Licensing contracts and the number of licenses under screening

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Abstract

This paper examines the licensing of an innovation—by a patent holder to one or more users—when the innovation's value (high or low) is known, after the contract is signed, by each user. In this setup, we analyze the patent holder's joint decision concerning the number of licenses and the type of contracts. Our first main finding is that, depending on how uncertain is the efficiency of users exploiting the innovation, both shut-down contracts and screening contracts can emerge in equilibrium. Second, shut-down contracts amount to fixed fees under exclusive licensing but are two-part contracts under non-exclusive licensing. Third, there is distorted production at the bottom of the innovation value's distribution under exclusive licensing as well as distortion at both the bottom and the top of that distribution under non-exclusive licensing. Fourth, asymmetric information favors the latter (i.e., issuing multiple licenses) *except* when the patent holder uses a screening contract, since then the need to distort production at both the bottom and the top renders non-exclusive licensing less profitable. Our final result is that the number of licenses issued by the patent holder is more likely to maximize aggregate surplus under asymmetric information than under symmetric information.

Keywords: exclusive and non-exclusive licensing, symmetric and asymmetric information, screening, fixed-fee and two-part contracts, welfare analysis

JEL classification: D43, D82, L24

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1. Introduction

The term *licensing* refers to transferring, in exchange for monetary compensation, the rights to commercialize a patent holder's technology. The possibility of technology transfer increases the incentives of inventors, and it enables innovations to be transformed into a new process or new product. Licensing has become more pronounced during the last few decades, as most firms in all sectors report the number of their licensing deals (as well as their licensing revenues) have increased over time. Cohen et al. (2000) reference the Carnegie Mellon Survey on Industrial R&D and report that, between 1983 and 1995, US patent awards grew by 78%; they also find that, in ten different industries, at least 40% of survey respondents specifically identify licensing revenues as a motive for patenting. Similarly, Gallardo et al. (2016) document a sharp increase in the number of patented fruit varieties developed by breeding programs at public universities in the United States. Finally, studies funded by the OECD (OECD, 2009) have gathered evidence confirming the greater volume and value of patent licensing in recent years—a phenomenon that the OECD finds is related to broad changes in globalization, strengthened market competition, and modes of innovation.

A central aim of the licensing literature is to identify contracts that are optimal from the patent owner's perspective. Factors that influence contract terms include whether the licensor is an outsider or instead a competitor (in the same product market) of the licensee, whether imitation is easy or difficult, whether patent protection is enforceable or imperfect, whether the innovation is drastic or merely incremental, and whether or not information asymmetries exist with regard to the patent's innovation value.

In its study of asymmetric information, the extant licensing literature focuses mainly on examining—conditional on only a single license being granted—the fixed fees and royalties that a patent holder can use as signaling or screening devices (Beggs, 1992; Macho-Stadler et al., 1996; Poddar and Sinha, 2002; Sen, 2005). Yet it is crucial to explore the outcomes resulting from multiple downstream licensees, because in that case the patent holder gains an additional signaling/screening tool: the number of licenses granted (Wu and Peng, 2015). And because the number of licenses sold is endogenously determined in an optimal licensing scheme, it follows that this instrument could affect the use of royalties and fixed fees (i.e., as a signaling or screening device). This topic is necessarily overlooked by research that fails to incorporate the patent holder's choice of how many licenses to grant. We remark that the possibility of granting several licenses is a real-world phenomenon; a patent holder must not only determine the licensing contract terms but also identify the buyers to whom licenses are sold. Li and Wang (2010) document several cases that illustrate the convention of using both exclusive and non-exclusive contracts, and a 2001 survey by the Association of University Technology Managers found a nearly even split between the number of exclusive versus non-exclusive licenses (Caballero-Sanz et al., 2005).

This paper jointly examines optimal contract design and the optimal number of licenses to grant, where in both cases "optimal" is defined from the patent owner's perspective. Thus we consider an inventor that owns an innovation whose cost of development is sunk. The inventor has obtained a patent, so the innovative technology cannot be legally replicated. The technology can be employed as an input to produce a homogeneous final good, but the patent owner has no production capacity and is therefore obliged to license the patent in exchange for monetary compensation. We assume that many firms—if given access to the technology—do have the needed production capacity. In what follows, *patent holder* refers to the inventor and *users* are the firms seeking the right to utilize its patent. In our setup, there can be either one user (in which case a monopoly market arises) or two users (in which case those users compete against each other as Cournot players).

Each user can be either efficient or inefficient at exploiting the innovation. An efficient or "good" user is one that has zero marginal cost when transforming the innovation into the final good; an inefficient or "bad" user is one that does incur a positive marginal cost. Since only the users—and not the patent holder—know their respective marginal costs, it follows that both the number of licenses and the form of licensing contracts are decided by assessing the likelihood of the innovation being exploited by an *inefficient* user.

The research reported here yielded three main findings. First, when there is non-exclusive licensing through separating menus of screening contracts, those contracts incorporate royalties irrespective of the user's efficiency; in this case, the contracts of both user types are distorted. This dynamic is in sharp contrast with the case of exclusive licensing, under which only the contract designed for an inefficient user is distorted. The reason is that, if the patent holder will grant only an exclusive license and is worried about screening, then the contract intended for the inefficient type is designed so as to be unattractive to the efficient type (because that minimizes the informational rents paid to the latter). Hence the patent holder distorts the contract intended for an inefficient type. Under non-exclusive licensing, however, a patent holder looking to induce self-selection can offer a menu of user-specific contracts—a procedure that can be viewed as a two-stage decision. In the first stage, the patent holder looks for the equivalent of a monopoly outcome (collusion effect), which requires that royalties be charged to both efficient and inefficient users. In the second stage, the situation resembles the exclusive licensing context; hence the menu of contracts that the patent holder offers to each user is such that the contract intended for the efficient type does not distort that user's production. At the same time, though, the contract intended for inefficient users allows for additional distortion so that it will be less attractive to efficient users. In sum, the presence of royalties in contracts intended for efficient types can be explained by the patent holder's incentives to replicate the monopoly outcome (collusion effect). The royalties present in contracts intended for inefficient types reflect two motivations: the patent holder's abiding incentive to replicate the monopoly outcome *and* its incentive to make the contract less appealing to efficient users.

The second finding concerns the impact of adverse selection as compared to symmetric information on the number of licenses granted. Non-exclusive licensing is more likely to be employed under adverse selection than under symmetric information. With regard to the former, we must distinguish screening from restricting licenses to only efficient users. If the patent holder chooses a contract that allows for screening the users, then it is better-off granting only one license. Yet if symmetric information prevails, then non-exclusive licensing would benefit the patent holder more because competition's negative effect on its profits can be offset by royalties (Antelo and Sampayo, 2016). However, when adverse selection is combined with competition, the patent holder must use royalties in order to induce screening. The use of royalties for screening is so detrimental to patent holder profits that exclusive licensing is preferred—even though it entails giving up the advantages of sampling.

The third main finding concerns how the number of licenses granted affects welfare (taking subsequent contracts and producer behavior as given). Namely: for a cost—of transferring the innovation to each user—at which aggregate welfare under exclusive licensing increases with information symmetry, non-exclusive licensing yields greater welfare under asymmetric information *unless* the patent holder uses a screening contract. With screening, welfare under exclusive licensing is greater even though consumer surplus is higher with non-exclusive licensing. The implication is that, under screening, the loss in consumer surplus that results from a switch to exclusive licensing is more than offset by the increased patent holder profits. This result emphasizes our second finding by showing how strong is a screening contract's negative effect—when combined with Cournot competition—on patent holder profits.

Both the form of licensing contracts and the number of licenses granted are examined in Li and Wang (2010), Doganoglu and Inceoglu (2014), and Wu and Peng (2015). Li and Wang (2010) show that the patent holder's preference (or not) for granting an exclusive license depends on both the licensing contract and the innovation's degree of novelty; they also establish that non-exclusive licensing may be welfare reducing. Doganoglu and Inceoglu (2014) study the case of a monopolist who owns a drastic innovation that enables the creation of differentiated products. They find that a monopolist remaining outside the industry can replicate multiproduct monopoly profits by way of two-part licensing contracts. Finally, Wu and Peng (2015) show that an innovator with privileged information about its technology uses a higher but still suboptimal royalty as a signal of "type" when it must license to all downstream firms in an oligopolistic industry. These authors also demonstrate that, if the number of licenses sold is determined endogenously, then the innovator should use the number of licenses sold—rather than a higher royalty—to signal its type. So, in the separating equilibrium that maximizes the innovator's payoff, the efficient innovator grants fewer licenses than does the inefficient one.

Our setting is characterized by asymmetric information, unlike the perfect information setting of Li and Wang (2010) and Doganoglu and Inceoglu (2014). Our paper differs also from Wu and Peng (2015) in that they consider a signaling game in which (i) the innovator is better informed than the users about the innovation's value and (ii) these users are equally efficient at exploiting the innovation.

The remainder of the paper proceeds as follows. In Section 2 we introduce the basic framework for developing our analysis, and Section 3 addresses the model's equilibrium under symmetric information. In Section 4 we introduce asymmetric information—and allow for screening—toward the end of identifying the equilibrium number and form of licensing contracts. Section 5 focuses on deriving the total surplus that results from granting one versus two licenses (while assuming that all other aspects of Section 4's equilibrium solution remain unchanged). We conclude in Section 6 with summary remarks and a proposed extension.

2. The model

An inventor owns an innovation with sunk development costs. Because this inventor has secured a patent, competitors cannot (legally) replicate the innovation. Although the innovation can be used to produce a homogeneous final good, the patent holder is not capable of production for market. Hence the inventor licenses the innovation in exchange for monetary compensation. It is typical of real-life scenarios that the patent holder can choose from among many suitors seeking access to the innovation. However, we ensure computational feasibility—and highlight the mechanisms at play—by restricting our attention to just two cases: when the patent holder contracts with one user (exclusive licensing) or with two users (non-exclusive licensing).

When licensing the innovation, the patent holder acts as a Stackelberg leader facing users. Note that any user accepting the patent holder's licensing contract will discover its own profitability from putting the innovation into practice. In contrast, third parties—that is, the patent holder (under exclusive licensing) or the patent holder and the potential rival user (under non-exclusive licensing)—have only a prior probability about others' profits.

Each user can be efficient or inefficient when exploiting the innovation. Whereas an efficient (good) user does not incur a marginal cost, $c = 0$, when transforming the innovation into the final good an inefficient (bad) user does incur a positive marginal cost, $c > 0$. The random variable $\tilde{c}$ representing marginal cost is distributed as follows:

$$\tilde{c} = \begin{cases} 0 & \text{with Prob } \mu \\ c & \text{with Prob } 1 - \mu \end{cases} \tag{1}$$

here $0 < \mu < 1$. There is a cost $S > 0$, which is borne by the patent holder, of transferring the innovation to each user.[1] All players in this licensing game are risk neutral, and no player discounts the future.

As regards the market for any innovation-based product, there is a unit-sized continuum of symmetric and homogeneous consumers. Each consumer has preferences given by the quasi-linear utility function $u(q, m) = U(q) + m$, where $U(q)$ is the utility derived from consuming the product and m is the numéraire. We shall consider the quadratic utility function $U(q) = q - \frac{1}{2}q^2$, under which market demand may be written as

$$p(q) = \begin{cases} 1 - q & \text{if } q < 1 \\ 0 & \text{if } q \geq 1 \end{cases} \tag{2}$$

where q measures the quantity produced by either one or two users; when there are two users, they are assumed to produce an identical good.

Finally, to ensure regularity we set the following upper bound on the inefficient user's marginal cost c when transforming the innovation into the final product.

Assumption 1. $0 < c < \frac{1}{2}$

Under this assumption, if licensing is non-exclusive and if information is symmetric then both users will generate positive output—that is, regardless of the differences (if any) in their respective levels of transformation efficiency.

3. Symmetric information

As a benchmark for comparison, we start by assuming that all players in the licensing game share the same information about the patented innovation's value. Thus the timing is as follows. The patent holder, who does not know for certain the innovation's true value, offers a license to one or two users in a take-it-or-leave manner. For any commonly held belief about uncertainty, users accept or reject the offer. Once a contract is accepted, the user's type (or the users' types when there are two users) becomes common knowledge. Finally, users produce the final good as a monopoly or Cournot duopoly according as whether one or two licenses were granted. In this context, a patent holder's optimal behavior is formalized by the following proposition.

[1] This may be understood as the cost of producing an input that is used by licensees.

Proposition 1 (Antelo and Sampayo, 2016). *If information about the innovation's value is uncertain but symmetric, then there is a threshold* $S^*(\mu, c) \equiv \frac{1}{4}[\mu(1-\mu)(2-c)c]$ *of the per-license issuing cost such that the following statements hold.*

(i) *If* $S < S^*(\mu, c)$, *then the patent holder prefers non-exclusive licensing and sells each license via either a two-part contract (when the users have the same type) or a fixed-fee contract (when they have different types).*

(ii) *If* $S > S^*(\mu, c)$, *then the patent holder prefers exclusive licensing through a fixed-fee contract.*

(iii) *If* $S = S^*(\mu, c)$, *then the patent holder is indifferent between* (i) *and* (ii).

Under exclusive licensing, the revenue from licensing is maximized when the user's market behavior is not distorted. In this case, the patent holder offers a fixed-fee contract that extracts all of the user's expected profit. The same thing happens under non-exclusive licensing, provided that the two users have different types. In this latter case, the per-unit royalty charged to the inefficient user is so high that it is better-off rejecting the contract; the efficient user is then left as the market's unique producer—a monopolist. However, if licensing is non-exclusive and users are identical, then the patent holder benefits from contracts that include royalties; in this way, the innovator can soften the competition between potential users to such an extent that the monopoly outcome can be achieved.

The threshold per-license issuing cost at which the patent holder is indifferent between exclusive versus non-exclusive licensing increases with c and is concave in μ. Granting two licenses is more likely as c increases because competition then becomes less intense. Likewise, if there is no uncertainty (i.e., if $\mu = 0$ or $\mu = 1$) then the patent holder will license a single user to put the innovation into practice (i.e., no matter the value of *S*). Yet under maximal uncertainty ($\mu = 1/2$), the innovation is most likely licensed to two users—especially when *c* is high.

4. Asymmetric information

We now examine two topics in the context of adverse selection: the patent holder's optimal number of licenses to grant and the optimal contract for it to offer in the two licensing cases considered.

4.1. Exclusive licensing

Exclusive licensing under asymmetric information leads to a licensing-screening game whose timing is as follows. The patent holder, before knowing the innovation's value, offers one license to the user in exchange for a fixed fee plus contracted royalty payments. The user, which already knows the value it can obtain from transforming the innovation into a final good (this value determines the user's “type”), either accepts or rejects the offered contract. If the contract is accepted, then the accepting user’s type becomes publicly known. Finally, the user produces and markets the final good.

With exclusive licensing, the patent holder has three options. First, it can offer a non-screening contract under which both user types (efficient and inefficient) can produce; in this case, revised beliefs concerning the innovation value are equivalent to priors. Second, the patent holder can offer what is known as a shut-down contract: a contract whose fixed fee is so high that production will occur only if the user is efficient at transforming the innovation—in which case the innovation might *never* be embedded in a final good. Third, a patent holder can devise two contracts—one geared to efficient users and the other to inefficient users—and thereby allow the innovation to become manifest and marketed regardless of users’ respective abilities.

Our next proposition addresses the screening of innovation value. Recall that, for efficient (resp. inefficient) users, the marginal cost c is zero (resp. positive); hence we use subscript 0 or c to index users who transform the innovation in, respectively, an efficient or inefficient manner. All proofs are given in the Appendix.

Proposition 2. *Let* $c_1(\mu) = \frac{1}{1+\mu^2}\left(1 - \sqrt{\mu(1-\mu-\mu^2)}\right)$ *be a given efficiency level of a bad user. Then, under exclusive licensing and asymmetric information, we have the following possibilities.*

(i) *(Separating licensing) If the menu of contracts* $\{(F_0, r_0), (F_c, r_c)\} = \left\{\left(\frac{1}{4} - \left(\frac{1}{2} - \frac{(1+\mu)c}{4(1-\mu)}\right)c, 0\right), \left(\left(\frac{1-\mu-c}{2(1-\mu)}\right)^2, \frac{\mu}{1-\mu}c\right)\right\}$ *is offered to a bad user of efficiency less than* $1-\mu$*, then the innovation is always transformed and marketed no matter how inefficient the user is.*

(ii) *(Shut-down licensing) If the menu of contracts*

$$(F, r) = \begin{cases} \left(\dfrac{(1-(1+\mu)c)^2}{4}, \mu c\right) & \text{if } c \leq c_1(\mu) \\ \left(\dfrac{1}{4}, 0\right) & \text{if } c \geq c_1(\mu) \end{cases}$$

is offered to a bad user of efficiency greater than $1-\mu$*, then the innovation will not be transformed and marketed unless it is of high value.*

According to part (i), an innovation’s true value can be inferred from a revealing or “separating” contract offer. In particular, the contract that identifies a high-value innovation is a fixed-fee contract (“no distortion at the top”) whereas the contract that identifies a low-value innovation is a two-part contract with a fixed fee and a per-unit royalty. The royalty, whose amount depends on the proportion of efficient users, is intended to distort the bad user’s production decision (“distortion at the bottom”) and thereby make this contract unappealing to the good user, thus minimizing the latter’s informational rents. This finding parallels results obtained in the literature on screening licenses (Macho-Stadler et al., 1996; Poddar and Sinha, 2002).

Part (ii) of the proposition describes the problem of a patent holder that will license only to an efficient user, in which case information asymmetry has no part to play. The prescribed shut-down contract is simply a fixed-fee payment that the inefficient user rejects; hence the innovation becomes manifest only if licensed to an efficient user, which occurs with probability μ. In a separating contract, the distortion at the bottom would be so great (provided μ and c satisfy $c > 1 - \mu$) that the inefficient user has no incentive to produce. In this event, the patent holder will switch from a screening contract to a shut-down contract under which only efficient production is allowed and there are no informational rents to be paid.

Having examined how the patent holder can screen innovation value, we are now in a position to formalize the patent holder's optimal licensing scheme as follows.

Proposition 3. *Let* $c_2(\mu) = \min\left\{\frac{1}{2}, \frac{1-\mu}{1-\mu+\mu^2}\left(1 - \sqrt{\mu(1-\mu)}\right)\right\}$ *be a given efficiency level of a bad user, where* $c_2(\mu) \in \left(0, \frac{1}{2}\right)$ *for all* $\mu \in (0,1)$. *In this case, if information is asymmetric then the following statements hold.*

(i) *If* $c \leq c_2(\mu)$, *exclusive licensing is chosen via a revealing contract and the innovation is always put into practice.*

(ii) *For* $c \geq c_2(\mu)$, *exclusive licensing is chosen via a shut-down contract and the innovation is put into practice only by an efficient user.*

Provided the probability μ of licensing to a good user remains below 0.28—which is the value that solves $\frac{1}{2} = \frac{1-\mu}{1-\mu+\mu^2}\left(1 - \sqrt{\mu(1-\mu)}\right)$—a patent holder wants the innovation to be transformed and marketed irrespective of the user's (in)efficiency. A patent holder in these circumstances prefers not to offer a shut-down fixed-fee contract that a bad user would reject; the reason is that finding a good user to accept such a contract is most unlikely. If $\mu \geq 0.28$ then the patent holder offers a separating contract when the efficiency of bad users is less than $c_2(\mu)$.

In contrast, if users can be so inefficient that they exceed the threshold determined by $c_2(\mu)$ then two situations must be considered. First, if $c_2(\mu) \leq c \leq 1 - \mu$ then the patent holder could still implement a separating contract. Yet the required distortion at the bottom would then diminish the patent holder's benefits due to the inefficient user, in which case those benefits no longer compensate for the patent holder's reduced benefits due to the efficient user. The second possibility is that $c \geq 1 - \mu$; here the patent holder cannot implement a separating contract.

Proposition 3's results are plotted in Figure 1. In the green area the patent holder offers a screening/separating contract and both user types manifest the innovation in a product. In the red area, information asymmetry is of no consequence because the patent holder offers a shut-down contract and so its innovation will be marketed only with probability μ. Beneath (resp., above) the red-dashed locus, the separating contract as compared with a shut-down contract where both user types (resp., only efficient users) produce.

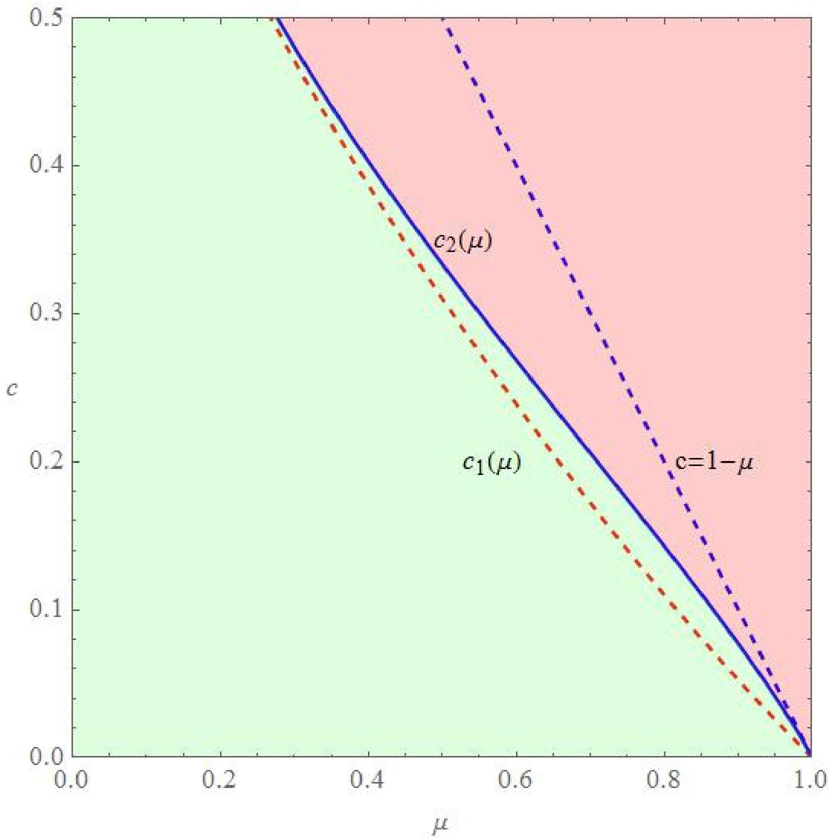

Figure 1. Contracts that are optimal for the patent holder under exclusive licensing.

4.2. Non-exclusive licensing

Now we consider the case in which the patent holder employs non-exclusive licensing. The licensing game proceeds much as in the single-user case. The patent holder offers to each user either a screening pair of contracts—of which one is expected to attract the efficient user and the other to attract the inefficient firm—or a single contract. In the latter case, the patent holder can offer a contract that allows both user types to produce or a (shut-down) contract that allows only efficient users to produce. Contracts are simultaneously offered to each user *before* their respective types (good or bad) are publicly known. Next, each user either accepts or rejects the offered contract. The user's type is revealed by acceptance provided that different user types prefer different types of contracts. Finally, users produce a homogeneous good and compete as Cournot players.

Our next proposition characterizes the optimal contract—from the patent holder's perspective—when licensing is non-exclusive. As before, subscripts 0 and *c* signify an efficient and an inefficient user (respectively).

Proposition 4. *Under non-exclusive licensing and asymmetric information, the following statements hold.*

(i) *(Separating licensing) Let* $c_3(\mu) = \min\left\{\frac{1}{2}, \frac{1-\mu}{1+\mu}\right\}$ *be a level of efficiency in exploiting the innovation, for all* $\mu \in (0,1)$. *If the menu of contracts* $\{(F_0, r_0), (F_c, r_c)\} = \left\{\left(\frac{1-\mu-(2-2\mu-c-7\mu c)c}{16(1-\mu)}, \frac{1-c}{4}\right), \left(\left(\frac{1-\mu-c-\mu c}{4(1-\mu)}\right)^2, \frac{1-\mu-c+5\mu c}{4(1-\mu)}\right)\right\}$ *is offered to each user and if the bad user's efficiency is less than* $c_3(\mu)$, *then the innovation will always be put into practice.*

(ii) *(Shut-down licensing) Let* $c_4(\mu) = \min\left\{\frac{1}{2}, \frac{1}{1+2\mu^2}\left(1 - \sqrt{\frac{2\mu(1-\mu+\mu^2)}{1+\mu}}\right)\right\}$ *be a level of efficiency in exploiting the innovation, for all* $\mu \in (0,1)$. *If the unique contract*

$$(F, r) = \begin{cases} \left(\left(\dfrac{1-r}{2+\mu}\right)^2, \dfrac{\mu}{2+2\mu}\right) & \text{if } c \leq c_4(\mu) \\ \left(\left(\dfrac{2-2c-2r-\mu c}{6}\right)^2, \dfrac{1}{4} + \left(\mu - \dfrac{1}{4}\right)c\right) & \text{if } c \geq c_4(\mu) \end{cases}$$

is offered to each user and if a bad user's efficiency is less than $c_4(\mu)$, *then the innovation is always produced no matter how inefficient the users are. Otherwise, the innovation will be produced only by efficient users.*

Thus the optimal form of non-exclusive licensing contracts differs from that of exclusive licensing contracts. First, per part (i), a royalty is part of the contract regardless of whether it is intended to attract users that are efficient or inefficient at transforming the innovation into a final good. Hence we observe distortion at the bottom and also at the top. The reason is that—in contrast to exclusive licensing, under which contracts are designed to elicit revelation—under non-exclusive licensing the patent holder retains that goal yet seeks also to replicate the monopoly outcome in the market for the final good. So when designing the menu of contracts for each user, the patent holder tries to replicate the monopoly outcome and therefore includes a royalty rate both for efficient and for inefficient users. Then, once royalties are set such that users behave as a monopoly, the patent holder faces a situation that resembles exclusive licensing. Its response is to offer a menu of contracts to each user, where contracts intended for efficient users are not distorted beyond the "optimal" level previously charged whereas contracts intended for inefficient users are *more* distorted than the "optimal" level previously charged. That extra distortion is meant to reduce this contract's attractiveness to efficient users.

According to part (ii) of Proposition 4, shut-down contracts under non-exclusive licensing are no longer (i.e., as under exclusive licensing) pure fixed-fee contracts. Instead, shut-down contracts are two-part contracts even when only the efficient types produce—which is the relevant case in equilibrium, as Proposition 5 will show. When the patent holder allows only efficient users to produce, asymmetric information is irrelevant and so the patent holder's sole aim is to reproduce, via royalties, the monopoly outcome.

Proposition 5. *Let* $c_5(\mu) = \min\left\{\frac{1}{2}, \frac{1-\mu}{1-\mu+2\mu^2}\left(1-\sqrt{\frac{2\mu(1-\mu)}{1+\mu}}\right)\right\}$ *be a level of inefficiency in exploiting the innovation, where* $0 < c_4(\mu) < c_5(\mu) \leq \frac{1}{2}$ *for all* $\mu \in (0,1)$. *Under asymmetric information, non-exclusive licensing is implemented as follows*:

(i) *through a separating menu of contracts if the efficiency of bad users is no bigger than* $c_4(\mu)$; *or*

(ii) *through a two-part shut-down contract, given by* $(F, r) = \left(\left(\frac{2-2c-2r-c\mu}{6}\right)^2, \frac{1}{4} + \left(\mu - \frac{1}{4}\right)c\right)$, *if the efficiency of bad users is no lower than* $c_5(\mu)$.

In Figure 2 we plot the different regions of the (μ, c)-space where the various licensing contracts are optimal for the patent holder. The farther below the $c_5(\mu)$ locus, the better is a separating contract for the patent holder (below the $c_4(\mu)$ locus, as compared with a single contract under which all users produce; above that locus, as compared with a single contract under which only efficient users produce). In the red area, the patent holder benefits more from offering a single contract under which only efficient users produce. Observe that, above the $c_3(\mu)$ locus, no comparison is needed because the separating contract is not profitable there.

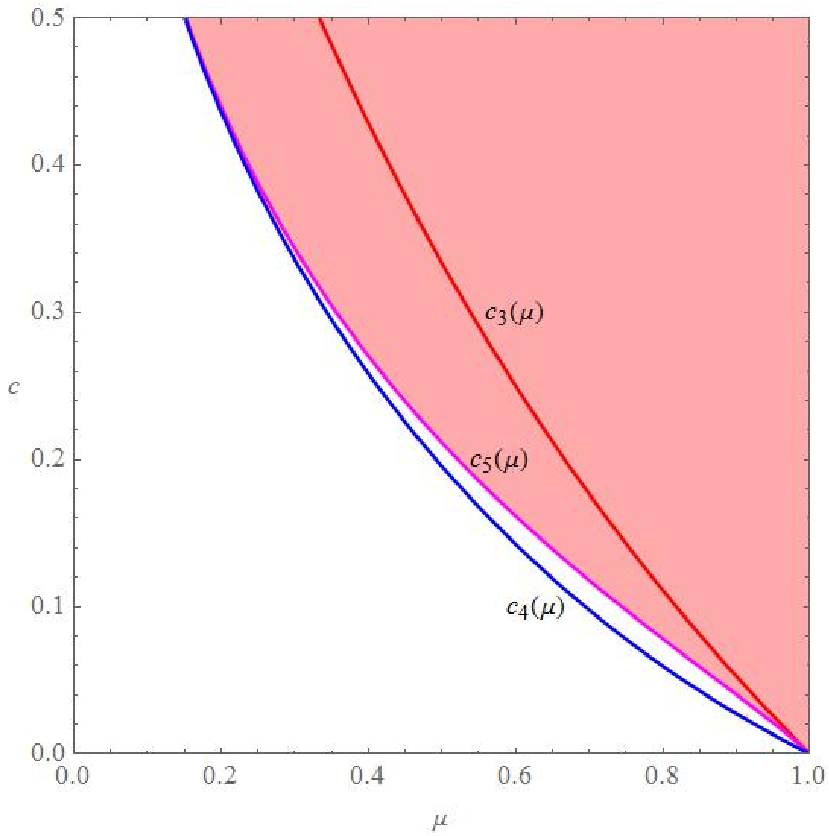

Figure 2. Contracts optimal for the patent holder under non-exclusive licensing.

Propositions 1, 3, and 5 allow us to compare the number of licenses that the patent holder grants under symmetric information and under screening. We can then identify the scenarios in which each license is more likely to be used.

Proposition 6. *Let $S^*(\mu, c)$ be the per-license issuing cost at which the patent holder is indifferent between exclusive or non-exclusive licensing under symmetric information (as defined in Proposition 1). In a screening context, at this cost the patent holder prefers non-exclusive licensing in the region of parameters $\Gamma_1 \cup \Gamma_2 \cup \Gamma_3$ for*

$$\Gamma_1 = \left\{(\mu, c) \in (0,1) \times \left(0, \tfrac{1}{2}\right) \middle| c_6(\mu) \le c \le c_5(\mu)\right\},$$

$$\Gamma_2 = \left\{(\mu, c) \in (0,1) \times \left(0, \tfrac{1}{2}\right) \middle| \max\{c_7(\mu), c_5(\mu)\} \le c \le c_2(\mu)\right\},$$

$$\Gamma_3 = \left\{(\mu, c) \in (0,1) \times \left(0, \tfrac{1}{2}\right) \middle| c_2(\mu) \le c \le c_8(\mu)\right\},$$

where $c_6(\mu) = \frac{2-4\mu+2\mu^2}{1-\mu+\mu^2}$, $c_7(\mu) = \min\left\{\frac{1}{2}, \frac{1-2\mu+2\mu^2-\mu^3}{1-2\mu+3\mu^2-\mu^3} - (1-\mu)\sqrt{\frac{\mu[1-\mu(1-\mu)(2+\mu^2)]}{(1+\mu)(1-2\mu+3\mu^2-\mu^3)}}\right\}$, *and* $c_8(\mu) = \min\left\{\frac{1}{2}, 1 - \sqrt{\frac{\mu}{1+\mu}}\right\}$.

Proposition 6 states that non-exclusive licensing—and the resulting Cournot competition—is more likely to be used under asymmetric than symmetric information. For an issuing cost at which the marginal effect of a second license on the patent holder's profit is zero under *symmetric* information, that effect is positive for a wide range of the (μ, c)-space under *asymmetric* information. For high values of the μ and c parameters, the patent holder chooses shut-down contracts for both exclusive and non-exclusive licensing. Competition, then, does not reduce the profits of a patent holder operating under a shut-down contract because the royalties charged have two benefits: they replicate the monopolistic outcome; and they make it more likely that the producer will be efficient (sampling effect), which becomes more valuable as uncertainty increases. This sampling effect is high enough to compensate for the costs of issuing two licenses, but only for intermediate values of μ and c. Thus non-exclusive licensing through shut-down contracts is of no interest to the patent holder when both μ and c take high values, because in that case the sampling effect tends to be lower and the issuing costs higher. For low values of μ and c, the patent holder prefers separating contracts under exclusive and non-exclusive licensing both. In this case, the patent holder is obliged to pay informational rents to the efficient user; hence it cannot replicate the monopoly outcome when issuing several licenses. Since, moreover, the sampling effect is also low for low values of μ, it follows that monopoly profits are preferred by the patent holder despite the low issuing costs in this case.

Figure 3 depicts the region in which, under asymmetric information, the patent holder prefers non-exclusive to exclusive licensing even though, under symmetric information, it would be indifferent between them. Observe that this region excludes nearly all of the region where separating contracts are used with non-exclusive licensing (i.e., the area under $c_5(\mu)$) and a large part of the region where a separating contract is used with exclusive licensing (the area under $c_2(\mu)$).

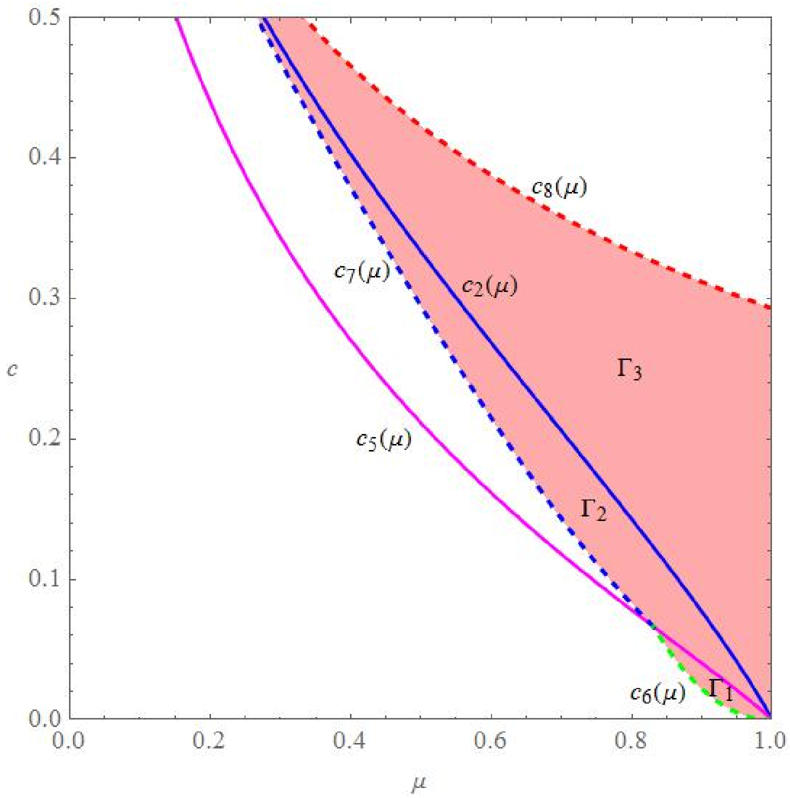

Figure 3. Region $\Gamma_1 \cup \Gamma_2 \cup \Gamma_3$. In this region, the patent holder strictly prefers non-exclusive licensing under asymmetric information despite being indifferent (between exclusive vs. non-exclusive licensing) under symmetric information.

5. Number of licenses and welfare

In this section we examine the social welfare resulting from patent holder behavior—with regard to choosing the number of licenses—under symmetric and also under asymmetric information. Toward that end, we define *expected aggregate welfare* as the sum of expected consumer surplus and expected profits achieved by the innovation user(s).

Thus we examine how the number of licenses granted affects welfare. To do it, we assume that a planner always chooses the number of licenses (here, one or two) that increases welfare *and* allows the patent holder (the actual licensor) to set the terms of licensing contracts and the users to decide their production strategies as in the equilibrium described above. We therefore suppose that (a) the patent holder takes the planner-determined number of licenses as given and then decides (as in the equilibria described in previous sections) about forms of contract and (b) either a single user behaves as a monopolist or two users behave as Cournot duopolists (also as in the preceding text). By comparing the number of licenses in this framework with that in the equilibrium where the *patent holder* determines the number of licenses, we establish that the area of the (μ, c)-space where, in equilibrium, the patent holder chooses a socially inefficient number of licenses (one license) is much smaller under asymmetric information than under symmetric information. The difference depends on three factors: μ, c, and the cost S of issuing a license. As a starting point we consider the issuing cost $S^*(\mu, c)$ at which, under symmetric information, the patent holder is indifferent between granting one or two licenses (cf. Proposition 1). We shall use $\tilde{S}(\mu, c)$ to denote the per-license issuing cost that would yield, again under symmetric information, the same total surplus regardless of whether one or two licenses were granted.

Proposition 7. *Let $S^*(\mu, c)$ be the per-license issuing cost (as defined in Proposition 1) at which patent holder profits are the same under exclusive and non-exclusive licensing when information is symmetric. Let $\tilde{S}(\mu, c) = \frac{3}{8}\mu(1-\mu)(2-c)c$ be the per-license issuing cost at which, under symmetric information, aggregate welfare is the same under exclusive and non-exclusive licensing. Then the following statements hold.*

(i) *Under symmetric information, if the issuing cost S satisfies $S^*(\mu, c) \leq S \leq \tilde{S}(\mu, c)$ then non-exclusive licensing is socially efficient—although exclusive licensing always prevails in equilibrium.*

(ii) *Under asymmetric information, if the issuing cost S satisfies $S^*(\mu, c) \leq S \leq \tilde{S}(\mu, c)$ then either non-exclusive or exclusive licensing can be socially efficient, depending on the parameters μ and c; moreover, there exist (μ, c) values for which the number of licenses chosen by the patent holder is equal to the socially efficient number.*

Two main conclusions emerge from Proposition 7. Part (i) states that, for the range of issuing costs considered, total welfare under symmetric information is always higher under non-exclusive than under exclusive licensing; hence, from a social welfare perspective, there is unequivocally insufficient diffusion of the innovation. However, the same cannot always be said under asymmetric information. In this case, that suboptimal welfare outcome would obtain only for a sufficiently high probability of the innovation being transformed by good (rather than bad) users (see Figures A4 and A5 in the Appendix). Note that this statement holds even though expected consumer surplus is always greater under non-exclusive than exclusive licensing, since in the former case higher quantities are produced and selling prices are lower. So when the proportion of efficient users is low enough, the reduction in consumer surplus that would follow a move from non-exclusive to exclusive licensing is outweighed by the subsequent increase in user profits. In short, the combination of higher prices and lower quantities in the market for the final good—as would follow from a reduced number of licenses—does not result in an *overall* loss of welfare.

The second finding is based on part (ii) of the proposition and follows from comparing the patent holder's optimal number of licenses with the socially optimal number of licenses (again, see Figures A4 and A5). When information is asymmetric, it becomes more likely that—in equilibrium—the patent holder's chosen number of licenses is equal to the number that a social planner would choose to maximize aggregate surplus. To emphasize the role of asymmetric information in increasing the welfare efficiency of the number of licenses in equilibrium, we prove Proposition 7 for the case of an issuing cost per license at which the equilibrium number of licenses under symmetric information is inefficient. If $S < S^*(\mu, c)$ then we would already have non-exclusive licensing in the symmetric information equilibrium. Yet if $S > \tilde{S}(\mu, c)$, asymmetric information does increase welfare efficiency (by increasing the number of licenses granted in equilibrium). This effect is offset as the issuing cost increases, however, because the equilibrium number of licenses suffers more from higher issuing costs than it benefits from the positive effect stipulated in Proposition 7(ii).

6. Conclusions

We analyzed an innovation licensing game in which the patent holder must transfer the innovation to users capable of turning it into a final good. Each potential user is privately informed about its own marginal costs in the event it does use the innovation to produce a final good. We considered the patent holder's problem of simultaneously designing two-part tariff licensing contracts (which are fairly common in practice) and deciding how many licenses it should grant.

The option to choose the number of licenses affects the form of screening contracts to the extent that, whereas only the contract intended for an inefficient user includes a royalty (distortion at the bottom) under exclusive licensing, if licensing is instead non-exclusive then royalties are included also in contracts intended for efficient types (distortion at the top). This first result is partly driven by the patent holder's incentive to replicate the monopoly outcome even when selling two licenses, which explains the inclusion of royalties in contracts intended for each type (efficient and inefficient) of oligopolistic user. Royalties also serve, just as under exclusive licensing, the patent holder's purpose of making contracts intended for inefficient users less attractive to efficient ones.

Second, we find that non-exclusive licensing is more likely to be employed under asymmetric information. With regard to such adverse selection situation, however, we must distinguish screening from the granting of licenses only to efficient users. A patent holder offering contracts that screen the users is better-off under exclusive than non-exclusive licensing. Yet if information is symmetric then, ceteris paribus, non-exclusive licensing would benefit the patent holder more (than exclusive licensing) because the negative effect of competition on the patent holder's profits can be offset by requiring royalty payments. We establish that, when both competition and asymmetric information apply, the patent holder must use royalties as a screening device. However, using royalities for screening purposes reduces the patent holder's profits so much that it prefers exclusive licensing to the sampling advantages of non-exclusive licensing.

Our third main result concerns how welfare is affected by the number of licenses granted—taking as given the contracts subsequently offered by the patent holder and also the behavior of potential users (producers). If the cost of transferring the innovation to multiple producers is such that aggregate welfare under exclusive licensing is greater when information is symmetric than asymmetric, then non-exclusive licensing can yield greater welfare under asymmetric information *except* when the patent holder uses a screening contract. With screening, total welfare under exclusive licensing is greater even though consumer surplus is higher under non-exclusive licensing. Hence we conclude that, under screening, the consumer surplus lost in a move from non-exclusive to exclusive licensing is more than offset by the patent holder's gains. This result emphasizes our second finding by showing the extent of screening's negative effect, when combined with Cournot competition, on patent holder profits.

Summing up, our paper contributes to the licensing literature by adding to the scarce research on licensing behavior when the patent holder must determine not only the contract's optimal form but also the optimal number of licenses to grant. We believe that the results reported here can serve as a solid benchmark for the more general case of *more* than one or two competing users. Indeed, a most valuable extension of our model would be to consider n potential users (each, as here, capable of transforming the innovation into a final product) and then derive the optimal number of licenses granted under symmetric and also asymmetric information structures.

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APPENDIX

To facilitate the exposition, throughout this appendix we use masculine and feminine pronouns when referencing (respectively) the patent holder and any users.

Proof of Proposition 2

(i) The problem of a patent holder who offers a menu of screening contracts can be written as follows:

$$\left.\begin{array}{l} \max_{(F_0,r_0,F_c,r_c)} \mu(F_0+r_0q_0)+(1-\mu)(F_c+r_cq_c) \\ \text{s.t.} \begin{cases} \frac{(1-r_0)^2}{4}-F_0\geq 0 & (\mathrm{PC}_0) \\ \frac{(1-c-r_c)^2}{4}-F_c\geq 0 & (\mathrm{PC}_c) \\ \frac{(1-r_0)^2}{4}-F_0\geq \frac{(1-r_c)^2}{4}-F_c & (\mathrm{IC}_0) \\ \frac{(1-c-r_c)^2}{4}-F_c\geq \frac{(1-c-r_0)^2}{4}-F_0 & (\mathrm{IC}_c) \end{cases} \end{array}\right\} \tag{A.1}$$

Here the subscripts 0 and c denote, respectively, the efficient and inefficient user; $q_0=\frac{1-r_0}{2}$ and $q_c=\frac{1-r_c-c}{2}$ are the profit-maximizing quantities for the respective user's type when acting as a naïve monopolist. The two first inequalities in problem (A.1)'s conditions are the participation constraints (PC) for the efficient and inefficient user, respectively, and each inequality states that the net profit of a user choosing the contract intended for her is no less than zero. The final two inequalities are the incentive compatibility (IC) conditions for the efficient and inefficient user, respectively. The first of these constraints is that the efficient user's monopoly profit when agreeing to contract (F_0, r_0), which is aimed at her, is greater than from agreeing to the contract (F_c, r_c)—and thus misrepresenting herself as an inefficient user. The second IC constraint serves the same purpose but concerns the inefficient user.

We shall first prove that if (PC_c) holds then (PC_0) must also hold and so we can ignore it. Note that the right-hand side (RHS) of (IC_0) is greater than the left-hand side (LHS) of (PC_c) while the LHS of

(IC_0) is just the LHS of (PC_0). Therefore, if (PC_c) holds then (PC_0) also holds and hence can be ignored.

Second, we prove that the solution verifies (IC_0) with equality and verifies (IC_c) with strict inequality, as is typical of such incentive problems. Suppose for the moment that instead we solve the problem while *ignoring* both IC constraints. In that case, it is easy to see that the solution would verify both participation constraints with equality. Yet the efficient user would then be better-off misrepresenting the innovation as one of low value: if she speaks the truth, her profits will be zero; if she lies, her profits will amount to $\frac{(1-r_c)^2}{4} - \frac{(1-r_c-c)^2}{4} > 0$. But given that $F_c = \frac{(1-r_c-c)^2}{4}$, those profits are equal to the RHS of (IC_0) and so the inequality would not hold. In contrast, an inefficient user has no incentive to misrepresent the innovation's perceived value: if she tells the truth then her profits are zero, but if she lies then her profits are $\frac{(1-r_0-c)^2}{4} - \frac{(1-r_0)^2}{4} < 0$. Therefore, it is safe to ignore (IC_c).

Now we can prove that both (IC_0) and (PC_c) will be verified as equalities in the solution. If (IC_0) were not so verified then the patent holder could always raise F_0 and thereby increase his profits (note that this action would not affect (PC_c)). In turn, (PC_c) will also be verified with equality; for if not, the patent holder could raise F_c to increase his profits. Such a raise has the additional effect of diminishing the RHS of (IC_0), which in turn allows the patent holder to increase F_0 still further. Hence $F_c = \frac{(1-c-r_0)^2}{4}$, from which it follows that

$$F_0 = \frac{(1-r_0)^2}{4} - \frac{(1-r_c)^2}{4} + F_c = \frac{(1-r_0)^2}{4} - \frac{(1-r_c)^2}{4} + \frac{(1-c-r_c)^2}{4} \tag{A.2}$$

The patent holder's problem given in (A.1) can be now rewritten as

$$\max_{\{F_0, r_0, F_c, r_c\}} \mu\left(F_0 + r_0 \frac{1-r_0}{2}\right) + (1-\mu)\left(F_c + r_c \frac{1-r_c-c}{2}\right)$$

$$= \frac{1}{4}(1-c)^2 + \frac{c}{2}\mu r_c - \frac{1}{4}(1-\mu)r_c^2 - \frac{1}{2}\mu r_0^2 \tag{A.3}$$

the solution of which is

$$r_0 = 0, \quad r_c = \frac{\mu c}{1-\mu}; \quad F_c = \frac{(1-\mu-c)^2}{4(1-\mu)^2}, \quad F_0 = \frac{1}{4} - \frac{c}{2}\left(1 - \frac{(1+\mu)c}{2(1-\mu)}\right) \tag{A.4}$$

The user produces $q_0 = \frac{1}{2}$ if she exploits the innovation efficiently or produces $q_c = \frac{1-\mu-c}{2(1-\mu)}$ if her exploitation is inefficient; here $q_c > 0$ (and $F_c > 0$) whenever $c < 1-\mu$. Finally, expected profits of the patent holder (PH) are

$$E[\pi^{\text{PH}}] = \frac{1}{4}\left(1 - 2c - \mu c^2 + \frac{c^2}{1-\mu}\right) \tag{A.5}$$

(ii) If problem (A.1) is solved while ignoring (IC_0) and (IC_c), then the patent holder cannot screen the user's type and so will offer a single contract (F, r). However, he must still assess the benefits (if any) from allowing a bad user to produce the innovation. Suppose that F is such that only the good user produces. Then $F = \frac{(1-r)^2}{4}$; that is, F is equal to the profits of a monopolist producing $q_0 = \frac{1-r}{2}$ (and an inefficient user will market no product based on this innovation). Therefore, the patent holder's optimal per-unit royalty solves

$$\max_r \mu\left(\frac{(1-r)^2}{4} + r\frac{1-r}{2}\right) \tag{A.6}$$

the solution is $r = 0$ and so $F = \frac{1}{4}$. The user produces $q_0 = \frac{1}{2}$ only if she exploits the innovation efficiently, in which case the patent holder's expected profits are

$$E[\pi^{\mathrm{PH}}_{\mathrm{G,NS}}] = \frac{\mu}{4} \tag{A.7}$$

here the subscripts G and NS mark the good (zero–marginal cost) user and the "no screening" case. If the fixed fee required by the patent holder allows the user to produce according to her type, then the participation constraints imply that $F = \frac{(1-c-r)^2}{4}$; this is also the profit of a bad (positive–marginal cost) user producing $q_c = \frac{1-c-r}{2}$. Yet because an efficient user must pay the same fixed fee, she will produce $q_0 = \frac{1-r}{2}$. In this case, the optimal per-unit royalty is the one that solves

$$\max_r \mu\left(\frac{(1-c-r)^2}{4} + r\frac{1-r}{2}\right) + (1-\mu)\left(\frac{(1-c-r)^2}{4} + r\frac{1-c-r}{2}\right) \tag{A.8}$$

From (A.8) it follows that $r = \mu c$ and hence that $F = \frac{(1-(1+\mu)c)^2}{4}$. Production of the good and bad user are then $q_0 = \frac{1-\mu c}{2}$ and $q_c = \frac{1-(1+\mu)c}{2}$, respectively. We remark that both Assumption 1 and the inequalities $0 < \mu < 1$ imply that $\mu c < c(1+\mu) < 1$; hence q_0, q_c, and F are each positive. When all these factors are taken into account, the patent holder's expected profits (under no screening) when both user types produce are

$$E[\pi^{\mathrm{PH}}_{\mathrm{G+B,NS}}] = \frac{1}{4}(1 - 2c + c^2 + \mu^2 c^2) \tag{A.9}$$

where the subscript $\mathrm{G+B}$ indicates that both good and bad (efficient and inefficient) users are licensed to produce. It is easy to check that $E[\pi^{\mathrm{PH}}_{\mathrm{G+B,NS}}] < E[\pi^{\mathrm{PH}}_{\mathrm{G,NS}}]$ if and only if

$$c_1(\mu) = \frac{1}{1+\mu^2}\left(1 - \sqrt{\mu(1-\mu-\mu^2)}\right) \leq c \leq \frac{1}{1+\mu^2}\left(1 + \sqrt{\mu(1-\mu-\mu^2)}\right) \tag{A.10}$$

However, since $c < \frac{1}{2}$ (by Assumption 1) it follows that (A.10)'s upper bound on c is irrelevant because $\frac{1}{1+\mu^2}\left(1+\sqrt{\mu(1-\mu-\mu^2)}\right) > \frac{1}{2}$ for all $\mu \in (0,1)$. So when the patent holder chooses to offer a shut-down contract, the following statements hold.

(a) If $0 \le c \le c_1(\mu) = \frac{1}{1+\mu^2}\left(1-\sqrt{\mu(1-\mu-\mu^2)}\right)$, then the contract offered will be $(F,r) = \left(\frac{(1-(1+\mu)c)^2}{4}, \mu c\right)$ and the user will produce irrespective of her type; in this case, the patent holder's expected profits are given by (A.9).

(b) If $c_1(\mu) = \frac{1}{1+\mu^2}\left(1-\sqrt{\mu(1-\mu-\mu^2)}\right) \le c < \frac{1}{2}$, then the contract offered will be $(F,r) = \left(\frac{1}{4}, 0\right)$ because the patent holder wants production only by an efficient user; in this case, the patent holder's expected profits are given by (A.7).

The pink area in Figure A1 corresponds to the region in (μ, c)-space where the separating contract described in Proposition 2 is well-defined.

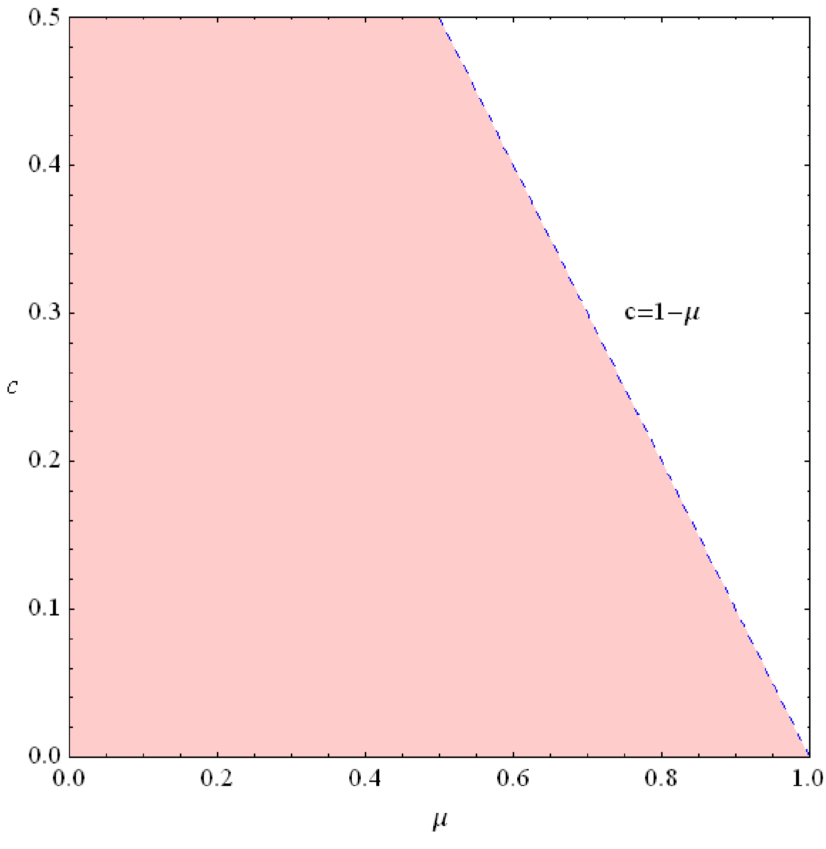

Figure A1. Region in (μ, c)-space where the separating contract of Proposition 2 may occur.

■

Proof of Proposition 3

The result follows in a straightforward way from comparing (A.5), (A.7), and (A.9).

Figure A2 illustrates the results of Proposition 3. There we depict both the region where the user produces irrespective of her type (green area) as well as the region where only efficient users employ the innovation to produce a marketable good (blank area).

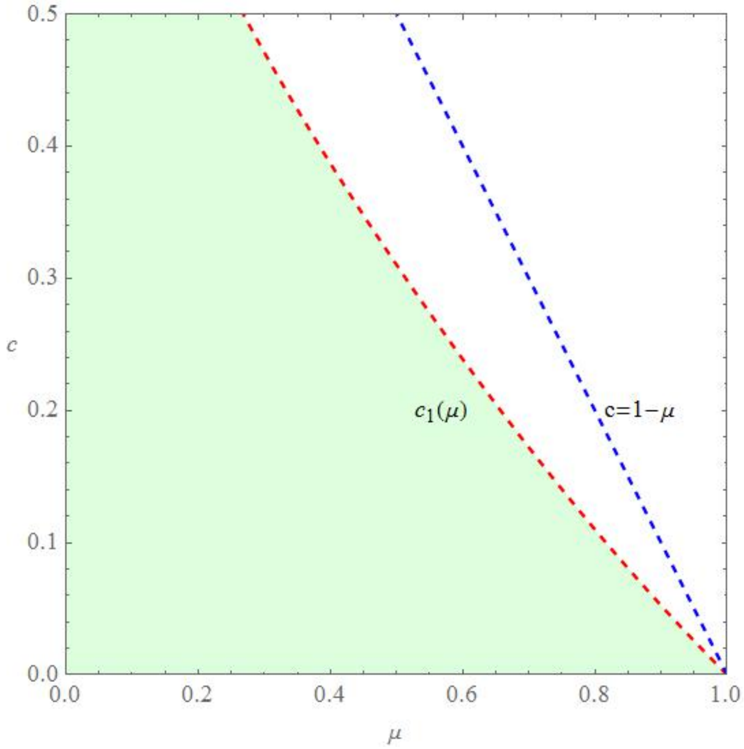

Figure A2. Contracts described by Proposition 3. In the green area, the patent holder benefits most from a shut-down contract under which either user type can produce. In the blank area, the patent holder offers a shut-down contract such that the innovation is marketed only by the efficient user.

■

Proof of Proposition 4

(i) If the patent holder offers a menu of screening contracts $\{(F_0, r_0), (F_c, r_c)\}$ to each potential user, then he solves the following problem:

$$\left.\begin{array}{l} \max\limits_{\{(F_0,r_0),(F_c,r_c)\}} \{2\mu^2(F_0 + r_0 q_0) + 2\mu(1-\mu)(F_0 + r_0 q_0 + F_c + r_c q_c) + 2(1-\mu)^2(F_c + r_c q_c)\} \\ \text{s.t.} \begin{cases} E[\pi^i(0;0)] - F_0 \geq 0, & (\mathrm{PC}_0) \\ E[\pi^i(c;c)] - F_c \geq 0, & (\mathrm{PC}_c) \\ E[\pi^i(0;0)] - F_0 \geq E[\pi^i(0;c)] - F_c, & (\mathrm{IC}_0) \\ E[\pi^i(c;c)] - F_c \geq E[\pi^i(c;0)] - F_0. & (\mathrm{IC}_c) \end{cases} \end{array}\right\} \tag{A.12}$$

In the incentive and participation constraints of this problem, $E[\pi^i(\tilde{c};\tilde{c}^M)$ are the expected profits of user i under a Cournot equilibrium, where $\tilde{c}=\{0,c\}$ is i's true type and $\tilde{c}^M=\{0,c\}$ is user i's self-reported type—assuming that user j tells the truth about her type. If $\tilde{c}^M=\tilde{c}$, then $E[\pi^i(\tilde{c};\tilde{c})]$ are i's expected profits when all users tell the truth about their type; if $\tilde{c}^M\neq\tilde{c}$, then $E[\pi^i(\tilde{c};\tilde{c}^M)]$ denotes i's expected profits when user i lies about her type but user j does not. Those profits are computed as follows.

First suppose that $\tilde{c}^M=\tilde{c}$. Then, for $\tilde{c}\in\{0,c\}$ and each user $i\in\{1,2\}$, we compute $E[\pi^i(\tilde{c};\tilde{c})]$ by solving the problem

$$\max_{q^i_{\tilde{c}}}\left(1-r_{\tilde{c}}-\tilde{c}-q^i_{\tilde{c}}-\mu q^j_0-(1-\mu)q^j_c\right)q^i_{\tilde{c}}\,,\quad j\in\{1,2\},\ j\neq i \tag{A.13}$$

where the first-order condition is

$$1-r_{\tilde{c}}-\tilde{c}-2q^i_{\tilde{c}}-\mu q^j_0-(1-\mu)q^j_c=0 \tag{A.14}$$

For $i\in\{1,2\}$, solving the four equations defined by (A.14) yields

$$q^i_0=\tfrac{1}{6}(2+(1-\mu)(c+r_c)-(3-\mu)r_0) \tag{A.15}$$

$$q^i_c=\frac{1}{6}(2-(2+\mu)(c+r_c)+\mu r_0) \tag{A.16}$$

the resulting users' expected profits are then

$$E[\pi^i(0;0)]=\tfrac{1}{36}(2+(1-\mu)(c+r_{\mathrm{B}})+(\mu-3)r_{\mathrm{G}})^2 \tag{A.17}$$

$$E[\pi^i(c;c)]=\frac{1}{36}(2(1-c-r_{\mathrm{B}})+\mu(r_{\mathrm{G}}-c-r_{\mathrm{B}}))^2 \tag{A.18}$$

(recall from (A.9) that the subscripts G and B denote, respectively, good and bad users). Now suppose that $\tilde{c}^M\neq\tilde{c}$. To compute $E[\pi^i(\tilde{c};\tilde{c}^M)]$ for each user $i\in\{1,2\}$ and for $\tilde{c},\tilde{c}^M\in\{0,c\}$ when $\tilde{c}^M\neq\tilde{c}$, we solve the problem

$$\max_{q^i_{\tilde{c},\tilde{c}^M}}\left(1-r_{\tilde{c}^M}-\tilde{c}-q^i_{\tilde{c},\tilde{c}^M}-\mu q^j_0-(1-\mu)q^j_c\right)q^i_{\tilde{c},\tilde{c}^M} \tag{A.19}$$

here q^j_0 and q^j_c ($j\in\{1,2\}, j\neq i$) are given by (A.15) and (A.16), respectively. The solutions to these two problems are

$$q^i_{0,c}=\frac{1}{6}(2+(1-\mu)c-(2+\mu)r_c+\mu r_0), \tag{A.20}$$

$$q^i_{c,0}=\frac{1}{6}(2-(2+\mu)c+(1-\mu)r_c-(3-\mu)r_0) \tag{A.21}$$

from which follow

$$E[\pi^i(0;c)] = \tfrac{1}{36}(2 + (1-\mu)c + \mu r_0 - (2+\mu)r_c)^2, \tag{A.22}$$

$$E[\pi^i(c;0)] = \frac{1}{36}(2 + (\mu - 3)r_0 - (1-\mu)r_c - (2+\mu)c)^2 \tag{A.23}$$

for each user i. Now substituting expressions (A.15)–(A.18) as well as (A.22) and (A.23) into (A.12), we can write the patent holder's problem as

$$\begin{aligned}\max_{\{(F_0,r_0),(F_c,r_c)\}} \Big\{ & 2\mu^2 \left(F_0 + r_0 \tfrac{1}{6}(2 + (1-\mu)(c + r_c) - (3-\mu)r_0)\right) \\ & + 2\mu(1-\mu)\left(F_0 + r_0\tfrac{1}{6}(2 + (1-\mu)(c + r_c) - (3-\mu)r_0) + F_c \right. \\ & \qquad \left. + r_c \tfrac{1}{6}(2 - (2+\mu)(c + r_c) + \mu r_0)\right) \\ & + 2(1-\mu)^2 \left(F_c + r_c\tfrac{1}{6}(2 - (2+\mu)(c+r_c) + \mu r_0)\right)\Big\}\end{aligned} \tag{A.24}$$

subject to

$$\frac{1}{36}(2 + (1-\mu)(c + r_c) - (3-\mu)r_0)^2 - F_0 \geq 0, \tag{PC$_0$}$$

$$\frac{1}{36}(2(1 - c - r_c) + \mu(r_0 - c - r_c))^2 - F_c \geq 0, \tag{PC$_c$}$$

$$\begin{aligned}&\frac{1}{36}(2 + (1-\mu)(c + r_c) - (3-\mu)r_0)^2 - F_0 \\ &\qquad \geq \frac{1}{36}(2 + (1-\mu)c + \mu r_0 - (2+\mu)r_c)^2 - F_c,\end{aligned} \tag{IC$_0$}$$

$$\begin{aligned}&\frac{1}{36}(2(1 - c - r_c) + \mu(r_0 - c - r_c))^2 - F_c \\ &\qquad \geq \frac{1}{36}(2 - (3-\mu)r_0 - (1-\mu)r_c - (2+\mu)c)^2 - F_0\end{aligned} \tag{IC$_c$}$$

We can use the same arguments as in the proof of Proposition 2 to show that, in the solution to problem (A.24), (PC$_0$) and (IC$_c$) are verified with strict inequality (and so can be ignored) whereas (PC$_c$) and (IC$_0$) are verified with equality. So if we now substitute (PC$_c$) and(IC$_0$) into (A.24) *as equalities*, then the problem can be written as

$$\begin{aligned}\max_{\{r_0,r_c\}} \Big\{ & \frac{1}{18}\mu(\mu - 9)r_0^2 + \frac{1}{18}(7\mu + \mu^2 - 8)r_c^2 \\ & + \frac{1}{9}\mu\big(2 - c(2+\mu)\big)r_0 + \frac{1}{9}(2 - 2c - 2\mu + 10c\mu + c\mu^2)r_c \\ & + \frac{1}{9}\mu(1-\mu)r_0 r_c + \frac{1}{18}(4 - 8c + 4c^2 - 4c\mu + 4c^2\mu + c^2\mu^2)\Big\}\end{aligned}$$

Provided that $0 < c \leq c_3(\mu) = \min\left\{\frac{1}{2}, \frac{1-\mu}{1+\mu}\right\}$, the solutions are $r_0 = \frac{1-c}{4}$ and $r_c = \frac{1-\mu-c+5\mu c}{4(1-\mu)}$. But if $c \geq c_3(\mu)$ then, under the computed fees for r_0 and r_c, the production q_0 and q_c as given by (A.15) and (A.16) would be zero. Figure A3 illustrates the region wherein a separating contract is profitable for the patent holder.

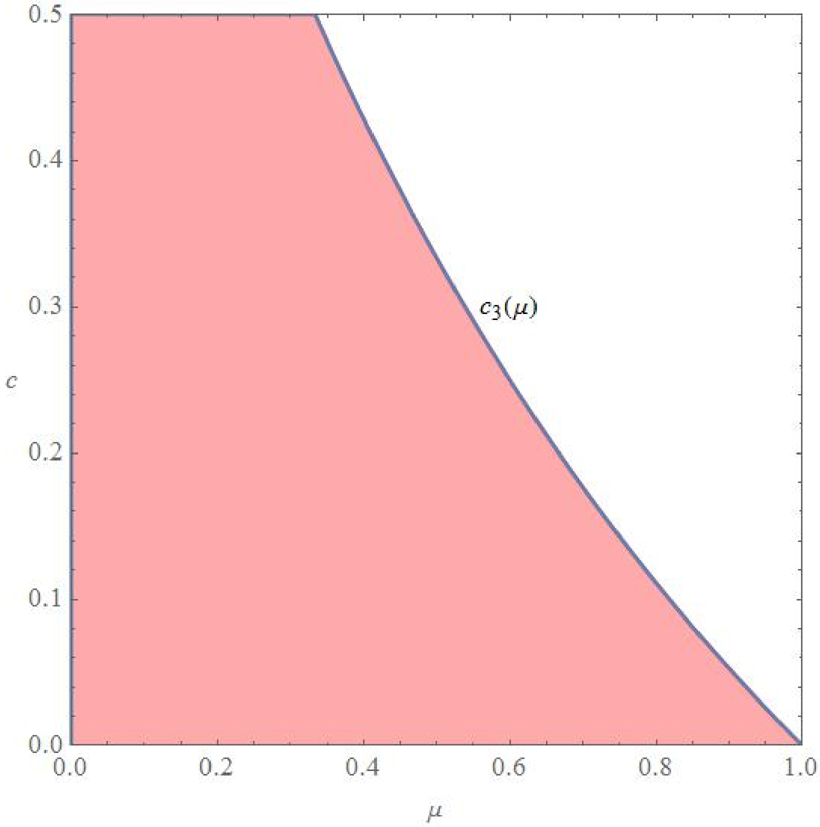

Figure A3. The region (outlined in pink) in which the patent holder profits from a separating contract.

If $0 < c \leq c_3(\mu)$, then substituting the values obtained for r_0 and r_c into the constraints (PC_0) and (IC_c) of problem (A.24)—when written as equalities—yields

$$F_c = \frac{(1-\mu-c-\mu c)^2}{16(1-\mu)^2} \tag{A.27}$$

$$F_0 = \frac{1-\mu-(2-c-2\mu-7\mu c)c}{16(1-\mu)} \tag{A.28}$$

If we now substitute the solution into the objective function of (A.24), then the patent holder's expected profits when $0 < c \leq c_3(\mu)$ are

$$E[\pi^{\text{PH}}] = \frac{1-\mu-2(1-\mu)c+(1-\mu+2\mu^2)c^2}{4(1-\mu)} \tag{A.29}$$

(ii) If the patent holder does not screen potential users, then he can offer the same contract (F, r) to both user types by ignoring the IC constraints. That being said, the patent holder has two options: offer contracts under which only efficient users are encouraged to produce; or offer contracts under which both good and bad users find it attractive to produce. In the former case, the patent holder solves the problem

$$\max_{F_0, r_0} 2(\mu^2 + \mu(1-\mu))(F + rq) \quad \text{s.t.} \;\; E[\pi^i(0)] \geq F \tag{A.30}$$

here $E[\pi^i(0)]$ are the expected profits for user i in a Cournot equilibrium where only efficient users produce. To compute $E[\pi^i(0)]$, each user takes the rival's production as given and solves the problem

$$\max_{q^i}(1-r-q^i-\mu q^j)q^i, \quad j \neq i \tag{A.31}$$

The Cournot solution for this problem is $q^i = \frac{1-r}{2+\mu}$, and users' expected profits are

$$E[\pi^i(0)] = \left(\frac{1-r}{2+\mu}\right)^2 \tag{A.32}$$

It follows that, in the solution to the patent holder's problem (A.30), the fixed fee is given by

$$F = \left(\frac{1-r}{2+\mu}\right)^2 \tag{A.33}$$

hence the problem can be rewritten as

$$\max_r 2\mu\left(\left(\frac{1-r}{2+\mu}\right)^2 + r\frac{1-r}{2+\mu}\right) \tag{A.34}$$

The solution to (A.34) is

$$r = \frac{\mu}{2+2\mu} \tag{A.35}$$

and the patent holder's expected profits are

$$E[\pi^{\mathrm{PH}}] = \frac{\mu}{2+2\mu} \tag{A.36}$$

In the alternative scenario under which the patent holder allows both user types to produce, his problem becomes

$$\max_{(F,r)} 2\mu^2(F+rq_0) + 2\mu(1-\mu)\big(2F + r(q_0+q_c)\big) + 2(1-\mu)^2(F+rq_c) \tag{A.37}$$
$$\text{s.t.} \quad E[\pi^i(0)] \geq F \text{ and } E[\pi^i(c)] \geq F$$

In the restrictions to this problem, $E[\pi^i(\tilde{c})]$ ($i \in \{1,2\}$, $\tilde{c} \in \{0,c\}$) is derived by solving the problem

$$\max_{q^i_{\tilde{c}}}(1-r-q^i_{\tilde{c}}-\mu q^j_0-(1-\mu)q^j_c)q^i_{\tilde{c}} \tag{A.38}$$

whose solution in a Cournot equilibrium is the same as the solution to problem (A.13) when $r_0 = r_c = r$—namely,

$$q_c = \frac{1}{6}(2-2c-2r-\mu c) \tag{A.39}$$

Using these quantities to compute $E[\pi^i(\tilde{c})]$ in problem (A.38) now gives

$$E[\pi^i(0)] = \frac{1}{36}(2 + c - 2r - \mu c)^2 > \frac{1}{36}(2 - 2c - 2r - \mu c)^2 = E[\pi^i(c)] \tag{A.40}$$

Therefore, the solution to problem (A.37) must verify

$$F = E[\pi^i(c)] = \frac{1}{36}(2 - 2c - 2r - \mu c)^2 \tag{A.41}$$

a problem that can be recast as

$$\max_r \frac{1}{18}(4(c - 2r - 1)(c + r - 1) + 4\mu c(c + 4r - 1) + \mu^2 c^2) \tag{A.42}$$

The solution to (A.42) is

$$r = \frac{1}{4} + \left(\mu - \frac{1}{4}\right)c \tag{A.43}$$

which applies as long as $c \leq \min\left\{\frac{1}{2}, \frac{1}{1+2\mu}\right\}$; otherwise, production would not be positive.

Taking the solution to the objective function in (A.37), we obtain

$$E[\pi^{\mathrm{PH}}] = \frac{1}{4}(1 + (c + 2\mu^2 c - 2)c) \tag{A.44}$$

When $c \geq \min\left\{\frac{1}{2}, \frac{1}{1+2\mu}\right\}$, the patent holder will choose the contract—given by (A.32) and (A.35)—that allows only efficient users to produce; this strategy yields him the profits given in (A.36). To see which option is better for the patent holder when $c \leq \min\left\{\frac{1}{2}, \frac{1}{1+2\mu}\right\}$, we must compare his profits as given by (A.36) and as given by (A.44). Thus we find that both user types should be allowed to produce if and only if

$$c \leq c_4(\mu) = \min\left\{\frac{1}{2}, \frac{1}{1+2\mu^2}\left(1 - \sqrt{\frac{2\mu(1-\mu+\mu^2)}{1+\mu}}\right)\right\} \tag{A.45}$$

in which case the patent holder offers the contract given by (A.41) and (A.43). Otherwise, if $c \geq c_4(\mu)$, he is better-off allowing only efficient users to produce—in which case he offers the contract given by (A.33) and (A.35). Figure A4 shows the regions in which the patent holder benefits from offering two different contracts.

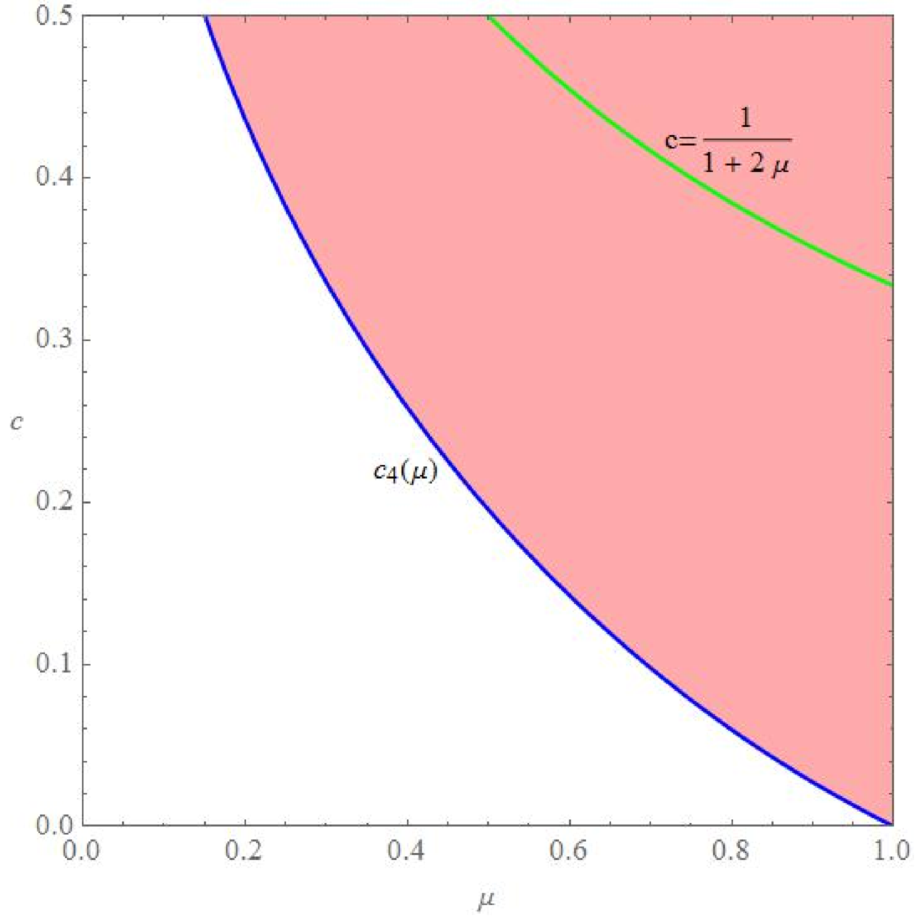

Figure A4. The entire region beneath the blue locus is where the patent holder prefers a single contract under which both user types produce. In the pink area, the patent holder is better-off with a single contract under which only efficient users produce. (No such comparisons are relevant *above* the green locus because a contract under which all user types produce is not profitable in that region.)

■

Proof of Proposition 5

To identify the patent holder's preferred contract, we need to compare his profits under the contracts established via parts (i) and (ii) of Proposition 4. When inequality (A.45) is satisfied, it is easy to see that the patent holder's profits under the single contract given by (A.44) are less than his profits under a separating contract as given by (A.29). However, when c is not less than the RHS of (A.45), a separating contract is better than a single contract intended for efficient users—in other words, the profits given by (A.29) are greater than those given by (A.36)—if and only if

$$c \le c_5(\mu) = \min\left\{\frac{1}{2}, \frac{1-\mu}{1-\mu+2\mu^2}\left(1-\sqrt{\frac{2\mu(1-\mu)}{1+\mu}}\right)\right\} \qquad \text{(A.46)}$$

Finally, the patent holder's profits are given by (A.29) when $0 < c \le 5(\mu)$ and by (A.36) when $c_5(\mu) \le c < 0.5$. ■

Proof of Proposition 6

If licensing is exclusive then we know from Proposition 2 that, above the locus $\mu + c = 1$, the patent holder will offer a shut-down contract whereby only the efficient user produces. In the region where a separating contract is profitable (green area in Figure 1; this region includes the area under the blue locus in Figure 2), we must determine which is better for the patent holder: the separating contract or the corresponding shut-down contract as follows:

1. If $0 \le c \le c_1(\mu) = \min\left\{\frac{1}{2}, \frac{1}{1+\mu^2}\left(1 - \sqrt{\mu(1-\mu+\mu^2)}\right)\right\}$, then the patent holder's profits under a shut-down contract are given by (A.9). It is easy to check that (A.5) is always greater than (A.9), so the patent holder will choose the separating contract given in Proposition 2(i).

2. If $c_1(\mu) = \min\left\{\frac{1}{2}, \frac{1}{1+\mu^2}\left(1 - \sqrt{\mu(1-\mu+\mu^2)}\right)\right\} \le c < \min\left\{\frac{1}{2}, 1-\mu\right\}$, then the patent holder's profits under a shut-down contract are given by (A.7). A comparison of (A.5) and (A.7) reveals that the separating contract is better for the patent holder if and only if

$$\begin{aligned} c_1(\mu) = \min\left\{\frac{1}{2}, \frac{1}{1+\mu^2}\left(1 - \sqrt{\mu(1-\mu+\mu^2)}\right)\right\} &\le c \le c_2(\mu) \\ = \min\left\{\frac{1}{2}, \frac{1-\mu}{1-\mu+\mu^2}\left(1 - \sqrt{\mu(1-\mu)}\right)\right\} \end{aligned}$$

   Otherwise, the patent holder will choose the shut-down contract and receive the profits given by (A.7).

We can use the previous reasoning to conclude that, under exclusive licensing, patent holder profits are given by (A.5) as long as $c \le c_2(\mu)$ and otherwise are given by (A.7):

$$E[\pi_1^{\mathrm{PH}}] = \begin{cases} \frac{1}{4}\left(1 - 2c - \mu c^2 + \frac{c^2}{1-\mu}\right) & \text{if } 0 < c \le c_2(\mu) \\ \frac{\mu}{4} & \text{if } c_2(\mu) \le c < \frac{1}{2} \end{cases} \tag{A.47}$$

With non-exclusive licensing, the proof of Proposition 5 implies that

$$E[\pi_2^{\mathrm{PH}}] = \begin{cases} \frac{1-\mu-2(1-\mu)c+(1-\mu+2\mu^2)c^2}{4(1-\mu)} & \text{if } 0 < c \le c_5(\mu) \\ \frac{\mu}{2+2\mu} & \text{if } c_5(\mu) \le c < \frac{1}{2} \end{cases} \tag{A.48}$$

Subscripts "1" and "2" mean "one license" and "two licenses", respectively. Comparing (A.47) and (A.48) now yields the stated result. ■

Proof of Proposition 7

With *exclusive* licensing, expected consumer net surplus is given by

$$E[CS_1] = \frac{1}{2}(\mu(1-p_0)q_0 + (1-\mu)(1-p_c)q_c) \tag{A.49}$$

for $p_0 = 1 - q_0$ and $p_c = 1 - q_c$. Similarly, expected poducer net surplus (before issuing costs) are

$$E[PS_1] = \mu(1 - q_0)q_0 + (1 - \mu)(1 - c - q_c)q_c \tag{A.50}$$

With *non-exclusive* licensing, the expected consumer surplus is given by

$$E[CS_2] = \frac{1}{2}(\mu^2(1 - p_0)2q_0 + 2\mu(1 - \mu)(1 - p_{0c})(q_0 + q_c) + (1 - \mu)^2(1 - p_c)2q_c) \tag{A.51}$$

(here $p_0 = 1 - 2q_0$, $p_{0c} = 1 - (q_0 + q_c)$, and $p_c = 1 - 2q_c$) and the expected producer net surplus is

$$E[PS_2] = \mu^2(1 - 2q_0)2q_0 + 2\mu(1 - \mu)\big(1 - (q_0 + q_c)\big)(q_0 + q_c) + \ (1 - \mu)^2(1 - 2q_c)2q_c \tag{A.52}$$

(i) Under *symmetric* information, the monopoly quantities produced by a single user are $q_0 = \frac{1}{2}$ and $q_c = \frac{1-c}{2}$. Hence the expected consumer surplus is

$$E[CS_1^{SI}] = \frac{\mu+(1-\mu)(1-c)^2}{8} \tag{A.53}$$

Here the superscript "SI" stands for "symmetric information". The user's profits are equal to $\frac{1}{4}$ or to $\frac{(1-c)^2}{4}$ according as whether she is a good or bad producer. Therefore, from (A.50) it follows that the user's gross expected surplus (before issuing costs) are equal to

$$E[PS_1^{\text{SI}}] = \frac{\mu+(1-\mu)(1-c)^2}{4} \tag{A.54}$$

Under non-exclusive licensing, the Cournot quantities produced by users are $q_0^i = \frac{1}{4}$ and $q_c^i = \frac{1-c}{2}$ for (respectively) good and bad users of the patented innovation. Hence it follows from (A.49) that the expected consumer surplus is

$$E[CS_2^{\text{SI}}] = \frac{1-(1-\mu)^2(2-c)c}{8} \tag{A.55}$$

and from (A.50) that the expected porducers net surplus is

$$E[PS_2^{\text{SI}}] = \frac{1-\ (1-\mu)^2(2-c)c}{4} \tag{A.56}$$

Equations (A.53)–(A.56) establish that the total surplus from non-exclusive licensing is greater than that from exclusive licensing if and only if $S \leq \tilde{S}(\mu, c)$, where $\tilde{S}(\mu, c)$ solves

$$\frac{\mu+(1-\mu)(1-c)^2}{8} + \frac{\mu+(1-\mu)(1-c)^2}{4} - \tilde{S}(\mu, c) = \frac{1-\ (1-\mu)^2(2-c)c}{8} + \frac{1-\ (1-\mu)^2(2-c)c}{4} - 2\tilde{S}(\mu, c) \tag{A.57}$$

therefore,

$$\tilde{S}(\mu, c) = \frac{3}{8}\mu(1-\mu)(2-c)c \tag{A.58}$$

It is easy to show that $\tilde{S}(\mu, c) > S^*(\mu, c)$ for all $(\mu, c) \in (0,1) \times \left(0, \frac{1}{2}\right)$. Moreover, if $S^*(\mu, c) \leq S \leq \tilde{S}(\mu, c)$ then the patent holder will choose exclusive licensing—although aggregate expected welfare would be greater under non-exclusive licensing.

(ii) Under *asymmetric* information, the expected consumer net surplus from exclusive licensing is given by

$$E[CS_1^{\mathrm{AI}}] = \begin{cases} \frac{(1-c)^2 - \mu(1-2c)}{8(1-\mu)} & \text{if } c < c_2(\mu) \\ \frac{\mu}{8} & \text{if } c > c_2(\mu) \end{cases} \tag{A.59}$$

where "AI" stands for "asymmetric information". (There is no surplus defined when $c = c_2(\mu)$ because in that case production is not continuous.) Equation (A.59) is obtained from (A.49) as follows. Quantities q_0 and q_c are the monopoly quantities produced by (respectively) the good and bad user. According to Proposition 2, a self-selection (screening) contract results in $q_0 = \frac{1}{2}$ and $q_c = \frac{1-c-\mu}{2(1-\mu)}$ whereas a shut-down contract—under which only the efficient user produces—results in $q_0 = \frac{1}{2}$ and $q_c = 0$. Furthermore, Proposition 3 shows that the patent holder will choose the former contract if $c \leq c_2(\mu)$ and will otherwise choose the latter. Substituting these values into (A.49), our expression for the consumer surplus when the profit-maximizing patent holder chooses exclusive licensing, now yields (A.59).

Under exclusive licensing, we obtain expected producer surplus (before issuing costs) is by substituting the quantities from Propositions 2 and 3 into (A.50). The result is

$$E[PS_1^{\mathrm{AI}}] = \begin{cases} \frac{1-\mu-2(1-\mu)^2 c + (1-2\mu)c^2}{4(1-\mu)} & \text{if } c < c_2(\mu) \\ \frac{\mu}{4} & \text{if } c > c_2(\mu) \end{cases} \tag{A.60}$$

Total expected welfare $E[W_1^{\mathrm{AI}}]$ under exclusive licensing and asymmetric information is therefore

$$E[W_1^{\mathrm{AI}}] = E[CS_1^{\mathrm{AI}}] + E[PS_1^{\mathrm{AI}}] = \begin{cases} \frac{3(1-\mu)-(6-10\mu+4\mu^2)c+(3-4\mu)c^2}{8(1-\mu)} & \text{if } c < c_2(\mu) \\ \frac{3\mu}{8} & \text{if } c > c_2(\mu) \end{cases} \tag{A.61}$$

When licensing is non-exclusive, we can use (A.51) and the results of Propositions 4 and 5 to derive the expected consumer net surplus as

$$E[CS_2^{\mathrm{AI}}] = \begin{cases} \frac{1-\mu+(c+\mu(2+c)-2)c}{8(1-\mu)} & \text{if } c < c_5(\mu) \\ \frac{1}{4(1+\mu)} & \text{if } c > c_5(\mu) \end{cases} \tag{A.62}$$

and the expected porducers net surplus as

$$E[PS_2^{\mathrm{AI}}] = \begin{cases} \frac{1-\mu-(3+2\mu(\mu-3))c+(2-\mu-2\mu^2)c^2}{8(1-\mu)} & \text{if } c < c_5(\mu) \\ \frac{\mu(1+2\mu)}{2(1+\mu)^2} & \text{if } c > c_5(\mu) \end{cases} \tag{A.63}$$

Then total expecte surplus amounts to

$$E[W_2^{\mathrm{AI}}] = E[CS_2^{\mathrm{AI}}] + E[PS_2^{\mathrm{AI}}] = \begin{cases} \frac{3+\left(5(c-2)+\frac{2}{1-\mu}+4\mu(1+c)\right)c}{8} & \text{if } c < c_5(\mu) \\ \frac{\mu(3+5\mu)}{4(1+\mu)^2} & \text{if } c > c_5(\mu) \end{cases} \tag{A.64}$$

Now suppose that the issuing costs $S^*(\mu, c)$ are such that the patent holder is indifferent between exclusive and non-exclusive licensing. In this case, non-exclusive licensing yields higher total expected welfare (than does exclusive licensing) if and only if

$$E[W_2^{\mathrm{AI}}] - E[W_1^{\mathrm{AI}}] \geq S^*(\mu, c) \tag{A.65}$$

After substituting the expressions for $E[W_1^{\mathrm{AI}}]$, $E[W_2^{\mathrm{AI}}]$, and $S^*(\mu, c)$, we see that this inequality holds as long as

$$\begin{cases} \min\left\{\frac{1}{2}, c_2(\mu)\right\} \leq c & \text{if } 0 < \mu \leq 0.29 \\ \frac{-3+7\mu-6\mu^2+2\mu^3}{-3+6\mu-4\mu^2+2\mu^3} - \sqrt{\frac{3\mu+\mu^2-13\mu^3+21\mu^4-34\mu^5+34\mu^6-16\mu^7+4\mu^8}{(1+\mu)^2(-3+6\mu-4\mu^2+2\mu^3)^2}} \leq c & \text{if } 0.29 \leq \mu \leq 0.49 \\ \frac{2-8\mu^2+4\mu^3}{2+5\mu-8\mu^2+2\mu^3} \leq c \leq c_5(\mu) & \text{if } 0.47 \leq \mu \leq 0.49 \\ \max\left\{\frac{2-8\mu^2+4\mu^3}{2+5\mu-8\mu^2+2\mu^3}, 0\right\} \leq c & \text{if } 0.49 \leq \mu < 1 \end{cases} \tag{A.66}$$

Comparing this region with the region (defined in Proposition 6 and shown in Figure 3) in which the patent holder grants two licenses, we see that the latter is nearly—but not completely—subsumed by the former; see Figure A5. The implication is that, under asymmetric information, if issuing costs are given by $S^*(\mu, c)$ then the number of licenses (either one or two) granted by the patent holder is socially efficient for a wide range of parameters. There is also a tiny region (the blank area above the locus $c_7(\mu)$) in which the patent holder chooses non-exclusive licensing even though aggregate welfare is higher under exclusive licensing.

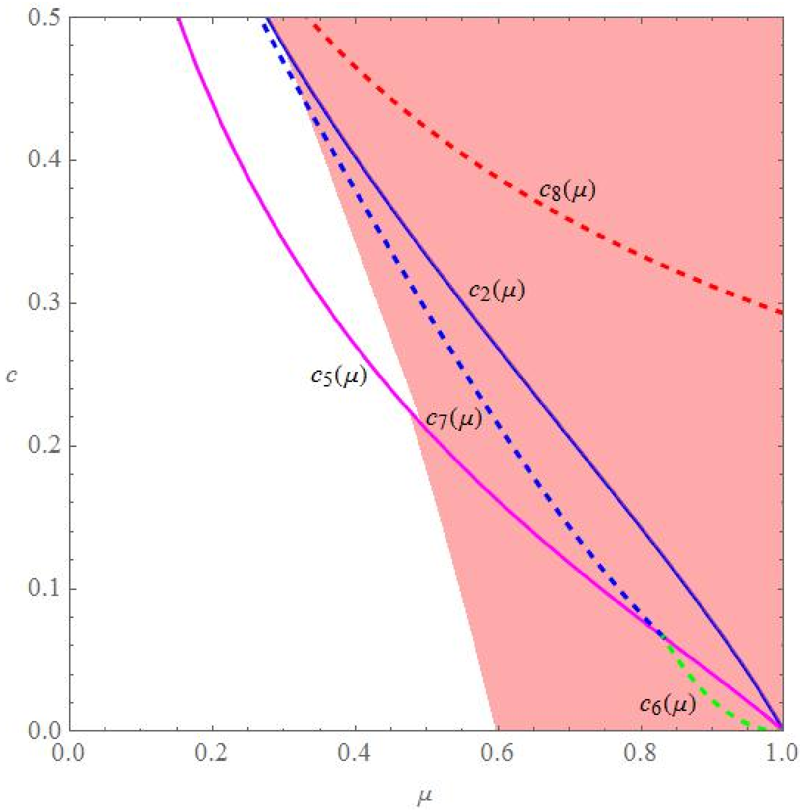

Figure A5. When the issuing cost is given by $S^*(\mu, c)$, non-exclusive licensing yields a greater surplus in the red area; in the blank area, a greater surplus results from exclusive licensing. The patent holder chooses non-exclusive licensing in the area delimited by $c_6(\mu)$, $c_7(\mu)$, and $c_8(\mu)$.

Next we assume that the issuing cost is given instead by $\tilde{S}(\mu, c)$. In this case, the region in which the patent holder chooses non-exclusive licensing is defined as in Proposition 6 but while using $\tilde{S}(\mu, c)$ (rather than $S^*(\mu, c)$) and with $c_6(\mu)$, $c_7(\mu)$, and $c_8(\mu)$ replaced by

$$c_6'(\mu) = \frac{2(3-6\mu+3\mu^2)}{3-4\mu+3\mu^2} \tag{A.67}$$

$$c_7'(\mu) = \frac{5\mu-6\mu^2+3\mu^3}{-2+5\mu-8\mu^2+3\mu^3} - \sqrt{\frac{2\mu-11\mu^2+25\mu^3-34\mu^4+36\mu^5-27\mu^6+9\mu^7}{(1+\mu)(5\mu-8\mu^2+3\mu^3-2)^2}} \tag{A.68}$$

$$c_8'(\mu) = 1 - \sqrt{\frac{1+3\mu}{3(1+\mu)}} \tag{A.69}$$

respectively. Now repeating the computations for total surplus but with $\tilde{S}(\mu, c)$ instead of $S^*(\mu, c)$, we find that non-exclusive licensing leads to a higher aggregate surplus if and only if

$$c \geq \begin{cases} \min\left\{\frac{1}{2}, c_1(\mu)\right\} & \text{if } 0 < \mu \leq 0.42 \\ \frac{-3+8\mu-8\mu^2+3\mu^3}{-3+7\mu-6\mu^2+3\mu^3} - \sqrt{\frac{7\mu^2-13\mu^3+14\mu^4-33\mu^5+46\mu^6-30\mu^7+9\mu^8}{(1+\mu)^2(7\mu-6\mu^2+3\mu^3-3)^2}} & \text{if } 0.42 \leq \mu \leq 0.54 \\ \max\left\{\frac{2+2\mu-12\mu^2+6\mu^3}{2+6\mu-10\mu^2+3\mu^3}, 0\right\} & \text{if } 0.54 \leq \mu < 1 \end{cases} \quad \text{(A.70)}$$

The patent holder chooses non-exclusive licensing in the region defined by (A.67)–(A.69). Yet that area is contained within the region, defined by (A.70), where aggregate surplus (under non-exclusive licensing) is higher; see Figure A6.

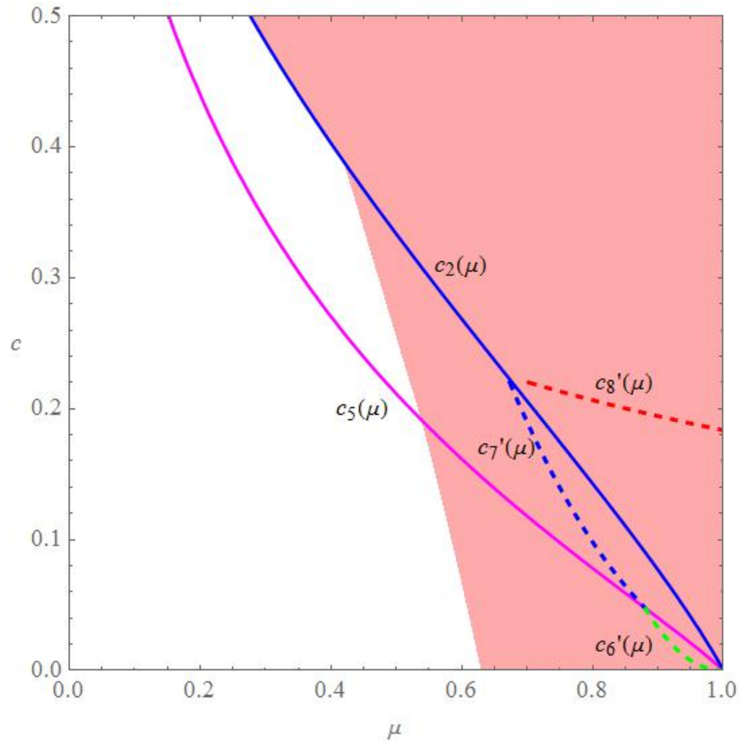

Figure A6. When the issuing cost is $\tilde{S}(\mu, c)$, non-exclusive licensing yields a greater surplus in the red area; in the blank area, a surplus results from exclusive licensing. The patent holder chooses non-exclusive licensing in the area delimited by $c_6'(\mu)$, $c_7'(\mu)$, and $c_8'(\mu)$.

■