Multi-product and Multi-region Marketing

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Abstract:
Firms could position themselves to compete within the same industry in different ways. They try to get their competitive advantage, which is defined as the ability to earn a higher rate of economic profit than the average of economic profit of other firms competing within the same markets (Besanko et al, 2013). Michael Porter (1980) coined generic strategies for firms to compete in the markets they serve, i.e. cost leadership, benefit leadership, and focus. Besanko et al (2014) noted three possible how it could happen in three different ways: (1) the cost leader can get the benefit parity by producing products with the same benefit (B) but at lower cost (C); (2) the cost leader can get benefit proximity, which involves offering a benefit (B) that is not much less than those of competitors; (3) the cost leader might offer a product that is qualitatively different from that its competitors. Benefit and Cost leadership closely relates to the crucial issue of how the firm will create the higher competitive advantage or economic value created compared to its rivals. The other important issue is where to create higher economic value. More specifically, will the firm seek to create economic value across broad of regional markets segments (broad coverage strategy), or will it focus only on narrow set of segments (focus strategy)? Yet, Porter (1980) had not given any mathematical formula to analyze the performance of sales which is related to the strategic positioning. Therefore, firstly this paper is addressed to derive a mathematical formula for analyzing the performance of sales in the cases of multi-product and multi-markets. In the real all markets, now a firm could produce more than one product (multi-product) and sell the products in more than one markets (multi-markets). It is very useful for the firms to know the determinants of the changes of their sales. Are they affected by the products or the markets? The mathematical formula derived in this paper offers the answer. Every firm needs this information to formulate the suitable markets policies or strategies. For sure, the mathematical formula requires detail information on sales by products, markets and competitors. Secondly, due to the unavailability of empirical data, the formula is simulated by using hypothetical data.

JEL: M21.M31
Keywords: Constant Market Share, Marketing, Multi-region.
1. Introduction

Firms could position themselves to compete within the same industry in different ways. They try to get their competitive advantage, which is defined as the ability to earn a higher rate of economic profit than the average of economic profit of other firms competing within the same markets (Besanko et al., 2010:362). Economic profit (\(\pi\)) basically is the difference between total revenue (TR) and total cost (TC), or (\(\pi=TR-TC\)). A firm’s profitability depends simultaneously on the economics of its markets and its success in generating value than its competitor. This means that the amount of value the firm generates compared to competitors is influenced by its cost (C) and benefit (B) position relative to the competitors.

Economic value is created when a firm of a producer could combine inputs or production factors, such as capital, labor, and raw materials as well as purchased components, to make a product whose received benefit (B) is greater than the cost (C) incurred in making the product. Like the definition of economic profit, the economic value created (in short, value-created, VC) is defined as the difference between benefit and cost, B-C). Value-created (VC) consists of two components, i.e. consumer surplus (CS) and producer surplus (PS) as presented in the Figure 1. The demand curve (D) basically represents the willingness to pay (WTP) of the consumer and the supply curve (S) represents the marginal cost (MC). Therefore, the consumer surplus is defined as the difference between WTP and the markets price (P*), CS=WTP-P*. It shows the portion of the value created that the consumer captures. While, the producer surplus is the difference between the markets price and the marginal cost, PS=P*-MC. It represents the producer’s margin or the portion of the valued-value created that the producer captures. Value-created is the sum up of consumer surplus and producer surplus, VC=CS+PC= (WTP-P*)+(P*-MC)=WTP-MC.

2. Generic Strategic Positioning

Michael Porter (1980) coined generic strategies for firms to compete in the markets they serve, i.e. cost leadership, benefit leadership, and focus as described in Figure 2. A firm that applies a strategy cost leadership could get more value (B-C) by offering lower cost than its competitors, shown by Supply S’ (MC’) in Figure 1. It is easy to see that with lower S’ the firm could have higher value-created. Besanko et al. (2010:379) noted three possible how it could happen in three different ways: (1) the cost leader can get the benefit parity by producing products with the same benefit (B) but at lower cost (C); (2) the cost leader can get benefit proximity, which involves offering a benefit (B) that is not much less than those of competitors; (3) the cost leader might offer a product that is qualitatively different from that its competitors. While, a producer that applies a strategy of benefit leadership generates more value that its rivals by offering products that have a higher benefit (B) than its competitors, shown by Demand D’ (WTP’) in Figure 1. It is clear to see that with higher D’ the firm could have higher value-created. This could happen in three ways, i.e. (1) the benefit leader could achieve benefit parity by making products with the same cost (C) but at higher benefit (B) than its competitors; (2) the benefit leader could attain cost proximity, which entails a cost (C) that is not too much higher than the rivals; (3) the firm might offer significantly higher benefit (B) and cost (C).
Figure 2. Porter’s Generic Strategies

Benefit and Cost leadership closely relates to the crucial issue of how the firm will create the higher competitive advantage or economic value created compared to its rivals. The other important issue is where to create higher economic value. More specifically, will the firm seek to create economic value across broad of markets segments (broad coverage strategy), or will it focus only on narrow set of segments (focus strategy), as depicted by Figure 2?
Porter (1980) did not give any mathematical formula to analyze the performance of sales which is related to the strategic positioning. Therefore, firstly this study aims to derive a mathematical formula for analyzing the performance of sales in the cases of multi-product and multi-markets. In the real all markets, now a firm could produce more than one product (multi-product) and sell the products in more than one markets (multi-markets), as depicted in Figure 3. It is very useful for the firms to know the determinants of the changes of their sales. Are they affected by the products or the markets? The mathematical formula derived in this study offers the answer. Every firm needs this information to formulate the suitable markets policies or strategies. For sure, the mathematical formula requires detail information on sales by products, markets and competitors. Secondly, due to the unavailability of empirical data, the formula is simulated by using hypothetical data.

Figure 3. The Determinants of the Changes in Sales
3. The Constant Markets Shares (CMS)

This paper applies the logics of the CMS in international economics and apply it in business economics. The different analysis is that the CMS in international economics deals with relationships between countries to consumers in different countries (export and import activities), but the CMS in business economics deals with the relationships between firms and consumers in markets. In international economics, Tyszynski (1951) firstly proposes the CMS method. However, Leamer and Stern (1970) give a more detailed discussion of the method and possible applications. They also propose a version of the method, where the changes in exports can be caused by (a) the general rise in world exports, (b) the commodity composition, (c) the market distribution and (d) the competitiveness. Richardon (1971a, 1971b) points out that the commodity composition and market distribution effects are interdependent (the order of their calculation matters), and that the values and signs may change if the final, instead of the initial, year of the period under consideration is used as the base year.

Fagerberg and Sollie (1987) develop their version of the method, which can explicitly give the interpretation of the competitiveness effect. They find five effects instead of Leamer and Stern’s three effects. The two additional effects reflect a firm’s ability to adapt its exports structure to changes in the commodity and markets composition of the world exports. Widodo (2008, 2010) finds that there are different points of view between the two first and the third. Leamer and Stern (1970) as well as Richardson (1971a, 1971b) focuses their analysis on factors underlying a country’s changes in exports. Meanwhile, Fagerberg and Sollie (1987) explain factors underlying country’s changes in shares in the world export. Secondly, by combining the original concept in country’s change in exports and change in share in the world exports by Fagerberg and Sollie (1987), Widodo (2008, 2010) proposes a new version of the CMS which breakdown the change in a country’s export into six effect instead of two effects (by Tyszynki (1951)) or four effects (by Leamer and Stern (1970) and Richardson (1971a, 1971b). The six effects are (1) general changes in world exports, (2) market share effects, (3) commodity composition effect, (4) market composition effect, (5) commodity adaptation effect, (6) market adaptation effect.

This part describes the derivation of CMS for business economics which is suitable for analyzing strategic positioning of a firm marketing its many various products in many different regional market.

3.1. The constant-share norm

The CMS method is derived from the constant-share norm. Suppose, there are two competitive firms A and B selling their commodity to a particular markets. Demand from the two competing suppliers may be shown by the following expression:

\[ \frac{q_A}{q_B} = f\left(\frac{p_A}{p_B}\right) \]  

(1)

where \( q_A \) and \( q_B \) refer to quantity sold by A and B, respectively. Meanwhile, \( p_A \) and \( p_B \) represent price of the commodity from firms A and B, respectively. By multiplying the both right-hand and left-hand sides of (1) with \( p_A/p_B \), the following expression is obtained:

\[ \frac{p_A q_A}{p_B q_B} = \frac{p_A}{p_B} f\left(\frac{p_A}{p_B}\right) \]  

(2)

The firm A’s share of sales is expressed as follows:
\[
\frac{p_Aq_A}{p_Aq_A + p_Bq_B} = \left( 1 + \frac{p_Bq_B}{p_Aq_A} \right)^{-1} \\
= \left\{ 1 + \left[ \frac{p_Af(p_A/p_B)}{p_A} \right]^{-1} \right\}^{-1} \\
= h \left( \frac{p_A}{p_B} \right)
\]

Equation (3) implies that firm A’s share of the markets in question \(\frac{p_Aq_A}{p_Aq_A + p_Bq_B}\) will be unchanged except as the price ratio \(\frac{p_A}{p_B}\) changes. This refers to the validity of the constant-share norm. It also shows that the difference between growths of sales may be indicated by the price changes. The aggregate markets share of a firm will be the same if its markets share in individual commodity groups have also remained constant (hypothetical). It refers to the difference between the hypothetical and the initial markets shares as the changes in markets share, which is caused by the structural changes in the markets. The residual—the difference between the final and the hypothetical markets shares—is due to the changes in competitiveness. This method is called as “constant markets shares” (CMS) analysis.

The discrepancy between the constant-share norm and actual shares as the “competitiveness effect”. If a firm is fail to maintain its share in the markets, the competitiveness term will be negative. It also indicates that the firm’s prices increase relatively higher than that of the competitors as shown in equation (3). However, this is the case if we impose an additional assumption of the elasticity of substitution exceeding one in absolute value.

### 3.2. The levels of analysis

Figure 4 illustrates firms’ sales for the two periods 0 and t. It is used to explain the CMS method. Suppose, there are a number of firms (z) and markets (k). Firm A is a firm in question. The definitions and notations used here are firstly determined:

- \(V_{i0}^w\) = value of the total markets sales commodity i in period 0
- \(V_{it}^w\) = value of the total markets sales commodity i in period t
- \(V_{i0}^w\) = value of the total markets sales to markets j in period 0
- \(V_{ij}^w\) = value of the total markets sales to markets j in period t
- \(V_{ij0}^w\) = value of the total markets sales of commodity i to markets j in period 0
- \(V_{ijt}^w\) = value of the total markets sales of commodity i to markets j in period t
- \(V_{i*0}^w\) = value of the total markets sales in period 0
- \(V_{i*}^w\) = value of the total markets sales in period t
- \(V_{i0}^{AO}\) = value of the firm A’s sales of commodity i in period 0
- \(V_{it}^{AO}\) = value of the firm A’s sales of commodity i in period t
- \(V_{ij0}^{AO}\) = value of the firm A’s sales to markets j in period 0
\[ V_{ij}^{At} = \text{value of the firm A's sales to markets j in period t} \]
\[ V_{ij}^{A0} = \text{value of the firm A's sales of commodity i to markets j in period 0} \]
\[ V_{ij}^{At} = \text{value of the firm A's sales of commodity i to markets j in period t} \]

### Firms A’s Sales to Total Markets

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Sales Value</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( V_{ij}^{A0} )</td>
<td>( V_{ij}^{At} )</td>
</tr>
<tr>
<td>1</td>
<td>( V_{ij}^{A0} )</td>
<td>( V_{ij}^{At} )</td>
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<tr>
<td>n</td>
<td>( V_{ij}^{A0} )</td>
<td>( V_{ij}^{At} )</td>
</tr>
<tr>
<td>Total</td>
<td>( V_{ij}^{A0} )</td>
<td>( V_{ij}^{At} )</td>
</tr>
</tbody>
</table>

### Firms Z’s Sales to Total Markets

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<th>Sales Value</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
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<td>( V_{iz}^{Z0} )</td>
<td>( V_{iz}^{Z1} )</td>
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<tr>
<td>1</td>
<td>( V_{iz}^{Z0} )</td>
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<td>n</td>
<td>( V_{iz}^{Z0} )</td>
<td>( V_{iz}^{Z1} )</td>
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<tr>
<td>Total</td>
<td>( V_{iz}^{Z0} )</td>
<td>( V_{iz}^{Z1} )</td>
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### Total Sales: All firms’ Sales to All Markets

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Sales Value</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
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<td>( V_{wi}^{W0} )</td>
<td>( V_{wi}^{W1} )</td>
</tr>
<tr>
<td>1</td>
<td>( V_{wi}^{W0} )</td>
<td>( V_{wi}^{W1} )</td>
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<td>...</td>
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<tr>
<td>N</td>
<td>( V_{wi}^{W0} )</td>
<td>( V_{wi}^{W1} )</td>
</tr>
<tr>
<td>Total</td>
<td>( V_{wi}^{W0} )</td>
<td>( V_{wi}^{W1} )</td>
</tr>
</tbody>
</table>

\[ r = \text{percentage increase in total sales} \]
\[ r = \frac{V_{ij}^{W1} - V_{ij}^{W0}}{V_{ij}^{W0}} \]
\[ r_i = \text{percentage increase in all markets of commodity i} \]
\[ r_i = \frac{V_{ij}^{W1} - V_{ij}^{W0}}{V_{ij}^{W0}} \]

---

**Figure 4. Illustration of Sales Flows**

\[ r = \text{percentage increase in total markets sales} \]
\[
\begin{align*}
\delta &= \frac{V_{i}^{Wt} - V_{i}^{Wo}}{V_{i}^{Wo}} \\
\delta_{i} &= \text{percentage increase in total markets sales of commodity } i; \\
\delta_{ij} &= \text{percentage increase in total markets sales of commodity } i \text{ to markets } j;
\end{align*}
\]

From above definitions and notations, the firm A’s total sales values for commodity \( i \) and for destination markets \( j \) for period 0 can be written as:

\[
\sum_{j} V_{ij}^{A0} = V_{i}^{A0} \quad \text{and} \quad \sum_{i} V_{ij}^{A0} = V_{j}^{A0}
\]

and similarly for the period \( t \). In addition, the value of firm A’s sales in the period 0 is given by:

\[
\sum_{i} \sum_{j} V_{ij}^{A0} = \sum_{i} V_{i}^{A0} = \sum_{j} V_{j}^{A0} = V_{.}^{A0}
\]

There are three levels of CMS analysis, which depend on how we treat markets and commodities (Leamer and Stern, 1970). First, it may be assumed that commodities can be treated as a single and completely undifferentiated product. In addition, sales destinations can be treated as a single market. In short, sales may be treated as a single good destined for a single market. If firm A maintains its share in this markets, its sales would simply increase by \( \delta V_{i}^{A0} \), and the following identity is obtained:

\[
V_{i}^{At} - V_{i}^{A0} \equiv \delta V_{i}^{A0} + (V_{i}^{At} - V_{i}^{A0} - \delta V_{i}^{A0}) \tag{6}
\]

Equation (6) is called a “one level” analysis. It implies that the change in A’s sales \((V_{i}^{At} - V_{i}^{A0})\) can be divided into two parts i.e. (a) a part related with the general increase in total markets sales \((\delta V_{i}^{A0})\) and (b) an unexplained part, the competitiveness effect \((V_{i}^{At} - V_{i}^{A0} - \delta V_{i}^{A0})\).

Second, it may be assumed that commodities are quite diverse sets of goods. For a specific commodity (say \( i \)), an analogous identity may be written as follows:

\[
V_{i}^{At} - V_{i}^{A0} \equiv \delta_{i} V_{i}^{A0} + (V_{i}^{At} - V_{i}^{A0} - \delta_{i} V_{i}^{A0}) \tag{7}
\]

Taking the aggregate equation (7), the following expression is obtained:

\[
V_{i}^{At} - V_{i}^{A0} \equiv \sum_{i} \delta_{i} V_{i}^{A0} + \sum_{i} (V_{i}^{At} - V_{i}^{A0} - \delta_{i} V_{i}^{A0})
\]

\[
\equiv (\delta V_{.}^{A0}) + \sum_{i} (\delta_{i} - r)V_{i}^{A0} + \sum_{i} (V_{i}^{At} - V_{i}^{A0} - \delta_{i} V_{i}^{A0}) \tag{8}
\]

Equation (8) is called a “two level” analysis. The change in A’s sales \((V_{i}^{At} - V_{i}^{A0})\) is broken down into three components regarding with: (a) the general rise in total markets sales \((\delta V_{.}^{A0})\), (b) the commodity composition of A’s sales in the period 0 \(\left(\sum_{i} (\delta_{i} - r)V_{i}^{A0}\right)\); and (c) an unexplained residual (the competitiveness effect) \(\left(\sum_{i} (V_{i}^{At} - V_{i}^{A0} - \delta_{i} V_{i}^{A0})\right)\). The difference between the “one level” and “two level” analysis is in the existence of the commodity composition effect, \(\sum_{i} (\delta_{i} - r)V_{i}^{A0}\). If the total markets sales of commodity \( i \) increase by more than the total markets
average for all commodities, \((r_i - r) > 0\), the sales of commodity \(i\) contribute to the increase in firm A’s sales. Therefore, the sum represented by \(\sum (r_i - r)V_{iA}^0\) would be positive if A has concentrated on the sales of commodities whose markets are growing relatively faster and would be negative if A has concentrated in slowly growing commodity markets. Third, it may be assumed that sales are differentiated by destinations as well as commodity types. In this case, sales of a particular commodity for a particular destination are considered. Therefore, the analogous identity can be written as follows:

\[
V_{ij}^A - V_{ij}^{A0} = r_j V_{ij}^{A0} + (V_{ij}^A - V_{ij}^{A0} - r_j V_{ij}^{A0})
\]  

Taking the aggregate equation (9) yields:

\[
V_{..}^A - V_{..}^{A0} = \sum_i \sum_j r_j V_{ij}^{A0} + \sum_i \sum_j (V_{ij}^A - V_{ij}^{A0} - r_j V_{ij}^{A0})
\]

\[
= rV_{..}^{A0} + \sum_i (r_i - r)V_{i..}^{A0} + \sum_i \sum_j (r_j - r_j)V_{ij}^{A0} + \sum_i \sum_j (V_{ij}^A - V_{ij}^{A0} - r_j V_{ij}^{A0})
\]

Expression (10) shows a “three level” analysis. The change in firm A’s sales \((V_{..}^A - V_{..}^{A0})\) can be divided into four components associated with: (a) the total markets sales, \((rV_{..}^{A0})\); (b) the commodity composition of firm A’s sales, \(\left(\sum_i (r_i - r)V_{i..}^{A0}\right)\); (c) the markets distribution of firm A’s sales, \(\left(\sum_i \sum_j (r_j - r_j)V_{ij}^{A0}\right)\); and (d) an unexplained residual (the competitiveness effect), \(\left(\sum_i \sum_j (V_{ij}^A - V_{ij}^{A0} - r_j V_{ij}^{A0})\right)\). The markets distribution effect \(\sum_i \sum_j (r_j - r_j)V_{ij}^{A0}\) will be positive if firm A has concentrated its sales in markets with relatively rapid growth. It is important to note that whether the commodity effect (b) follows the markets distribution effect (c), or vice versa. Therefore, equation (10) can be described in another way:

\[
V_{..}^A - V_{..}^{A0} = rV_{..}^{A0} + \sum_j (r_j - r)V_{..}^{A0} + \sum_i \sum_j (r_i - r_i)V_{ij}^{A0} + \sum_i \sum_j (V_{ij}^A - V_{ij}^{A0} - r_j V_{ij}^{A0})
\]

Now, the increase of firm A’s sales \((V_{..}^A - V_{..}^{A0})\) can be divided into four components associated with: (a) the general rise in total markets sales \((rV_{..}^{A0})\); (b) the markets distribution of firm A’s sales \(\left(\sum_i (r_i - r)V_{i..}^{A0}\right)\); (c) the commodity composition of firm A’s sales \(\left(\sum_i \sum_j (r_j - r_j)V_{ij}^{A0}\right)\);
and (d) an unexplained residual (the competitiveness effect) \( \left( \sum_{i,j} (V_{ij}^{A} - V_{ij}^{A0} - r_{ij}V_{ij}^{A0}) \right) \). The equation (10) can be normalized by dividing \( V_{ij}^{A0} \) (Laspeyres index) or \( V_{ij}^{At} \) (Paasche index)\(^1\):

\[
\text{Laspeyres Index: } \frac{V_{ij}^{At} - V_{ij}^{A0}}{V_{ij}^{A0}} = r_{ij} + \frac{\sum (r_{ij} - r_{ij})V_{ij}^{A0}}{V_{ij}^{A0}} + \frac{\sum (V_{ij}^{At} - V_{ij}^{A0} - r_{ij}V_{ij}^{A0})}{V_{ij}^{A0}} (12)
\]

\[
\text{Paasche Index: } \frac{V_{ij}^{At} - V_{ij}^{A0}}{V_{ij}^{A0}} = \frac{\sum (t_{ij} - t_{ij})V_{ij}^{A0}}{V_{ij}^{A0}} + \frac{\sum (V_{ij}^{At} - V_{ij}^{A0} - r_{ij}V_{ij}^{A0})}{V_{ij}^{A0}} (13)
\]

3.3. Changes in the share of sales

The interpretation of competitiveness effect (d) in the identity (10) is not as straightforward as the other terms. There are many other things beside the relative prices affecting a firm’s competitiveness such as (a) the differential rates of price inflation, (b) differential rates of quality improvement and the development of new products, (c) differential rates of improvement in the efficiency of marketing or in the terms of financing the sale of goods, (d) differential changes in the ability for prompt fulfillment of orders (Leamer and Stern, 1970). More recently, Fagerberg and Sollie (1987) develop another version of the CMS method by Tyszynski (1951). This version gives much more explanation on the competitiveness effect.

The change in share of sales depends on how we treat markets and commodities in our analysis (Fagerberg and Sollie, 1987). To give clear explanation, two cases will be described separately, i.e. ‘several commodities – one markets’ and ‘several commodities – several markets’ cases\(^2\). The following symbols and definitions will be used:\(^3\):

- \( V \) = value of sales;
- \( i \) = commodities
- \( j \) = sales (destinations) markets
- \( n \) = number of commodities;
- \( k \) = number of countries (K is the last sales markets)

---

\(^1\) Tyszynski (1951) actually employs \( \frac{V_{ij}^{At} - V_{ij}^{A0}}{V_{ij}^{W0}} = \left( \frac{\sum (r_{ij} - t_{ij})V_{ij}^{A0}}{V_{ij}^{W0}} \right) + \frac{\sum (V_{ij}^{At} - V_{ij}^{A0} - r_{ij}V_{ij}^{A0})}{V_{ij}^{W0}} \)

\(^2\) We will use variable (data) on sales only. This is slightly different with that of Fagerberg and Sollie (1987). They use term sales of specific firm. However, for markets destination they employ “total sale of a firm” instead of “all markets’ sales to the firm”. Theoretically, the two terms must be the same i.e. the “total sales” value of a firm is the same with the “all market sales” to the firm. In practice, since sales are calculated based on cost-insurance-freight (CIF) meanwhile sales are calculated based on free-on-board (FOB), the use of only sales can therefore avoid misleading.

\(^3\) The symbols and definitions are different with those of Fagerberg and Sollie (1987). This is to accommodate our comparison analysis among the versions of CMS method.
0, t = subscripts which refer to the initial year and to the final year of the comparison, respectively;
A = firm in question
W = all markets
$S^A$ = markets share of firm A in all markets sales (the ratio of A’s total sales and the all markets total sales);

$$S^A = S^{A1} + S^{A2} + \ldots + S^{AK} = \sum_{i} \sum_{j} V^A_{ij}$$

$s^A$ = macro share of firm A in all markets sales (the ratio of A’s total sale and all markets total sale in each markets); row vector of dimension K:

$$s^A = [s^{A1} \quad s^{A2} \quad \ldots \quad s^{AK}] = \left[ \begin{array}{c} \sum_{i} V^A_{1i} \\ \sum_{i} V^A_{i2} \\ \vdots \\ \sum_{i} V^A_{iK} \end{array} \right]$$

$\alpha^{Aj}$ = markets share, by commodity, of firm A (macro share of firm A) in the all markets sales to markets j (the ratio of firm A’s and the all markets’ sales of commodity i to markets K); matrix of dimension Kxn:

$$\alpha^{Aj} = \left[ \begin{array}{cccc} \alpha^{A1}_{1j} & \alpha^{A1}_{2j} & \ldots & \alpha^{A1}_{nj} \\ \alpha^{A2}_{1j} & \alpha^{A2}_{2j} & \ldots & \alpha^{A2}_{nj} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^{AK}_{1j} & \alpha^{AK}_{2j} & \ldots & \alpha^{AK}_{nj} \end{array} \right] = \left[ \begin{array}{cccc} V^A_{11} & V^A_{12} & \ldots & V^A_{1n} \\ V^A_{21} & V^A_{22} & \ldots & V^A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ V^A_{K1} & V^A_{K2} & \ldots & V^A_{Kn} \end{array} \right]$$

$\beta^{Wi}$ = commodity shares of the all markets sales to firm j to the all markets total sales (the ratio of all markets’ specific commodity sales and total all markets’ sales to firm K); matrix of dimension nxK:

$$\beta^{Wi} = \left[ \begin{array}{c} \beta^{W1}_{1j} \\ \beta^{W2}_{1j} \\ \vdots \\ \beta^{WK}_{1j} \\ \beta^{W1}_{2j} \\ \beta^{W2}_{2j} \\ \vdots \\ \beta^{WK}_{2j} \\ \vdots \\ \beta^{W1}_{nj} \\ \beta^{W2}_{nj} \\ \vdots \\ \beta^{WK}_{nj} \end{array} \right] = \left[ \begin{array}{c} \sum_{i} V^W_{11} \\ \sum_{i} V^W_{i2} \\ \vdots \\ \sum_{i} V^W_{iK} \\ \sum_{i} V^W_{11} \\ \sum_{i} V^W_{i2} \\ \vdots \\ \sum_{i} V^W_{iK} \\ \vdots \\ \sum_{i} V^W_{1n} \\ \sum_{i} V^W_{i2} \\ \vdots \\ \sum_{i} V^W_{iK} \end{array} \right]$$

$\delta^{wj}$ = firm shares of the all markets sales (the ratio of the all markets sales to firm j and the all markets sales); column vector of dimension K:

$$\delta^{wj} = \left[ \begin{array}{c} \delta^{W1} \\ \delta^{W2} \\ \vdots \\ \delta^{WK} \end{array} \right] = \left[ \begin{array}{c} \sum_{i} V^W_{11} \\ \sum_{i} V^W_{i2} \\ \vdots \\ \sum_{i} V^W_{iK} \end{array} \right] / \left[ \begin{array}{c} \sum_{i} V^W_{ij} \end{array} \right]$$

11
The ‘several commodities – one markets’ case

In the case of ‘several commodities – one markets’, it is assumed that firm A in question sales several commodities (n) in only one markets, say markets K (i.e. j=K). In Figure 4, it is depicted by the last column. Based on the definitions and symbols, the macro share of firm A ($S_{AK}$) can be written as the inner product of the vector of its micro share ($\alpha_{AK}$) and the vector of commodity share in total all markets sale to county K ($\beta_{WK}$), as follows:

$$S_{AK} = \alpha_{AK} \beta_{WK} = \left[ \frac{V_{1K}^A}{V_{1K}^W}, \frac{V_{2K}^A}{V_{2K}^W}, ..., \frac{V_{nK}^A}{V_{nK}^W} \right] \left[ \frac{V_{1K}^W}{V_{1K}^W}, \frac{V_{2K}^W}{V_{2K}^W}, ..., \frac{V_{nK}^W}{V_{nK}^W} \right]$$

(14)

The change in macro share of firm A ($\Delta S_{AK}$) between the two periods $t$ and 0 can be obtained:

$$\Delta S_{AK} = S_{AK}^t - S_{AK}^0 = \alpha_{AK}^t \beta_{WK}^t - \alpha_{AK}^0 \beta_{WK}^0$$

(15)

$$= \left[ \frac{V_{1K,t}^A}{V_{1K,t}^W}, \frac{V_{2K,t}^A}{V_{2K,t}^W}, ..., \frac{V_{nK,t}^A}{V_{nK,t}^W} \right] \left[ \frac{V_{1K,t}^W}{V_{1K,t}^W}, \frac{V_{2K,t}^W}{V_{2K,t}^W}, ..., \frac{V_{nK,t}^W}{V_{nK,t}^W} \right] - \left[ \frac{V_{1K,0}^A}{V_{1K,0}^W}, \frac{V_{2K,0}^A}{V_{2K,0}^W}, ..., \frac{V_{nK,0}^A}{V_{nK,0}^W} \right] \left[ \frac{V_{1K,0}^W}{V_{1K,0}^W}, \frac{V_{2K,0}^W}{V_{2K,0}^W}, ..., \frac{V_{nK,0}^W}{V_{nK,0}^W} \right]$$

If either the Laspeyres or Paasche indices are employed for the whole calculation, a third (residual) term necessarily appears. This is because neither Laspeyres nor Paasche index passes the factor reversal test$^4$ (Fagerberg and Sollie, 1987). Therefore, the residual term appears as shown as follows (Laspeyres index is used):

$$\Delta S_{AK} = \Delta S_{AK}^\alpha + \Delta S_{AK}^\beta + \Delta S_{AK}^\theta$$

(16)

where:

$$\Delta S_{AK}^\alpha = (\alpha_{1K}^t - \alpha_{0K}^0) \beta_{WK}$$

(17)

$$= \left[ \frac{V_{1K,t}^A}{V_{1K,t}^W}, \frac{V_{2K,t}^A}{V_{2K,t}^W}, ..., \frac{V_{nK,t}^A}{V_{nK,t}^W} \right] \left[ \frac{V_{1K,0}^W}{V_{1K,0}^W}, \frac{V_{2K,0}^W}{V_{2K,0}^W}, ..., \frac{V_{nK,0}^W}{V_{nK,0}^W} \right]$$

$^4$ The factor reversal test requires that multiplying a price index and a volume index of the same type should be equal to the proportionate change in the current values.
\[
\Delta S_{A^K} = \alpha_0^{A^K} (\beta_i^{A^K} - \beta_0^{A^K})
\]

\[
\Delta S_{\alpha_0} = (\alpha_i^{A^K} - \alpha_0^{A^K}) (\beta_i^{A^K} - \beta_0^{A^K})
\]

\[
= \left[ \begin{array}{cccc}
V_{1K,0}^{A} & V_{1K,0}^{A} & \cdots & V_{nK,0}^{A} \\
V_{2K,0}^{A} & V_{2K,0}^{A} & \cdots & V_{nK,0}^{A} \\
\vdots & \vdots & \ddots & \vdots \\
V_{nK,0}^{A} & V_{nK,0}^{A} & \cdots & V_{nK,0}^{A}
\end{array} \right] - \left[ \begin{array}{cccc}
\sum_{i=1}^{n} V_{1K,i}^{W} / \sum_{i=1}^{n} W_{1K,i}^{W} \\
\sum_{i=1}^{n} V_{2K,i}^{W} / \sum_{i=1}^{n} W_{2K,i}^{W} \\
\vdots \\
\sum_{i=1}^{n} V_{nK,i}^{W} / \sum_{i=1}^{n} W_{nK,i}^{W}
\end{array} \right] \tag{18}
\]

The first term \((\Delta S_{\alpha_0}^{A^K})\) is the effect of changes in micro shares (micro share effect), the second term \((\Delta S_{\alpha_0}^{A^K})\) is the commodity composition effect. The third (residual) term \((\Delta S_{\alpha_0}^{A^K})\) is the inner product of a vector of changes in micro shares and a vector of changes in commodity composition. Fagerberg and Sollie (1987) argue that the residual term has economic meaning, since its sign and value depend on the correlation between the changes in micro shares of the firm and the change in commodity composition of the markets. A formal proof on this matter is given below (for simplicity reason, the superscripts of firm A and markets K are omitted):

\[
\Delta S_{\alpha_0} = (\alpha_i - \alpha_0) (\beta_i - \beta_0) \tag{20}
\]

The correlation coefficient between the changes in micro shares \((\alpha_i - \alpha_0)\) and the changes in commodity shares \((\beta_i - \beta_0)\), which is denoted by \(r_{\alpha_0}\), is formulated as:\n
\[
r_{\alpha_0} = \frac{(\alpha_i - \alpha_0 - \bar{\alpha}_0 + \bar{\alpha}_0)(\beta_i - \beta_0 - \bar{\beta}_0 + \bar{\beta}_0)}{\sqrt{(\alpha_i - \alpha_0 - \bar{\alpha}_0 + \bar{\alpha}_0)(\alpha_i - \alpha_0 - \bar{\alpha}_0 + \bar{\alpha}_0)(\beta_i - \beta_0 - \bar{\beta}_0 + \bar{\beta}_0)(\beta_i - \beta_0 - \bar{\beta}_0 + \bar{\beta}_0)}} \tag{21}
\]

The symbol \((\cdot)^T\) denotes transposition, while \(\bar{\alpha}_0, \bar{\alpha}_0, \bar{\beta}_0\) and \(\bar{\beta}_0\) are vectors of means, defined by:

\[
\bar{\alpha}_i = (1/n) \sum_{i=1}^{n} \alpha_i \\
\bar{\alpha}_0 = (1/n) \sum_{i=1}^{n} \alpha_0 \\
\bar{\beta}_i = (1/n) \sum_{i=1}^{n} \beta_i
\]

\[
^5 \text{From the standard statistics, correlation between two variables X and Y with n observations is formulated as:}
\]

\[
r_{XY} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2 \sum_{i=1}^{n} (Y_i - \bar{Y})^2}} \text{ where } \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \text{ and } \bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}.
\]

13
\[ \bar{\beta}_0 = (1/n)u' \beta_0 u = (1/n)u \]

where \( u \) is vector of one and \( u' \) denotes transposition of \( u \). It follows from equations 21-25 that:

\[ r_{\alpha\beta} \sqrt{(\alpha_i - \alpha_0 - \bar{\alpha}_i + \bar{\alpha}_0)(\alpha_i - \alpha_0 - \bar{\alpha}_i + \bar{\alpha}_0)(\beta_i - \beta_0)(\beta_i - \beta_0)} = (\alpha_i - \alpha_0 - (1/n)\alpha_0 u' + (1/n)\alpha_0 u' \beta_i - \beta_0) \]  

By rearranging, equation (26) can be simplified as follows:

\[ r_{\alpha\beta} \sqrt{(\alpha_i - \alpha_0 - \bar{\alpha}_i + \alpha_0)(\alpha_i - \alpha_0 - \bar{\alpha}_i + \alpha_0)(\beta_i - \beta_0)(\beta_i - \beta_0)} = (\alpha_i - \alpha_0)(\beta_i - \beta_0) - (1/n)(\alpha_i - \alpha_0) u'(\beta_i - \beta_0) \]

Since the sum of the commodity shares is always equal to one, it follows that:

\[ u'(\beta_i - \beta_0) = 0 \]

Therefore, it is:

\[ r_{\alpha\beta} \sqrt{(\alpha_i - \alpha_0 - \bar{\alpha}_i + \alpha_0)(\alpha_i - \alpha_0 - \bar{\alpha}_i + \alpha_0)(\beta_i - \beta_0)(\beta_i - \beta_0)} = (\alpha_i - \alpha_0)(\beta_i - \beta_0) \]

By substituting equation (29) into equation (20) the residual can be expressed as the product of the correlation between the changes in micro shares and the change in commodity shares, and two terms which are necessarily non-negative. The first of these terms is a measure of the spread of the changes in micro shares, while the second is a measure of the changes in commodity shares (superscript are reintroduced):

\[ \Delta S^A_{\alpha\beta} = (\alpha_i^{AK} - \alpha_0^{AK})(\beta_i^{WK} - \beta_0^{WK}) \]

\[ = r_{\alpha\beta} \sqrt{(\alpha_i^{AK} - \alpha_0^{AK} - \bar{\alpha}_i^{AK} + \alpha_0^{AK})(\alpha_i^{AK} - \alpha_0^{AK} - \bar{\alpha}_i^{AK} + \alpha_0^{AK})(\beta_i^{WK} - \beta_0^{WK})(\beta_i^{WK} - \beta_0^{WK})} \]

Therefore, the third effect shows to what degree a firm has succeeded in adapting the commodity composition of its sales to the changes in the commodity composition of the markets. Fagerberg and Sollie (1987) name it as the ‘relative commodity adaptation effect’ or just simply ‘commodity adaptation effect’. If the commodity adaptation effect equals zero, it does not necessarily mean that no adaptation takes place, but that the firm adapts its sale structure at exactly the same rate as the average of all countries selling to the markets in question.

**The ‘several commodities – several markets’ case**

This sub-section explains the CMS method in the case of ‘several commodities – several markets’. For example, we want to analyze firm A that sell n commodities to all k countries (sale destinations) as depicted in Figure 4. The markets share of firm A in all markets sale \( (S^A) \) can be written as the inner product of the vector of its macro share \( (S^A) \) and the vector of firm shares of all markets sales \( (\delta^{W}) \):

\[ S^A = S^A \delta^{W} = \left[ \sum_i \sum_i V^{A}_{ii} / \sum_i \sum_i V^{A}_{ii}, \sum_i \sum_i V^{A}_{i2} / \sum_i \sum_i V^{A}_{i2}, \ldots, \sum_i \sum_i V^{A}_{ik} / \sum_i \sum_i V^{A}_{ik} \right] \]

\[ = \left[ \sum_i \sum_i V^{W}_{ii} / \sum_i \sum_i V^{W}_{ii}, \sum_i \sum_i V^{W}_{i2} / \sum_i \sum_i V^{W}_{i2}, \ldots, \sum_i \sum_i V^{W}_{ik} / \sum_i \sum_i V^{W}_{ik} \right] \]

(31)
The change in $\Delta S^A$ between the periods 0 and $t$ is:

$$\Delta S^A = S_t^A - S_0^A$$  \hspace{1cm} (32)

or

$$\Delta S^A = \Delta S_0^A + \Delta S_s^A + \Delta S_{s0}$$

where:

$$\Delta S_s^A = (s_t^A - s_0^A) \delta_0^W$$

and

$$\Delta S_0^A = \sum \left[ \begin{array}{c}
\sum V_{i,t,0}^A \\
\sum V_{i,t,0}^A \\
\sum V_{i,t,0}^A
\end{array} \right] - \sum \left[ \begin{array}{c}
\sum V_{i,t,0}^W \\
\sum V_{i,t,0}^W \\
\sum V_{i,t,0}^W
\end{array} \right]$$

The change in the markets share can be broken down into three effects:

$$\Delta S^A = \Delta S_s^A + \Delta S_s^A + \Delta S_{s0}^A$$  \hspace{1cm} (34)

The first effect is the changes in the macro shares weighted by firm shares in the initial year, while the second effect is the changes in the firm shares weighted by macro shares in the initial year. Thus, the second effect measures the effect on the markets share of a firm in the all markets of changes in the composition of the markets. It is named the markets composition effect. The third effect can be interpreted as the degree of success of the firm in adapting the markets composition of its sales to the changes in the firm composition of all markets sales. Therefore, following the argument described in the previous sub-section, it is named the markets adaptation effect. A formal proof on this matter is given below. Let $r_{sA}^A$ denotes the correlation...
coefficient between the changes in macro shares and the changes in firm shares, and let $A_0$, $A_t$, $\delta_0$ and $\delta_t$ be vectors of means. The correlation coefficient between the changes in micro shares $(s - s_0)$ and the changes in commodity shares $(\delta_t - \delta_0)$, which is symbolized by $r_{s\delta}$, is formulated as:

$$r_{s\delta} = \frac{(s_t - s_0 - \tilde{s}_t + \tilde{s}_0)(\delta_t - \delta_0 - \tilde{\delta}_t + \tilde{\delta}_0)}{\sqrt{(s_t - s_0 - \tilde{s}_t + \tilde{s}_0)(s_t - s_0 - \tilde{s}_t + \tilde{s}_0)(\delta_t - \delta_0 - \tilde{\delta}_t + \tilde{\delta}_0)(\delta_t - \delta_0 - \tilde{\delta}_t + \tilde{\delta}_0)}}$$  (38)

The symbol $\tilde{s}_t, \tilde{s}_0, \tilde{\delta}_t$ and $\tilde{\delta}_0$ are vectors of means, defined by:

$$\tilde{s}_t = (1/n)s_uu'$$  (39)
$$\tilde{s}_0 = (1/n)s_0uu'$$  (40)
$$\tilde{\delta}_t = (1/n)u'\delta_uu = (1/n)u$$  (41)
$$\tilde{\delta}_0 = (1/n)u'\delta_0u = (1/n)u$$  (42)

It follows from equations 38-42 that:

$$r_{s\delta}\sqrt{(s_t - s_0 - \tilde{s}_t + \tilde{s}_0)(s_t - s_0 - \tilde{s}_t + \tilde{s}_0)(\delta_t - \delta_0)(\delta_t - \delta_0)} = (s_t - s_0 - (1/n)s_0uu' + (1/n)s_0uu')\tilde{\delta}_t$$  (43)

By rearranging, we get:

$$r_{s\delta}\sqrt{(s_t - s_0 - \tilde{s}_t + \tilde{s}_0)(s_t - s_0 - \tilde{s}_t + \tilde{s}_0)(\delta_t - \delta_0)(\delta_t - \delta_0)} = (s_t - s_0)(\delta_t - \delta_0) - (1/n)(s_t - s_0)u'\delta_0u(\delta_t - \delta_0)$$  (44)

Since the sum of the firm shares is always equal to one, it follows that:

$$u'(\delta_t - \delta_0) = 0$$  (45)

Therefore

$$r_{s\delta}\sqrt{(s_t - s_0 - \tilde{s}_t + \tilde{s}_0)(s_t - s_0 - \tilde{s}_t + \tilde{s}_0)(\delta_t - \delta_0)(\delta_t - \delta_0)} = (s_t - s_0)(\delta_t - \delta_0)$$  (46)

And

$$\Delta S^A_s = r_{s\delta}\sqrt{(s_t^A - s_0^A - \tilde{s}_t + \tilde{s}_0)(s_t^A - s_0^A - \tilde{s}_t + \tilde{s}_0)(\delta_t^w - \delta_0^w)(\delta_t^w - \delta_0^w)}$$  (47)

By taking into account equation 15-19 and the definition of $s^A$, $\Delta S^A_s$ may be written as the sum of three effects:

$$\Delta S^A_s = \Delta S^A_{\alpha} + \Delta S^A_{\beta} + \Delta S^A_{\alpha\beta}$$  (48)

$$\Delta S^A_{\alpha} = \sum_j (\alpha^A_i - \alpha^A_0)\beta^w_0\delta^w_0$$  (49)

$$\Delta S^A_{\beta} = \sum_j (\alpha^A_0 \beta^w_i - \beta^w_0)\delta^w_0$$  (50)

$$\Delta S^A_{\alpha\beta} = \sum_j (\alpha^w_i - \alpha^w_0)\beta^w_0\delta^w_0$$  (51)

The first effect ($\Delta S^A_{\alpha}$) is the effect of changes in the micro shares of county A in each markets weighted by the commodity composition of each markets and the firm composition of total all markets sales in the initial year. Following the argument of the previous section, this is labeled ‘the markets share effect’. By the same token, the second effect ($\Delta S^A_{\beta}$) is labeled ‘the
commodity composition effect’ and the third \(\Delta S_{a\beta}^A\) is labeled ‘the commodity adaptation effect’. Since the proof and interpretation in the latter case is quite analogous to the previous case, the result of the proof is simply stated here:

\[
\Delta S_{a\beta}^A = \sum_t \delta_t \left( \alpha_t^A - \alpha_t^A_0 - \bar{\alpha}_t^A + \bar{\alpha}_t^A_0 \right) \left( \beta_t^W - \beta_t^W_0 \right) \left( \beta_t^s - \beta_t^s_0 \right)
\]

(52)

To sum up, the change in the firm’s markets share in total all markets sales may be split into five effects:

- \(\Delta S_{\alpha}^A\) = the markets share effect;
- \(\Delta S_{\beta}^A\) = the commodity composition effect;
- \(\Delta S_{\delta}^A\) = the markets composition effect;
- \(\Delta S_{a\beta}^A\) = the commodity adaptation effect;
- \(\Delta S_{a\delta}^A\) = the markets adaptation effect;

so that

\[
\Delta S^A = \Delta S_{\alpha}^A + \Delta S_{\beta}^A + \Delta S_{\delta}^A + \Delta S_{a\beta}^A + \Delta S_{a\delta}^A
\]

(53)

### 3.4. The Proposed New Version of CMS

After describing comprehensively, the two fundamental methods of CMS proposed by Leamer and Stern (1970) and Fagerberg and Sollie (1987), we argue that the concepts have different focuses. Leamer and Stern focus on factors underlying the changes in sales \(V_{\ast\ast}^A - V_{\ast\ast}^A_0\), which also may be represented as the growth of sales, either using Laspeyres index \(\frac{V_{\ast\ast}^A - V_{\ast\ast}^A_0}{V_{\ast\ast}^A_0}\) or Paasche index \(\frac{V_{\ast\ast}^A - V_{\ast\ast}^A_0}{V_{\ast\ast}^A}\). They conclude that the change (growth) in sales may be caused by (a) the general rise in all markets sales; (b) the markets distribution of firm A’s sales; (c) the commodity composition of firm A’s sales; and (d) an unexplained residual (the competitiveness effect).

Meanwhile, Fagerberg and Sollie examine factors causing the changes in shares of sales or the change in markets share \(\frac{V_{\ast\ast}^A - V_{\ast\ast}^A_0}{V_{\ast\ast}^W - V_{\ast\ast}^W_0}\). They conclude that the change in markets share can be caused by (a) the markets share effect; (b) the commodity composition effect; (c) the markets composition effect; (d) the commodity adaptation effect; (e) the markets adaptation effect. Since the markets share shows the competitiveness, we argue that Fagerberg and Sollie (1987) actually focus on factors underlying the change in firm’s competitiveness, not the change in sales as described by Leamer and Stern (1970).

We derive the following new version of the CMS method by combining the two methods by Leamer and Stern (1970) and Fagerberg and Sollie (1987) previously discussed. The paragraphs below explain the derivation of the new version. The increase in the markets share implies the increase in competitiveness. The share of sales of a given firm is a function of the firm’s relative “competitiveness” (Richardson, 1971a):

\[
S^A = \frac{V_{\ast\ast}^A}{V_{\ast\ast}^W} = f\left( \frac{c}{C} \right)
\]

(54)
where \( f'(0) > 0 \). \( S^A \) is the sale share of the focus firm A; \( V^A_\cdot \) and \( V^W_\cdot \) are total sales of the focus firm A and the all markets, respectively; \( c \) and \( C \) are “competitiveness” of the focus firm and the all markets, respectively. Taking the derivative with respect to time \( (t) \) of equation (54) will result:

\[
\frac{dV^A_\cdot}{dt} = S^A \frac{dV^W_\cdot}{dt} + V^W_\cdot \frac{dS^A}{dt} = S^A \frac{dQ}{dt} + V^W_\cdot \frac{df}{C} \tag{55}
\]

or

\[
V^\circ_\cdot = S^A V^\circ W_\cdot + V^W_\cdot S^\circ A \tag{56}
\]

A dotted \( (\circ) \) variable represents that the derivative of the variable with respect to time \( (t) \).

In this simplest CMS model, a firm’s total sale growth \( (V^A_\cdot) \) is explained by (a) all markets growth effect \( (S^A V^\circ W_\cdot) \) and (b) competitive effect \( (V^W_\cdot S^\circ A) \). The former represents the firm’s growth in sales would have been if it had maintained its sales share and the later represents any additional sales growth due to changes in relative competitiveness. In term of the discrete time, equation (56) can be written as:

\[
\Delta V^A_\cdot = S^A \Delta V^W_\cdot + V^W_\cdot \Delta S^A \tag{57}
\]

Substituting \( \Delta S^A \) with equation (31), a new version of the CMS method is obtained:

\[
\Delta V^A_\cdot = S^A \Delta V^W_\cdot + V^W_\cdot \left( \Delta S^A_\alpha + \Delta S^A_\beta + \Delta S^A_\delta + \Delta M^A_\alpha + \Delta M^A_\beta + \Delta M^A_\delta \right) \tag{58}
\]

Where

- \( \Delta V^A_\cdot \) = change of firm A’s sales
- \( S^A \Delta V^W_\cdot \) = change in A’s sales due to the general rise of all markets’ sales
- \( V^W_\cdot \Delta S^A_\alpha \) = the markets share effect
- \( V^W_\cdot \Delta S^A_\beta \) = the commodity composition effect
- \( V^W_\cdot \Delta S^A_\delta \) = the markets composition effect
- \( V^W_\cdot \Delta M^A_\alpha \) = the commodity adaptation effect
- \( V^W_\cdot \Delta M^A_\beta \) = the markets adaptation effect

If fully written, the equation (58) will as follows:

\[
V^A_\cdot - V^{A0}_\cdot = \frac{V^{A0}}{V^W_\cdot} \left( V^W_\cdot - V^{A0}_W \right) + V^W_\cdot \left( \frac{V^A_\cdot - V^{A0}_\cdot}{V^W_\cdot - V^{A0}_W} \right) \tag{59}
\]

If the first effect is calculated by using initial year \( (0) \) then the second effect must necessarily be calculated by using final year \( (t) \), vice versa. This implies \( V^A_\cdot - V^{A0}_\cdot = \frac{V^{A0}}{V^W_\cdot} \left( V^W_\cdot - V^{A0}_W \right) + V^W_\cdot \left( \frac{V^A_\cdot - V^{A0}_\cdot}{V^W_\cdot - V^{A0}_W} \right) \) or

\[
V^A_\cdot - V^{A0}_\cdot = \frac{V^{A0}}{V^W_\cdot} \left( V^W_\cdot - V^{A0}_W \right) + V^W_\cdot \left( \frac{V^A_\cdot - V^{A0}_\cdot}{V^W_\cdot - V^{A0}_W} \right). \tag{59}
\]
\[ \Delta V^A = S^A \Delta V^W + V^w \sum_j (\alpha^A_i - \alpha^A_j) \beta^j \delta^w_j \]

(a) \[ + V^w \sum_j \alpha^A_i (\beta^w_j - \beta^w_0) \delta^w_j + V^w s^A (\delta_i - \delta_0) \]

(b) \[ + V^w \sum_j (\alpha^w_j - \alpha^w_0) (\beta^w_j - \beta^w_0) \delta^w_j + V^w (s^A - s^A_0) (\delta^A - \delta^A_0) \]

(c) \[ (59) \]

(d) \[ + V^w \sum_j (\alpha^w_j - \alpha^w_0) (\beta^w_j - \beta^w_0) \delta^w_j + V^w (s^A - s^A_0) (\delta^A - \delta^A_0) \]

(e) \[ (59) \]

(f) \[ + V^w \sum_j (\alpha^w_j - \alpha^w_0) (\beta^w_j - \beta^w_0) \delta^w_j + V^w (s^A - s^A_0) (\delta^A - \delta^A_0) \]

Equation (59) implies that the change in firm A’s sales can be caused by (a) the general changes in the all markets’ sales, (b) the markets share effect, (c) the commodity composition effect, (d) the markets composition effect, (e) the commodity adaptation effect, (f) the markets adaptation effect.

There are three main differences between the new version (59) and Leamer and Stern’s (1970) version. First, the problem of subjectivity in choosing the markets distribution effect or the commodity composition effect to be calculated first in the CMS version by Leamer and Stern (1970) is avoided in this new version. Second, the new version gives six effects instead of Leamer and Stern’s four effects. In the new version the markets adaptation and commodity adaptation effects are introduced instead of Leamer and Stern’s residual effect. Clear economic interpretation of the two effects is also given. Third, Laspeyres index is employed throughout the calculations. Therefore, lack of comparability due to differences in weighting procedures is avoided (Fagerberg and Sollie, 1987).

4. Simulations

Due to the unavailability of empirical data, the formula is simulated by using hypothetical data. In real world, researchers who want to apply this formula need data on the specific firms and its competitors’ sales by products and market. The formula can be applied for any industries, for examples automotive, dairy products, banking, insurance, manufacture industries etc. Based on our formulas, change in sales of a specific firm can be caused by (a) the general changes in the all markets’ sales, (b) the markets share effect, (c) the commodity composition effect, (d) the markets composition effect, (e) the commodity adaptation effect, (f) the markets adaptation effect. In real world, researchers who want to apply this formula need data on the specific firms and its competitors’ sales by products and market.

Accordingly, Equation (59) alternatively can be written as:
\[ \Delta V^A = S^A \Delta V^W + V^w \sum_j (\alpha^w_i - \alpha^w_j) \beta^j \delta^w_j \]

(a) \[ + V^w \sum_j \alpha^w_i (\beta^w_j - \beta^w_0) \delta^w_j + V^w s^A (\delta_i - \delta_0) \]

(b) \[ + V^w \sum_j (\alpha^w_j - \alpha^w_0) (\beta^w_j - \beta^w_0) \delta^w_j + V^w (s^A - s^A_0) (\delta^A - \delta^A_0) \]

(c) \[ (59) \]

(d) \[ + V^w \sum_j (\alpha^w_j - \alpha^w_0) (\beta^w_j - \beta^w_0) \delta^w_j + V^w (s^A - s^A_0) (\delta^A - \delta^A_0) \]

(e) \[ (59) \]

(f)
## Case 1. Monopoly

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**Note:** M is market, C is commodity, TC is Total Commodity

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Case 2. Duopoly Cournot Model

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Note: M is market, C is commodity, TC is Total Commodity

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Case 3. Duopoly Cournot Model: Strategic Complement

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Note: M is market, C is commodity, TC is Total Commodity

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Case 4. Duopoly Cournot Model: Change in Commodity

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Note: M is market, C is commodity, TC is Total Commodity

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## Case 5. Perfect Competition

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Note: M is market, C is commodity, TC is Total Commodity

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### Case 6. Perfect Competition

#### Firm A

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#### Sales Change 2014-2015

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Total | 155,989,910,885 |

Note: M is market, C is commodity, TC is Total Commodity
6. Conclusion

This paper analyzes the factors underlying firms’ changes in sales using the Constant Market Share (CMS) method. Firstly, the CMS concepts are comprehensively described, especially works by Leamer and Stern (1970), Richardson (1971a, 1971b) and Fagerberg and Sollie (1987). Secondly, by combining the original concept in firm’s change in sales and change in share in the total sales, this paper proposes a new version of the CMS which breakdown the change in a firm’s sales into six effect instead of two effects (by Tyszynski (1951)) or four effects (by Leamer and Stern (1970) and Richardson (1971a, 1971b). The six effects are (1) general changes in total sales, (2) market share effects, (3) commodity composition effect, (4) market composition effect, (5) commodity adaptation effect, (6) market adaptation effect. Thirdly, from the simulations this paper concludes the constant share norm seems powerful in explaining a firm’s sales.

References


