Can Trade Unions Increase Social Welfare? An RD Model with Cash-in-Advance Constraints

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Abstract

Economic growth crucially depends on the level of R&D investment, as well as on the existing labour market institutions (LMI); the latter might shape the amount of profit obtained by each firm and its incentives to continuously innovate. This paper proposes a novel analysis combining a Schumpeterian growth model with cash-in-advance (CIA) constraints on R&D to study the impact of trade unions on economic growth and social welfare. Two main results arise: one the one hand, economic growth is always decreasing in trade union's markup and interest rate. However, in terms of social welfare, although Friedman rule appears to be optimal across all the considered scenarios, free labour market can be suboptimal below a specific threshold level of economic growth, depending on whether there is over or underinvestment in R&D. Hence, by demanding a wage above the perfect competition equilibrium, trade unions can have a positive impact on welfare through a reallocation of labour among sectors. This relationship seems to be stronger for countries with lower labour share and higher rents in the intermediate sector. This latter case highlights the redistributive effect of trade unions, contributing for a decrease in inequality between monopolists and workers. Therefore, for the case of the Eurozone, a "common" labour market setting might be more "inefficient" than a common monetary policy.

JEL classification: E24, J51, O40, O42.
Keywords: employment, trade unions, economic growth, R&D.

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1 Introduction

The essential role of trade unions on society is widely recognised and has often been a prominent issue in policy debates.[1] From a macroeconomic viewpoint, however, their effect may not be so clear; as indicated in Figure 1 (a), there seems to be no clear relationship or a slightly negative one between a country's labour union density and its macroeconomic growth rate. This is not obviously consistent with another fact given in Figure 1 (d) that a higher trade union density appears to be associated with more patent applications, which should positively affect growth provided that the economy grows with inventions resulting from research and development (R&D) activities.[2] In addition, Figure 1 (b) shows that a higher trade union density relates to a higher employment rate. This itself is natural, but there may still remain a mystery because the high employment rate typically contributes to economic growth. All in all, data may only provide a mixed view on the macroeconomic consequences of trade unions.

In line with this casual observation, the theory also seems controversial. Although relatively less attention has been paid to the issue on trade unions and growth (Neto and Silva, 2013), the results obtained so far can basically be split into two separate ways. For example, Chang and Hung (2016) and Palokangas (2004) find that unions can contribute positively to economic growth, whereas Dobbelaere and Mairesse (2013) and Barbosa and Faria (2011) argue that unions have a negative effect on growth.[3]

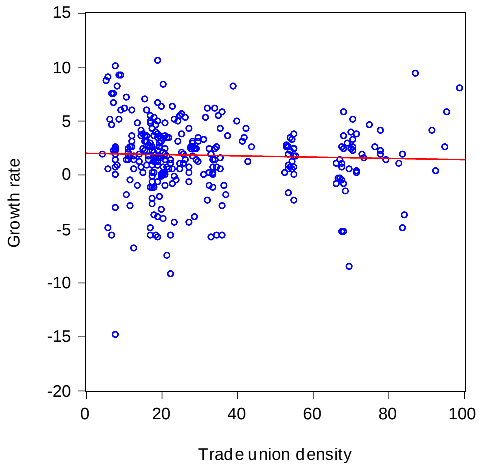
Hence, what are the effects of unions on employment, R&D, and growth? All the above provide us with the motivation to conduct a deeper study on this question. In dealing with R&D in a growth-theoretic environment, we explicitly incorporate an empirically supported idea that investing in R&D crucially depends on the nominal interest rate, for instance, through cash-in-advance (CIA) constraints (Chu and Lai, 2013; Aghion et al., 2012; Brown et al., 2012; and Chu, 2010). In modeling unions and employment, we naturally go with a standard method based on wages and unit labour costs (e.g., Neto et al., 2017; Chang and Hung, 2016; and Chu et al., 2016).

Therefore, we propose a novel analysis combining a Schumpeterian growth model with CIA constraints on R&D, and trade unions. To the best of our knowledge, this is the first study to combine these three features into a single, integrated, endogenous growth model. Besides answering our main research question, this framework also allows to study: (a) the optimality of Friedman rule; (b) the existence of an optimal trade union's markup. Regarding the literature, we rely on Chu and Cozzi (2014) to build our theoretical framework.

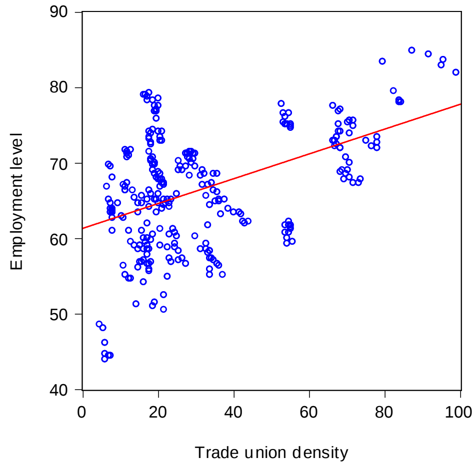
[1]For example, some recent Reuter News articles argue that "[t]he German government and unions have agreed on a two-stage wage increase of 4.75 percent over this year and next for more than 2 million public sector employees at the federal and municipal level" (http://www.reuters.com/article/us-germany-strike-idUSKCN0XQ14E); "[F]rench students and trade unions staged protest marches across the country (...) against far-reaching labor reforms" (http://www.reuters.com/article/us-france-protests-idUSKCN0WB0GW); and "US unions plan attack on Donald Trump in attempt to derail presidential bid" (http://www.theguardian.com/us-news/2016/apr/26/us-unions-donald-trump-us-election-2016). Hence, they can bargain wages, promote labour reforms, and even influence the political agenda.

[2]Endogenous growth theory identifies inventions coming out from R&D investments as a main source of long-run growth; see, for example, Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

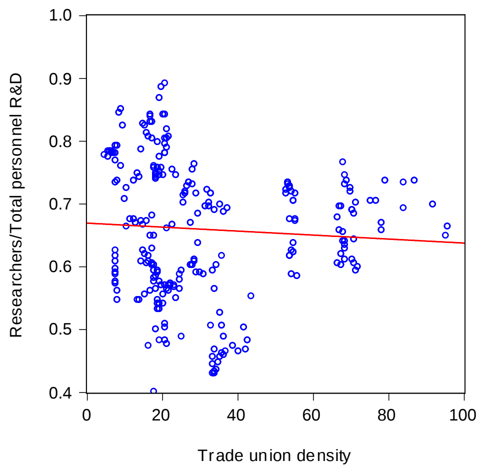
[3]A recent study by Chu et al. (2016) demonstrates a more moderate result that depending on whether the trade union is wage (employment)-oriented, they can have a negative (positive) effect on economic growth.



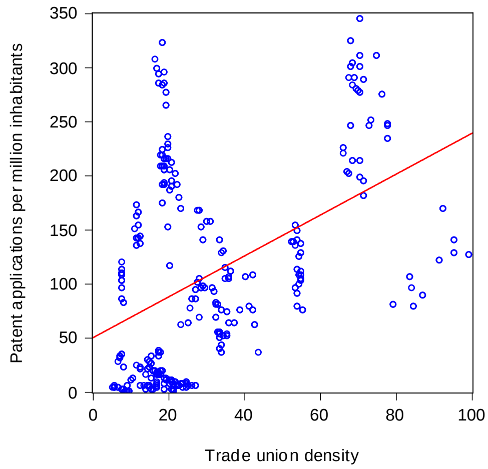
(a) Growth rate vs Trade union density



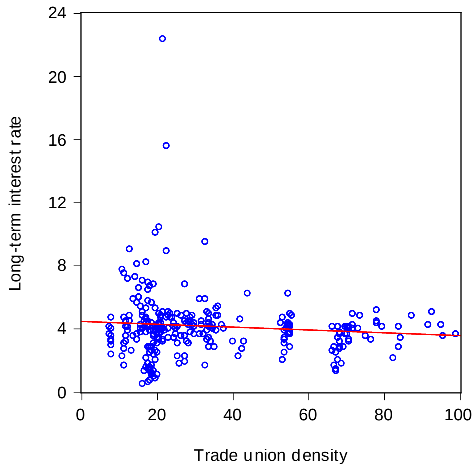
(b) Employment rate



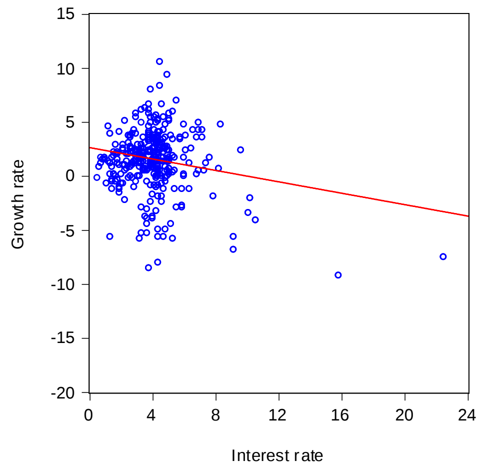
(c) R and D personnel



(d) PCT patents applications



(e) Long-term interest rate



(f) Growth rate vs Long-term interest rate

Figure 1: The relationship between Trade Union Density and Main Economic Variables
Source: OECD on-line database, tables: (1) Short-Term Labour Market Statistics - Employment rates and Trade union density; (2) R and D personnel total, as percentage of employment; (3) PCT patents applications per 10.000 inhabitants; (4) Long-term interest rate, available at http://stats.oecd.org (accessed on May 2016). Notes: Annual data (2002-2014), 29 OECD countries, Pooled OLS, N=377.

However, despite the richness of their framework, introducing labour market imperfections in the final-good sector (i.e., a trade union) allows us to complement their analysis and to further study the impact of trade unions on economic growth and welfare, as well as the relationship between monetary policy and different types of wage-setting framework. Additionally, this paper also relates with Chang and Hung (2016) and Chu et al. (2016). Nevertheless, the role of CIA and its impact on R&D, combined with a different wage setting approach, are some of the features that are present in our approach and help to complement and enhance the analysis of the previous papers.

Our main conclusions can be summarised as follows. First, economic growth is decreasing in both interest rate and trade union's markup; nevertheless, regarding welfare, although Friedman rule seems to be optimal for all the considered cases, a competitive labour market framework can be suboptimal below a specific threshold level of economic growth, depending on whether there is over or underinvestment of R&D; hence, by demanding a wage above the perfect competition equilibrium, trade unions can have a positive impact on welfare through a reallocation of labour among sectors; second, this relationship seems to be stronger for countries with lower labour share and higher rents in the intermediate sector. This latter case highlights the redistributive effect of trade unions, contributing for a decrease in inequality between monopolists and workers. Third, there seems to exist an interdependency between monetary policy and the macroeconomic role of trade unions: a lower (higher) interest rate increases (decreases) the effectiveness of trade union's markup on R&D labour and social welfare. Finally, these results have strong policy implications concerning monetary policy and labour market frameworks. If Friedman rule seems to be suitable for a wide range of parameters (i.e., different type of countries), this is certainly not the case for the labour market framework. Hence, for the case of the Eurozone, a "common" labour market setting might be more "inefficient" than a common monetary policy. A correct work of trade unions through determination of wages (i.e., the markup's size) might be the answer to promote economic growth and innovation.

The rest of this study proceeds as follows. Section 2 introduces a general description of the model. Section 3 describes the equilibrium and its main properties. Section 4 analyses the optimal trade union's markup, and Section 5 studies the optimal monetary policy. Section 6 provides a quantitative exercise. Finally, Section 7 states the main conclusions.

2 A Monetary Schumpeterian Growth Model with Trade Unions

2.1 Final good sector

Final good for consumption is homogenous and is produced by competitive firms that combine labour and intermediate goods, as follows:

$$y_t = \frac{1}{1-\beta}\left(\int_0^1 x_t\left(j\right)^{1-\beta} dj\right) L_{y,t}^{\beta}, \tag{1}$$

where $x_t\left(j\right)$ denotes the quantitive of intermediate goods $j \epsilon \left[0,1\right]$ and L_y corresponds to the labour used in final goods production. From profit maximisation, the demand function for $x_t\left(j\right)$ and $L_{y,t}$ are, respectively,

$$x_t(j) = \frac{L_{y,t}}{p_{x,t}^{1/\beta}(j)}, \tag{2}$$

$$L_{y,t}^D = w_t^{-\frac{1}{1-\beta}} \left[\frac{\beta}{1-\beta} \left(\int_0^1 x_t(j)\, dj \right) \right], \tag{3}$$

where $p_{x,t}(j)$ is the price of $x_t(j)$, and w_t is the wage.

2.2 Labour market framework: Monopoly trade union

As one of the main novelties of this paper, we embed an imperfect competitive labour market into an endogenous growth model. Imperfect competition is modeled through including a monopoly trade union, firstly proposed by Dunlop (1944) and Ross (1948).[4] Within this framework, the union decides unilaterally the level of wages, leaving firms to choose the level of employment afterwards. The monopoly trade union operates exclusively in the final-good sector and its utility function is assumed to have the following Stone-Geary form:

$$U^{MU} = (w_t - \bar{w}_t)^{1-v} \left(L_{y,t}^D \right)^v, \tag{4}$$

where w_t corresponds to the demanded wage and $\bar{w}_t$ to the perfect competition wage. The value of v states whether the union is relatively more employment-oriented or more wage-oriented. Since wages are set by the union previously to the firm's decision on the employment level, it can anticipate the impact of their wage claims on the employment level. Replacing (3) into (4), and maximising U^{MU} in order to w_t, we have:

$$w_t = \phi \bar{w}_t, \tag{5}$$

where $\phi = \frac{1}{\left(1 - \frac{1-v}{v} \frac{1}{\varepsilon_{L,w}}\right)} > 1$ can be interpreted as a markup over the perfect competition wage, and $\varepsilon_{L,w}$ is the elasticity of labour demand. We believe this markup can be a proxy of the trade union density discussed in Sector 1, in a sense that countries with high trade union density would present a higher markup due to higher bargaining power. Although set in the final good sector, wages apply to workers in the other productive sectors.

2.3 Intermediate-good sector

There is a unit-continuum of industries producing j-different quality-enhancing intermediate goods. Each industry is dominated by a temporary industry leader until the arrival of the next innovation, and the owner of the new innovation becomes the next industry leader. This dynamics captures the so called Arrow replacement effect.[5] The production function for the leader in industry j is:

$$x_t(j) = z^{q_t(j)} L_{x,t}(j), \tag{6}$$

[4] For a survey regarding the labour market framework see, among others, Lawson (2011) and Kaufman (2002).

[5] See Cozzi (2007) for a survey.

where the parameter $z > 1$ is the step-size of a productivity improvement, and $q_t(j)$ is the number of productivity improvements that have occurred in industry j as of time t. $L_{x,t}(j)$ is the production labour in industry j. Following Chu and Cozzi (2014), we adopt a cost-reducing view of vertical innovation as in Peretto (1998). This implies that, given $z^{q_t(j)}$, the (perfect competitive) marginal cost of production for the industry leader in industry j is $mc_t(j) = \frac{\bar{w}_t}{z^{q(j)}}$, and the amount of monopolistic profit is given by:

$$\Pi_t(j) = p_{x,t}(j)\, x_t(j) - \phi \bar{w}_t L_{x,t}(j). \tag{7}$$

We explicitly separate the union's markup and the perfect competitive marginal cost to better study the relationship between trade unions and profits. The standard Bertrand price competition leads to a profit-maximizing price $p_{x,t}(j)$ determined by a markup $\mu = \frac{p_{x,t}(j)}{mc_t(j)}$ over the perfect competitive marginal cost. As in Grossman and Helpman (1991), the markup is assumed to equal the step size z of innovation.

Following Acemoglu (2009), we normalise the prices to unity, i.e., $p_{x,t}(j) = 1$. Hence, recalling (2), we get:

$$\Pi_t = \frac{(\mu - \phi)}{\mu} L_{y,t}. \tag{8}$$

Finally, production-labour income is given by: $w_t L_{x,t}(j) = \frac{\phi}{\mu} L_{y,t}$. Since w_t corresponds to the monopoly wage, through using (5), we get:

$$\bar{w}_t L_{x,t}(j) = \frac{1}{\mu} L_{y,t}. \tag{9}$$

2.4 R&D-good sector

Denote $v_t(j)$ as the value of the monopolistic firm in industry j. From (8), we know that $\pi_t(j) = \pi_t,\ j \epsilon [0,1]$. Combining this fact with the symmetry of the profit-sharing rule across industries implies that $v_t(j) = v_t,\ j \epsilon [0,1]$. In other words, the arrival rate of innovation across industries is the same. Hence, the standard no-arbitrage condition for v_t is given by

$$r_t v_t = \Pi_t + \dot{v}_t - \lambda_t v_t. \tag{10}$$

The left-hand side is the risk-free return of holding v_t as an asset. The right-hand side represents the expected asset return, which corresponds to the sum of (a) the monopolistic profit π_t, (b) potential capital gains $\dot{v}_t$, and (c) expected per capita loss, $\lambda_t v_t$, due to creative destruction; λ_t stands for the arrival rate of the next innovation.

There is a unit continuum of R&D firms indexed by $k \epsilon [0,1]$, hiring R&D-labour, $L_{r,t}(k)$, for innovation. The labour cost for R&D is $w_t L_{r,t}(k)$. Following Chu and Cozzi (2014), we assume that the entrepreneur faces a CIA constraint and needs to borrow a specific amount of money from households, $B_t(k)$, subject to the nominal interest rate, i. For simplicity, we also assume that $B_t(k) = w_t L_{r,t}(k)$, which implies that firms need to borrow the entire wage bill, and the total cost of R&D per unit of time is $(1 + i_t)\, \phi \bar{w}_t L_{r,t}(k)$.[6] Note that the trade union's markup and the CIA constraint on R&D gives the trade union and the monetary authority the ability to influence and affect the equilibrium allocation

[6]For a detailed explanation, see Chu and Cozzi (2014).

of resources across sectors through the markup and the nominal interest rate, respectively. The zero-expected-profit condition of firm k is given by:

$$v_t \lambda_t(k) = (1 + i_t) \phi \bar{w}_t L_{r,t}(k), \tag{11}$$

where $\lambda_t(k) = \bar{\varphi} L_{r,t}(k)$ corresponds to the *firm-level* innovation arrival rate per unit time. $\bar{\varphi} = \frac{\varphi}{N_t}$ captures the dilution effect that removes scale effects (Laincz and Peretto, 2006), where φ is the R&D productivity parameter. Therefore, the *aggregate* arrival rate of innovation is:

$$\lambda_t = \int_0^1 \lambda_t(k)\, dk = \frac{\varphi}{N_t} L_{r,t} = \varphi l_{r,t},$$

where $l_{r,t} = \frac{L_{r,t}}{N_t}$ represents the intensity of R&D (measured as R&D employment per capita). Similarly, $l_{x,t} = \frac{L_{x,t}}{N_t}$ and $l_{y,t} = \frac{L_{y,t}}{N_t}$ stand for employment intensity (per capita) in the intermediate and final-goods sectors, respectively.

2.5 Households

There is a unit continuum of identical households, who have a life time utility function given by

$$U = \int_0^\infty e^{-\rho t} \left[\ln c_t + \theta \ln (1 - l_t)\right] dt, \tag{12}$$

where c_t is per capita consumption of final goods and l_t is the supply of labour per person at time t. The parameters $\theta \geq 0$ and $\rho > 0$ determine, respectively, the leisure preferences and the subjective discount rate. At time t, the population size equals N_t, and grows at an exogenous rate $n \geq 0$.[7] Each household maximises (12) subject to the following asset-accumulation equation:

$$\mathring{a}_t + \mathring{m}_t = (r_t - n) a_t + w_t l_t + \tau_t - c_t - (\pi_t + n) m_t + i_t b_t. \tag{13}$$

r_t is the real interest rate and $\mathring{a}_t$ is the real value of assets owned by each household in the form of equity shares in monopolistic intermediate goods firms. Each household supplies labour l_t (inelastically if $\theta = 0$) to earn a real wage rate, w_t, which is set by the trade union. The cost of holding money is given by π_t, the inflation rate, and τ_t is a a lump-sum transfer that households take as given. m_t is the per capita real money balance held to enable lending to the R&D sector. The CIA constraint is given by $b_t \leq m_t$, where b_t is the amount of money borrowed from each household by entrepreneurs to finance R&D investment. The return on b_t is i_t.

The optimality condition for consumption is:

$$\frac{1}{c_t} = \eta_t, \tag{14}$$

where η_t is the Hamiltonian co-state variable. The optimality condition for labour supply is:

[7] We assume that the utility function is based on per capita utility. Alternatively, one can assume that the utility function is based on aggregate utility, in which case the effective discount rate simply becomes $\rho - n$.

$$w_t(1-l_t) = \theta c_t, \tag{15}$$

and the intertemporal optimality condition is given by:

$$-\frac{\mathring{\eta}_t}{\eta_t} = r_t - \rho - n. \tag{16}$$

Finally, from the dynamic optimisation, we also derive a no-arbitrage condition

$$i_t = r_t + \pi_t. \tag{17}$$

2.6 Monetary authority

M_t denotes the nominal money supply. The aggregate real money balance is $m_t N_t = \frac{M_t}{P_t}$, where P_t denotes the price of final goods. The central bank is assumed to exogenously set i_t and the inflation rate is endogenously determined accordingly to $\pi_t = i_t - r_t$. Then, the growth rate of nominal money supply is also endogenously determined accordingly to $\frac{\mathring{M}_t}{M_t} = \frac{\mathring{m}_t}{m_t} + \pi_t + n$. Finally, the monetary authority returns the seigniorage revenue as a lump transfer $\tau_t N_t = \frac{\mathring{M}_t}{P_t} = [\mathring{m}_t + (\pi_t + n) m_t] N_t$ to households.

3 Closing the model

In this section we (a) define the decentralised equilibrium; (b) derive the balanced-growth path; and (c) find the socially optimal allocation of resources.

3.1 Decentralised Equilibrium

The equilibrium is a path of allocations $\{c_t, m_t, l_t,, y_t, x_t(j), L_{y,t}, L_{x,t}(j), L_{r,t}(k)\}$ and a path of prices $\{p_{x,t}(j), w_t, r_t, i_t, v_t\}$. Also, at each instance of time:

- households choose $\{a_t, c_t\}$ to maximise utility taking $\{i_t, r_t, w_t\}$ as given;
- competitive final-goods firms produce $\{y_t\}$ and choose $\{L_{y,t}\}$to maximise profit taking $\{p_{x,t}(j), w_t\}$ as given;
- trade union chooses $\{w_t\}$to maximise utility taking $\left\{L^D_{y,t}\right\}$ as given;
- monopolistic intermediate-goods firms produce $\{x_t(j)\}$ and choose $\{L_{x,t}(j), p_{x,t}(j)\}$ to maximise profits according to the Bertrand price competition and taking $\{w_t\}$ as given;
- R&D firms choose $\{L_{r,t}(k)\}$ to maximise expected profit taking $\{i_t, w_t, v_t\}$ as given;
- the market-clearing condition for labour holds such that $L_{y,t} + L_{x,t} + L_{r,t} = l_t N_t$;
- the market-clearing condition for final goods holds such that $y_t = c_t N_t$;
- the value of monopolistic firms adds up to the value of households' assets such that $v_t = a_t N_t$;
- the amount of money borrowed by R&D entrepreneurs is $w_t L_{r,t} = b_t N_t$.

Balanced-growth path

Substituting (6) into (1), we find the aggregate production function

$$y_t = \frac{1}{1-\beta} Z_t^{1-\beta} L_{x,t}^{1-\beta} L_{y,t}^{\beta}, \tag{18}$$

where aggregate technology Z_t is defined as

$$Z_t = \exp\left(\int_0^1 q_t\left(j\right) dj \ln z\right). \tag{19}$$

Applying the law of large numbers on the previous equation yields

$$Z_t = \exp\left(\int_0^1 \lambda_s ds \ln z\right). \tag{20}$$

Finally, differentiating the log of (20) with respect to t gives the growth rate of aggregate technology as:

$$g_t \equiv \frac{\mathring{Z}_t}{Z_t} = \lambda_t \ln z = \left(\varphi \ln z\right) l_{r,t}. \tag{21}$$

THEOREM 1. *Given a constant nominal interest rate i and a markup over perfect competitive wages, the economy immediately jumps to a unique and saddle-point stable balanced growth path along which each variable grows at a constant (possible zero) rate.*

PROOF. See Appendix I.

□

On the balanced growth path, the equilibrium labour allocation is stationary. Recalling $r_t = \frac{\pi_t + \dot{v}_t - \lambda_t v_t}{v_t}$ and $r = g + \rho + n$, imposing balanced growth path on (10) gives $v_t = \frac{\Pi_t}{\rho+\lambda}$. Therefore, combining the next four equations,

$$\left\{\begin{array}{c} v_t = \frac{\Pi_t}{\rho+\lambda} \\ v_t \lambda_t\left(k\right) = \left(1+i_t\right) \phi \bar{w}_t L_{r,t}\left(k\right) \\ \lambda_t = \varphi l_{r,t} \\ \Pi_x = \frac{\left(\mu-\phi\right)}{\mu} L_{y,t} \end{array}\right. ,$$

we can derive the first of four equations needed to find the equilibrium labour allocation across sectors.

$$\left(\mu - \phi\right) l_x = \left(1+i\right) \phi \left(\frac{\rho}{\varphi} + l_r\right). \tag{22}$$

Notice that, since we assume $p_{x,t} = 1$, we also know that $y = \frac{1}{1-\beta} L_y$. Recalling (9), we get $y = \frac{1}{1-\beta}\left[\bar{w}\mu L_x\right]$. Thus, combining:

$$\left\{\begin{array}{c} \phi \bar{w}\left(1-l\right) = \theta c \\ y = c = \frac{1}{1-\beta}\left[\bar{w}\mu l_x\right] \end{array}\right. ,$$

we obtain the second equation:

$$l = 1 - \frac{\theta}{\phi}\left(\frac{1}{1-\beta}\right)\mu l_x. \tag{23}$$

Moreover, we also know that the perfect competition wages corresponds to $\bar{w} = \frac{\beta}{1-\beta}$. Combined this result with (9), we get the third equation:

$$\frac{\beta}{1-\beta}l_x = \frac{1}{\mu}l_y. \tag{24}$$

Finally, we just need to define the per capita version of the labour-market-clearing condition:

$$l = l_y + l_x + l_r. \tag{25}$$

Combining the previous four equations, and after some mathematical manipulation, we find:

$$l_x = \frac{(1-\beta)(1+i)\phi}{\mu\beta(1+i)\phi + (1-\beta)(i\phi+\mu) + \theta(1+i)\mu}\left(1+\frac{\rho}{\varphi}\right). \tag{26}$$

Obtaining the remaining employment levels across sectors is straightforward:

$$l_r = \frac{(\mu-\phi)(1-\beta)}{\mu\beta(1+i)\phi + (1-\beta)(i\phi+\mu) + \theta(1+i)\mu}\left(1+\frac{\rho}{\varphi}\right) - \frac{\rho}{\varphi}, \tag{27}$$

$$l_y = \frac{\mu\beta(1+i)\phi}{\mu\beta(1+i)\phi + (1-\beta)(i\phi+\mu) + \theta(1+i)\mu}\left(1+\frac{\rho}{\varphi}\right), \tag{28}$$

$$l = \frac{\mu\beta(1+i)\phi + (1-\beta)(i\phi+\mu)}{\mu\beta(1+i)\phi + (1-\beta)(i\phi+\mu) + \theta(1+i)\mu}\left(1+\frac{\rho}{\varphi}\right) - \frac{\rho}{\varphi}. \tag{29}$$

Equations (26) and (28) show that the shares of intermediate-goods' labour, l_x, and final-goods' labour, l_y, increase with trade union's markup, ϕ, and with nominal interest rate, i, under both elastic ($\theta > 0$) and inelastic ($\theta = 0$) labour supply.[8] On the other hand, l is increasing in ϕ but decreasing in i. Finally, (27) shows that R&D-labour, l_r, is decreasing in the trade union's markup, ϕ, and in nominal interest rate, i, under both elastic and inelastic labour supply.

PROPOSITION 1. *Labour supply is increasing in trade union's markup but decreasing in interest rate, whereas R&D labour is decreasing in both variables. Furthermore, both final-goods' labour and intermediate-goods' labour are increasing both in trade union's markup and interest rate.*

PROOF. Proven in the text.

□

[8] Note that, under $\theta = 0$, $l_y = \frac{\mu\beta(1+i)\phi}{\mu\beta(1+i)\phi+(1-\beta)(i\phi+\mu)}\left(1+\frac{\rho}{\varphi}\right)$, $l_x = \frac{(1-\beta)(1+i)\phi}{\mu\beta(1+i)\phi+(1-\beta)(i\phi+\mu)}\left(1+\frac{\rho}{\varphi}\right)$, $l_r = \frac{(\mu-\phi)(1-\beta)}{\mu\beta(1+i)\phi+(1-\beta)(i\phi+\mu)}\left(1+\frac{\rho}{\varphi}\right) - \frac{\rho}{\varphi}$, and $l = 1$.

Interestingly, by the definition of ϕ, a more employment-oriented union (a higher v) means a lower markup ϕ. By Proposition 1, this implies that a more employment-oriented union results in a smaller amount of the total labor supply, l, which seems rather paradoxical but understandable within the model as follows. On the one hand, under this framework, companies within the intermediate sector employ all workers willing to provide labour due to the existing constant markup. In other words, firms will have a "fixed" margin of revenues. On the other hand, taking into account the standard positive effect of wages on labour supply, (15), a more wage-oriented union can increase the amount of workers available in the market. Hence, combining this with the constant markup and its inter-connection with the final sector, a more wage(employment)-oriented union results in a larger(smaller)-amount of production labour, l_y and l_x, as well as in the total labour supply, l. Concerning R&D labour, since a higher ϕ implies a higher innovation cost, without a constant markup R&D firms will need to decrease their level of employment, l_r.

Furthermore, the previous results imply that economic growth given by $g = (\varphi \ln z)\, l_r$ also decreases with ϕ and i. Note that:

$$g = (\varphi \ln z)\, l_r = \left[\frac{(\mu - \phi)(1-\beta)(\varphi + \rho)}{\mu\beta(1+i)\phi + (1-\beta)(i\phi + \mu) + \theta(1+i)\mu} - \rho\right] \ln z. \tag{30}$$

The negative effect of i on g is in line with the empirical evidence provided by Brown et al. (2011) and Becker and Pain (2008), where a higher(lower) interest rate implies a lower(higher) level of R&D expenditure due to the increase in investment costs. Chu and Lai (2013) states a negative relationship between interest rate and R&D through inflation. This effect is also present in our paper through (16) and (17). On the other hand, Doucouliagos and Laroche (2013), Barbosa and Faria (2011) and Lingens (2009) document a negative relationship between trade unions and economic growth, which is in line with (30).[9] Furthermore, both theoretical effects are in line with the empirical motivation provided in Figure 1 (a,e).

PROPOSITION 2. *Economic growth is decreasing in trade union's markup and nominal interest rate.*

PROOF. Proven in the text.

□

An advantage of the model is that we have both money and unions in a single setup. Hence, it is possible to study the interaction of i and θ by studying the sign of $\frac{\partial}{\partial i}\left|\frac{\partial l_r}{\partial \phi}\right|$. If negative, the negative effect of ϕ on R&D l_r (or the positive effect of a more employment-oriented union on R&D) is magnified by a lower nominal interest rate, which highlights the interdependency between monetary policy (based on a CIA constraint) and the macroeconomic role of trade unions. Through some mathematical manipulations, we get

$$\frac{\partial^2 l_r}{\partial i \partial \phi} \equiv -\frac{\mu(1-\beta)(1-\beta+\beta\mu+\theta)}{(\zeta i + (\zeta + (1-\beta)\mu))^3}\left[(1-\beta)\mu - (1+i)\zeta - (2+i)(1-\beta)\phi\right],$$

[9]Nevertheless, according to Chu et al. (2016), Mukherjee and Wang (2013) and Menezes-Filho et al. (1998), the effect of trade unions on economic growth might be positive if both trade unions and firms bargaining over wages and employment, which is not the case in this paper.

where $\zeta \equiv \mu(\beta\phi + \theta)$. This expression of $\frac{\partial^2 l_r}{\partial i \partial \phi}$ is strictly positive for any $i \geq 0$ and $\phi > 1$ if $1 - 2\beta < \theta$, which holds for the range of empirically plausible parameters (see Section 6).[10] Since $\frac{\partial^2 l_r}{\partial i \partial \phi} > 0$ means $\frac{\partial}{\partial i}\left|\frac{\partial l_r}{\partial \phi}\right| < 0$ due to $\frac{\partial l_r}{\partial \phi} < 0$, the negative effect of the union's markup (the positive effect of an employment-oriented union) on R&D and growth is magnified by a smaller nominal interest rate, i. Conversely, as i is higher, $\phi' s$ negative effect on l_r is smaller. This result crucially relates with our motivation - indeed, under a low-interest scenario or an autonomous monetary policy, a correct work of trade unions through determination of wages (i.e., the markup's size) might be the answer to promote economic growth and innovation.

PROPOSITION 3: *There is an interdependency between monetary policy and the macroeconomic role of trade unions: a lower (higher) interest rate increases (decreases) the effectiveness of trade union's markup on R&D labour and economic growth.*

PROOF. Proven in the text.

$\square$

3.2 Socially Optimal Allocation

Following Chu and Cozzi (2014), it is possible to derive the socially optimal allocation of the model. Imposing balanced growth on (12), we get:

$$U = \frac{1}{\rho}\left[\ln(c_0) + \frac{g}{\rho} + \theta \ln(1 - l)\right], \tag{31}$$

where $c_0 = Z_0^{1-\beta} l_x^{1-\beta} l_y^{\beta}$, $g = \lambda \ln z = (\varphi \ln z) l_r$ and the exogenous Z_0 is normalized to unity. Maximizing the previous equation subject to $l = l_y + l_x + l_r$, we obtain the first-best allocation, here denoted with a superscript *:

$$l_y^* = \beta \frac{\rho}{\varphi \ln(z)}, \tag{32}$$

$$l_x^* = (1 - \beta) \frac{\rho}{\varphi \ln(z)}, \tag{33}$$

$$l_r^* = 1 - \frac{\rho(1 + \theta)}{\varphi \ln(z)}, \tag{34}$$

$$l^* = 1 - \frac{\rho\theta}{\varphi \ln(z)}. \tag{35}$$

As in Chu and Cozzi (2014), we restrict the parameter space to ensure that $l_r^* > 0$, which, in turn, implies that $l^* > 0$.

[10]Since $(1-\beta)\mu - (1+i)\zeta - (2+i)(1-\beta)\phi \equiv \Phi$ is decreasing in $\phi > 1$ and $i \geq 0$, we have $\sup_{(\phi,i)} \Phi \equiv (1 - 2\beta - \theta)\mu - 2(1-\beta)$, noting $\zeta \equiv \mu(\beta\phi + \theta)$. This supremum is always negative when $1 - 2\beta - \theta < 0$.

4 Optimal Trade Union's markup

Taking into account the social optimal allocation, in this section we provide a detailed analysis regarding the optimal trade union's markup, assuming an exogenous constant interest rate. As we stated in Section 1, this corresponds to one of the main novelties of our analysis since, to best of our knowledge, we are the first to assess the implications of an optimal trade union's markup within a quality-ladder R&D model.

We first analyse the case of inelastic labour supply (Section 4.1); then, we consider the general case of elastic labour supply (Section 4.2). We denote ϕ^* as the optimal union's markup, referring to the trade union's markup that maximises social welfare, regardless of whether or not it enables the first-best socially optimal allocations $\{l_y, l_x, l_r, l\}$. It might be the case that the optimal trade union's markup fails to optimally allocate labour across sectors. We will discuss this possibility under both scenarios.

4.1 Optimal trade union's markup under inelastic labour supply

Under inelastic labour supply, by choosing the optimal trade union's markup, ϕ^* (from $\frac{\partial U}{\partial \phi} = 0$), the first-best allocation $\{l_Y^*, l_x^*, l_r^*\}$ may be enabled as follows:

$$\phi^* = \left[\frac{(1-\beta)\,\mu}{(1+i)\left[(1-\beta)+\beta\mu\right]\left[\ln(z)\left(\frac{\varphi}{\rho}+1\right)-1\right]+(1-\beta)}, 1 \right]. \tag{36}$$

Since trade union's markup cannot be lower than 1 (corresponding to the perfect competitive wage), we impose $\phi^* \geq 1$. Hence, if $\phi^* = 1$, a perfect competitive labour market is optimal, but the first-best allocation may not be attained, unless $\phi^* = 1$ holds exactly and it is not binding. On the other hand, if $\phi^* > 1$, a competitive labour market is inefficient (in a sense that $\phi^* = 1$ does not correspond to the optimal markup) and the first-best allocation is attained with $\phi = \phi^*$.

Moreover, it is important to note that, due to some specific characteristics of the quality-ladder models, such as the intertemporal spillover effect and the business-stealing effect (Chu and Cozzi, 2014), and the monopoly distortion effect (Denicolò and Zanchettin, 2014; Aghion and Howitt, 1992), the equilibrium with $\phi = 1$ may feature either overinvestment or underinvestment in R&D. Over(Under)investment is associated with l_r *versus* l_r^*. Indeed, comparing l_r^* with l_r under $\theta = 0$, it is possible to prove that $\phi^* > 1$ if and only if the decentralised equilibrium l_r, evaluated at $\phi = 1$, is greater than optimal l_r^*. Therefore, R&D overinvestment in equilibrium is a necessary and sufficient condition for a competitive labour market to be suboptimal.

PROPOSITION 4. *If and only if R&D overinvestment occurs under the competitive labour market equilibrium, the optimal trade union's markup would be strictly positive; in this case, a competitive labour market is suboptimal. If and only if the optimal trade union's markup is positive, then* ϕ^* *enables the first-best allocation* $\left\{l_y^*, l_x^*, l_r^*\right\}$.

PROOF. Impose $\theta = 0$ on l_r^* and compare it with l_r.

$$l_r|_{\phi=1} > l_r^* \Leftrightarrow \frac{(\mu-1)(1-\beta)}{\mu\beta(1+i)+(1-\beta)(i+\mu)}\left(1+\frac{\rho}{\varphi}\right) - \frac{\rho}{\varphi} > 1 - \frac{\rho}{\varphi ln(z)}.$$

A few mathematical manipulations show that the previous equation is equivalent to:

$$(1+i)\left[(1-\beta)+\beta\mu\right]\left[ln\left(z\right)\left(\frac{\varphi}{\rho}+1\right)-1\right]+(1-\beta)<(1-\beta)\,\mu,$$

Thus, $l_r|_{\phi=1} > l_r^*$ if and only if $(1+i)\left[(1-\beta)+\beta\mu\right]\left[ln\left(z\right)\left(\frac{\varphi}{\rho}+1\right)-1\right]+(1-\beta) < (1-\beta)\,\mu$, which corresponds to the numerator in (36). Hence, $l_r|_{\phi=1} > l_r^* \Leftrightarrow \phi^* > 1$.

□

Regarding the comparative statics of ϕ^* when it is strictly positive, it is interesting to note that ϕ^* is decreasing with i^*, which is in line with the empirical motivation provided in Figure 1,f). Intuitively, a larger interest rate decreases R&D through the increase in R&D costs, which implies that R&D overinvestment (underinvestment) is less (more) likely to occur.

On the other hand, ϕ^* is increasing in μ, meaning that a larger markup in the intermediate sector (which can be interpreted as a larger patent breadth) increases R&D - in this case, overinvestment is more likely to occur. Note that, from other point of view, a higher markup in the intermediate sector also allows the trade union to demand a higher wage for its workers. In this case, unions work to decrease inequality between monopolists and workers. Furthermore, ϕ^* is decreasing in β, the parameter that accounts for the labour-intermediate goods ratio share in the final good sector. One possible interpretation relates with the fact that when β increases, the labour share in the final-good sector increases and the share of intermediate goods decreases, which negatively affects the amount of labour available for the R&D sector, leading to a decrease in R&D activity - in this case, underinvestment is now more likely to occur. Finally, ϕ^* is increasing in ρ and decreasing in φ, and z due to the over(under)investment mechanism.

4.2 Optimal trade unions' markup under elastic labour supply

Taking into account (29), under elastic labour supply ($\theta > 0$), labour supply l is decreasing in ϕ. In this case, the trade union's market power entails a distortionary effect on the consumption-leisure decision, implying that the optimal trade union's markup can no longer achieve the first-best allocation. Since the optimal trade union's markup rate is given by $\frac{\partial U}{\partial \phi}=0$, after a few steps of mathematical manipulation, one can get:

$$\phi^*=\left[\frac{-\left\{\Psi\Phi\left(1-\theta\right)-\Theta\left(\frac{\varphi}{\rho}+1\right)\right\}\pm\sqrt{\left\{\Psi\Phi\left(1-\theta\right)-\Theta\left(\frac{\varphi}{\rho}+1\right)\right\}^2+4\left(\theta\Phi^2\right)\Psi^2}}{-2\theta\Phi^2},1\right], \tag{37}$$

with $\Theta=(1-\beta)\,\mu\,(1+i)\left\{\theta+\mu\beta+(1-\beta)\right\}\ln\left(z\right)$, $\Psi=\mu\left[\theta\left(1+i\right)+(1-\beta)\right]$, and $\Phi=(1+i)\,\mu\beta+(1-\beta)\,i$.

In this case, it might be possible that the optimal trade union's markup include two different solutions, conditional on $\phi^* > 1$. In Section 6.2, below, we numerically simulate in detail its dynamics and the main implications of ϕ^* under elastic labour supply.

5 Optimal Monetary Policy and the Friedman Rule

This section closely follows Chu and Cozzi (2014) and could be considered as a general case. Indeed, adding the assumption of a constant trade union's markup, we take into account the two possible scenarios of inelastic and elastic labour supply. Denoting i^* as the optimal interest rate, it might also be the case that the optimal interest rate fails to optimally allocate labour across sectors. Following the same approach as in Section 4, the optimal nominal interest rate under elastic and elastic labour supply are given by, respectively,

$$i^* = max\left[\frac{\mu\left[(1-\beta)+\beta\phi\right]-\left[\mu\beta+(1-\beta)\right]\phi\left[\ln(z)\left(\frac{\varphi}{\rho}+1\right)\right]}{\left[\mu\beta+(1-\beta)\right]\phi\left[\ln(z)\left(\frac{\varphi}{\rho}+1\right)-1\right]},0\right] \tag{38}$$

and

$$i^* = \left(\frac{\mu-\Theta}{\Theta-\phi},0\right), \tag{39}$$

where Θ is a composite parameter defined as:

$$\Theta = \left\{\frac{\left[\mu\beta\phi+(1-\beta)\phi+\theta\mu\right]}{(1+\theta)(1-\beta)}\left(\frac{\varphi}{\rho}+1\right)\ln(z)\right\}-\frac{\mu\beta\phi+\theta\mu}{(1-\beta)}. \tag{40}$$

As in Chu and Cozzi (2014), R&D overinvestment in equilibrium is a necessary and sufficient condition for the Friedman rule to be suboptimal. Nevertheless, setting $i^* = i$ under the elastic labour supply case does not yield the first-best allocation of manufacturing and labour supply, but yields the first-best allocations for R&D labour. Finally, the relationship between i^* and ϕ is identical to the one identified in the previous section - i^* is decreasing in ϕ if $i^* > 0$.

6 Quantitative Analysis

In this section, we provide a numerical simulation on the optimality of the trade union's markup (Section 6.1) and the Friedman rule (Section 6.2), under both inelastic and elastic labour supply. Under the former case, we have the following set of parameters $\{\rho, z, \mu, \varphi, \beta, i_{LR}\}$,where i_{LR} corresponds to the long-run nominal interest rate, which is set to 0.08 (Chu and Cozzi, 2014). Note that, for the analysis of the optimal monetary rule, we also need to specific the (exogenous) value of ϕ. We follow Acemoglu and Akcigit (2012) and Barlevy (2007) to set the discount rate ρ to 0.05 (meaning that a unit of time in the model corresponds to a year) and the step size z of innovation to 1.05. Regarding the markup, we set $\mu = 1.275$, a slightly higher value than Chu and Cozzi (2014), but within the empirical estimates reported by Jones and Williams (2000) and the calibrated values in Reis and Sequeira (2007). In regards to β, Karabarbounis and Neiman (2014) reports a global decline of the labor share mostly since the early 1980s. Hence, we consider two scenarios: $\beta = 0.3$ and $\beta = 0.6$. These values aim to represent a lower and a higher bound, respectively, and are in line with the empirical estimates reported in Samaniego and Sun (2015) and Karabarbounis and Neiman (2014), as well as with the calibrated values in Barlevy (2007) and Reis and Sequeira (2007). Furthermore, by considering a higher and a lower value for β, we can also take into account the reported estimates by

OECD (2015) and Guerriero (2012), capturing that developing countries exhibit lower labor shares than developed ones.

Following Chu and Cozzi (2014), we fix $g = 0.02$, which corresponds to the long-run economic growth rate of the United States. However, we also take into consideration the argument that long-run economic growth might not be entirely driven by domestic R&D investment (Comin, 2004). According to Chu (2010), for the US economy, the fraction f of long-run economic growth driven by domestic R&D investment is approximately 0.4. Hence, we analyse the optimal interest rate and trade union's markup for $f\,[0.3, 1]$. Since considering lower values of f is equivalent to consider lower economic growth rates, we can extend our analysis to other countries with different R&D growth patterns and lower growth rates, respectively. Hence, the R&D productivity parameter (φ) is calibrated to match the value of $g = (\varphi \ln z)\, lr$, where lr depends on the elasticity of the labour supply. In the analysis of optimal interest rate, ϕ is exogenous and set at unity ensuring a perfect competitive labour market.

ρ	z	μ	g	i_{LR}	β_{Low}	β_{High}	ϕ	θ
0.05^a	1.05^a	1.275^b	0.02^c	0.08^c	0.30^d	0.60^d	1.00	2.30^c

Table 1: Baseline calibration

a) Acemoglu and Akcigit (2012) and Barlevy (2007).

b) Chu and Cozzi (2014), Reis and Sequeira (2007) and Jones and Williams (2000).

c) Chu and Cozzi (2014).

d) Samaniego and Sun (2015),Karabarbounis and Neiman (2014), Barlevy (2007), Reis and Sequeira (2007).

In the case of elastic labour supply, we have one extra parameter, θ. Chu and Cozzi (2014) fixes $l = 0.3$ and choose the value of θ accordingly. In our case, we fix $\theta = 2.3$ (in line with Chu and Cozzi, 2014) while l adjusts freely. This approach enables the analysis of employment. Table 1 reports the values for each parameter in the benchmark case, mainly based on the United States (US) economy estimates.

6.1 Optimal markup

In this case, note that there is an extra relationship since the optimal trade union's markup, ϕ^*, depends on the R&D productivity parameter, φ, as follows:

$$\begin{cases} \phi^* &= \dfrac{-\left\{\Psi\Phi(1-\theta)-\Theta\left(\frac{\varphi}{\rho}+1\right)\right\}\pm\sqrt{\left\{\Psi\Phi(1-\theta)-\Theta\left(\frac{\varphi}{\rho}+1\right)\right\}^2-4(\theta\Phi^2)\Psi^2}}{-2\theta\Phi^2} \\ \varphi &= \left[\frac{g}{\ln z}+\rho\right]\frac{[\phi\{\mu\beta(1+i)+(1-\beta)i\}+\mu\{(1-\beta)+\theta(1+i)\}]}{(\mu-\phi)(1-\beta)}-\rho \end{cases}.$$

After some mathematical manipulation (see Appendix II), we find the optimal markup associated with the existing R&D productivity parameter in the economy:

$$\phi^* = \left[\frac{[\xi + \Lambda\mu + \Gamma\chi] \pm \sqrt{[\xi + \Lambda\mu + \Gamma\chi]^2 - 4\Lambda\mu\xi}}{2\Lambda}, 1\right]. \tag{41}$$

Table 2 and Table 3 report the simulation results under inelastic and elastic labour supply, respectively. Regarding the first case, a competitive labour market framework works better in all the considered scenarios, i.e., there seems to be no room for trade unions to improve social welfare since the optimal markup is one ($\phi^* = 1$). However, under elastic labour supply, this is certainly not the case since there is an exception that breaks the optimality of the competitive labour market framework, which arises under the lowest considered economic growth rate ($f = 0.3$) and $\beta = 0.3$. This implies that a trade union's markup greater than one, meaning a wage above the perfect competition scenario, might actually be socially desired if the growth rate is below a particular level - in this case below approximately 0.8%. The mechanism is the following: under a low growth rate and a competitive labour market framework, there is an excess of R&D labour, l_r, in terms of social welfare, due to the low productivity of R&D sector (φ). Hence, by setting $\phi^* > 1$, the trade union can increase social welfare through a reallocation of labour force due to an increase in labour supply and wages. The latter effect has a negative impact on the demand for R&D labour but a positive effect on the productivity of R&D, under a fixed economic growth rate. Finally, the additional available labour supply through the former effect will be employed in the final and intermediate sector .

Both tables also report the welfare gains from reducing the long-run nominal interest rate from 8% to 0%.[11,12] Interestingly, the welfare gains crucially depend on the value of f and β, as well as on the elasticity of labour supply. The higher welfare gains are achieved under elastic labour supply with the combination $\beta = 0.6$ and $f = 1$. On one hand, a higher β implies a lower i^*. On the other hand, a higher f implies that monetary policy can have a higher impact on the R&D policy and, ultimately, on economic growth. Hence, the decrease in interest rate from 8% to 0% brings higher welfare gains within this scenario.

f	1	0.9	0.8	0.7	0.6	0.5	0.4	0.35	0.3
g	2.00%	1.80%	1.60%	1.40%	1.20%	1.00%	0.80%	0.70%	0.60%
φ	3.20	2.91	2.62	2.33	2.04	1.75	1.46	1.32	1.17
ϕ^*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ΔU	1.94%	1.67%	1.39%	1.11%	0.84%	0.57%	0.29%	0.16%	0.02%

(a) $\beta = 0.3$

f	1	0.9	0.8	0.7	0.6	0.5	0.4	0.35	0.3
g	2.00%	1.80%	1.60%	1.40%	1.20%	1.00%	0.80%	0.70%	0.60%
φ	5.67	5.16	4.65	4.14	3.63	3.12	2.61	2.36	2.10
ϕ^*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ΔU	2.67%	2.37%	2.08%	1.78%	1.48%	1.18%	0.89%	0.74%	0.59%

(b) $\beta = 0.6$

Table 2: Optimal monetary policy - simulation under inelastic labour supply

[11]Note that, to calibrate the R&D productivity parameter, φ, we set the long-run interest rate to 8%.
[12]We report the welfare gains as the standard equivalent variation in consumption.

f	1	0.9	0.8	0.7	0.6	0.5	0.4	0.35	0.3
g	2%	1.8%	1.6%	1.4%	1.2%	1%	0.8%	0.7%	0.6%
φ	10.77	9.81	8.84	7.88	6.91	5.95	4.98	4.50	4.15
ϕ^*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01
$\triangle U$	2.33%	2.02%	1.71%	1.40%	1.09%	0.78%	0.47%	0.32%	0.21%

(a) $\beta = 0.3$

f	1	0.9	0.8	0.7	0.6	0.5	0.4	0.35	0.3
g	2.00%	1.80%	1.60%	1.40%	1.20%	1.00%	0.80%	0.70%	0.60%
φ	18.91	17.22	15.53	13.84	12.15	10.46	8.77	7.93	7.08
i^*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ΔU	2.90%	2.58%	2.26%	1.94%	1.62%	1.31%	0.99%	0.83%	0.68%

(b) $\beta = 0.6$

Table 3: Optimal trade union's markup - simulation under elastic labour supply

6.2 Optimal interest rate

Regarding the optimal interest rate, the Friedman rule is confirmed for all the considered scenarios, which implies that the results from Table 2 and Table 3 apply almost fully, except for the case where $\phi^* > 1$.[13] The optimality of the Friedman rule is in line with Chu and Cozzi (2014) except for one case ($g = 0.4$), where the previous authors found an optimal positive interest rate. The explanation relates with the specification of the final-goods sector. In our paper, labour is considered as an input in the production function, whereas in Chu and Cozzi (2014) labour can only be used in intermediate and R&D sectors. Hence, since a higher β implies a lower i^*, and in Chu and Cozzi (2014) $\beta = 0$ for any considered case, i^* needs necessarily to be lower in our paper.

Finally, it is interesting to note that the welfare gains reported in the previous subsection where $\phi^* > 1$ ($\Delta U = 0.21\%$) are higher than the welfare gains obtained for the same circumstances but with $\phi^* = 1$ ($\Delta U = 0.17\%$).[14] Note that, in terms of employment, $l_{\phi^*>1} > l_{\phi=1}$, meaning that unions can actually rise the level of employment if $\phi^* > 1$. These efficiency gains on R&D and on the employment level with markup greater than 1 are in line with the empirical motivation introduced in Figure 1 (b,d), supporting, therefore, our theoretical analysis.

6.3 Sensitivity analysis

In this section, we provide a sensitivity analysis regarding three main parameters $\{\mu, z, i\}$. Taking into account the results of the previous section, where the Friedman rule seems to be optimal across all cases, we focus our analysis only on the optimal trade union's markup. We first study the implications of changing the markup value (μ) under three different scenarios: $\mu = 1.4$ (Jones and Williams, 2000); $\mu = 1.6$ (Abraham et al., 2009; Banerjee and Russell, 2005; and Rotember and Woodford, 1991); and $\mu = 1.8$ (Martins et al., 1996; and Rotember and Woodford, 1991). Furthermore, we decrease the step size of a productivity improvement to $z = 1.025$ (Stokey, 1995), to take into account less innovative countries. Finally, since the Friedman rule seems to correspond to

[13] In this case, $\varphi = 4.02$.

[14] Calculations available upon request.

the optimal interest rate, we also consider the impact of decreasing the long-run nominal interest rate to $i_{LR} = 0\%$. Table 4 and Table 5 report the simulation results.

	Parameter	Threshold	ϕ^*_{max}
μ	1.4	$g = 0.80\%$	1.08
	1.6	$g = 1.40\%$	1.22
	1.8	$g = 1.80\%$	1.36
z	1.025	$g = 0.80\%$	1.03
i_{LR}	0	$g = 0.80\%$	1.01

(a) $\beta = 0.3$

	Parameter	Threshold	ϕ^*_{max}
μ	1.4	–	1.00
	1.6	–	1.00
	1.8	$g = 0.70\%$	1.07
z	1.025	–	1.00
i_{LR}	0	–	1.00

(b) $\beta = 0.6$

Table 4: Sensitivity analysis (inelastic labour supply)

	Parameter	Threshold	ϕ^*_{max}
μ	1.4	$g = 1.00\%$	1.10
	1.6	$g = 1.60\%$	1.24
	1.8	$g = 2.00\%$	1.37
z	1.025	$g = 0.80\%$	1.04
i_{LR}	0	$g = 0.80\%$	1.02

(a) $\beta = 0.3$

	Parameter	Threshold	ϕ^*_{max}
μ	1.4	–	1.00
	1.6	$g = 0.70\%$	1.05
	1.8	$g = 1.00\%$	1.13
z	1.025	–	1.00
i_{LR}	0	–	1.00

(b) $\beta = 0.6$

Table 5: Sensitivity analysis (elastic labour supply)

The *threshold* column corresponds to the growth rate below which the optimal trade union's markup is positive. The results can be summarised as follows. Firstly, under $\beta = 0.3$, all the considered scenarios show a positive trade union's markup for a specific threshold level. Hence, countries with a lower labour share benefit from an active role of trade unions in the form of higher markups. More specifically, under elastic labour supply, $\mu = 1.8$ implies a positive trade unions' markup across all the considered growth rates ($g = [2\%, 0.8\%]$). Hence, there might be cases where competitive labour markets appear to be systematically inefficient since social welfare would increased with $\phi^* > 1$. Intuitively, a higher markup in the intermediate sector allows the trade union to demand a higher wage for its workers. In this case, unions work to decrease inequality between monopolists and workers. Secondly, under $\beta = 0.6$, a higher μ leads to a $\phi^* > 1$, extending therefore the results from the previous section, where competitive labour markets were always efficient. Thus, countries with a higher labour share might also benefit from higher trade union density, conditioned on higher μ. Furthermore, bringing i_{LR} to 0 does not have any impact on the ϕ^* under $\beta = 0.6$, but implies $\phi^* > 1$ under $\beta = 0.3$. Note that, as in the previous section, the welfare gains from reducing the nominal interest rate from 8% to 0% are also higher with $\phi^* > 1$. Finally, lowering the step-size R&D innovation has only an impact on ϕ^* in the low labour share scenario. Our simulations are supported empirically by Abraham et al. (2009) and Dobbelaere (2004), who report unions bargaining power between $[0.08, 0.18]$ and $[0.244, 0.285]$, respectively.

7 Conclusions

In this paper, we built a novel framework combining a Schumpeterian growth model with CIA constraints and a labour union. We aimed to test the optimality of the competitive labour market hypothesis, as well as of the Friedman rule. More specifically, our goal was to understand whether trade unions could actually increase social welfare by demanding a wage higher than the perfect competition equilibrium. Although nominal interest rate and trade union's markup can be considered as "distortions" within the standard economic growth model, both implying lower economic growth rates, this might not be the case in terms of welfare. Indeed, regarding the trade union's markup, we found that a competitive labour market framework might be suboptimal below a specific threshold level of economic growth, depending on whether there is over or underinvestment of R&D. Hence, trade unions can actually increase social welfare through a reallocation of labour among sectors. Interestingly, this relationship seems to be stronger for countries with lower labour share and higher rents in the intermediate sector. This latter case highlights the redistributive effect of trade unions, contributing for a decrease in inequality between monopolists and workers. Furthermore, there seems to exist an interdependency between monetary policy and the macroeconomic role of trade unions: a lower (higher) interest rate increases (decreases) the effectiveness of trade union's markup on R&D labour and social welfare. Therefore, these results have strong policy implications concerning monetary policy and labour market frameworks. If the Friedman rule seems to be suitable for a wide range of parameters (i.e., different type of countries), this is certainly not the case for the labour market framework. Hence, for the case of the Eurozone, a "common" labour market setting might be more "inefficient" than a common monetary policy. A correct work of trade unions through determination of wages (i.e., the markup's size) might be the answer to promote economic growth and innovation.

Appendix I

Proof of Theorem 1: Define a transformed variable $\Psi_t = \frac{y_t}{v_t}$. Its law of motion is given by

$$\frac{\mathring{\Psi}_t}{\Psi_t} = \frac{\mathring{y}_t}{y_t} - \frac{\mathring{v}_t}{v_t}. \tag{42}$$

Combining the resource constraint $y_t = c_t N_t$ with (16) and (14), the law of motion for y_t is

$$\frac{\mathring{y}_t}{y_t} = \frac{\mathring{c}_t}{c_t} + n = r_t - \rho, \tag{43}$$

because $i_t = i$ for all t. From (10), the law of motion for v_t is

$$\frac{\mathring{v}_t}{v_t} = r_t + \lambda_t - \frac{\Pi_t}{v_t}. \tag{44}$$

Notice that: $\lambda_t = \varphi l_{r,t}$ and $\Pi_x = \frac{(\mu-\phi)}{\mu} L_{y,t} = \frac{(\mu-\phi)}{\mu} (1-\beta) y_t$. Hence, substituting (43) and (44) into (42), we obtain:

$$\frac{\mathring{\Psi}_t}{\Psi_t} = \left(\frac{(1-\beta)(\mu-\phi)}{\mu} \right) \Psi_t - \rho - \varphi l_{r,t}. \tag{45}$$

In order to find a relationship between $l_{r,t}$ and Ψ_t, we first need to combine (9) with (11) to obtain:

$$l_{x,t} = \frac{\phi(1+i)(1-\beta)}{\varphi\mu}\Psi_t. \tag{46}$$

The next step is to combine (9) and (15), yielding:

$$l_t = 1 - \frac{\theta(1+i)}{\phi(1-\beta)}\mu l_{x,t} \tag{47}$$

Hence, combining (46),(47), (24) and $l_t = l_{y,t} + l_{x,t} + l_{r,t}$, we derive

$$l_{r,t} = 1 - \left[1 + \frac{\theta(1+i) + \phi\beta}{\phi(1-\beta)}\mu\right]\frac{(1+i)\phi(1-\beta)}{\varphi\mu}\Psi_t \tag{48}$$

Finally, substituting (48) into (45) we obtain an autonomous dynamic system of Ψ_t.

$$\frac{\mathring{\Psi}_t}{\Psi_t} = \left\{\left(\frac{(1-\beta)(\mu-\phi) + (1+i)[1+\mu[\theta(1+i)+\phi\beta]]}{\mu}\right)\right\}\Psi_t - (\varphi+\rho). \tag{49}$$

Thus, the dynamics of Ψ_t is characterised by saddle-point stability such that Ψ_t jumps immediately to its interior steady-state given by

$$\Psi_t = \frac{(\varphi+\rho)}{(1-\beta)(\mu-\phi) + (1+i)\{1+\mu[\theta(1+i)+\phi\beta]\}}. \tag{50}$$

$\square$

Appendix II

Regarding the optimal trade union's markup, (41), we have two equations:

$$[\theta\mu(1+i)(1-\beta\phi) + (1-\beta)(\mu - i\phi\theta)][\mu\beta(1+i)\phi + (1-\beta)(i\phi+\mu) + \theta(1+i)\mu] =$$
$$\phi\left\{(1-\beta)\mu(1+i)\{\theta+\mu\beta+(1-\beta)\}\left(\tfrac{\varphi}{\rho}+1\right)\ln(z)\right\}$$

$$g = \left[\frac{(\mu-\phi)(1-\beta)(\varphi+\rho)}{\mu\beta(1+i)\phi + (1-\beta)(i\phi+\mu) + \theta(1+i)\mu} - \rho\right]\ln z$$

Defining:
$\chi = (1-\beta)\mu(1+i)\{\theta+\mu\beta+(1-\beta)\}\ln(z)$
$\psi = \mu[\theta(1+i) + (1-\beta)]$
$\omega = (1+i)\mu\beta + (1-\beta)i$
We get the first equation as:

$$[\psi - \omega\theta\phi][\omega\phi+\psi] = \chi\left(\frac{\varphi}{\rho}+1\right)\phi$$

$$\varphi = \left[\frac{[\psi-\omega\theta\phi][\omega\phi+\psi]}{\chi\phi} - 1\right]\rho$$

Rearranging the second equation, we obtain:

$$\varphi = \left[\frac{g}{\ln z} + \rho\right] \frac{\left[\phi \left\{\mu\beta\left(1+i\right) + \left(1-\beta\right)i\right\} + \mu\left\{\left(1-\beta\right) + \theta\left(1+i\right)\right\}\right]}{\left(\mu-\phi\right)\left(1-\beta\right)} - \rho \tag{51}$$

Defining $\Gamma = \left[\frac{g}{\ln z} + \rho\right]$, we can rewrite the previous equation as:

$$\varphi = \frac{\Gamma\left[\omega\phi + \psi\right]}{\left(\mu-\phi\right)\left(1-\beta\right)} - \rho \tag{52}$$

Hence, combing 51 with 52, and after some mathematical manipulation, we got:

$$\mu\rho\left(1-\beta\right)\psi - \phi\left[\rho\left(1-\beta\right)\psi + \left(1-\beta\right)\omega\theta\rho\mu + \Gamma\chi\right] + \phi^2\left(1-\beta\right)\omega\theta\rho = 0$$

Finally, defining $\xi = \rho\left(1-\beta\right)\psi$ and $\Lambda = \left(1-\beta\right)C\theta\rho$, we find the optimal markup as:

$$\phi^* = \frac{\left[\xi + \Lambda\mu + \Gamma\chi\right] \pm \sqrt{\left[\xi + \Lambda\mu + \Gamma\chi\right]^2 - 4\Lambda\mu\xi}}{2\Lambda}$$

□

References

Abraham, F., J. Konings, and S. Vanormelingen (2009). The effect of globalization on union bargaining and price-cost margins of firms. *Review of World Economics 145*(1), 13–36.

Acemoglu, D. (2009). *Introduction to modern economic growth.* Princeton University Press.

Acemoglu, D. and U. Akcigit (2012). Intellectual property rights policy, competition and innovation. *Journal of the European Economic Association 10*(1), 1–42.

Aghion, P., E. Farhi, and E. Kharroubi (2012, May). Monetary policy, liquidity, and growth. NBER Working Paper No. 18072.

Aghion, P. and P. Howitt (1992). A model of growth through creative destruction. *Econometrica 60*(2), 323–351.

Banerjee, A. and B. Russell (2005). Inflation and measures of the markup. *Journal of Macroeconomics 27*(2), 289–306.

Barbosa, N. and A. P. Faria (2011). Innovation across Europe: how important are institutional differences? *Research Policy 40*(9), 1157–1169.

Barlevy, G. (2007). On the cyclicality of research and development. *American Economic Review 97*(4), 1131–1164.

Becker, B. and N. Pain (2008). What determines industrial R&D expenditure in the UK? *The Manchester School 76*(1), 66–87.

Brown, J. R., G. Martinsson, and B. C. Petersen (2012). Do financing constraints matter for R&D? *European Economic Review 56*(8), 1512–1529.

Brown, M., S. Ongena, A. Popov, and P. Yeşin (2011). Who needs credit and who gets credit in Eastern Europe? *Economic Policy 26*(65), 93–130.

Chang, J.-j. and H.-w. Hung (2016). Trade unions, unemployment, economic growth, and income inequality. *Macroeconomic Dynamics 20*(1), 404–428.

Chu, A. C. (2010). Effects of patent length on R&D: A quantitative DGE analysis. *Journal of Economics 99*(2), 117–140.

Chu, A. C. and G. Cozzi (2014). R&D and economic growth in a cash-in-advance economy. *International Economic Review 55*(2), 507–524.

Chu, A. C., G. Cozzi, and Y. Furukawa (2016). Unions, innovation and cross-country wage inequality. *Journal of Economic Dynamics and Control 64*, 104–118.

Chu, A. C. and C.-c. Lai (2013). Money and the welfare cost of inflation in an R&D growth model. *Journal of Money, Credit and Banking 45*(1), 233–249.

Comin, D. (2004). R&D: A small contribution to productivity growth. *Journal of Economic Growth 9*(4), 391–421.

Cozzi, G. (2007). The Arrow Effect under competitive R&D. *The B.E. Journal of Macroeconomics 7*(1), 1–20.

Denicolò, V. and P. Zanchettin (2014). What causes over-investment in R&D in endogenous growth models? *The Economic Journal 124*(581), 1192–1212.

Dobbelaere, S. (2004). Estimation of price-cost margins and union bargaining power for belgian manufacturing. *International Journal of Industrial Organization 22*(10), 1381–1398.

Dobbelaere, S. and J. Mairesse (2013). Panel data estimates of the production function and product and labor market imperfections. *Journal of Applied Econometrics 28*(1), 1–46.

Doucouliagos, H. and P. Laroche (2013). Unions and innovation: new insights from the cross-country evidence. *Industrial Relations: A Journal of Economy and Society 52*(2), 467–491.

Dunlop, J. (1944). *Wage Determination Under Trade Unions*. New York: Macmillan.

Grossman, G. M. and E. Helpman (1991). Quality ladders in the theory of growth. *The Review of Economic Studies 58*(1), 43–61.

Guerriero, M. (2012). The labour share of income around theworld: evidence from a panel dataset. Development Economics and Public Policy Working Paper no. 32. Manchester: Institute for Development Policy and Management (IDPM), University of Manchester.

Jones, C. I. and J. C. Williams (2000). Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth 5*(1), 65–85.

Karabarbounis, L. and B. Neiman (2014). The global decline of the labor share. *The Quarterly Journal of Economics 129*(1), 61–103.

Kaufman, B. (2002). Models of union wage determination: what have we learned since Dunlop and Ross? *Industrial Relations 41*(1), 110–158.

Laincz, C. A. and P. F. Peretto (2006). Scale effects in endogenous growth theory: an error of aggregation not specification. *Journal of Economic Growth 11*(3), 263–288.

Lawson, N. (2011). Is collective bargaining pareto efficient? A survey of the literature. *Journal of Labor Research 32*(3), 282–304.

Lingens, J. (2009). Union wage bargaining and the incentive to innovate. *Journal of Institutional and Theoretical Economics 165*(2), 260–280.

Martins, J. O., S. Scarpetta, and D. Pilat (1996). Mark-up ratios in manufacturing industries: estimates for 14 OECD countries. OECD Economics Department Working Papers, No. 162, OECD Publishing.

Menezes-Filho, N., D. Ulph, and J. V. Reenen (1998). The determination of R&D: empirical evidence on the role of unions. *European Economic Review 42*(3–5), 919–930.

Mukherjee, A. and L. F. Wang (2013). Labour union, entry and consumer welfare. *Economics Letters 120*(3), 603–605.

Neto, A., O. Afonso, and S. T. Silva (2017). How powerful are trade unions? A skill-biased technological change approach. *Macroeconomic Dynamics forthcoming*.

Neto, A. and S. T. Silva (2013). Growth and unemployment: a bibliometric analysis on mechanisms and methods. FEP Working Papers 498, Universidade do Porto, Faculdade de Economia do Porto.

OECD (2015). The labour share in G20 economies. Technical report, International Labour Organization and Organisation for Economic Co-operation and Development.

Palokangas, T. (2004). Union–firm bargaining, productivity improvement and endogenous growth. *LABOUR 18*(2), 191–205.

Peretto, P. F. (1998). Technological change and population growth. *Journal of Economic Growth 3*(4), 283–311.

Reis, A. B. and T. N. Sequeira (2007). Human capital and overinvestment in R&D. *The Scandinavian Journal of Economics 109*(3), 573–591.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy 98*(5), S71–S102.

Ross, A. (1948). *Trade union wage policy*. Berkeley: University of California Press.

Rotember, J. J. and M. Woodford (1991). Markups and the business cycle. In *NBER Macroeconomics Annual*, Volume 6. MIT Press.

Samaniego, R. M. and J. Y. Sun (2015). Technology and contractions: evidence from manufacturing. *European Economic Review 79*, 172–195.

Stokey, N. L. (1995). R&D and economic growth. *The Review of Economic Studies 62*(3), 469–489.