Nowcasting Slovak GDP by a Small Dynamic Factor Model

Tóth, Peter

National Bank of Slovakia

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by

Peter Tóth[1]

Research Department, National Bank of Slovakia[2]

Abstract

The aim of this paper is to estimate a small dynamic factor model (DFM) for nowcasting GDP growth in Slovakia. The model predicts the developments of real activity based on monthly indicators, such as sales, employment, employers' health care contributions, export and foreign surveys. The forecast accuracy of the model prevails over naive models that ignore monthly data. This result holds especially on the shortest horizon of one quarter ahead and on the evaluation period including the crisis of 2008-2009. Thus we may conclude that our small DFM is a valuable indicator of business cycle turning points in Slovakia. Further, the model allows for frequent and automatic updates of the GDP forecast each time new monthly data becomes available. This makes it useful for institutions which monitor the developments of monthly indicators of real activity.

Keywords: dynamic factor model, real activity, short-term forecasting

JEL codes: C52, C53, E23, E27

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[2] Most of this paper was written while employed at the Institute for Financial Policy of the Ministry of Finance of the Slovak Republic. The model developed in this project had been implemented and used on a regular basis at the Institute under the name MRKVA (Model pRe Krátkodobý Výhľad ekonomickej Aktivity). Earlier versions of this text are available in Slovak in Tóth (2014a,b).

1. Introduction

National statistical offices publish quarterly national accounts data with a substantial delay. The first release of Slovak GDP is available after nine weeks, while an early estimate of it is published seven weeks after the end of each quarter. At the same time, numerous monthly indicators are released much sooner, i.e. immediately after the end of each month or, at most, six weeks later. Some of these are regarded as coincident indicators of GDP, which macroeconomic analysts at central banks, finance ministries or the private sector follow and comment on regularly. Apart from the expert view that monthly data are useful, several studies of GDP nowcasting[3] (see e.g. the review paper by Bańbura et al., 2013) have shown that timely information inherent in monthly indicators help reduce the out-of-sample forecast errors of GDP models. As a further advantage, the sequential publication of new monthly data allows updating of the GDP estimate on about a weekly basis. This includes updates well before the official flash estimate.

At the same time, the combination of mixed frequency data (quarterly and monthly) in the same model can be challenging. An additional issue, specific to the context of GDP nowcasting, emerges from ragged edges of data. This means uneven endpoints of time series due to differences in publication lags. Fortunately, dynamic factor models (DFM) are able to deal with those issues. In what follows we provide a brief non-technical description of factor models available in the literature, which are commonly grouped into three model generations.

The first generation of DFMs are also called strict or exact factor models[4]. In the context of GDP nowcasting, the DFM links output growth to the developments of a few monthly indicators and at the same time can handle mixed frequencies and ragged edges of data. The model is set up in state-space and is estimated via maximum likelihood on monthly frequency. The latent common factor is filtered using the Kalman filter. In this setup, GDP is observed only in one month of each quarter and it is treated as missing in the remaining two months. Missing values are typically filled in by random i.i.d. draws from the normal distribution, as suggested by Mariano and Murasawa (2003). Alternatively, GDP can be interpolated from quarterly to monthly frequency using standard statistical techniques.

[3] The term *nowcasting* refers to a situation, when the last GDP figure available at the time of producing the forecast refers to the preceding quarter according to the calendar.
[4] Appropriate estimation methods were suggested by Dempster et al. (1977), Shumway and Stoffer (1982), Watson and Engle (1983) and Stock and Watson (1989). Applications include Engle and Watson (1981), Mariano and Murasawa (2003), Auroba et al. (2009) and Camacho and Pérez-Quirós (2010, 2011).

As the main disadvantage of the first generation approach, the total number of indicators that can be included in the model is somewhat limited. The main reason is that the maximum likelihood method cannot reliably estimate a large number of parameters of such a model[5]. For this reason, a small set of indicators was also considered in the applications of Camacho and Pérez-Quirós (2010, 2011), who included less than ten variables. However, as the above authors show, using just a few good predictors of real activity can be a fruitful strategy. According to their findings, their small-sized model for the euro area beats most of its large-scale competitors in a real-time out-of-sample forecasting exercise. Further, the above authors also argue that oversampling monthly indicators of the same type (i.e. different sub-sectors or essentially very similar series) may increase the cross-correlation of idiosyncratic shocks across series. This could lead to biased estimates of the common factor, therefore including more variables is not necessarily a better approach.

The second generation is also known as approximate or static factor models[6]. Estimating such a model usually takes two steps. First, the common factors of the indicators are estimated via principal components analysis[7]. Second, the variable to be forecasted is linked to the components from the first step by a linear OLS equation. This approach was introduced in order to handle large datasets comprised of tens or hundreds of indicators. However empirical studies generally find that including more than 30-40 variables does not usually improve the model's forecasting accuracy[8]. Among the main disadvantages of this approach we can mention the static specification of factors and the way mixed frequencies and ragged edges of data are treated. The latter problem is typically solved by aggregation and realignment of the series, which may distort the relationship between GDP and monthly series.

The third generation of factor models aims to benefit from the advantages of the first two generations. This means that static factors are first estimated by principal components on the balanced subsample of the data. Next, the above initial values of factors are treated as observed when estimating the parameters of a large-scale DFM in state-space. The final step consists of iterating between smoothing the factors by the Kalman filter while taking the

[5] Poncela and Ruiz (2012) show via Monte Carlo simulations that including more variables decreases the uncertainty of estimating the common factor at the cost of increasing parameter uncertainty. For a sample size of 100 time periods, the authors find the optimal number of included variables is equal to roughly ten.

[6] The main references include Chamberlain and Rothschild (1983), Forni and Reichlin (1998), and Stock and Watson (2002a,b).

[7] An alternative method of approximating common factors relates to spectral analysis and aims to estimate dynamic principal components in the frequency domain of indicators instead of the time domain. This approach was developed in a series of papers by Forni et al. (2000, 2003, 2004, 2005).

[8] See for example Boivin and Ng (2006), Bańbura and Rünstler (2011) and Alvarez et al. (2012).

parameters from the previous iteration as given, and re-estimating the parameters while taking the factors from the previous iteration as given[9]. The advantages of the third generation can be summarised as the ability to handle large datasets of mixed frequencies and with uneven publication lags.

Apart from factor models, there are additional methods one could adopt for the purposes of forecasting GDP on short horizons. Popular and successful approaches include bridge equations[10], mixed data sampling[11] (MIDAS) and mixed-frequency vector autoregressions[12] (MF-VAR). As the present paper is an application of factor models, the above models are not reviewed here in more detail. An interested reader may refer to excellent survey articles on both factor models and other approaches to GDP forecasting[13].

In the present paper we apply a first generation small DFM following the work of Camacho and Pérez-Quirós (2010, 2011), with some minor modifications[14]. Our choice was motivated by the advantages of a state-space model, which can deal with missing values arising from publication lags and can combine different data frequencies in an elegant way. This feature is beneficial if a forecasting practitioner aims to update GDP nowcasts frequently. Given the set of variables we included, GDP nowcasts from our model can be updated on about a weekly basis each time new monthly data become available. This represents a clear time advantage compared to GDP flash estimates of the statistical office.

Another factor prompting us to pick a small model was the set of monthly indicators available in Slovakia. Note that large DFMs in the literature typically include at least 30-40 variables. In our case, compiling such a big dataset of unique indicators which are relevant for the developments of real activity would be quite challenging. However, as already noted above, oversampling indicators from the same class could lead to biased estimates of the common factor. Therefore, considering data limitations in Slovakia, a small DFM with less than ten variables included seems the most feasible approach.

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[9] This methodology was introduced by Giannone et al. (2008) and Doz et al. (2011, 2012). Subsequent applications include Angelini et al. (2011), Bańbura and Modugno (2014), Bańbura and Rünstler (2011), and Schumacher and Breitung (2008), for example.

[10] Pioneered by Ingentino and Trehan (1996), Runstler and Sedillot (2003), Baffigi et al. (2004) and Diron (2008).

[11] Introduced by Ghysels et al. (2004, 2007) and extended by Marcellino and Schumacher (2010). For a literature review on MIDAS see also Andreou et al. (2011).

[12] First proposed by Zadrozny (1990).

[13] See Bańbura et al. (2011, 2013), Barhoumi et al. (2014), Camacho et al. (2013) and Stock and Watson (2011).

[14] See section 2 for details.

Our paper is the first to estimate a first-generation small DFM on Slovak data. Unlike some of the related studies, we use historical versions of the GDP series in our out-of-sample forecast evaluation. This makes the evaluation more realistic. Similar papers from Slovakia and the Central and Eastern European (CEE) region have been published before, however those are based on different types of factor models. Huček et al. (2015) and Kľúčik and Juriová (2010) use methods similar to the second generation of DFM using Slovak data. From neighbouring countries, with comparable data limitations to the Slovak case, we can mention Arnoštová et al. (2011), Franta et al. (2014), Rusnák (2016) using Czech data, a Slovenian study by Radovan (2017) and a Latvian paper by Bessonovs (2015)[15]. Cross-country studies from the CEE region include Feldkircher et al. (2015), Havrlant et al. (2016) and Rünstler et al. (2009). The above listed authors mostly estimate second or third generation large scale factor models, as well as various non-factor models. They generally find that incorporating information from monthly data in the GDP forecast improves the out-of-sample forecasting accuracy compared to benchmark models neglecting monthly data. This result is also in line with our findings.

The paper is organised as follows. The next section specifies the small dynamic factor model. The third section describes the dataset used for estimation. Section 4 discusses estimation issues. Section 5 presents estimation results. The sixth section evaluates the accuracies of out-of-sample forecasts by the DFM and naive benchmarks. The final section summarises the main findings.

2. A Small Dynamic Factor Model

Our small DFM is estimated on monthly data frequency. It links the quarter-on-quarter growth rate of GDP (y_{it}) to month-on-month growth rates of indicators i (x_{it}), where y_{it} is observed only in the third month of each quarter. All time series considered are seasonally adjusted and are stationary according to the ADF test after their transformation to growth rates. To shrink the number of parameters to be estimated, it is common in the related literature to normalise all time series to have a zero mean and unit variance. We also adhere to the latter approach. As regards the structure of the model, let us assume that the developments

[15] Similar applications to other emerging markets include Kabundi et al. (2016) for South Africa, Liu et al. (2011) for Latin American countries, Modugno et al. (2016) for Turkey, Porshakov et al. (2016) for Russia, Wang et al. (2009) and Yiu and Chow (2011) for China.

of y_{it} and x_{it} are driven by a common monthly factor f_t, which is an unobserved variable. Next we can define our model in a state-space form as follows:

$$x_{it} = \alpha_i f_{t+K_i} + \varepsilon_{it} \quad (1)$$

$$y_t = \beta f_t + \omega_t \quad (2)$$

$$f_t = \varphi f_{t-1} + u_t \quad (3)$$

, where (1) and (2) are signal equations of observed variables. Note the time index K_i in equation (1), which defines the monthly lead of indicator x_{it} ahead of the common factor and output growth. The relationship expressed in (3) is the state equation and defines the dynamics of the unobserved state variable f_t. The normally distributed error terms ε_{it}, ω_t and u_t have zero means and variances $\sigma^2_{\varepsilon i}$, σ^2_{ω} and σ^2_{u} respectively. The error terms may be autocorrelated and weakly cross-correlated, but are assumed to be uncorrelated with factor f_t. The dynamic factor model defined by (1)-(3) can be understood as the decomposition of the variance of x_{it} and y_{it} into a common and an idiosyncratic component. The model can be extended to include N monthly indicators, when index i of x_{it} ranges from $1,..,N$.

A standard specification of a small dynamic factor model in the literature differs slightly from ours. This concerns mainly equations (1) and (2), where typically the specification of Mariano and Murasawa (2003) is used. The authors mentioned specify the particular equations as:

$$x_{it} = \alpha_i f_t + \varepsilon_{it} \quad (4)$$

$$y_t = \beta(f_t + 2f_{t-1} + 3f_{t-2} + 2f_{t-3} + f_{t-4}) + \omega_t \quad (5)$$

, which de facto predetermines the partial correlation of y_t with all five lags of x_{it-L} ($i=1,..N$ and $L=0,..,4$) at the same time. However, our pre-screening of indicators x_{it} suggested a statistically significant partial correlation between x_{it} and y_t only for one or two lags. So for simplicity we further assumed only one lag of f_t to enter equations (1) and (2). In this case it suffices to set time index K_i appropriately in equation (1) and skip lagged values of f_t in (5).

3. Data

In the case of GDP we used chain-linked volumes seasonally adjusted by the Statistical Office and considered the vintage published in September 2013 as the last one. We took the quarter-on-quarter growth rates of the series to reach stationarity according to the ADF test performed on the whole sample. Finally we normalised the growth rate to have zero mean and unit variance. Monthly indicators were downloaded on the 6th of November in 2013 immediately after the new release of retail sales data. Initially we considered a full set of 21 time series, where we took the seasonally adjusted versions of the data provided by the publishing institutions whenever possible[16]. Next, we transformed the series to month-on-month growth rates and normalised them to have zero mean and unit variance. All these monthly growth rates were found to be stationary according to the ADF test performed on the whole sample.

As regards monthly variables, we focused on six data categories, which are also similar to those favoured by Camacho and Pérez-Quirós (2010, 2011). First, we included real activity indicators, such as indexes of production (industry, manufacturing and construction) and real sales (retail trade, car sales, industry, manufacturing and construction). Second, from labour market indicators we added the series of employment in selected industries, flows to employment and free vacancies. The third category was represented by employers' health care contributions, which approximate household income. The fifth group relates to international trade and is comprised of monthly exports, imports and the producer price index (PPI) in exporting industries. The final group is associated with surveys, such as the economic sentiment indicator (ESI) of Slovakia and other foreign soft indicators from surveys (ESI of the Eurozone and Germany, the IFO index from Germany, the Eurozone's Purchasing Managers Index and the ZEW institute's survey of economic conditions in Germany).

When choosing from similar alternatives on the list of indicators, first we tried to focus on more aggregated versions (e.g. sales in total industry rather than the manufacturing sub-sector). By this we aimed to minimise the impact of volatility from idiosyncratic shocks specific to a smaller sub-sector of the economy, which would make it more difficult to identify the common factor. Second, we decided to omit production indexes as the weights of the volume index before and after 2008 were not consistent. Bridging the two versions of the series would be possible only under the assumption of constant weights of industry sub-sectors. However, the reason why the Slovak Statistical Office revised those weights was to reflect the latest trends in the relative developments of industrial subsectors.

[16] Seasonally adjusted versions were not available for the producer price index (PPI) in exporting industries and health care contributions of employers. In these two cases we used the X12 method for seasonal adjustment.

In case of labour market variables we only considered indicators related to employment. A potentially useful indicator left out by this restriction is the unemployment rate. However we preferred not to bring in additional volatility originating from changes in labour force participation. Further, we included health care contributions due to the short publication lag of this indicator in comparison with other proxies of household income.

Further, the advantage of survey-based soft indicators lies in their timeliness, as they are published mostly by the end of the month in which the survey is conducted. Early availability however comes at the cost of somewhat higher volatility of these series. Finally, we totally excluded price indexes (except the PPI of exporters), as our simple model would not be able to differentiate between supply and demand shocks to prices, each having the opposite effect on output. We also ignored financial variables, such as exchange rates, interest rates, asset prices and commodity prices. Their main disadvantage is their volatility and merely indirect connection to real activity. Similar considerations were made by Camacho and Pérez-Quirós (2010, 2011) as well.

In Table 1 below we summarise the full list of monthly variables considered for inclusion in the DFM. The third column shows the approximate day of the month in which new data is typically released for the particular series. The fourth column reports the month to which the new data point corresponds, where *m* is the month in which the release occurs and, for example, *m-1* is the month preceding the month of the release. The last column reports data sources. Data are generally downloadable from the websites of the respective institutions. The only exception is employers' health care contributions, which are available upon request from the Institute for Financial Policy of the Ministry of Finance of the Slovak Republic.

Table 1 – List of monthly indicators

	Monthly indicator	Category	Approx. release day	Month	Avail. from	Source*
1	Index of production – industry	production	10.	m-2	2008 m01	SO SR
2	Index of production – manufact.	production	10.	m-2	2008 m01	SO SR
3	Index of production – construct.	production	10.	m-2	1998 m01	SO SR
4	Retail sales	sales	4.	m-2	2000 m01	SO SR
5	Car sales	sales	4.	m-2	2000 m01	SO SR
6	Sales in industry + construct.	sales	11.	m-2	2000 m01	SO SR
7	Sales in industry	sales	11.	m-2	2000 m01	SO SR
8	Sales in manuf.	sales	11.	m-2	2000 m01	SO SR
9	Employment in selected industries	labour market	11.	m-2	2002 m01	SO SR

10	Flows to employment	labour market	20.	m-1	2004 m01	SO SR
11	Free vacancies	labour market	20.	m-1	2002 m01	SO SR
12	Export	foreign trade	9.	m-2	1998 m01	SO SR
13	Import	foreign trade	11.	m-2	1998 m01	SO SR
14	PPI in exporting industries	foreign trade	28.	m-1	2003 m01	SO SR
15	Health c. contrib. of employers	income	30.	m-1	2000 m01	MF SR
16	ESI Slovakia	surveys	28.	m	1998 m01	Eurostat
17	ESI Eurozone	surveys	30.	m	1998 m01	Eurostat
18	ESI Germany	surveys	30.	m	1998 m01	Eurostat
19	IFO Germany	surveys	25.	m	1998 m01	CESifo
20	PMI Eurozone	surveys	23.	m	1998 m07	Markit
21	ZEW Germany	surveys	17.	m	1998 m01	ZEW

Notes: * SOSR – Statistical office of the Slovak Republic, MF SR – Ministry of Finance of the Slovak Republic, CESifo – Center for Economic Studies - Ifo Institute Munich, Markit – Markit Economics Financial Information Services Ltd., ZEW – Centre for European Economic Research, Mannheim (Germany).

Source: institutions listed in the last column of the table and the author's own considerations.

As suggested by previous applications of the small DFM (Camacho and Pérez-Quirós 2010, 2011) and found by the Monte Carlo study of Poncela and Ruiz (2012), the maximum number of variables that can be included in the model is limited to about ten. Otherwise the set of parameters to be estimated grows too large and maximum likelihood is not able to estimate the system. In our case this means we need to reduce the list of indicators in Table 1 by more than half. To achieve this, we picked one or two alternatives from each category listed in column 2 of Table 1, mostly based on correlations with GDP growth.

The final selection of variables includes (1.) retail sales, which approximates domestic consumption according to the expenditure approach to measuring GDP. The next variable, (2.) sales in industry and construction approximate GDP from the production point of view. Its advantage compared to other sales sub-aggregates stems from its broad coverage of a large set of subsectors. The third series, (3.) employment in selected industries, represents an important factor of production and reflects developments on the labour market. Employment was preferred to flows into employment and free vacancies because it is a somewhat less volatile alternative and is better correlated with GDP. The fourth variable, (4.) exports, covers foreign demand for Slovak output, which complements domestic demand in the expenditure approach to measuring GDP. We favoured exports to imports and exporters' PPI due to the higher correlation of exports with GDP. The fifth indicator, (5.) health care contributions of employers, mimics the income approach to measuring output and is included to obtain a more heterogeneous indicator base for estimating the common factor.

The choice was somewhat more difficult in case of soft indicators, as their correlation with GDP was high in all cases. Furthermore, their correlations with GDP were in some cases significant for several monthly lags at the same time, especially for ESI and IFO. In contrast, the eurozone PMI and the German ZEW index were significantly correlated with GDP only in one month. We found that the above correlation pattern, i.e. concentration in one month, is similar to the first five groups of “hard” data. Therefore, in order to keep consistency with the rest of the variables in the model, we included the Eurozone PMI. The latter indicator slightly dominated the ZEW index in terms of correlation with GDP. Eurozone PMI can also be viewed as superior to ZEW due to its broader geographical coverage.

4. Estimation

State-space models such as (1)-(3) are commonly estimated by the maximum likelihood method. The unobserved state f_t is subsequently filtered by the Kalman filter. For this approach one needs to set the initial value of f_t in period $t=0$ and specify starting values for parameters α_i, β, φ, $\sigma^2_{\varepsilon i}$, σ^2_{ω} and σ^2_u. A typical assumption are diffuse priors, which is to set zeros for all starting values. However, in the case of more complicated model structures or small sample sizes one can assist the estimation process by setting somewhat more informative priors.

For the initial state of the factor we assumed a diffuse prior meaning $f_{t=0} = 0$ and $u_{t=0} = 0$. Starting values for α_i, β a φ were set to 0.5, since if considering x_{it} and y_t transformed to normalised values, we expected the estimates to come from the interval of 0 to 1. Next, we were forced to calibrate the starting values for variances $\sigma^2_{\varepsilon i}$, σ^2_{ω} and σ^2_{u}, as their estimation failed even with starting values. During calibration we reflected information from the Hodrick-Prescott (HP) filter as a rule of thumb. Namely, we computed the variances of the gaps of x_{it} and y_t, where the gap comes from the HP filter with parameter $\lambda=1$[17]. In most cases we used the above HP gap variances in the calibration (see Table 2), except for the variance of the idiosyncratic term of x_6, i.e. the Eurozone PMI index. Here we significantly increased the calibrated variance due to the high volatility of this monthly indicator. Next, we also slightly increased the calibrated variance of the idiosyncratic term in (2), σ^2_{ω}, so that coefficient β turns less than one as a result. Finally, as for lags K_i in the last columns of Table

[17] Parameter λ was set to 1 judgementally with the aim to filter some of the volatility and to keep only the main trends in each series.

2, we slightly modified the lags suggested by correlations between x_{it} and y_t in most cases to keep α_i in the range from 0 to 1.

Table 2 – Calibrated parameters

Equation		Calibrated value		implied by HP gaps	Lag K'_i (correlations)	Lag K_i (calibrated)
Retail sales	x_1	$\sigma^2_{\varepsilon 1}$	0.6	0.6	2	0
Sales in indust. & constr.	x_2	$\sigma^2_{\varepsilon 2}$	0.4	0.4	4	2
Employment in sel. ind.	x_3	$\sigma^2_{\varepsilon 3}$	0.3	0.3	0	0
Export	x_4	$\sigma^2_{\varepsilon 4}$	0.6	0.6	4	2
Health care contributions	x_5	$\sigma^2_{\varepsilon 5}$	0.7	0.7	2	0
Eurozone PMI	x_6	$\sigma^2_{\varepsilon 6}$	1.5	0.3	4	3
GDP	y_t	σ^2_{ω}	0.2	0.1	-	-
Factor	f_t	σ^2_{u}	0.6	-	-	-

Source: the author's own calculations

5. Results

This section summarises estimation results of the state-space model defined in (1)-(3). Table 3 below reports parameter estimates. The model was estimated on the time interval from the beginning of 2002 to September 2013 and data was downloaded onn November 6, 2013. All parameter estimates in Table 3 are statistically significant at 5% and fall into the expected range of 0 to 1. Not surprisingly, the largest partial correlation with factor f_t is found in case of the GDP series. In contrast, the partial correlation of the factor with monthly indicators is somewhat diminished for employment, retail sales and sales in industry and construction, while the same coefficient is quite small for the Eurozone PMI and health care contributions (below 0.2). This means that idiosyncratic variance seems to dominate their evolution and so the common factor shows a diminished explanatory power for the latter two variables.

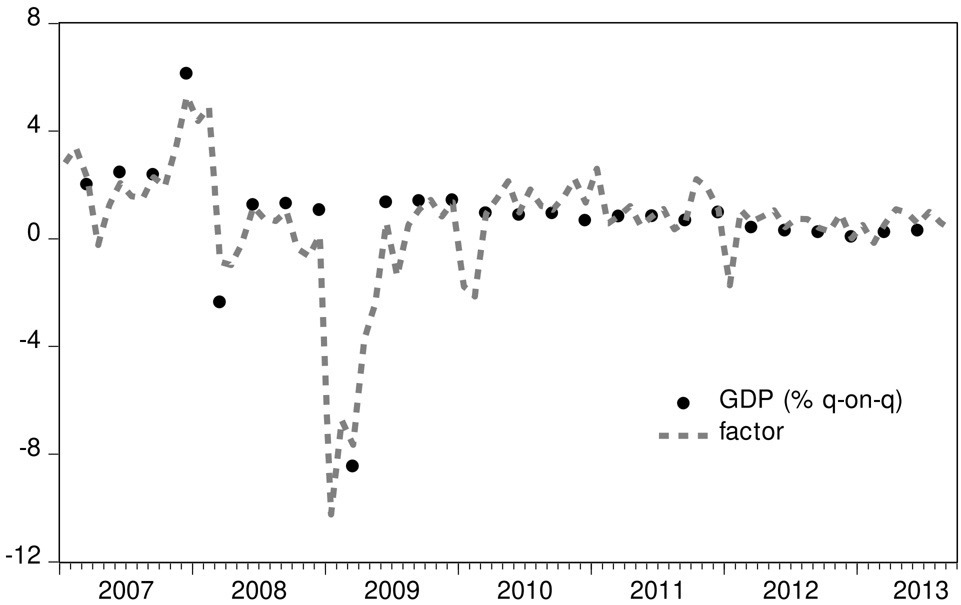
Table 3 – Estimates

Equation			Coefficient		Standard error
1.	Retail sales	x_1	α_1	0.53	(0.04)***
2.	Sales in industry & constr.	x_2	α_2	0.36	(0.04)***
3.	Employment in sel. indust.	x_3	α_3	0.81	(0.04)***
4.	Export	x_4	α_4	0.32	(0.05)***
5.	Health care contributions	x_5	α_5	0.12	(0.04)***
6.	Eurozone PMI	x_6	α_6	0.18	(0.07)**
7.	GDP	y_t	β	0.99	(0.05)***
8.	Factor	f_t	φ	0.63	(0.06)***
Number of observations:		141			
Estimation interval:		2002m01 : 2013m09			

Notes: *, **, *** denote statistical significance at 10%, 5% and 1%.

Source: the author's own calculations.

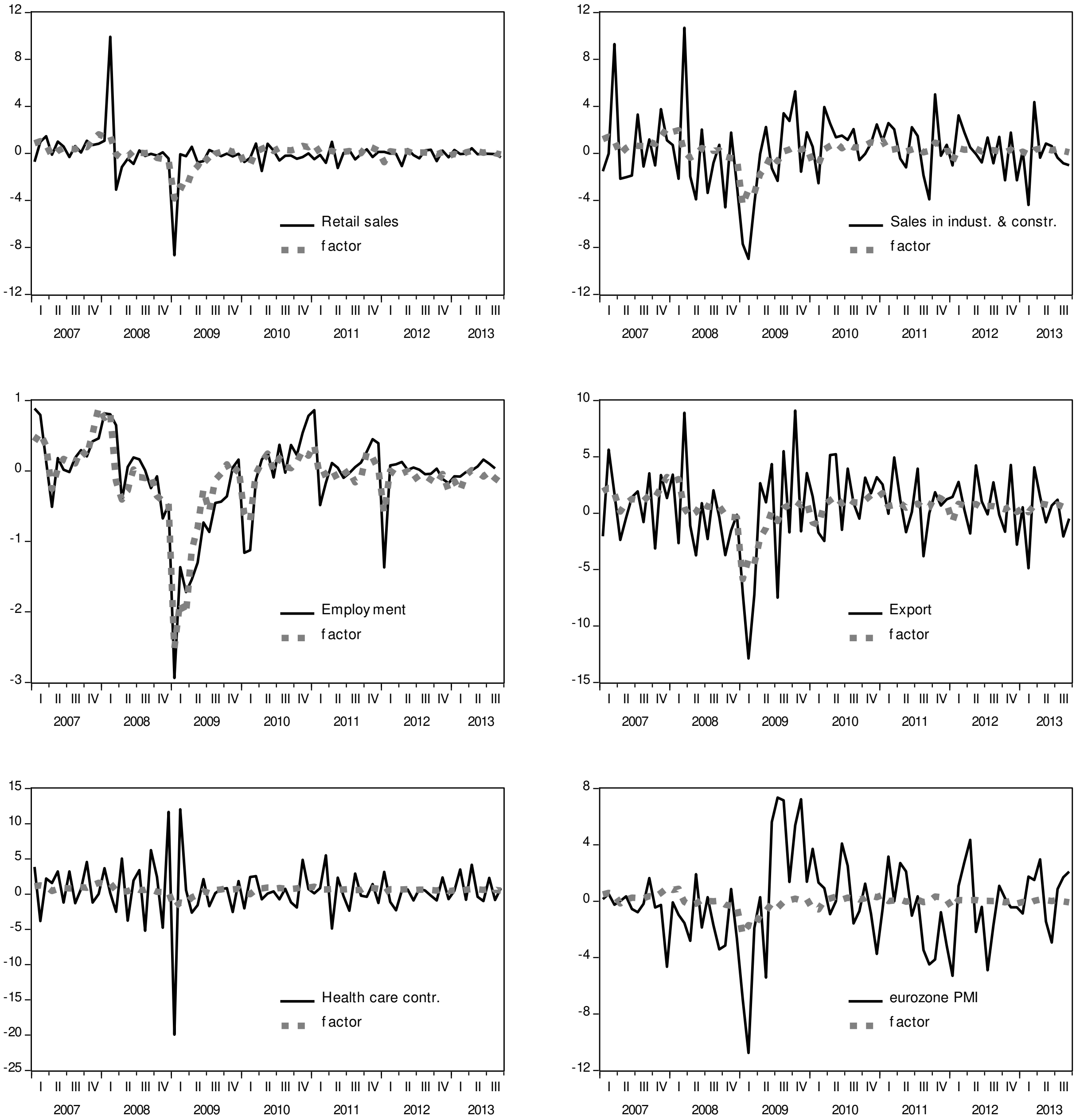
Figure 1: GDP and the common factor (%, q-o-q growth)



Source: Statistical Office of the Slovak Republic and the author's own calculations

Figure 1 above depicts quarterly GDP growth together with the evolution of the monthly factor. GDP growth is treated as observed in the third month of each quarter and is assumed to be unobserved otherwise. In contrast, the estimated monthly factor is continuously observed in every month. In this figure, the factor is expressed as a contribution to GDP growth, meaning the product βf_t. In Figure 2 we show a similar comparison for the six monthly indicators included in the model. This means the respective subfigures illustrate a monthly indicator and its component that is shared with the rest of the indicators including GDP growth. The common component is again understood as the product $\alpha_i f_{t+Ki}$. Looking at Figure 2, we can infer that important structural breaks, such as the impact of the global financial crisis in 2008-2009, are apparent in all monthly indicators as well as in the development of the common factor. On the contrary, the higher idiosyncratic volatility characteristic for some series (especially the Eurozone PMI and health care contributions) does not enter their sub-component represented by the common factor.

Figure 2 – Monthly indicators and the common factor (%, m-o-m growth)



Source: Data publishing institutions listed in Table 1 and the author's own calculations

6. Evaluation of Forecast Accuracies

In order to evaluate the accuracy of our small DFM we performed a so-called out-of-sample forecasting exercise in pseudo-real time. First we restricted the end of the estimation interval to the end of 2007, corresponding with the pre-crisis period. Next we gradually added new observations of monthly data and GDP mimicking the publication calendar of the series. In each step we re-estimated the model and recorded its out-of-sample forecast for GDP growth on the horizon of up to three quarters ahead. For each GDP quarter from 2008 to 2.Q of 2013 we made 12 forecast simulations. This means 4 estimates for each month within the quarter adding up to 264 forecasts in total.

In the above exercise we used historical versions of the GDP series, which were available at the time corresponding to each simulation. In other words, the last observation of GDP in each of the data vintages was the first release of that quarter by the statistical office. Forecast errors of the model were also computed with respect to the first release. Monthly data were used in their last available versions as of November 2013, as we did not have a vintage dataset at our disposal for those variables. However, monthly data are typically not subject to such significant revisions as quarterly national accounts.

After recording forecast errors of the DFM on the horizon of up to three quarters ahead we compared them to the forecast accuracy of so-called naive univariate benchmark models. Following the related literature[18], we used the AR(1) and random walk models of GDP for this purpose. The above benchmarks became standard in the nowcasting literature, as they are simple to estimate and at the same time difficult to beat by other models due to the notable persistence in GDP growth.

To compare the performance of the models mentioned, we looked at out-of-sample root-mean-squared errors (RMSE) over two evaluation intervals (see Table 4 below). The first interval, reported in the left part of the table, focused on a less volatile period of 2010-2013 with a smooth GDP growth series. The second interval was somewhat wider as it included the crisis year of 2008 and reached to the end of our sample in 2013 (right half of the table). RMSEs of the DFM by the three horizons in columns of Table 4 are simple average RMSEs of twelve forecast updates. The twelve updates in total come from four weekly updates in each of the three months of a quarter.

[18] See for example the survey articles by Bańbura et al. (2011, 2013) and Stock and Watson (2011).

Table 4 – Root Mean Squared Errors (RMSE) of out-of-sample forecasts

Interval 2010-2013

RMSE	+1Q	+2Q	+3Q
AR(1)	0.42	0.40	0.41
RW	0.38	0.37	0.53
DFM	0.36	0.39	0.41

Interval 2008-2013

RMSE	+1Q	+2Q	+3Q
AR(1)	2.74	2.77	2.76
RW	4.12	3.94	4.09
DFM	2.31	2.71	2.74

Source: the author’s own calculations.

Results in Table 4 above imply that the DFM is more accurate than its benchmarks one quarter ahead in both evaluation intervals. This result holds especially in the wider period including the crisis. However, on longer horizons of two or three quarters ahead we must note that forecast errors differ only slightly, while the DFM performs slightly better than its alternatives in most cases. Hence we may conclude that the DFM seems especially valuable on the shortest horizon and around business cycle turning points. The latter result and also the finding that models using monthly data tend to outperform simple univariate models is in line with the conclusions of related studies[19].

7. Conclusion

This paper is the first to estimate a small dynamic factor model for nowcasting GDP growth in Slovakia. Unlike some of the related studies from Slovakia and the CEE region, we use historical vintages of the GDP series in our out-of-sample forecast evaluation. This makes the evaluation more realistic. Our findings indicate that the DFM tends to outperform its univariate benchmarks in forecast precision. This result holds especially on the shortest horizon of one quarter ahead and on the evaluation period including the crisis of 2008-2009. The above finding is consistent with other related studies of GDP nowcasting. Hence we may conclude that our small DFM incorporating monthly data is a valuable indicator of business cycle turning points in Slovakia. As its further advantages, the DFM allows for frequent updates of the forecast that take into account new data releases in an automatic way. These features make it suitable for institutions, which monitor monthly indicators of the real economy on a regular basis.

[19] Note that a more detailed quantitative comparison of our results with other studies from Slovakia is not feasible. This is because Feldkircher et al. (2015), Huček et al. (2015) and Kľúčik and Juriová (2010) did not use vintage data of GDP and considered different evaluation periods. Quantitative results from other countries are not directly comparable for similar reasons.

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