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Junichi Haraguchi and Toshihiro Matsumura

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Junichi Haraguchi†
Graduate School of Economics, The University of Tokyo
and
Toshihiro Matsumura‡
Institute of Social Science, The University of Tokyo

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Abstract

We discuss government-leading welfare-improving collusion in a mixed duopoly. We formulate an infinitely repeated game in which a welfare-maximizing firm and a profit-maximizing firm coexist. The government proposes welfare-improving collusion and this is sustainable if both firms have incentives to follow it. We compare two competition structures—Cournot and Bertrand—in this long-run context. We find that Cournot competition yields greater welfare when the discount factor is sufficiently large, whereas Bertrand competition is better when the discount factor is small.

JEL classification numbers: L41, L13

Key words: repeated game, public collusion, Cournot-Bertrand welfare comparison

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†Corresponding author: Junichi Haraguchi, Graduate School of Economics, The University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. Phone: (81)-5841-4932. Fax: (81)-5841-4905. E-mail: 7299572051@mail.ecc.u-tokyo.ac.jp

‡E-mail:matusmur@iss.u-tokyo.ac.jp
1 Introduction

Collusion among profit-maximizing firms raises prices, and thus, is harmful for consumer and economic welfare. However, if some firms are concerned with social welfare in the market, welfare-improving and consumer-benefiting collusion may be formed. In this study, we analyze an infinitely repeated game under complete information in a market in which a welfare-maximizing firm competes with a profit-maximizing firm.\(^1\) The government proposes welfare-improving collusion and this is sustainable if incentive compatibility is satisfied for both firms.\(^2\) We compare two competition structures—Cournot and Bertrand—in this long-run context. We find that Cournot competition (the quantity-setting model) yields greater welfare when the discount factor is sufficiently large, whereas Bertrand competition (the price-setting model) is better when the discount factor is small.

We show that the deviation incentive from welfare-improving collusion (one-shot gain of deviating from collusion) is greater under Cournot than Bertrand competition, in contrast to profit-maximizing private collusion. For this effect, it is more difficult for the government to form welfare-improving collusion under Cournot competition, and this is harmful for welfare. However, the punishment for the deviation is stricter under Cournot competition, again in contrast to a private duopoly. This punishment effect makes the collusion more stable. Therefore, it is easier to form welfare-improving collusion under Cournot competition, and this is beneficial for welfare. The former effect dominates when the discount factor is small, while the latter effect dominates when the discount factor is large. This leads to the above result.

In the literature on mixed oligopolies, Cournot–Bertrand comparisons are popular.\(^3\)

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\(^1\)One natural interpretation of this market is that one firm is a state-owned public firm, which is adopted in the literature on mixed oligopolies. For the examples of mixed oligopolies and recent development of this field, see Ye (2016). Another interpretation is that one firm is concerned with corporate social responsibility (Ghosh and Mitra, 2014; Matsumura and Ogawa, 2014).

\(^2\)For the reality of welfare-improving collusion in a mixed oligopoly, see Wen and Sasaki (2001). The government’s intervention in collusion and competition occurs often in Japan and is discussed intensively in the context of industry policies. See Itoh et al. (1991).

\(^3\)Another popular topic in the literature is private oligopolies. It is well known that under moderate conditions, price competition is stronger, yielding lower profits and greater welfare than in the case of quantity
Ghosh and Mitra (2010), Matsumura and Ogawa (2012), and Haraguchi and Matsumura (2014) showed that Bertrand competition yields larger profit in the private firm, and Scrimitore (2014) and Haraguchi and Matsumura (2016) showed that profit ranking can be reversed. However, these works showed that Bertrand competition yields greater welfare than Cournot competition under moderate conditions, whereas our study suggests that Cournot competition can be better for social welfare. More importantly, no study has discussed this problem in the context of long-run competition (an infinitely repeated game).

While Colombo (2016) discussed an infinitely repeated game in a mixed oligopoly, he discussed profit-maximizing partial collusion among private firms and investigated how the degree of privatization of the outsider (the public firm) affects the stability of private collusion. Thus, his analysis is completely different to ours.

Wen and Sasaki (2001) is the most closely related to our study. They also discussed welfare-improving collusion and showed that the public firm’s idle capacity stabilizes the collusion. However, they did not discuss a comparison between Bertrand and Cournot competition.

2 The Model

We adopt a standard duopoly model with differentiated goods and linear demand (Dixit, 1979). The quasi-linear utility function of the representative consumer is:

\[ U(q_0, q_1, y) = \alpha(q_0 + q_1) - \frac{\beta}{2}(q_0^2 + 2q_0q_1 + q_1^2) + y, \quad (1) \]

where \( q_0 \) is the consumption of good 0 produced by the public firm, \( q_1 \) is the consumption of good 1 produced by the private firms, and \( y \) is the consumption of an outside good that

\(^4\)Nakamura (2015) investigated the bargaining between managers and owners in this context.

\(^5\)For the discussion on the stability collusion among non-profit-maximizers, see also Matsumura and Matsushima (2012).

\(^6\)For long-run analysis not based on infinitely repeated game in mixed oligopolies, see Ishibashi and Matsumura (2006) and Nishimori and Ogawa (2002, 2005).

\(^7\)This demand function is popular in the literature on mixed oligopolies. See Bárcena-Ruiz (2007), Ishida and Matsushima (2009), Matsumura and Shimizu (2010), and Haraguchi and Matsumura (2014, 2016).
is competitively provided, with a unitary price. Parameters $\alpha$ and $\beta$ are positive constants and $\gamma \in (0, 1)$ represents the degree of product differentiation: a smaller $\gamma$ indicates a larger degree of product differentiation. The inverse demand functions for goods $i = 0, 1$ with $i \neq j$ are

$$p_i = \alpha - \beta q_i - \beta \gamma q_j,$$

where $p_i$ is the price of firm $i$.

The marginal cost of production is constant for both firms. Let us denote with $c_i$ the marginal cost of firm $i$, assuming $\alpha > c_i$. Firm 0 is a state-owned public firm whose payoff is the social surplus (welfare). This is given by:

$$SW = (p_0 - c_0)q_0 + (p_1 - c_1)q_1 + \left[ \alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\gamma q_0 q_1 + q_1^2)}{2} - p_0 q_0 - p_1 q_1 \right].$$

Firm 1 is a private firm and its payoff is its own profit:

$$\pi_1 = (p_1 - c_1)q_1.$$

Firms engage in an infinitely repeated game. Let $\delta$ denote the discount factor between periods. Along the punishment path, the firms are assumed to use the grim trigger strategy of Friedman (1971).\footnote{This punishment strategy is not optimal (Abreu, 1988). We use the grim trigger strategy for simplicity and tractability. We believe that this is a very realistic punishment strategy because of its simplicity. Many works adopt this strategy when analyzing stability of agreements. See, among others, Deneckere (1983), Gibbons (1992), Maggi (1999), Gupta and Venkatu (2002), and Matsumura and Matsushima (2005).}

We consider government-leading welfare-improving collusion. The government proposes a pair of outputs ($q_0^C, q_1^C$) in the quantity competition case and a pair of prices ($p_0^C, p_1^C$) in the price competition case, where the superscript $C$ denotes collusion. Both firms accept the proposal if it is sustainable in the infinitely repeated game under the grim trigger strategy.
3 Results

3.1 Bertrand case

First, we consider a competitive situation in which firms face a one-shot game. Let \( a_i := \alpha - c_i \). The first-order conditions of firms 0 and 1 are

\[
\frac{\partial SW}{\partial p_0} = \frac{c_0 - p_0 - \gamma c_1 + \gamma p_1}{\beta(1 - \gamma^2)} = 0, \tag{5}
\]

\[
\frac{\partial \pi_1}{\partial p_1} = \frac{c_1 - 2p_1 + \alpha + \gamma p_0 - \alpha \gamma}{\beta(1 - \gamma^2)} = 0, \tag{6}
\]

respectively. The second-order conditions are satisfied. Let \( R_i(p_j) \) \((i = 0, 1, i \neq j)\) be the reaction function of the one-shot game (stage game). From the above first-order conditions, we obtain

\[
R_0(p_1) = c_0 + \gamma(p_1 - c_1), \tag{7}
\]

\[
R_1(p_0) = \frac{c_1 + \alpha + p_0 \gamma - \alpha \gamma}{2}. \tag{8}
\]

The equilibrium price, resulting profit of firm 1, and welfare are

\[
p_0^N = \frac{\alpha \gamma - \alpha \gamma^2 + 2c_0 - c_1 \gamma}{2 - \gamma^2}, \tag{9}
\]

\[
p_1^N = \frac{\alpha - \alpha \gamma + c_1 + c_0 \gamma - c_1 \gamma^2}{2 - \gamma^2}, \tag{10}
\]

\[
\pi_1(p_0^N, p_1^N) = \frac{(a_1 - \gamma a_0)^2}{\beta(1 - \gamma^2)(2 - \gamma^2)^2}, \tag{11}
\]

\[
SW(p_0^N, p_1^N) = \frac{(2 \gamma^4 - 5 \gamma^2 + 4)a_0^2 + (\gamma^4 - 3 \gamma^2 + 3) - 2\gamma(\gamma^4 - 3 \gamma^2 + 3)a_0 a_1}{2\beta(1 - \gamma^2)(2 - \gamma^2)^2}, \tag{12}
\]

where the superscript \( N \) denotes one-shot Nash equilibrium.

Next, we consider collusion in the infinitely repeated game. Both firms accept the government proposal \((p_0^C, p_1^C)\) if the following two inequalities are satisfied.

\[
\frac{SW(p_0^C, p_1^C)}{1 - \delta} \geq SW(R_0(p_1^C), p_1^C) + \frac{\delta SW(p_0^N, q_1^N)}{1 - \delta}, \tag{13}
\]

\[
\frac{\pi_1(p_0^C, p_1^C)}{1 - \delta} \geq \pi_1(p_0^C, R_1(p_0^C)) + \frac{\delta \pi_1(p_0^N, p_1^N)}{1 - \delta}. \tag{14}
\]
Sustainable pairs of prices must not yield smaller welfare than that of the one-shot Nash equilibrium because otherwise, the public firm never accepts them. Because the price of the private firm at one-shot Nash equilibrium is too high for social welfare and that of the public firm is optimal given $p^C_1$, $p^C_1 \leq p^N_1$ must hold.

Sustainable pairs of prices must not yield smaller profit in the private firm than that of the one-shot Nash equilibrium because otherwise, the private firm never accepts them. Given $p_0^C$ ($< p^N_1$) yields smaller profit in firm 1 than that of the one-shot Nash equilibrium. Thus, to compensate the private firm’s profit, $p_0^C > p_0^N$ must hold when $p_1^C < p_1^N$. These lead to the following lemma (see Figure 1 for Lemma 1-ii).

**Lemma 1**

(i) $(p_0^C, p_1^C)$ is sustainable only if $p_0^C > p_0^N$ and $p_1^C < p_1^N$ or $(p_0^N, p_1^N) = (p_0^N, p_1^N)$. (ii) If $p_0^C > p_0^N$ and $p_1^C < p_1^N$, $p_0^C > R_0(p_1^C)$ and $p_1^C < R_1(p_0^C)$.

Lemma 1(i) presents a necessary (but not sufficient) condition for sustainable prices. Lemma 1(ii) states that firm 0 (res. firm 1) prefers a lower (res. higher) price than the collusive price given the rival’s price.
3.2 Cournot case

First, we consider a competitive situation in which firms face a one-shot game. The first-order conditions of firms 0 and 1 are

\[
\frac{\partial SW}{\partial q_0} = a_0 - \beta q_0 - \beta \gamma q_1 = 0, \tag{15}
\]

\[
\frac{\partial \pi_1}{\partial q_1} = a_1 - 2\beta q_1 - \beta \gamma q_0 = 0, \tag{16}
\]

respectively. The second-order conditions are satisfied. Let \( R_i(q_j) \) \( (i = 0, 1, i \neq j) \) be the reaction function of the one-shot game (stage game). From the above first-order conditions, we obtain

\[
R_0(q_1) = \frac{a_0 - \beta \gamma q_1}{\beta},
\]

\[
R_1(q_0) = \frac{a_1 - \beta \gamma q_0}{2\beta}.
\]

The equilibrium output, resulting profit of firm 1, and welfare are

\[
q_0^N = -\frac{2a_0 - \gamma a_1}{\beta(2 - \gamma^2)}, \tag{17}
\]

\[
q_1^N = \frac{a_1 - \gamma a_0}{\beta(2 - \gamma^2)}, \tag{18}
\]

\[
\pi_1(q_0^N, q_1^N) = \frac{(a_1 - \gamma a_0)^2}{\beta(2 - \gamma^2)^2}, \tag{19}
\]

\[
SW(q_0^N, q_1^N) = \frac{(4 - \gamma^2)a_0^2 + (3 - \gamma^2)a_1^2 - 2\gamma(3 - \gamma^2)a_0a_1}{2\beta(2 - \gamma^2)^2}. \tag{20}
\]

Next, we consider collusion in the infinitely repeated game. Both firms accept the government proposal \((q_0^C, q_1^C)\) if the following two inequalities are satisfied.

\[
\frac{SW(q_0^C, q_1^C)}{1 - \delta} \geq SW(R_0(q_1^C), q_1^C) + \frac{\delta SW(q_0^N, q_1^N)}{1 - \delta}, \tag{21}
\]

\[
\frac{\pi_1(q_0^C, q_1^C)}{1 - \delta} \geq \pi_1(q_0^C, R_1(q_0^C)) + \frac{\delta \pi_1(q_0^N, q_1^N)}{1 - \delta}. \tag{22}
\]

Similar discussions as for Lemma 1 lead to the following lemma.

**Lemma 2** \((q_0^C, q_1^C)\) is sustainable only if \(q_0^C < q_0^N\) and \(q_1^C > q_1^N\) or \((q_0^C, q_1^C) = (q_0^N, q_1^N)\).
Lemma 2 presents a necessary but not sufficient condition for sustainable outputs. The private (public) firm increases (decreases) its output expecting that the public (private) firm decreases (increases) its output.

3.3 Comparison

Before presenting the main results, we present a well-known result in the literature.⁹

Result 1 \( \pi_1(p_0^N, p_1^N) > \pi_1(q_0^N, q_1^N) \) and \( SW(p_0^N, p_1^N) > SW(q_0^N, q_1^N) \).

In contrast to a private oligopoly, Bertrand competition yields larger profit in the private firm when the rival firm is a welfare maximizer.

We now present our main results. As mentioned in Subsection 3.1, the price of the private firm is too high for social welfare, and the government wants to decrease it. Thus, the government sets \( p_1^C < p_1^N \). It sets \( p_0^C > p_0^N \) because otherwise, firm 1 never accepts the collusion.

Although we cannot solve the optimal \( p_1^C \) and \( q_1^C \) explicitly, we derive a key property of the collusion. We show that the deviation incentive from the collusion is greater under the quantity case than under the price case, in contrast to the case of profit-maximizing collusion among profit-maximizing firms.

Proposition 1 Suppose that \( p_1^C = \alpha - \beta q_1^C - \beta \gamma q_1^C \). Suppose that \( p_0^C > p_0^N \) and \( p_1^C < p_1^N \). Then \( SW(R_0(p_1^C), q_1^C) < SW(R_0(q_1^C), q_1^C) \) and \( \pi_1(p_0^C, R_1(p_0^C)) < \pi_1(q_0^C, R_1(q_0^C)) \).

Proof Let \( p_1^D := R_1(p_0^C) \), and let \( q_1^D \) be the resulting output of firm i when \((p_0, p_1) = (p_0^C, p_1^D)\). Consider the Cournot case. Suppose that firm 1 deviates from the collusion and chooses \( q_1 = q_1^D \) given \( q_0 = q_0^C \). Its profit is \( \pi_1(q_0^C, q_1^D) \). Because \( q_1^D \neq R_1(q_0^C) \), \( \pi_1(q_0^C, q_1^D) < \pi_1(q_0^C, R_1(q_0^C)) \).

From Lemma 1(ii) we obtain \( p_1^D > p_1^C \). We obtain \( q_0^D > q_0^C \) because \( q_0 \) is increasing in \( p_1 \). Because \( \pi_1(q_0, q_1) \) is decreasing in \( q_0 \), \( \pi_1(q_0^D, q_1^D) > \pi_1(q_0^D, q_1^C) = \pi_1(p_0^D, p_1^C) \). These imply that \( \pi_1(p_0^C, R_1(q_0^C)) < \pi_1(q_0^C, R_1(q_0^C)) \).

⁹See Ghosh and Mitra (2010).
A similar principle applies to the deviation incentive for firm 0. ■

We explain the intuition behind the result that the one-shot gain of the deviation is greater in the Cournot case than in the Bertrand case. If the private firm were to maximize current profit and not care about future profits, it would raise its price in the Bertrand case and reduce its output in the Cournot case. In the Bertrand case, the rival’s price is given exogenously. Thus, the deviation increases the resulting output of the rival and is harmful for the private firm. By contrast, in the Cournot case, the rival’s output is given exogenously, and thus, the abovementioned harmful effect does not exist. Therefore, the private firm obtains a larger profit from the deviation in the Cournot case.

If the public firm were to maximize current welfare and not care about future welfare, it would reduce its price in the Bertrand case and increase its output in the Cournot case. In the Bertrand case, the rival’s price is given exogenously. Thus, the deviation decreases the resulting output of the rival and is harmful for welfare. By contrast, in the Cournot case, the rival’s output is given exogenously, and thus, the abovementioned harmful effect does not exist. Therefore, the public firm has a stronger incentive to deviate in the Cournot case, too.

Proposition 1 is in sharp contrast to the result in private oligopolies, in which one-shot gain of the deviation from a joint-profit-maximizing collusion is greater in the Bertrand case than in the Cournot case (Deneckere 1983, Gibbons, 1992).

Next, we investigate welfare implications. The following results state that Bertrand competition yields greater welfare than Cournot competition does when $\delta$ is sufficiently small (Proposition 2)¹⁰, while the opposite result is obtained when $\delta$ is sufficiently large (Proposition 3).¹¹

**Proposition 2** If $\delta$ is close to 0, Bertrand competition yields greater welfare than Cournot

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¹⁰This result does not depend on the assumption of grim trigger strategy because we use only Proposition 1 to derive this result.

¹¹In the case of profit-maximizing collusion among private firms, both types of competition yield the same economic welfare when $\delta$ is sufficiently large because both yield the monopoly outcome.
competition.

Proof Suppose that $\delta$ is sufficiently close to 0. Suppose that $(q_0^C, q_1^C)$ is sustainable and yields greater welfare than $SW(p_0^N, p_1^N)$. Because the deviation incentive is stronger under Cournot competition (Proposition 1), $(p_0^C, p_1^C) := (\alpha - \beta q_0^C - \beta \gamma q_1^C, \alpha - \beta q_1^C - \beta \gamma q_0^C)$ must be sustainable under Bertrand competition. Thus, Cournot competition never yields greater welfare than Bertrand competition.

Suppose that $(p_0^C, p_1^C) := (\alpha - \beta q_0^C - \beta \gamma q_1^C, \alpha - \beta q_1^C - \beta \gamma q_0^C)$ is sustainable and yields the greatest welfare among the sustainable outcomes. Then, either (13) or (14) is satisfied with equality because otherwise, a slight decrease in $p_1$ improves welfare, ensuring that (13) and (14) are satisfied. Under these conditions, $(q_0^C, q_1^C)$ must not be sustainable because the deviation incentive is stronger under Cournot competition and either (21) or (22) is not satisfied. Thus, Bertrand competition can yield strictly greater welfare than Cournot.

Proposition 3 If $\delta$ is close to 1, Cournot competition yields greater welfare than Bertrand competition.

Proof Suppose that $\delta$ is sufficiently close to 1. Suppose that $(p_0^C, p_1^C) := (\alpha - \beta q_0^C - \beta \gamma q_1^C, \alpha - \beta q_1^C - \beta \gamma q_0^C)$ is sustainable and yields greater welfare than $SW(p_0^N, p_1^N)$. Because the punishment for the deviation is more severe under Cournot competition (Result 1), $(q_0^C, q_1^C)$ must be sustainable under Cournot competition. Thus, Cournot competition never yields greater welfare than Bertrand competition.

Suppose that $(q_0^C, q_1^C)$ is sustainable and yields the greatest welfare among the sustainable outcomes. Then, either (21) or (22) is satisfied with equality because otherwise, a slight increase in $q_1$ improves welfare, ensuring that (21) and (22) are satisfied. Under these conditions, $(p_0^C, p_1^C) := (\alpha - \beta q_0^C - \beta \gamma q_1^C, \alpha - \beta q_1^C - \beta \gamma q_0^C)$ must not be sustainable because the punishment for the deviation is more severe under Cournot competition and either (13) or (14) is not satisfied. Thus, Cournot competition can yield strictly greater welfare than Bertrand competition.
On one hand, the deviation incentive is stronger under Cournot (Proposition 1) and this makes the collusion less stable. Therefore, it is more difficult for the government to form welfare-improving collusion under Cournot competition and this is harmful for welfare. On the other hand, the punishment effect is stricter under Cournot competition and this makes the collusion more stable. Therefore, it is easier for the government to form welfare-improving collusion under Cournot competition and this is beneficial for welfare. The former effect dominates when δ is small, while the latter effect dominates when δ is large. This leads to Propositions 2 and 3.

4 Concluding Remarks

In this study, we discuss welfare-improving collusion in mixed duopolies. We find that the deviation incentive is stronger under Cournot competition than under Bertrand competition. This leads the government to form welfare-improving collusion more easily under Bertrand competition, and thus, Bertrand competition can yield greater welfare. However, in a mixed duopoly, competition is more severe, and thus, the punishment for deviation is stricter under Cournot competition. This leads the government to form collusion more easily under Cournot competition, and thus, Cournot competition can yield greater welfare. The latter effect outweighs the former effect when the discount factor is large, and thus, Cournot competition is better for social welfare when firms are sufficiently patient.

In this study, we assume that a private firm is domestic. In the literature on mixed oligopolies, ownership of the private firm often matters. Our results, however, hold when the private firm is foreign. In this sense, our results are robust.

Our results may be dependent on the assumption of duopoly. As Haraguchi and Matsumura (2016) showed, Bertrand competition yields larger profit than Cournot competition as long as the number of private firms is equal to or smaller than four. However, they showed that Bertrand competition may yield smaller profit than Cournot competition if the num-

ber of private firms is equal to or larger than five, and always yields larger profit when the number of private firms is sufficiently large. Thus, if the number of private firms is large, the punishment effect becomes stricter under Bertrand competition for each private firm, whereas it remains weaker for the public firm, and therefore, the result becomes ambiguous. Moreover, if the number of private firms is sufficiently large, on one hand, it is more difficult to form collusion under both Bertrand and Cournot cases, and on the other hand, the welfare gain of collusion is small because competition yields an outcome close to the first-best outcome. Thus, in such a case, it might not be natural to discuss such welfare-improving collusion.\textsuperscript{13}

\textsuperscript{13}By contrast, in profit-maximizing collusion, the profit gain of collusion is greater when the number of firms is larger because more severe competition yields smaller profits.
References


