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Abstract

We analyze individual decisions of when to face difficult tasks. Although threatening, difficult tasks provide better economic outcomes than easy ones. We argue how individual dispositions, i.e., the expression of some non-cognitive dimensions, might drive timing decisions. Specifically, when experiencing low dispositions, individuals get trapped into low value easy tasks while when experiencing high dispositions, they are willing to always deal with high value difficult tasks. Also, when outcome achievements motivate individuals, they move from low value easy tasks to high value difficult tasks. This finding is interpreted as individuals preparing themselves to cope with difficulties.

Keywords: individual dispositions, task difficulty, avoidance behavior
JEL classification: D83, D84

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1 Introduction

One of the reasons as to why individuals tend to avoid difficult tasks is because they do not feel able enough to confront with them. Not coping with them might, however, imply foregoing the opportunity of getting better economic outcomes, not available otherwise. As Liebow (1967) documents in his study of the Negro male community of Washington inner city:

"Convinced of their inadequacies, not only do they not seek out those few better-paying jobs which test their resources, but actively avoid them, gravitating in a mass to the menial, routine jobs which offer no challenge -and therefore posse not threat- to the already diminished images they have of themselves(...). Thus, the man's low self-esteem generates a fear of being tested and prevents him from accepting a job with responsibilities or, once on a job, form staying with it if responsibilities are thrust on him, even if wages are commensurably higher."

The story above offers two interesting insights. The first one is that individual dispositions might dramatically influence decisions of huge economic relevance. When documenting the relationship between the *achievement motive* and upward mobility patterns in the United States, Atkinson and Feather (1966) highlight how, despite of the fact that education is the main determinant of upward mobility, individual dispositions should not be neglected. In fact, 65% of the people who exhibited upward mobility patterns at a higher extent, only had high school education or less.[1] The second one is the trade-off between task difficulty and economic outcomes. While a routine job is probably more easily developed than a very demanding one, better economic outcomes, as higher wages or promotion opportunities, might only be available in the latter.[2]

Our purpose in this paper is to understand and highlight the role played by individual dispositions in shaping avoidance behavior. We interpret individual dispositions as an expression of non-cognitive abilities.[3] Examples of non-cognitive abilities are emotional stability, that manifests, among others aspects, in self-confidence and self-esteem, or conscientiousness, that manifests, among others aspects, in persever-

[1] To establish comparisons between the prestige of occupations of parents and sons, private households populated by people older than 21 were interviewed. Specific measures related to individual dispositions to strive for success (i.e., the achievement motive) were collected.

[2] Also, as Atkinson and Feather (1966) suggest, high prestige occupations are perceived as being more difficult to attain than low prestige occupations. This hierarchy can be seen as a series of tasks in which the outcome value comes together with difficulty.

[3] We will interchangeably use the term state, disposition or simply ability when referring to the non-cognitive ability level that the individual enjoys.

ance.[4] In order to do it, we develop a tractable model in which the decision maker, henceforth DM, who is characterized by a disposition (that is, a non-cognitive abilities level), decides the optimal time to deal with difficulties.

More specifically at each point in time the DM might experience two states, namely, the full capacity state and the deteriorated capacity state, with a constant positive probability. Experiencing the full capacity state means that she enjoys high dispositions while experiencing the deteriorated capacity state means that her dispositions are low. Tasks are of two types, easy and difficult ones. On the one hand, getting good economic outcomes is less likely under difficult than under easy tasks but on the other hand, outcomes associated to difficult tasks are more valuable than outcomes associated to easy tasks. We consider that states and economic performance are positively related, specifically, the higher the DM's disposition the higher the probability of being successful when developing a task, either easy or difficult. It is worth mentioning that no effort decision is analyzed here. As we previously stated, the only decision of the DM is when to confront with difficulties. We assume that once she decides to confront with them, she sticks at this decision forever.

We also consider an extension of the model in which we analyze the role of motivation. That is, we study the case in which individuals' dispositions are sensitive to outcome achievements. Formally, the probabilities of experiencing the full capacity state and the deteriorated capacity state are not constant over time. We assume that their value at a given period depends on their value and on the likelihood of good economic outcomes in the previous period.[5]

Our results are as follows. We find how a low disposition DM will avoid difficulties forever while a high disposition DM will cope with them since the beginning. Thus, individuals with poor abilities get trapped into low value easy tasks. However, when motivation plays a role, the achievement of good economic outcomes out of easy tasks leaves the DM with the disposition of coping with difficulties from some point in time on.[6] In line with this finding it is worth mentioning the results of a program carried out in West Bengal, by the indian microfinance institution *Bandhan*, consisting on providing extremely poor individuals with productive assets. The authors observed how people ended up working 28% more hours, mostly on activities not related to the assets they were given and that their mental health had improved. The program

[4]See John and Srivastava (1999) for the Big Five domains of non-cognitive abilities, their traits and facets.

[5]In particular we assume that probabilities evolve according to a Markov process.

[6]This is consistent with Ali (2011), a model in which the long-run self, the planner, decides at each point in time whether to allow the short-run self, the doer, to face a menu in which a tempting alternative is available. The planner does so whenever the doer experiences high self-control.

was considered to have injected a dose of motivation, that pushed people to start new economic activities.[7]

Our proposal is closely related to the branch of literature that links poverty and psychology. For instance, Dalton et al. (2014) discuss the importance of aspirations failure in the perpetuation of poverty. This paper, as ours, highlights the role of internal constraints as a source of behaviors that might preclude individuals from getting high welfare achievements. Their research question is, however, different from ours, whereas they focus in one particular bias we analyze non-cognitive abilities. We also find relations with the literature of addiction and self-control. Specifically, Bernheim and Rangel (2004) study patterns of addictive behavior of a DM that operates in two modes, namely, *cold* and *hot*. When in the cold mode the DM selects her most preferred alternative whereas when in the hot mode, choices and preferences may diverge because the DM losses cognitive control. This paper presents a theory of addiction whereas ours focuses on the effects of non-cognitive abilities in the decision of facing onerous but valuable tasks. Also, Ozdenoren et al. (2012) exhaustively account for the dynamics of self-control performance of a DM that has to choose her optimal consumption path. We depart from this paper since we focus on outcome achievement and motivation, and not on capacity exhaustion, as the mechanism determining decisions.

The paper is organized as follows. Section 2 presents the baseline version of the model. In this version, the probability of experiencing the full capacity state is time independent. Section 3 presents the extension of the model. In it, the probability of experiencing the full capacity state is sensitive to outcome achievements. The dynamics of its evolution is therefore outlined. In both sections, optimal strategies and their associated utility gains are presented. Section 4 concludes. Section 5 contains technical proofs.

2 A model on avoidance behavior

Let s_1 and s_2 be the two states that the DM might experience. When experiencing s_2 the DM is in the full capacity state and enjoys high abilities. When experiencing s_1 the DM is in the deteriorated capacity state, meaning that she executes her abilities poorly. At every point in time, $t \in \mathbb{Z}_+$, she has a probability $q \in [0, 1]$ of experiencing s_2. Thus, she experiences s_1 with probability $1 - q$. Tasks are of two types, (e)asy ones, denoted d_1, and (d)ifficult ones, denoted d_2.

The likelihood of getting good economic outcomes is denoted p_{ij}, with $i, j = 1, 2$.

[7] The complete article is available at http://www.economist.com/node/21554506.

Subscript i refers to the DM's state, that is, either s_1 or s_2, whereas subscript j refers to the difficulty of the task, that is, either d_1 or d_2. Probabilities are as follows: first, fixing difficulty, the likelihood of good economic outcomes increases with the DM's state. There is, in fact, a large amount of literature posing non-cognitive abilities as one of the factors determining performance and outcomes, for instance, in the domains of education and in the labor market.[8] Second, fixing the DM's state, the likelihood of good economic outcomes decreases with task's difficulty. Table 1 presents these probabilities. They increase as we move downward and to the right:

Table 1: Probabilities of getting good economic outcomes.

$$\mathrm{P} \equiv \begin{array}{c|cc} & s_1 & s_2 \\ \hline d_2 & p_{12} & p_{22} \\ d_1 & p_{11} & p_{21} \end{array}$$

Notice that no assumption is made about the relationship between p_{11} and p_{22}. Finally, good economic outcomes are worth just 1 unit when they are the result of developing easy tasks and $K > 1$ units when they come out of developing difficult tasks.

We make an assumption regarding the probabilities of success. It captures the idea that individuals with low dispositions are more vulnerable than individuals with high dispositions to the characteristics of the tasks they deal with. For high disposition individuals, task's difficulty is less relevant than for low disposition individuals in determining their chances of success.[9] We formally express it as:

Assumption 1: $p_{11} - p_{12} > p_{21} - p_{22}$.

The second assumption is related to the strategies among which the DM chooses:

Assumption 2: once the DM decides to face difficulties, she commits to this decision forever.

Regarding this assumption there exists evidence showing that in many situations individuals become locked into costly courses of action and a cycle of escalating commitment arises. The justification of previous decisions, the necessity to comply with norms or a desire for decision consistency in the decision making process, might

[8]See Heckman et al. (2006) and Balart and Cabrales (2014) for an illustration of this relationship.

[9]On the domain of cognitive abilities Gonzalez (2005) provides experimental evidence on how increasing task difficulty, understood as workload, was more detrimental for individuals with low abilities.

encourage commitment.[10] The strategies available to the DM therefore comprise choosing the point in time in which to face difficulties. We denote by (∞) the *always avoiding difficulties* strategy and by (0) the *facing difficulties since the beginning* strategy. A strategy consisting on facing difficulties from a point in time $0 < t < \infty$ on, is denoted (t).[11]

The DM behaves as an expected utility maximizer. She determines her optimal path of action at the initial point in time, taking into account her disposition, that is, the point-wise probability q of being in the full capacity state. We consider that the DM is risk neutral. We then focus on the role of dispositions without dealing with risk aversion issues.[12] Thus, the current expected utility of developing an easy task at time t is $qp_{21} + (1-q)p_{11}$ and the current expected utility of developing a difficult task at time t is $K(qp_{22} + (1-q)p_{12})$. Furthermore, let $\delta \in (0,1)$ denote the discount factor of the stream of pay-offs. We now formally state the DM's problem. It is as follows:

When experiencing the full capacity state, s_2, with probability q, the DM decides, at $t = 0$, the point in time t to face difficult tasks, in order to maximize her long-run expected utility. Specifically, she solves:

$$\underset{t}{Max}\ u((t)) = \underset{t}{Max} \sum_{i=0}^{t-1} \delta^i (qp_{21} + (1-q)p_{11}) + K \sum_{i=t}^{\infty} \delta^i (qp_{22} + (1-q)p_{12})$$

2.1 Results

It seems intuitive that individuals who enjoy better dispositions perform tasks better. In fact, it is common that people tend to avoid difficulties when they do not feel prepared to face them. Results in this section capture this idea. When the DM experiences the full capacity state with high enough probability, she will opt for difficulties since the beginning. In contrast, when the probability of experiencing the full capacity state is low enough, she will prefer to avoid them forever.

We build results in Theorem 1 by means of a function $\lambda : [0,1]^4 \times \mathbb{R} \longrightarrow \mathbb{R}$, that depends on the primitives of the model, defining the environment in which the DM maker her decision, namely, the probabilities of good economic outcomes and their value. It defines a domination threshold between the strategy of facing difficulties

[10] See Staw (1981) for the concept of *Escalation of Commitment*. See also Arkes and Blumer (1985) and Thaler (1980) for a justification of this phenomenon based on the *sunk cost effect*.

[11] It is worth to stress how the explicit introduction of time does not aim to describe the evolution of abilities along the life cycle. Time only captures that, at each point in time, the DM develops a task.

[12] See Tanaka et al. (2010) for a paper studying the relationship between poverty and risk and time preferences.

since the beginning and the strategy of postponing them for one period, that is, between (0) and (1).[13] For values of q higher or equal than this threshold, (0) is preferred to (1) and for values of q smaller than it, (1) is preferred to (0). This information is enough to identify the optimal strategy.

Before stating the result it is worth highlighting that whenever outcomes out of difficult tasks do (respectively do not) compensate the decrease in probability of successfully dealing with them, that is, whenever $p_{11}/p_{12} \leq K$ (respectively $K \leq p_{21}/p_{22}$), the DM finds optimal to always face (respectively to always avoid) difficulties, even if $q = 0$ (respectively $q = 1$). We then focus on the interesting case in which $p_{21}/p_{22} < K < p_{11}/p_{12}$. Theorem 1 is as follows:

Theorem 1. *The DM's optimal strategy is to face difficulties since the beginning whenever she enjoys the full capacity state with high enough probability (that is, whenever $\lambda \leq q$) and to always avoid them whenever she experiences the full capacity state with low enough probability (that is, whenever $q < \lambda$).*

Notice that optimal paths of action are characterized by extreme behaviors. Facing difficulties from an intermediate point in time is never optimal. Furthermore, assumption 2 does not impose any restriction in this case, that is, going back from difficult to easy tasks is never considered. Notice also that if for the DM never (respectively always) postponing difficulties is the best thing to do, this will also be the case for other DM characterized by a higher (respectively by a lower) disposition (that is, a lower q). We interpret the always avoiding difficulties strategy as procrastination on onerous tasks.[14]

The following example aims to clarify the elements of the model and the first result:

Example. Consider a DM who is deciding which type of job to look for or to accept. Jobs are of two types, easy or routine jobs, that give the DM a payoff (wage) of 1, and high responsibility jobs with payoff of $K = 1.3$. The probabilities of properly dealing with either job are higher when the DM is in the full capacity state than when she is in the deteriorated capacity state, specifically:

	s_1	s_2
d_2	0.5	0.7
d_1	0.7	0.8

[13] Specifically, $\lambda = \dfrac{Kp_{12} - p_{11}}{(p_{21} - p_{11}) - K(p_{22} - p_{12})}$. It relates the long-run expected utility of (0) versus (1) under the deteriorated capacity state, that is, when $q = 0$ and the long-run expected utility of (0) versus (1), under the full capacity state, that is, when $q = 1$. See the proof of Theorem 1.

[14] See O'Donoghue and Rabin (2001) and O'Donoghue and Rabin (2008) for two references on procrastination.

In this case $\lambda = 0.31$. Thus if the DM is of low enough disposition (that is, if $q < 0.31$), she will find optimal to always postpone the acceptance of the high responsibility job, whereas if she is of high enough disposition (that is, if $q \geq 0.31$), she will find optimal to deal with the high responsibility job since the beginning.

We plot below the ranking of long-run expected utilities under the strategies the DM chooses among. The left figure illustrates the case in which always avoiding difficulties is optimal whereas the right figure illustrates the case in which facing them since the beginning is optimal:

Figure 1: (∞) is the optimal strategy

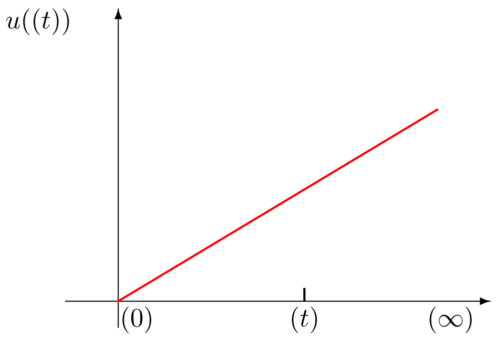
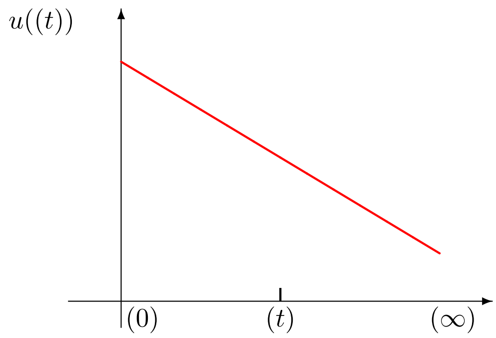


Figure 2: (0) is the optimal strategy



The following remark discusses the comparative statics on the primitives of the model, that define the environment in which the DM performs, namely, the probabilities of success and the value of good economic outcomes under difficult tasks:

Remark. The threshold λ is decreasing in p_{22} and p_{12}, increasing in p_{21} and p_{11}, and decreasing in K.

Higher chances of successfully developing either easy or difficult tasks make the DM more prone to choose each of them. In the same vein, an increase in the value of good economic outcomes out of difficult tasks incentivizes the DM to face them. An increase in K directly raises the utility of facing difficult tasks. This happens despite of the fact that the probability of achieving good economic outcomes in these circumstances is systematically lower. Theorem 1 and this remark are summarized as follows:

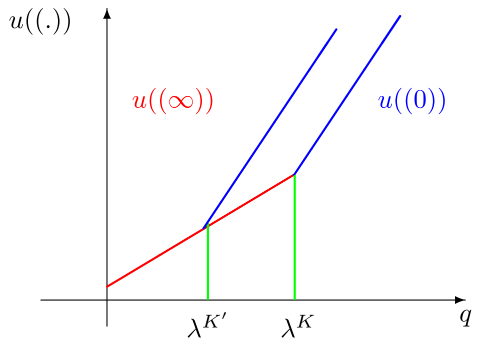
Corollary 1. *The range of probabilities $[q, 1]$ such that the DM's optimal strategy is to face difficulties since the beginning increases (respectively decreases) with the probability of success under difficult (respectively easy) tasks. It also increases with the value of economic outcomes out of difficult tasks.*

We also describe utility gains under the aforementioned optimal strategies and analyze the effect of a marginal boost in dispositions. We assume that a marginal increase in dispositions, does not affect the originally optimal strategy. Proposition 1 is as follows:

Proposition 1. *The long-run expected utility of any optimal strategy is monotonically increasing and linear q. Its value is $\dfrac{K(qp_{22}+(1-q)p_{12})}{1-\delta}$ whenever (0) is optimal and $\dfrac{qp_{21}+(1-q)p_{11}}{1-\delta}$ whenever (∞) is optimal. Moreover, the marginal return of an increase in q is higher when $q \geq \lambda$ than when $q < \lambda$.*

A marginal increase in the DM's disposition, increases the utility of any path of action and in particular, of the optimal one. Moreover, the higher the DM's disposition the higher this marginal return. The following figure illustrates the statement. The horizontal axis represents the probability of experiencing the full capacity state and the vertical axis represents the long-run expected utility of the optimal strategies, either (∞) or (0), according to this probability:

Figure 3: utility as a function of q, with $K' > K$



Observe how the threshold λ, which decreases with K, generates a kink, making utility convex (specifically, piece-wise linear) in q.

To conclude this section, it is worth mentioning the possibility of carrying out a welfare assessment analysis. The intuition is as follows: consider two individuals. One of them, the disadvantaged individual, has low abilities and always avoids difficulties, the other, the advantaged individual, has high abilities and always faces difficulties. It turns out that the marginal return of boosting abilities is higher for the advantaged individual than for the disadvantaged individual. Suppose that a social planner has one unit of resources, devoted to improve abilities. If it is the case that the planner only cares about maximizing total returns, he might allocate this unit on the advantaged individual. If he also has equity concerns, he will have to take into account that the utility gap between the advantaged and the disadvantaged individual will exacerbate. In this case, the planner might be willing to allocate resources on the disadvantaged individual.

3 The role of motivation

In this section we analyze how success and failure might affect the manifestation of non-cognitive abilities, for instance, self-esteem, self-confidence or perseverance.[15]

We assume that the probability of experiencing the full capacity state varies over time according to a Markovian process. This modeling aims to capture the idea that success may boost the manifestation of the non-cognitive abilities while failure may deteriorate them. Formally, the probability of experiencing the full capacity state at t, depends on its value at $t-1$, and on the probability of getting good economic outcome in that period. Let $q^{(t)} \in [0,1]$ denote this probability. Thus, $1-q^{(t)}$ denotes the probability of experiencing the deteriorated capacity state at t. Let the probabilities of getting good outcomes be the ones in the previous section. The following expression accounts for the evolution of the probabilities of experiencing either state:

$$[q^{(t-1)} \quad 1-q^{(t-1)}] \begin{bmatrix} p_{2j} & 1-p_{2j} \\ p_{1j} & 1-p_{1j} \end{bmatrix} = [q^{(t)} \quad 1-q^{(t)}] \tag{1}$$

where $j=\{1,2\}$ accounts for task's difficulty.[16] Consider that the DM experiences s_2 at $t-1$ with probability $q^{(t-1)}$. Then, at t she will experience s_2 with the probability with which she was successful in the previous period. This is captured by the first column in the matrix above. Similarly, at t she will experience s_1 with the probability with which she failed in the previous period. This is captured by the second column the matrix above. Let denote $q^{(0)}$, the DM's initial probability of experiencing the full capacity state. The current expected utility of developing an easy task at time t is $q^{(t)}p_{21} + (1-q^{(t)})p_{11}$ and the current expected utility of developing a difficult task at time t is $K(q^{(t)}p_{22} + (1-q^{(t)})p_{12})$.

Notice that, as a consequence of $q^{(t)}$ evolving according to a Markovian process,

[15] As Mruk (2006) points out, the demands of life are not constant, so self-esteem levels will fluctuate depending on what is happening in a persons life. Redundancy, bereavement, illness, studying, gaining a qualification, parenthood, poverty, being a victim of crime, divorce, promotion at work will all have an impact on our self-esteem levels. Self-esteem levels go up and down and can change over time. Also, as Bénabou and Tirole (2002) point out, motivation helps individuals to persevere in the presence of setbacks.

[16] Notice that $q^{(t)}$ depends on the chosen strategy. If the DM decides to face difficult tasks from a point in time $t^*=5$ on, $q^{(4)}$ is the resulting probability of having opted for easy tasks for four periods. If the DM decides to face difficult tasks from $t^*=3$ on, $q^{(4)}$ is the resulting probability of having faced easy tasks for two periods and difficult ones from the third period on.

two stationary probabilities arise. These are, the one related to always facing easy tasks, denoted q^e, and the one related to always facing difficult tasks, denoted q^d. We interpret them as the average long-run frequencies with which the DM experiences the full capacity state when she always faces easy or difficult tasks, respectively. Since the likelihood of success is higher in easy tasks, we have that $q^e > q^d$.[17] The DM's problem is as follows:

When experiencing the full capacity state, s_2, with probability $q^{(0)}$, she decides, at $t = 0$, the point in time t, to face difficult tasks, in order to maximize her long-run expected utility. Formally, she solves:

$$\underset{t}{Max}\ u((t)) = \underset{t}{Max} \sum_{i=0}^{t-1} \delta^i (q^{(i)} p_{21} + (1 - q^{(i)}) p_{11}) + K \sum_{i=t}^{\infty} \delta^i (q^{(i)} p_{22} + (1 - q^{(i)}) p_{12})$$

Results are summarized in the following section.

3.1 Results

Theorem 2 summarizes optimal strategies. As in the previous section our analysis relies on a function $\mu : [0,1]^4 \times \mathbb{R} \longrightarrow \mathbb{R}$, that depends on K and P.[18] It defines a domination threshold between strategy the strategy of facing difficulties since the beginning and the strategy of postponing them for one period, that is, between (0) and (1). For values of $q^{(0)}$ higher or equal than this threshold, (0) is preferred to (1) and for values of $q^{(0)}$ smaller than it, (1) is preferred to (0). This threshold, together with the stationary probabilities, determine the chosen strategy. In the following result we focus on the case in which assumption 2 does not play a role, that is, the case in which DM's optimal strategy is, in fact, within the class of strategies prescribed by assumption 2. We comment on the remaining cases afterwards. Theorem 2 is as follows:

Theorem 2. *The DM's optimal strategy is to face difficulties since the beginning whenever she always enjoys the full capacity state with high enough probability (that is, whenever $\mu \leq q^d < q^e, q^{(0)}$), to always avoid them whenever she always experiences the full capacity state with low enough probability (that is, whenever $q^{(0)}, q^d < q^e \leq \mu$) and to face them from an intermediate point in time whenever she gets*

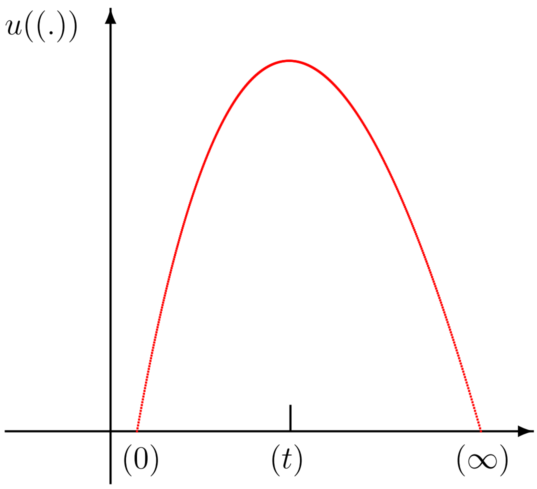
[17] Let $\mathbb{T}^k$, with $k = e, d$, denote the transition matrices out of always facing either easy or difficult tasks, respectively. In getting q^e and q^d we solve $[q^k, 1 - q^k]\mathbb{T}^k = [q^k, 1 - q^k]$. The determinants of the transition matrices are denoted $T^d = p_{22} - p_{12}$ and $T^e = p_{21} - p_{11}$ for difficult and easy tasks, respectively. We have that $q^d = (p_{12})(1 - T^d)^{-1}$ and $q^e = (p_{11})(1 - T^e)^{-1}$. Suppose that $q^e < q^d$. This implies that $p_{11}(1 - T^d) < p_{12}(1 - T^e)$ or $p_{11}(1 - p_{22}) < p_{12}(1 - p_{21})$ which cannot hold since $p_{11} > p_{12}$ and $(1 - p_{22}) > (1 - p_{21})$. Thus $q^e > q^d$ holds.

[18] The interpretation of μ is parallel to the one of λ.

motivated through outcome achievements associated to easy tasks (that is, whenever $q^{(0)} < \mu \leq q^d < q^e$*).*

In contrast with results in the previous section, jumping into difficult tasks at some point in time can be optimal here. We interpret this strategy as one in which the DM prefers to first deal with easy tasks, because performing properly motivates her to deal with difficult but more rewarding tasks. The figure below plots the ranking of utilities in this case. The DM exhibits single-peaked preferences on the optimal time to face difficulties, with the peak corresponding to the aforementioned intermediate strategy:

Figure 4: (t) is the optimal strategy



The question of when to do the hard stuff arised in Quora, an internet knowledge market, in which people discuss about a specific given topic. The topic was: Is it better to do easy tasks first and then move on to harder ones, or vice versa?.[19] One of the answers, that accurately illustrate our statement, was:

"Important is to evaluate, which are the harder tasks and which the easy tasks. Out of this it becomes clear, how long it will take to do them. (...) The rest has more psychological character and is strongly depending on the personality. I personally like to mix it. This gives the success feeling, if you do the easy tasks and motivates, to continue with the harder tasks, to make the overall project the success."

If individuals indeed behave this way, there will be chances of improving individual achievements that have to do with motivation. A model of human capital

[19]See http://www.quora.com/Is-it-better-to-do-easy-tasks-first-and-then-move-on-to-harder-ones-or-vice-versa.

accumulation in which individuals build their skills by developing easy tasks up to the point that it is optimal for them to face difficulties, might offer the same type of results. However we truly think that the human capital accumulation story is essentially different from the motivation story. This difference relies on the following reasoning: while individuals build their human capital in the actual process of developing a task, motivation results when outcomes are achieved. We think that this is a crucial distinction, that would posse different policy implications.

It is also worth mentioning that if $q^d = q^{(0)}$ when facing difficulties since the beginning is optimal or $q^e = q^{(0)}$ when always avoiding them is optimal, we are back to the optimal behavior in the previous section. Notice that the DM enjoys the full capacity state with the average long-run frequencies q^d and q^e, respectively.

We now briefly comment on some cases in which assumption 2 plays a role. That is, the DM has to choose the optimal strategy among the class of strategies prescribed by assumption 2, regardless of whether other path of action would have delivered higher utility. Under $q^d < q^e < \mu \leq q^{(0)}$ the DM would have preferred to switch to easy tasks after have been dealing with difficulties for a while. Within the class of strategies she can choose among due to assumption 2, the DM exhibits single deep preferences on the optimal time to face difficulties. The deep corresponds to an intermediate strategy and the peaks correspond to the extreme strategies or either never dealing with difficulties or facing them since the beginning. In this case, either of the two is chosen. The same happens under $q^d < \mu \leq q^e, q^{(0)}$. Among the available strategies prescribed by assumption 2, the DM ends up dealing with difficulties since the beginning. Finally under $q^{(0)}, q^d \leq \mu < q^e$ the DM would have also preferred to switch to easy tasks after have been dealing with difficulties for a while. As a result of assumption 2, the DM ends up performing an intermediate strategy.[20]

The following result deals with the properties of the utility gains under the three types of optimal strategies. It also describes the returns of a boost in the DM's initial disposition, i.e., $q^{(0)}$. We assume that a marginal increase in the initial disposition, does not affect the originally optimal strategy. Specifically, for the case in which an intermediate strategy is optimal we consider that marginal increase in the initial disposition, does not affect the particular point in time to face difficulties.[21] Before stating the results let us denote by $mr((.))$, the marginal return of a increase in the DM's initial disposition, under any optimal strategy. Proposition 2 is as follows:

Proposition 2. *The long-run expected utility is monotonically increasing and linear*

[20] See the proof of Theorem 2.

[21] Formally, $t|_{q^{(0)}} = t|_{q^{(0)}+\epsilon}$.

in $q^{(0)}$ under any optimal strategy. Moreover, $mr((0)) > mr((t-1)) > mr((t)) > mr((\infty))$.

Results in Proposition 2, as the ones in Proposition 1, capture the idea that individuals with better abilities perform better and achieve higher utility. It is also the case that advantaged individuals, those with high $q^{(0)}$, benefit more from a marginal increase in their abilities. As the table below illustrates, as $q^{(0)}$ increases within a row, everything else equal, that is, as the DM is of higher initial dispositions, she moves from finding (∞) or (t) optimal to finding (0) optimal.

Furthermore, figure 3 in the previous section illustrated how the utility of high disposition individuals that always confront with difficulties, is higher than the utility of low disposition individuals that always avoid difficulties. We also carry such an analysis in this framework. For this purpose we make use of the table below. In it we list the three combinations in which μ, q^e and q^d may appear. Within each combination we consider that μ, q^e and q^d remain unaltered. However, they might be different across combinations. For the ease of exposition we only consider strict inequalities. Thus, within each combination we present the four alternatives in which $q^{(0)}$ relates with the previous values.[22] We also present the optimal strategies in each of the combinations. Table 2 is as follows:

Table 2: Optimal strategies

Case (i)	$q^{(0)} < q^d < q^e < \mu$ (∞)	$q^d < q^{(0)} < q^e < \mu$ (∞)	$q^d < q^e < q^{(0)} < \mu$ (∞)	$q^d < q^e < \mu < q^{(0)}$ (0) or (∞)
Case (ii)	$q^{(0)} < \mu < q^d < q^e$ (t)	$\mu < q^{(0)} < q^d < q^e$ (0)	$\mu < q^d < q^{(0)} < q^e$ (0)	$\mu < q^d < q^e < q^{(0)}$ (0)
Case (iii)	$q^{(0)} < q^d < \mu < q^e$ (t)	$q^d < q^{(0)} < \mu < q^e$ (t)	$q^d < \mu < q^{(0)} < q^e$ (0)	$q^d < \mu < q^e < q^{(0)}$ (0)

Let us focus on optimal strategies in the row corresponding to case (ii), that is, either (t) or (0). In this case assumption 2 does not play a role. This allow us to make neat analysis. Recall that $q^{(0)}$ increases as we move from left to right within a row. We can then make direct comparison of the utility gains under these two optimal strategies. The following lemma summarizes the findings. It is as follows:

Lemma 1. *Consider a DM characterized by $q^{(0)}$. The optimal strategy of facing difficulties since the beginning (that is, whenever $\mu < q^{(0)}, q^d < q^e$) yields higher utility than the optimal strategy of facing them from an intermediate point in time (that is, whenever $q^{(0)} < \mu < q^d < q^e$).*

[22] For the complete analysis, see the proof of Theorem 2

Also, in order to make the intermediate strategy (t) in case (ii) and the always avoiding strategy (∞) in case (i) comparable, we consider, for the latter, the specific situation in which $q^{(0)} < q^d < q^e < \mu$. Notice that this is the only situation in case (i) in which $q^{(0)} < q^d$. Let $q^{(0)}$ be the same in both scenarios. Let us also focus on the case in which the only difference between case (i) and case (ii) is that we increase μ, from case (ii) to case (i), by decreasing K.[23] Since q^e and q^d do not depend on K, they remain unaltered. Lemma 2 is as follows:

Lemma 2. *Consider a DM characterized by $q^{(0)}$. The optimal strategy of facing difficulties from an intermediate point in time (that is, whenever $q^{(0)} < \mu < q^d < q^e$) yields higher utility than the optimal strategy of always avoiding difficulties (that is, whenever $q^{(0)} < q^d < q^e < \mu$).*

With these two lemmas we conclude that optimal strategies involving the choice of difficulties at some point in time, yield higher utility than optimal strategies in which individuals avoid difficulties forever.[24]

4 Conclusions

Non-cognitive abilities have an impact in determining performance in dimensions of huge economic relevance, as labor market entry or search decisions or educational attainments. We link, in a dynamic setting, non-cognitive abilities to the decision of when to deal with difficult but valuable tasks. We show how low disposition individuals always avoid difficulties and forego better economic opportunities while high disposition individuals are willing to deal with difficulties. The behavior of individuals that always avoid dealing with onerous tasks resembles procrastination results, without relying on the hyperbolic discounting assumption.[25] Also, individuals that get motivated by outcome achievements find optimal to jump into difficult tasks at some point in time.

5 Appendix

Before proceeding we set some useful definitions. Let us denote by $u^{1,(0)}$ and $u^{2,(0)}$, the DM's long-run expected utility when she only experiences the deteriorated ca-

[23] See the proof of Theorem 2, step 1.

[24] Also consider the optimal intermediate strategy (t) associated to $q^{(0)}$, as described in case (ii) in table 2. Consider a DM with initial disposition $q^{(0)'} > q^{(0)}$ such that her optimal strategy is $(t-1)$ (as $q^{(t)'} > q^{(t)}$ for every t, the DM with initial disposition $q^{(0)'}$ will reach μ before than the DM with initial disposition $q^{(0)}$. See Claim 2 and the proof of Theorem 2). Then, $u((t-1)) > u((t))$ for the DM characterized by $q^{(0)'}$. As utility increases with dispositions, then $u((t))$ for the DM with initial disposition $q^{(0)'}$ has to be also higher than $u((t))$ for the DM with initial disposition $q^{(0)}$.

[25] See Rubinstein (2003) for a discussion on this assumption.

pacity state s_1, that is when $q = 0$, and when she only experiences the full capacity state s_2, that is when $q = 1$, respectively, under strategy (0). Similarly, let us denote by $u^{1,(1)}$ and $u^{2,(1)}$, the DM's long-run expected utility when she only experiences the deteriorated capacity state s_1, that is when $q = 0$, and when she only experiences the full capacity state s_2, that is when $q = 1$, respectively, under strategy (1).

Let us define functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ as $f(\lambda) = \lambda u^{2,(1)} + (1-\lambda)u^{1,(1)}$ and $g(\lambda) = \lambda u^{2,(0)} + (1-\lambda)u^{1,(0)}$, respectively. For $\lambda \in [0, 1]$, these functions are the convex combination of the DM's long-run expected utilities, when she experiences s_2 with probability $q = 1$ and $q = 0$, out of strategies (1) and (0), respectively. Furthermore, we say that a strategy (t) dominates strategy $(t + 1)$ whenever the long-run expected utility of (t) is higher than the one of $(t + 1)$. Let $(t) > (t + 1)$ denote this domination relationship. Recall that (t) denotes any strategy such that $0 < t < \infty$. We also say that strategy (0) dominates strategy (1) whenever the long-run expected utility of (0) is higher than the one of (1). Let $(0) > (1)$ denote this domination relationship.

5.1 Proofs of section 2

Proof of Theorem 1. It is composed by two steps. In Step 1 we derive the threshold λ such $f(\lambda)$ and $g(\lambda)$ equalize. For such a λ, (1) and (0) yield the same long-run expected utility. For values higher or equal than λ then $(0) > (1)$. For values lower than λ then $(1) > (0)$. In step 2 we argue how this information is enough to set the optimal strategy, depending on the values of λ and q.

Step 1. If the DM experiences s_2 with probability $q = 1$, the long-run expected utility of strategy (0) is $u^{2,(0)} = Kp_{22} + \delta u^{2,(0)}$. If she experiences s_1 with probability $1 - q = 1$, the long-run expected utility of strategy (0) is $u^{1,(0)} = Kp_{12} + \delta u^{1,(0)}$. Similarly, when she experiences s_2 with probability $q = 1$, the long-run expected utility of strategy (1) is $u^{2,(1)} = p_{21} + \delta u^{2,(0)}$ whereas when she experiences s_1 with probability $1 - q = 1$, then $u^{1,(1)} = p_{11} + \delta u^{1,(0)}$. From previous definitions, $f(\lambda) = \lambda(u^{2,(1)} - u^{1,(1)}) + u^{1,(1)}$ and $g(\lambda) = \lambda(u^{2,(0)} - u^{1,(0)}) + u^{1,(0)}$. Solving $f(\lambda) = g(\lambda)$ we get $\lambda((u^{2,(1)} - u^{1,(1)}) - (u^{2,(0)} - u^{1,(0)})) = u^{1,(0)} - u^{1,(1)}$. Notice that $u^{2,(0)} - u^{1,(0)} = \dfrac{K(p_{22} - p_{12})}{1-\delta}$ and $u^{1,(0)} = \dfrac{Kp_{12}}{1-\delta}$. Thus, $\lambda((p_{21} - p_{121}) - K(p_{22} - p_{12})) = Kp_{12} - p_{11}$ or $\lambda = \dfrac{Kp_{12} - p_{11}}{(p_{21} - p_{11}) - K(p_{22} - p_{12})}$. Assumption 1 implies that $p_{22} - p_{12} > p_{21} - p_{11}$. Thus the denominator of λ is always different from zero, meaning that λ is a real number. Since by assumption 1, the denominator is negative, for values lower than λ then $(1) > (0)$ and for values higher than it, $(0) > (1)$. For values equal to λ we assume that (0) > (1) as well. Assumption 1 also implies that $p_{11}/p_{12} > p_{21}/p_{22}$.[26] Thus, $\partial\lambda/\partial K = \dfrac{p_{21}p_{12} - p_{22}p_{11}}{((p_{21} - p_{11}) - K(p_{22} - p_{12}))^2} < 0$. When $K = \dfrac{p_{21}}{p_{22}}$ then $\lambda = 1$ and when $K = \dfrac{p_{11}}{p_{12}}$ then $\lambda = 0$. Thus, when $K < \dfrac{p_{21}}{p_{22}}$ then $\lambda > 1$, and no matter q, (1) > (0). On the contrary, when $K > \dfrac{p_{11}}{p_{12}}$ then $\lambda < 0$, and no matter q, (0) > (1). We focus on the interesting case such that $\dfrac{p_{11}}{p_{12}} < K < \dfrac{p_{11}}{p_{12}}$ and $\lambda \in (0, 1)$. We conclude that when $q \geq \lambda$ then (0) > (1) and when $q < \lambda$ then (1) > (0).

Step 2. We set here optimal strategies. Two cases arise depending on the position of q relative to λ.

(i) Consider that $q \geq \lambda$. By step 1, (0) > (1). Compare now any pair of strategies (t) and $(t + 1)$. We have that $u((t)) = \sum_{i=0}^{t-1} \delta^i (qp_{21} + (1 - q)p_{11}) + K \sum_{i=t}^{\infty} \delta^i (qp_{22} + (1 - q)p_{12})$ and $u((t + 1)) = \sum_{i=0}^{t-1} \delta^i (qp_{21} + (1 - q)p_{11}) + \delta^t (qp_{21} + (1 - q)p_{11}) + K \sum_{i=t+1}^{\infty} \delta^i (qp_{22} + (1 - q)p_{12})$. Up to $t - 1$, (t) and $(t + 1)$ yield the same utility. From time t on the comparison is between (0) and (1) evaluated from the point of view of time t. Since for every t, $q \geq \lambda$, by step 1, it follows that $(t) > (t + 1)$ for every t. It is useful to observe that $\lim_{i\longrightarrow\infty} u((t + i)) = u((\infty))$. We then conclude that $(0) > (1) > ... > (t) > (t + 1) > ... > (\infty)$ holds.

(ii) Consider that $q < \lambda$. We use a similar reasoning as above, and thus omit it here, to conclude that $(\infty) > ... > (t + 1) > (t) > ... > (1) > (0)$. In this case (∞) is optimal.

The result is then established.

■

Proof of the Remark. See the proof of Theorem 1 for the relation between λ and K. We analyze here how λ varies with the probabilities of success. We omit the denominator in the subsequent derivatives since it it always positive.[27] We have that: $\partial\lambda/\partial p_{11} = Kp_{22} - p_{21}$, $\partial\lambda/\partial p_{12} = K(p_{21} - Kp_{22})$, $\partial\lambda/\partial p_{21} = p_{11} - Kp_{12}$ and $\partial\lambda/\partial p_{22} = K(Kp_{12} - p_{11})$. Since $K \in (p_{21}/p_{22}, p_{11}/p_{12})$ then $\partial\lambda/\partial p_{i1} > 0$ and $\partial\lambda/\partial p_{i2} < 0$ with $i = 1, 2$. The result is then established.

■

Proof of Proposition 1. Notice that $u((0)) = K \sum_{i=0}^{\infty} \delta^i (qp_{22}+(1-q)p_{12}) = (K(qp_{22}+(1 - q)p_{12})(1 - \delta)^{-1}$. Similarly $u((\infty)) = \sum_{i=0}^{\infty} \delta^i (qp_{21} + (1 - q)p_{11}) = (qp_{21} + (1 -$

[26] By assumption 1, $p_{11} - p_{12} > p_{21} - p_{22}$ This implies that $\dfrac{p_{11} - p_{12}}{p_{12}} > \dfrac{p_{21} - p_{22}}{p_{22}}$ or equivalently $p_{11}/p_{12} > p_{21}/p_{22}$.

[27] Its value is $(p_{21} - p_{11} - K(p_{22} - p_{12}))^2$.

$q)p_{11})(1-\delta)^{-1}$. Since $\partial u((0))/\partial q = K(p_{22}-p_{12})(1-\delta)^{-1} > 0$ and $\partial u((\infty))/\partial q = (p_{21}-p_{11})(1-\delta)^{-1} > 0$, utilities are increasing and linear in q. Finally, assumption 1 implies that $p_{22}-p_{12} > p_{21}-p_{11}$, then the marginal return of an increase in q is higher under (0) than under (∞). The result is then established. ■

5.2 Proofs of section 3

Before the proof of Theorem 2 let us set two useful claims.

Claim 1. *For every t, $q^{(0)} > q^{(t)} > q^{(t+1)} > q^e$ whenever $q^{(0)} > q^e$ and $q^{(0)} < q^{(t)} < q^{(t+1)} < q^e$ whenever $q^{(0)} < q^e$.*

Proof of Claim 1. The proof of Claim 1 is by induction. We focus on the case in which $q^{(0)} > q^e$. We prove that for $t = 1$, $q^{(0)} > q^{(1)} > q^e$ holds. We set the induction part afterwards.

Step 1. We prove that $q^{(0)} > q^{(1)} > q^e$. In showing that $q^{(0)} > q^{(1)}$, we compare the initial probability of s_2, with its first perturbation, after having decided to face an easy task. Consider expression (1) in the main body:

$$[q^{(0)} \quad 1-q^{(0)}]\begin{bmatrix} p_{21} & 1-p_{21} \\ p_{11} & 1-p_{11} \end{bmatrix} = [q^{(1)} \quad 1-q^{(1)}]$$

we have that $q^{(1)} = q^{(0)}p_{21} + (1-q^{(0)})p_{11}$. Since $q^{(0)} > q^e$, replacing q^e by its value we have that $q^{(0)} > \dfrac{p_{11}}{1-p_{21}+p_{11}}$ or equivalently $q^{(0)}(1-p_{21}+p_{11}) > p_{11}$. This expression becomes $q^{(0)} > q^{(0)}p_{21} + (1-q^{(0)})p_{11}$. The RHS of this expression is exactly $q^{(1)}$. In showing that $q^{(1)} > q^e$ we proceed by contradiction. Suppose that $q^{(0)} > q^e$ and $q^{(1)} < q^e$ holds. We have that $q^{(0)}p_{21} + (1-q^{(0)})p_{11} < q^e$. This is equivalent to $q^{(0)}p_{21}+(1-q^{(0)})p_{11} < \dfrac{p_{11}}{1-p_{21}+p_{11}}$ or $(p_{11}+q^{(0)}(p_{21}-p_{11}))(1-p_{21}+p_{11}) < p_{11}$. Rearranging terms it becomes $p_{11} - p_{11}(p_{21}-p_{11}) + q^{(0)}(p_{21}-p_{11})(1-p_{21}+p_{11}) < p_{11}$. This is equivalent to $q^{(0)} < \dfrac{p_{11}}{1-p_{21}+p_{11}} = q^e$, contradicting our initial assumption. Thus, $q^{(0)} > q^{(1)} > q^e$ holds.

Step 2. If for an arbitrary t, $q^{(t)} > q^{(t+1)} > q^e$ holds, for $q^{(t+1)}$ we have:

$$\left[q^{(t+1)} \quad 1-q^{(t+1)}\right]\begin{bmatrix} p_{21} & 1-p_{21} \\ p_{11} & 1-p_{11} \end{bmatrix} = [q^{(t+2)} \quad 1-q^{(t+2)}].$$

Following the same reasoning than in the first step we conclude that $q^{(t+1)} > q^{(t+2)} > q^e$. We conclude that $q^{(0)} > q^{(t)} > q^{(t+1)} > ... > q^e$ holds. The result is then established.

The case in which $q^{(0)} < q^e$ relies on the same argument. The same analysis goes through for describing the relation between $q^{(t)}$ and q^d.. We thus omit these proofs here. ■

Claim 2. *$q^{(i)} = q^{(0)}(T^k)^i + p_{1k}\sum_{j=0}^{i-1}(T^k)^j$ with $k = e, d$.*

Proof of Claim 2. Recall that $T^d = p_{22} - p_{12}$ and $T^e = p_{21} - p_{11}$. By expression (1) in the main body, $q^1 = q^{(0)}T^k + p_{1k}$. Also $q^2 = q^{(1)}T^k + p_{1k} = (q^{(0)}T^k + p_{1k})T^k + p_{1k} = q^{(0)}(T^k)^2 + p_{1k}T^k + p_{1k}$. In general $q^{(i)} = q^{(0)}(T^k)^i + p_{1k}(T^k)^{i-1} + ... + p_{1k}T^k + p_{1k}$ or $q^{(i)} = q^{(0)}(T^k)^i + p_{11}\sum_{j=0}^{t-1}(T^k)^j$. To conclude that $q^{(t+i)} = q^{(0)}(T^k)^i + p_{1k}\sum_{j=0}^{i-1}(T^k)^j$ we follow a similar reasoning. We therefore omit it here. ■

Proof of Theorem 2. We follow the same steps than in the proof of Theorem 1.

Step 1. Consider expression (1) in the main body. When the DM experiences s_2 with probability $q^{(0)} = 1$, the long-run expected utility out of strategy (0) is $u^{2,(0)} = Kp_{22} + \delta(p_{22}u^{2,(0)} + (1 - p_{22})u^{1,(0)})$. When she experiences s_1 with probability $1 - q^{(0)} = 1$, the long-run expected utility of strategy (0) is $u^{1,(0)} = Kp_{12} + \delta(p_{12}u^{2,(0)} + (1 - p_{12})u^{1,(0)})$. Solving for $u^{1,(0)}$ and $u^{2,(0)}$ we get $u^{2,(0)} = \dfrac{K(p_{22} - \delta T^d)}{(1-\delta)(1-\delta T^d)}$ and that $u^{1,(0)} = \dfrac{Kp_{12}}{(1-\delta)(1-\delta T^d)}$. Thus, $u^{2,(0)} - u^{1,(0)} = \dfrac{KT^d}{1-\delta T^d}$. When she experiences s_2 with probability $q^{(0)} = 1$, the long-run expected utility out of strategy (1) is $u^{2,(1)} = p_{21} + \delta(p_{21}u^{2,(0)} + (1 - p_{21})u^{1,(0)})$. Similarly, when she experiences s_1, with probability $1 - q^{(0)} = 1$, the long-run expected utility out of strategy (1) can is $u^{1,(1)} = p_{11} + \delta(p_{11}u^{2,(0)} + (1 - p_{11})u^{1,(0)})$. Now, $f(\mu) = \mu(u^{2,(1)} - u^{1,(1)}) + u^{1,(1)}$ and $g(\mu) = \mu(u^{2,(0)} - u^{1,(0)}) + u^{1,(0)}$ Solving $f(\mu) = g(\mu)$ we get $\mu = \dfrac{u^{1,(0)} - u^{1,(1)}}{(u^{2,(1)} - u^{1,(1)}) - (u^{2,(0)} - u^{1,(0)})}$. With respect to the numerator, $u^{1,(0)} - u^{1,(1)} = (1-\delta)u^{1,(0)} - p_{11} - \delta p_{11}(u^{2,(0)} - u^{1,(0)})$ or equivalently $u^{1,(0)} - u^{1,(1)} = \dfrac{(1-\delta)Kp_{12}}{(1-\delta)(1-\delta T^d)} - p_{11} - \dfrac{\delta p_{11}KT^d}{1-\delta T^d} = \dfrac{K(p_{12} - \delta p_{11}T^d) - (p_{11} - \delta p_{11}T^d)}{1-\delta T^d}$. Regarding the denominator, $u^{2,(1)} - u^{1,(1)} = T^e + \delta T^e(u^{2,(0)} - u^{1,(0)})$ and $u^{2,(1)} - u^{1,(1)} - (u^{2,(0)} - u^{1,(0)}) = T^e - (1 - \delta T^e)(u^{2,(0)} - u^{1,(0)})$. This is equivalent to $\dfrac{T^e(1-\delta T^d) - KT^d(1-\delta T^e)}{(1-\delta T^d)}$. Thus, $\mu = \dfrac{K(p_{12} - \delta p_{11}T^d) - (p_{11} - \delta p_{11}T^d)}{T^e(1-\delta T^d) - KT^d(1-\delta T^e)}$.[28] Since by assumption 1, $T^d > T^e$ the denominator is different from zero, hence μ is a real number. Since by assumption 1, the denominator is negative, for values lower than μ then (1) > (0) and for values higher it, (0) > (1). For values

[28]When the DM does not care about the future, that is, when $\delta = 0$, $\lambda = \mu$. That q does not vary over time is conceptually equivalent to think about a DM making one period decisions without consequences on her subsequent states. Additionally, $\partial\mu/\partial\delta = K(K-1)(p_{11}p_{22} - p_{21}p_{22}) > 0$, meaning that the more the DM cares about the future the more she postpones difficult tasks, where good outcomes are less frequent.

equal to μ we assume that $(0) > (1)$ as well. Also by assumption 1, $\partial\mu/\partial K = \dfrac{(1-\delta T^d)(p_{21}p_{12}-p_{11}p_{22})}{(T^e(1-\delta T^d)-KT^d(1-\delta T^e))^2} < 0$. Furthermore, when $K = \dfrac{p_{11}-\delta p_{11}T^d}{p_{12}-\delta p_{11}T^d}$ then $\mu = 0$ and when $K = \dfrac{p_{21}-\delta p_{21}T^d}{p_{22}-\delta p_{21}T^d}$ then $\mu = 1$. Then, $\dfrac{p_{21}-\delta p_{21}T^d}{p_{22}-\delta p_{21}T^d} < \dfrac{p_{11}-\delta p_{11}T^d}{p_{12}-\delta p_{11}T^d}$. Furthermore, when $K > \dfrac{p_{11}-\delta p_{11}T^d}{p_{12}-\delta p_{11}T^d}$, then $\mu < 0$ and $(0) > (1)$ no matter $q^{(0)}$ and when $K < \dfrac{p_{21}-\delta p_{21}T^d}{p_{22}-\delta p_{21}T^d}$, then $\mu > 1$ and $(1) > (0)$ no matter $q^{(0)}$. The interesting case is such that $K \in \left(\dfrac{p_{21}-\delta p_{21}T^d}{p_{22}-\delta p_{21}T^d}, \dfrac{p_{11}-\delta p_{11}T^d}{p_{12}-p_{11}\delta T^d}\right)$ and $\mu \in (0,1)$. We conclude that when $q^{(0)} \geq \mu$ then $(0) > (1)$ and when $q^{(0)} < \mu$ then $(1) > (0)$.

Step 2. We identify here optimal strategies.

Consider that $\mu \leq q^{(0)}$. By step 1, $(0) > (1)$ from the point of view of $q^{(0)}$. Three cases arise:

(i) Suppose that $\mu \leq q^d < q^e, q^{(0)}$. Let us compare any pair (t) and $(t+1)$. We evaluate $u((t)) = \sum_{i=0}^{t-1}(q^{(i)}p_{21}+(1-q^{(i)})p_{11})+K\sum_{i=t}^{\infty}\delta^i(q^{(i)}p_{22}+(1-q^{(i)})p_{12})$ versus $u((t+1)) = \sum_{i=0}^{t-1}(q^{(i)}p_{21}+(1-q^{(i)})p_{11}) + \delta^t(q^{(t)}p_{21}+(1-q^{(t)})p_{11}) + K\sum_{i=t+1}^{\infty}\delta^i(q^{(i)}p_{22}+(1-q^{(i)})p_{12})$. Up to $t-1$ both expressions yield the same utility. From time t on, the comparison is between (0) and (1), from the point of view of $q^{(t)}$. Notice that $q^{(t)}$ is the result of have been dealing with easy tasks up to period $t-1$. Thus, by Claim 1, $q^{(t)}$ is higher than μ. This implies, by step 1, that from the point of view of $q^{(t)}$, $(0) > (1)$. As a consequence, for any pair of strategies, $(t) > (t+1)$. Recall that $\lim_{i\longrightarrow\infty} u((t+i)) = u((\infty))$. This observation applies to the remaining cases, hence we omit it in subsequents proofs. We then establish that $(0) > (1)... > (t) > (t+1) > ... > (\infty)$.

Notice that assumption 2 does not play any role in case (i), that is, the DM optimal strategy is within the class of strategies that it prescribes. However, assumption 1 plays a role in cases (*ii*) and (*iii*) below. In both cases the DM would have find optimal to start with difficult tasks and to switch to easy ones when $q^{(t)}$ crosses μ. We look for the optimal strategies within the ones prescribed by assumption 2.

(ii) Suppose that $q^d < \mu \leq q^e, q^{(0)}$. Let us compare (t) and $(t+1)$ as above. We then evaluate $u((t))$ and $u((t+1))$ as defined in case (*i*). The relevant comparison is between (0) and (1) from the point of view of $q^{(t)}$. Notice that $q^{(t)}$ is the result of have been dealing with easy tasks up to period $t-1$. By Claim 1, every $q^{(t)}$ is higher than μ. Thus, from the point of view of $q^{(t)}$, $(0) > (1)$ and, as a consequence, $(t) > (t+1)$. Then, $(0) > (1) > ... > (t) > (t+1) > ... > (\infty)$ and, by the same reasoning as above, (0) is optimal.[29]

(iii) Suppose that $q^d < q^e \leq \mu \leq q^{(0)}$. As above, the analysis relies on comparing intermediate strategies in which the DM first deals with easy tasks. By Claim 1, for some strategies (t), there will exist a point in time such that $q^{(t)}$ will exceed μ while approaching q^e. Let $t^* - 1$ denote the last point in time such that $q^{(t^*-1)} \geq \mu$. Consider points in time $t \leq t^* - 1$ and compare (t) and $(t+1)$. That is, let us evaluate $u((t))$ and $u((t+1))$ as defined in case (i). As above, the relevant comparison is between (0) and (1) from the point of view of $q^{(t)}$. Since $t \leq t^* - 1$, by Claim 1, $q^{(t)} \geq \mu$. Thus, from the point of view of $q^{(t)}$, $(0) > (1)$ and, as a consequence, for every $t \leq t^* - 1$, $(t) > (t+1)$. In particular, $(t^* - 1) > (t^*)$. Let us compare now $u((t^*))$ and $u((t^* + 1))$. Notice that, $q^{(t^*)} < \mu$. Thus, from the point of view of $q^{(t^*)}$, $(0) < (1)$ and, as a consequence, $(t^* + 1) > (t^*)$. By Claim 1, $q^{(t)} < \mu$ for every $t \geq t^*$. In this case for any pair of strategies, $(t+1) > (t)$. Summing up we have that $(0) > (1) > ... > (t^* - 1) > (t^*) < (t^* + 1) < ... < (\infty)$. The optimal strategy might be either (0) or (∞).[30]

Consider that $q^{(0)} < \mu$. By step 1, $(1) > (0)$ from the point of view of $q^{(0)}$. Three cases arise:

(i) Suppose that $q^{(0)}, q^d < q^e \leq \mu$. Let us compare any pair (t) and $(t+1)$. We evaluate $u((t))$ and $u((t+1))$ as defined in case (i) above. Up to $t-1$ both expression yield the same utility. From t on the comparison is between (0) and (1), from the point of view of $q^{(t)}$ which is the result of have been dealing with easy tasks up to period $t-1$. Thus, by Claim 1, $q^{(t)} < \mu$. This implies, by step 1, that $(1) > (0)$ from the point of view of $q^{(t)}$. As a consequence, for any pair of strategies, $(t+1) > (t)$. Thus, $(0) < (1)... < (t) < (t+1) < ... < (\infty)$ and (∞) is optimal.

Similar arguments are used in the remaining cases. We go briefly over them.

(ii) Suppose that $q^{(0)} < \mu \leq q^d < q^e$. By Claim 1 and by the same argument as in case (iii) above, there exists a last point in time t^*-1 such that $q^{(t^*-1)} < \mu$. The comparison between any pair of strategies (t) and $(t+1)$ relies on comparing

[29] In the particular case in which $q^e = \mu = q^{(0)}$, when evaluating (t) versus $(t+1)$ the relevant comparison is between (0) and (1) from the point of view of $q^{(t)}$. But $q^{(t)}$ remains equal to q^e because is the result of developing easy tasks for $t-1$ periods. We assume that when $\mu = q^{(t)}$ then $(0) > (1)$ and we maintain this assumption all along the proof. Thus, $(t) > (t+1)$. In this case (0) is optimal.

[30] See case (ii) for the situation in which $q^e = \mu = q^{(0)}$.

(0) and (1) from the point of view of $q^{(t)}$. Consider points in time such that $t \leq t^* - 1$. By Claim 1, $q^{(t)} < \mu$ and thus $(1) > (0)$ from its point of view. We then have that $(t+1) > (t)$ and in particular that $(t^*) > (t^*-1)$. In contrast, at time $t \geq t^*$ then $q^{(t)} \geq \mu$. As now $(0) > (1)$ from the point of view of $q^{(t)}$, then $(t^*) > (t^*+1)$. In general, $(t) > (t+1)$ for any pair of strategies. Summing up we have that $(0) < (1) < ... < (t^*-1) < (t^*) > (t^*+1) > ... > (\infty)$ and the optimal strategy is of type (t).

Assumption 2 does not play a role in cases (i) and (ii). However, it does the following case. In it the DM would have preferred to switch from difficult to easy at some point. We look for the optimal strategies within the ones prescribed by assumption 2.

(iii) Suppose that $q^{(0)}, q^d \leq \mu < q^e$. Whenever $t \leq t^* - 1$, $(t+1) > (t)$ and in particular $(t^*) > (t^*-1)$. At t^*, it follows that $(t^*) > (t^*+1)$. In general, whenever $t > t^*$, $(t) > (t+1)$. We have that $(0) < (1) < ... < (t^*-1) < (t^*) > (t^*+1) > ... > (\infty)$ and the optimal strategy is of type (t).

The result is then established.

■

Proof of Proposition 3. We have three cases depending on the optimal strategy.

Case 1. (0) is optimal. We have $u((0)) = K\sum_{i=0}^{\infty}\delta^i(q^{(i)}p_{22} + (1-q^{(i)})p_{12})$. By Claim 2, $q^{(i)} = q^{(0)}(T^d)^i + p_{12}\sum_{j=0}^{i-1}(T^d)^j$. Thus, we rewrite $u((0)) = K\sum_{i=0}^{\infty}\delta^i(q^{(i)}T^d + p_{12})$ or $KT^dq^{(0)}\sum_{i=0}^{\infty}(\delta T^d)^i + p_{12}T^dK\sum_{i=0}^{\infty}\delta^i\sum_{j=0}^{i-1}(T^d)^j + Kp_{12}\sum_{i=0}^{\infty}\delta^i$. Then $\partial u((0))/\partial q^{(0)} = KT^d(1-\delta T^d)^{-1} > 0$.

Case 2. (∞) is optimal. We have $u((\infty)) = \sum_{i=0}^{\infty}\delta^i(q^{(i)}p_{21} + (1-q^{(i)})p_{11})$. We follow exactly the same reasoning than in case 1, and thus omit it here, to conclude that $\partial u((\infty))/\partial q^{(0)} = T^e(1-\delta T^e)^{-1} > 0$.

In both cases utility is increasing and linear in $q^{(0)}$. Since $T^d > T^e$, the marginal return of an increase in $q^{(0)}$ is higher under (0) than under (∞).

Case 3. (t) is optimal. We have $u((t)) = \sum_{i=0}^{t-1}\delta^i(q^{(i)}p_{21} + (1-q^{(i)})p_{11}) + K\sum_{i=t}^{\infty}\delta^i(q^{(i)}p_{22} + (1-q^{(i)})p_{12})$. Let us focus first on $\sum_{i=0}^{t-1}\delta^i(q^{(i)}p_{21} + (1-q^{(i)})p_{11})$. By Claim 2, $q^{(i)} = q^{(0)}(T^e)^i + p_{11}\sum_{j=0}^{i-1}(T^e)^j$. Thus, $\sum_{i=0}^{t-1}\delta^i(q^{(i)}T^e + p_{11}) = \sum_{i=0}^{t-1}\delta^i((q^{(0)}(T^e)^i + p_{11}\sum_{j=0}^{i-1}(T^e)^j)T^e + p_{11})$. This expression is equivalent to $q^{(0)}T^e\sum_{i=0}^{t-1}\delta^i(T^e)^i + T^e\sum_{i=0}^{t-1}\delta^ip_{11}\sum_{j=0}^{i-1}(T^e)^j + \sum_{i=0}^{t-1}\delta^ip_{11}$. Its derivative with respect to $q^{(0)}$ is $T^e\sum_{i=0}^{t-1}(\delta T^e)^i > 0$. Consider now, $K\sum_{i=t}^{\infty}\delta^i(q^{(i)}p_{22} + (1-q^{(i)})p_{12})$. By Claim 2 $q^{(t+i)} = q^{(t)}(T^d)^i + p_{12}\sum_{j=0}^{i-1}(T^d)^j$. Thus, we rewrite the previous expression as $K(\sum_{i=0}^{\infty}\delta^{t+i}((q^{(t)}(T^d)^i + p_{12}\sum_{j=0}^{i-1}(T^d)^j)T^d + p_{12}))$. This is equivalent to $K(q^{(t)}T^d \sum_{i=0}^{\infty} \delta^{t+i}(T^d)^i + T^d \sum_{i=0}^{\infty} \delta^{t+i} p_{12} \sum_{j=0}^{\infty}(T^d)^j + \sum_{i=0}^{\infty} \delta^{t+i} p_{12}))$. The part that depends on $q^{(t)}$ is equivalent to $q^{(t)} K\delta^t T^d/(1-\delta T^d)^{-1}$. By Claim 2 we express it as $(q^{(0)}(T^d)^t + p_{12}\sum_{i=0}^{t-1}(T^d)^i)K\delta^t T^d/(1-\delta T^d)^{-1}$. Taking derivatives w.r.t $q^{(0)}$ we get $K\delta^t(T^d)^{t+1}/(1-\delta T^d)^{-1} > 0$. Thus, we have $\partial u((t))/\partial q^{(0)} = T^e \sum_{i=0}^{t-1}(\delta T^e)^i + K\delta^t(T^d)^{t+1}/(1-\delta T^d)^{-1} > 0$.

Notice that we can express $u((0)) = K\sum_{i=0}^{t-1}\delta^i(q^{(i)}T^d + p_{12}) + K\sum_{i=t}^{\infty}\delta^i(q^{(i)}T^d + p_{12})$ and $u((\infty)) = \sum_{i=0}^{t-1}\delta^i(q^{(i)}T^e + p_{11}) + \sum_{i=t}^{\infty}\delta^i(q^{(i)}T^e + p_{11})$. Similar algebra as in the case in which (t) is optimal yields $\partial u((0))/\partial q^{(0)} = KT^d\sum_{i=0}^{t-1}(\delta T^d)^i + K\delta^t(T^d)^{t+1}/(1-\delta T^d)^{-1}$ and $\partial u((\infty))/\partial q^{(0)} = T^e\sum_{i=0}^{t-1}(\delta T^e)^i + \delta^t(T^e)^{t+1}/(1-\delta T^e)^{-1}$, respectively. Since $T^d > T^e$, then $\partial u((0))/\partial q^{(0)} > \partial u((t))/\partial q^{(0)}$ and $\partial u((t))/\partial q^{(0)} > \partial u((\infty))/\partial q^{(0)}$ holds. We also compare the marginal return of any pair of intermediate strategies $(t-1)$ and (t). In this case $t-1 \geq 1$. We have that $\partial u((t-1))/\partial q^{(0)} = T^e\sum_{i=0}^{t-2}(\delta T^e)^i + K\delta^{t-1}(T^d)^t/(1-\delta T^d)^{-1}$ and $\partial u((t))/\partial q^{(0)} = T^e\sum_{i=0}^{t-1}(\delta T^e)^i + K\delta^t(T^d)^{t+1}/(1-\delta T^d)^{-1}$. Subtracting the second expression from the first we get $\delta^{t-1}(K(T^d)^t - (T^e)^t)$ which is positive since $T^d > T^e$. ■

Proof of Lemma 1. Consider case (ii) in table 2 in the main body. Let us denote by $q^{(0)'}$ the initial probability in any of the cases in which (0) is optimal. Let us also denote by $q^{(0)}$ the initial probability in the case in which (t) is optimal. Notice that $q^{(0)'} > q^{(0)}$. Consider the utility of (t) when the DM is characterized by $q^{(0)'}$, that is, when (0) is optimal. It has to be higher than the utility of (t) when the DM is characterized by $q^{(0)}$, that is, when precisely, (t) is optimal. To see this, notice that up to $t-1$ easy tasks are chosen. But since $q^{(0)'} > q^{(0)}$, by Claim 2, $q^{(i)'} > q^{(i)}$ for every $i \leq t-1$. Consider now time t on. By Claim 1, iterations on $q^{(i)}$ approach q^d from below without exceeding it. Notice, also by Claim 1, that iterations on $q^{(i)'}$ may approach q^d from below or from above, without exceeding it.[31] In the former case, also by Claim 2, $q^{(i)'} > q^{(i)}$ for every $i \geq t$. In the latter case, by Claim 1, $q^{(i)'} > q^d > q^{(i)}$ for $i \geq t$. Since current expected utility increases with $q^{(0)}$, it has to be that $u((t))$ is higher under $q^{(0)'}$ than under $q^{(0)}$. Also, under $q^{(0)'}$, $u((0)) > u((t))$ by optimality. Summing up u((0)) under $q^{(0)'}$ is higher than $u((t))$ under $q^{(0)'}$ and $u((t))$ under $q^{(0)'}$ is higher utility than $u((t))$ under $q^{(0)}$. The result is then established.

■

Proof of Lemma 2. Consider that the DM is characterized by $q^{(0)}$. By the proof of

[31] The behavior of $q^{(i)'}$ depends on whether in approaching q^e, q^d is exceeded or not. In the cases in which $q^{(i)'} > q^d$, by Claim 2, the stationary state is approached from above.

Theorem 2, under $q^{(0)} < q^d < q^e < \mu'$, (∞) is optimal whereas under $q^{(0)} < \mu < q^d < q^e$, (t) is optimal. By the proof of Theorem 2, μ is decreasing in K, thus $\mu < \mu'$ is associated to $K' < K$. By optimality of (t) we have that $\sum_{i=0}^{t-1} \delta^i (q^{(i)} T^e + p_{11}) + K \sum_{i=t}^{\infty} \delta^i (q^{(i)} T^d + p_{12}) > \sum_{i=0}^{t-1} \delta^i (q^{(i)} T^e + p_{11}) + \sum_{i=t}^{\infty} \delta^i (q^{(i)} T^e + p_{11})$. Since we consider the case in which $q^{(0)}$ as well as probabilities of success affecting q^d and q^e are the same, when either (∞) or (t) are optimal, the RHS of this expression brings exactly the same utility that when, indeed, (∞) is optimal, that is, when $q^{(0)} < q^d < q^e < \mu'$. Thus, $u((t))$ under $q^{(0)} < \mu < q^d < q^e$ is higher than $u((\infty))$ under $q^{(0)} < q^d < q^e < \mu'$.[32] ■

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[32] We can also reach the same conclusion by the following reasoning. By optimality of (∞), $\sum_{i=0}^{t-1} \delta^i (q^{(i)} T^e + p_{11}) + \sum_{i=t}^{\infty} \delta^i (q^{(i)} T^e + p_{11}) > \sum_{i=0}^{t-1} \delta^i (q^{(i)} T^e + p_{11}) + K' \sum_{i=t}^{\infty} \delta^i (q^{(i)} T^d + p_{12})$ with. Since $K' < K$, this does not imply that $\sum_{i=0}^{t-1} \delta^i (q^{(i)} T^e + p_{11}) + \sum_{i=t}^{\infty} \delta^i (q^{(i)} T^e + p_{11}) > \sum_{i=0}^{t-1} \delta^i (q^{(i)} T^e + p_{11}) + K \sum_{i=t}^{\infty} \delta^i (q^{(i)} T^d + p_{12})$. By the proof of Theorem 2, this inequality would hold in a scenario in which $q^{(t)}$ was lower than μ, the threshold associated to K. But precisely iteration $q^{(t)}$ is higher or equal than μ, and as a consequence it brings us the optimality of (t) in case (ii). Thus, it cannot be that optimal (∞) in case (i) yields higher utility than optimal (t). It has to yield lower utility than optimal (t) in case (ii).

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