducing the loan size b^h and the number of financial claims on the construction firms x_t on their balance sheets. This fire sales create the negative externalities as it further pushes down the v_t and bankers' net worth.

To see the whole process clearly, we start from the equation (28) and impose the equilibrium condition $x_t = \overline{H}$:

$$R_t^x = \frac{(q_t^h + v_t)\pi_t}{v_{t-1}}$$

The above equation decomposes the return on the financial claim into two components: the real payoff q_t^h and the capital gain when reselling these financial instrument v_t. When substituting it into (8), we have:

$$\frac{v_t}{c_t} = \beta E_t \left[\frac{q_{t+1}^h + v_{t+1}}{c_{t+1}} \right] + (1-\kappa)\mu_t \tag{42}$$

Solving this Euler equation forward and impose the transversality condition, the price v_t of the financial claim x_t is:

$$v_t = E_t \left[\sum_{j=1}^{\infty} \beta^j \left(\frac{c_t}{c_{t+j}} \right) q_{t+j}^h \right] + (1-\kappa) E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{c_t}{c_{t+j}} \right) \mu_{t+j} \right] \tag{43}$$

Similarly, we can solve forward for the price q_t of government bonds from (10):

$$q_t = E_t \left[\sum_{j=1}^{\infty} \beta^j \left(\frac{c_t}{c_{t+j}} \right) \right] + E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{c_t}{c_{t+j}} \right) \mu_{t+j} \right] \tag{44}$$

From (43) and (44), we can express the price v_t as the function of government bond's price, the discounted housing price from future and the discounted of the shadow price of capital constraint:

$$v_t = \underbrace{q_t}_{\text{Govt bonds' price}} + \underbrace{E_t\left[\sum_{j=1}^{\infty}\beta^j\left(\frac{c_t}{c_{t+j}}\right)(q^h_{t+j}-1)\right]}_{\text{Adjusted discounted house prices}} - \underbrace{\kappa E_t\left[\sum_{j=0}^{\infty}\beta^j\left(\frac{c_t}{c_{t+j}}\right)\mu_{t+j}\right]}_{\text{Shadow price of capital constraint}} \tag{45}$$

The conventional monetary policy can push up the price of government bonds; and therefore, increasing the price of other assets. However, with a big housing demand shock, the decline of the second term outweighs the increase in the first term. When the capital constraint starts binding, the contribution of the increase in the shadow price of capital constraint declines v_t further. Even when the central bank pushes R^f_t to its lower bound (the interest rate paid on reserves R^n), they can not mitigate the decline of v_t (Figure 2d).

It is worthy noting that the capital constraint creates the negative externalities when bankers fire-sell their assets x_t, as μ_t is a function of v_t. When bankers make decision about selling x_t, they treat the price v_t as given and do not take into account their action on v_t. As a result, v_t further declines, tightening more banker's net worth, pushing up the shadow price μ_t, which in turn has a feedback effect on v_t through (45).

As the supply of x_t is fixed at $\overline{H}$, bankers have to cut the amount of loans to households b^h_t to satisfy the capital requirement. This is also the main mechanism that creates the sharp drop in output and investment in our model.

6.3.2 Deleveraging and Shock Transmission to Production Sector

In a common representative agent framework, the decline in housing demand usually expands the consumption good sector as the agent will switch from spending on housing into on consumption goods. However, in our model, the sharp decline in the housing price makes the bankers fall into the capital constraint trap, and the shock is transmitted from the banking sector to the production sector. To relax the capital constraint, bankers have to actively cut loans to household. The positive of the shadow price μ_t pushes up the real loan rate. In a deterministic setting, let r^h_t be defined as:

$$r^h_t = \frac{\delta_b}{r_t} + \frac{(1-\delta_b)r_{t+1}}{r_t} \tag{46}$$

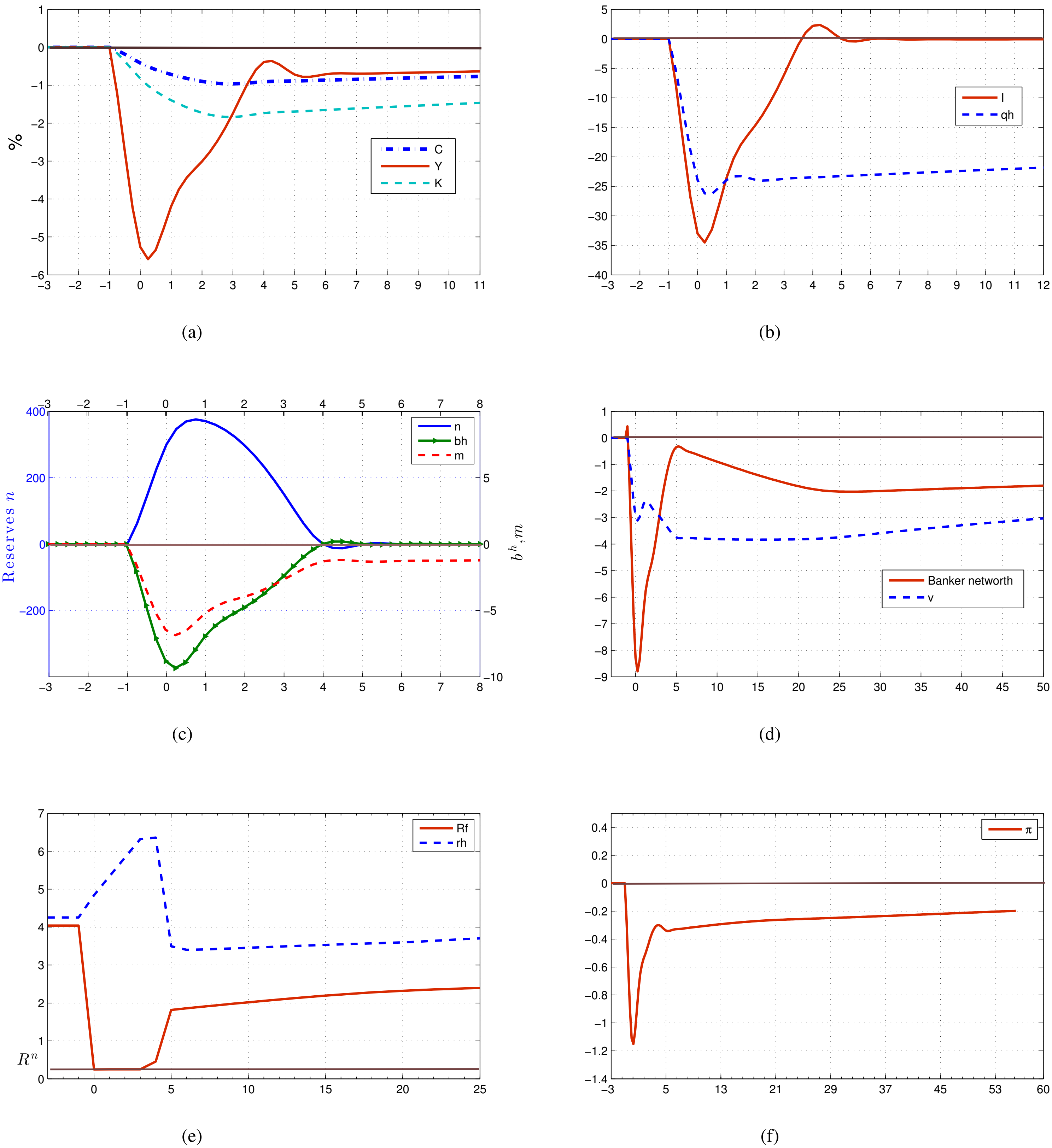


Figure 2: The impulse response under -3σ deviation of housing demand shock

Then we can rewrite the equation (25) in the deterministic setting as:

$$\frac{1}{\widetilde{c}_t} = \widetilde{\beta} r_t^h \frac{1}{\widetilde{c}_{t+1}} + \omega_t$$

The above equation implies that the real rate r_t^h can be considered roughly as the real prime rate that bankers lend out to households in the normal one period loan. Figure (2e) shows that the prime rate sharply increases at the beginning of the crisis even though the central bank already pushes the federal funds rate to the lower bound R^n. Figure (2c) shows that the amount of loan is cut by 10 percent at the beginning of the crisis.

The deleverage process in the banking sector leads to the situation where the economy lacks liquidity. The decline of ZMDs tightens the households' cash-in-advance constraint, forcing them to cut the level of investment quickly to smooth their consumption. The investment is a key variable causing the sharp decline and the slow recovery of the aggregate output (Figure 3). The fall in the aggregate demand, in turn, creates a sharp prolonged deflation episode. Figure (2f) shows that the deflation can last for a long time under the housing crisis.

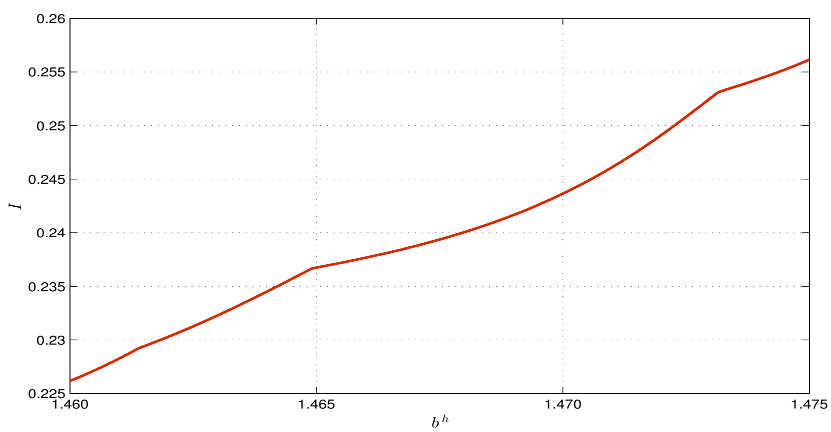


Figure 3: The response of investment I to loan size b^h

Our model partially gives the answer to the productivity puzzle in the Great Recession. Even though outputs in US contracted up to 6.8% in 2008Q4, the data shows that the productivity increased slightly 0.2% during the Great Recession (Bureau of Labor Statistics Data). It is clear that a big negative productivity shock cannot explain this phenomenon. In our model, the shock is transmitted to the production sector from the banking sector. These shocks look like the credit shocks, reducing the private investment sharply. When the capital stock is destroyed, our model can generate a jump in the labor productivity at the beginning of the crisis. The interesting part in the shock transmission mechanism in our model. The housing demand shock comes from the households, but it mainly affects to the bankers, which in turn, has big feedback effect on households.

6.3.3 Reserves Skyrockets but Money Supply Declines

Even with the counter-factual experiment that the central bank only conducts the conventional monetary policy, we can still generate the big jump in the level of reserves, which is nearly four times as high as the steady state level (Figure 2c). Most of these reserves are certainly excess reserves. By injecting a massive amount of reserves into the banking system, the reserves constraint is no longer binding $\lambda_t = 0$, the federal funds rate is at the lower bound $R_t^f = R^n$.

One of puzzle in the Great Recession is that the sharp increase in the level of reserves was not accompanied by the increase in the money supply. The insight to understand this phenomenon is pretty clear in our model. A negative shock to the bankers' balance sheet forces them to cut down their liability side, which is the zero maturity deposits. As money in our model is inside money issued by the commercial bank, the total money supply must decline. Even though the central bank injects a lot of reserves, the main concern now of the banker is the capital constraint. The money multiplier model predicts that the increase of money supply is accompanied by the level of reserves based on only the reserves requirement constraint, while ignoring the more important one - the capital constraint. Hence, it cannot predict what happened during the Great Recession

The banking crisis is special in the sense that the central bank might lose the control of the money supply in the short run. That feature in our model is totally different from the New Keynesian literature where the total money supply is always controlled directly by the central bank. It is nearly impossible to generate the decline in the money supply in the New Keynesian framework.

6.3.4 Large Shock vs Small Shock

Due to the appearance of three occasionally binding constraints, the economy responds differently to the small shock and large shock. For the small shocks, the conventional monetary policy is enough to push the price of x_t through lowering the federal funds rate. Bank loans are not cut; therefore, the economy does not witness the long deflation episode. On the other hand, with the large housing demand shock, the capital constraint is binding, pushing R_t^f to the lower bound R^n is not enough to prevent the deleveraging process. Figure 4 shows the decision rule for inflation as the function of housing demand shock ξ. This graph assume that other state variables are in the deterministic steady state level.

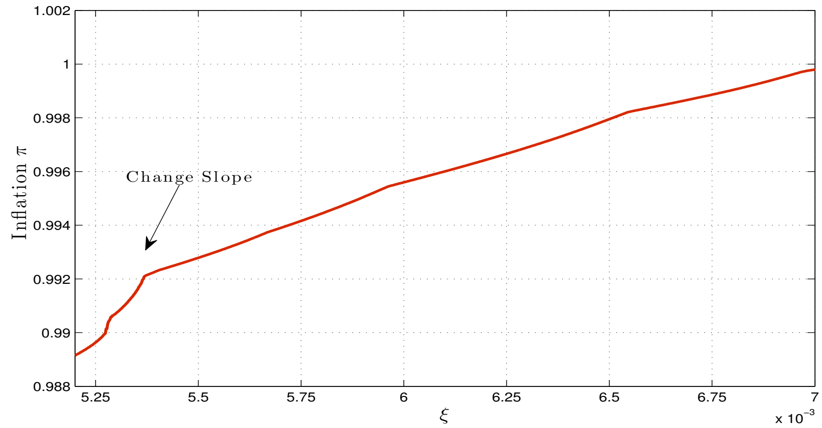


Figure 4: The response of inflation π to ξ

It is clear that there is a kink in the response of π to ξ. Once the shock ξ go over the critical level, the inflation responds more strongly to the decline in ξ. This emphasizes the importance of taking into account the nonlinearity in the dynamic general equilibrium model.

6.4 Distressed Asset Purchases - Unconventional Monetary Policy

Now we do an experiment that the central bank will purchase the distressed asset x_t, rather than the government bonds b^g from bankers in the crisis. In fact, this experiment is similar to what the Fed did in the QE1 when they purchases the massive amount of agency mortgaged-backed-security from commercial banks. We still assume that there is a huge negative housing demand shock at time $t = 0$. In the first experiment, the central bank immediately responds to the crisis by purchasing x_t from time $t = 0$ (QE since $t = 0$). In the second experiment, the central bank still uses the conventional monetary policy at time $t = 0$ and only unexpectedly switches to the quantitative easing in period $t = 1$ (QE since $t = 1$). In both experiment, the central bank will purchase the financial claim x_t following this rule:

$$n_t = \rho_n n_{t-1} + (1-\rho_n)\left(\bar{n} + \left(E_t\left[R^x_{t+1}\right] - \overline{R^x}\right)\right) + \varepsilon_{n,t} \tag{47}$$

$$b^g_t = \overline{b^g}, \quad \forall t \geq 1 \tag{48}$$

$$\begin{cases} \varepsilon_{n,0} = \alpha_n \bar{x}\bar{v} > 0, & \varepsilon_{n,t} = 0 \quad \forall t \geq 1, \quad \text{QE since } t = 0 \\ \varepsilon_{n,1} = \alpha_n \bar{x}\bar{v} > 0, & \varepsilon_{n,t} = 0 \quad \forall t \geq 2, \quad \text{QE since } t = 1 \end{cases} \tag{49}$$

We can interpret these above conditions as the following. As b_t^g is fixed at the steady state level, it means that the central bank will target the level of reserves by purchasing and selling the financial claims on the construction firm x_t. We can see it more clearly from rewriting the equation (1) under the equilibrium with $b_t^g = \overline{b^g}$:

$$\frac{n_{t-1}}{\pi} = n_t + v_t(x_t - x_{t-1})$$

We also assume that either at time $t = 0$ or at $t = 1$, there is unexpected shock $\varepsilon_n > 0$. It means the central bank will suddenly purchase α_n fraction of the total value of distress assets. We calibrate $\alpha_n = 0.02$, means that the central bank will purchase around 2 percent of total value (in the steady state) of x_t in the market. After that, there is no other shocks. Central bank will simply slowly sells these asset back to the market. The parameter ρ_n is set to equal 0.98, means that the level of reserves is very persistent after the unconventional monetary policy, or the central bank will keep these assets in their balance sheet for a long time.

When the central bank purchases a large amount of long term assets, reserves will rise sharply, and of course, the reserve requirement is no longer binding $\lambda = 0$ and the federal funds rate will be equal to the interest rate paid on reserves $R_t^f = R^n$. For this section, we temporarily assume that the central bank does not use the interest rate paid on reserves R^n to adjust the federal funds rate when the banking system is awash of reserves. In the next section, we will discuss the situation when the central bank use R^n to adjust the interbank rate and its influence on the inflation and output path.

For these experiments, we do not calculate the recursive competitive equilibrium as the function of the state variables like in the conventional monetary policy. We simply calculate the trajectory of the economy under the perfect foresight condition. It still contains all the interesting switches between the reserves requirement constraint and the capital constraint. Like in the conventional monetary policy, we assume that economy is back to the steady state at time $T = 1200$. As the system is big and very nonlinear, we have to use the homotopy method in combination with Ipopt to solve this big system.

6.5 Effects on bankers' balance sheet and asset price

When the central bank suddenly purchases a large amount of the distressed asset, the price v_t of the financial claim on the construction firms goes up immediately. If the amount of purchased asset is large enough, the central bank can push the banker's out of the capital constraint quickly. Figure 5 compares the effects when the central bank purchases the government bonds in the conventional monetary policy and when it purchases the distressed assets x_t.

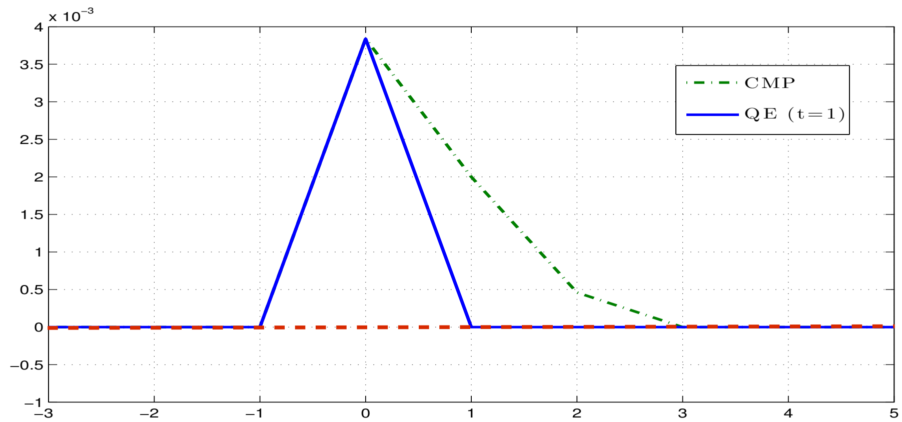
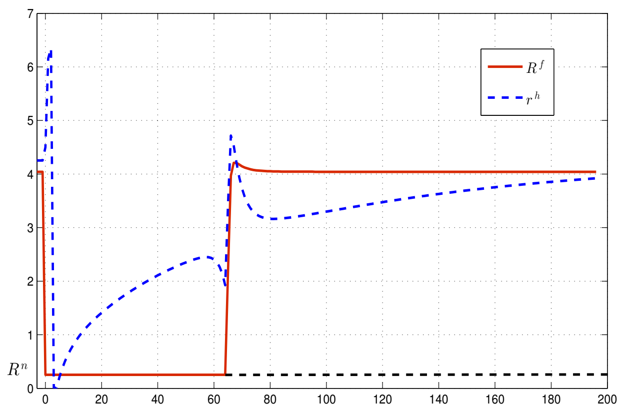


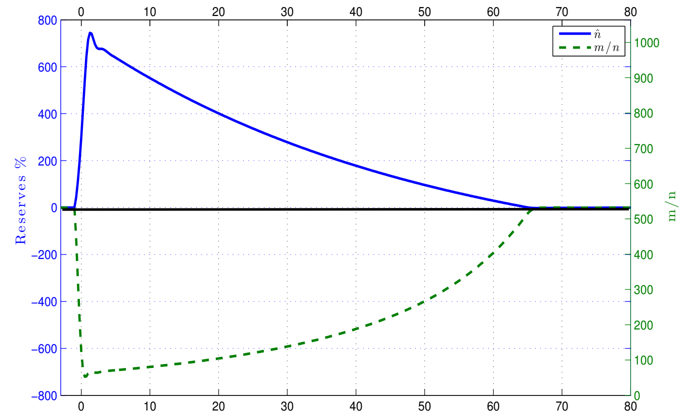
Figure 5: The shadow price of capital constraint

The unconventional monetary policy is very effective to push the bankers out of the capital constraint. When the housing demand shock is realized, as v_t declines, the capital constraint is binding, and the shadow price μ_t is positive. With the conventional monetary policy, it takes three quarters for the capital constraint is slack again while the unconventional policy can help banks out of the capital constraint immediately in our model. In our experiment 2, the central bank only responds from $t = 1$ so the capital constraints is still bound at $t = 0$.

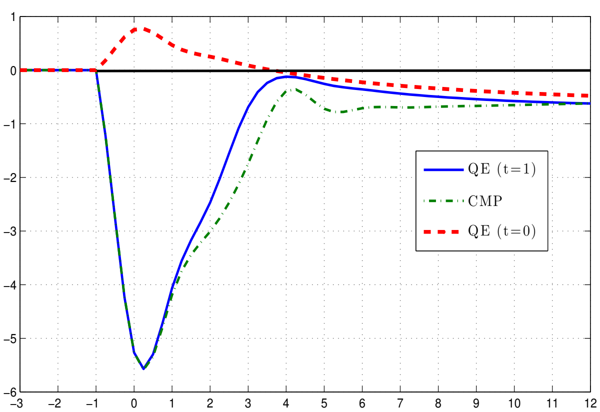
It is worthy noting that both conventional and unconventional monetary policy help to increase the asset price in the economy and improve the bankers' net worth; however, their mechanisms are different. The conventional monetary policy pushes down the yield of government bonds to affect the yields of x_t and its price v_t, whereas the unconventional one directly shifts the demand of x_t to the right to push down its yield and push up its price. Of course, theoretically, there is a risk that the central bank might suffer the loss by conducting this program. However, through the lens of our model, we know that v_t is lower than its "fair value" due to the fire sales and the binding of the capital constraint. In fact, our model generate that the central bank gains a lot of profit from conducting this large scale asset purchase program, even though profit is not the central bank's target.



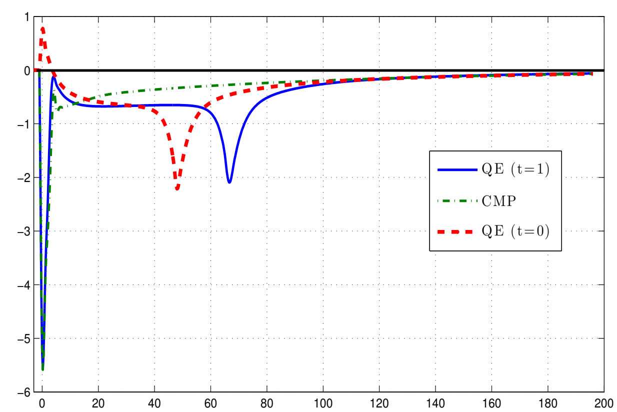
(a) Federal Funds Rate and Prime Rate in QE (t=1)



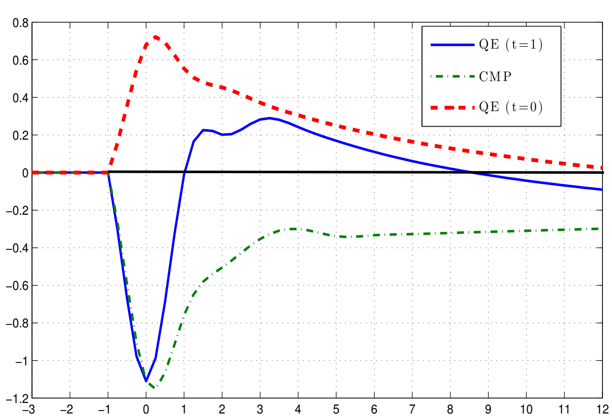
(b) Reserves and Money Multiplier in QE (t=1)



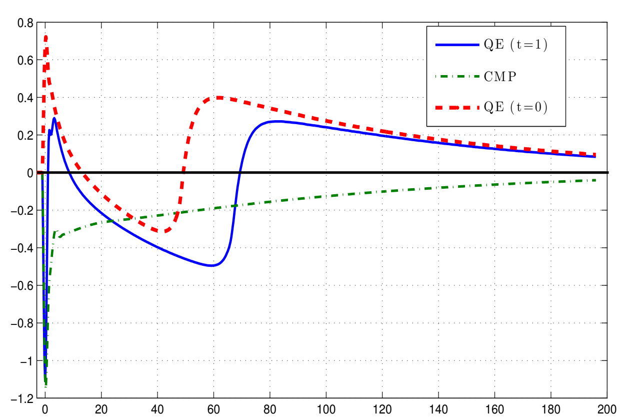
(c) Compare Output in Short Run



(d) Compare Output in Long Run



(e) Compare Inflation in Short Run



(f) Compare Inflation in Long Run

Figure 6: Compare Quantitative Easing (QE) and Conventional Monetary Policy (CMP)

6.6 Effect on Output and Inflation in Short-Run

In our experiment, R^n is not adjusted, so when the central bank conducts the quantitative easing, R_t^f is fixed at the the lower bound for 65 quarters (more than 15 years) (Figure 6a). In the deterministic setting with the assumption of constant R^n, the quantitative easing also imply the forward guidance to the public that the federal funds rate will be at the lower bound R^n for a long time. This has a big effect on the output and inflation in the short-run as the prime rate r_t^h declines sharply.

Figure (6c) compares the effect of quantitative easing and the conventional monetary policy on outputs in the short run. By putting the federal funds rate and the prime rate to the low level, quantitative easing can quickly push up the output level to the steady state level. The credit flow is recovered more quickly as the commercial bank is no longer in the capital constraint. If the central bank conducts quantitative easing since $t = 0$, the crisis will not even happen in our model. The interesting part of our model is the housing demand shock itself only affects the asset price, and crisis is transmitted through the financial market and bank lending channel. By purchasing x_t itself since $t = 0$, the central bank can prevent the chain effect of crisis. If QE is conducted at $t = 1$, it is still very effective in comparison to the conventional monetary policy. Output can be 1.2 percent higher in the unconventional monetary policy in the short run.

Inflation is even recovered more quickly under the unconventional monetary policy in the short run, even though our model still features the price stickiness in the same line as New Keynesian literature. By purchasing a massive amount of asset x_t, the central bank can create inflation immediately. As R^n is not adjusted, the prime rate r^h decreased sharply following QE. This stimulates the borrowing from households. The investment, the aggregate demand and the inflation go up as the cash-in-advance constraint is relaxed. For our first experiment, deflation does not appear in the short run. For the second experiment, after the sharp deflation in the period $t = 0$ when the commercial banks cut loans, the inflation goes back immediately when the central bank purchases x_t.

It is, in fact, a challenging task for any New Keynesian models to generate the sharp decline, then immediately, the sharp rise in the inflation within two periods. Del Negro, Giannoni and Schorfheide (2015) explain the missing deflation by the rise in the expected marginal cost in the future. Christiano, Eichenbaum and Trabandt (2015) build a model featuring a working capital channel. The inflation does not decline much in their model as the higher spread feeds directly on the marginal cost. On the other hand, the result in our model is the combination of two unexpected shocks. The first unexpected shock is the housing demand shock ξ_t at time $t = 0$, and the second one is the unexpected shock on the central bank's policy

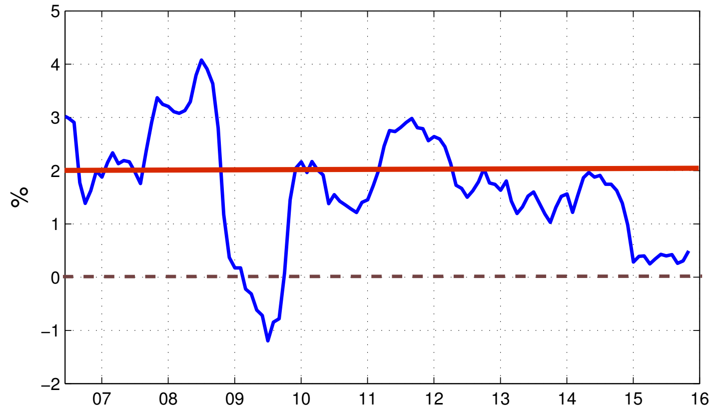


Figure 7: Personal Consumption Expenditure Index (Percent Change). **Source**: BEA

$t = 1$ when they switch from the conventional monetary policy to the unconventional one. In one period, inflation goes up by more than 1 percent in the quantitative easing. The economy is out of the deflation episode immediately while this will last in a long time if the central bank only conducts the conventional monetary policy.

6.7 Is QE inflationary or deflationary ?

In the short run, the answer is clear that quantitative easing causes the inflation. By purchasing the distressed asset, the central bank not only injects a lot of reserves in the banking system but also signal to the market that the federal funds rate will be near zero for a long time. As the result of that, the credit market recovers, bank issues more loans, and households are willing to borrow more as the real cost of borrowing goes down. Due to the combination of all these effect, the total amount of money supply ZMDs goes up and creates inflation. The data from Bureau of Economic Analysis also shows that after quantitative easing, inflation in US rose up very quickly from -1 percent to its target level 2 percent (Figure 7).

However, our model shows that, in the medium run, if the Fed does not adjust the interest rate paid on reserves R^n, quantitative easing will cause deflation. Using the definition of the prime rate r_t^h in the previous section, we can rewrite (22) and (25) in the deterministic setup as:

$$\frac{1}{\widetilde{c}_t} = \widetilde{\beta}\frac{R_t^m}{\widetilde{c}_{t+1}\pi_{t+1}} + \gamma_t \tag{50}$$

$$\frac{1}{\widetilde{c}_t} = \widetilde{\beta}\frac{r_t^h}{\widetilde{c}_{t+1}} + \omega_t \tag{51}$$

From the two above equations, we can rewrite:

$$\frac{R_t^m}{\pi_{t+1}} = r_t^h + \frac{\widetilde{c}_{t+1}(\omega_t - \gamma_t)}{\widetilde{\beta}} \tag{52}$$

As the households will smooth their consumption and there is no big change in the liquidity premium of money as well as the shadow price of borrowing constraint, it can be said that the real rate return of money (R_t^m/π_{t+1}) will move in the same direction with the real prime rate r_t^h in the long run. When the central bank conducts the quantitative easing, the nominal interest rate on zero maturity deposits is equal to the interest rate paid on reserves, $R_t^m = R_t^f = R^n$. The tricky part is the real prime rate in our model as the bank loan is modeled different from the New Keynesian literature. As each period, households only pay a fraction of the loan they borrow before, bankers have to take into account the interest path in the future. In the perfect foresight setting, if banker knows that the federal funds rate will go up after 15 year, the loan rate they offer to households at the current moment already reflects partially that expectation. Figure 6a show that after the sharp decline, the real prime rate r_h^t slowly increases. As the R_t^m is fixed at the interest rate paid on reserves R^n, the inflation has to go down to satisfy the equation (52).

Another way of thinking why quantitative easing causes deflation in the medium run is to use the aggregate demand to explain. When the central bank suddenly purchases a large amount of assets from the bankers' balance sheet, the wealth is transferred from the households to bankers. What is the mechanism behind this transfer? Bankers hold zero maturity deposits on the liability side and hold real long-term assets (loan) on the asset side. So after the short-run effect of quantitative easing, the cost of liability side reduces substantially while the return on the asset side starts picking up as agents expect the interest will rise again in the future. In our assumption, the discount rate of banker is higher than the one of households, so aggregate demand will decline when wealth is transferred from households to bankers, resulting in the deflation in the medium run.

6.8 The interest rate paid on reserves as a monetary policy tool: $R^n \to R_t^n$

When the banking system is awash of reserves, the central bank can still control the federal funds rate by adjusting the interest rate paid on reserves. As we do not assume that R^n as a constant in this section, we put a subscript t into the notation to show that the central bank can change that. Before using the numerical result to weigh the effect of adjusting R_t^n, we ask two fundamental questions about using R_t^n as the policy tool.

6.8.1 Is adjusting the interbank rate by R_t^n similar to the one adjusted by the open market operation?

Even though both tools can change the federal funds rate, they are not exactly identical. When the central bank conducts the open market purchase, and the reserves requirement is still binding $\lambda > 0$, it not only reduces the federal funds rate but also increase the level of reserves, and therefore directly increases the level of money supply (m_t/R_t^m) in the economy through the constraint (4). On the other hand, until the central bank uses payoffs they get from holding the financial instruments on the asset side to pay the interest rate on reserves for bankers, adjusting the interbank rate by R_t^n does not change the level of reserves. It only changes the money supply indirectly through stimulating the credit flows between bankers and households.

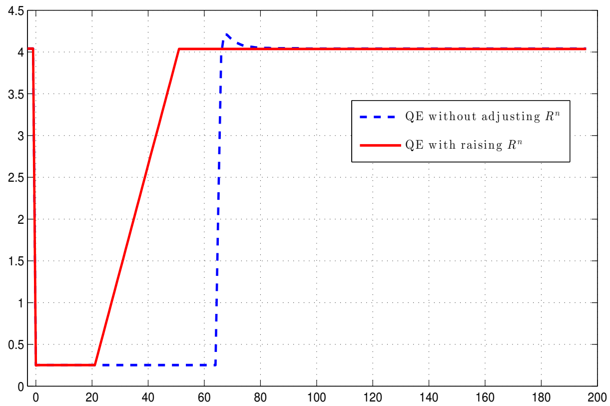
6.8.2 Do banker earn a lot of profit from this policy ?

Under the assumption of our model that banking market is perfectly competitive, the answer is no. When R_t^n goes up, the return on reserves increases. How do the bankers get reserves? The two most basic ways for bankers to have more reserves in our model is through either borrowing from the interbank market or receiving more zero maturity deposits. So we can think the interbank rate R_t^f and the zero maturity deposits rate R_t^m are the cost of obtaining more reserves. In the perfectly competitive market environment when banking system is awash of reserves, we have $R_t^f = R_t^m = R_t^n$, so raising the R_t^n does not transfer wealth from households to bankers as households also receive their fair share through the increase in R^m.

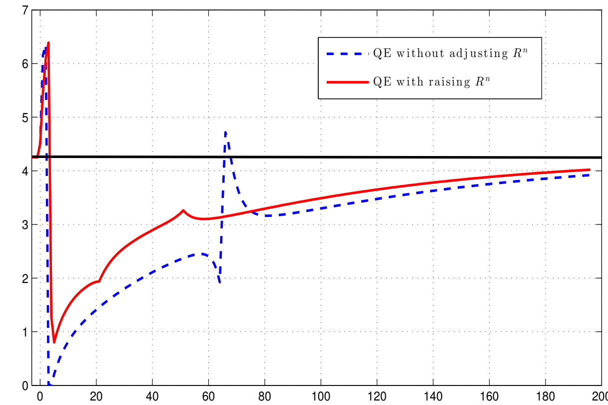
6.8.3 The effect of raising R_t^n after QE

In our last experiment in the unconventional monetary policy section, we know that, without adjusting the federal funds rate by using R_t^n, quantitative easing, even though pushes up inflation and output in the short run, might cause a period of deflation in the medium run as the federal funds rate is near zero for a too long time. In this section, we do one experiment that central bank will use the interest rate on reserves R_t^n to adjust the R_t^f after QE. We assume that after 20 periods (around 5 years) since the date of housing crisis ($t = 0$), the central bank will slowly increase R_t^n from 25 basis points ($t = 20$) to 414 basis points-the long run level ($t = 50$).

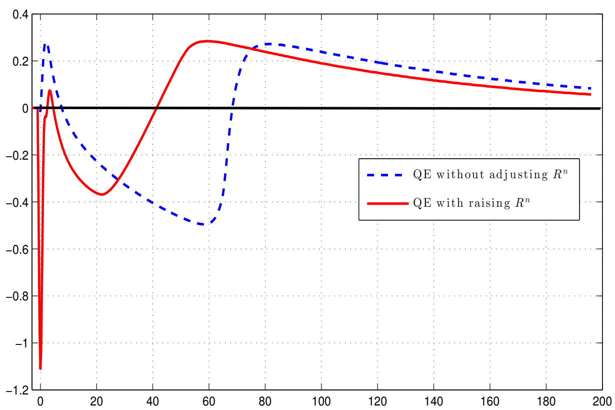
There is a trade-off between the short run and the long run when the central bank raises the interest paid



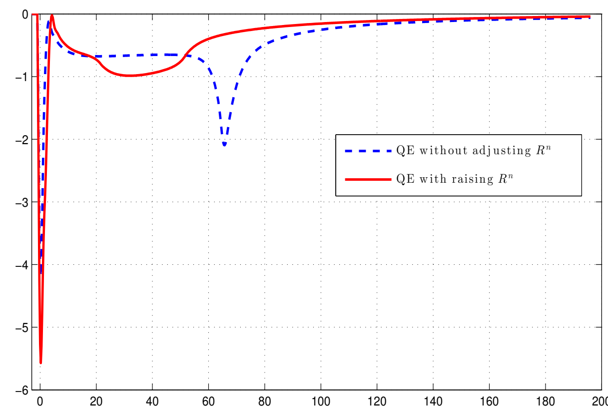
(a) Interbank Rate R^f



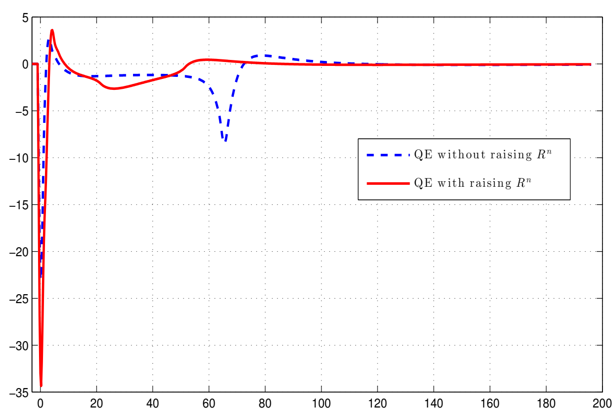
(b) Prime Rate r^h



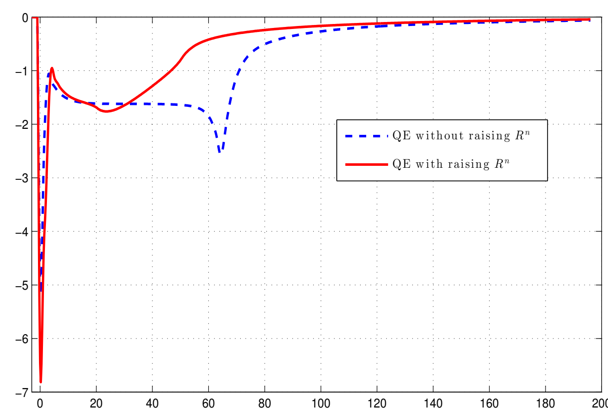
(c) Inflation π



(d) Ouput y



(e) Investment I



(f) Real money balance m

Figure 8: Comparison QE with and without raising R^n in the period $20th$

on reserves after conducting the quantitative easing. In the deterministic model, raising the R_t^n sooner will reduce the effect of the large scale asset purchase program in the short run. During the period of raising R_t^n, the economy will go through a slight recession period. However, in the long run period, both inflation and output are smoother than in the case of quantitative easing without adjusting R_t^n.

6.9 Raising money supply and federal funds rate at the same time. Why not ?

In the Keynesian theory, it is impossible to raise the federal fund rate and money supply at the same time. With electronic money, this task is possible and should be discussed thoroughly as it might be an effective policy to get out the zero lower bound. As electronic money, in the form of checkable deposits and saving deposits, earn the nominal interest rate the follows the federal funds rate, theoretically and practically, central bank can increase the interest rate paid on reserves and money supply at the same time.

For simplicity, assume that the Fed conducts the helicopter money by sending free checks to households and raises the interest rate on reserves simultaneously[5]. The balance sheet of the central bank, banks and private sector will look like Table 7.

The Fed		Banks		Public	
	Reserves:+M	Reserves:+M	Deposits: +M	Deposits:+M	Net worth: +M
	Net worth:-M				

Table 7: Helicopter Money

In our paper, the central banks hold the indexed government bonds on the asset side, so helicopter money will not create any troubles as the values of bonds will be adjusted to the inflation. We still assume that there is an unexpected housing price shock ($t = 0$), and an unexpected policy shock at $t = 1$ when central banks send free checks to everybody (only at $t = 1$. Simultaneously, the central bank raises R_t^n from 25 basis points to 350 basis points during 200 periods before returning back to 25 basis points.

For comparison with the previous case, we set the total amount of money drop as $\tau_1 = 0.02\bar{x}\bar{v}$. The new set of equations in equilibrium (for $t \geq 1$) will be:

$$\frac{R_t^n n_{t-1}}{\pi_t} + \tau_t = n_t + q_t(b_t^g - b_{t-1}^g) \tag{53}$$

[5]Legally, the Fed cannot conduct this policy directly. We should think the helicopter money as Money-Financed Fiscal Program.

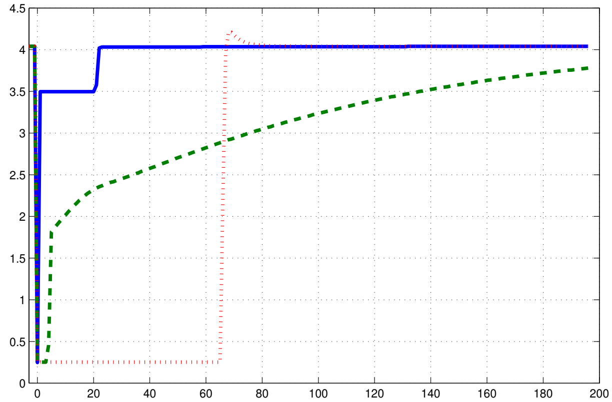
$$\frac{m_t}{R_t^m} = \frac{m_{t-1}}{\pi_t} + f\left(\frac{s_t}{b_{t-1}^h}\right) + \nu b_t^h + r_t s_t - \delta_b b_{t-1}^h + c_t - x_{t-1} r_t^x + d_t - b_{t-1}^g - (R_t^n - 1)\frac{n_{t-1}}{\pi} + \tau_t \tag{54}$$

$$n_t = \rho_n n_{t-1} + (1-\rho_n)\left(\bar{n} + (E_t\left[R_{t+1}^x\right] - \overline{R^x})\right) \tag{55}$$

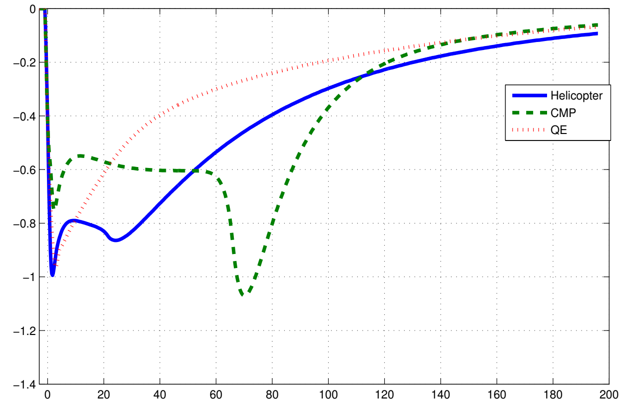
$$x_t = 1 \tag{56}$$

The dynamics of the economy is shown in the Figure 9. The inflation responds much stronger in the helicopter case as the balance sheet of the public expands. By raising the interest rate paid on reserves, the Fed can partially control the magnitude of the inflation by restricting the flows of credits.

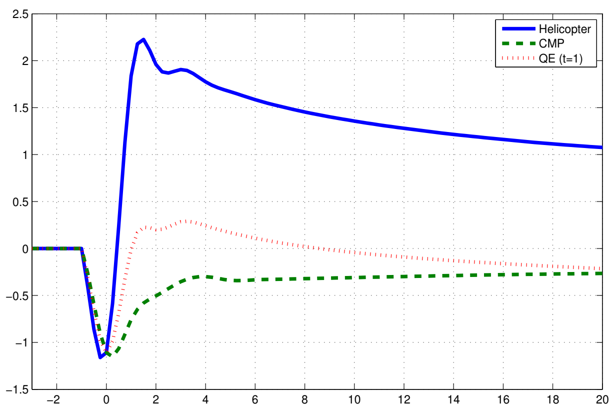
The essential message here is that money supply and the federal funds rate can be controlled by the Fed at the same time (at least in the short run), which is contradict with the Keynesian theory. Failing to model the whole banking sector with electronic money gives a incorrect picture about the relationship between money supply and the interest rate.



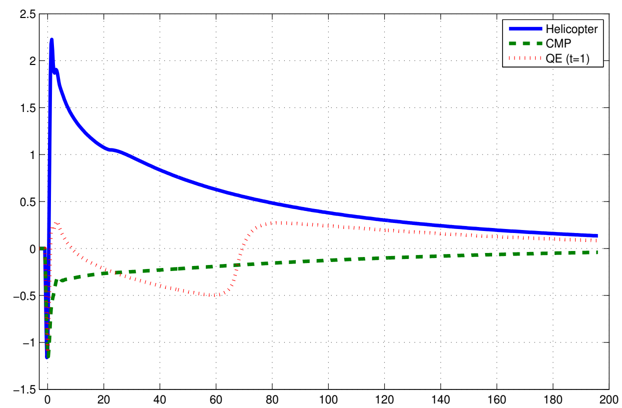
(a) Interbank Rate R^f



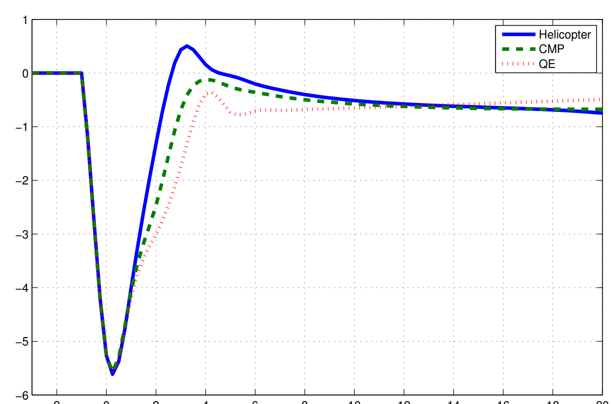
(b) Aggregate Consumption



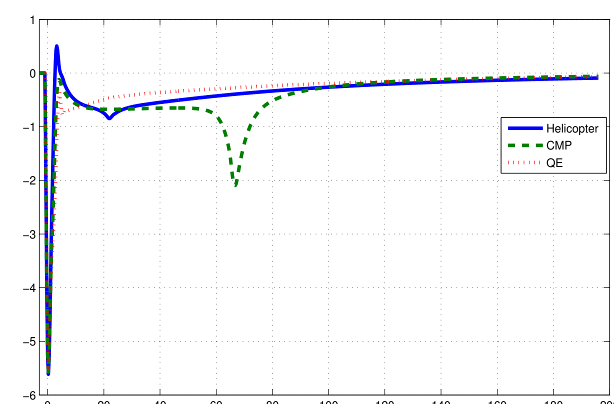
(c) Inflation π in SR



(d) Inflation π in LR



(e) Output y in SR



(f) Output y in LR

Figure 9: Compare Helicopter and raise IROR with previous cases

7 Conclusion

E-money, as an inside money issued by commercial banks, play a vital role to understand the modern monetary system. Our objective in this paper is to build a model linking the central bank's policy instruments (reserves level, interbank rate, interest rate paid on reserves) to the monetary aggregate (M1) and the real economic activities. Through the lens of our model, we contribute to the discussion about the Great Recession and the unconventional monetary policy.

The first important remark is that the financial crisis is extremely dangerous as the central bank might lose the control of money supply in the short run. A big negative shock to the asset side of banks' balance sheets will lead to the deleveraging process, where the conventional monetary policy, by only adjusting the interbank rate to the lower bound, might not be enough. The private sector will be in the liquidity constraint, leading to the sharp cut in investment and output.

We find that a quantitative easing program is very powerful to push the economy out of trouble in the short run. Through the asset price channel, it helps banks out of the capital constraint. Through the bank lending channel, it helps the private sectors get the credits with the low cost. It can push up inflation immediate in the short run.

However, quantitative easing is a tricky tool that might create deflation and the slow recovery of the economy in the medium run. They key mechanism for this deflation is the mismatch in the maturity between money and loan. If the public believes that interest rate will rise in the future, deflation will appear to balance the real rate return of money to the real rate of loan. Raising the interest rate paid on reserves is necessary to stabilize the economy in the long run. To avoid the negative effect from raising the interest rate, the central should simultaneously increase the money supply. This policy is impossible in the Keynesian theory, but is totally feasible in a new world where electronic money is the main form of payments.

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A Banking Data

Table 8: High Yield Checking Account (Source: Bankrate.com , Mar 2006)

Institution Name	APY%	Balance Cap ($)
Amegy Bank	0.20	25000
American Bank of Huntsville	0.99	15000
Arizona Bank & Trust	1.51	20000
Atlantic Coast Bank	0.02	20000
Avidia Bank	1.06	25000
Bank of Blue Valley	1.25	15000
Bank of the Wichitas	1.50	10000
BankGloucester	1.51	25000
BTH Bank	2.25	7000
Capital Bank	0.75	10000
Citizens Community Federal	2.00	25000
City National Bank	0.20	None
Community Bank of Pleasant Hill	1.50	10000
Community Bank of Raymore	1.50	10000
Coulee Bank	2.55	15000
Cross Keys Bank	2.05	10000
Farmers Bank & Trust	2.01	25000
First American Bank	1.00	15000
First Bank & Trust	1.01	25000
First Security Bank & Trust	1.05	15000
Heartland Bank	2.00	15000
Heritage Bank	1.26	25000
Jeff Davis Bank	3.25	10000
KANZA Bank	2.00	10000
Lee Bank	2.50	15000
Lee County Bank & Trust, N.A.	1.10	15000
Liberty Bank	1.01	10000
Meredith Village Savings Bank	0.30	25000
NorStates Bank	0.50	None
North Country Savings Bank	1.05	25000
Northpointe Bank	5.00	5000
Ouachita Independent Bank	3.01	15000
Premier Valley Bank	0.31	25000
Royal Banks of Missouri	1.85	24999
Security Bank	2.05	15000
The State Bank of Toledo	0.90	25000
Sutton Bank	1.77	25000
The Bank of Fayetteville	1.00	15000
The Provident Bank (Green Checking)	1.00	1000
The Provident Bank (Smart Checking)	1.51	15000
TriSummit Bank	1.11	25000
West Texas National Bank	1.26	25000
Westfield Bank	2.01	15000
Bank of America	0.02	50000
Citi Bank	0.03	50000
Well Fargo	0.05	50000
JP Morgan Chase	0.01	15000

B The Sparse Grid

We follow closely Brumm and Scheidegger (2015) to build the "Clenshaw-Curtis" sparse grid. We transform the domain of our state variables into $\Omega = [0,1]^d$, where d is the dimensionality of the problem.
Let $\vec{l} = (l_1, ..., l_d) \in \mathbb{N}$ and $\vec{i} = (i_1, ..., i_d) \in \mathbb{N}$ denote multi-indexes representing the grid refinement level and the spatial position of a d-dimensional grid point $\vec{x}_{\vec{l},\vec{i}}$. Using this notation, we can define the full grid $\Omega_{\vec{l}}$ on Ω with mesh size:

$$h_{\vec{l}} := (h_{l_1}, ..., h_{l_d}) = 2^{-\vec{l}} := \left(2^{-l_1}, ..., 2^{-l_d}\right) \tag{B.1}$$

and generic grid point:

$$\vec{x}_{\vec{l},\vec{i}} := (x_{l_1,x_1}, ..., x_{l_d,x_d}) \tag{B.2}$$

where

$$x_{l_t,i_t} := i_t 2^{-l_t}, \text{ with } i_t \in \{0, 1, ..., 2^{l_t}\}, \text{ and } t \in \{1, ..., d\}$$

Next, we describe about the basis functions. For each grid point $\vec{x}_{\vec{l},\vec{i}}$, there is an associated piecewise d-linear basis function $\phi_{\vec{l},\vec{i}}(\vec{x})$ defined as the product of the one-dimensional basis functions :

$$\phi_{\vec{l},\vec{i}}(\vec{x}) := \prod_{t=1}^{d} \phi^{C,C}_{l_t,i_t}(x_t) \tag{B.3}$$

where $\phi^{C,C}_{l_t,i_t}$ is defined as:

$$\phi^{C,C}_{l_t,i_t}(x_t) = \begin{cases} 1 & \text{if } l = 1 \text{ and } i = 1 \\ \begin{cases} 1 - 2x_t & \text{if } x_t \in [0, \frac{1}{2}] \\ 0 & \text{else} \end{cases} & \text{if } l = 2 \text{ and } i = 0 \\ \begin{cases} 2x_t - 1 & \text{if } x_t \in [\frac{1}{2}, 1] \\ 0 & \text{else} \end{cases} & \text{if } l = 2 \text{ and } i = 2 \\ \phi_{l_t,i_t}(x_t) & \text{else,} \end{cases}$$

and ϕ_{l_t,i_t} is the hat function such that:

$$\phi_{l_t,i_t}(z) = \begin{cases} 1 - \left| \dfrac{z - i_t.2^{-(l_t-1)}}{2^{-(l_t-1)}} \right|, & \text{if } z \in [(i_t-1).2^{-(l_t-1)}, (i_t+1).2^{-(l_t-1)}] \\ 0, & \text{else} \end{cases}$$

Now, we are ready to define the sparse grid informally. We want to choose a set of points in the grid, and as each point represents one basis function (both are denoted by the same pair $(\vec{l},\vec{i})$), it is equivalent to choose a set of basis functions to approximate our function. So assume we want to approximate the banker's consumption rule $c(.)$ as the function of state variables $\{n, b^g, m, b^h, K, \xi\}$, we can approximate $c(.)$ as the linear combination of our basis functions. The difficult question is, given the fixed number of grid points and number of basis functions, how to choose its location in an optimized way.

Given our refinement level n, Clenshaw-Curtis sparse grid will choose all points indexed by $(\vec{l},\vec{i})$ that satisfy the following two criteria:

(i) $\sum_{t=1}^{d} l_t \leq n + d - 1$

(ii) $i_t \in I_{l_t}$ for $1 \leq t \leq d$

where the set I_l is defined as:

$$I_l := \begin{cases} \{i : i = 1\}, & \text{if } l = 1 \\ \{i : 0 \leq i \leq 2,\ i \text{ even}\}, & \text{if } l = 2 \\ \{i : 1 \leq i \leq 2^{l-1} - 1,\ i \text{ odd}\}, & \text{else} \end{cases} \tag{B.4}$$

The more formal introduction to the sparse grid and its application in economics can be found in Malin, Krueger and Kubler (2011), Judd et al. (2014), Brumm and Scheidegger (2015).

In our paper, we build a sparse grid in the level 4, then adaptively add more grid points into the high curvature region. Basically, each point in the grid has two children in each dimension, so if the coefficient attached to the basis function is big (greater than 0.05 in our paper), we add the children of this grid point to refine this region. Brumm and Scheidegger (2015) provides a thorough introduction to the adaptive sparse grid and its application in economics.

C System of Equations in Equilibrium

Bankers:

$$\frac{1}{c_t} = \beta R_t^f E_t\left[\frac{1}{c_{t+1}\pi_{t+1}}\right] + \mu_t \tag{C.1}$$

$$\frac{1}{c_t} = \beta R_t^m E_t\left[\frac{1}{c_{t+1}\pi_{t+1}}\right] + \mu_t + \varphi\lambda_t \tag{C.2}$$

$$\frac{1}{c_t} = \beta E_t\left[\frac{R_{t+1}^x}{c_{t+1}\pi_{t+1}}\right] + (1-\kappa)\mu_t \tag{C.3}$$

$$\frac{1}{c_t} = \beta R^n E_t\left[\frac{1}{c_{t+1}\pi_{t+1}}\right] + \lambda_t + \mu_t \tag{C.4}$$

$$\frac{1}{c_t} = \beta E_t\left[\frac{1+q_{t+1}}{q_t c_{t+1}}\right] + \mu_t \tag{C.5}$$

$$\frac{n_{t-1}}{\pi_t} = n_t + q_t(b_t^g - b_{t-1}^g) \tag{C.6}$$

$$\frac{m_t}{R_t^m} = \frac{m_{t-1}}{\pi_t} + \nu b_t^h + r_t s_t - \delta_b b_{t-1}^h + c_t - b_{t-1}^g - q_t^h \overline{H} - (R^n - 1)\frac{n_{t-1}}{\pi_t} \tag{C.7}$$

$$0 \leq \lambda_t \perp \left(n_t - \varphi\frac{m_t}{R_t^m}\right) \geq 0 \tag{C.8}$$

$$0 \leq \mu_t \perp \left[n_t + q_t b_t^g + (1-\kappa)\left(b_t^h + H_t v_t\right) - \frac{m_t}{R_t^m}\right] \geq 0 \tag{C.9}$$

$$b_t^h = (1-\delta_b)b_{t-1}^h + s_t \tag{C.10}$$

$$\frac{1}{c_t}\left(\nu + r_t + \frac{f_t'}{b_{t-1}^h}\right) = E_t\left[\frac{\beta\delta_b}{c_{t+1}}\right] + E_t\left[\frac{\beta s_{t+1} f_{t+1}'}{c_{t+1}(b_t^h)^2}\right] + E_t\left[\frac{\beta(1-\delta_b)(r_{t+1}b_t^h + f_{t+1}')}{c_{t+1}b_t^h}\right] + (1-\kappa)\mu_t \tag{C.11}$$

Housing Sector:

$$i_t^h = H_{t-1} \tag{C.12}$$

$$R_t^x = \frac{[q_t^h i_t^h + H_t v_t]\pi_t}{v_{t-1}H_{t-1}} \tag{C.13}$$

Households:

$$\frac{1}{\widetilde{c}_t} - \gamma_t - \Lambda_t = 0 \tag{C.14}$$

$$\Lambda_t = \widetilde{\beta} R_t^m E_t\left[\frac{1}{\widetilde{c}_{t+1}\pi_{t+1}}\right] \tag{C.15}$$

$$\frac{q_t^h}{\widetilde{c}_t} = \frac{\xi_t}{\tilde{h}_t} + \widetilde{\beta}(1-\delta^h)E_t\left[\frac{q_{t+1}^h}{\widetilde{c}_{t+1}}\right] \tag{C.16}$$

$$\chi \widetilde{l}_t^{\eta+1} = (1-\alpha)\Lambda_t p_t^m y_t \tag{C.17}$$

$$\frac{r_t}{\widetilde{c}_t} = \widetilde{\beta}\delta_b E_t\left[\frac{1}{\widetilde{c}_{t+1}}\right] + \widetilde{\beta}(1-\delta_b)E_t\left[\frac{r_{t+1}}{\widetilde{c}_{t+1}}\right] + \omega_t \tag{C.18}$$

$$\frac{1}{\widetilde{c}_t} = \widetilde{\beta}(1-\delta)E_t\left[\frac{1}{\widetilde{c}_{t+1}}\right] + \widetilde{\beta}E_t\left[\frac{\alpha\Lambda_{t+1}p_{t+1}^m y_{t+1}}{K_t}\right] \tag{C.19}$$

$$0 \leq \omega_t \perp [\overline{b^h} - b_t^h] \geq 0 \tag{C.20}$$

Retail Firm:

$$1 - \iota\left(\pi_t - 1\right) + \iota\widetilde{\beta}E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\pi_{t+1}-1\right)\pi_{t+1}\frac{y_{t+1}}{y_t}\right] = (1-p_t^m)\theta \tag{C.21}$$

$$y_t = K_{t-1}^{\alpha} l_t^{1-\alpha} \tag{C.22}$$

Central bank:

$$R_t^f = \overline{R^f} + \phi_n\left(\frac{n_t - (1-\rho_n)\bar{n} - \rho_n n_{t-1}}{\bar{n}}\right) + \phi_\pi(\pi_t - \bar{\pi}) + \phi_y(\log(y_t^s) - \log(\overline{y^s})) \tag{C.23}$$

Market Clearing:

$$y_t = c_t + \widetilde{c}_t + I_t + \nu b_t^h + f\left(\frac{s_t}{b_{t-1}^h}\right) + \frac{\gamma}{2}\left(\pi_t - 1\right)^2 y_t \tag{C.24}$$

$$H_t = \overline{H} \tag{C.25}$$

$$h_t = \frac{\overline{H}}{\delta_h} \tag{C.26}$$

$$K_t = (1-\delta)K_{t-1} + I_t \tag{C.27}$$

$$\log(\xi_t) = (1-\rho_x)\log(\overline{\xi}) + \rho_\xi \log(\xi_{t-1}) + \varepsilon_t \tag{C.28}$$