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Abstract: Estimates of the welfare costs of moderate inflation are generally modest or small. This paper, by shedding light on increasing returns to production specialization, obtains a substantial welfare cost of 8% in an endogenous growth model of monopolistic competition with endogenous entry. Analytically, we show that the effect of inflation is aggravated (resp. alleviated) by a price markup if the degree of increasing returns to production specialization is relatively high (resp. low). Accordingly, our quantitative analysis indicates that the welfare cost of inflation exhibits an inverted U-shaped relationship with the price markup. This non-monotone is sharply in contradiction to the conventional notion. Nonetheless, the welfare cost of inflation is unambiguously increasing in the degree of increasing returns to production specialization.

JEL Classification: E31, O42, L16

Keywords: Welfare cost of inflation, price markup, increasing returns to production specialization.

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1 Introduction

The welfare cost of inflation has been a fundamental issue in monetary economics and macroeconomics. In the literature, estimates of the welfare costs of moderate inflation are generally modest or small. By shedding light on the *increasing returns to production specialization* (hereafter IRPS), this study obtains a substantial welfare cost in a simple endogenous growth model in which (i) households are subject to a cash-in-advance (CIA) constraint and (ii) firms operate in a monopolistically competitive market with *endogenous entry*.

By focusing on CIA constraint models, a seminal study is that by Cooley and Hansen (1989) who report a welfare cost of inflation of around 0.5% (increasing inflation from the Friedman-rule value of deflation of -4% to 10%) in a Neoclassical growth model.[1] In an endogenous growth model with an endogenous labor-leisure trade-off, Gomme (1993) also obtains a small welfare cost: a 10% money growth rate results in a welfare cost of less than 0.03% of output. Dotsey and Ireland (1996) consider the services of a financial intermediary and find that the figure is about 3.3%.[2] By highlighting labor market imperfections (labor frictions), Heer (2003) obtains a welfare cost of 0.65% (increasing inflation from the Friedman optimal deflation level of -1% to 1.3%).[3] By highlighting product market imperfections, Wu and Zhang (2000) and Chu et al. (2012) estimate the welfare cost of inflation and examine the role played by a price markup. In an RBC model of monopolistic competition with increasing returns to scale, Wu and Zhang (2000) show that the welfare cost is around 2.7% when inflation increases from the Friedman optimal deflation level of -4% to 10% and the cost is increasing in the price markup. In an endogenous-growth model of R&D, Chu et al. (2012) obtain a corresponding figure of around 1.76%, while the positive relationship between the welfare cost and the markup is numerically confirmed.

IRPS have been shown to have their practical importance. Romer (1987) indicates that an expansion in the number of firms leads the same assortment of commodities to be manufactured in specialized firms. This renders a positive externality on the firms' production, raising aggregate

[1] With regard to money-in-the-utility-function models, the reader can refer to, for example, Lucas (2000) and Chu and Lai (2013).

[2] Furthermore, Wen (2015) finds a relatively substantial welfare cost of inflation (about 9.6%) when the liquidity function of money and the precautionary motive of money demand are taken into account so as to capture the buffer-stock-insurance value of cash.

[3] Recently, Lagos and Wright (2005) and Chu et al. (2014) obtained a substantial welfare cost of inflation by developing a search model of monetary exchange.

output more than proportionally.[4] Nowadays, many products are becoming more modular over time and this development is often associated with a change in industry structure towards higher degrees of specialization.[5] This in turn has contributed to specific activities becoming more suitable and has attracted a large number of *de novo* entrants (see Sanchez and Mahoney, 1996 and Langlois, 2002). Due to their importance, IRPS have been introduced in order to provide a possible explanation for the real-wage and fiscal puzzle (Devereux et al., 1996a), the international trade puzzle (Krugman, 2009) and the properties of the business cycle (Chang et al., 2011 and Pavlov and Weder, 2012), as well as to modify the Solow residual by remeasuring technology shocks (Devereux et al., 1996b). In this paper, we will show that the existence of IRPS plays a crucial role in terms of governing the magnitude of the welfare cost of inflation and its relationship with the price markup.

We analytically show that in the balanced-growth-path equilibrium an expansionary monetary policy raises inflation, which retards economic growth and lowers social welfare. The latter implies that the optimal inflation follows Friedman's rule. More importantly, the negative effects on growth and welfare are amplified by a price markup if the degree of IRPS is relatively high. By contrast, the price markup can mitigate these unfavorable effects if the degree of IRPS is relatively low. Numerically, we offer a significantly high welfare cost of 8.796% given that endogenous entry leads to IRPS. Moreover, our quantitative analysis indicates that the welfare cost of inflation exhibits an inverted U-shaped relationship with the price markup. When the *status quo* price markup is relatively low (less than 1.3 in our parameterization), the welfare cost of inflation is more pronounced in a less competitive market, but when the *status quo* price markup is relatively high, it turns out to be less pronounced in a less competitive market. The welfare cost of inflation reaches a maximum of 8.801%, when the price markup is 1.3. This non-monotone is sharply in contradiction to the conventional notion, as in Wu and Zhang (2000) and Chu et al. (2012). Besides, we find that the welfare cost of inflation is unambiguously increasing in the degree of IRPS.

[4] Conceptually, IRPS are similar to the classical notion of economies of scope (see Holtz-Eakin and Lovely, 1996 and more recently Blancard et al., 2011).

[5] Specialization is also often used to explain higher productivity in the US. See Rapping (1965) and Balassa (1967) for the details.

2 The Model

Consider an economy consisting of households, firms, and a government. Households derive utility from consumption, but incur disutility from work. There are two types of goods: a homogeneous final good which is the numéraire and differentiated intermediate goods indexed by $i = 1, ..., N_t$, where N_t is the number of intermediate goods at time t. The final good is produced by competitive firms, while intermediate goods are produced by monopolistically competitive firms. A government representing the fiscal and monetary authorities, on the one hand, balances its budget and, on the other hand, sets the optimal money growth rate.

2.1 Households

There is a unit measure of identical, infinitely lived households. By following Hansen's (1985) specification of indivisible labor, the lifetime utility function of the representative household is given by:

$$U = \int_0^\infty e^{-\rho t}(\ln C_t - BH_t)dt, \tag{1}$$

where C_t is consumption, H_t are hours worked, $\rho(> 0)$ is the time preference rate, and $B(> 0)$ is the preference weight on leisure (or work).[6] Given the initial capital K_0, households maximize their lifetime utility (1) subject to the following budget and cash-in-advance (CIA) constraints:

$$\dot{K}_t + \dot{M}_t = W_t H_t + R_t K_t + \Omega_t + T_t - C_t - \pi_t M_t - \delta K_t. \tag{2}$$

$$\xi C_t \leq M_t, \ 0 < \xi < 1. \tag{3}$$

where K_t is capital, M_t is the real money balance, π_t is inflation, W_t is the real wage rate, R_t is the real interest rate, δ is the depreciation rate, Ω_t is the aggregate profit, and T_t is the real lump-sum transfer from the government. Equation (3) indicates that in the Lucasian CIA constraint economy, households hold money M_t in order to facilitate the final good purchases.[7]

Let $\lambda_{1,t}$ and $\lambda_{2,t}$ be the Lagrangian multiplier associated with the household's budget constraint and CIA constraint, respectively. Thus, the necessary conditions for the household's optimization

[6] Our result is robust to a standard RBC utility function, $\ln C_t - B\frac{H_t^{1+\epsilon}-1}{(1+\epsilon)}$.

[7] The qualitative results of this paper do not change in alternative models of money, such as in the money-in-the-utility-function and transactions cost models.

problem are:

$$C_t: \quad \frac{1}{C_t} = \lambda_{1,t}\left(1+\xi\frac{\lambda_{2,t}}{\lambda_{1,t}}\right), \tag{4}$$

$$H_t: \quad B = \lambda_{1,t}\, W_t, \tag{5}$$

$$M_t: \quad -\lambda_{1,t}\,\pi_t + \lambda_{2,t} = -\dot{\lambda}_{1,t} + \lambda_{1,t}\,\rho, \tag{6}$$

$$K_t: \quad \lambda_{1,t}(R_t-\delta) = -\dot{\lambda}_{1,t} + \lambda_{1,t}\,\rho, \tag{7}$$

$$\lim_{t\to\infty}\frac{1}{C_t}K_t e^{-\rho t} = 0 \text{ and } \lim_{t\to\infty}\frac{1}{C_t}M_t e^{-\rho t} = 0. \tag{8}$$

Equation (4) equalizes the marginal benefit and marginal cost of consumption, with the latter being the sum of the shadow price of the real money balances and the shadow price of the CIA constraint on consumption. Equation (5) is the household's labor supply. Equations (6) and (7) refer to the optimal conditions for money and capital holdings, respectively. To ensure that (2) can be transformed into an infinite-horizon, present-value budget constraint, the transversality conditions in (8) have to be met. Moreover, combining (6) and (7) yields the no arbitrage condition between capital and real money balances: $\frac{\lambda_{2,t}}{\lambda_{1,t}} = R_t - \delta + \pi_t$. With that, (4) and (5) allow us to derive

$$\frac{1}{BC_t} = \frac{[1+\xi(R_t-\delta+\pi_t)]}{W_t}, \tag{9}$$

implying that the consumption-leisure tradeoff is affected by the portfolio between capital and real money balances.

2.2 Firms

The production side is built on the Dixit-Stiglitz (1977) model of monopolistic competition with endogenous entry which leads to increasing/decreasing returns to production specialization.

2.2.1 Final-good firms

The final-good sector is perfectly competitive. By following Bénassy (1996) and Pavlov and Weder (2012), the final good Y_t is produced by simply using a continuum of intermediate inputs $y_t(i)$ for $i \in [0, N_t]$, based on the following generalized form of production function:

$$Y_t = N_t^{1+\upsilon-\frac{1}{\theta}}\left[\int_0^{N_t} y_t^{\theta}(i)di\right]^{\frac{1}{\theta}}, \tag{10}$$

where $\theta \in (0, 1)$ measures the degree of substitution between intermediate goods and v measures the returns to production specialization. The production function (10) displays a generalized form of increasing/decreasing returns to production specialization in the sense that the larger the number of intermediate firms is, the higher (lower) is the amount of final output obtained. Under a symmetric equilibrium in which all intermediate goods are hired in the same quantity y_t, final output is given by $Y_t = N_t^{1+v} y_t$. Accordingly, an expansion in the number of firms raises the final goods production more (less) than proportionally if $v > 0$ ($v < 0$). This implies that there are constant returns to the quantities employed of a fixed variety of intermediate goods, but either increasing or decreasing returns to an expansion in such a variety, while holding the quantity employed of each intermediate good fixed. Aghion and Howitt (1998, p. 407) argue that "[f]or while having more products definitely opens up more possibilities for specialization, and of having instruments more closely matched with a variety of needs, it also makes life more complicated and creates greater chance of error...". The former statement refers to the so-called production-enhancing effect, while the latter refers to the so-called production-complexity effect (Bucci, 2013). The case where $v > 0$ in our model corresponds to a situation in which the production-enhancing effect dominates the production-complexity effect, and vice versa.

It is important to note that the specification of Dixit and Stiglitz (1977) and Devereux et al. (1996a, 2000) refers to $N_t^{\frac{1}{\theta}} y_t$, indicating that monopoly power and increasing returns to specialization (to an expansion in variety) are characterized by the same parameter θ. Thus, it is difficult to distinguish what arises due to market imperfections and what is due to increasing returns. To overcome this shortcoming, we specify (10) in order to clearly separate increasing/decreasing returns from imperfect competition, so that both effects can be fully disentangled, as in Bénassy (1996) and Pavlov and Weder (2012). At the same time, we can consider both increasing returns to production specialization (IRPS) and decreasing returns to production specialization (DRPS) in a unified model.

With (10), we can derive the final good producer's demand for intermediate goods as follows:

$$p_t(i) = \left(\frac{Y_t}{y_t(i)}\right)^{1-\theta} N_t^{\left(v+1-\frac{1}{\theta}\right)\theta}, \tag{11}$$

indicating that the price elasticity of demand for the ith intermediate good is $\frac{1}{1-\theta}$. A larger θ implies a higher price elasticity of demand for the intermediate good and, accordingly, the intermediate-good sector is more competitive.

2.2.2 Intermediate-good firms

The intermediate-good sector is monopolistically competitive. Each intermediate-good firm employs capital $k_t(i)$ and labor $h_t(i)$ to produce intermediate goods according to the following production technology:

$$y_t(i) = Z_t \left[k_t(i)\right]^{\alpha} \left[h_t(i)\right]^{1-\alpha} - \phi, \tag{12}$$

where Z_t is a production externality, α $(1-\alpha)$ is the capital (labor) share, and ϕ is an overhead cost that is paid in units of the intermediate good. To generate perpetual growth, we assume that $Z_t = \bar{Z} K_t^{\sigma}$, where $\bar{Z}$ is the technology parameter, K_t is the economy-wide capital stock, and σ measures the degree of the production externality.

With the demand function (11) and production function (12), the intermediate-good firm maximize its profits

$$\omega_t(i) = p_t(i) y_t(i) - W_t h_t(i) - R_t k_t(i), \tag{13}$$

by choosing capital $k_t(i)$ and labor $h_t(i)$. The corresponding first-order conditions are respectively given as follows:

$$R_t = \left(\frac{\alpha}{\eta}\right) p_t(i) \frac{y_t(i) + \phi}{k_t(i)}, \tag{14}$$

$$W_t = \left(\frac{1-\alpha}{\eta}\right) p_t(i) \frac{y_t(i) + \phi}{h_t(i)}. \tag{15}$$

Accordingly, we obtain the price markup of the intermediate-good firms $\eta = \frac{1}{\theta}$.

2.3 Government

Given that the nominal money supply is S_t and P_t is the price of the final good, the real money balance is given by: $M_t \equiv \frac{S_t}{P_t}$. Thus, the evolution of real money balances is expressed as:

$$\frac{\dot{M}_t}{M_t} = \mu_t - \pi_t, \tag{16}$$

where $\mu = \frac{\dot{S}_t}{S_t}$ denotes the growth rate of nominal money supply, which is the monetary authority's policy instrument. To balance the government's budget, the seigniorage that the government receives from money growth is rebated to households in a lump-sum manner, i.e.:

$$T_t = \frac{\dot{S}_t}{P_t} = \mu_t M_t = \dot{M}_t + \pi_t M_t. \tag{17}$$

2.4 Balanced-Growth-Path (BGP) Equilibrium

We confine the analysis to a symmetric equilibrium where $k_t(i) = k_t$, $h_t(i) = h_t$, $y_t(i) = y_t$, $p_t(i) = p_t$, $K_t = N_t k_t(i)$, and $H_t = N_t h_t(i)$ for all $i \in [0, N_t]$. Perfect competition in the final-good sector implies $p_t = N_t^v$. Moreover, free entry leads to a zero-profit condition for the intermediate-good sector. By substituting (14) and (15) into (13), the zero-profit condition is given by $y_t = \frac{\phi}{\eta - 1}$. Combining these resulting relations with (12) yields the number of firms:

$$N_t = \left(\frac{\eta - 1}{\eta \phi} \right) \bar{Z} K_t^{\alpha + \sigma} H_t^{1-\alpha}. \tag{18}$$

Equation (18) indicates that a larger markup, η, raises the firm's profits which attract new entrants that in turn increase the number of firms, N_t. With (18), under a symmetric equilibrium $Y_t = N_t^{1+v} y_t$, we can use (10) to derive the aggregate output of the final good (i.e., gross domestic product):

$$Y_t = \frac{1}{\eta} \left(\frac{\eta - 1}{\eta \phi} \right)^v A [K_t^{\alpha + \sigma} H_t^{1-\alpha}]^{1+v}, \tag{19}$$

where $A \equiv \bar{Z}^{1+v}$. We impose:

Assumption 1. (Capital Externality) $\sigma = \frac{1}{1+v} - \alpha$.

This assumption indicates that the degree of the capital externality must satisfy $\sigma = \frac{1}{1+v} - \alpha$ so that the aggregate production in (19) exhibits constant returns to scale for capital, which guarantees substained growth.

Accordingly, we can rewrite (14) and (15) as follows:

$$R_t K_t = \alpha Y_t \text{ and } W_t H_t = (1 - \alpha) Y_t. \tag{20}$$

Let $\Omega_t = \int_0^{N_t} \omega_t(i) di = N_t \omega_t$. Given (20), putting the budget constraints of household (2) and government (17) together yields the following aggregate resource constraint:

$$\dot{K}_t = Y_t - C_t - \delta K_t. \tag{21}$$

A competitive equilibrium is defined as a set of market clearing prices and quantities such that: (i) the representative household maximizes its lifetime utility, i.e., (4)-(7); (ii) the final-good and intermediate-good firms maximize their profits, i.e., (11), (14), and (15); and the government balances its budget constraint, i.e., (17). Thus, a non-degenerate BGP equilibrium is a tuple of paths such that output Y_t, consumption C_t, capital K_t and the real money balance M_t, grow at a positive constant rate and the inflation rate π_t is a positive constant.

We now characterize the dynamic system of the model. By using (5), (20), and (19), we first derive

$$B = \lambda_{1,t} \left(\frac{1-\alpha}{\eta} \right) \left(\frac{\eta - 1}{\eta \phi} \right)^{v} A K_t H_t^{\beta - 1}, \tag{22}$$

where $\beta \equiv v + \sigma(1+v)$ is a composite parameter.

Assumption 2. (Labor Externality) $\beta \equiv v + \sigma(1+v) < 1$, or equivalently $v < \frac{\alpha}{1-\alpha}$ (given $\sigma = \frac{1}{1+v} - \alpha$).

Assumption 2 is a sufficient condition for ensuring the steady-state determinacy. This echoes the finding of Chang et al. (2011) whereby indeterminacy occurs if there exist IRPS ($v > 0$) and their degree is substantially large ($v > \frac{\alpha}{1-\alpha}$) in our terminology.

Differentiating (22) with respect to time t leads to

$$\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}} + \frac{\dot{K}_t}{K_t} + (\beta - 1) \frac{\dot{H}_t}{H_t} = 0.$$

Define the transformed variable $X_t \equiv C_t / K_t$. By combining the above equation with (7), (20), (19), and (21), we then obtain the following differential equation:

$$\frac{\dot{H}_t}{H_t} = \frac{1}{1-\beta} \left[\frac{1-\alpha}{\eta} \left(\frac{\eta - 1}{\eta \phi} \right)^{v} A H_t^{\beta} - X_t + \rho \right]. \tag{23}$$

From (9), (20), and (19), we derive the relationship of inflation:

$$\pi_t = \frac{1}{\xi} \left[\frac{\frac{1-\alpha}{\eta} \left(\frac{\eta-1}{\eta\phi} \right)^{v} A H_t^{\beta-1}}{B X_t} - 1 \right] - \frac{\alpha}{\eta} \left(\frac{\eta - 1}{\eta \phi} \right)^{v} A H_t^{\beta} + \delta.$$

With this, (16) and (21) allow us to obtain another differential equation as follows:

$$\frac{\dot{X}_t}{X_t} = X_t - \frac{1}{\xi} \left[\frac{\frac{1-\alpha}{\eta} \left(\frac{\eta-1}{\eta\,\phi} \right)^{v} A H_t^{\beta-1}}{B X_t} - 1 \right] - \frac{1-\alpha}{\eta} \left(\frac{\eta - 1}{\eta \phi} \right)^{v} A H_t^{\beta} + \mu. \tag{24}$$

Based on (23) and (24), the dynamic system can be reduced to a 2×2 one in terms of H_t and X_t.

In the steady state, the values of hours worked and the consumption-capital ratio are stationary and satisfy the equilibrium conditions $\dot{H}_t = 0$ and $\dot{X}_t = 0$. In what follows, we denote the steady-state value by leaving out the variable's time subscript. Thus, we have:

Theorem 1 (Existence and Uniqueness of Equilibrium) *Under the condition* $HX < \frac{1-\beta}{\xi\beta B}$, *there exists a nondegenerate, unique BGP equilibrium, which is locally determinate.*

Proof. All proofs are relegated to the Appendix. ■

In the BGP equilibrium, the economy exhibits common growth in which consumption C_t, capital K_t, output Y_t, and real money balances M_t all grow at the same rate g. It follows from (4), (6), and (7) that the familiar Euler equation: $\frac{\dot{C}_t}{C_t} = R_t - \delta - \rho$ holds true. Accordingly, we can easily obtain the balanced-growth rate:

$$g = \frac{\dot{C}_t}{C_t} = \frac{\alpha}{\eta}\left(\frac{\eta-1}{\eta\phi}\right)^{v} AH^{\beta} - \delta - \rho. \tag{25}$$

3 Growth, Welfare, and Welfare Cost of Inflation

In this section, we examine the growth and welfare effects of the money growth rate, μ, along the BGP equilibrium. In particular, we discuss the role played by the price markup, η, and IRPS/DRPS, v. To examine the welfare effect, we need to calculate social welfare based on the household's utility (1). Along the BGP, the initial consumption-capital ratio refers to $C_0 = XK_0$, where $X = \frac{1-\alpha}{\eta}\left(\frac{\eta-1}{\eta\phi}\right)^{v} AH^{\beta} + \rho$. By normalizing the initial capital stock K_0 to unity, social welfare can be computed as:

$$U = \frac{1}{\rho}\left(\ln C_0 + \frac{g}{\rho} - BH\right) = \frac{1}{\rho}\left(\ln X + \frac{g}{\rho} - BH\right). \tag{26}$$

3.1 Growth and Welfare Effects

Based on (25) and (26), we derive the effects of inflation (or the money growth rate) on economic growth and social welfare, which lead to the following proposition.

Proposition 1 (Growth and Welfare Effects) *In the presence of monopolistic competition with endogenous entry,*

(i) *a higher growth rate of money supply,* μ, *decreases the balanced-growth rate,* g;

(ii) *social welfare U is monotonically decreasing in the money growth rate, μ, implying that the optimal monetary policy follows Friedman's rule with a zero nominal interest rate;*

(iii) *the negative effects on growth and welfare are amplified (resp. attenuated) by the price markup, η, if there exist IRPS, $v > 0$, and the degree of IRPS is relatively high, $v > \eta - 1$ (resp. low, $v < \eta - 1$). If the economy exhibits DRPS, the price markup unambiguously alleviates the negative effect of the money growth rate on growth and welfare.*

A higher money growth rate is associated with a higher inflation rate, which raises the cost of consumption relative to leisure. Therefore, households, on the one hand, decrease their consumption (hence the consumption-capital ratio X falls), and on the other hand, increase their leisure (hence hours worked H decrease). A decrease in consumption lowers the intermediate-good firms' profits, which decreases the equilibrium number of firms and hence the aggregate marginal product of capital, $\frac{\alpha Y_t}{K_t}$, where $Y_t = N_t^{1+v} y_t$. A reduction in hours worked decreases the firms' demand for capital, which slows down capital accumulation. Both result in a fall in the balanced-growth rate g.

Of particular interest, the growth effect of inflation hinges upon the market competition intensity or markup, η. In the presence of a higher markup, intermediate-good firms will exercise their monopoly power to raise prices through cutting their output. This, on the one hand, leads the firms to demand less labor, and on the other hand, lowers the elasticity of the *aggregate* demand for labor. With endogenous entry, a higher markup, however, may increase the aggregate demand for labor. Higher monopoly power increases the firms' profits, creating an incentive for new firms to enter the market. If the presence of endogenous entry leads to IRPS ($v > 0$), the increase in the number of firms will generate a strong external effect, expanding the aggregate labor demand and its elasticity with respect to wages. It turns out that if $v > \eta - 1$, the latter effect dominates, and as a result, a higher markup is associated with a stronger demand for labor with a higher elasticity. In the presence of a stronger and more elastic labor demand, the negative labor effect of an expansionary monetary policy becomes more pronounced when inflation discourages households from supplying labor. Therefore, the negative impact on growth is reinforced by the price markup. By contrast, if $v < \eta - 1$, the price markup will alleviate, rather than aggravate, the negative effect of inflation on growth.

In response to an expansionary monetary policy, while households' leisure increases, social welfare decreases given that inflation slows down growth and lowers the consumption-capital ratio. Thus, social welfare is monotonically decreasing in the money growth rate. This implies that the Friedman rule is socially optimal in our model in the sense that an optimal rate of inflation is negative, referring to a zero nominal interest rate. More importantly, since the negative welfare effect of inflation stemming from growth (and the consumption-capital ratio) dominates its positive effect stemming from leisure, the influence of a price markup on welfare is similar to that on growth. Therefore, as shown in Proposition 1(iii), the markup aggravates the effect of inflation if $v > \eta - 1$, but alleviates the effect of inflation if $v < \eta - 1$. As is evident, if the economy exhibits DRPS ($v < 0$), the condition $v < \eta - 1$ is true. Under such a situation, the negative effects on growth and welfare unambiguously become less pronounced in a less competitive market.

3.2 Welfare Cost of Inflation: Quantitative Analysis

The analytical study above has shown that the price markup and returns to production specialization both play an important role in terms of governing the growth and welfare effects of inflation. It is important to numerically calculate the welfare costs of inflation, and accordingly, quantify the importance of the price markup in terms of the welfare effect of inflation. To this end, we use data for the US from 1959-2012 to calibrate the endogenous growth model of monopolistic competition with endogenous entry.

In the benchmark, each structural parameter is either set a conventional value or matched to an empirical moment in the US. For simplicity, the discount rate ρ is set to 0.04. The price markup is set as $\eta = 1.28$, which corresponds to an intermediate value of the empirical estimates reported in Jones and Williams (2000). The IRPS parameter is set to $v = 0.3$, which is the value of the empirical estimates in Paul and Siegel (1999). In line with Devereux et al. (2000), we set the unit overhead cost as $\phi = 0.1$. Accordingly, we calibrate the parameter $\sigma = 0.47$ in order to match the capital share of output of 0.3. The leisure parameter B is set to be 2.58, which satisfies a standard ratio of labor supply of 0.33. As for the capital depreciation rate, we calibrate $\delta = 0.05$ by matching the investment-capital ratio of 0.08. We set the consumption-CIA parameter as $\xi = 0.26$, which matches the ratio of M1 to consumption. We consider the initial money growth rate $\mu = 6.8\%$, so that the annual inflation rate is 3.6%. Finally, the technology parameter is set as $A = 1.11$ (i.e.,

$\bar{Z} = 1.08$), so that the output growth rate is 3.2%. Thus, the value of social welfare is $U = -29.76$ in the benchmark. These parameter values are summarized in Table 1 below.

Table 1: Calibrated parameter values

α	ρ	η	ξ	ϕ	σ	μ	υ	δ	A	B
0.30	0.04	1.28	0.26	0.10	0.47	0.068	0.30	0.05	1.11	2.58

Under these selected and calibrated parameters, we then compute the effects of the money growth rate on growth and welfare. We lower μ from 0.068 to -0.04 so that the nominal interest rate declines and approaches zero (i.e., the Friedman rule holds). We consider a variety of price markups $\eta \in [1.18, 1.38]$ to highlight the importance of a price markup. The changes in g are expressed in percentage points, and the changes in U are expressed in the usual equivalent variations in annual consumption, i.e., the welfare costs of inflation. The results are reported in Table 2 which allows us to establish Result 1 as follows:

Result 1 (Welfare Cost and Markup) *In the presence of monopolistic competition with endogenous entry and IRTS,*

(i) *in the benchmark with a price markup* $\eta = 1.28$*, the welfare gain from reducing inflation to the optimal level (welfare cost of inflation) is* 8.796%*, which is associated with an increase in growth of* 0.346%*;*

(ii) *the welfare cost of inflation exhibits an inverted U-shaped relationship with the price markup;*

- **a.** *if* $\eta < 1.3$*, the welfare cost of inflation is increasing in the markup;*
- **b.** *if* $\eta = 1.3$*, the welfare cost of inflation reaches a maximum at* 8.801%*;*
- **c.** *if* $\eta > 1.3$*, the welfare cost of inflation is decreasing in the markup.*

In the literature, estimates of the welfare costs of moderate inflation are generally modest or small, as noted in the Introduction. For appropriate comparisons, we simply compare Result 1 with Wu and Zhang (2000) and Chu et al. (2012) who also calculate the welfare cost in a CIA model with product market imperfections. In an RBC model of monopolistic competition with increasing returns to scale, Wu and Zhang (2000) show that the welfare cost is around 2.7%. In an endogenous-growth model of R&D, Chu et al. (2012) obtain a welfare cost of 1.76%. By shedding light on IRPS, Result 1 shows a significantly higher welfare cost: the figure is 8.796% in an endogenous growth model of monopolistic competition with *endogenous entry*.

There are two monetary policy objectives: (a) the Friedman rule and (b) price stability (or equivalently, zero inflation) which are commonly analyzed in the literature (see Dotsey and Ireland, 1996). As an alternative, one may also estimate the welfare cost under the zero-inflation objective. Under our parametrization, the corresponding welfare cost is 2.72%. This figure is only one-third of that under the Friedman rule, but it is still nonnegligible. Dotsey and Ireland (1996) find that reducing inflation from 4% to 0% leads to a welfare gain of about 1% of output. In a search-theoretic model of monetary exchange, Lagos and Wright (2005) find that the welfare gain from reducing inflation from 10% to price stability is around 3% to 5% in terms of consumption. By developing a search-and-matching endogenous growth model, Chu et al. (2014) obtain a welfare gain of about 1.6% of consumption through a reduction in inflation from 4% to 0%.

Table 2: Growth and welfare effects of monetary policy under $\eta \in [1.18,1.38]$

η	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.36	1.38
Δg (%)	0.336	0.340	0.342	0.344	0.345	0.346	0.347	0.346	0.345	0.344	0.343
ΔU(%)	8.547	8.637	8.703	8.749	8.780	8.796	8.801	8.797	8.784	8.764	8.738

Wu and Zhang (2000) show that the greater the monopoly power the firms have (the less the intensity of competition among firms), the higher the welfare cost is. However, our estimates refer to a non-monotonic relationship between the intensity of competition (or markup) and the welfare cost of inflation. Proposition 1(iii) has indicated that the welfare effect of inflation could be either positively or negatively related to the price markup. Table 2 further shows that the welfare cost of inflation exhibits an inverted U-shaped relationship with the price markup. This implies that the *status quo* product market competition (or markup) plays a crucial role in terms of governing the inflation cost caused by an inappropriate monetary policy. Our numerical analysis reveals that when the *status quo* price markup is relatively low, $\eta < 1.3$, the cost of inflation is more pronounced in a less competitive market. By contrast, when the *status quo* price markup is relatively high, $\eta > 1.3$, the cost of inflation turns out to be less pronounced in a less competitive market. The welfare cost of inflation is not necessarily decreasing in the intensity of product market competition. One may note that if the economy exhibits DRPS ($v < 0$), the relationship between the price markup and the welfare effect of inflation becomes unambiguously negative (as indicated in Proposition 1) and hence the non-monotonic relationship between the markup and welfare cost of inflation will disappear as well.

Next, we turn to a discussion regarding the relationship between the welfare cost and IRPS. In a variety of *IRPS* $v \in [0.2, 0.4]$, the results are shown in Table 3. The relevant results are summarized as follows:

Result 2 (Welfare Cost and IRPS) *In the presence of monopolistic competition with endogenous entry and IRPS, the welfare cost of inflation is unambiguously increasing in the degree of IRPS.*

The intuition is straightforward. Higher inflation discourages households from consumption. With endogenous entry, a decrease in consumption lowers the intermediate-good firms' profits, which decreases the number of firms. Under a symmetric equilibrium with $Y_t = N_t^{1+v} y_t$, the reduction in the equilibrium number of firms retards economic growth and in turn decreases social welfare. Given a lower number of firms, the higher that the degree of IRTS, v, is, the more that the balanced-growth rate g falls. As a result, welfare U also decreases by a larger magnitude. Thus, the cost of inflation is increasing in the degree of IRPS, as shown in Figure 3.

Table 3: Growth and welfare effects of monetary policy under $v \in$ [0.2,0.4]

v	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40
Δg (%)	0.306	0.314	0.322	0.330	0.338	0.346	0.353	0.360	0.367	0.374	0.380
ΔU(%)	7.303	7.610	7.913	8.211	8.506	8.796	9.083	9.366	9.646	9.922	10.195

4 Concluding Remarks

Estimates of the welfare costs of moderate inflation are generally modest or small. By shedding light on IRPS, this paper has shown that there is a substantial welfare cost in an endogenous growth model of monopolistic competition with endogenous entry.

We have shown that the negative effects of inflation on growth and welfare are amplified by a price markup if IRPS exist in the economy and their degree is relatively high. As a result, the welfare cost of inflation exhibits an inverted U-shaped relationship with the price markup.

When the *status quo* price markup is relatively low, the cost of inflation is more pronounced in a less competitive market. Otherwise, the cost of inflation is less pronounced in a less competitive market. This implies that the welfare cost of inflation is not necessarily decreasing in the intensity of product market competition. This result is in contrast to the conventional notion, as in Wu and Zhang (2000), which predicts that the less the intensity of competition among firms, the higher the welfare cost. In addition, we have also found that the cost of inflation rises with the degree of IRPS.

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Appendix

Proof of Theorem 1. Linearizing (23) and (24) around the steady-state equilibrium, we have:

$$\begin{bmatrix} \dot{H}_t \\ \dot{X}_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} H_t - H \\ X_t - X \end{bmatrix}, \tag{A1}$$

where

$$a_{11} = \frac{\beta}{1-\beta}\left(\frac{1-\alpha}{\eta}\right)\left(\frac{\eta-1}{\eta\phi}\right)^{v} AH^{\beta}, \ \ a_{12} = -\frac{H}{1-\beta},$$

$$a_{21} = \frac{1-\alpha}{\eta}\left(\frac{\eta-1}{\eta\phi}\right)^{v}\left(\frac{1-\beta}{B\xi} - \beta XH\right) AH^{\beta-2}, \ \ a_{22} = \frac{\left(\frac{1-\alpha}{\eta}\right)\left(\frac{\eta-1}{\eta\,\phi}\right)^{v} AH^{\beta-1}}{B\xi X} + X.$$

Let κ_1 and κ_2 be two characteristic roots of this dynamic system. According to the Jacobian matrix, we further obtain:

$$Tr(J) = \kappa_1 + \kappa_2 = \frac{\beta}{1-\beta}\left(\frac{1-\alpha}{\eta}\right)\left(\frac{\eta-1}{\eta\phi}\right)^{v} AH^{\beta} + \frac{\left(\frac{1-\alpha}{\eta}\right)\left(\frac{\eta-1}{\eta\,\phi}\right)^{v} AH^{\beta-1}}{B\xi X} + X, \tag{A2}$$

$$Det(J) = \kappa_1\kappa_2 = \frac{\left(\frac{1-\alpha}{\eta}\right)\left(\frac{\eta-1}{\eta\,\phi}\right)^{v} AH^{\beta-1}}{B\xi\,(1-\beta)\,X}\left[\beta\left(\frac{1-\alpha}{\eta}\right)\left(\frac{\eta-1}{\eta\phi}\right)^{v} AH^{\beta} + (1-\beta)\,X\right]. \tag{A3}$$

It is clear from (A2) and (A3) that the there are two roots with positive real parts in the dynamic system, provided that $\beta < 1$. Given that both H_t and X_t are jump variables, this implies that the steady-state equilibrium is locally determinate.

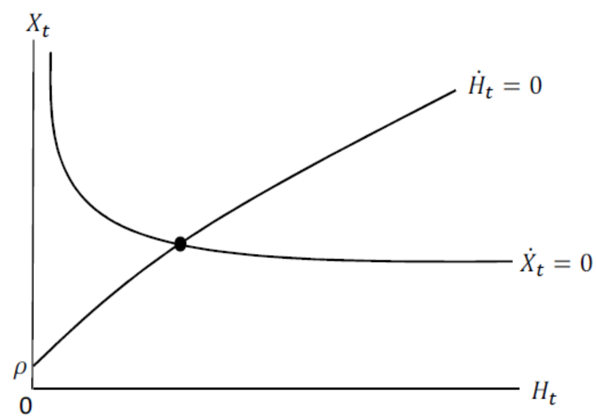

Figure A: The Steady-State Equilibrium

It follows from (23) that the $\dot{H}_t = 0$ locus intersects the X_t-coordinate at $\lim_{H_t \to 0} X_t = \rho > 0$ and is upward sloping, i.e.:

$$\left(\frac{\partial X}{\partial H}\right)_{\dot{H}_t=0} = \frac{1-\alpha}{\eta}\left(\frac{\eta-1}{\eta\,\phi}\right)^{v} A\beta H^{\beta-1} > 0.$$

From (24), we derive $\lim_{H_t \to 0} X_t = \infty$ and the slope of the $\dot{X}_t = 0$ locus is negative, i.e.,

$$\left(\frac{\partial X}{\partial H}\right)_{\dot{X}_t=0} = \frac{\frac{1-\alpha}{\eta}\left(\frac{\eta-1}{\eta\,\phi}\right)^v AH^{\beta-1}\left(\frac{\beta-1}{B\xi}\frac{1}{XH}+\beta\right)}{\left[1+\frac{1}{\xi}\frac{\frac{1-\alpha}{\eta}\left(\frac{\eta-1}{\eta\,\phi}\right)^v AH^{\beta-1}}{BX^2}\right]} < 0,$$

provided that $XH < \frac{1-\beta}{\xi\beta B}$. Accordingly, the existence and uniqueness of the steady-state equilibrium can be guaranteed, as shown in Figure A above. ■

Proof of Proposition 1. Based on (23) and (24) with $\dot{H}_t = 0$ and $\dot{X}_t = 0$, it is easy to derive:

$$\frac{\partial g}{\partial \mu} = -\left(\frac{\alpha}{1-\alpha}\right)\frac{B\xi\beta XH}{\Phi} < 0, \tag{A4}$$

$$\frac{\partial U}{\partial \mu} = -\frac{B\xi\Psi H}{\rho^2[1+\xi\,(\mu+\rho)]\Phi} < 0, \tag{A5}$$

where $\Phi \equiv (1-\beta) + [1+\xi\,(\mu+\rho)]\beta BH > 0$ and $\Psi \equiv \rho\xi\beta\,(\mu+\rho)\left[1+\left(\frac{\alpha}{1-\alpha}\right)\frac{X}{\rho}\right] + \rho v + \alpha(1+v)(X-\rho) > 0$. It follows from (16) and (25) that along the BGP, the following relationship must hold:

$$g = \mu - \pi = R - \delta - \rho.$$

Given that the nominal interest rate is $i = R - \delta + \pi$, we can easily obtain:

$$i = \mu + \rho.$$

Social welfare U is decreasing in μ, but the nominal interest rate i is increasing in μ. Therefore, social welfare is maximized as the nominal interest rate approaches zero and the Friedman rule holds.

In addition, from (A4) and (A5), we further obtain:

$$\frac{\partial}{\partial \eta}\left|\frac{\partial g}{\partial \mu}\right| = \frac{B\xi\beta H}{(1-\alpha)\Phi^2}\left\{\frac{\eta\rho\,(1-\beta)\left(\frac{\eta\,\phi}{\eta-1}\right)^v X}{(\Phi X+\rho\beta)\,AH^\beta} + \Phi\,(1-\alpha)\right\}\Theta \gtrless 0;\ \text{if } v \gtrless \eta-1,$$

$$\frac{\partial}{\partial \eta}\left|\frac{\partial U}{\partial \mu}\right| = \frac{B\xi H}{[1+\xi\,(\mu+\rho)]\,(\rho\Phi)^2}\left\{\frac{\eta\rho\,(1-\beta)\left(\frac{\eta\,\phi}{\eta-1}\right)^v \Psi}{\alpha\,(\Phi X+\rho\beta)\,AH^\beta} + \frac{\beta\left(\frac{\eta-1}{1-\alpha}\right)[1+\xi\,(\mu+\rho)]}{H}\right\}\Theta \gtrless 0;\ \text{if } v \gtrless \eta-1.$$

where $\Theta = \frac{\alpha}{\eta^2(\eta-1)}\left(\frac{\eta-1}{\eta\phi}\right)^v\left(1+\frac{\rho\beta}{\Phi X}\right)AH^\beta\,[v-(\eta-1)] \gtrless 0$ if $v \gtrless \eta-1$. ■