Efficiency inducing taxation for polluting oligopolists: the irrelevance of privatization

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Abstract

This paper examines the optimal environmental policy in a mixed oligopoly when pollution accumulates over time. Specifically, we assume quantity competition between several private firms and one partially privatized firm. The optimal emission tax is shown to be independent of the weight the privatized firm puts on social welfare. The optimal tax rule, the accumulated stock of pollution, firms' production paths and profit streams are identical irrespective of the public firm's ownership status.

Introduction

In recent years, the theoretical analysis of environmental policy under imperfect competition has received large attention(see Requate , 2005, for an excellent survey). A first strand of the literature focuses on the regulation of flow pollutants. In this context, it has been shown that the optimal tax policy should be designed so as to balance two effects of taxation. On the one hand, taxation increases social welfare by reducing polluting emissions and thus environmental damage. But, on the other hand, it is harmful because it induces private firms to reduce their already suboptimal output level. This trade-off was first disclosed by Buchanan (1969) and then formally studied by Barnett (1980) and Misiolek (1980). They found that the optimal tax is less than marginal external damage under monopolistic competition. Ebert (1991), Katsoulacos and Xepapadeas (1996) and Lee (1999) proved that this result remains valid under oligopoly. However, when additional externalities are taken into account (such as endogenous entry, Katsoulacos and Xepapadeas , 1995, or inter-firm externalities Yin , 2003), the optimal tax rate may exceed marginal damage.

A second strand of the literature examines the regulation of stock pollutants. Since pollution accumulation generates present as well as future damages, inter temporal externalities must be taken into account in analysing the optimal environmental policy.

A natural way to do this is to model the interaction between the regulator and the industry as a Stackelberg differential game with the regulator as the leader. Building on the literature on efficiency-inducing taxation[1], Benchekroun and Van Long (1998) studied corrective taxation for polluting oligopolists. They showed that there exists a time-independent tax rule that induces firms to follow the socially optimal production path. The parameters of the optimal markovian tax rule depend on whether firms use open-loop or closed-loop strategies. However, in both cases, the tax rule exhibits intuitively appealing properties. First, since the tax rule is time independent, it satisfies strong time-consistency requirements; i.e., subgame perfectness. Therefore, even if the government is unable to commit to the entire time-path of taxation, the announced tax rule is credible and cannot be manipulated by the firms. Second, the tax rate is increasing in the pollution stock. Thus, it conforms with the idea that the marginal rate of taxation should increase as the environmental problem becomes more stringent.

When the market is competitive enough, a pollution tax is always optimal. However, as a result of the tradeoff between pollution (which generates environmental damages) and market power (which reduces social welfare because of higher prices and lower output), it may be optimal to subsidize production in the initial time period when there are just a few firms and the stock of pollution is low. Furthermore, and quite surprisingly, it may still be optimal to subsidize production when the *laissez-faire* output level exceeds the socially optimal one at all points of time. The reason for this is simple. A reduction in current industrial production induces positive intertemporal externalities in the form of reduced future environmental damages. In order to capture these positive externalities, the corrective tax rule may consist initially in a subsidy that decreases as industry output increases and turns into a tax when the stock of pollution becomes large. In that case, the progressive nature of the corrective tax system provide firms with an incentive to reduce their current outputs in order to keep the benefits of the subsidy and postpone the coming of the tax.

Most analyses of the regulation of polluting firms have assumed private firms[2]. This assumption ignores an important feature of a number of regulatory settings: the active role of public and (partially) privatized firms as providers of goods and services. As a result of the process of market liberalization in Western Europe (through which private firms are allowed into markets that were previously monopolized by state-owned enterprises) and of the transition process in the countries comprising the former Soviet Union, Eastern Europe and Asia (of which privatizing state-owned enterprises is an essential part), mixed market structures are becoming increasingly common. Actually, public firms compete with private firms in many highly polluting sectors such as energy supply, transportation, iron and steel, chemicals and petrochemicals. They are responsible for releasing large amounts of toxic compounds that accumulates in the environment causing present as well as long-term environmental damages. Thus, the issue of the environmental regulation of mixed markets deserves important consideration.

The purpose of this paper is to analyse efficiency-inducing taxation when the market is served by private and public (or partially privatized) providers. In this connection, we would like to address two questions. First, we want to understand how the mixed market structure affects the design of the optimal corrective tax. Second, we are interested in the welfare effect of privatization when optimal taxation is used before and after

privatization. With these purposes in mind, we introduce a partially privatized firm in the model studied by Benchekroun and Van Long (1998). In line with the literature on mixed oligopolies, we assume that firms with different ownership structures differ in their aims. Since the privatized firm is partly privately owned and partly state-owned, it cannot be assumed to be either exclusively profit oriented or exclusively welfare oriented. Rather, its objective should reflect the different interests of its public and private shareholders. Following Bös (1991) and Matsumura (1998) we describe the payoff function of the privatized firm as a weighted average of social welfare and its own profit[3]; i.e., $f = (1 - \theta)w + \theta\pi$. In this formulation, the weight $\theta \in [0, 1]$ measures the extent of privatization.

We obtain an irrelevance result that might seem counter-intuitive at first glance. Namely, we find that the optimal linear markovian tax rule which decentralizes the social optimum as an open-loop Nash equilibrium of the differential oligopoly game is independent of θ, the degree of privatization of the public firm. Thus, the optimal environmental policy tells us that technologically identical firms must be taxed the same whatever their ownership status. Furthermore, this result is robust to changes in the information structure of the differential oligopoly game considered[4]. Indeed, the optimal tax rule remains independent of the extent of privatization if we assume that oligopolists use closed-loop strategies. Turning to the welfare effects of privatization, we prove that welfare is unchanged by privatization when the optimal tax rule is used. This result stems directly from the fact that the social optimum is independent of θ and thus unique.

Our irrelevance result suggests that mixed oligopolies and private oligopolies should not differ substantially in terms of economic and environmental performance if pollution charge programs are correctly designed. This result seems consistent with experience and empirical evidence which indicate that the economic and environmental consequences of privatization reforms are mixed and vary substantially across sectors and countries[5]. Privatization conveys promises of increased productive efficiency and more efficient use of resources, improved access to capital markets and greater investments in cleaner technologies, better management practices and easier access to markets for environmentally friendly goods and services. However, it also involves costs. For example, the decrease in supply as a result of the exercise of increased market power may result in a larger economic deadweight loss. In most cases analysed to date, the quality of environmental regulations and commercial pressure have been playing a preeminent role in the successes and failures of privatization reforms. Environmental tax exemptions or lax environmental regulations have resulted in poor environmental performance, whatever the ownership structure of the industry (e.g., Lovei and Gentry , 2002).

This paper contributes to the literature on the interaction between privatization policy and other policy instruments. Starting with White (1996) a number of irrelevance results has been established in the context of static mixed oligopoly models. In the linear-quadratic case, Poyago-Theotoky (2001) showed that the optimal output subsidy is identical and profits, output and social welfare are also identical irrespective of whether (i) a public firm moves simultaneously with n private firms or (ii) it acts as a Stackelberg leader or (iii) all firms, public and private, behave as profit maximizers. Myles (2002) extended this result to general inverse demand and cost functions and

Sepahvand (2004) to mixed markets open to foreign competition. Claude and Hindriks (2006) proved that the irrelevance result suggested by White (1996) remains valid when partial privatization is explicitly allowed. When several private firms with profit objectives compete with one partially privatized firm maximizing a weighted average of social welfare and its own profit, the optimal subsidy is identical irrespective of the weight the privatized firm puts on social welfare. The unicity of the social optimum implies that profits, output levels and social welfare are also identical irrespective of whether (i) the partially privatized firm moves simultaneously with n private firms or (ii) it acts as a Stackelberg leader or (iii) all firms, public and private, behave as profit maximizers. This paper shows that a similar irrelevance result obtains when production generates polluting emissions which accumulate over time and the optimal environmental tax rule is used to regulate pollution. To the best of our knowledge, it is the first irrelevance result obtained in an explicitly dynamic regulatory setting.

The remainder of this paper is organized as follows. Section 1 describes the basic model and characterizes the social optimum. Optimal corrective tax rules are derived for open-loop and closed-loop mixed markets in sections 3 and 4, respectively. Section 5 concludes the paper.

1 The model

Consider a mixed market consisting of one public firm (indexed by 0) and n identical private firms (indexed by $1, 2, ..., n$). Market competition takes place à la Cournot-Nash over the continuous time period $[0, \infty[$. In each period, firms face a downward sloping inverse demand function $p = P(Q)$ where $Q \equiv \sum_{i=0}^{n} q_i$ with q_i denoting the quantity produced by firm i. Let the total cost function of firm i be $C_i(q_i)$ with $C_i(0) = 0$, $C_i'(q_i) > 0$ and $C_i''(q_i) \geq 0$. We assume that technology is identical across private firms; i.e., $C_i(q) = C_1(q), \forall q > 0$ and $\forall i = 1, 2, \ldots, n$. However, we leave open the possibility of a cost-asymmetry between public and private firms by assuming that $C_0(q) \geq C_1(q), \forall q > 0$. There is no capacity constraint and entry by additional firms is supposed to be effectively blocked.

Production of good q generates polluting emissions, which accumulates over time in the ambient environment. Without loss of generality, we assume that firm i's level of polluting emission is $e_i = q_i$. Furthermore, no pollution abatement technology is available so that firms can only reduce emissions by reducing output. Assuming a constant rate of decay, the dynamics of the accumulated stock of pollution S is described by

$$\frac{dS(t)}{dt} \equiv \dot{S}(t) = Q(t) - \delta S(t), \qquad S(0) = S_0 \geq 0, \tag{1}$$

where the coefficient $\delta > 0$ reflects the environment's self-cleaning capacity and S_0 is the initial size of the pollution stock.

The welfare of society at time t depends on the current vector of production decisions $q(t) = (q_0(t), q_1(t), \ldots, q_n(t))$ and the current stock of pollution $S(t)$. It is measured by the sum of consumers' and producers' surplus less environmental damages. At time t

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social welfare is given by

$$w(t) = \int_0^{q_0(t)+\sum_{i=1}^n q_i(t)} P(u)du - C_0(q_0(t)) - \sum_{i=1}^n C_i(q_i(t)) - D(S(t)), \qquad (2)$$

where the damage function $D(.)$ measures the economic loss resulting from the current stock of pollution $S(t)$. It is assumed that the function $D(S)$ satisfies the following restrictions: $D(0) = 0$, $D'(0) = 0$, and $D'(S) > 0$, $D''(S) > 0, \forall S > 0$. Furthermore, we assume that $P(0) > C_i'(0), \forall i = 0, 1, \ldots, n$.

In an ideal regulatory setting where the environmental regulator has direct control over the production level of each firm, it can achieve the social optimum by choosing time-paths of production for each firm so as to maximize social welfare. Let r denote the social rate of discount. The optimal allocation of production $q^*(t) = (q_0^*(t), q_1^*(t), \ldots, q_n^*(t))$ is found by solving

$$\max_{q(t) \geq 0} W = \int_0^\infty w(t)e^{-rt}dt, \qquad (3)$$

subject to the stock dynamics described by Equation (1).

In actual practice however the environmental regulator lacks the authority to enforce the social optimum directly. Therefore, we assume that it seeks to implement the social optimum indirectly by relying on fiscal policy. Following Benchekroun and Van Long (1998), we suppose that the regulator uses linear Markov tax rules to regulate pollution. Namely, we assume that each firm is charged a tax $\tau_i[S(t)]$ per unit of output, where the unit tax depends only on the current pollution stock $S(t)$.

The timing of the environmental regulation game is as follows. Prior to market competition, the regulator announces the markovian tax scheme $\tau(S) = (\tau_0(S), \tau_1(S), \ldots, \tau_n(S))$ that will be applicable to the firms. Then, firms engage in Cournot competition at each subsequent instant of time $t \in [0, \infty[$ taking as given the tax policy followed by the regulator.

In the remainder of this section, we define firms' objective functions, specify the information structure of the dynamic oligopoly game and state the problem that the environmental regulator must solve to characterize the optimal tax scheme.

The polluting oligopoly

Let us assume that the environmental regulator imposes a tax $\tau_i(S)$ on each unit of pollution produced by firm i $(i = 0, 1, 2, \ldots, n)$. Then, firm i's instantaneous profit level is

$$\pi_i(t) = P[q_i + Q_{-i}(t)]q_i - C_i[q_i(t)] - \tau_i[S(t)]q_i(t), \qquad (4)$$

where $Q_{-i}(t) \equiv -q_i(t) + \sum_{i=0}^n q_i(t)$. In this paper we abstract from agency problems between the regulator, private shareholders and the management of the public firm in order to concentrate on the difference between private and public firms' objectives. Private firms are considered to be profit maximizers while the privatized firm is assumed to behave differently. Following Bös (1991), we assume that the privatized firm's ob-

jective reflects the conflicting interests of its public and private shareholders. A natural way to formalize this idea is to assume that the privatized firm's instantaneous objective is a weighted average of social welfare and its own profit

$$f_0(t) = (1 - \theta)w(t) + \theta \, \pi_0(t), \tag{5}$$

where $\theta \in [0, 1]$. In this formulation, the weight $(1 - \theta)$ describes the extent to which the government is able to control the behavior of the public firm through the shares that it has retained [6]. If $\theta = 1$ the privatized firm behaves as a private oligopolist; i.e., it is exclusively profit oriented. If $\theta = 0$ the privatized firm behaves as a welfare-maximizing public firm and strictly adheres to the objective of the environmental regulator. In the remainder of this paper θ is assumed to be exogenously given and readily observable by the firms.

Firms long-term objectives are as follows. Each private firm i seeks to maximize its aggregated profit, defined as the value Π_i of its stream of discounted short-run profits:

$$\Pi_i = \int_0^\infty \pi_i(t)e^{-rt}dt. \tag{6}$$

By contrast, the privatized firm seeks to maximize the value F_0 of its stream of discounted short-run payoffs:

$$F_0 = \int_0^\infty f_0(t)e^{-rt}dt. \tag{7}$$

The specific sets of strategies that are available to the firms depend on the information structure of the game. In this paper we restrict our attention to open-loop and closed-loop information structures. Under an open-loop information structure, firms are unable to observe the current state of the game. Consequently, they condition their strategies only on time. Namely, each firm i $(i = 1, \ldots, n)$ uses an open-loop strategy; i.e., a decision rule of the form $q_i(t) = \phi_i(t)$. By contrast, under a closed-loop information structure, firms are able to observe the current state of the game and use this information to revise their strategies at each point of time. Each firm i $(i = 1, \ldots, n)$ uses a closed-loop strategy; i.e., a decision rule of the form $q_i(t) = \phi_i(S(t))$. Whatever the information structure considered, the relevant equilibrium concept for the analysis of the dynamic oligopoly game is the Nash equilibrium. Let us recall that an open-loop (closed-loop) Nash equilibrium is a profile of open-loop (closed-loop) strategies that are mutual best responses.

The environmental regulator

At a prior stage the environmental regulator determines the system of linear Markov tax rules $\tau(S) = (\tau_0(S), \ldots, \tau_n(S))$ to regulate pollution. Having determined firms' optimal behaviors in the oligopoly subgame, it selects the tax scheme $\tau(S)$ so as to maximize

social welfare. Formally, the optimal tax scheme $\hat{\tau}^e(S)$ is obtained by solving

$$\max_{\tau(S)} W^e = \int_0^\infty w^e(t) e^{-rt} dt, \tag{8}$$

$$\text{s.t. } \dot{S}(t) = Q(t) - \delta S(t), S(0) = S_0 \geq 0, q(t) \geq 0. \tag{9}$$

where the superscript $e \in \{\text{ol}, \text{cl}\}$ indicates whether variables are evaluated at the open-loop or closed-loop Nash equilibrium of the underlying dynamic oligopoly game. Note that taxes appear in expression (8) both as a revenue for the state and as an expenditure for the firms. Thus, the direct effect of taxation on social welfare is zero. However, taxation has an indirect effect on aggregated social welfare through its effect on firms' equilibrium output levels.

2   The Social Optimum

Before analysing the environmental regulation game, it is useful to characterize the social optimum where firms can be directly controlled by the regulator. This solution provides a relevant benchmark against which the outcome of the environmental regulation game will be evaluated. It can be obtained by solving the infinite-horizon optimal control problem (3) with the stock of pollution $S(t)$ as state variable and individual output levels $q_i(t)$ as control variables. First we derive the necessary and sufficient conditions for optimality. Second we characterize the steady state solution.

The current value Hamiltonian for this problem is defined as [7]

$$H_r = \int_0^Q P(u)du - C_0(q_0) - \sum_{i=1}^n C_i(q_i) - D(S) + \lambda_r (Q - \delta S) \tag{10}$$

where λ_r denotes the costate (or adjoint) variable associated with $\dot{S}$. Assuming interior solutions, the maximum principle implies the following necessary and sufficient optimality conditions

$$\frac{\partial H_r}{\partial q_i} = 0, \quad -\lambda_r = P(Q) - C_i'(q_i), \quad \forall i = 0, \ldots, n, \tag{11}$$

along with the adjoint equation

$$\dot{\lambda}_r = \lambda_r r - \frac{\partial H_r}{\partial S} = \lambda_r(r + \delta) + D'(S), \tag{12}$$

the dynamic process of pollution accumulation (1) and the transversality condition

$$\lim_{t \to \infty} e^{-rt} \lambda_r(t) S(t) = 0. \tag{13}$$

From the short-run optimality conditions (11), the costate variable λ_r is negative: it can be interpreted as the shadow cost of the accumulated pollution stock. Furthermore,

solving these conditions yields

$$C_i'(q_i) = C_j'(q_j), \quad \forall i = 0, 1, \ldots, n. \tag{14}$$

Optimality requires that the aggregate output be produced at least cost. Therefore, optimality requires that marginal costs of the last unit of output be equal across firms. Using (11) and (14) to eliminate the social shadow cost from (12), we obtain the following system of equations

$$P'(Q)\dot{Q} - C_i''(q_i)\dot{q}_i = (r+\delta)[P(Q) - C_i'(q_i)] - D'(S), \quad \forall i = 0, 1, \ldots, n. \tag{15}$$

We now look for a steady state solution to the dynamical system defined by (1) and (15), i.e., a vector $(\hat{S}^\infty, \hat{q}_0^\infty, \hat{q}_1^\infty, \ldots, \hat{q}_n^\infty)$ such that the pollution stock S and individual output levels $(q_0, q_1, \ldots, q_n)$ do not change over time. The steady state solution is obtained by setting $\dot{S} = 0$, and $\dot{q}_i = 0, \forall i = 0, 1, \ldots, n$ in the system (1) and (15), summing over all i and solving for $(\hat{S}^\infty, \hat{q}_0^\infty, \hat{q}_1^\infty, \ldots, \hat{q}_n^\infty)$. There exists a unique optimal steady state stock of pollution and industry output level and it is defined by:

$$(\hat{q}_0^\infty + \sum_{i=1}^n \hat{q}_i^\infty) = \hat{Q}^\infty = \delta \hat{S}^\infty \tag{16}$$

where $\hat{S}^\infty$ satisfies the following equation

$$P(\delta \hat{S}^\infty) = C_i'(\hat{q}_i^\infty) + \frac{D'(\hat{S}^\infty)}{r+\delta} \tag{17}$$

and the respective share of each firm in the steady state industry output is given by the condition $C_i'(\hat{q}_i^\infty) = C_j'(\hat{q}_j^\infty), \ \forall i, j (i \neq j) \in \{0, 1, \ldots, n\}$.

Condition (17) establishes that the socially optimum output should be chosen so that marginal benefits equal marginal production costs plus the present value of marginal external damages. It can be clearly seen from above that firms' ownership structure is immaterial from the point of view of the social planner. Indeed, social optimality requires exclusively that allocative efficiency and cost efficiency conditions be satisfied. The optimal control rule can be expressed as a function of the optimal level of accumulated pollution S. The so-called feedback control rule $\hat{Q}(S)$ determines the current optimal aggregate level of production Q as a function of the current stock of pollution. As an illustration, we consider the following linear-quadratic specification of the model with linear inverse demand, quadratic damage cost and quadratic production costs

$$P(Q) = \alpha - \beta Q, \ D(S) = \frac{\gamma}{2}S^2, \ C_0(q_0) = \frac{c_0}{2}q_0^2, \ C_i(q_i) = \frac{c_1}{2}q_i^2, \forall i \neq 0, \tag{18}$$

where α, β, γ, c_0 and c_1 are positive constants. The steady-state is

$$\hat{S}^\infty = \frac{\alpha(r+\delta)}{\gamma + \delta(r+\delta)(\beta + c_0 c_1/(nc_0 + c_1))} \text{ and } \hat{Q}^\infty = \delta \hat{S}^\infty. \tag{19}$$

The feedback control rule $\hat{Q}(S)$ is

$$\hat{Q}(S) = \hat{Q}^\infty + (\rho_e + \delta)[S - \hat{S}^\infty] = (\rho_e + \delta)S - \rho_e \hat{S}^\infty, \tag{20}$$

where ρ_e is the negative root of the characteristic equation

$$\rho^2 - r\rho - J\gamma - r\delta - \delta^2 = 0, \text{ with } J = \frac{1}{(\beta + c_0 c_1/(c_1 + nc_0))}. \tag{21}$$

Finally, we have

$$\hat{q}_0(S) = \frac{c_1}{(nc_0 + c_1)}\hat{Q}(S), \quad \hat{q}_i(S) = \frac{c_0}{(nc_0 + c_1)}\hat{Q}(S), \forall i \neq 0. \tag{22}$$

3 Open-loop mixed oligopoly

In this section we assume that firms are unable to revise their production paths once they have made their choices; i.e., we assume an open-loop information structure. An open-loop Nash equilibrium is a profile of open-loop strategies such that no firm wishes to revise its strategy choice given the strategy choices of its rivals. From this definition, it follows that each private firms i $(i = 1, \ldots, n)$ chooses its time-path of production $q_i(.)$ so as to solve (6) taking as given the production paths of all other players and the tax rule $\tau_i(S)$. Similarly, the partially privatized firm chooses the time path of production $q_0(.)$ which solves problem (5) taking as given the production paths of all other players and the tax rule $\tau_0(S)$. Under an open-loop information structure, current value Hamiltonians for the firms are given by

$$H_i = \pi_i + \lambda_i(q_i + Q_{-i} - \delta S), \quad \forall i = 1, \ldots, n, \tag{23}$$
$$H_0 = (1 - \theta)W + \theta\pi_0 + \lambda_0(q_0 + Q_{-0} - \delta S). \tag{24}$$

The open-loop Nash equilibrium requires that the optimality conditions of the $(n + 1)$ optimal control problems hold simultaneously. Assuming interior solutions, the necessary conditions for optimality are given by

$$\frac{\partial H_i}{\partial q_i} = P'(Q)q_i + P(Q) - C_i'(q_i) - \tau_i(S) + \lambda_i = 0, \forall i \neq 0, \tag{25}$$

$$\frac{\partial H_0}{\partial q_0} = P(Q) - C_0'(q_0) + \lambda_0 + \theta\left[-\tau_0(S) + q_0 P'(Q)\right] = 0, \tag{26}$$

$$\dot{\lambda}_i = \lambda_i r - \frac{\partial H_i}{\partial S} = \lambda_i(r + \delta) + \tau_i'(S)q_i, \quad \forall i \neq 0, \tag{27}$$

$$\dot{\lambda}_0 = \lambda_0 r - \frac{\partial H_0}{\partial S} = \lambda_0(r + \delta) + \theta\tau_0'(S)q_0 + (1 - \theta)D'(S), \tag{28}$$

together with (1) and the $(n + 1)$ transversality conditions

$$\lim_{t \to \infty} e^{-rt}\lambda_i(t)S(t) = 0, \forall i = 0, 1, \ldots, n. \tag{29}$$

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Using (25) to eliminate the shadow cost λ_i from (27) the following conditions obtain,

$$P''\dot{Q}\, q_i + P'\dot{q}_i + P'\dot{Q} - C_i''\, \dot{q}_i - \tau_i'\dot{S} = (r+\delta)[P'q_i + P - C_i' - \tau_i] - \tau_i'q_i, \qquad (30)$$

$\forall i = 1,\ldots,n$ where $P = P(Q)$, $C_i' = C_i'(q_i)$ and $\tau_i = \tau_i(S)$. Similarly, from (26) and (28), the following condition obtains

$$C_0''\dot{q}_0 - P'\dot{Q} + \theta\left[\tau_0'\dot{S} - P''\dot{Q}q_0 - P'\dot{q}_0\right] = (1-\theta)D' + \theta\tau_0'q_0$$
$$+ (r+\delta)\left[C_0' - P + \theta\left(\tau_0 - P'q_0\right)\right] \quad (31)$$

We proceed with the stability analysis of the system defined by (30) and (31) together with (1). We look for a steady state solution where the stock of pollution $S(.)$ and individual output decisions $q = (q_0(.), q_1(.), \ldots, q_n(.))$ remain constant over time. Steady state conditions for an open-loop Nash equilibrium are obtained by setting $\dot{Q} = 0$ and $\dot{q}_0 = \dot{q}_1 = \cdots = \dot{q}_n = 0$ in (30) and (31). It comes that the open-loop Nash equilibrium steady state pollution stock S_{ol}^{∞} must satisfy the following system of $(n+1)$ equations

$$(r+\delta)(C_i' - P) = (P'q_i - \tau_i)(r+\delta) - \tau_i'q_i, \quad \forall i = 1,\ldots,n, \qquad (32)$$
$$(r+\delta)(C_0' - P) + D' = \theta\left[(P'q_0 - \tau_0)(r+\delta) - \tau_0'q_0 + D'\right]. \qquad (33)$$

where arguments have been omitted for sake of brevity.

Now we are in a position to study how the environmental regulator can decentralize the social optimum as an open-loop Nash equilibrium of the dynamic game played by the firms. The regulator designs the tax scheme $\tau(S)$ so that firms optimality conditions match the socially optimal conditions. To begin with, we restrict our attention to open-loop Nash equilibrium conditions. First we derive the condition that $\tau_i(S)$ must satisfy in order to induce a given private firm i to behave in accordance with the social optimum. By comparison of (30) with (15) the following condition obtains

$$\tau_i'\dot{S} - P''\dot{Q}q_i - P'\dot{q}_i - (r+\delta)(\tau_i - P'q_i) + D' - \tau_i'q_i = 0 \qquad (34)$$

where arguments have been omitted to save space. Second we derive the corresponding condition for an optimal regulation of the partially privatized firm. Comparing conditions (31) and (15) yields

$$\tau_0'\dot{S} - P''\dot{Q}q_0 - P'\dot{q}_0 - (r+\delta)\left(\tau_0 - P'q_0\right) + D' - \tau_0'q_0 = 0. \qquad (35)$$

Now, we proceed by considering steady state conditions for an open-loop Nash equilibrium. From section 2, we know that $(r+\delta)(C_i' - P(\delta\hat{S}_\infty)) + D'(\hat{S}_\infty) = 0$. Therefore, conditions (32) and (33) rewrites as

$$P(\delta\hat{S}^\infty) = C_i' + \tau_i(\hat{S}^\infty) + q_i(\hat{S}^\infty)\left[\frac{\tau_i'(\hat{S}^\infty)}{(r+\delta)} - P'(\delta\hat{S}^\infty)\right], \qquad (36)$$

$\forall i = 0,\ldots,n$. The profile of markovian tax rules $\tau(S)$ must satisfy conditions (34) and (35) and $\tau(\hat{S}_\infty)$ must satisfy the $n+1$ steady state conditions for an open-loop Nash equilibrium (36). Clearly, these conditions are independent of θ. We thus obtain the

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irrelevance result stated in the following proposition:

Proposition 1 *When the environmental regulator uses efficiency inducing taxation in order to regulate a polluting oligopoly, the optimal linear-Markov taxation scheme, the time-path of pollution accumulation, firms' time-paths of production and profit streams are identical irrespective of whether i) all $(n+1)$ firms behave as profit maximizers or ii) a partially privatized firm competes in quantities with n private firms.*

Remark 1 *Let us observe the formal relationship that relates the long-run optimal tax to the optimal corrective policy that would obtain in a static setting. Assuming an unitary emission output ratio, environmental damage can be writen as a function of the aggregate industry output so that social welfare becomes $W = \int_0^Q P(u)\,du - \sum_{i=0}^n C_i(q_i) - D(Q)$ and the socially optimal allocation is now determined by*

$$P(Q) - C_i'(q_i) - D'(Q) = 0, \quad \forall i = 0, 1, \ldots, n. \tag{37}$$

Suppose that firms are charged a tax τ_i per unit of emissions so that firm i's tax bill is $\tau_i q_i$. Under Cournot competition, the firms' first order conditions are then given by

$$P(Q) - C_0'(q_0) - (1-\theta)D'(Q) + \theta[-\tau_0 + P'(Q)q_0] = 0, \tag{38}$$
$$P(Q) + P'(Q)q_i - C_i'(q_i) - \tau_i = 0, \forall i = 1, 2, \ldots, n. \tag{39}$$

Straightforward comparisons of the firms first-order conditions with the social optimum reveal that the optimal tax is

$$\tau_i = D'(Q) + P'(Q)q_i, \quad \forall i = 0, 1, \ldots, n. \tag{40}$$

A similar expression could have been derived from equation (36) by writing off the actualisation parameter $(r = 0)$, assuming that polluting emissions are instantaneously assimilated by the environment $(\delta = 1)$ and replacing the tax rule $\tau_i(S)$ by a per unit tax τ_i. It is important to note that proposition 1 not only shows that the optimal tax rule is independent of θ at the steady state–as intuitions from the static model would suggest–but also, and more surprisingly, all over the planning period.

The basic intuition for proposition 1 is simple. To begin with, we restrict our attention to the two limiting cases: the regulation of a private oligopoly and that of a pure mixed oligopoly. The first is obtained by setting $\theta = 1$ in the objective of the public firm. In this case, the public firm is a profit maximizer and the problem boils down to the regulation of a private polluting oligopoly. From Benchekroun and Van Long (1998), we know that there exists an optimal tax rule which induces firms to follow the socially optimal production path. The second is obtained by setting $\theta = 0$. In this case, the privatized firm maximizes aggregated social welfare. Corrective taxation does not affect the output decision of the public firm directly; the tax only affects the behavior of the public firm through its effect on private firms' output levels. Now, suppose that the regulator uses the tax rule obtained in the private oligopoly case to regulate the pure mixed oligopoly. Then, private firms follow the optimal production path. Since the public firm seeks to maximize social welfare, its best response to the behavior of

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private firms is also to follow the socially optimal production path.

Now let us consider intermediate cases; i.e., $\theta \in]0,1[$. In these cases, the partially privatized firm deviates from strict welfare maximization. However, its behavior is not exclusively profit oriented. Suppose that the regulator uses the tax rule obtained in the private oligopoly case to regulate the mixed market. Now, corrective taxation affects the behavior of the public firm directly since it appears in its profits. Public and private owners of the public firm have an common interest in following the socially optimal production path. Indeed, it would be the policy chosen by the public shareholders if they were the unique owners of the privatized firm and the choice of private shareholders if they were the unique owners of the privatized firm. As an illustration, we characterize the linear-Markov tax policy in two special cases: first, under the assumption that all firms (public and private) use the same technology; and second, under the linear-quadratic specification introduced in section 2.

Identical Firms

Consider the special case where all firms are identical in costs and technology so that $C_i(q_i) = C(q_i), \forall i = 0, 1, \dots, n$. Productive efficiency requires that all firms choose identical output levels at each instant of time: $q_i(t) = q(t), \forall i = 0, 1, \dots, n$. Let us assume at the outset that technologically identical firms face the same tax treatment; i.e., $\tau_i(S) = \tau(S), \forall i = (0, 1, \dots, n)$. By summing equations (34) and (35), we get:

$$(n+1)\tau' \dot{S} - P'' \dot{Q} Q - P' Q - (r+\delta)\left((n+1)\tau - P'Q\right) + (n+1)D' - \tau' Q = 0 \quad (41)$$

Substituting $d\hat{S}(t)/dt$ for $\hat{Q}(S) - \delta S$ and $d\hat{Q}(S)/dt$ for $\hat{Q}'(S)[\hat{Q}(S) - \delta S]$ in equation 41 yields a first-order linear differential equation in τ which can be written as

$$\tau(S) + A(S)\tau'(S) = B(S) \quad (42)$$

where

$$A(S) = \frac{\left((n+1)\delta S - nQ(S)\right)\tau'(S)}{(n+1)(r+\delta)} \quad (43)$$

$$B(S) = -\frac{D'(S)}{(r+\delta)} - \frac{Q(S)}{(n+1)}P' + \frac{Q'(S)\left(Q(S) - \delta S\right)\left(P' + Q(S)P''\right)}{(n+1)(r+\delta)} \quad (44)$$

Summing the $n+1$ equations (36) yields the boundary condition

$$P(\delta \hat{S}^{\infty}) = C' + \tau(\hat{S}^{\infty}) + \frac{\delta \hat{S}^{\infty}}{(n+1)}\left[\frac{\tau'(\hat{S}^{\infty})}{(r+\delta)} - P'(\delta \hat{S}^{\infty})\right]. \quad (45)$$

The optimal tax rule $\tau(S)$ is obtained as the general solution of equation (42):

$$\tau(S) = K \exp\left[\int_0^S -A(u)du\right] + \tau^p(S) \quad (46)$$

where $\tau^p(S)$ is a particular solution of (42) and K is a constant determined by (45).

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Linear-quadratic specification

Now, we proceed by solving for the optimal tax rule under the linear quadratic specification introduced in section 2 for which analytical solutions can be obtained. Under linear demand, quadratic damage cost and quadratic production costs, the optimal taxation scheme $\tau(S) = (\eta_0^* + \sigma_0^* S, \eta_1^* + \sigma_1^* S)$ is given by

$$\sigma_0^* = \frac{\gamma(1 + \beta JK)}{(r + \delta) + K(\delta + \rho_e) - \rho_e}, \eta_0^* = \frac{\rho_e \left[\sigma_0^*(K-1) + K\beta(r - \rho_e)\right]\hat{S}^\infty}{(r + \delta)} \tag{47}$$

$$\sigma_1^* = \frac{\gamma(1 + \beta JL)}{(r + \delta) + L(\delta + \rho_e) - \rho_e}, \eta_1^* = \frac{\rho_e \left[\sigma_1^*(L-1) + L\beta(r - \rho_e)\right]\hat{S}^\infty}{(r + \delta)} \tag{48}$$

where $K = c_1/(nc_0 + c1)$, $L = c_0/(nc_0 + c1)$ and $J = 1/(\beta + c_0 K)$, decentralizes the social optimum as an OLNE. Note that $K = L$ if $c_0 = c_1$. Obviously, the optimal system of tax rules requires that public and private firms be taxed the same if they use the same technology. In this case, it equalizes partially privatized and private firms' production.

4 Closed-loop mixed oligopoly

The analysis of subsection 3 has confined itself to oligopolistic situations in which firms make use of open-loop strategies. By focusing exclusively on open-loop solution concepts, it excludes strategic interactions between firms through the evolution of the state variable over time and the associated adjustment in controls. We now proceed by considering the broader class of closed-loop strategies in order to prove that our irrelevance result is not contingent upon assumptions regarding the informational structure of the game. Since optimality conditions for private firms are independent of θ, we may restrict our attention to the behavior of the partially privatized firm. Now, each firm assumes that the strategies used by its competitors are a function of the accumulated stock. Accordingly, firm 0 chooses the output path $q_0^*(t)$ which maximizes its discounted payoff F_0 subject to (1) and its current value Hamiltonian is

$$
\begin{aligned}
H_0 &= (1 - \theta)\left[\int_0^{q_0 + Q_{-0}(S)} P(u)du - C_0(q_0) - \sum_{i=1}^n C_i(\phi_i(S)) - D(S)\right] \\
&\quad + \theta\left[P(q_0 + Q_{-0}(S))q_0 - C_0(q_0) - \tau_0(S)q_0\right] + \lambda_0\left[q_0 + Q_{-0}(S) - \delta S\right]
\end{aligned}
\tag{49}
$$

where $Q_{-0}(S) = \sum_{i=1}^n \phi_i(S)$. Assuming interior solutions, the necessary and sufficient conditions are

$$\lambda_0 = (C_0' - P) + \theta\left[\tau_0 - q_0 P'\right], \tag{50}$$

$$
\begin{aligned}
\dot{\lambda}_0 &= (1 - \theta)\left[\sum_{i=1}^n C_i' \phi_i' + D' - PQ_{-0}'\right] + \theta q_0\left[\tau_0' - P'Q_{-0}'\right] \\
&\quad + \lambda_0\left[(r + \delta) - Q_{-0}'\right],
\end{aligned}
\tag{51}
$$

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and $\lim_{t\to\infty} e^{-rt}\lambda_0(t)S(t) = 0$, along with the dynamic process of pollution accumulation (1). Assuming identical private firms and following the same steps as in section 3, one obtains the following condition on $\tau_0^*(S)$:

$$\Lambda + Q'_{-0}\left(\tau_0 + (C'_i - P)\right) = 0 \tag{52}$$

where $\Lambda = \tau_0'\dot{S} - P''\dot{Q}q_0 - P'\dot{q}_0 - (r+\delta)\left(\tau_0 - P'q_0\right) + D' - \tau_0'q_0$ is the bracketed term in (35). Again, we observe that this equation will be satisfied or not regardless of the value of θ provided that it is different from zero. Following the same steps as in section 3, it is straightforward to show that the corresponding steady state condition for a closed-loop Nash equilibrium is independent of θ. The system of linear Markov tax rules must satisfy a system of differential equations that is independent of θ and thus the following proposition obtains:

Proposition 2 *When the environmental regulator uses efficiency inducing taxation in order to regulate a polluting oligopoly, the optimal linear-Markov taxation scheme, the time-path of pollution accumulation, firms' time-paths of production and profit streams are identical irrespective of whether i) all $(n+1)$ firms behave as profit maximizers or ii) a partially privatized firm competes in quantities with n private firms.*

As in section 4, the general characterization of the optimal markovian tax scheme for the symmetric model and explicit solutions for the linear-quadratic model can be easily derived[8].

5 Conclusion

We considered efficiency-inducing taxation for a polluting mixed market in which a partially privatized firm competes with private firms. The analysis of this paper provided some answers to hitherto neglected questions about the interaction between privatization and environmental taxation. Assuming that the partially privatized firm maximizes a weighted average of social welfare and its own profit, we proved that the optimal corrective tax scheme is independent of the weight the privatized firm puts on its own profit; i.e., the extent of privatization. This result tells us that technologically identical privatized and private firms should be taxed the same even if they have different incentives to produce. It was shown that this conclusion holds with respect to the regulation of both open-loop and closed-loop polluting oligopolies.

Turning to the welfare effect of privatization, we proved that social welfare is unchanged by privatization when the optimal pollution tax rule is used. Actually, our analysis showed that the optimal tax rule guides polluting oligopolists to achieve the socially optimal production path. Since the social optimum is unique and the optimal tax rule is independent of the extent of privatization, the same level of aggregate welfare is achieved irrespective of the ownership status of the public firm.

Acknowledgements

Notes

[1] See, for example, Bergstrom et al. (1981); Karp and Livernois (1992, 1994)

[2] An exception is Barcena-Ruiz and Garzon (2002) who examined effluent taxation in a mixed oligopoly with a welfare maximizing public firm and several private firms.

[3] See also Sasaki and Wen (2001); Lee and Hwang (2003); Matsumura and Kanda (2005); Sun, Zhang and Li (2005).

[4] As will be shown this result also holds in a static setting.

[5] For a recent survey of empirical studies of privatization, see Megginson and Netter , 2001. Environmental implications of privatisation are extensively reviewed in Lovei and Gentry , 2002.

[6] Alternatively, this objective function can be interpreted in terms of strategic delegation (See for example Vickers (1985) and Fershtman and Judd (1987)). In this second interpretation, the government delegates the control of the public firm to a private manager and θ measures the extent of the delegation

[7] We will omit the time argument t whenever this does not cause confusion.

[8] Detailed derivations are available upon request from the authors.

Appendix A

Social optimum in the linear-quadratic model

In this appendix we characterize the social optimum under the linear quadratic specification (18). As a first step, we solve for the steady state level of the pollution stock. From (16) and (17), the steady state level of pollutant stock is

$$\hat{S}^\infty = \frac{(nc_0 + c_1)\alpha(r + \delta)}{\delta\left[(r + \delta)(\beta(nc_0 + c_1) + c_0 c_1) + \gamma(nc_0 + c_1)\right]}, \tag{53}$$

$$\hat{Q}^\infty = \delta\hat{S}^\infty, \quad \hat{q}_0^\infty = \frac{c_1}{(nc_0 + c_1)}\hat{Q}^\infty, \quad \hat{q}_1^\infty = \frac{c_0}{(nc_0 + c_1)}\hat{Q}^\infty \tag{54}$$

We now proceed with the characterization of the unique trajectory which satisfies all necessary conditions for optimality and ensures the convergence of $S(t)$ to the steady state. Under the linear

quadratic specification, short run conditions (11) become

$$\lambda_r = -\alpha + \beta Q + c_i q_i = 0, \quad \forall i = \{0, 1\}. \tag{55}$$

From proposition (1), we have $c_0 q_0 = c_1 q_1$. Accordingly, we have $q_0 = (c_1/c_0)q_1$, $q_0 = \frac{c_1}{nc_0+c_1}Q$ and $q_i = \frac{c_0}{nc_0+c_1}Q$. Then, using this piece of information, the system (55) reduces to a unique equation

$$Q = \frac{1}{\beta + (c_0 c_1)/(nc_0 + c_1)}(\lambda_r + \alpha) \tag{56}$$

which can be differentiated with respect to time to get

$$\dot{Q} = \frac{1}{\beta + (c_0 c_1)/(nc_0 + c_1)}\dot{\lambda}_r. \tag{57}$$

Using the adjoint equation (12), we rewrite as follows

$$\dot{Q} = \frac{1}{\beta + (c_0 c_1)/(nc_0 + c_1)}(\lambda_r(r + \delta) + \gamma S). \tag{58}$$

Finally, substituting (53) into (58) gives

$$\dot{Q} = \frac{1}{\beta + (c_0 c_1)/(nc_0 + c_1)}\left(-\alpha + \left(\beta + \frac{(c_0 c_1)}{(nc_0 + c_1)}\right)(r + \delta)Q + \gamma S\right). \tag{59}$$

Therefore, the Hamiltonian differential system reduces to a system of first order linear differential equations

$$\dot{S}(t) = Q(t) - \delta S(t), \tag{60}$$
$$\dot{Q}(t) = Q(t)(r + \delta) + \gamma J S(t) - \alpha J, \tag{61}$$

which can be rewritten as $\dot{y} = Ay(t) + B$ where $y(t) = (S(t), Q(t))'$,

$$A = \begin{pmatrix} -\delta & 1 \\ \gamma J & (r + \delta) \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 \\ -\alpha J \end{pmatrix}. \tag{62}$$

The characteristic equation of (61) is defined by $det(\rho I - A) = 0$ where I is the identity matrix:

$$-(J\gamma) - r\delta - \delta^2 - r\rho + \rho^2 = 0 \tag{63}$$

The roots of the characteristic equation are

$$\rho_e = \frac{r - \sqrt{r^2 + 4J\gamma + 4r\delta + 4\delta^2}}{2}, \; \rho_d = \frac{r + \sqrt{r^2 + 4J\gamma + 4r\delta + 4\delta^2}}{2} \tag{64}$$

Note that these two roots are real and of opposite sign, confirming a saddle point solution. The positive root ρ_d corresponds to a diverging branch of the saddle point and is ruled out by the transversality condition. It follows that there exists a unique solution of the Hamiltonian system that converges to the saddle point for every initial stock of pollution S_0. This solution corresponds to the negative root ρ_e.

On the basis of ρ_e we can proceed to the characterization of equilibrium quantities. The optimal evolution of S is

$$\hat{S}(t) = (S_0 - \hat{S}^\infty)e^{\rho_e t} + \hat{S}^\infty. \tag{65}$$

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Since $Q = \dot{S} + \delta S$, firms' equilibrium strategies are

$$\hat{q}_0(t) = \frac{c_1}{(nc_0 + c_1)} \left[(S_0 - \hat{S}^\infty)(\rho_e + \delta)e^{\rho_e t} + \delta \hat{S}^\infty \right], \ \hat{q}_1(t) = \frac{c_0}{c_1}\hat{q}_0(t) \tag{66}$$

From (65) and (66) the socially optimal feedback control rule $\hat{Q}(S)$ is

$$\hat{Q}(S) = \hat{Q}^\infty + (\rho_e + \delta)[\hat{S} - \hat{S}^\infty] = (\rho_e + \delta)\hat{S} - \rho_e \hat{S}^\infty, \tag{67}$$

and

$$\hat{q}_0(S) = \frac{c_1}{(nc_0 + c_1)}\hat{Q}(S), \qquad \hat{q}_1(S) = \frac{c_0}{(nc_0 + c_1)}\hat{Q}(S). \tag{68}$$

Markovian taxation in the open-loop linear-quadratic model

In this appendix, we extend the analysis conducted in Appendix 1 and solve for the optimal tax scheme $\tau(S) = (\tau_0(S), \tau_1(S), \ldots, \tau_1(S))$ in the open-loop scenario. With this intention in mind, we prove that differential equations (34) and (35) have solutions of the form $\tau_i(S) = \eta_i + \sigma_i S$. Replacing dS/dt by $\hat{Q}(S) - \delta S$, $d\hat{Q}(S)/dt$ by $\hat{Q}'(S)(\hat{Q}(S) - \delta S)$, $\hat{q}_1(S)$ by $(c_0/(nc_0 + c_1))\hat{Q}(S)$ and $\hat{q}_0(S)$ by $(c_1/(nc_0 + c_1))\hat{Q}(S)$, we obtain two independent first order linear differential equations

$$\tau_0(S) + A_0(S)\tau_0'(S) = B_0(S), \tag{69}$$
$$\tau_1(S) + A_1(S)\tau_1'(S) = B_1(S), \tag{70}$$

where

$$A_0(S) = \frac{1}{(r+\delta)}\left[\delta S + \hat{Q}\left(\frac{c_1}{(nc_0 + c_1)} - 1 \right) \right], \tag{71}$$

$$B_0(S) = \frac{D'}{(r+\delta)} + \frac{c_1}{(nc_0 + c_1)}\left[P'\hat{Q} - \frac{1}{(r+\delta)}\left(P''\hat{Q} + P' \right)\hat{Q}'(\hat{Q} - \delta S) \right] \tag{72}$$

$$A_1(S) = \frac{1}{(r+\delta)}\left[\delta S + \hat{Q}\left(\frac{c_0}{(nc_0 + c_1)} - 1 \right) \right], \tag{73}$$

$$B_1(S) = \frac{D'}{(r+\delta)} + \frac{c_0}{(nc_0 + c_1)}\left[P'\hat{Q} - \frac{1}{(r+\delta)}\left(P''\hat{Q} + P' \right)\hat{Q}'(\hat{Q} - \delta S) \right] \tag{74}$$

Replacing $\tau_0(S)$ by $\eta_0 + \sigma_0 S$, $\tau_1(S)$ by $\eta_1 + \sigma_1 S$, and $\hat{Q}(S)$ by expression (20) and collecting all terms that have S as a common factor, equations (69) and (70) can be rewritten as

$$r_0(\sigma_0, \eta_0)S + s_0(\sigma_0, \eta_0) = 0, \tag{75}$$
$$r_1(\sigma_1, \eta_1)S + s_1(\sigma_1, \eta_1) = 0. \tag{76}$$

where

$$r_0(\sigma_0, \eta_0) = \frac{(r+\delta)\eta_0 + \rho_e\left[\sigma_0(1 - K) + \beta K(\rho_e - r)\right]\hat{S}^\infty}{(r+\delta)} \tag{77}$$

$$r_1(\sigma_1, \eta_1) = \frac{(r+\delta)\eta_1 + \rho_e\left[\sigma_1(1 - L) + \beta L(\rho_e - r)\right]\hat{S}^\infty}{(r+\delta)} \tag{78}$$

$$s_0(\sigma_0, \eta_0) = \frac{\left[-\gamma + \sigma_0(r+\delta - \rho_e) + K(\sigma_0 + \beta(r+\delta - \rho_e))(\delta + \rho_e)\right]S(t)}{(r+\delta)} \tag{79}$$

$$s_1(\sigma_1, \eta_1) = \frac{\left[-\gamma + \sigma_1(r+\delta - \rho_e) + L(\sigma_1 + \beta(r+\delta - \rho_e))(\delta + \rho_e)\right]S(t)}{(r+\delta)} \tag{80}$$

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Each of these equations must be satisfied for all $S \geq 0$. Accordingly, for the system of linear tax rules $\tau(S) = (\eta_0^* + \sigma_0^* S, \eta_1^* + \sigma_1^* S, \ldots, \eta_1^* + \sigma_1^* S)$ to be a solution of (69-70), it must hold that

$$r_0(\sigma_0^*, \eta_0^*) = 0, \quad s_0(\sigma_0^*, \eta_0^*) = 0, \tag{81}$$

$$r_1(\sigma_1^*, \eta_1^*) = 0, \quad s_1(\sigma_1^*, \eta_1^*) = 0. \tag{82}$$

Solving this system we obtain the required expressions for $\tau_0^*(S)$ and $\tau_1^*(S)$.

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