Comparing Welfare and Profit in Quantity and Price Competition within Stackelberg Mixed Duopolies

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20 March 2017

Comparing Welfare and Profit in Quantity and Price Competition within Stackelberg Mixed Duopolies*

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March 20, 2017

Abstract

We compare welfare and profits under price and quantity competition in mixed duopolies, wherein a state-owned public firm competes against a private firm. It has been shown that price competition yields larger profit for the private firm and greater welfare if the two firms move simultaneously, regardless of whether the private firm is domestic or foreign. We investigate welfare and profit rankings under Stackelberg competition. Under public leadership, the profit and welfare rankings have common features with the simultaneous-move game, regardless of the nationality of private firms. By contrast, under private leadership, the result depends on the nationality of the private firm. When the private firm is domestic, welfare is greater under quantity competition, while the result is reversed when the private firm is foreign. However, regardless of nationality, private firms earn more under price competition. Introducing the nonnegative profit constraint in the public firm improves welfare and increases the private firm's profit, and price competition yields a higher profit for private firms regardless of nationality and which firm is the leader. However, this constraint affects the welfare ranking. Under private leadership, quantity competition yields greater welfare regardless of the nationality of the private firm. These results indicate that profit ranking is fairly robust to the time structure in mixed Stackelberg duopolies, but welfare ranking is not.

JEL classification numbers: H42, H44, L13, L32

Keywords: public leadership, private leadership, mixed markets, Cournot–Bertrand comparison

*We acknowledge financial support from JSPS KAKENHI Grant Numbers (15K03347, 16J04589) and the Zengin Foundation for Studies on Economics and Finance. Needless to say, we are responsible for any remaining errors.

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1 Introduction

The literature contains extensive comparisons between price and quantity competition. In oligopolies of private firms, price competition is stronger, yielding lower profits and greater welfare than in the case of quantity competition.[1] Ghosh and Mitra (2010) revisited the comparison between price and quantity competition in a mixed duopoly in which a welfare-maximizing public firm competes against a profit-maximizing private firm.[2] They showed that price competition yields larger profit for the private firm and greater welfare than quantity competition. In other words, welfare ranking is common with private duopolies but profit ranking is the opposite.

The literature on Cournot–Bertrand comparison in mixed oligopolies has become rich and diverse recently. Haraguchi and Matsumura (2014) showed that Ghosh and Mitra's (2010) result holds, regardless of the nationality of the private firm. Scrimitore (2014) adopted Matsumura's (1998) partial privatization approach and considered the optimal degree of privatization. Her findings showed that under optimal privatization policies, Cournot competition could yield higher profits for private firms than under Bertrand competition. The optimal degree of privatization is lower under Bertrand competition, and a lower degree of privatization leads to stronger competition. Here, the profit ranking can be reversed under optimal privatization policies, while welfare ranking is not (Bertrand yields greater welfare). Haraguchi and Matsumura (2016) investigated an oligopoly model and showed that the profit-ranking can be reverted if the number of private firms is large, whereas the welfare-ranking is not. Ghosh and Mitra (2014) discussed a case in which both firms are concerned with welfare and showed that Cournot competition could yield higher profits for the firms than under Bertrand competition. The aforementioned studies, however, assumed that firms play simultaneous-move games.

[1] See Shubik and Levitan (1980) and Vives (1985). Introducing some externality effects can undoubtedly reverse the welfare ranking. For example, if we introduce a negative externality associated with production, a lower output level under quantity competition can yield greater welfare. In this study, we completely neglect this type of technological externality.

[2] Most countries have state-owned public firms with substantial influence on their market competitors. Such mixed oligopolies occur in various industries, such as airlines, steel, automobile, railway, natural gas, electricity, postal services, education, hospital, home loan, and banking. Analyses of mixed oligopolies date back to Merrill and Schneider (1966). Their study and many others in the field assume that a public firm maximizes welfare (consumer surplus plus firm profits), while private firms maximize profits. For examples of mixed oligopolies and recent developments in research in this field, see Ishida and Matsushima (2009), Tomaru and Kiyono (2010), and Cato and Matsumura (2012).

Another strand of the literature related to this study is on Stackelberg mixed oligopolies. The literature on mixed oligopolies contains intense discussions of both public leadership and private leadership models.[3] Wang and Mukherjee (2012) and Wang and Lee (2013) considered public leadership in a homogeneous product market and showed that the public leadership benefits total social surplus but is less beneficial for consumer welfare than public monopoly.[4] Gelves and Heywood (2013) found that mergers of public and private firms can improve welfare under public leadership. Pal (1998) showed that welfare is greater in the private leadership game than in the public leadership and simultaneous-move games, and Ino and Matsumura (2010) showed it in a free-entry market. Poyago-Theotoky (2001) and Myles (2002) showed that White's (1996) privatization neutrality theorem holds under public leadership, and Tomaru and Saito (2010) showed it under private leadership. Matsumura and Okumura (2015, 2017) illustrated the same under two sequential move games by output-floor regulation in non-free-entry markets and free-entry markets. However, all of these studies assumed quantity competition and did not discuss the price–quantity comparison.

In this study, we consider two sequential-move games, public leadership and private leadership games, and revisit the price–quantity comparison in mixed duopolies. In private duopolies with sequential move, Boyer and Moreaux (1987) showed that quantity competition is more profitable and price competition provides higher total welfare, whatever the role (leader, follower). Thus, we can naturally suppose that in mixed duopolies, the two sequential-move games also provide the same welfare and profit ranking. This supposition is, however, not correct. The two sequential-move games provide the same profit ranking

[3]Both public leadership and private leadership models are important for analyzing Japanese financial markets, which are typical examples of mixed oligopolies. Until the 1980s, public enterprises played a leading role in the Japanese economy. It was believed that lending by public financial institutions (e.g., the Development Bank of Japan) had a pump-priming effect on private bank lending. Furthermore, public financing occupied an important position in Japanese financial markets for over 40 years (Horiuchi and Sui, 1993). The Koizumi Cabinet (April 2001–September 2006) changed this by declaring that public firms should play a complementary role to private firms, with the latter leading the markets rather than the former. Consequently, major public institutions were substantially downscaled. This situation can be described by the private leadership model (Ino and Matsumura, 2010, Matsumura and Ogawa, 2017). However, public institutions recently begun to lead Japanese markets once more. Newly established public financial institutions such as the Industrial Revitalization Corporation of Japan, the Enterprise Turnaround Initiative Corporation of Japan, and the Regional Economy Vitalization Corporation of Japan play leading roles in financial markets (Matsumura and Ogawa, 2017). The public leadership model is also a useful means to investigate this situation.

[4]They established another great contribution by showing that each private firm's profit can be increasing with the number of private firms in their mixed oligopolies. For this discussion, see also Matsumura and Sunada (2013).

as the simultaneous-move game, but the private leadership game yields a different welfare ranking from that of the simultaneous-move and public leadership games.

First, we analyze a model in which the private (public) firm is the leader (follower). In the context of mixed oligopolies, the public firm's acceptance of the follower role often improves welfare (Pal, 1998; Matsumura, 2003a; Ino and Matsumura, 2010; Matsumura and Ogawa, 2017). Many stated that the public firm should play a complementary role to private firms, and the role of follower is adequate from this viewpoint. In this study, we show that when the public firm is the follower, quantity competition is stronger than price competition, resulting in a smaller profit for the private firm. This result is in accordance with that in the simultaneous-move game. However, the welfare ranking can be reversed. When the private firm is domestic (foreign), quantity (price) competition yields greater welfare than price (quantity) competition. In other words, the welfare ranking is crucially dependent on the nationality of the private firm.[5]

Next, we analyze a model in which the public (private) firm is the leader (follower). We find the same profit and welfare rankings as in the simultaneous-move game, that is, price competition yields higher profit for the private firm and greater welfare, regardless of the nationality of the private firm.

We then introduce the nonnegative profit constraint for the public firm. The literature on mixed oligopolies shows that the public firm's welfare-maximizing behavior may yield negative profits for the public firm.[6] However, as Estrin and de Meza (1995), Ishida and Matsushima (2009), and Wang and Tomaru (2015) discussed, we often observe that the nonnegative profit constraint is imposed on public firms. We find that the welfare and profit ranking remains unchanged under public leadership. By contrast, under private leadership, the nonnegative profit constraint affects the welfare ranking. This constraint improves welfare with quantity competition, while welfare remains unchanged with price competition. As a result, quantity competition more likely yields greater welfare than price competition.

[5]The nationality of private firms is often crucial in shaping mixed oligopolies; refer to the literature starting with Corneo and Jeanne (1994) and Fjell and Pal (1996). For recent developments in this field, refer to Bárcena-Ruiz and Garzón (2005a,b), Han and Ogawa (2008), Lin and Matsumura (2012), and Cato and Matsumura (2015).

[6]See the studies mentioned in footnote 5.

Finally, we discuss the endogenous competition structure (endogenous price-quantity choice) as discussed by Singh and Vives (1984). They formulated a two-stage game. In the first stage, each firm simultaneously chooses a price or quantity contract. In the second stage, after observing the rival's choice in the first stage, each firm simultaneously chooses price or quantity according to the first stage choice. They investigated a private duopoly and showed that both firms choose the quantity contract. Cournot competition therefore appears in equilibrium. Matsumura and Ogawa (2012) investigated this endogenous competition structure in a mixed duopoly and showed that both firms choose the price contract. Bertrand competition therefore appears in equilibrium.[7]

We investigate this problem in public and private leadership games.[8] We consider two time lines: one where two firms simultaneously choose a price or quantity contract before facing Stackelberg competition, and the other in which the leader chooses between price or quantity first, before the follower chooses after observing the leader's price or quantity. We find that in both timelines, under both public and private leadership, price competition appears in equilibrium, regardless of whether quantity or price competition yields greater welfare. This indicates that equilibrium competition structure can be inefficient under private leadership.

2 Model

We adopt a standard duopoly model with differentiated goods and linear demand (Dixit, 1979).[9] The quasi-linear utility function of the representative consumer is

$$U(q_0, q_1, y) = \alpha(q_0 + q_1) - \frac{\beta}{2}(q_0^2 + 2\delta q_0 q_1 + q_1^2) + y, \tag{1}$$

[7]However, not all studies on mixed oligopolies support this result. Chirco and Scrimitore (2013), Chirco *et al.* (2014), Scrimitore (2013), and Haraguchi and Matsumura (2016) suggested that price competition may fail to be an equilibrium outcome.

[8]Because our primary aim is to revisit price–quantity comparisons in Stackelberg mixed duopolies, we do not endogenize the timing of their action. Although many papers discussed endogenous timing in mixed oligopolies, endogenizing the timing is out of scope of this study. Pal (1998) began the discussion of endogenous timing in mixed oligopolies. Many papers, such as those by Matsumura (2003a), Capuano and De Feo (2010), and Matsumura and Ogawa (2010) showed that private leadership is more likely to appear in mixed oligopolies. However, Matsumura (2003b) and Matsumura and Ogawa (2017) showed examples in which public leadership is more likely to appear in equilibrium.

[9]This demand function is popular in the literature on mixed oligopolies. Refer to Bárcena-Ruiz (2007), Ishida and Matsushima (2009), and Matsumura and Shimizu (2010).

where q_0 is the consumption of good 0 produced by the public firm, q_1 is the consumption of good 1 produced by the private firm, and y is the consumption of an outside good provided competitively (with a unitary price). Parameters α and β are positive constants and $\delta \in (0,1)$ represents the degree of product differentiation, where a smaller δ indicates a larger degree of product differentiation. The inverse demand functions for goods $i = 0, 1$ with $i \neq j$ are

$$p_i = \alpha - \beta q_i - \beta \delta q_j, \tag{2}$$

where p_i is the price of firm i.

The marginal cost of production is constant for both firms. We denote the marginal cost of firm i with c_i, assuming $\alpha > c_0 \geq c_1$. In addition, we assume that α is sufficiently large and that $c_0 - c_1$ is not too large to assure interior solutions in the following games. Firm 0 is a state-owned public firm whose payoff is the domestic social surplus (welfare). This is given by

$$SW = (p_0 - c_0)q_0 + (1-\theta)(p_1 - c_1)q_1 + \left[\alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2} - p_0 q_0 - p_1 q_1\right], \tag{3}$$

where $\theta \in [0,1]$ is the ownership share of foreign investors in firm 1, which is potentially affected by policymakers acting on capital liberalization. Firm 1 is a private firm and its payoff is its own profit:

$$\pi_1 = (p_1 - c_1)q_1. \tag{4}$$

3 Private leadership: Public firm as the Stackelberg follower

In this section, we analyze a model in which firm 0 (1) is the follower (leader). First, we discuss quantity competition. Firm 1 chooses its quantities and then firm 0 moves after observing q_1. The first-order condition for firm 0 is[10]

$$\frac{\partial SW}{\partial q_0} = \alpha - c_0 - \beta(q_0 + (1-\theta)\delta q_1) = 0. \tag{5}$$

From (5), we obtain the following reaction function for firm 0:

$$R_0(q_1) = \frac{\alpha - c_0 - \beta(1-\theta)\delta q_1}{\beta}. \tag{6}$$

[10] All of the second-order conditions in this study are satisfied.

Firm 1 maximizes its profit, $\pi_1(q_1, R_0(q_1))$, with respect to q_1. The first-order condition for firm 1 is

$$\alpha - c_1 - \delta(\alpha - c_0) - 2\beta(1 - (1-\theta)\delta^2)q_1 = 0. \tag{7}$$

Let the superscript "FQ" denote the equilibrium outcome of this game, where "F" means public followership (private leadership) and "Q" means quantity competition. From (7), we obtain

$$q_1^{FQ} = \frac{(\alpha - c_1) - (\alpha - c_0)\delta}{2\beta(1 - (1-\theta)\delta^2)}. \tag{8}$$

Substituting it into (6), we obtain

$$q_0^{FQ} = \frac{(\alpha - c_0)(2 - (1-\theta)\delta^2) - (\alpha - c_1)(1-\theta)\delta}{2\beta(1 - (1-\theta)\delta^2)}. \tag{9}$$

Substituting these into firm 1's profit and domestic welfare functions, we obtain

$$\pi_1^{FQ} = \frac{(\alpha - c_1 - (\alpha - c_0)\delta)^2}{4\beta(1 - (1-\theta)\delta^2)}, \tag{10}$$

$$SW^{FQ} = \frac{H_1}{8\beta(1 - (1-\theta)\delta^2)^2}, \tag{11}$$

where the Appendix reports H_1 and other constants.

Next, we discuss price competition. The direct demand for good i is given by

$$q_i = \frac{\alpha - \alpha\delta - p_i + \delta p_j}{\beta(1 - \delta^2)} \ (i = 1, 2, i \neq j). \tag{12}$$

After observing p_1, the follower, firm 0, chooses p_0. The first-order condition is

$$\frac{\partial SW}{\partial p_0} = \frac{c_0 - p_0 + \delta(1-\theta)(p_1 - c_1)}{\beta(1-\theta)} = 0. \tag{13}$$

From (13), we obtain the following reaction function of firm 0.

$$R_0(p_1) = c_0 + \delta(1-\theta)(p_1 - c_1). \tag{14}$$

The leader, firm 1, maximizes its profit, $\pi_1(p_1, R_0(p_1))$. The first-order condition of firm 1 is

$$\frac{\alpha(1-\delta) + \delta c_0 + (1 - 2(1-\theta)\delta^2)c_1 - 2(1 - \delta^2 + \theta\delta^2)p_1}{\beta(1-\theta)} = 0. \tag{15}$$

Let the superscript "FP" denote the equilibrium outcome of this game, where "P" indicates price competition. From (15), we obtain

$$p_1^{FP} = \frac{\alpha(1-\delta) + \delta c_0 + (1 - 2(1-\theta)\delta^2)c_1}{2 - 2(1-\theta)\delta^2}. \tag{16}$$

Substituting it into (14), we obtain

$$p_0^{FP} = \frac{(1-\theta)\delta(\alpha(1-\delta) - c_1) + (2 - (1-\theta)\delta^2)c_0}{2 - 2(1-\theta)\delta^2}. \tag{17}$$

Substituting these into firm 1's profit and domestic welfare functions, we obtain

$$\pi_1^{FP} = \frac{(\alpha(1-\delta) + \delta c_0 - c_1)^2}{4\beta(1-\delta^2)(1-(1-\theta)\delta^2)}, \tag{18}$$

$$SW^{FP} = \frac{H_2}{8\beta(1-\delta^2)(1-(1-\theta)\delta^2)^2}. \tag{19}$$

We now compare the welfare and profit levels in these two games. We address how the leader–follower structure affects the profit and welfare rankings in the mixed duopoly.

Proposition 1 *Consider Stackelberg competition with private leadership. (i) Quantity competition yields greater welfare than price competition if and only if the foreign ownership share in the private firm is below the threshold value* $\bar{\theta}(\delta) \in (0,1)$*. (ii) The private firm obtains greater profit under price competition than under quantity competition regardless of* θ*.*

Proof. Comparing social surplus under price competition and quantity competition, we obtain

$$SW^{FQ} - SW^{FP} = \frac{(\alpha - c_1 - (\alpha - c_0)\delta)^2\delta^2\left(1 - \theta^2\delta^2 + 2\theta\left(\delta^2 - 1\right) - \delta^2\right)}{8\beta(1-\delta)(1+\delta)\left(1-(1-\theta)\delta^2\right)^2}.$$

This shows that $SW^{FQ} > SW^{FP}$ holds if $0 < \theta < \bar{\theta}(\delta) = \left(\sqrt{(1-\delta^2)} - (1-\delta^2)\right)/\delta^2$. Similarly, straightforward computations show

$$\pi_1^{FP} - \pi_1^{FQ} = \frac{\delta^2(\alpha - c_1 - (\alpha - c_0)\delta)^2}{4\beta(1-\delta^2)(1-(1-\theta)\delta^2)} > 0. \quad \blacksquare$$

As Haraguchi and Matsumura (2014) showed, price competition yields greater welfare and profit for the private firm than quantity competition, regardless of the nationality of the private firm in the

simultaneous-move game. Proposition 1(i) is in sharp contrast to the result of the simultaneous-move game, whereas Proposition 1(ii) suggests that the profit ranking is in accordance with it. We explain the intuition behind Proposition 1(i).

As the Stackelberg leader, the private firm has an incentive to make the public firm less aggressive (choosing a smaller output or higher price) to increase its profit. Thus, under quantity (price) competition, the private firm chooses a larger output (higher price) than in the simultaneous-move case. The larger output (higher price) of the private firm improves (reduces) the consumer surplus. Therefore, welfare is greater under quantity competition than under price competition when the private firm is domestic.

However, when θ is positive, the private firm's higher output increases its profit, thus increasing the outflow to foreign investors, which cancels the welfare gain under quantity competition. This effect is more significant when the foreign ownership share in the private firm is larger. Therefore, the welfare ranking is again reversed in this case.

4 Public leadership: Public firm as the Stackelberg leader

In this section, we analyze a Stackelberg model in which firm 0 (1) is the leader (follower). First, we discuss quantity competition. Firm 1 maximizes its own profit given q_0. The first-order condition for firm 1 is given by

$$\frac{\pi_1}{\partial q_1} = \alpha - c_1 - \beta(\delta q_0 + 2q_1) = 0. \tag{20}$$

From (20), we obtain the following reaction function of firm 1:

$$R_1(q_0) = \frac{\alpha - c_1 - \beta\delta q_0}{2\beta}. \tag{21}$$

Considering the reaction function $R_1(q_0)$, firm 0 maximizes domestic welfare. The first-order condition for firm 0 is

$$\frac{1}{4}(4(\alpha - c_0) - (\alpha - c_1)(3 - 2\theta)\delta - \beta(4 - 3\delta^2 + 2\theta\delta^2)q_0) = 0. \tag{22}$$

Let the superscript "LQ" denote the equilibrium outcome of this game, where "L" indicates public leadership (private followership) and "Q" represents quantity competition. From (22), we obtain

$$q_0^{LQ} = \frac{4(\alpha - c_0) - (\alpha - c_1)(3 - 2\theta)\delta}{\beta(4 - (3 - 2\theta)\delta^2)}. \tag{23}$$

Substituting it into (21), we obtain

$$q_1^{LQ} = \frac{2((\alpha - c_1) - \delta(\alpha - c_0))}{\beta(4 - (3 - 2\theta)\delta^2)}. \tag{24}$$

Substituting these equilibrium quantities into firm 1's profit and domestic welfare, we obtain

$$\pi_1^{LQ} = \frac{4((\alpha - c_1) - \delta(\alpha - c_0))^2}{\beta(4 - (3 - 2\theta)\delta^2)^2}, \tag{25}$$

$$SW^{LQ} = \frac{H_3}{2\beta\left(4 - (3 - 2\theta)\delta^2\right)}. \tag{26}$$

Next, we discuss price competition. After observing p_0, firm 1 chooses p_1 to maximize its own profit. The first-order condition for firm 1 is

$$\frac{\partial \pi_1}{\partial p_1} = \frac{\alpha(1 - \delta) + c_1 + \delta p_0 - 2p_1}{\beta(1 - \delta^2)} = 0. \tag{27}$$

From (27), we obtain the following reaction function for firm 1:

$$R_1(p_0) = \frac{\alpha(1 - \delta) + c_1 + \delta p_0}{2}. \tag{28}$$

The leader, firm 0, maximizes domestic welfare with respect to p_0. The first-order condition for firm 0 is

$$\frac{\delta(1 - 2\theta)(\alpha(1 - \delta) - c_1) + 2c_0\left(2 - \delta^2\right) - (2\delta^2\theta - 3\delta^2 + 4)p_0}{\beta\left(\delta^2 - 1\right)} = 0. \tag{29}$$

Let the superscript "LP" denote the equilibrium outcome of this game. From (29), we obtain

$$p_0^{LP} = \frac{\delta(1 - 2\theta)(\alpha(1 - \delta) - c_1) + 2c_0\left(2 - \delta^2\right)}{4 - \delta^2(3 - 2\theta)}. \tag{30}$$

Substituting it into (28), we obtain

$$p_1^{LP} = \frac{\left(2 - \delta^2\right)\left(\alpha(1 - \delta) + c_0\delta\right) + 2c_1\left(1 - (1 - \theta)\delta^2\right)}{4 - \delta^2(3 - 2\theta)}. \tag{31}$$

Substituting these equilibrium prices into firm 1's profit and domestic welfare, we obtain

$$\pi_1^{LP} = \frac{\left(2-\delta^2\right)^2\left((\alpha-c_1)-(\alpha-c_0)\delta\right)^2}{\beta(1-\delta^2)\left(4-(3-2\theta)\delta^2\right)^2}, \tag{32}$$

$$SW^{LP} = \frac{H_4}{2\beta(1-\delta^2)\left(4-(3-2\theta)\delta^2\right)}. \tag{33}$$

We now compare welfare and profit levels in these two games.

Proposition 2 *Consider Stackelberg competition with public leadership. (i) Price competition yields greater welfare than quantity competition. (ii) The private firm obtains greater profit under price competition than under quantity competition.*

Proof. Comparing the social surplus and private profit under both types of competition, we obtain

$$SW^{LP} - SW^{LQ} = \frac{\delta^2((\alpha-c_1)-(\alpha-c_0)\delta)^2}{2\beta(1-\delta^2)\left(4-(3-2\theta)\delta^2\right)} > 0$$

$$\pi_1^{LP} - \pi_1^{LQ} = \frac{\delta^4((\alpha-c_1)-(\alpha-c_0)\delta)^2}{\beta(1-\delta^2)\left(4-(3-2\theta)\delta^2\right)^2} > 0. \quad \blacksquare$$

As the Stackelberg leader, the public firm has an incentive to make the private firm more aggressive (choosing larger output or lower price) to improve welfare. Thus, under quantity (price) competition, the public firm chooses a smaller output (lower price) than in the simultaneous-move case. Therefore, the private firm's profit is larger (smaller) than in the simultaneous-move case under quantity (price) competition. Thus, the public firm's strategic behavior as the Stackelberg leader reduces the profit advantage of price competition. Nevertheless, profit ranking is not reversed. A public firm's lower price reduces the resulting output of the private firm, and thereby reduces welfare. Therefore, the public firm still sets a higher price under price competition than the resulting price under quantity competition, and price competition thus yields a larger profit for the private firm.

We now explain the intuition of Proposition 2(i). Under quantity competition, the private firm's larger output improves welfare, while (given the output of the private firm) a smaller output from the public firm reduces welfare. Under price competition, the lower prices of both the public and private firms

improve welfare. Thus, the welfare-improving effect of the public firm's strategic behavior is stronger under price competition than under quantity competition. Therefore, the welfare ranking is not reversed.

5 Nonnegative profit constraint

In the previous sections, we allowed the public firm to choose the price or quantity without any constraint. As a result, the public firm's equilibrium profit can be negative. However, we often observe that the nonnegative profit constraint is imposed on public firms. The equilibrium profit is positive under private leadership if firms face price competition, and this constraint is not binding. By contrast, the equilibrium profit of the public firm is negative under private leadership when firms face quantity competition and $\theta > 0$. In addition, the equilibrium profit of the public firm is negative under public leadership if $\theta > 1/2$, regardless of whether the firms face price or quantity competition.

We find that under public leadership, introducing the nonnegative constraint in the public firm's profit affects neither the profit nor welfare ranking between price and quantity competition (i.e., welfare is greater and the private firm's profit is larger under price competition than under quantity competition).[11] However, under private leadership, introducing this constraint increases the private firm's profit and improves welfare when firms face quantity competition, whereas both remain unchanged when firms face price competition. Therefore, imposing the nonnegative constraint on the public firm's profit strengthens the profit and welfare advantage of quantity competition.

We now proceed to the formal analysis of private leadership. First, we consider quantity competition. Given q_1, firm 0 chooses its quantity to maximize social welfare subject to the nonnegative profit condition:

$$\max_{q_0} SW \quad \text{subject to } \pi_0 \geq 0.$$

[11] The formal proof is available upon request from the authors.

We then obtain the following reaction function:

$$R_0(q_1) = \begin{cases} \dfrac{\alpha - c_0 - \beta\delta q_1}{\beta} & \text{if} \quad 0 \le q_1 \le \overline{q}_1 := \dfrac{\alpha - c_0}{\beta\delta(1-\theta)} \quad (34) \\ \dfrac{(\alpha - c_0 - \beta\delta(1-\theta)q_1}{\beta} & \text{if} \quad \overline{q}_1 < q_1. \quad (35) \end{cases}$$

(34) is derived from the zero profit condition (i.e., the constraint is binding) and (35) is derived from the first-order condition without constraint (i.e., the constraint is not binding). Intuitively, when firm 1 chooses a smaller output ($q_1 \le \overline{q}_1$), firm 0 produces a larger output to improve social welfare, resulting in a negative profit if there is no constraint. However, the nonnegative profit constraint applies to firm 0 and cannot choose the optimal output. Therefore, the constraint determines firm 0's output.

Firm 1 maximizes its profit, $\pi_1(q_1, R_0(q_1))$, with respect to q_1. We obtain

$$\overline{q}_1 - q_1^{FQ} = \frac{(\alpha - c_0)(2 - \delta^2(1-\theta)) - (\alpha - c_1)\delta(1-\theta)}{2\beta\delta(1-(1-\theta)\delta^2)(1-\theta)} > 0.$$

Because $\pi_1(q_1, R_0(q_1))$ is concave with respect to q_1 in the private leadership game without the nonnegative profit constraint, it is decreasing in q_1 for $q_1 \ge q_1^{FQ}$. This implies that $q_1 \le \overline{q}_1$ in equilibrium because $q_1^{FQ} < \overline{q}_1$. Under the nonnegative profit condition, firm 0's reaction function is the former case. Thus, the first-order condition of firm 1 is

$$(\alpha - c_1) - \delta(\alpha - c_0) - 2q_1\beta(1-\delta^2) = 0. \quad (36)$$

Let the superscript "FQcon" denote the equilibrium outcome in the private leadership quantity competition game with the nonnegative profit constraint. From (36), we obtain

$$q_1^{FQcon} = \frac{\alpha - c_1 - \delta(\alpha - c_0)}{2\beta(1-\delta^2)}. \quad (37)$$

Substituting this into (34), we obtain

$$q_0^{FQcon} = \frac{(\alpha - c_0)(2-\delta^2) - (\alpha - c_1)\delta}{2\beta(1-\delta^2)}. \quad (38)$$

Substituting these equilibrium quantities into firm 1's profit and domestic welfare functions, we obtain

$$\pi_1^{FQcon} = \frac{((\alpha - c_1) - (\alpha - c_0)\delta)^2}{4\beta(1-\delta^2)} \quad (39)$$

$$SW^{FQcon} = \frac{H_5}{8(1-\delta^2)}. \quad (40)$$

Next, we consider price competition. From (14), we obtain $R_0(p_1) \geq c_0$ as long as $p_1 \geq c_1$. Because firm 1 never chooses $p_1 < c_1$, firm 0's profit is never negative, even without the nonnegative profit constraint, and this constraint is therefore not binding. Comparing social surplus and the private firm's profit under both types of competition with the nonnegative profit condition, we obtain

$$SW^{FQcon} - SW^{FP} = \frac{\delta^2\left(1-2\delta^2\theta^3-\left(1-3\delta^2\right)\theta^2-\delta^2\right)\left((\alpha-c_1)-(\alpha-c_0)\delta\right)^2}{8\beta(1-\delta)(1+\delta)\left(1-(1-\theta)\delta^2\right)^2} > 0 \tag{41}$$

$$\pi_1^{FP} - \pi_1^{FQcon} = \frac{((\alpha-c_1)-(\alpha-c_0)\delta)^2\delta^2(1-\theta)}{4\beta(1-\delta^2)(1-(1-\theta)\delta^2)} > 0. \tag{42}$$

These discussions lead to the following result.

Proposition 3 *Suppose that firm 0 cannot choose an output that yields a negative profit. Under private leadership with quantity competition, the private firm's profit is smaller and welfare is greater when firms face quantity competition than when firms face price competition, regardless of* θ.

As we stated above, under private leadership, the public firm's profit is negative when firms face quantity competition. Imposing the nonnegative profit constraint makes the public firm less aggressive. Expecting this less aggressive behavior, the private leader expands its output, resulting in a welfare gain. Under quantity competition, the private firm's profit increases due to the less competitive situation brought about by the nonnegative profit constraint. The profit ranking, however, does not change.

Proposition 3 has an important policy implication. Under private leadership, it is beneficial to impose a nonnegative profit constraint on the public firm if firms face quantity competition. However, when firms face price competition, this constraint does not matter.

6 Endogenous competition structure

In this section, we endogenize the choice of strategic variable (either price or quantity contract). We consider the following timeline:

(a) First, both the leader and the follower choose a price or quantity contract. After observing the price-quantity choices, they face Stackelberg competition.

(b) First, the leader chooses either the price or quantity level. After observing the leader, the follower chooses price or quantity.

In other words, the roles of the leader and the follower are given exogenously but firms can choose a price or quantity contract.

In both scenarios, given the leader's price or quantity, the follower's demand function that maps the follower's price to its quantity is fixed. Therefore, the follower's choice of price and quantity does not affect the equilibrium outcome and thus the follower is indifferent between the two. However, the leader's choice significantly affects the equilibrium outcome. As Singh and Vives (1984) showed, the firm's demand elasticity is higher when the rival chooses the price than when it chooses the quantity. In our context, the follower's demand is more sensitive to its price when the leader chooses the price. We explain the intuition. Given the leader's price, a reduction in the follower's price reduces the leader's output; by definition, the price remains unchanged. Given the leader's quantity, a reduction in the follower's price reduces the leader's price; by definition, the output remains unchanged. Therefore, given the leader's quantity, the follower's price reduction automatically reduces the rival's price and thus, the follower's demand is less sensitive to its own price given the leader's quantity.

If the leader chooses the price (quantity), price (quantity) competition occurs because, as we discuss above, the follower's choice between the price and quantity does not matter. We showed earlier that price competition always provides the private firm with a higher profit. Therefore, under private leadership, the private firm chooses the price, resulting in price competition. We also showed that price competition yields greater welfare under public leadership. Therefore, under public leadership, the public firm chooses the price, resulting in price competition. Under these conditions, price competition occurs if we endogenize the competition structure, regardless of public or private leadership. This suggests that under private leadership, it is possible that the equilibrium competition structure will be inefficient.[12]

[12]In the simultaneous-move games, price competition yields greater welfare in both private and mixed duopolies, and the equilibrium outcome is price competition. In this sense, the outcome is efficient in mixed duopolies. Our result clearly shows that this is not true under private leadership.

7 Concluding remarks

In this study, we revisit the welfare and profit comparison between price and quantity competition in mixed duopolies. We consider sequential-move games and find that welfare can reverse when the public firm is the follower, while price competition is always better for welfare when the public firm is the leader. In addition, we find that foreign ownership share plays an important role when the public firm is the follower. Finally, we endogenize the competition structure and find that price competition appears regardless of whether the public or private firm is the leader. We do not consider any government strategic policies such as tax-subsidies, privatization, and trade policies in this study. These policies are intensively discussed in the literature and incorporating these into our analysis remains for future research.[13]

[13]For recent developments in studies of these policies in mixed markets, see Cato and Matsumura (2015) and the works cited therein.

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