Optimal Privatization and Uniform Subsidy Policies: A Synthesis

Lin, Ming Hsin and Matsumura, Toshihiro

Osaka University of Economics, The University of Tokyo

4 March 2017

Optimal Privatization and Uniform Subsidy Policies: A Synthesis*

Ming Hsin Lin†
Faculty of Economics, Osaka University of Economics
and
Toshihiro Matsumura‡
Institute of Social Science, The University of Tokyo

March 4, 2017

Abstract

The privatization neutrality theorem states that the share of public ownership in an enterprise does not affect welfare (i.e., any degree of privatization is optimal) under optimal uniform tax-subsidy policy. We revisit this neutrality result. First, we investigate the case in which the private enterprise is domestic. We show that this neutrality result does not hold unless public and private enterprises have the same cost function. In addition, we show that the optimal degree of privatization is zero regardless whether the public or private firm has a cost advantage under the optimal subsidy policy. Next, we investigate a case in which the private enterprise is owned by both domestic and foreign investors. We show that the optimal degree of privatization is never zero, and thus, the neutrality result does not hold even when there is no cost difference between public and private enterprises.

JEL classification numbers: H42, L13

Keywords: mixed oligopoly, mixed ownership, subsidy policy, partial privatization, optimal degree of privatization

---

*We acknowledge financial support from the Japan Society for the Promotion of Science (JSPS) KAKENHI (Grant Numbers 15K03347 and 16K03681) and Zengin Foundation for Studies on Economics and Finance. We are responsible for any errors.

†Corresponding author. Faculty of Economics, Osaka University of Economics, 2-2-8, Osumi, Higashiyodogawa-ku, Osaka, 533-8533, Japan. E-mail: linmh@osaka-ue.ac.jp

‡E-mail: matsumur@iss.u-tokyo.ac.jp

1 Introduction

Studies on mixed oligopolies in which public enterprises compete with private enterprises have recently attracted more attention and have become increasingly popular.[1] Despite a wave of privatization being prevalent, state-owned public enterprises still exist and are active. Some public enterprises are monopolists in natural monopoly markets. However, many public enterprises compete with private enterprises in a wide range of industries. Such mixed oligopolies are still important in many developed, developing, and formerly communist transitional countries. In addition, some influential firms facing financial problems have been nationalized, thereby creating new mixed oligopolies.[2]

Because production levels are often suboptimal for social welfare under imperfect competition, public enterprises play an important role in making up for underproduction by private enterprises. In the literature on mixed oligopolies, it is mostly assumed that public enterprises maximize social surplus (the sum of consumer surplus and enterprises' profits), whereas private enterprises maximize their own profits. Because public enterprises take into consideration consumers' benefits as well as their own profits, public enterprises produce more than private enterprises do. As a result, the existence of public enterprises mitigates the inefficiency due to underproduction in the markets. Although the existence of public enterprises increases total output, it reduces the output of private enterprises, which might yield another inefficiency in the form of production allocation. The literature on mixed oligopolies has showed that the welfare loss of mixed oligopolies can be larger than that of private oligopolies, and thus, privatization of public enterprises can be welfare improving.[3]

Since Merrill and Schneider (1966), many studies of mixed oligopolies have investigated

[1] See Ishida and Matsushima (2009), Bose *et al.* (2014), and papers cited therein.

[2] Examples are Tokyo Electric Power Corporation, Japan Airlines, General Motors, Federal Home Loan Mortgage Corporation, Anglo Irish Bank, Northern Rock, and Bradford & Bingley Plc.

[3] See Matsumura and Shimizu (2010) and papers cited therein.

the situation wherein the government seeks to control public enterprises inside the market as an instrument of regulation, instead of using industrial policies from outside the markets. In many mixed markets, however, governments intervene using subsidies. Typical examples are medical care, education, energy, finance, and international trade. The subsidy policy might mitigate the problem of inefficient allocation of production among the public and private enterprises, as mentioned above. Thus, if we were to consider subsidy policy explicitly, the implication of the privatization policy discussed in the literature on mixed oligopoly might change drastically.

White (1996) made an important contribution on this issue. He showed that a uniform production subsidy yields the first-best outcome in both mixed and private oligopolies. In other words, the optimal uniform subsidy policy removes inefficient production allocation, regardless of whether public enterprises are privatized. His result has rich policy implications. Privatization does not affect welfare under the optimal uniform subsidy policy (privatization neutrality theorem).

Many studies following White (1996) have proved that this result is robust. As for competition structures, Poyago-Theotoky (2001) and Myles (2002) considered a Stackelberg model in which the public enterprise is the Stackelberg leader. They showed that the resulting welfare is the same as that for mixed and private Cournot oligopolies under optimal uniform subsidy policies. Tomaru and Saito (2010) showed that their result holds when the public enterprise is the Stackelberg follower.[4] Hashimzade *et al.* (2007) showed that the privatization

[4]Fjell and Heywood (2004) showed a different result from those of Poyago-Theotoky (2001), Myles (2002), and Tomaru and Saito (2010). Fjell and Heywood (2004) assumed that private firms move sequentially in their private duopoly, whereas the others assumed that all firms move simultaneously after privatization. As Hamilton and Slutsky (1990) showed, in a private duopoly, simultaneous-move outcome appears in equilibrium in the endogenous timing game. By contrast, in some mixed duopolies, sequential-move outcomes appear in equilibrium (Pal, 1998; Matsumura, 2003a,b; Tomaru and Kiyono, 2010) and in other mixed duopolies, simultaneous-move might appear (Matsumura and Ogawa, 2010; Bárcena-Ruiz and Garzón, 2010; and Capuano and De Feo, 2010). Therefore, we believe that the time structure in Poyago-Theotoky (2001), Myles (2002), and Tomaru and Saito (2010) is more plausible than that in Fjell and Heywood (2004).

neutrality theorem holds in a differentiated product market. Tomaru (2006) adopted the partial privatization approach formulated by Matsumura (1998) and showed that a degree of privatization does not matter under optimal subsidy policy. Kato and Tomaru (2007) considered nonprofit-maximizing private enterprises and showed that the theorem holds under various payoff functions of private enterprises. These works have demonstrated that the privatization neutrality theorem is robust.[5]

In this study, we revisit this theorem. We adopt the partial privatization approach formulated by Matsumura (1998) and investigate the conditions under which the privatization neutrality theorem (any degree of privatization is optimal under optimal **uniform** subsidy policy) holds.[6] We discuss the combination of optimal subsidy policy and privatization policy under fairly general demand and cost functions.

First, we investigate the case in which the private enterprise is domestic. We show that this neutrality result does not hold unless public and private enterprises have the same cost function. In addition, we show that the optimal degree of privatization is zero regardless whether the public or private enterprise has a cost advantage. Our nonneutrality result is in contrast to the above mentioned results.[7] In addition, the result that the optimal degree

[5]However, Matsumura and Tomaru (2013) showed that the privatization neutrality theorem does not hold if excess burden of taxation exists. However, this study, as well as many other studies, such as White (1996), Fjell and Heywood (2004), and Tomaru (2006), have adopted a popular but very specific model formulation (linear demand and quadratic or linear costs).

[6]Needless to say, the restriction of uniform subsidy policy is quite important. Suppose that n firms compete in a market, the government knows all of the cost and demand conditions, and it can set different subsidy-tax rates for each of n-firms. Then, it is absolutely obvious that the first best is achieved by such a nonuniform tax-subsidy policy in any market (both in mixed and private oligopolies, regardless whether the private firms maximize their profits, or whether firms move simultaneously or sequentially). This is a special case of the Tinbergen theorem. However, such discriminatory and discretional tax-subsidy policy is far from realistic. Such an unfair tax-subsidy policy is quite difficult to implement for political reasons. Therefore, in this study we restrict our attention to the uniform tax-subsidy case, following most studies in this field and all studies mentioned in this paper.

[7]Although we discuss subsidy policy only, our principle can apply also to the discussion of other policy tools. For example, Matsumura and Okumura (2013) showed that an optimal output-floor regulation leads to privatization neutrality regardless whether firms move sequentially or simultaneously. We can show, however, that their neutrality result does not hold if there is a cost difference between the two firms or the private firm is foreign.

of privatization is zero is in sharp contrast to that of Matsumura (1998), who showed that the optimal degree of privatization is never zero without subsidy policy under moderate conditions.

Next, we investigate the case in which the private enterprise is at least partially owned by foreign investors.[8] We show that even when there is no cost difference between public and private enterprises, the optimal degree of privatization is never zero. This result again implies that the neutrality result does not hold, and that the nationality of the private enterprise affects the optimal privatization policy even when the government uses a subsidy policy, too.[9] This result is also in sharp contrast to the result without subsidy. Without subsidy, the optimal degree of privatization is decreasing in the share of foreign ownership in private enterprises (Lin and Matsumura, 2012). However, under optimal subsidy policy, this result does not hold because the optimal degree of privatization is zero without foreign ownership and it is strictly positive with foreign ownership.

Our two results suggest that under optimal uniform subsidy policy, introduction of foreign ownership of private firms changes the incentive of the government to participate less in the public firm. This has rich policy implications.[10]

The rest of this study is organized as follows. Section 2 formulates the duopoly model. Section 3 investigates the case with a domestic private enterprise. Section 4 investigates the

[8]The nationality of private enterprises is often crucial in shaping mixed oligopolies because it affects the objective function (domestic welfare) of the public enterprise. See the literature starting with Corneo and Jeanne (1994) and Fjell and Pal (1996). See also Pal and White (1998) and Bárcena-Ruiz and Garzón (2005a,b).

[9]Matsumura and Tomaru (2012) also investigated the relationship between privatization and subsidy policies. They showed that privatization affects welfare under foreign penetration. However, they used specific demand and cost functions (linear demand and quadratic cost functions). More importantly, they allowed only full privatization and full nationalization in an enterprise (i.e., they did not allow partial privatization), and thus, they did not discuss the optimal degree of privatization.

[10]A similar result is obtained in Cato and Matsumura (2012), who showed that the optimal degree of privatization increases with foreign ownership share in private firms. However, these authors derived this result for a free-entry market and they emphasized that the result does not hold if the number of firms were given exogenously. For the privatization neutrality theorem in free-entry markets, see Cato and Matsumura (2013).

case in which the private enterprise is owned by both domestic and foreign investors. Section 5 concludes.

2 The Model

Firm 0 and firm 1 produce perfectly substitutable commodities for which the (inverse) demand function is given by $p(Q) : \mathbb{R}_+ \mapsto \mathbb{R}_+$. We assume that $p(Q)$ is twice continuously differentiable and $p'(Q) < 0$ for all Q as long as $p > 0$. Firm 0 is a (semi) public firm that is jointly owned by both public and private sectors, and firm 1 is a pure private firm. We assume that $p'(Q) + p''(Q)q_i < 0$ as long as $p > 0$, where q_i is firm i's $(i = 0, 1)$ output. In other words, we assume that the strategy of the private firm is a strategic substitute under quantity competition. A sufficient, but not necessary, condition is $p'' \leq 0$.

Firm 1's cost is $c_1 = c(q) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ and firm 0's cost is $c_0(q) = kc(q)$ where k is a positive constant. $k >\ (=, <)\ 1$ implies that the public firm is less efficient than (as efficient as, more efficient than) the private firm.[11] We assume that $c' \geq 0$ and $c'' > 0$.[12]

Firm i's profit π_i is $p(Q)q_i - c_i(q_i) + sq_i$ where q_i $(i = 0, 1) \in \mathbb{R}_+$ is firm i's output quantity, $Q := q_0 + q_1$ is total output, and $s \in \mathbb{R}$ is the production subsidy (if s is negative, it is a production tax).

Following a common assumption in the literature, we assume that the subsidy is financed from taxes imposed on other industries unrelated to the industry under study. The social

[11] Some readers might consider the public firm less efficient than the private firm. However, not all empirical studies support this view. See Megginson and Netter (2001) and Stiglitz (1988). In addition, see Martin and Parker (1997), who suggested that the change in corporate performance went both ways after privatization in the UK. For a discussion of endogenous cost differences between public and private firms, see Matsumura and Matsushima (2004).

[12] If $c'' = 0$, we can show that the privatization neutrality theorem holds if and only if $k = 1$ or k is sufficiently large or small in which the more efficient firm becomes the monopolist, regardless of α under optimal subsidy policy. In the former case, our main results hold. In the latter case, it is absolutely nonsense to discuss a mixed oligopoly. Therefore, we do not discuss this case.

surplus W is the sum of consumer surplus and producer surplus

$$\begin{aligned} W(q_0, q_1) &= \int_0^Q p(q)dq - pQ + \pi_0(q_0, q_1) + \pi_1(q_1, q_0) - sQ \\ &= \int_0^Q p(q)dq - kc(q_0) - c(q_1). \end{aligned} \tag{1}$$

Note that s does not appear in (1) because the subsidy is income transfer from the government to the firms and does not yield social surplus. However, s affects the output of each firm, and thus, affects welfare.

Firm 1 maximizes its profit, while firm 0 maximizes the weighted average of social surplus and its own profit, $\alpha\pi_0 + (1-\alpha)W$, where $\alpha \in [0, 1]$ indicates the degree of privatization. If $\alpha = 0$, firm 0 is fully nationalized. If $\alpha = 1$, firm 0 is fully privatized.[13]

The game proceeds as follows. In the first stage, the government chooses α and s to maximize W. In the second stage, two firms simultaneously choose their outputs.

Before solving this game, we briefly discuss what would happen if the government could control q_0 and q_1 directly. The first-order conditions are

$$p(q_0 + q_1) - kc'(q_0) = 0, \quad p(q_0 + q_1) - c'(q_1) = 0. \tag{2}$$

The second-order conditions are satisfied.[14] Let a pair of (q_0^*, q_1^*) be the first-best outputs. Obviously, $q_0^* > \ (=, <)\ q_1^*$ if $k < \ (=, >)\ 1$.

We assume that the solution is interior (i.e., $q_0^* \neq 0$ and $q_1^* \neq 0$). A sufficient condition for it is that k is neither too large nor too small. Another sufficient condition is $c'(0) = 0$.[15]

[13]This is a standard model formulation of partial privatization in the literature on mixed oligopoly (Matsumura, 1998). We do not allow the government to nationalize both firms. As pointed out by Merrill and Schneider (1966), the most efficient outcome is achieved by the nationalization of both firms in the case in which nationalization does not change the firms' costs. The need for an analysis of mixed oligopoly lies in the fact that it is impossible or undesirable, for political or economic reasons, to nationalize an entire sector. For example, without a competitor, public firms might lose the incentive to improve their costs, resulting in a loss of social welfare. Thus, we neglect the possibility of nationalizing both firms.

[14]Because we assume $p' < 0$ and $c'' > 0$, we obtain $p' - kc'' < 0$, $p - c'' < 0$ and $(p' - kc'')(p - c'') > (p')^2$.

[15]If $c'(0) = 0$, then $kc'(0) = 0$. Because $c'' > 0$, the corner solution is never optimal for social welfare.

3 Equilibrium

We now solve the game by backward induction. First, we discuss the second-stage game given α and s. The first-order conditions for firm 0 and firm 1 are

$$p + \alpha p' q_0 - kc' + \alpha s = 0, \tag{3}$$

$$p + p' q_1 - c' + s = 0, \tag{4}$$

respectively. The assumption $p' + p'' q_i < 0$ ensures that the strategies of both firms are strategic substitutes regardless of α, and that the second-order and stability conditions are satisfied. Let $q_i^E(\alpha, s)$ be the equilibrium output of firm i ($i = 0, 1$) in the second-stage subgame.

In the first stage, the government chooses α and s to maximize W. By definition, if a pair of (α, s) yields $(q_0^E, q_1^E) = (q_0^*, q_1^*)$ in equilibrium, W is maximized by the policy.

Let $s^* := -p'(q_0^* + q_1^*)q_1^*$. Substituting $s = s^*$ and $(q_0, q_1) = (q_0^*, q_1^*)$ into the left-hand side in (4), we obtain the result that the first-order condition for firm 1 is satisfied. Note that $p(q_0^* + q_1^*) - c'(q_1^*) = 0$. Because the left-hand side in (4) is decreasing in s given $(q_0, q_1) = (q_0^*, q_1^*)$, firm 1 chooses $q_1 = q_1^*$ given $q_0 = q_0^*$ if and only if $s = s^*$. This yields the following result.

Lemma 1 *The first best (i.e., $(q_0, q_1) = (q_0^*, q_1^*)$) is achieved only if $s = s^*$.*

We now discuss the optimal privatization policy. Substituting $\alpha = 0$ and $(q_0^E, q_1^E) = (q_0^*, q_1^*)$ into (3), we obtain the result that the first-order condition for firm 0 is satisfied. This implies that a pair $(\alpha, s) = (0, s^*)$ yields the first-best outcome (Proposition 1).

Proposition 1 (An optimal policy) $(\alpha, s) = (0, s^*)$ *yields the first-best outcome regardless of k.*

The optimal subsidy policy $s = s^*$ induces the private firm to choose the first-best output

q_1^* given $q_0 = q_0^*$. Given $q_1 = q_1^*$, the welfare maximizer (firm 0 with $\alpha = 0$) also chooses the first-best output q_0^*.

We now discuss whether any other policies yield the first-best outcome. Suppose $k = 1$. Substituting $s = s^*$ and $(q_0, q_1) = (q_0^*, q_1^*)$ into the left-hand side in (3), we obtain that the first-order condition for firm 0 is satisfied regardless of α (note that $q_0^* = q_1^*$ when $k = 1$). This implies the following result.

Proposition 2 (Privatization neutrality result) *Suppose $k = 1$. Any $\alpha \in [0, 1]$ is optimal as long as $s = s^*$.*

This result is shown in White (1996) in the model in which only full privatization and full nationalization are allowed, and in Tomaru (2006) in the model in which partial privatization is allowed. However, in their models, linear demand and quadratic cost functions are assumed. Proposition 2 states that the results of these studies hold under general demand and cost functions.

We now show that this neutrality result does not hold under any cost difference between public and private firms.

Proposition 3 (Nonneutrality result) *Suppose $s = s^*$. Suppose $\alpha = \alpha^{**}$ is one of the optimal privatization policies. Then*

$$\left.\frac{dq_0^E}{d\alpha}\right|_{\alpha=\alpha^{**}} = 0$$

if and only if $k = 1$.

Proof See the Appendix.

Proposition 3 is one of our main result. We have already shown that $(\alpha, s) = (0, s^*)$ yields the first-best outcome (Proposition 1), and that the first-best outcome is achieved only when $s = s^*$ (Lemma 1). Thus, the privatization neutrality theorem (any $\alpha \in [0, 1]$ is one of the optimal privatization policies) holds only when α does not affect q_0^E. Therefore, Proposition 3 implies that the privatization neutrality theorem does not hold unless the public and private firms are equally efficient (i.e., $k = 1$).

We explain the intuition behind Propositions 2 and 3. When the public and private firms are equally efficient, the optimal output is the same for both firms. The government induces the private firm to produce the optimal output by choosing the optimal subsidy rate s^*. Given that $q_1 = q_1^*$, $q_0 = q_0^* = q_1^*$ is the best for a welfare maximizer (when $\alpha = 0$) and for a profit maximizer (when $\alpha = 1$), any $\alpha \in [0, 1]$ yields the optimal behavior of firm 0 as long as $s = s^*$ (Proposition 2).

When the private firm is more or less efficient than the public firm, the optimal output of firm 0 is different from that of firm 1. When firm 0 is less (more) efficient, the optimal output of firm 0 is smaller (larger) than that of firm 1, and s^* is too high (low) to induce firm 0 to produce efficient output unless firm 0 is a pure welfare maximizer (Proposition 3).

In our analysis, the uniform subsidy rate is crucial. Needless to say, if we were to allow different subsidy rates between firms 0 and 1 (i.e., if the government can set s_0 and s_1 for firms 0 and 1, respectively), it would be obvious from the classical Tinbergen theorem that the first best is achieved by the optimal subsidy policy, regardless of α. We believe that such a discriminatory and discretional subsidy policy is far from realistic. Such an unfair tax-subsidy policy is quite difficult to be implemented for political reasons. If there were two firms, such an unfair policy might be possible. However, if there were n firms, n subsidy rates would be required, and the policy would be far from realistic when n is large. Therefore, we believe that the assumption of a uniform subsidy rate imposed in the literature is plausible and relevant.

4 Foreign Private Firm

In the previous section, we show that the privatization neutrality theorem holds only when there is no cost difference between public and private firms. However, we assume that the private firm is a domestic firm (i.e., is owned by domestic investors only). In this section, we allow the private firm to be owned by both domestic and foreign investors. We show that even when there is no cost difference between public and private firms, the privatization neutrality theorem does not hold unless the private firm is purely domestically owned. Henceforth, we assume $k = 1$.[16]

Let β denote the foreign ownership share in firm 1. Domestic surplus is given by

$$\begin{aligned} W(q_0, q_1) &= \int_0^Q p(q)dq - pQ + \pi_0(q_0, q_1) + (1-\beta)\pi_1(q_1, q_0) - sQ \\ &= \int_0^Q p(q)dq - c(q_0) - (1-\beta)c(q_1) - \beta(p+s)q_1. \end{aligned} \tag{5}$$

In contrast to (1), s appears in (5) because a part of the subsidy flows out to foreign investors.

We now solve the game by backward induction. We discuss the second-stage game given α and s. The first-order condition for firm 1 is given by (4), and that for firm 0 is given by

$$p + \alpha p' q_0 - (1-\alpha)\beta p' q_1 - c' + \alpha s = 0. \tag{6}$$

We assume that $|p'|$ is sufficiently large relative to $|p''|$ or c'' is sufficiently large. This ensures that the second-order and stability conditions are satisfied.

In the first stage, the government chooses α and s to maximizes W. We now present the nonneutrality result.

Proposition 4 $\alpha = 0$ *is optimal only when* $\beta = 0$.

Proof See the Appendix.

[16] Proposition 4 holds even when $k \neq 1$. A sufficient but not necessary condition is that the welfare-maximizing public firm (i.e., $\alpha = 0$) produces more than the profit-maximizing private firm does in equilibrium, and a sufficient condition for it is $k = 1$. This holds unless k is too large.

Proposition 4 states that unless the private firm is purely domestic (i.e., $\beta = 0$), full nationalization is never optimal (i.e., either partial or full privatization is optimal). This implies that the privatization neutrality theorem does not hold unless $\beta = 0$. Proposition 4 is in sharp contrast to Proposition 1, which states that $\alpha = 0$ is optimal when the private firm is domestic. Without a subsidy policy, the optimal degree of privatization is decreasing in β (Lin and Matsumura, 2012). Thus, Proposition 4 suggests that introducing a subsidy policy significantly affects the relationship between β and the optimal degree of privatization.

We explain the intuition behind Proposition 4. When firms are domestic, the subsidy is income transfer from the government to the firms, and thus, there is no direct welfare loss. Thus, the government chooses s to induce the optimal output level of the private firm. When the private firm is owned by foreign investors, the subsidy to the private firm increases the outflow to the foreign investors, and causes direct welfare loss. Therefore, the government has a lesser incentive to raise s and to stimulate production of the private firm. As a result, the equilibrium output level of firm 1 must be lower than that of the optimal output level for domestic welfare (underproduction of the private firm). A marginal increase in α from 0 makes the public firm less aggressive and reduces q_0, and through strategic interaction, it increases q_1. Because firm 0 is a welfare maximizer before the increase of α, a marginal decrease in q_0 affects domestic welfare by the second order (envelope theorem), while a marginal increase in q_1 improves domestic welfare by the first order. Thus, a marginal increase in α from 0 always improves domestic welfare when $\beta > 0$.

5 Concluding Remarks

In this study, we revisit the privatization neutrality theorem (a degree of privatization does not affect welfare under optimal subsidy policy). We find that the neutrality result does not hold unless there is no cost difference between public and private firms, and the private firm is owned by domestic investors only. Our result suggests that the privatization neutrality theorem is far from robust.

In addition, we find that the optimal privatization policy is crucially dependent on the nationality of the private firm. When the private firm is domestic, the optimal degree of privatization is zero, while it is never zero if the private firm is even partially owned by foreign investors.

In this study, we do not explicitly derive the optimal degree of privatization and the optimal subsidy rate under foreign penetration of the private firm. It is quite difficult to derive these without specifying the demand and cost functions. Deriving the optimal degree of privatization and investigating the relationship between foreign ownership share, the optimal degree of privatization, and the optimal subsidy rate by specifying the demand and cost functions remains for future research.

In this study, we assume that demand and cost functions are given exogenously. In addition, we assume that foreign ownership share is given exogenously. However, advertising or R & D activities might affect these functions, and regulation policy might affect foreign ownership share in private firms. Endogenizing these might affect optimal subsidy and privatization policies.[17] We believe that this is a promising future research topic.

[17] For a discussion of advertising or demand-creating activities in mixed oligopolies, see Han and Ogawa (2012) and Matsumura and Sunada (2013). For discussions on R & D competition in mixed oligopolies, see Matsumura and Matsushima (2004), Ishibashi and Matsumura (2006), and Gil-Molto *et al.* (2011).

Appendix

Proof of Proposition 3

By differentiating (3) and (4), we obtain

$$H \begin{pmatrix} dq_0 \\ dq_1 \end{pmatrix} = - \begin{pmatrix} p'q_0 + s \\ 0 \end{pmatrix} d\alpha, \tag{7}$$

where

$$H := \begin{pmatrix} (1+\alpha)p' + \alpha p'' q_0 - kc'' & p' + \alpha p'' q_0 \\ p' + p'' q_1 & 2p' + p'' q_1 - c'' \end{pmatrix}. \tag{8}$$

Because $\alpha \in [0, 1]$, $p' + p''q_i < 0$, and $c'' > 0$, we obtain $\det H > 0$.

By applying Cramer's rule to (7), we obtain

$$\frac{dq_0}{d\alpha} = -\frac{(p'q_0 + s)(2p' + p''q_1 - c'')}{\det H}. \tag{9}$$

From (9), $dq_0/d\alpha$ is zero if and only if $p'q_0 + s = 0$. Suppose that $(\alpha, s) = (\alpha^{**}, s^*)$ is one of the best policies. Because we have already shown that $(\alpha, s) = (0, s^*)$ yields the first-best outcome and $s = s^*$ is a necessary condition to yield the first-best outcome, $\alpha = \alpha^{**}$ is one of the best policies only when it also yields the first-best outcome (i.e., $(q_0, q_1) = (q_0^*, q_1^*)$) given $s = s^*$. By definition of s^*, $p'q_1^* + s^* = 0$, and thus, $p'q_0^* + s^* = 0$ if and only if $q_0^* = q_1^*$. This holds if and only if $k = 1$. **Q.E.D.**

Proof of Proposition 4

We prove Proposition 4 by contradiction. We assume that $\alpha = 0$ is one of the optimal policies.

By differentiating (6) and (4), we obtain

$$G \begin{pmatrix} dq_0 \\ dq_1 \end{pmatrix} = - \begin{pmatrix} \alpha \\ 1 \end{pmatrix} ds, \tag{10}$$

where

$$G := \begin{pmatrix} (1+\alpha)p' + \alpha p'' q_0 - (1-\alpha)\beta p'' q_1 - c'' & (1-\beta+\alpha\beta)p' + \alpha p'' q_0 - (1-\alpha)\beta p'' q_1 \\ p' + p'' q_1 & 2p' + p'' q_1 - c'' \end{pmatrix}. \tag{11}$$

Under the assumptions made, we obtain $\det G > 0$.

By applying Cramer's rule to (10), we obtain

$$\begin{aligned} \frac{dq_1}{ds} &= -\frac{((1+\alpha)p' + \alpha p'' q_0 - (1-\alpha)\beta p'' q_1 - c'') - \alpha(p' + p'' q_1)}{\det G} \\ &= -\frac{p' + \alpha p''(q_0 - q_1) - (1-\alpha)\beta p'' q_1 - c''}{\det G} > 0. \end{aligned} \tag{12}$$

Note we assume that $|p'|$ is sufficiently large relative to $|p''|$ or c'' is sufficiently large.

The government maximizes W with respect to s. The first-order condition is

$$\begin{aligned} \frac{dW}{ds} &= \frac{\partial W}{\partial q_0}\frac{dq_0}{ds} + \frac{\partial W}{\partial q_1}\frac{dq_1}{ds} + \frac{\partial W}{\partial s} \\ &= (p - \beta p' q_1 - c')\frac{dq_0}{ds} + ((1-\beta)(p-c') - \beta(p' q_1 + s))\frac{dq_1}{ds} - \beta q_1 = 0. \end{aligned} \tag{13}$$

From (6), we obtain $\partial W/\partial q_0 = p - \beta p' q_1 - c' = 0$ when $\alpha = 0$. Because $dq_1/ds > 0$, from (13) we obtain

$$(1-\beta)(p-c') - \beta(p' q_1 + s) > 0. \tag{14}$$

From (14), we obtain that either $(p - c')$ or $-\beta(p' q_1 + s)$ must be positive. From (4), we obtain $p - c' = -(p' q_1 + s)$, and thus, both $p - c'$ and $-(p' q_1 + s)$ are positive.

We now show that welfare is improved if the government slightly increases α from 0. We show that

$$\begin{aligned} \left.\frac{dW}{d\alpha}\right|_{\alpha=0} &= \frac{\partial W}{\partial q_0}\frac{dq_0}{d\alpha} + \frac{\partial W}{\partial q_1}\frac{dq_1}{d\alpha} \\ &= (p - \beta p' q_1 - c')\frac{dq_0}{d\alpha} + ((1-\beta)(p-c') - \beta(p' q_1 + s))\frac{dq_1}{d\alpha} > 0. \end{aligned} \tag{15}$$

Because $\partial W/\partial q_0 = p - \beta p' q_1 - c' = 0$ and $\partial W/\partial q_1 = (1-\beta)(p-c') - \beta(p' q_1 + s) > 0$, we have to show only $dq_1/d\alpha > 0$.

By differentiating (6) and (4), we obtain

$$G \begin{pmatrix} dq_0 \\ dq_1 \end{pmatrix} = - \begin{pmatrix} \beta p' q_1 + p' q_0 + s \\ 0 \end{pmatrix} d\alpha. \tag{16}$$

By applying Cramer's rule to (16), we obtain

$$\frac{dq_1}{d\alpha} = \frac{(\beta p' q_1 + p' q_0 + s)(p' + p'' q_1)}{\det G}. \tag{17}$$

From (17), $dq_1/d\alpha > 0$ if $\beta p' q_1 + p' q_0 + s < 0$.

We now show that $\beta p' q_1 + p' q_0 + s < 0$ and thus, $dq_1/d\alpha > 0$. From (4) and (6), we obtain $q_0 > q_1$ when α is sufficiently close to zero. Note we have already shown that $p' q_1 + s < 0$. Thus, $\beta p' q_1 + p' q_0 + s < \beta p' q_1 + p' q_1 + s$, and it is negative because $\beta p' q_1 < 0$ and $p' q_1 + s < 0$.

Therefore, a marginal increase in α from 0 improves welfare, which contradicts the result that $\alpha = 0$ is one of the optimal policies. Q.E.D.

References

Bárcena-Ruiz, Carlos, Juan, and Begona Maria Garzón (2005a) 'Economic integration and privatization under diseconomies of scale.' *European Journal of Political Economy* 21(1), 247–67.

Bárcena-Ruiz, Carlos, Juan, and Begona Maria Garzón (2005b) 'International trade and strategic privatization.' *Review of Development Economics* 9(4), 502–13.

Bárcena-Ruiz, Carlos, Juan, and Begona Maria Garzón (2010) 'Endogenous timing in a mixed oligopoly with semipublic firms.' *Portuguese Economic Journal* 9(2), 97–113.

Bose, Arup, Debashis Pal, and David E. M. Sappington (2014) 'The impact of public ownership in the lending sector.' *Canadian Journal of Economics* 47(4), 1282–311.

Capuano, Carlo, and Giuseppe De Feo (2010) 'Privatization in oligopoly: the impact of the shadow cost of public funds.' *Rivista Italiana Degli Economisti* 15(2), 175–208.

Cato, Susumu, and Toshihiro Matsumura (2012) 'Long-run effects of foreign penetration on privatization policies.' *Journal of Institutional and Theoretical Economics* 168(3), 444–54.

Cato, Susumu, and Toshihiro Matsumura (2013) 'Long-run effects of tax policies in a mixed market.' *FinanzArchiv* 69(2), 215–40.

Corneo, Giacomo, and Olivier Jeanne (1994) 'Oligopole mixte dans un marche commun.' *Annales d'Economie et de Statistique* 33, 73–90.

Fjell, Kenneth, and John S. Heywood (2004) 'Mixed oligopoly, subsidization and the order of firm's moves: the relevance of privatization.' *Economics Letters* 83(3), 411–6.

Fjell, Kenneth, and Debashis Pal (1996) 'A mixed oligopoly in the presence of foreign private firms.' *Canadian Journal of Economics* 29(3), 737–43.

Gil-Molto, Maria Jose, Joanna Poyago-Theotoky, and Vasileios Zikos (2011) 'R&D subsidies, spillovers, and privatization in mixed markets.' *Southern Economic Journal* 78(1), 233–55.

Hamilton, Jonathan H., and Steven M. Slutsky (1990) 'Endogenous timing in duopoly games: Stackelberg or Cournot equilibria.' *Games and Economic Behavior* 2(1), 29–46.

Han, Lihua, and Hikaru Ogawa (2012) 'Market-demand boosting and privatization in a mixed duopoly.' *Bulletin of Economic Research* 64(1), 125–34.

Hashimzade, Nigar, Hassan Khodavaisi, and Gareth Myles (2007) 'An irrelevance result with differentiated goods.' *Economics Bulletin* 8(2), 1–7.

Ishibashi, Ikuo, and Toshihiro Matsumura (2006) 'R&D competition between public and private sectors.' *European Economic Review* 50(6), 1347–66.

Ishida, Junichiro, and Noriaki Matsushima (2009) 'Should civil servants be restricted in wage bargaining? A mixed-duopoly approach.' *Journal of Public Economics* 93(3–4), 634–46.

Kato, Kazuhiko, and Yoshihiro Tomaru (2007) 'Mixed oligopoly, privatization, subsidization and the order of firms' moves: several types of objectives.' *Economics Letters* 96(2), 287–92.

Lin, Ming Hsin, and Toshihiro Matsumura (2012) 'Presence of foreign investors in privatized firms and privatization policy.' *Journal of Economics* 107(1), 71–80.

Martin, Stephen, and David Parker (1997) *The Impact of Privatisation. Ownership and Corporate Performance in the UK*, London and New York: Routledge.

Matsumura, Toshihiro (1998) 'Partial privatization in mixed duopoly.' *Journal of Public Economics* 70(3), 473–83.

Matsumura, Toshihiro (2003a) 'Endogenous role in mixed markets: a two production period model.' *Southern Economic Journal* 70(2), 403–13.

Matsumura, Toshihiro (2003b) 'Stackelberg mixed duopoly with a foreign competitor.' *Bulletin of Economic Research* 55(3), 275–87.

Matsumura, Toshihiro, and Noriaki Matsushima (2004) 'Endogenous cost differentials between public and private enterprises: a mixed duopoly approach.' *Economica* 71(284), 671–88.

Matsumura, Toshihiro, and Akira Ogawa (2010) 'On the robustness of private leadership in mixed duopoly.' *Australian Economic Papers* 49(2), 149–60.

Matsumura, Toshihiro, and Yasunori Okumura (2013) 'Privatization neutrality theorem revisited.' *Economics Letters* 118(2), 324–26.

Matsumura, Toshihiro, and Daisuke Shimizu (2010) 'Privatization waves.' *Manchester School* 78(6), 609–25.

Matsumura, Toshihiro, and Takeaki Sunada (2013) 'Advertising competition in a mixed oligopoly.' *Economics Letters* 119(2), 183–5.

Matsumura, Toshihiro, and Yoshihiro Tomaru (2012) 'Market structure and privatization policy under international competition.' *Japanese Economic Review* 63(2), 244–58.

Matsumura, Toshihiro, and Yoshihiro Tomaru (2013) 'Mixed duopoly, privatization, and subsidization with excess burden of taxation.' *Canadian Journal of Economics* 46(2), 526–54.

Megginson, William L., and Jeffry M. Netter (2001) 'From state to market: a survey of empirical studies on privatization.' *Journal of Economic Literature* 39(2), 321–89.

Merrill, William C., and Norman Schneider (1966) 'Government firms in oligopoly industries: a short-run analysis.' *Quarterly Journal of Economics* 80(3), 400–12.

Myles, Gareth (2002) 'Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result for the general case.' *Economics Bulletin* 12(1) 1–6.

Pal, Debashis (1998) 'Endogenous timing in a mixed oligopoly.' *Economics Letters* 61(2), 181–5.

Pal, Debashis, and Mark D. White (1998) 'Mixed oligopoly, privatization, and strategic trade policy.' *Southern Economic Journal* 65(2), 264–81.

Poyago-Theotoky, Joanna (2001) 'Mixed oligopoly, subsidization, and the order of firm's moves: an irrelevance result.' *Economics Bulletin* 12(3), 1–5.

Stiglitz, Joseph E. (1988) *Economics of the Public Sector*, 2nd Edition, Norton, New York.

Tomaru, Yoshihiro (2006) 'Mixed oligopoly, partial privatization and subsidization.' *Economics Bulletin* 12(5), 1–6.

Tomaru, Yoshihiro, and Kazuharu Kiyono (2010) 'Endogenous timing in mixed duopoly with increasing marginal costs.' *Journal of Institutional and Theoretical Economics* 166(4), 591–613.

Tomaru, Yoshihiro, and Masayuki Saito (2010) 'Mixed duopoly, privatization and subsidization in an endogenous timing framework.' *Manchester School* 78(1), 41–59.

White, Mark D. (1996) 'Mixed oligopoly, privatization and subsidization.' *Economics Letters* 53(2), 189–95.