Tools and Techniques for Economic Decision Analysis

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Chapter 2
Application of Markowitz Portfolio Theory by Building Optimal Portfolio on the US Stock Market

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ABSTRACT

This chapter is focused on building investment portfolios by using the Markowitz Portfolio Theory (MPT). Derivation based on the Capital Asset Pricing Model (CAPM) is used to calculate the weights of individual securities in portfolios. The calculated portfolios include a portfolio copying the benchmark made using the CAPM model, portfolio with low and high beta coefficients, and a random portfolio. Only stocks were selected for the examined sample from all the asset classes. Stocks in each portfolio are put together according to predefined criteria. All stocks were selected from Dow Jones Industrial Average (DJIA) index which serves as a benchmark, too. Portfolios were compared based on their risk and return profiles. The results of this work will provide general recommendations on the optimal approach to choose securities for an investor's portfolio.

INTRODUCTION

Investing in capital markets is one of the main activities of large number of economic subjects. This activity was particularly driven by development of information technology as well as deregulation and globalization, which is typical of the current financial markets. The development of information technology has enabled even small retail investors, who generally do not have the appropriate knowledge and experience, to take advantage of the direct purchase or sale of securities on the capital market. Driven by different motives, investors allocate their available resources to the assets and through selected invest-

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ment strategies they seek to derive maximum value from invested funds and at the same time eliminate the threat of losses.

Different models for assets valuation describing the relationship between risk and return on the given investment can be used as a tool to support investment decision-making.

One of the most common methods in designing strategies and building portfolios is the Modern Portfolio Theory (MPT). Although it is based on simplifying assumptions, it can be successfully used in portfolio analysis for explaining the relationship between the return and risk of individual portfolio components. The Capital Market Theory, which is closely related to the MPT, then came up with the Capital Asset Pricing Model (CAPM), which extended the existing theory by an equilibrium view of the asset market. In spite of the fact that the capital asset pricing model rests on simplifying assumptions and has been tested many times since its inception in the 60s, but its general applicability was not confirmed, it is currently among the most widely used models and can be used to manage investment strategies and build investment portfolios. The model is based on the equilibrium between the risk and return, or more precisely the risk (represented by beta coefficient) of a specific title is directly proportional to the return achieved on the given investment.

It is these findings about this approach and the model, or its principle (i.e. the idea of equilibrium of return or loss stemming from the risk of a specific investment) that are the reasons for examining its functionality on real data and are used to achieve the objectives of this paper.

The aim of the present paper is to define, on a selected sample of US stocks, the most suitable method for optimal portfolio compilation using the Markowitz Portfolio Theory. That is meaning whether it is appropriate to favour stocks with high or low beta coefficient or whether it is preferable to use a random selection of each stock.

Therefore, the aim of this paper is to verify or answer the research question whether the optimal portfolio compiled in accordance with the Portfolio Theory brings investor an optimal ratio of return to the given risk. Within this basic research question, following research sub-questions can be set out regarding the assumptions and the basic idea of the CAPM model:

- High values of beta coefficient guarantee higher returns on stock titles.
- Random selection of securities in the portfolio provides satisfactory return at an acceptable level of risk.

Defined research questions or empirical analysis of functionality of the CAPM model is based upon knowledge as well as criticism of this issue, which is given in the following chapter. Achieved results of this paper support the arguments against the model and provide investors with recommendation on how to properly compile portfolio regarding its profitability and risk, and whether higher values of beta indicator actually “guarantee” higher valuation.

LITERATURE REVIEW

Just like other areas of economics, the theory of financial markets has a rich history. The firm foundation theory is an approach better known as the determination of the intrinsic value of stock, which is an output of fundamental analysis (Malkiel, 2012). Williams (1938) developed this technique and, thanks to the work by Graham & Dodd (2008), it founded its way even among investors on Wall Street. Although the fundamental analysis has been losing its importance recently and investors rather make their decisions based on subjective and psychological preferences, even instinctively, to a long-term investor (not a speculator) it is still crucial in combination with the modern portfolio theory.

Criticizers of the firm foundation theory focus on psychological analysis; they examine the investor as part of a collective investment game, in which the determinant of behaviour is human psyche. Keynes (1936) expressed the idea that in a way that it makes no sense to calculate the intrinsic value, but it is worth analysing the likely behaviour of a group of investors in the future. Investors can be divides into players and speculators, and investigates their behaviour in the short term, in which, prices are determined primarily by psychological reactions (Kostolany, 1989).

In a comprehensive form, the efficient market theory was introduced by Fama (1965). He concluded that asset markets behave randomly and there is no correlation between the current and past price movements. Asset prices react sharply, precisely and immediately on each new price-sensitive information. On such market, all investment strategies fail, and no investor is able to achieve any above-average long-term yield. Such described behaviour of markets is called the random walk. A distinction can be made between weak, moderate and strong form of efficiency4 according to how new messages are absorbed by the market. The theory, however, has been a subject to sharp criticism, Lo & MacKinlay (2002) demonstrated inertia of prices on the market or inadequate response to newly released reports. Stock prices react to unexpected information in an inadequate manner (over reactive capital market) and also mentions the occurrence of anomalies5 on the markets (Haugen, 1999). Musílek (2011) summarizes that the advanced liquid stock markets behave quite effectively. It is necessary to consider them economically efficient, as new information is not absorbed immediately; in the long run, however, above-average returns cannot be achieved. Less developed and illiquid markets, behave inefficiently, although this inefficiency gradually decreases (Musílek, 2011). Pástor & Veronesi (1999) point out that efficiency together with liquidity of stock markets changes and there are further significant changes in the investment environment, which may make the absorption of new information more difficult. Malkiel (2012) indicates that markets can be highly effective, even if errors occur. Fama himself said at the conference held at the University of Chicago that markets can somehow behave irrationally, thus admitted the non-existence of a perfect financial market (The Wall Street Journal, 2004). This position is now occupied by the vast majority of economists.

Harry Markowitz (1952) is at the origins of the modern portfolio theory, which is sometimes referred to as Markowitz portfolio theory. In 1990, the author was awarded the Nobel Prize in economics for his work. Such optimal portfolio can be build using the Markowitz model that will have less risk than the weighted average of risks of individual securities included in the portfolio, while preserving the given profitability. The risk was thus diversified between the assets constituting the portfolio. This portfolio is located along the efficient frontier, and other portfolios will be omitted by the investor.

Markowitz model assumed a selection of different securities, which are subject to the risk. In the theory of capital markets, the investor may also include a risk-free asset in the portfolio whose rate of return is certain. Musílek (2011) lists three methods for determining the rate of return on a risk-free asset. You can use a rate of return on treasury bills (T-bills) or long-term government bonds (duration of the bond should equal the duration of the intended security). The third method is to consider the current rate of return on T-bills for the first investment period; for future years the investment is based on forward rates according to the shape of the yield curve. For the expected rate of return of a portfolio it can be established that:

$$\overline{r_p} = w_z \overline{r_z} + (1 - w_z) r_f \,,$$

where:

w_z = weight of risk (equity) component of the overall portfolio,

$\overline{r_z}$ = mean profitability of risk (equity) component of portfolio,

r_f = rate of return of a riskless asset.

Furthermore, it is possible to formulate the variance (risk) as:

$$\sigma_p^2 = w_z^2 \sigma_z^2$$

where:

σ_z^2 = variance of risk component of portfolio.

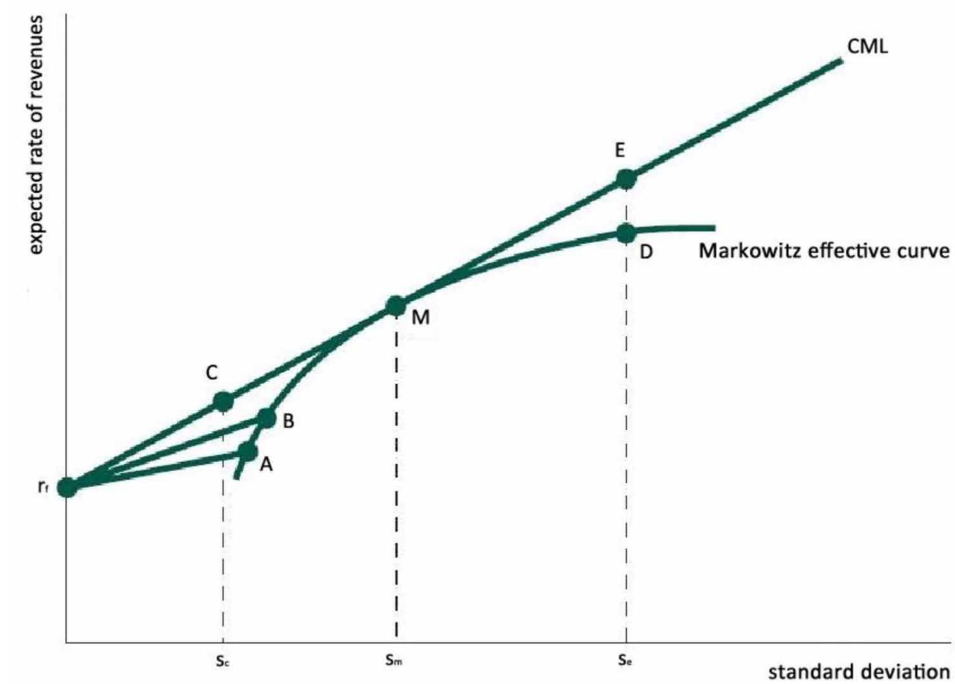
As Figure 1 shows, investors can combine efficient portfolios from the efficient frontier with a risk-free asset. Portfolios lying on the line joining points r_f and M represent the best achievable combinations of return and risk when investing in a portfolio of risk-free and risky assets, or the combination thereof. Risk-free asset utilization contributes to reduction the risk while maintaining the required rate of return. Portfolios lying on the intersection are also referred to as lending, as investors in T-bills lend money to the state. Borrowing portfolios, which in turn lie between points M and E, may be achieved by a less risk-averse investor so that he or she will borrow at the risk-free rate and the funds acquired in such way are invested in a risky portfolio. As mentioned by Reilly & Brown (2012), investors use leverage to achieve a higher rate of interest compared to the portfolio along the Markowitz's efficient frontier (the difference between the portfolios E and D).

Should it not be possible to borrow funds at a risk-free rate, the efficient frontier would be formed by the abscissa leading from r_f to M as well as curve from M to D. Sharpe (2000) calls the half line from r_f to E to r_f the Capital Market Line (CML).

The capital market line only provides an optimized relationship for the expected return and risk of efficient portfolio, and does not differentiate between unique and systematic risk of individual securities. To express the relationship between the expected rate of return and the systematic risk of an investment instrument or portfolio, we use the Capital Asset Pricing Model (CAPM), which was independently created by Sharpe (1964), Lintner (1965), Treynor (1962), Mossin (1966).

The basic idea behind the capital asset pricing model is that the overall risk may be split into a systematic (non-diversifiable) risk, which indicates the sensitivity of securities to the general market fluctuations, and unsystematic (diversifiable) risk, which is influenced by factors associated with a particular economic entity. In valuation of a security or portfolio, this theory does not take into account the overall risk, but only that part of the risk which cannot be eliminated by diversifying (Malkiel, 2012). Thus, if

Figure 1. Capital Market Line
Source: Reilly, Brown (2012)



the investor wants to achieve higher long-term profitability, he or she must increase the level of non-diversifiable portfolio risk. From the above mentioned facts, the sensitivity of investment instruments to market developments can be expressed by a coefficient beta. The relationship between the expected return and systematic risk is then expressed by the following equation:

$$\overline{r_i} = r_f + \beta_i(\overline{r_m} - r_f)$$

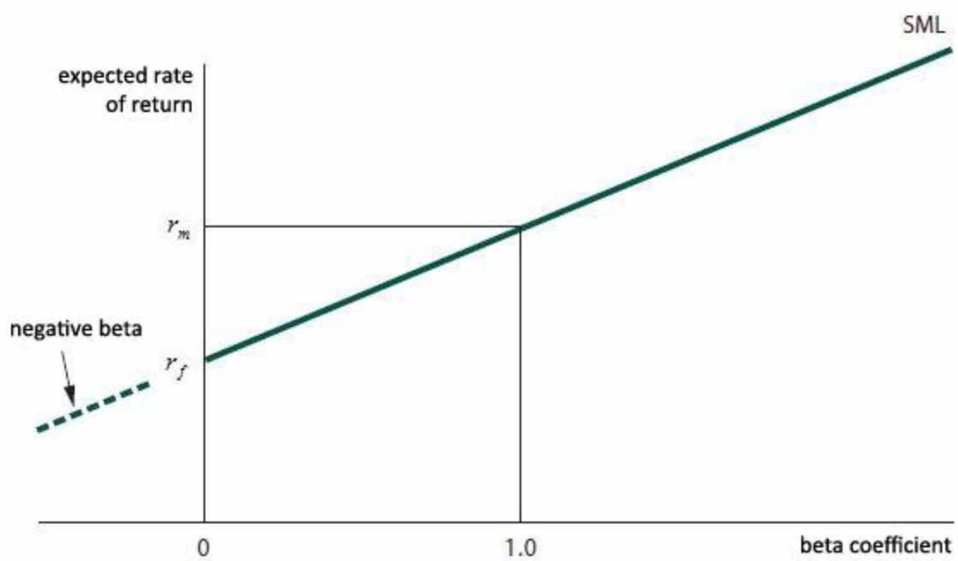
This relationship is also the equation of a security market line (SML), showing a positive correlation between expected rate of return and systematic risk (or beta factor).

Securities and portfolios from the security market line correspond to correctly priced securities with an equilibrium rate of return. The more the securities are placed to the right on the line, the greater the difference is between the rate of return of the instrument and the risk-free asset (Veselá, 2007). This difference is then a reward in the form of a premium for the systematic risk.

The difference between SML and CML is in the process of how the risk is measured, if bz using beta coefficient or standard deviation (Reilly & Brown, 2012). The consequence is that SML can be applied only to a fully diversified portfolio.

Beta factor expresses a comparison between the movement in prices of individual instruments and that of the whole market. As mentioned above, mathematically it expresses the size of a systematic risk. Broad market index is assigned a value of 1. The higher the beta of a security, the more its price changes on average compared to the rest of the market. The opposite is true for the low value of this

Figure 2. Security market line
Source: Reilly, Brown (2012)



factor. Securities with high beta values are sometimes also referred to as aggressive investments, while instruments with low beta can be called defensive investments.

The space above the line includes undervalued securities and portfolios that bring higher yields than the level of systematic risk. In other words, the real rate of return of investment instruments is higher than the equilibrium rate of return for a given risk.

Markowitz model showed how investors should behave when compiling optimal portfolios. The Capital Market Theory explains the valuation of assets using this model and is based on the concept of effective diversification. The CAPM was and is used due to its simplicity and clarity of the input data. This simplicity, or simplifying assumption in the model, but only those, is a frequent subject of criticism. For example, Veselá (2007) mentions the distortion in the calculated beta factor by selection of a time period. Beta factors of portfolios and especially individual securities also exhibit considerable volatility. Furthermore, she mentions problems in the derivation of the risk-free interest rate and the actual availability of a risk-free asset to all investors. The practical existence of a relationship between the return and risk was tested by Fama & French (2004). They concluded that higher risk measured by beta coefficient is not necessarily associated with a higher yield. Empirical security market line is then flatter than the theoretical line. The security line may take different shapes due to the existence of different borrowing and lending rates or transaction costs (Liška & Gazda, 2004). The fact that investors may be rewarded even for part of unsystematic risk, which is completely contrary to the spirit of CAPM (Fuller & Wong, 1988). The model also ignores industry factors, taxation, dividend yield or the book value of a company as relevant factors that contribute to the expected yield rate. The level of the risk premium varies not only between securities, but also between countries (Damodaran, 2011). The reason may be different national economic policies and development of national macroeconomic variables. Frequent criticism led many economists to modification of the simple version of CAPM.

The problem is, for example, the question of a risk-free asset. The classic version of CAPM considers the existence of a risk-free asset, which is available to all without exceptions at the same borrowing and lending rates. At the same time, we cannot consider the risk-free asset to be completely risk-free, even if these are government bonds. Because even the issuing state may get into serious trouble caused by inflation or exchange rate changes (Širůček, Šoba, & Něměček, 2014). Black (1972) developed the capital asset pricing model with a zero beta coefficient (zero-beta CAPM) that does not assume the existence of a risk-free asset. If such asset does not exist, then there must be portfolios with a zero-beta factor against the market portfolio. Instead of a risk-free asset, investors then combine portfolios not correlated to the market, which have the lowest risk.

Brennan (1970) contested another assumption - the non-existence of taxes. In his model he considered different tax rates for individual investors and even for income and capital gains. His tax-capital asset pricing model (T-CAPM) bases its calculation of expected rate of return on the beta factor of securities, their dividend yield, and different tax rates. Investors in higher tax groups then due to the impact of taxation may prioritize portfolios with lower dividend yield.

Merton (1973) extended the original CAPM to a multifactorial capital asset pricing model (multifactor CAPM or M-CAPM). In this model, investors do not take into account only risks associated with the expected rates of return, but also risks affecting the amount of future consumption, such as future income, relative prices of goods or investment opportunities. More beta coefficients then enter into the model, which determine the portfolio's sensitivity to off-market sources of risk

Amihud & Mendelson (1986) respond to the assumption of absence of transaction costs with their capital asset pricing model which contains a premium for illiquidity (capital asset pricing model with illiquidity premium - IP-CAPM). The authors divide the investment instruments into liquid and illiquid. Lower demand for illiquid instruments causes a decline in their price and vice versa the growth in returns. These instruments due to higher transaction costs bring investors a premium for illiquidity. In the short term, the investor should invest into liquid instruments, while illiquid, but more profitable instruments may be an appropriate choice for a long-term strategy.

Another approach is the downside CAPM (D-CAPM), which was tested, for example, by Estrada (2002). This approach stems from the fact that investors are not averse to high variance if the rate of return of a security grows. However, the opposite is true when the market downturns, when investors are highly averse to losses (Malkiel, 2012). Beta in D-CAPM therefore expresses the covariance-variance ratio in the market downturn.

As mentioned in the introduction, the CAPM model despite its criticism is extensively used due to its simplicity in equity analysis, as it is in the case of this paper, which focuses on appropriate ways to build equity portfolio in terms of return and risk.

DATA AND METHODS

The Dow Jones Industrial Average (DJIA) was selected as sample to be examined. This index was selected for the potential diversification across almost all traded sectors and moreover. This index may be viewed as a global market mood indicator (Širůček, 2013a). The reason for choosing the US market is also its share of the global market capitalization, accounts for about 42% of the global market capitalization (Širůček, 2013b). In a price-weighted index; high-priced stock has therefore a greater impact on the value of the index. In the event of a stock split, the split stock has a lower impact on the index value, while the impact of others slightly rises (Siegel, 2011). Compared to capitalization-weighted indices (Standard & Poor's 500), the relative size of the company (market capitalization) does not enter the calculation.

Individual DJIA index stocks were selected and optimal portfolio was set up so as to attain the set objective or evaluate the research questions, using the following four approaches:

- Setting up an optimum portfolio from all stocks included in the index, based on the CAPM model and its maximization task
- Setting up a portfolio of low-beta stocks (generally, beta factor < 1 is considered low)
- Setting up a portfolio of high-beta stocks (generally, beta factor > 1 is considered high)[1].
- Setting up a portfolio using a random stock picking[2].

To determine the weight of stocks selected for the portfolio can be used several approaches. One of the options is to use Lagrange multipliers while applying one or two constraints. The calculation, for example, can be run as a minimization task, while minimizing the risk of changes in the portfolio's rate of return. Use may also be made of the so-called "Wolfe's method", which is the method of quadratic programming. Its disadvantage is the rising number of variables in the calculation (Ševčíková, 2008). To derive the weights, also the CAPM model can be used. Derivation of optimal portfolio weights through the CAPM is based on the maximization task. This is to maximize the angle φ between the security market line (SML) and an imaginary line parallel with x-axis starting from point r_f. The task can be formulated as:

$$f(\overrightarrow{X}) = \tan\phi = \frac{\overline{r_p} - r_f}{\sigma_p} = \frac{\sum_{i=1}^{n} w_i(\overline{r_i} - r_f)}{\left[\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}\right]^{1/2}} \longrightarrow MAX$$

Then is necessary within the actual calculation to rank securities in a descending order according to their proportion on the right side of the following expression that reflects excess return on a security in relation to its beta coefficient (Čámský, 2007).

$$C_k < \frac{\overline{r_i} - r_f}{\beta_i}$$

In further derivation required for individual securities, following equation can be drawn:

$$C_i = \frac{\sigma_M^2 \sum_{j=1}^{n} \frac{\overline{r_j} - r_f}{\sigma_{\varepsilon_j}} \beta_j}{1 + \sigma_M^2 \sum_{j=1}^{n} \frac{\beta_j^2}{\sigma_{\varepsilon_j}^2}}$$

From the set of calculated C_i $(i \in k)$ are then selected those that meet the above condition of excess return. The last security meeting this condition is marked as C^* and becomes a portfolio constraint. Now we can proceed to calculate the weights:

$$Z_i = \frac{\beta_i}{\sigma^2_{\varepsilon_i}} \left(\frac{\overline{r_i} - r_f}{\beta_i} - C^* \right)$$

In order to express the actual weights of individual stocks in the portfolio, the following formula may be introduced:

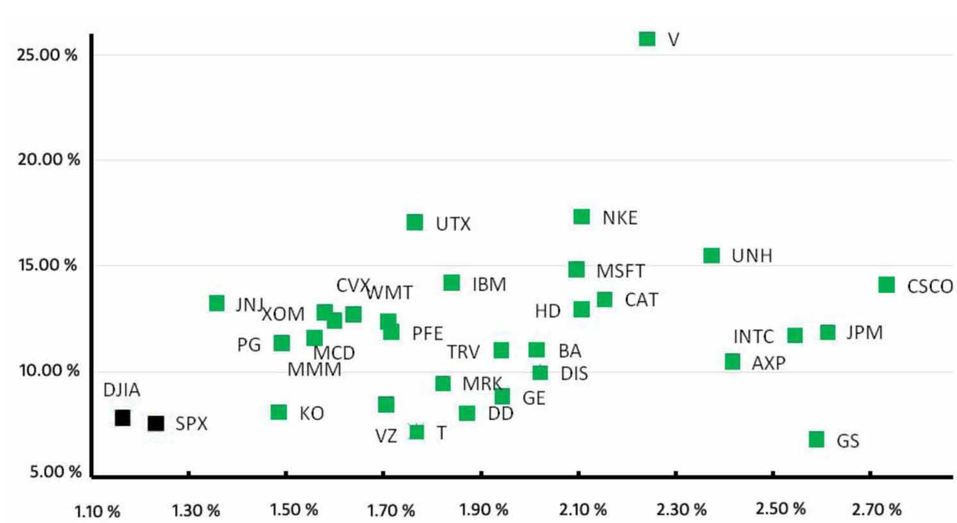
$$w_i = \frac{Z_i}{\sum_{i=1}^{k} Z_i}$$

Daily data from early 1995 to the end of March 2014 entered into calculations, with dividends and any eventual stock splits included. The considered risk-free rate (RFR) of return on three-month US treasury bills (US Government Treasury Bills 3-Month) is 0.05%.

RESULTS

Figure 3 shows annualised returns (vertical axis) and standard deviations (horizontal axis) of individual securities, rates of return the Dow Jones Industrial Average as well as Standard & Poor's 500 as potential benchmarks.

Figure 3. Risk and revenue profile of each asset
Source: Yahoo Finance



The inclusion of a particular stock title to the portfolio depends on the expected excess return on a security relative to beta coefficient of that security, riskiness of the stock and its beta coefficient. The calculation of the optimal portfolio is shown in the table below.

Stocks were ranked according to the expected excess return relative to their beta coefficients. The last title that meets this condition is Merck & Co. (MRK). Optimal market portfolio of DJIA index thus consists of thirteen titles (see Table 1). The rest of the companies were not included in the portfolio

Other possibilities how to build portfolios is to take into account beta coefficients of individual titles. Two portfolios with five securities each were set up, one consisting of stocks with low beta coefficient and the second consisting of high beta instruments. In the selection of individual titles their sectors were considered, too, so as to avoid, for example, three securities of the same sector within the portfolio. This should contribute to better allocation of risk.

For low-beta portfolio, securities listed in Table 2 were picked.

Given the fact that the portfolio is only made up of several instruments, the excess return condition was not met, and no stock was eliminated from the portfolio. Allocation of weights followed the proce-

Table 1. Weights of securities in an optimal market portfolio

Ticker	$(r_i - r_f)/\beta_i$	C_i	Z_i	w_i
VZ	2.791	0.0013	0.0271	11.47%
V	0.429	0.0479	0.0365	15.08%
MCD	0.353	0.0697	0.0280	11.87%
WMT	0.308	0.0847	0.0216	9.15%
PG	0.282	0.1019	0.0240	10.19%
NKE	0.258	0.1197	0.0170	7.21%
T	0.245	0.1229	0.0079	3.37%
UNH	0.242	0.1313	0.0101	4.27%
JNJ	0.235	0.1485	0.0297	12.59%
KO	0.230	0.1519	0.0116	4.92%
MSFT	0.214	0.1570	0.0097	4.09%
IBM	0.205	0.1619	0.0109	4.64%
MRK	0.178	0.1628	0.0027	1.16%

Source: Yahoo Finance, own calculation

Table 2. Portfolio consisting of stocks with low beta coefficient

Ticker	$(r_i - r_f)/\beta_i$	C_i	Z_i	w_i
VZ	2.7910	0.0013	0.0277	18.51%
V	0.4289	0.0479	0.0438	29.30%
MCD	0.3525	0.0678	0.0371	24.80%
IBM	0.2050	0.0932	0.0269	17.99%
MRK	0.1777	0.1009	0.0141	9.40%

Source : Yahoo Finance, own calculation

Table 3. Portfolio consisting of stocks with high beta coefficient

Ticker	$(r_i - r_f)/\beta_i$	C_i	Z_i	w_i
CVX	0.1171	0.0774	0.0367	71.14%
CSCO	0.1122	0.0821	0.0064	12.41%
BA	0.1058	0.0861	0.0084	16.45%
DIS	0.0757	0.0829		
JPM	0.0623	0.0744		

Source: Yahoo Finance, own calculation

dure specified in the methodology. A procedure identical in terms of the methodology was then applied to the portfolio consisting of high-beta stocks. The results are shown in Table 3.

Due to the high sensitivity to market developments, two securities were removed to optimize the portfolio. But portfolio made up of only three instruments does not provide adequate diversification.

A randomly selected portfolio or more precisely the simulation of randomness when building the portfolio had been put on the computer. To ensure sufficient diversification, the computer randomly selects a number of securities in the portfolio. The minimum possible number of stocks is set at five and the maximum possible number is set at twelve (higher number of stocks does reduce the risk, but diminishes returns). The individual stocks are picked up by computer at random; the method used is combination without repetition. The computer simulation is ensured by the RANDBETWEEN feature. This method provided nine securities in a random portfolio (see Table 4).

To increase its explanatory power, the simulation of a random portfolio setup was repeated. This time the computer picked up eleven stocks. The process of optimization and calculation of weights is shown in Table 5.

Table 4. Random portfolio I

Ticker	$(r_i - r_f)/\beta_i$	C_i	Z_i	w_i
PG	0.2821	0.0308	0.0305	22.95%
UNH	0.2418	0.0519	0.0142	10.67%
JNJ	0.2352	0.0921	0.0429	32.27%
IBM	0.2051	0.1085	0.0192	14.49%
PFE	0.1513	0.1169	0.0080	6.01%
UTX	0.1479	0.1305	0.0181	13.60%
AXP	0.0937	0.1268		
MMM	0.0929	0.0981		
GE	0.0735	0.0970		

Source: Yahoo Finance, own calculation

Table 5. Random portfolio II

Ticker	$(r_i - r_f)/\beta_i$	C_i	Z_i	w_i
NKE	0.2583	0.0397	0.0261	23.34%
T	0.2449	0.0469	0.0129	11.51%
UNH	0.2418	0.0649	0.0166	14.84%
KO	0.2299	0.0754	0.0204	18.28%
HD	0.1447	0.0897	0.0091	8.15%
XOM	0.1368	0.1076	0.0199	17.79%
CVX	0.1171	0.1114	0.0066	5.92%
CSCO	0.1122	0.1115	0.0002	0.17%
TRV	0.0960	0.1088		
DIS	0.0757	0.1017		
GS	0.0343	0.0710		

Source: Yahoo Finance, own calculation

DISCUSSION AND CONCLUSION

In the context of this paper, several portfolios were built and weights of the selected stocks in respective portfolios identified. However, every investor would be interested in the expected rate of return and risk entailed by individual portfolio options. The expected future return may be easily calculated as a scalar product of weights and rates of return of included instruments as follows:

$$\overline{r_p} = \sum_{i=1}^{n} w_i \overline{r_i}$$

where:

w_i = weights of individual securities,

$\overline{r_i}$ = average rates of return on individual securities.

Standard deviation of a multi-component portfolio may be obtained by extracting the root of the matrix product as follows:

$$\sigma_P^2 = (w_1\sigma_1 \quad w_2\sigma_2 \quad \cdots \quad w_n\sigma_n) x \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{pmatrix} x \begin{pmatrix} w_1\sigma_1 \\ w_2\sigma_2 \\ \vdots \\ w_n\sigma_n \end{pmatrix} = (w\sigma) x C x (w\sigma)^T$$

Table 6. Rates of return and risk of resulting portfolios

Portfolio	r_p	σ_p
P1 – Optimal portfolio (according to the CAPM)	14.20%	0.87%
P2 – Portfolio made up from low beta stocks	15.63%	1.20%
P3 – Portfolio made up from high beta stocks	12.60%	1.46%
P4 – Random portfolio 1	13.62%	0.92%
P5 – Random portfolio 2	12.76%	0.89%

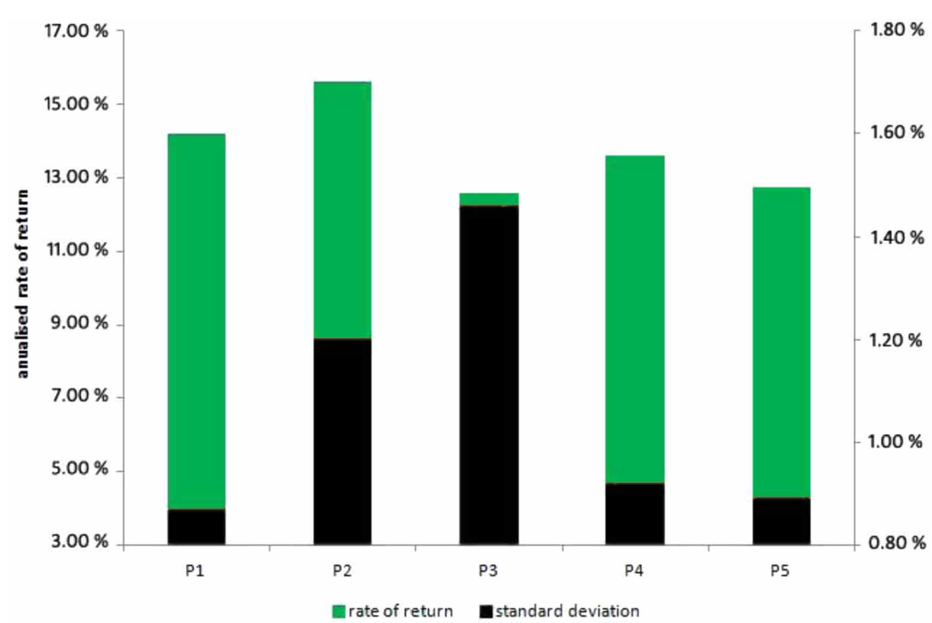
Source: Yahoo Finance, FRED, NASDAQ, own calculation

Table 6 shows the calculated expected historic rates of return and risk with a size of one standard deviation.

Results from Table 6 are also shown in Figure 4. The left axis shows the achieved rate of return, the right axis shows the risk incurred.

The optimal portfolio made up of stocks from the entire index range reached the lowest risk rate and, simultaneously, the average rate of return within the examined portfolios. Portfolio made up of securities selected based on their beta factors returned significantly different results. The use of low-beta securities resulted in the second highest rate of return; however, the risk of this portfolio increased, too, being even the second highest amongst the five examined portfolios. The portfolio consisting of high-beta securities generated the lowest returns while incurring high risk. The first random portfolio features the third lowest rate of return and also the third lowest risk rate. Both characteristics of the second random portfolio were slightly lower as compared to the first one, mainly in terms of the rate of return.

Figure 4. Risk and rate of return of selected portfolios
Source: Yahoo Finance, FRED, NASDAQ



Building a portfolio from securities around the entire index under a restrictive condition that arose in derivation of the maximizing function using the CAPM model, offers one of the best alternatives analysed in this work. The choice of securities is not prejudiced by any subjective views of the investor on individual stock corporations or sectors, and, therefore, the factors decisive for inclusion of a security in the portfolio are solely its historic excess return expressed by the proportion shown on the right side of the aforesaid equation and the risk rate. This result complies with the opinion of Malkiel (2012) and Kohout (2008) who recommend that investors should buy portfolios copying as best as possible the selected index without trying to attempt at active investment strategy, although such portfolio by its broadness somehow lags behind the index. This approach appears to be suitable for common investors who wish to participate in the capital market returns, but find it difficult to go through financial statements of the companies and monitor market trends and events.

One of the research questions examined above was the effect of beta factors of securities on the portfolio‘s rate of return. Specifically, the idea behind this was that securities with high beta factor should generate higher profits for the investor. Over the period examined in this study and covering close to twenty years, the stocks in the United States did experience two major slumps, but have presently hit their historic highs and despite the ongoing and gradual monetary policy tightening there is still moderate optimism prevailing in the markets. A similar situation may also be encountered in some of the European stock exchanges. Thus, it could be expected that stocks more sensitive to market trends will generate higher rates of return. Still, the results obtained run counter to these assumptions.

The rate of return of the portfolio consisting of low-beta securities outperforms by a large margin the return of the portfolio made up of high-beta securities, by more than three per cent. Still, the risk rate of the former was considerably lower than the risk borne by the second portfolio. A major drop in the rate of return accompanied by a rise in the risk rate disfavours the choice of securities with high beta factors in the investor‘s portfolio. Similar results were previously arrived at by Fama & French (2004), who argue that higher risk measured by beta coefficient may not be a guarantee for higher returns, or Black, Jensen, Scholes (1972). In their examined sample of stocks, the low-beta stocks achieved better result, which again contradicts the traditional form of the capital assets pricing model. Širůček, Šoba, Němeček (2014) confirm that the link between beta coefficient and return proved to be very weak. According to the results, the model significantly undervalued low beta stocks, which reached considerably higher returns than would match their beta. In the case of stocks in the group with higher beta factor, both an undervaluation and very often also overvaluation was demonstrated.

Based on the above findings the choice of high-beta securities may not be recommended to an investor who wishes to allocate available funds in capital markets over long-term investment horizons. Nevertheless, instruments with high beta factors may be an interesting choice for speculators who can only settle for a few selected companies given the expected market growth and excess return. Investment horizon of these investors would probably reach a maximum of several years and the success of this strategy would furthermore depend on the correct estimates of the periods of partial growth in the stock markets. However, a study undertaken by Pioneer Investments and Ibbotson Associates (2013) goes to show that the portfolio performance is largely, by as much as 92%, affected by assets allocation, which means by ensuring sufficient diversification. Other factors such as market timing, choice of stocks and other similar efforts made by the investors have an almost negligible share in the overall portfolio performance. The study also mentions the portfolio re-balancing as an important factor playing a major role in active risk reduction. Even though this has been based on historic rates of return, which might introduce a certain error into the following recommendations, securities with high beta factors may not be taken for a secure choice if the investor wishes to achieve high returns. Conversely, investors thinking about long-term investments should select low-beta stocks.

The first random portfolio returned slightly worse results both in terms of the expected returns and risk compared to the first calculated tangency portfolio. The second random portfolio generated very similar results. To start with, it should be stressed that no investor should follow this way of managing the securities. If need be, for example in the event of a major market slump and a necessity to reduce the positions or rebalance the positions if the prices of some of the stocks go up, managing such a portfolio would be most likely highly difficult, and as a result of unformulated strategy the investor would have to do with a passive investment strategy.

As regards a random portfolio, the investor does not have guaranteed quality stocks that could possibly be obtained through a detailed analysis and no optimal diversification is guaranteed, either. Thus, such a random choice may result in excessive allocation of invested funds, for example, into just two sectors of the entire economy, which may strongly impact on the profitability and in particular, elevate the risk rate of such a portfolio. Although this does not happen very often on average, there may be portfolios in specific cases composed of, for instance, high-beta securities, which were mentioned above as portfolios suitable for speculators and not for investor thinking more along long-term lines. Furthermore, as stated by Cohen & Pogue (1968), an investor managing a random portfolio composed of a large number of securities would be forced to make a huge number of partial calculations despite the sophisticated computer technologies available today, but by buying the entire index, the investor could easily avoid this problem. Nevertheless, Malkiel (2012) offers several examples demonstrating that the average fund performance did not differ too much from the returns of randomized portfolios.

The results of portfolios developed in this study by a random selection from amongst the available stocks compared to the tangency portfolio from the entire index indicate a relatively high performance of US stock markets. However, this conclusion may not be adopted on the basis of two created portfolios, as this result might be explained as a stroke of luck. The conducted analyses did not aim to confirm nor refute efficient market behaviour. Still, the random portfolio setup did have another major benefit. Although these were randomly selected securities, parameters of the resulting portfolio did not differ too much from the other portfolios. The random portfolios particularly feature a lower risk rate. These results positively show that the final portfolio parameters are to the largest extent affected by the optimal allocation of assets carried out in this study on the basis of a calculation derived from the capital assets pricing model. Other factors such as an active selection of individual stocks and market timing have a substantially lower effect on the portfolio performance.

For the purpose of further analyses, it would be advisable to change the sample stocks and conduct similar calculations in a market other than the American stock market. As repeatedly highlighted above, the stock prices in the United States reach their maxima, similarly to European markets. Therefore, it might be interesting to set up a portfolio from stock corporations listed for example in any of the developing markets, or, potentially, combine securities from several countries.

SUMMARY

The paper deals with the application of Markowitz's portfolio theory on a specific sample of stocks. American DJIA index was selected for the analysis. As part of applying the Markowitz's theory, several options to select available stocks for the portfolios were used. Individual examined portfolio options were built upon the defined research questions, which were based on the literature review and criticism of the model. Thus, a summary tangency portfolio was set up, which could include all securities contained in the index. As part of the maximization task under the given constraint thirteen stocks were included in the portfolio. Crucial parameters determining whether the relevant security should be included in the portfolio are its risk rate and the amount of excess return. This portfolio was the broadest amongst the examined portfolios and was also characterized by the lowest risk rate; its rate of return could be taken as average compared to the other options.

To verify the research question whether securities with higher beta coefficients bring higher returns, two portfolios were set up. The first one included low-beta stocks, the second, on the contrary, high-beta stocks. The portfolio consisting of low-beta stocks generated satisfactory returns; nevertheless, its risk rate was above average compared to the other portfolios. Thus, selecting the lowest-beta stocks entailed a relatively high risk rate of the entire portfolio. As the portfolio was made up of five stocks, the investor would be likely to achieve a better rate of return/risk profile by conducting an in-depth analysis and replacing some of the securities. Conversely, a portfolio made up of higher-beta stocks achieved the worst results compared to the other options. Once optimized, the resulting portfolio consisted of three stocks only, which is definitely not an optimal option. Thus, the aforesaid research question was not confirmed; high-beta securities do not guarantee higher return for the investor.

In the next steps, random portfolios were developed. Naturally, this approach may not be recommended to investors in principle. Such portfolios face a risk of the investor including highly risky and, at the same time, no-yield securities in the portfolio through a random choice. Likewise, there is a potential allocation of funds into the same or similar sectors, which further increases the risk. Such portfolio would also become very hard to manage as the investor would have no investment strategy at hand and the choice of stocks would be left up to the random draw made by the computer. The results of random portfolios did not differ too much from the first examined tangency portfolio of the entire index. The greatest benefit of developing a random portfolio is the demonstration of optimal assets allocation as the crucial factor affecting the resulting characteristics of each portfolio in the greatest extent.

Results of the paper can be summarized into two findings. First, the relationship between the expected rate of return and the beta coefficient is not so prominent and straightforward as Sharpe and Lintner expected. Second, beta coefficient alone is not sufficient to explain the expected returns, or high beta coefficient does not guarantee higher returns and also other indicators should be taken into account, such as market capitalization or the ratio of book value to market value of stocks (B/M) as indicated by Fama and French (2004) or P/E ratio as indicated by Širůček, Šoba, and Němeček (2014).

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ENDNOTES

[1] These two approaches are based on criticism of CAPM model by Fama and French (2004).

[2] The approach is based on the theory of efficient markets, where asset prices react to new price-sensitive information and investment strategies fail.