Dynamic collective bargaining. Frictional effects under open-shop industrial relations

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by

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A dynamic Stackelberg game analyzes collective bargaining between a trade union (leader) and a firm (follower) in a monopoly union model. Frictional effects (FE) for the firm encompass symmetric adjustment costs linked to the number of hired and fired workers, plus a wage-dependent term (assuming wage-dependent hiring costs and wage discrimination against newcomers). The union faces marginally increasing costs in firings and marginally decreasing benefits from hirings. The two-part FE for the firm, the FE for the union, or both jointly considered differently affect employment and wages. Interestingly, standard adjustment costs increase hirings, even while the union reduces wages. (JEL: J5, J23, C73, C61)

Keywords: Dynamic labor demand, collective wage bargaining, monopoly union model, adjustment costs, Stackelberg differential game.

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1 Introduction

The collective bargaining between a representative firm and a trade union[1] is analyzed in this paper from a dynamic perspective. The relationship between these two agents is defined in a dynamic labor market, assuming a fixed workforce but a time-varying demand for labor. In particular, the focus is on the progressive expansion or contraction of the set of workers employed within the firm, and the evolution of the wage they earn. The dynamic aspects of this interaction are based on the existence of adjustment costs or, more precisely, frictional effects (FE).[2] That is, on how the firm's profits and the union's welfare are increased or reduced by the hiring of new employees or the firing of existing ones.[3] These FE crucially influence the speed of convergence towards the medium-run equilibrium, and even determine the levels reached by the main variables at the medium-run equilibrium.

We introduce two mechanisms which help dynamize the collective bargaining process and which, to the best of our knowledge, have not been previously addressed by the literature. Firstly, on the firm's side a frictional effect depending on the wage rate is added to the conventional adjustment costs which are exclusively dependent on the number of employees being hired or laid off. These two-part FE will jointly drive the dynamic behavior of the firm. Secondly, we also assume that the union's behavior is also affected by frictional effects. It is worth mentioning that this second mechanism is different from the union membership dynamics, often treated in the literature (see, for example the pioneer work by Kidd and Oswald 1987 and Jones 1987 and the more recent Dittrich and Schirwitz 2011a), because we do not take into account a closed-shop but an open-shop system of industrial relations. Thus, we assume that workers benefit from the union's achievements regardless of whether they are members or not. Put differently, the union represents members and non-members identically. These kinds of industrial/labor relations are typical in some countries in Continental Europe[4].

The model developed in this paper could be included within the insider-outsider

[1]As Booth (2014) points out, although labor economists' interest in trade unions has declined in recent years, trade unions are still important agents in many OECD countries. One of the reasons the author gives for this decline is the negligible role of trade unions in the US. Notwithstanding, our theoretical framework is thought to model some features of European labor markets.

[2]We explicitly avoid the term "adjustment costs" because, as will be shown later, the frictional effects are really a "cost" for firing and for specific levels of hiring, but can be considered "benefits" for specific levels of hiring.

[3]Our concept of FE encompasses some search frictions (e.g. those associated to the recruitment process for the firm) but is not restricted to them. For instance, firing costs are considered here as frictional effects, although they are not search frictions. A recent example of a collective bargaining model mainly focused on search frictions is Dobbelaere and Luttens (2016).

[4]For instance, Cahuc *et al.* (2014, p. 404) point out: "*(...) in France and Spain collective agreements do not have the right to discriminate between union members and non-unionized workers*".

literature from a broad perspective. The four central assumptions on which an insider-outsider model rests, (Lindbeck and Snower, 2001) are fulfilled: "*(1) Firms face labor turnover costs that they cannot entirely pass on to their employees. (2) Insiders have some market power. (3) If entrants remain with a firm long enough, they become associated with the same labor turnover costs as the insiders, and have an opportunity to renegotiate their wage. (4) Employment decisions are made unilaterally by the firms.*" We consider our theoretical approach as compatible, even complementary, with an insider-outsider framework. Nonetheless, it has not been conceived to satisfy all the requirements of such a view. In the end, the bargaining process described in this work seeks to describe the functioning of several European labor markets and for this reason it shares the main characteristics of insider-outsider models.

To sum up, in this paper we model the bargaining process between a trade union and a representative firm as a dynamic macroeconomic labor market. The dynamic analysis accounts for the frictional effects that hiring and firing decisions have on the firm and the trade union. The steady state equilibrium in the model can be thought of as a medium-run equilibrium,[5] in the sense described in macroeconomics textbooks (e.g. Blanchard 2005). The short-run macroeconomic equilibrium, which would be driven by aggregate demand forces, is not explicitly modeled. However, some short-run "information" is still taken into consideration through the initial conditions in our state variable. The long-run equilibrium would depend primarily on capital accumulation which is out of the scope of the paper.

The rest of the paper is organized as follows. The second section discusses and introduces, in a non-formal way, the frictional effects that we claim play an important role in the bargaining process; and locates them within the existing literature on collective bargaining in the labor market. In the third section, the formal model is built. Section 4 analyzes the the baseline scenario with no FE, and two intermediate scenarios with FE only, either for the union or for the firm. Section 5 solves the general Stackelberg game with frictional effect for both players, and develops a sensitivity analysis of some parameters which describe the FE. Conclusions are presented in the last section.

2 *Frictional effects for the firms and the union*

2.1 *Frictional effects for the firm*

The literature on dynamic labor demand has frequently considered symmetric adjustment costs in hiring and firing.[6] These adjustment costs are usually represented by a strictly convex function, or sometimes simply by a piecewise linear function in hiring and firing (see, for example, Nickell, 1987). This formulation does not distinguish between hiring and firing costs, although the empirical literature has frequently stressed

[5]We analyze the steady-state equilibria, although considering the capital stock as an exogenous constant. Therefore, we will talk about medium-run equilibria, given that only employment adjusts, and not the capital stock.

[6]Despite the fact that one of the first representations was not symmetric (Holt et al. 1960), this has been a usual assumption ever since Eisner and Strotz (1963).

that these costs differ to a certain degree. It is generally admitted that hiring costs are higher than firing costs in countries like the United States (see Hamermesh, 1996, for a review of some studies), whereas the opposite is true for continental Europe (e.g. Abowd and Kramarz, 2003, and Goux *et al.*, 2001, for French data).

For this reason, some authors departed from the assumption of symmetric quadratic adjustment costs in the late 80s and early 90s, introducing asymmetries (an excellent survey on this literature is Hamermesh and Pfann, 1996). Within the category of convex costs, we can identify authors who still consider continuous differentiability (e.g. Pfann and Palm, 1993) and those who consider a discontinuity in their adjustment cost function (e.g. Chang and Stefanou, 1988, and Jaramillo *et al.*, 1993). Alternatively, another convenient specification for the adjustment costs is a continuous piecewise linear function, although not continuously differentiable (see the seminal works by Bentolila and Bertola 1990, Bertola 1990, Bentolila and Saint-Paul 1994, and Bertola and Rogerson 1997), which treats hiring and firing costs differently. Finally, some authors have addressed the existence of lump-sum costs, such as those that arise in the search for certain categories of personnel (hiring); or the administrative costs of collective dismissals in many European countries (firing). These costs are independent, to a certain extent, of the number of employees hired or laid off. These fixed costs explain why, under certain circumstances, firms have an interest in hiring and firing in groups. Two representative works of this strand of literature are Hamermesh (1995) and Abowd and Kramarz (2003).

We align ourselves with those who consider asymmetric and continuously differentiable FE for the firm. Specifically, we focus on the case in which firing costs are more important than hiring costs. This is typically the case in continental Europe and particularly in Southern Europe.[7] Although hiring is associated with costs like recruiting and training, we claim that it also represents an opportunity for wage savings, if we further assume wage discrimination is in favor of incumbent employees and against newcomers. Incumbent employees are insiders, while newly-hired workers, who were outsiders in the instant immediately before (whom the union is less concerned about) receive a lower wage while they become insiders. The existence of this wage savings for the firm can be additionally explained based on the existence of a payroll tax subsidy to newly-hired employees (which is a common economic policy in the European countries. Thus, the net effect of hiring on the firm's accounts would depend on the relative size of these two opposing FE. To take in this asymmetry, the FE of hiring and firing are defined

[7]Cahuc *et al.* (2014, p.120) stated that: "*(...) in countries where strong legal measures are in place to enhance job security, the costs of separation outstrip recruitment costs".* A well documented example of this empirical regularity is France (Abowd and Kramarz, 2003, and Goux *et al.*, 2001). Nonetheless, it is also well-known that not only France but the rest of Southern European countries score high in the employment protection legislation (EPL) index. In order to illustrate this point, Table 10.1 in Boeri and van Ours (2013, p. 278) shows how France (3.0), Greece (3.0), Portugal (3.2), Italy (2.6), and Spain (3.1) exhibit extremely high levels in the overall EPL index. In contrast, English-speaking countries like Australia (1.4), Canada (1.0), Ireland (1.4), New Zealand (1.2), the United Kingdom (1.1), and the United States (0.9) tend to show the lowest levels for the overall EPL index.

in two parts. A first component, exclusively dependent on the number of hired or fired employees, collects the standard assumption of symmetric convex costs. A second part is explicitly dependent on the wage rate. We assume a positive relationship between firing costs and wages, because severance payments typically amount to a number of weeks of wage for every year that the employee has remained within the company. On the other hand, the negative effect of wages on hiring costs reflects the stylized fact in many countries that newcomers are initially paid only a fraction of the wage obtained by current employees and, in consequence, the greater the wage rate, the greater the wage savings from hiring. Thus, taking into account this wage-dependent second component, a higher wage rate increases firing costs, but conversely reduces hiring costs, rendering FE asymmetric. In fact, for small recruitment numbers, hiring represents a net benefit for the firm.

Finally, we hypothesize that countries with higher firing costs (in weeks of wages) also show a wider salary gap. This assumption is made for tractability, although it is not far from the reality shown by empirical data. Dias da Silva and Turrini (2015) point out that there is a direct relationship between the level of firing costs (measured through various dimensions of the EPL) and the permanent-temporary wage gap across countries. The theoretical reason for that relies on the different bargaining power of workers with an open-ended contract versus those with a fixed-term contract. In other words, a stronger employment protection for the permanent workers allows them, at the same time, to obtain a higher wage differential with respect to their temporary counterparts. These authors also show some empirical evidence supporting the previous statement. If we consider that the newcomers are typically hired with a fixed-term contract, the previous argument and the empirical evidence attached to it give credit to our way of modeling this stylized fact.

2.2 Frictional effects for the union

According to previous research, the dynamic modelisation of collective bargaining can be undertaken through three channels. One obvious way is through capital accumulation as, for example, in the seminal work by van der Ploeg (1987) and in Palokangas (1992, 1997). These models are especially adequate for analyzing long-run questions. A more recent example of this kind of literature on the long-run effects of collective bargaining is Chang *et al.* (2007), who put the emphasis on a political trade union and the conflict between the preferences of the leaders and the members. Nevertheless, as pointed out, long-run equilibria are not the aim of this paper.

A second option to dynamize the collective bargaining process is to focus on the labor input instead of the capital input. The cost (or the benefits if there exist) associated with employment adjustment will prevent immediate or instantaneous adjustment, inducing a dynamic behavior to maximizing firms. Thus, introducing frictional effects like those described in the previous section into the bargaining process will also yield dynamic effects. This approach is undertaken by Lockwood and Manning (1989), who analyze, in a discrete setting, a dynamic model that takes into account quadratic adjustment costs for the firm. They consider the right-to-manage theoretical framework in which the wage is jointly settled by the firm and the trade union (according to their bargaining power), and then the firm unilaterally determines employment. They conclude that the speed of adjustment is increased with respect to the competitive equilibrium case. An alternative approach, in Modesto and Tomas (2001), is based on the monopoly union model in which the firm chooses the optimal employment level, and the union determines the optimal wage rate, knowing the labor demand fixed by the firm. The non-cooperative equilibrium is confronted with the equilibrium under the cooperative or dynamically efficient bargaining model. The monopoly union model is also analyzed in Palokangas (1992, 1997), who considers a Stackelberg differential game with the union acting as the leader.[8]

A third way of introducing a dynamic perspective into the collective bargaining process is by assuming that FE also directly affect the union's welfare. The dynamic behavior of the trade union in collective bargaining has been analyzed by the literature on union membership. Two influential papers in this literature, Kidd and Oswald (1987) and Jones (1987), consider endogenous membership and analyze a monopolistic trade union that cares about current and future members. Hiring decisions of the firm have an effect on the union, since we are assuming that the union is concerned about all employees (or alternatively, under the assumption that new recruitment immediately joins the union). Correspondingly, the layoff of current employees induces welfare losses to the union.[9]

The analysis of union membership is relevant for a closed-shop union system of industrial relations. That could be the case for some countries, particularly in the Anglo-Saxon System,[10] but not all. We deviate from the the literature on union membership and consider an open-shop system of industrial relations in which the union negotiates on behalf of all employees (members or not members). The union focuses mainly on incumbent employees or insider privileged employees, while outsider workers will face less favorable employment conditions in the case of being hired, in particular, the wage discrimination mentioned already.

Frictional effects for the trade union are of different nature when new workers are hired or current employees are fired. We assume that firings reduce the union's welfare at an increasing rate. This assumption seems to be uncontroversial just by looking at the way unions operate in the real world, posing a strong opposition if the firm fires a large number of workers in one go, but much less so when firings occur gradually and

[8]Both, the right-to-manage model and the monopoly union model are studied in Koba (2003), who analyzes the effect of deregulation on employment and wages.

[9]Some interesting works in this strand of research are Chang and Lai (1997) and, more recently, Dittrich and Schirwitz (2011a), Dittrich and Schirwitz (2011b), or Kazanas and Miaouli (2014).

[10]The union membership models usually capture some stylized facts of the industrial relations systems of English-speaking countries. As already pointed out, in France and Spain practically all workers are covered under a collective bargaining agreement, independently of whether they are union members or not. Moreover, in Nordic countries (and more generally in those countries under the Ghent system) the incentives to become a union member are rather different from those described in the canonical union membership model.

are restricted to few employees. Correspondingly, the union welcomes the recruitment of new employees. Nonetheless, taking into account organizational aspects, the arrival of new employees might give rise to a growing number of problems in the organization of the union (i.e. an excessive inflow of workers could cause managerial problems in the union). In consequence, we will assume that the union values the recruitment of new employees positively, but at a decreasing rate.[11]

From this specification, one can conclude that the trade union is mainly concerned about current employed workers (insiders), whose dismissal would reduce the union's welfare. Moreover, given the FE, the union is also partially worried about current unemployed workers (outsiders) as they would be welcomed if hired.

3 The two actors in the collective bargaining process

Collective bargaining in a monopoly union model involves two agents: a firm, which acts as the follower, and a monopolistic trade union, which takes the role of the leader. The former has to determine the recruitment of new employees (or the layoff of current employees) at each instant. The latter determines the wage rate although, given its monopolistic role, it determines wages optimally, knowing the firm's demand for new employees.

Employment, $L(t)$, is not considered as a decision variable but as a stock variable. Therefore, the level of employment at time t, can be defined as the initial stock of workers within the firm, L_0, plus the accumulated flow of employees recruited, and not fired, and minus the accumulated flow of employees who voluntarily quit the firm from the start-up to the current time:

$$L(t) = L_0 + \int_0^t h(\tau)d\tau - \int_0^t \delta L(\tau)d\tau, \quad L_0 \geq 0,$$

with $\delta > 0$ the rate of voluntary quit, assumed constant for tractability. The flow variable, $h(t)$, can be positive, implying new recruitment, or negative, representing firings. Meanwhile, employment should not take a negative value, $L(t) \geq 0$. The evolution of employment can be alternatively defined by the differential equation:

$$\dot{L}(t) = h(t) - \delta L(t), \quad L(0) = L_0 \geq 0. \tag{1}$$

According to this specification, the firm optimally adjusts the level of employment, which is not the consequence of the attrition between actual and desired employment, as, for example, in Guerrazzi (2011).[12]

[11] A simple example may help to clarify these asymmetric FE. We claim that firing some current employees and replacing them with the same number of newcomers does not leave the union's welfare unaltered. The improvement in welfare associated with the arrival of new employees is more than offset by the strong decrease in the union's welfare from their dismissal.

[12] This work includes additional dynamics governing the marginal productivity of labor.

3.1 The employer

The main interest of the paper is to analyze to what extent the adjustment effects or frictions in the labor market can affect the employment level and the wage rate at the equilibrium, and the speed of convergence towards this equilibrium. The installed capital stock is taken as a given constant, and our analysis corresponds to a medium-run equilibrium. Since we assume a fixed amount of capital, production can be made dependent on only the total amount of labor $Y(L(t))$, characterized by a labor productivity which decreases in the employment level. For tractability, the specification considered here is a linear quadratic function in the level of employment, $L(t)$:

$$Y(L(t)) = aL(t) - \frac{L^2(t)}{2}, \tag{2}$$

with $a > 0$ the highest possible labor productivity (attained for $L(t) = 0$). This parameter collects, among other factors, the installed capital stock and the level of technology, both assumed to be time-invariant.

Under the unit price hypothesis the profits of the firm are defined as total output minus the wage bill paid to employees, $Y(L(t)) - w(t)L(t)$, with $w(t)$ the identical wage rate for all employees regardless of their seniority. Firm's finances are also affected by the frictions from the hiring or firing of employees. Hence, the firm cannot bring forth the desired employment level instantaneously and at no cost, but must develop it gradually. When recruiting new employees, the adjustment costs traditionally encompass the cost in the act of hiring (advertising and interviewing process) plus the expenses of incorporating new employees into the productive activity (training). The literature often assumes convex hiring costs.[13] Likewise, firing costs are also usually assumed to be increasing at the margin. The literature usually assumes strictly convex adjustment costs, and also symmetric hiring and firing cost functions, that is, $AC(h)$, with $AC'(h) > 0$ if $h > 0$, $AC'(h) < 0$ if $h < 0$, $AC''(h) \geq 0$ and $AC(h) = AC(-h)$. We consider that the adjustment costs are not only dependent on the number of employees being hired/fired, but also on the wage they receive. For fired employees, labor legislation in many countries includes a severance package, defined as a percentage (usually dependent on the number of years in work) of the wage earned by the incumbent worker. To the best of our knowledge, the relationship between FE and wages has not been addressed previously by the literature on dynamic collective bargaining. Thus, we define a twofold FE for the firm when hiring, composed of: the standard convex adjustment costs, exclusively dependent on the number of fired workers, plus a wage-dependent term, defined as a percentage, β,of the current wage times the number of employees being fired:

$$C_{\text{F}}(\text{firing}(t)) = AC(\text{firing}(t)) + \beta \cdot w(t) \cdot \text{firing}(t).$$

To analyze how the wage rate may affect hiring costs, and following the terminology in the insider-outsider theory of employment, we distinguish between incumbent employees

[13]This standard assumption is not exempt from criticism. Nickell (1987) states that for low levels of hiring it is hard to think of good reasons why hiring costs should be increasing at the margin.

and newly-hired employees, the latter with less favorable working conditions, reflected in a lower wage. This wage discrimination is exclusively based on institutional grounds. We do not include in the model differences in labor productivity associated with the level of expertise within the firm. Nor do we base the salary gap on the existence of hiring costs which disincentive labor turnover. We just assume that the union is less concerned about newly-hired employees, who were outsiders immediately before, and hence accepts that they receive a lower salary, $w_n < w$. Thus, the wage bill for the firm is directly dependent on the number of employees hired at each instant of time: $w(t)L(t) - (w(t) - w_n(t)) \cdot \text{hiring}(t)$. In consequence, hiring implies a direct cost for the firm, represented by the standard convex adjustment costs, but also wage savings, due to the wage discrimination against newcomers. So, the frictional effect of hiring for the firm can be defined as

$$C_{\text{F}}(\text{hiring}(t)) = AC(\text{hiring}(t)) - (w(t) - w_n(t)) \cdot \text{hiring}(t).$$

For simplicity, we assume that the wage gap remains constant over time in relative terms, i.e. $(w(t) - w_n(t))/w(t)$ is constant. Moreover, we also make the assumption that this constant is equal to β.[14] Thus, β is given a double interpretation, implying that a higher wage raises the marginal cost of firing by the same amount as it decreases the marginal cost of hiring (due to higher wage savings). This assumption has been made for tractability; however, we argue that it is not excessively far from the reality. As suggested by Dias da Silva and Turrini (2015), there seems to be an empirical correlation between firing costs and wage inequality. Under this assumption, it is possible to merge the two basic ideas of firing costs marginally increasing with wages and hiring costs marginally decreasing with wages, in a unique function to describe the FE. When the firm hires employees[15] ($h > 0$), the wage bill can be split in two parts, distinguishing between senior employees, $w(L - h)$, and newcomers, $w_n h$. And from the definition of β as the wage gap, the wage bill can be written as $wL - \beta wh$. The last term represents the wage savings (a profit) from hiring newcomers and must be subtracted from the quadratic standard adjustment costs $AC(h)$ to constitute what we denote as the frictional effect. When the firm fires current workers ($h < 0$), since there are no newcomers, the wage bill is simply wL. However, the frictional effect of firing presents two components: the standard quadratic costs, $AC(h)$, plus the term which relates firing costs and wages rates, $\beta w(-h)$. Then, assuming a quadratic specification for the standard convex costs, $AC(h)$, the FE of hiring and firing can be jointly represented by

$$C_{\text{F}}(h, w) = c\frac{h^2}{2} - \beta wh, \quad c, \beta > 0. \tag{3}$$

Notice, however, that this function does not imply symmetric hiring and firing costs, unless $\beta = 0$. As Figure 1 shows, when the firm fires workers, the standard adjustment costs are increased by the term which collects the effect of wages on firing costs. By

[14]Since it has been assumed that the wage paid to newcomers is strictly lower than the wage paid to current employees, then it must hold that $\beta \in (0, 1)$.

[15]Here and henceforth we omit the time argument when no confusion arises.

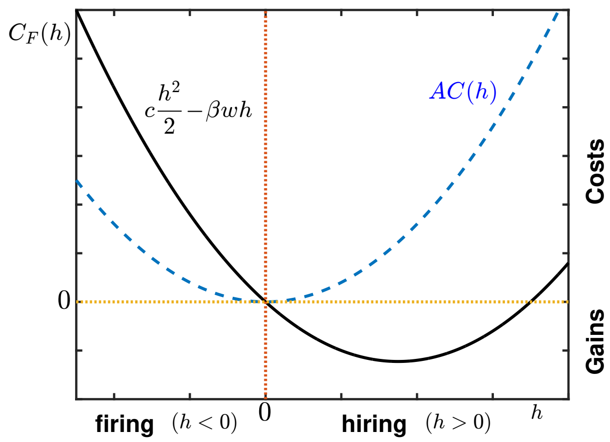

Figure 1
Frictional effects for the firm

contrast, when the firm hires new employees adjustment costs are reduced because of the wage savings. In fact, a firm which hires workers enjoys net gains if the number of new recruitments is not too large. For hiring rates below the upper bound, $2\beta w/c$, wage savings from newcomers more than offset hiring costs. It is worth noticing that at the steady-state equilibrium the firm hires exactly the fraction of employees that quits voluntarily. Because at the medium-run the firm hires new employees, from the double interpretation of β, what matters at the equilibrium is its role as a measure of the wage discrimination or wage gap between incumbent employees and newcomers. The interpretation of β as the part of the firing costs linked with wages may be relevant for short-term adjustment, if the initial situation is one of an excessively large level of employment and the firm has to fire some workers within a first period. But this interpretation is only valid temporally because, even in that situation, firings will stop at some point and, as the medium-run level of employment is approached, the firm starts hiring to replace the voluntary quit.

The profits of the firm are defined by the income from production, $Y(L)$, minus the wage bill. Moreover, it is also assumed that the FE associated with hiring and firing may increase or reduce income. Thus, a firm's profits can be written as

$$W_{\mathrm{F}}(h, w, L) = Y(L) - wL - C_{\mathrm{F}}(h, w),$$

with the production function given in (2) and the FE of hiring and firing in (3).

3.2 The trade union

This subsection analyzes the existence of FE of hiring and firing on the union's welfare within a monopoly union model. We depart from the standard dynamic model of union membership as presented in Jones (1987) and Kidd and Oswald (1987). By considering an open-shop system of industrial relations, the union represents all employees and not merely the union members. In particular, it is concerned about the insider incumbent employees already working in the firm. Taking into account an expected utility approach, the instantaneous union's utility reads: $W_{\text{U}}(w,L) = \left(\frac{L}{N}\right)u(w) + \left(1 - \frac{L}{N}\right)u(B)$, or by normalizing total population, N, to one,[16] $W_{\text{U}}(w,L) = Lu(w) + (1-L)\,u(B)$, with B the unemployment benefit, and $u(\cdot)$ a concave utility function.

Within this framework we introduce two additional assumptions, so giving entrance to the FE of hiring and firing on the trade union. The union is concerned about the excess utility, $u(w) - u(B)$, that the L current employees enjoy, associated with a wage, w, above the unemployment benefit, B, as well as the utility of unemployed workers. When a specific worker is fired, assuming the wage of all remaining employees remains unchanged, the union acknowledges a decrease in utility associated with the decrease in this worker's income (the opposite is true when an unemployed worker is hired). Additionally, we support the hypothesis that the union resents firings, more than the mere utility losses from a lower income. These welfare losses for the union, henceforth denoted firing costs (likewise as for the firm) can be based, for example, on the discontent among employees whose fellow workers are being fired, on the insecurity about their future within the firm, or the rise in their workload. We assume that the cost of firing is increasing at the margin. Correspondingly, the union welcomes the incorporation of new recruitment more than these workers' expected utility gains. In contrast to the firing costs, the marginal gain from hiring new employees decays with the number of newcomers. One possible explanation for this assumption is the organizational problems that the arrival of new employees may bring to either the firm or the union.

The hypotheses of increasing marginal costs from firing and decreasing marginal gains from hiring can both be encompassed by a single function describing the FE of hiring and firing for the union: $C_{\text{U}}(h) = dh(h-H)/2$, with $d > 0$ a measure of the size or relative importance of the FE with respect to the utilitarian part of the union's welfare. Parameter $H > 0$ is the recruitment level at which hiring stops being attractive for the union. More interestingly, $H/2$ is the recruitment level at which an additional employee starts not to be welcome by the union (i.e. represents a marginal disutility). The functions describing the FE and their marginal variations with h are depicted in Figure 2.

Finally, assuming a one-to-one utility function, the objective function for the union with FE would read

$$W_{\text{U}}(h,w,L) = Lw + (1-L)\,B - C_{\text{U}}(h).$$

4 Collective bargaining and frictional effects

To better understand the role of the adjustment costs linked to firing, and the adjustment gains that the firm and the union attach to hiring, this section characterizes the labor market equilibrium in successive stages. In the first stage we obtain the wage and the

[16] With a constant population equal to one, the expected utility approach is equivalent to considering a utilitarian union see, for example, Booth (1995).

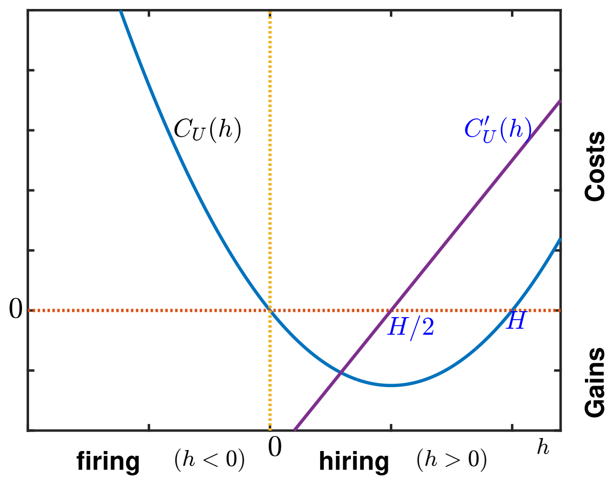

Figure 2
Frictional effects for the union

employment level when neither firm's profits nor union's welfare is influenced by hiring or firing decisions. This would be the equilibrium in a simple static monopoly union model. In a second step, we introduce the frictional effects for only one of the players, the firm or the union. In either case the model becomes dynamic and employment progressively adapts to its steady-state value. By comparing the steady-state equilibrium for these models against the equilibrium with no FE, it is possible to follow the trail of how the wage and the level of employment are affected by the frictions faced by either the firm or the union. In the last stage, the next subsection analyzes the differential game when both firm and union are affected by the frictions from hiring and firing.

4.1 A game with no FE

With no adjustment costs, the firm can instantaneously adapt the current level of employment to its desired level. It would fix the optimal level of employment at which wages and labor productivity equate, $Y'(L) = w$, which defines the static labor demand, $L = a - w$. The monopolistic union, knowing this labor demand function, would settle the wage which maximizes $W_{\text{U}}(0, w, a - w) = (a - w)(w - B) + B$. The wage and the employment level at this static equilibrium are[17]

$$w^s = \frac{a + B}{2}, \quad L^s = \frac{a - B}{2} \tag{4}$$

The wage, w^s, can be understood as a convex combination (the mean value) between the maximum labor productivity, a, and the unemployment benefit, B. That is, between the maximum and the minimum possible competitive wages. The employment level at the equilibrium, L^s, depends on the gap between these two wages. And likewise, it

[17]Superscript s refers to the static scenario or equivalently, the scenario without FE.

is a convex combination between the zero demand at the maximum wage, a, and the maximum demand, $a-B$, that would occur at the lower possible wage, or unemployment benefit, B. Since we have normalized total population to one, the equilibrium is feasible under condition $a \in [B, 2+B]$, assumed henceforth. In this static setting, the outcome is that a higher labor productivity, a, would increase wages and employment, while a more generous unemployment benefit would raise wages and reduce employment.

4.2 A game with FE only for the union

Facing no frictional effects, the firm would determine the level of employment by again equating the marginal productivity of labor to wages, hence setting the labor demand function $L = a - w$. By contrast, in this subsection firing costs and hiring gains come into the union's welfare. Given this demand function, it is clear for a monopoly union that changes in wages will be transformed into opposite changes in employment levels: $\dot{w} = -\dot{L}$. This equation, together with the dynamics of the employment level in (1), allows us to define the dynamic problem for the monopoly union considering the wage as a state variable and replacing the level of employment by the known labor demand function:

$$\begin{aligned} &\max_h \int_0^\infty \left[w(a-w) + (1-a+w)B - dh\frac{h-H}{2} \right] e^{-\rho t} dt, \\ &\text{s.t.: } \dot{w} = \delta a - h - \delta w, \quad w(0) = a - L_0. \end{aligned}$$

It seems more natural to define a trade union that, rather than taking hiring or firing decisions, chooses the change in the wage rate, u, (positive or negative) at any time, t. Thus, taking into account that $h = \delta(a-w) - \dot{w}$, the previous problem can be equivalently written as

$$\begin{aligned} &\max_u \int_0^\infty \left[w(a-w) + (1-a+w)B - d[\delta(a-w) - u]\frac{\delta(a-w) - u - H}{2} \right] e^{-\rho t} dt, \\ &\text{s.t.: } \dot{w} = u. \end{aligned}$$

This optimal control problem has a unique solution which converges towards the steady-state solution:[18]

$$(5) \qquad \bar{w}^{\text{AU}} = \frac{a + B + d(\rho+\delta)\left[a\delta - \frac{H}{2}\right]}{2 + d\delta(\rho+\delta)}, \quad \bar{L}^{\text{AU}} = \frac{a - B + d(\rho+\delta)\frac{H}{2}}{2 + d\delta(\rho+\delta)}, \quad \bar{h}^{\text{AU}} = \delta \bar{L}^{\text{AU}},$$

at the speed

$$\phi^{\text{AU}} = \frac{1}{2}\left\{ \rho - \sqrt{(\rho+2\delta)^2 + \frac{8}{d}} \right\} < 0.$$

By comparing this steady-state with the equilibrium in the scenario with no frictions, we observe that $\bar{L}^{\text{AU}} > L^s$, and equivalently $\bar{w}^{\text{AU}} < w^s$, if and only if $h^s < H/2$, where

[18]Superscript *AU* refers to the scenario with adjustment or frictional effects only for the union.

$h^s = \delta L^s$ denotes the hiring required to replace the voluntary quit in the equilibrium with no FE. To understand this condition, note that according to function $C_{\mathrm{U}}(h)$, while firing increasingly damages union's welfare, hiring represents a marginal increment in union's welfare only below $H/2$. Marginal increments in hiring above this level will reduce the union's welfare. Condition $h^s < H/2$ states that the number of employees that voluntarily quit if employment is at its static level (or equilibrium level with no FE), is lower than the value, $H/2$, which minimizes adjustment costs, i.e. maximizes the union's benefits from new recruitments. In consequence, if starting at the static employment level, L^s, the firm hired workers above those required to replace the voluntary quit, the union's welfare would be enhanced. This gives the union an incentive to fix a wage below the static wage in order to induce the firm to increase hirings. The steady-state equilibrium would be characterized by a higher employment level and therefore a greater number of workers who voluntarily quit and who are replaced by the firm, causing a rise in the union's welfare. Applying the opposite reasoning, if $h^s > H/2$ additional hirings damage union's welfare and in consequence, the union would fix a higher wage seeking to reduce employment. This can be summarized as

$$\bar{L}^{\mathrm{AU}} \gtreqqless L^s \Leftrightarrow \bar{w}^{\mathrm{AU}} \lesseqqgtr w^s \Leftrightarrow h^s \equiv \delta \frac{a-B}{2} \lesseqqgtr \frac{H}{2}. \tag{6}$$

From expression (6) it becomes immediately clear that the existence of FE for the union does not necessarily lead to greater wages and unemployment rates. This would be the case only if the size of these FE as measured by $H/2$, is small enough in comparison with the voluntary quit with no FE.[19] Therefore, a monopoly union that welcomed the entrance of new employees to the firm might fix a lower wage, so inducing a higher level of employment than a union only concerned about excess utility if i) the range of newly hired employees that raises union's welfare, H, is large; ii) the salary gap between the maximum feasible wage and the unemployment benefit, $a - B$, is small; or iii) the rate of voluntary quit is low. Conversely, if these conditions are not met, wages and unemployment would be higher.

4.3 A game with FE only for the firm

In a monopoly union model, if the firm, which acts as the follower, faces frictional effects, it would fix the hiring level in order to solve the dynamic problem:

$$\max_h \int_0^\infty \left[Y(L) - wL - C_{\mathrm{F}}(h,w)\right] e^{-\rho t} dt, \tag{7}$$

$$\text{s.t.: } \dot{L} = h - \delta L, \quad L(0) = L_0 \geq 0. \tag{8}$$

Given the linear-quadratic structure of the optimization problem, a linear-quadratic value function is conjectured[20] $V_{\mathrm{F}}^{\mathrm{AF}}(L) = a_{\mathrm{F}}^{\mathrm{AF}} L^2/2 + b_{\mathrm{F}}^{\mathrm{AF}} L + c_{\mathrm{F}}^{\mathrm{AF}}$, with $a_{\mathrm{F}}^{\mathrm{AF}}$, $b_{\mathrm{F}}^{\mathrm{AF}}$ and $c_{\mathrm{F}}^{\mathrm{AF}}$

[19]That is, if the range of h compatible with marginal gains from hirings, $H/2$, is small in comparison with the gap between the maximum possible wage and the unemployment benefit scaled by the rate of voluntary quit.

[20]Superscript AF refers to the scenario with FE only for the firm.

the unknowns to be determined. The firm would fix recruitment up to the point when the marginal frictional effect of an additional employee being hired or fired equates the marginal value of this additional employee for the firm, $(V_{\mathrm{F}}^{\mathrm{AF}})'(L)$. This optimality condition determines a hiring/firing function dependent on the wage rate and the employment level:

$$\hat{h}^{\mathrm{AF}}(w,L) = \frac{w\beta + (V_{\mathrm{F}}^{\mathrm{AF}})'(L)}{c}. \tag{9}$$

Notice that the marginal frictional effect when hiring is composed of the standard marginal adjustment costs, ch, plus the marginal benefits from the wage discount to newcomers, defined as a fraction, β, of the incumbents' wage, w. The greater the wage rate, the greater the wage savings from hiring new employees, and therefore the stronger the firm's incentive to hire, which explains the positive direct effect of wages on hirings.

The trade union acts as the leader and knows the reaction function of the firm in (9). It must determine wages in order to maximize its stream of discounted welfare, which assuming no FE would read

$$\max_{w} \int_0^\infty [wL + (1-L)B]\, e^{-\rho t} dt, \tag{10}$$

$$\text{s.t.: } \dot{L} = \hat{h}^{\mathrm{AF}}(w,L) - \delta L, \quad L(0) = L_0 \geq 0. \tag{11}$$

This linear optimization problem leads to a bang-bang solution for the union of the form

$$w^{\mathrm{AF}}(L) = \begin{cases} w_{\max} \equiv a & \text{if} \quad L + \dfrac{\beta}{c}(V_{\mathrm{U}}^{\mathrm{AF}})'(L) > 0, \\ w_{\min} \equiv B & \text{if} \quad L + \dfrac{\beta}{c}(V_{\mathrm{U}}^{\mathrm{AF}})'(L) < 0. \end{cases} \tag{12}$$

We consider here that the minimum possible wage fixed by the union is given by the unemployment benefit, B, and the maximum wage by the maximum productivity of labor, a. Because the union faces a linear problem we guess a linear value function, $V_{\mathrm{U}}^{\mathrm{AF}}(L) = b_{\mathrm{U}}^{\mathrm{AF}} L + c_{\mathrm{U}}^{\mathrm{AF}}$. Thus, the marginal value that the union assigns to employment, $(V_{\mathrm{U}}^{\mathrm{AF}})'(L) = b_{\mathrm{U}}^{\mathrm{AF}}$, is constant and independent of the actual level of employment, L.

Here we study the case in which the union fixes either the minimum wage, B, or the maximum wage, a, for the whole time period, since this allows us to obtain analytical results, which give us the intuition to explain the comparison with the case with no FE.[21] For brevity, we denote the constant wage as $\bar{w}$ (which may refer to either B or a). Under the assumption that no switches take place, the game can be easily solved, with[22]

$$a_{\mathrm{F}}^{\mathrm{AF}} = \frac{c(\rho + 2\delta) - \sqrt{\Delta^{\mathrm{AF}}}}{2} < 0, \quad \Delta^{\mathrm{AF}} = c^2(\rho + 2\delta)^2 + 4c. \tag{13}$$

[21]From (12), this assumption is equivalent to assuming a monotonously increasing employment starting from an initial level above $-\beta b_{\mathrm{U}}^{\mathrm{AF}}/c$, or a decreasing employment starting from an initial level below $-\beta b_{\mathrm{U}}^{\mathrm{AF}}/c$. In fact, we will show that the former is always true and that $w^{\mathrm{AF}}(L) = w_{\max}$ for any $t \geq 0$.

[22]The highly cumbersome expressions for $c_{\mathrm{F}}^{\mathrm{AF}}(\bar{w})$ and $c_{\mathrm{U}}^{\mathrm{AF}}(\bar{w})$ are not relevant and, hence, are not presented here. They are available from the authors on request.

$$(14)\qquad b_{\mathrm{F}}^{\mathrm{AF}}(\bar{w}) = \frac{c(a-\bar{w})+\beta a_{\mathrm{F}}^{\mathrm{AF}}\bar{w}}{c(\rho+\delta)-a_{\mathrm{F}}^{\mathrm{AF}}}, \quad b_{\mathrm{U}}^{\mathrm{AF}}(\bar{w}) = \frac{c(\bar{w}-B)}{c(\rho+\delta)-a_{\mathrm{F}}^{\mathrm{AF}}} > 0.$$

From (8), (9) and the expression of $a_{\mathrm{F}}^{\mathrm{AF}}$ the employment at the steady-state reads

$$(15)\qquad \bar{L}^{\mathrm{AF}}(\bar{w}) = \frac{a-[1-\beta(\rho+\delta)]\bar{w}}{1+c\delta(\rho+\delta)}$$

And the convergence towards this steady-state value is given by

$$\phi^{\mathrm{AF}} = \frac{1}{2}\left\{\rho - \sqrt{(\rho+2\delta)^2+\frac{4}{c}}\right\} < 0.$$

Under the assumption of a constant wage, the present value of the ongoing wages paid to an additional worker hired today (who is not fired) and who can voluntarily quit at a rate δ, is given by $\bar{w}/(\rho+\delta)$. Correspondingly, the instantaneous benefit from the hiring of this additional worker is given by the savings from his/her lower wage, $\beta\bar{w}$. From now on, we assume that current savings from hiring does not exceed the ongoing wage costs (otherwise it would be beneficial to hire unproductive workers), as is stated in the next condition.

Condition 1

$$(16)\qquad \beta < \frac{1}{\rho+\delta} \Leftrightarrow 1-\beta(\rho+\delta) > 0.$$

This Condition 1 is immediately obvious under the plausible assumption that the discount rate plus the rate of voluntary quit, $\rho+\delta$, does not surpass one, and provided that $\beta \in (0,1)$.

For the firm to be viable, the present value of the marginal productivity of labor when this is highest (in the case of a single worker), $a/(\rho+\delta)$, must exceed the present value of the wage cost of this single worker net of the initial wage discount when hired, $\bar{w}/(\rho+\delta)-\beta\bar{w}$. However, this condition is always satisfied, since $a \geq \bar{w} \geq \bar{w}(1-\beta(\rho+\delta))$. In consequence, a positive employment, $\bar{L}^{\mathrm{AF}}(\bar{w})$, in the medium-run is guaranteed.

As shown in (15), the employment in the medium-run depends on the wage charged by the monopoly union. To determine this wage, the union, acting as the leader, is aware of the positive link between wages and the recruitment policy settled by the firm. Thus, it acknowledges that a marginal increment in wages increases wage earnings, while at the same time induces the firm to enhance hirings at a rate β/c. Given that the marginal value of an additional employee for the union, $(V_{\mathrm{U}}^{\mathrm{AF}})'(L) = b_{\mathrm{U}}^{\mathrm{AF}}(\bar{w})$ is non-negative for any $\bar{w} \geq B$, then from equation (12) it follows that $w^{\mathrm{AF}}(L) = a, \forall t \geq 0$ is the optimal solution.[23] By contrast, with no FE for firm or union, higher wages lead the firm to reduce employment, and consequently the marginal gains for the union, reaching an

[23]This is true as long as L remains nonnegative, which is obvious from (8) and (9).

optimal value at $w^s = (a+B)/2$. Thus, the existence of adjustment costs for the firm lead the union to fix a higher wage than in the case without FE, $w^{\mathrm{AF}} = a > w^s = (a+B)/2$.

At this wage, from equation (9), the dynamics in (11) and the expressions for the coefficients of the value functions, it follows that the employment level converges towards its steady-state value:

$$\bar{L}^{\mathrm{AF}}(a) = \frac{\beta a(\rho+\delta)}{1+c\delta(\rho+\delta)} > 0. \tag{17}$$

Although wages are higher than in the case with no FE, this does not necessarily lead to lower employment levels. Next, we compare employment levels with and without FE for the firm.

Lemma 2 *If the wage rate with FE for the firm was given by its level in the case with no FE, $w^{AF} = w^s$, then $\bar{L}^{AF}(w^s) > L^s$ if and only if:*

$$(a-B)\delta c < (a+B)\beta. \tag{18}$$

Or equivalently, denoting by $\tilde{h}(\bar{w}) = \beta\bar{w}/c$ the recruitment which maximizes the benefits from wage savings at wage $\bar{w}$, then $\bar{L}^{AF}(w^s) > L^s$ if and only if:[24]

$$h^s \equiv \delta L^s < \tilde{h}(w^s). \tag{19}$$

The interpretation of this result is straightforward. If the hirings with no FE are lower than the value which maximizes wage savings, then the existence of FE for the firm gives it an incentive to increase hirings. In consequence, employment in the medium run would be higher than in the case with no FE, because then more employees would quit voluntarily, and correspondingly more workers would be hired at the steady state, so increasing the wage savings for the firm. The opposite reasoning applies if $h^s > \tilde{h}(w^s)$. Therefore, if the wage when the FE for the firm are included remained unchanged, the employment could be enhanced or reduced depending on whether condition (19) is fulfilled or not. However, the existence of FE for the firm does not leave the wage unchanged. It leads the union to fix a wage rate $w^{\mathrm{AF}} = a$ greater than w^s.

With FE for the firm, a higher wage rate has a double effect on hirings as collected in expression (9). It has a direct positive effect because a higher wage represents higher wage savings. Moreover, it has an indirect negative effect because a higher wage also represents a lower marginal valuation of employment by the employer. From (9) and (14) it is not hard to prove[25] that the latter, indirect, effect is stronger and hence that the net effect is negative under Condition 1:

$$\frac{\partial \hat{h}(w,L)}{\partial w} = \frac{\beta}{c} + \frac{(b_{\mathrm{F}}^{\mathrm{AF}})'(w)}{c} = -\frac{1-\beta(\rho+\delta)}{c(\rho+\delta)-a_{\mathrm{F}}^{\mathrm{AF}}} < 0.$$

[24]Recall that $h^s \equiv \delta L^s$ is the hiring required to replace the voluntary quit in the equilibrium with no FE.

[25]This negative relationship is also clear from the expression of $\bar{L}^{\mathrm{AF}}(\bar{w})$ in (15).

Condition (19) assumes a recruitment in the scenario with no-FE lower than the value which maximizes wage savings from hirings. In these circumstances, the existence of FE for the firm would lead to higher employment if the wage were the same as without FE, $\bar{L}^{\text{AF}}(w^s) > L^s$. However, FE also lead the union to raise the wage rate, which reduces hirings (and employment at the steady state), $\bar{L}^{\text{AF}}(a) < \bar{L}^{\text{AF}}(w^s)$. In consequence, the comparison between $\bar{L}^{\text{AF}}(a)$ and L^s is unclear. Conversely, if condition (19) is reversed, hirings in the case of no FE are too high. Thus, with no change in wages there would be an incentive to reduce employment. Further, since the wage rises, this incentive is enhanced. In consequence, in this case the existence of FE for the firm would lead to a smaller level of employment, $\bar{L}^{\text{AF}}(a) < L^s$.

To have a better insight into the effect of the FE on the equilibrium level of employment, in what follows we focus on parameter c, which defines the relative importance of the standard adjustment cost for the firm, and on parameter β, which represents both the effect of wages on the firing costs and, more importantly, the wage savings from new recruitments. Here and henceforth, we assume that $\rho + \delta < 1$, and hence Condition 1 is satisfied for all $\beta \in (0, 1)$. Depending on these parameters' values it is possible to characterize the regions at which the existence of FE for the firm implies a greater ($\bar{L}^{\text{AF}}(a) > L^s$) or a lower ($\bar{L}^{\text{AF}}(a) < L^s$) employment in the medium-term.

Proposition 3 *Assuming $\beta \in (0, 1)$, and $c > 0$, the steady-state level of employment with FE for the firm is greater/equal/lower than the employment with no FE, $\bar{L}^{AF}(a) \gtreqless L^s$, when parameters take values within regions:*

$$\bar{L}^{AF}(a) > L^s \;\Leftrightarrow\; \Omega = \{(\beta, c) \in (0,1) \times (0, \infty) | \beta > \phi(c)\}\,, \tag{20}$$

$$\bar{L}^{AF}(a) = L^s \;\Leftrightarrow\; \Omega^0 = \{(\beta, c) \in (0,1) \times (0, \infty) | \beta = \phi(c)\}\,, \tag{21}$$

$$\bar{L}^{AF}(a) < L^s \;\Leftrightarrow\; \Omega^c = \{(\beta, c) \in (0,1) \times (0, \infty) | \beta < \phi(c)\}\,. \tag{22}$$

$$\textit{with} \quad \phi(c) = \phi_0 + \phi_1 c = \frac{a - B}{2a} + \frac{(a - B)(\rho + \delta)\delta}{2a} c.$$

Regions Ω, Ω^0 and Ω^c are depicted in Figure 3 under the already mentioned assumption of $\rho + \delta < 1$. This figure shows that the FE for the firm will increase employment only if the wage saving from new recruitment, β, is sufficiently large. And this result becomes more unlikely the greater the standard part of the adjustment costs, c. In fact, for[26] $c > \tilde{c}$ or for $\beta < \phi_0$, it holds that $\bar{L}^{\text{AF}}(a)$ is always lower than L^s. Thus, the higher the standard quadratic adjustment costs, the more likely it is that the frictional effects will reduce employment. And, conversely, the stronger the wage discrimination to newcomers the more likely it is that FE for the firm increase labor.

A necessary condition for a non-empty set Ω is $\phi^0 < 1$, or equivalently,

$$a - B < 2a(\rho + \delta) \quad \Leftrightarrow \quad B > a(1 - 2(\rho + \delta)). \tag{23}$$

which requires a wage gap between employed and unemployed workers, that is not excessively wide. Under this condition, the area in region Ω can be interpreted as a

[26]Constant $\tilde{c}$ is the value of c which satisfies $\phi(\tilde{c}) = 1$: $\tilde{c} = [B - a(1 - 2(\rho + \delta))]/[(a - B)\delta(\rho + \delta)]$.

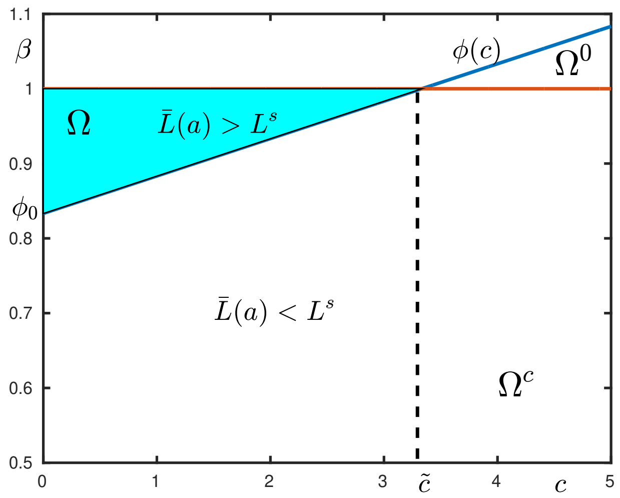

Figure 3
Comparison $\bar{L}^{\mathrm{AF}}(a)$ vs. L^s

measure of the likelihood that the FE for the firm increase the level of employment. This area reads

$$\Omega = \int_0^{\tilde{c}} 1 - \phi(c)dc = \frac{[B - a(1 - 2(\rho + \delta))]^2}{4a(a - B)\delta(\rho + \delta)^2}. \tag{24}$$

From this expression and condition (23) it is easy to show that $\Omega_{a-B} < 0$ and $\Omega_\rho > 0$. Thus, the region at which FE for the firm increase employment shrinks with the wage gap between employed and unemployed workers, $a - B$. Conversely, this region increases with the degree of impatience, ρ. However, the effect of the rate of voluntary quit on the size of Ω is unclear (Ω_δ can be positive or negative).

5 *A Stackelberg differential game*

5.1 A game with FE for the firm and the union

This section takes into account the existence of firing costs for the firm while at the same time considering that the flow of employees who unwillingly abandon the firm also reduces the union's welfare. In contrast, new recruitments enhance firm's profits and union's welfare at a decreasing rate.

The monopoly union model assumes that the union fixes wages knowing the hiring/firing decision taken by the firm. The solution concept considered in this dynamic game is the stagewise feedback Stackelberg solution (as it is called in Başar and Olsder (1982)). This type of solution considers a stagewise first-mover advantage for the trade union, which has an instantaneous advantage at each time. Thus, at each time, the union announces the wage to the firm, which then fixes recruitment. Knowing the recruitment decision of the firm for every wage rate, the union determines the optimal wage. As is usual in differential games with an infinite time horizon, we assume stationary strategies dependent on the level of employment within the firm but not explicitly on time (see, for example, Dockner *et al.*, 2000).

Thus, we characterize first the recruitment decision of the firm, which takes the role of the follower, as dependent on the wage fixed by the union. For the firm's optimization problem (7)-(8) the corresponding Hamilton-Jacobi-Bellman equation is

$$\rho V_{\mathrm{F}}(L) = \max_h \left\{ aL - \frac{L^2}{2} - wL + \beta wL - c\frac{h^2}{2} + V_{\mathrm{F}}(L)(h - \delta L) \right\}.$$

From the first order conditions one gets:

$$V_{\mathrm{F}}'(L) = ch - \beta w.$$

An additional hired employee represents a reduction in firing costs or an increment in hiring gains when $h < \beta w/c$; but, conversely, it implies an increment in hiring costs when $h > \beta w/c$. On the other hand, the marginal value of this additional employee for the firm is given by $V_{\mathrm{F}}'(L)$. The firm hires workers up to the point when the two effects balance. If the firm positively values employment, then it will hire a lot above the value $H/2$ and the standard convex adjustment costs surpass wage savings at the margin. The reverse is true if the firm negatively values employment. Thus, the reaction function for the firm reads

$$\hat{h}(w, L) = \frac{w\beta + (V_{\mathrm{F}})'(L)}{c}, \tag{25}$$

which is identical to the case with FE only for the firm in (9). The only difference is the coefficients of the quadratic value function, now defined as $V_{\mathrm{F}}(L) = a_{\mathrm{F}}L^2/2 + b_{\mathrm{F}}L + c_{\mathrm{F}}$.

Knowing the recruitment policy of the firm, the monopolistic union determines the wage rate in order to maximize:

$$\max_w \int_0^\infty \left[wL + (1 - L)B - d\hat{h}(w, L)\frac{\hat{h}(w, L) - H}{2} \right] e^{-\rho t} dt,$$
$$\dot{L} = \hat{h}(w, L) - \delta L, \quad L(0) = L_0.$$

The Hamilton-Jacobi-Bellman equation for this problem reads

$$\rho V_{\mathrm{U}}(L) = \max_w \left\{ wL + (1 - L)B - d\hat{h}(w, L)\frac{\hat{h}(w, L) - H}{2} + V_{\mathrm{U}}(L)(\hat{h}(w, L) - \delta L) \right\}.$$

And from the optimality conditions for this problem it must hold:

$$L = \frac{\partial \hat{h}(w, L)}{\partial w} \left[C_{\mathrm{U}}'(\hat{h}(w, L)) - V_{\mathrm{U}}'(L) \right] = \frac{\beta}{c} \left[d\left(h - \frac{H}{2} \right) - a_{\mathrm{U}}L - b_{\mathrm{U}} \right]. \tag{26}$$

where $V_{\mathrm{U}}(L) = a_{\mathrm{U}}L^2/2 + b_{\mathrm{U}}L + c_{\mathrm{U}}$ is the quadratic value function of the union. A marginal increment in wages increases the wage gains of employees (at rate L). It also raises the the wage discount to newcomers and hence the firm's incentive to hire, $\partial\hat{h}(w,L)/\partial w = \beta/c > 0$. More hirings have a double effect on the union. On the one hand, more hirings[27] may increase hiring gains ($C'_{\mathrm{U}}(h) < 0$ iff $h < H/2$), or they may reduce them ($C'_{\mathrm{U}}(h) > 0$ iff $h > H/2$). On the other hand, a marginal increment in hirings implies a rise in employment, which the union values as $V'_{\mathrm{U}}(L)$. The optimal wage balances the direct effect of higher wages on the rent earned by the collective of employees and the indirect effect on the union's welfare of more hirings associated with higher wages. From the first order condition (26), and the reaction function for the firm in (25) the feedback strategies for the union and the firm follow:

$$\phi_w(L) = \phi_w^0 + \phi_w^1 L = \frac{cb_{\mathrm{U}} - db_{\mathrm{F}}}{d\beta} + \frac{c}{\beta}\frac{H}{2} + \frac{c(\beta a_{\mathrm{U}} + c) - d\beta a_{\mathrm{F}}}{d\beta^2}L, \tag{27}$$

$$\phi_h(L) = \phi_h^0 + \phi_h^1 L = \frac{b_{\mathrm{U}}}{d} + \frac{H}{2} + \frac{\beta a_{\mathrm{U}} + c}{\beta d}L. \tag{28}$$

Note that the optimal recruitment decision of the firm $\phi_h(L)$, paradoxically does not depend on the marginal value of an additional employee for the firm $V'_{\mathrm{F}}(L)$. That is because the union settles wages so as to oblige the firm to hire at the rate which balances (26). If the marginal valuation of employment for the firm rises, $V'_{\mathrm{F}}(L)$, increasing the firm's willingness to hire (as shown in (25)), the union will correspondingly reduce wages to lower the wage savings that the firm obtains from newcomers and hence to reduce the incentive to hire for the firm. The two effects exactly cancels out and hiring and firing decisions of the firm, which is a Stackelberg follower, depend only on the value function of the Stackelberg leader trade union.

Plugging this optimal strategies into the Hamilton-Jacobi-Bellman equations we obtain a system of 6 Riccati equations in the unknowns a_{F}, b_{F}, c_{F}, $a_{\mathrm{U}}, b_{\mathrm{U}}, c_{\mathrm{U}}$. Two sets of solutions for the coefficients of the value functions are found analytically.[28]

Proposition 4 *A solution for this system of Riccati equations with a concave value function, $V_U(L)$, i.e. satisfying $a_U < 0$, is found under Condition 1 and sufficient condition $c \leq d$ (assumed henceforth).*

Proof See Appendix

Although no condition is found to characterize the sign of a_{F}, the numerical simulations[29] carried out for different parameter values support the hypothesis of a positive coefficient a_{F}, or a concave value function for the firm.

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[27] If $\hat{h}(w,L) < 0$, fewer firings will always represent lower firing costs.

[28] The solutions are obtained with the help of Mathematica. Due to their complexity, their expressions are not useful and we do not present them here.

[29] We have computed the coefficients of the value functions for parameters' values: $c = 0.1$, $\rho = 0.05$, $\delta = 0.15$, $d = 1$, $B = 0.1$, $H = 0.1$, $a = 0.77588$, $L0 = 0, \beta = 0.3$. The result is robust to 10 % up and down changes in each parameter's value (L_0 moving from 0 to 0.1), keeping all other parameters constant.

Proposition 5 *Under Condition 1, and sufficient condition $c \leq d$, $\phi_h'(L) = \phi_h^1 < 0$. Furthermore, if $a_F > 0$ (numerically shown) then $\phi_w'(L) = \phi_w^1 < 0$.*

Proof See appendix.

To have a better intuition of the results presented in Proposition 4, notice that an additional employee affects the optimal wage decision taken by the monopoly union in three ways:

i) It directly increases the level of employment, and with it the marginal gains from higher wages, hence increasing the union's willingness to rise wages.

ii) Since $a_{\mathrm{U}} < 0$, a greater employment level reduces the union's marginal valuation of employment, $V_{\mathrm{U}}'(L) < 0$. This makes the union less willing to accept a rise in hirings, and pushes it to reduce wages in order to induce the firm to reduce hirings.

iii) Since (numerically) $a_{\mathrm{F}} > 0$, a greater employment level increases the firm's marginal valuation of employment, $V_{\mathrm{F}}'(L) > 0$. Knowing the firm's higher willingness to hire, the union will be inclined to reduce wages in order to reduce the incentive to hire for the firm with lower wage savings to newcomers.

Proposition 4 proves that the unique positive effect i) is weaker than the negative effect ii).[30] Therefore, the wage rate decays as the employment grows, $\phi_w'(L) = \phi_w^1 < 0$.

To give an interpretation to the effect of employment on the optimal hiring decisions of the firm, recall that this decision is independent of the marginal value that the firm gives to employment. This is equivalent to saying that the positive effect that a greater employment has on the firm's marginal value of employment and hence on its willingness to hire, is exactly counterbalanced by a monopolistic trade union setting lower wages as stated in effect iii). In consequence, employment reduces hirings through the reduction in wages and the consequent decay in wage savings from the wage discount to newcomers, exclusively explained by effects i) and ii).

Remark 6 *Under Condition 1, and condition $c \leq d$, from equation (1) it follows that employment converges towards its steady-state value:*

$$\bar{L} = \beta \frac{b_U + d\frac{H}{2}}{\beta d\delta - (\beta a_U + c)}. \tag{29}$$

And the speed of convergence is given by

$$\phi = \left| \frac{\beta a_U + c}{d\beta} - \delta \right|. \tag{30}$$

If we assume an initial level of employment below its steady-state value, $L_0 < \bar{L}$, employment will increase steadily towards $\bar{L}$ as displayed in Figure 4 (right), at the speed

[30] And obviously weaker than the addition of the two negative effects ii) and iii).

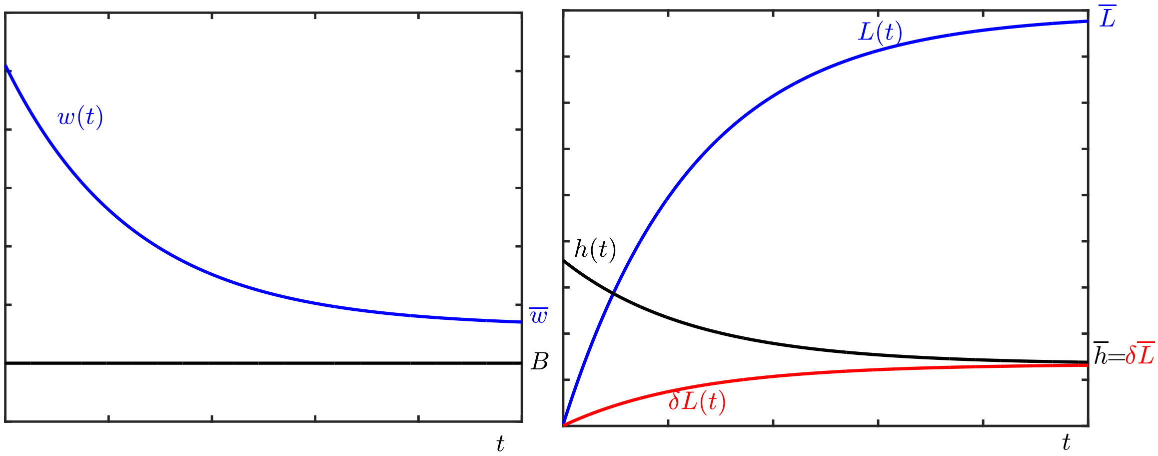

Figure 4
Optimal time path for wages (left); hirings and employment (right)

ϕ in (30). Correspondingly, as employment grows, the wage settled by the union decays towards a steady-state value greater than the unemployment benefit, B, as shown in Figure 4 (left). Likewise, the firm will hire fewer and fewer employees, while the number of employees who voluntarily quit the firm increases with the level of employment. As Figure 4 (right) shows, the two quantities converge in the medium run, and so employment remains constant. An opposite behavior of decreasing employment and increasing wages and recruitment rates would occur if the initial level of employment is above the steady state, $L_0 > \bar{L}$.

5.2 *Sensitivity analysis*

When FE are present for the firm as well as for the union, the influence on employment and wages of the different parameters defining the labor market can be numerically studied. The numerical analysis is carried out for the parameters' values in footnote 29. The results are presented in Table 1 which shows the effect of the main parameters on the labor market as described by the steady-state values of employment and wages, as well as the speed of adjustment (represented by the absolute value of ϕ). Results in this section are robust to 10 % up and down changes in each parameter's value (L_0 moving from 0 to 0.1), keeping all others parameters constant.

For parameter d, which defines the size of the FE for the union, it is found that the stronger these FE, the less faster is the convergence towards the steady-state equilibrium, characterized by a higher wage settled by the union and a lower level of employment. Low employment levels and higher wages are also attained if the unemployment benefit, B, is high, although this variable does not affect the speed of convergence. Finally, the effect of wages on firing costs, which is considered identical to the parameter describing the wage gap against newcomers, β, also has a negative effect on employment and raise wages. However, conversely to the size of the FE of the union, a wider salary gap to newcomers speeds up convergence.

	$\bar{w}$	$\bar{L}$	$\lvert\phi\rvert$
$\uparrow\ d$	$\uparrow\ w$	$\downarrow\ L$	$\downarrow$
$\uparrow\ B$	$\uparrow\ w$	$\downarrow\ L$	$\leftrightarrow$
$\uparrow\ \beta$	$\uparrow\ w$	$\downarrow\ L$	$\downarrow$
$\uparrow\ c$	$\downarrow\ w$	$\uparrow\ L$	$\uparrow$

Table 1
Sensitivity analysis

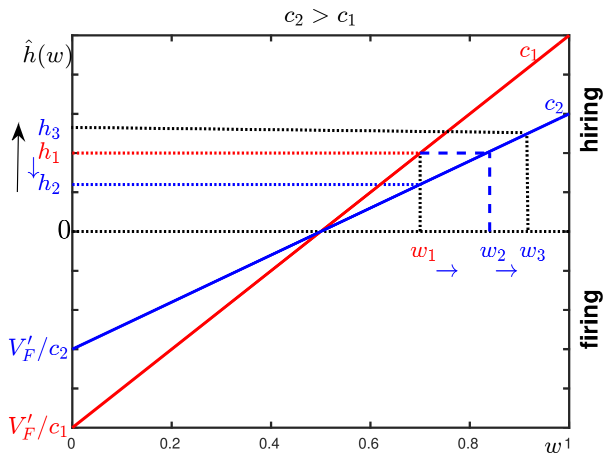

Figure 5
Direct effect of symmetric adjustment costs

This subsection centers its attention on the sensitivity analysis for parameter c, which defines the size of the symmetric quadratic adjustment costs for the firm, i.e. the adjustment costs dependent on the number of hired or fired employees and not on the wage rate. This analysis concludes the standard result that adjustment costs slow down convergence. However, at the same time it conclude the counterintuitive result that higher adjustment costs for the firm reduce wages and increase employment. These correspond to the standard adjustment costs that grow with both hiring and firing at an increasing rate. The effect of parameter c is illustrated assuming that it rises from c_1 to $c_2 > c_1$.

Given the reaction function of the firm in (25), the first direct effect of a higher c is a less sloped reaction function, which implies that firm's hiring respond less sharply to variations in wages. Assuming a fix level of employment, the reaction functions of the firm are plotted in Figure 5, for two different values of c, with the flatter line corresponding to the greater value, c_2. If the initial situation is a steady-state equilibrium (w, h) for c_1, a rise in c would reduce hirings from h_1 to h_2, so reducing marginal FE $C_{\text{U}}'(h_2) < C_{\text{U}}'(h_1)$. As long as the responsiveness and the marginal adjustment costs are lower, and according to the optimality condition (26), the union has an incentive to increase wages. An increment from w_1 to w_2, would induce the firm to raise hirings back

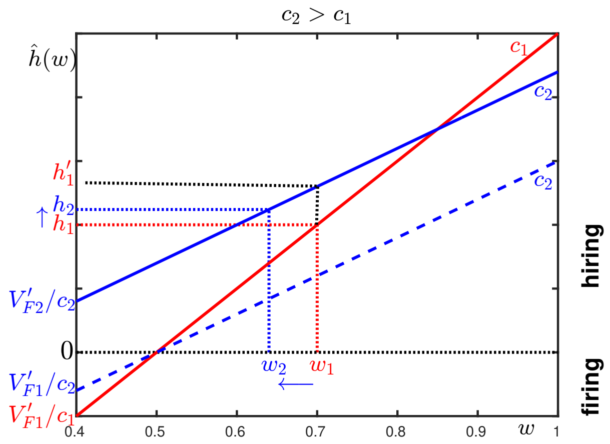

Figure 6
Net effect of symmetric adjustment costs

to h_1 and with them, the marginal adjustment costs back to their initial value. However, since the responsiveness of hirings to wage changes has decreased, further increments in wages towards w_3, and the corresponding increment in hirings towards h_3, are required to balance equation (26) again.

This direct effect ignores the possible effect of c on the marginal value given to the level of employment by the firm, $V'_{\text{F}}(L)$, (i.e. on the coefficients a_{F} and b_{F}). However, when adjustment costs are higher, an additional employee already within the firm does not need to be hired and for that reason is more valuable for the firm. Therefore, the marginal value for the firm of an additional employee, $V'_{\text{F}}(L)$, increases and the reaction function of the firm shifts up-left, as depicted in Figure 6 (the y intercept shifts up from $V'_{\text{F}1}/c_2$ to $V'_{\text{F}2}(L)/c_2$). Provided that this increment in the "shadow" value of employment for the firm is large enough, at the initial wage w_1, the firm will not reduce but increase hirings to $h'_1 > h_1$. Now, larger hirings increase marginal adjustment costs $C'_{\text{U}}(h_2) > C'_{\text{U}}(h_1)$, leading the union to decrease wages in order to induce a reduction in hirings. However, since the responsiveness of hirings to wages is lower, the reduction in wages should be soft enough not to decrease hirings back to their original level h_1, because in that case condition (26) would not be satisfied (the RHS would be too low). Hence, in the medium-run, wages are reduced, while hirings, and hence employment, are increased. In this situation, stronger adjustment costs would lead to lower wages and unemployment rates. A numerical simulation of this possible effect is presented in Figure 7, which displays the time paths of wages, hirings and employment for two values of parameter c, (0.1 and 0.3). A higher adjustment cost would lead to a steady-state equilibrium characterized by lower wages and higher employment. However, this result is not necessarily true along the transition. In fact, the opposite behavior takes place over an initial time interval.

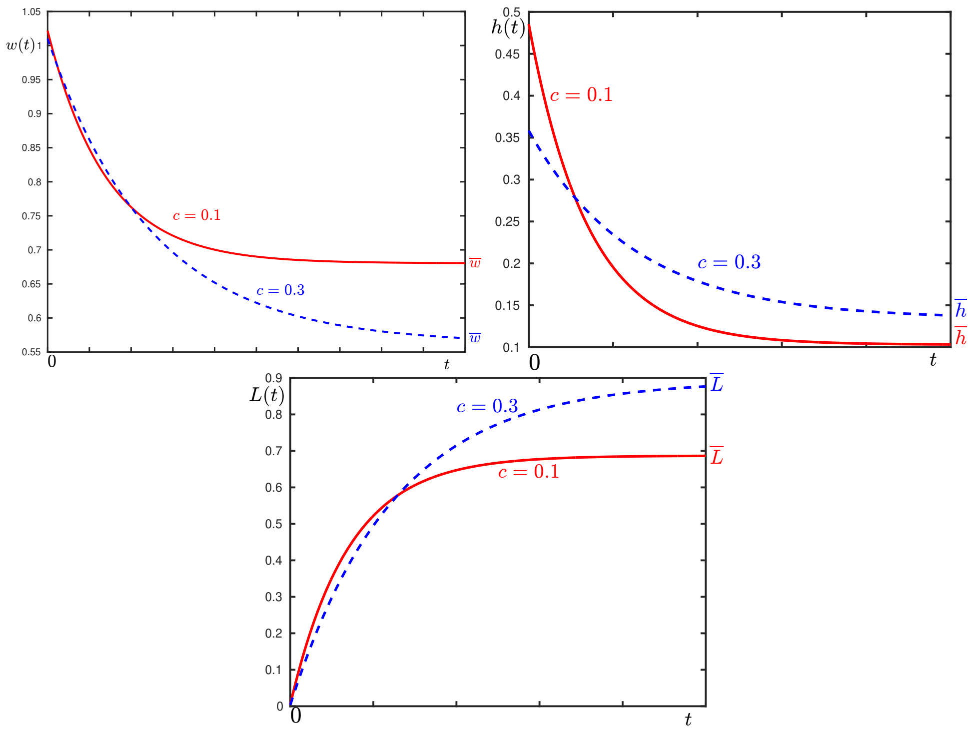

Figure 7
Direct effect of symmetric adjustment costs

6 Conclusions

This paper studies the collective bargaining process between a trade union and a firm, within the framework of the monopoly union model. We consider a dynamic perspective, according to which, the employment level within the firm progressively adjusts, converging towards a steady-state value. The employment can augment through the accumulation of a continuous flow of new recruitments, or diminish from a continuous flow of employees exiting the firm. This parsimonious process is based on the existence of FE for the firm, the union, or both. The collective action bargaining process is defined as a dynamic game played *à la* Stackelberg between the union (leader) and the firm (follower). The firm decides on the demand for newcomers or the layoff of current employees. The union, aware of this dynamic demand, would choose wages so as to manipulate the firm's recruitment decisions in its best interest.

Hiring new employees or firing current workers involves FE for the firm, which can be divided in two terms. First, the standard quadratic symmetric adjustment costs, exclusively dependent on the number of employees being hired or fired. And secondly, we incorporate a new term which makes firing costs also positively dependent on the wage earned by the employees laid off. Furthermore, we assume wage discrimination between incumbent and new recruited employees, which gives the firm the possibility for wage savings by hiring new employees. Indeed, when the number of hired employees is small, the wage savings from new recruitments outweigh the standard hiring costs and the firm experiences hiring benefits instead of hiring costs. Additionally, hiring and firing also influence the union's welfare. The union resents firings at an increasing rate, and welcomes hirings at a decreasing rate (because of organizational problems).

To analyze the influence of the FE on wages and employment, the scenario with no frictions is analyzed first. This is the baseline scenario in which employment and wages adjust instantaneously, and can be consequently identified as the static case. In this setting, the employment level and the wage rate depend only on the production technology and the unemployment benefit. By giving entrance to FE for the trade union, we observe that the wage increases and the employment decreases if the voluntary quit without frictional effect is large (larger than the hiring rate which maximizes the hiring benefits for the union); that is, if the voluntary quit (which defines hirings at the steady-state) lies within the range at which the FE for the union are negative at the margin. Conversely, if the union welcomes an additional employee up to very large recruitment levels (above the voluntary quit without FE), then the existence of hiring benefits leads the union to reduce wages, so inducing a rise in the number of newcomers and hence an increment in employment levels.

Secondly, we compare the static case with the scenario with FE only for the firm. In this scenario, raising wages is always rewarding for the union at the margin. Therefore, it chooses the maximum feasible wage. Again if the rate of voluntary quit without FE surpasses the hiring rate which maximizes wages savings from the salary gap to newcomers, then the FE for the firm will reduce the employment level. The opposite is not necessarily true. Since FE lead to fix a higher wage, a small rate of voluntary quit will not necessarily lead to higher employment levels. Nevertheless, for the parameters defining these FE, we have characterized the regions within which employment is either raised or reduced. Frictional effects lead to a higher level of employment if: the income gap between employed and unemployed workers is not too high; the size of the standard convex cost for the firm is not too large; and the wage discrimination against newcomers is strong.

We finally analyze the Stackelberg dynamic game with FE for both players. The solution to this game is characterized by a smooth convergence of employment towards its medium-run value from below if initially low. Under this assumption of initially low levels of employment, wages and hiring rates will be settled at initially large values, decreasing towards their steady-state values as employment rises. The opposite behavior applies for an excessively large initial level of employment.

For this general case, the sensitivity analysis concludes that employment decreases with the size of the FE for the union, or with the unemployment benefits. However, when focusing on the FE for the firm, a twofold conclusion is obtained. On the one hand, the wage gap against newly-hired employees which by definition is considered identical to the effect of wages on firing costs, reduces employment. Conversely, the standard symmetric costs only associated with the number of employees hired or fired increase employment. This controversial result is based on the fact that these costs not only reduce the positive responsiveness of the firm to a rise in wages, but also, and at the same time, raise the marginal value that the firm attaches to employees. This greater valuation increases hirings and employment. Knowing this, the monopolistic union will fix a lower wage in order to reduce the firm's benefits from the wage gap to newcomers, and hence reduce the firm's incentive to hire. However, this effect is not enough to offset the initial increment in hirings. In conclusion, greater symmetric standard costs for the firm raise employment while at the same time reducing wages.

Appendix

Proof of Proposition 4

The expression for a_{U} reads

$$a_{\mathrm{U}} = \frac{-[4c^2 - d\beta\Phi\Theta] - \sqrt{\Delta}}{2\beta\Theta},$$

with $\Phi = \rho + 2\delta$, $\Theta = 2(c+d) - d\beta\Phi$ and

$$\Delta = [4c^2 - d\beta\Phi\Theta]^2 - 4\Theta\left[c^2(2c - 2d - d\beta\Phi) - 2d^2\beta^2\right].$$

Under Condition 1, $\Theta > 0$, and hence a sufficient condition for $a_{\mathrm{U}} < 0$ would be $\Delta > [4c^2 - d\beta\Phi\Theta]^2$, which can be guaranteed under sufficient condition $c \leq d$.

Proof of Proposition 5

From the optimal feedback strategies in (28) and (27), one gets

$$\phi_w^1 = \frac{c(\beta a_{\mathrm{U}} + c) - d\beta a_{\mathrm{F}}}{d\beta^2}, \quad \phi_h^1 = \frac{\beta a_{\mathrm{U}} + c}{d\beta}.$$

We have numerically seen that $a_{\mathrm{F}} > 0$. Therefore, a necessary condition for $\phi_w^1 < 0$, and necessary and sufficient condition for $\phi_h^1 < 0$, is $\beta a_{\mathrm{U}} + c < 0$. This expression can be written as

$$\beta a_{\mathrm{U}} + c = \frac{-4c^2 + d\beta\Phi\Theta + 2c\Theta - \sqrt{\Delta}}{2\beta\Theta}.$$

Since $\Theta > 0$, a sufficient condition for a negative sign of this expression is $-4c^2 + d\beta\Phi\Theta + 2c\Theta < 0$, or equivalently, after some rearrangements

$$-(d\beta\Phi)^2 + 2d(d\beta\Phi) + 4cd < 0. \tag{31}$$

The LHS of this inequality can be interpreted as a second order polynomial in $d\beta\Phi$, with roots: $d \pm \sqrt{d^2 + cd}$. The inequation (31) holds true if $d\beta\Phi < d + \sqrt{d^2 + cd}$. And this condition is immediate under Condition 1.

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