# How much Keynes and how much Schumpeter? An Estimated Macromodel of the US Economy

Cozzi, Guido and Pataracchia, Beatrice and Ratto, Marco and Pfeiffer, Philipp

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Guido Cozzi[a] Beatrice Pataracchia[b] Philipp Pfeiffer[c]
Marco Ratto[d]

**Abstract**

The macroeconomic experience of the last decade stressed the importance of jointly studying the growth and business cycle fluctuations behavior of the economy. To analyze this issue, we embed a model of Schumpeterian growth into an estimated medium-scale DSGE model. Results from a Bayesian estimation suggest that investment risk premia are a key driver of the slump following the Great Recession. Endogenous innovation dynamics amplifies financial crises and helps explain the slow recovery. Moreover, financial conditions also account for a substantial share of R&D investment dynamics.

**JEL classification**: E3 · O3 · O4

**Keywords**: Endogenous growth · R&D · Schumpeterian Growth · Bayesian Estimation

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[a] University of St. Gallen, E-mail: guido.cozzi@unisg.ch

[b] European Commission, JRC, E-mail: beatrice.pataracchia@ec.europa.eu,

[c] Technische Universität Berlin, E-mail: philipp.pfeiffer@tu-berlin.de

[d] European Commission, JRC, E-mail: marco.ratto@ec.europa.eu.

# 1 Introduction

Medium-term growth prospects in the US and many developed countries have deteriorated substantially. In particular, the US Great Recession of 2008-2009 seems to have generated a persistent downward shift in the GDP trend displayed in Figure 1.

[Figure 1 about here.]

Since this has happened after the financial crisis of 2008, it suggests that the sudden unavailability of credit to investment resulting from the crisis has led to a not yet reverted GDP decline below its long-term trend. Can a temporary financial shock generate such a persistent effect? It can reduce physical capital accumulation during the crisis and the resulting credit crunch, but, according to standard macroeconomics - which assumes decreasing returns to physical capital - GDP should go back to trend after the financial trouble has ended. Alternatively, we could think of an adverse exogenous technology growth shock, perhaps occurred simultaneously with the financial crisis, or even causing it. However, the above evidence would suggest an unrealistically high persistence of such technological shock. We claim that the puzzle is solved, if we realize that technology is not exogenously evolving, but is affected by research and development (R&D). And in fact, R&D investment data feature a significant temporary negative deviation of R&D investment from its trend during the financial crisis.

It is then natural to conjecture that if TFP growth results from R&D-driven innovations, the drop in the right panel of Figure 1, by implying a permanent negative deviation of productivity from its pre-crisis trend, could be at least partially explain the GDP time series.

While this channel is certainly at work, others more common in standard New Keynesian models, could be important as well: demand shocks, price and wage distortions, etc. Hence, the correct question would be: how much of the observed GDP data of Figure 1 is due to an R&D or physical capital investment drop, persistently insufficient private or government demand, ineffective monetary policy close to the zero lower bound (ZLB) or labor and output market distortions?

If these factors are jointly at work they cannot be studied in isolation.

Therefore, a complete macroeconomic explanation of what has happened should include R&D, innovation, and growth in addition to the standard dynamic stochastic general equilibrium (DSGE) framework. Motivated by this need, we construct an integrated growth and business cycle medium scale macroeconomic model, which incorporates Schumpeterian creative destruction *à la* Aghion and Howitt (1992) and Nuño (2011) into a New Keynesian economic framework *à la* Smets and Wouters (2003) and Kollmann et al. (2016).[1] This setup allows us to estimate the model at quarterly frequency for the US economy in a period - 1995Q1 to 2015Q1 - rich in important events involving innovation and growth - burst of the bubble, the 2009 collapse in R&D - and in business cycles and monetary policy events - financial crisis build-up and explosion, unprecedented fiscal stimulus, ZLB hit by the Fed interest rate.

Despite the importance of such an analysis, the estimated models attempting to do it so far are rare. Most notably Bianchi and Kung (2014) estimate a model with R&D capital exerting a positive spillover on the economy like in Frankel (1962) and Romer (1986), while Anzoategui et al. (2016) adapt Comin and Gertler (2006) knowledge diffusion extension of Romer's 1990 expanding product variety growth mechanics to show how demand driven slumps lead to business cycle persistence via endogenous R&D activity. Varga et al. (2016) extend this framework to a semi-endogenous growth medium scale model calibrated to US and EA data. Guerron-Quintana and Jinnai (2015) highlight the role of financial frictions by deeply microfounding them. Despite their insightful contributions to an emerging integrated macroeconomics[2] literature, none of these papers considers creative destruction, which is the only approach consistent with the microeconometric evidence that innovation and growth correlates positively with firm entry *and* firm exit.[3] In fact, the variety expansion models predict firm exit to have a negative effect on growth, which is against industry evidence suggesting that reallocation from less productive exiting firms to

[1]Previous successful attempts at integrating endogenous growth and business cycle are Annicchiarico et al. (2011), Annicchiarico and Rossi (2013), and Annicchiarico and Pelloni (2014).

[2]Integrating growth and business cycle in a unified way.

[3]See for example Foster et al. (2001) influential evidence that the ongoing replacement of less productive with more productive plants hugely contributes to industry multifactor productivity.

more productive firms is an engine of productivity growth.[4]

In our model, growth is endogenously driven by Schumpeterian R&D entrepreneurs' activities and knowledge accumulation. In this formulation, each innovation is a new intermediate good of enhanced quality. Entrepreneurs collect funds from households to invest into R&D aimed at capturing monopoly rents. In each period they face a probability that the firm jumps to the technological frontier. If the innovation occurs, the entrepreneur earns monopoly profits until the firm is replaced by a new innovator. On aggregate, growth of the technological frontier is the outcome of positive knowledge spillovers from R&D activities. These spillovers are subject to shocks which alter the basic research content of applied R&D. Similarly to Varga et al. (2016), our main model assumes a semi-endogenous growth structure in the innovative frontier evolution, while allowing its adoption to follow a purely endogenous growth mechanism.

On the Keynesian side, we incorporate monopolistic competition in product and labor markets as well as price and wage stickiness. Despite the importance of creative destruction, only few papers have tried to join it with price stickiness. Prominent examples are Benigno and Fornaro (2016), Oikawa and Ueda (2015a,b,c), and Rozsypal (2016). Other standard aspects such as habit formation in leisure and consumption, flow adjustment costs in investment, capacity utilization, endogenous fiscal rules, and government debt accumulation help fit observed quantity dynamics and a rich set of macroeconomic shocks is used for the estimation. In addition, the central bank follows a Taylor rule. Our analysis also considers consequences of the ZLB constraint on the policy interest rate. To guarantee realistic features also on the growth mechanism, we eliminate the strong scale effect Jones (1995) and test our results both in a semi-endogenous and in a fully endogenous approach.

We allow the model to nest exogenous growth as a special case, leaving it to the data to decide. And indeed the data confirm that frontier growth in potential GDP is driven by endogenous R&D investment. As in Aghion and Howitt (1992) and Nuño (2011), innovations are the outcome of a patent-race in every sector, with each innovation improving upon ex-

[4] For the importance of reallocation as an engine of growth also see Acemoglu et al. (2013).

isting goods. Innovating firms replace the incumbent monopolist and earn higher profits until the next innovation occurs. Knowledge spillovers push the technological frontier further. Unlike existing stylized Schumpeterian growth models, the DSGE structure allows, for the first time in the endogenous growth literature, a sophisticated estimation of the main innovation and growth parameters. Semi-endogenous growth *à la* Jones (1995), Kortum (1997), and Segerstrom (1998) is confirmed, but knowledge spillover coefficient confidence intervals allow a very persistent effect of shocks affecting R&D.

More generally, our Bayesian estimation allows us to quantify the relative contribution of the various shocks in explaining the recent adverse growth experience. Investment dynamics emerges as a key driver of the Great Recession, alongside the consumer saving shock: their interaction characterizes the joint decline in physical capital, R&D investment, and consumption.[5]

The paper is intendedly standard, and indeed we constrain ourselves to putting together the already existing frameworks of New Keynesian and Schumpeterian growth theory. The next section are divided as follows: Section 2 describes the main model. Section 3 describes the Bayesian estimation approach and its results, and discusses numerical simulations. Section 4 shows the robustness of our main model results under important alternatives: a scale-free fully endogenous growth framework, and two hybrid versions which allow some degree of exogenous growth in either semi-endogenous and fully endogenous frameworks. Section 5 concludes.

# 2 Model

This section lays out the economic environment. Being ours a fairly rich medium-scale macroeconomic model, we will here provide only the main aspects of its components: households, manufacturing production, innovation, monetary and fiscal authorities, market clearing conditions, and

[5] Anzoategui et al. (2016) use a "liquidity" shock to generate the positive co-movement of consumption and investment during the crisis. We have also experimented with it obtaining similar results. However, off crisis the empirical feature of this shock is slightly less than that of two separate investment and saving shocks.

exogenous structural shocks.[6] To keep exposition lean, we relegate the most cumbersome details to Appendix B.

## 2.1 Households

There is a continuum of households indexed by $j \in [0,1]$. Households are split in two groups: Savers ("Ricardians", superscript $s$) who own the firms and hold government bonds, and constrained households ("rule-of-thumb" consumers, superscript $c$) whose only income is labor income and who do not save nor borrow. The share of savers in the population is $\omega^s$. The lifetime utility of a household of either type $r = s, c$ is defined as

$$U^r_{jt} = \sum_{q=t}^{\infty} \exp(\epsilon^C_t)\beta^{q-t} u^s\left(\cdot\right), \tag{1}$$

where $\beta$ denotes the discount factor, and $\epsilon^C_t$ is an exogenous savings shock distorting the household's discount factor. Both types of households enjoy utility from consumption $C^r_{jt}$ and incur disutility from labor $N^r_{jt}$.[7] In addition, the utility of Ricardian households depends on financial assets held.

### 2.1.1 Ricardian Households

Ricardians work, consume, own all firms, purchase risk-free bonds as well as government bonds and receive nominal transfers $T^s_{jt}$ from the government. They have access to financial markets and hold financial assets $FA_{jt}$. Total financial wealth consists of government bonds $B^g_{jt}$, private risk-free bonds $B^{rf}_{jt}$, and shares $P^S_t S_{jt}$. $P^S_t$ is the nominal price of shares in $t$ and $S_{jt}$ the number of shares held by household $j$:

$$FA_{jt} = B^g_{jt} + B^{rf}_{jt} + P^S_t S_{jt}. \tag{2}$$

[6]The present model shares standard features with Kollmann et al. (2016), with the major distinction of introducing endogenous innovation in place of fully exogenous technological progress.

[7]At any given moment of time $N^r_t$ households of type $r$ work. We assume perfect consumption insurance across household types so that in the equilibrium all households within a given type consume the same amount of consumption goods.

We define the gross nominal return of an asset $S_t$ as $i_t^s$. Therefore, the budget constraint of a saver household $j$ can be written as:

$$P_t(1+\tau^c)C_{jt}^s + FA_{jt} = \left(1-\tau^N\right) W_t N_{jt}^s + \left(P_t^S + P_t d_t\right) S_{jt-1} + (1+i_{t-1}^g) B_{jt}^g + (1+i_{t-1}^{rf}) B_{jt}^{rf} + T_{jt}^s + \Pi_t - tax_{jt}^s \exp(\epsilon^{TAX}), \quad (3)$$

where $P_t$ denotes the GDP deflator and $\tau^c$ is a consumption (VAT) tax. $\tau^N$ is the tax rate levied on wages $W_t$, $d_t$ are dividends from intermediate good producers, $i_{t-1}^g$ is the interest rate of governmental bonds and $i_{t-1}^{rf}$ is the risk-free rate. $T_{jt}^s$ are government transfers to savers. $\Pi_t$ denotes the profits all the firms other than intermediate goods producers. $tax_{jt}^s$ are lump-sum taxes paid by savers subject to a tax shock $\epsilon^{TAX}$.

Each Ricardian household $j$ maximizes its lifetime utility:

$$u^s\left(C_{jt}^s, N_{jt}^s, \frac{U_{jt-1}^A}{P_t}\right) = \frac{1}{1-\theta}\left(C_{jt}^s - hC_{t-1}^s\right)^{1-\theta} - \frac{\omega^n \exp(\epsilon_t^U)}{1+\theta^N}\left(C_t^s\right)^{1-\theta}\left(N_{jt}^s - h^N N_{t-1}^s\right)^{1+\theta^N} + \left(C_{jt}^s - hC_{t-1}^s\right)^{-\theta}\frac{U_{jt-1}^A}{P_t}, \quad (4)$$

with $C_t^s = \int_0^{\omega_s} C_{jt}^s dj$. $h$ and $h^N \in (0,1)$ measure the strength of external habits in consumption and labor, and $\omega^n$ is the relative weight of labor in the utility function. $\epsilon_t^U$ is a labor preference shock common in the real business cycle literature. To allow for realistic spreads, we microfound the risk premium shock as in Kollmann et al. (2016) by assuming that savers' preferences for real financial wealth, $(FA_{jt})/(P_t)$, may fluctuate as well according to

$$U_{jt-1}^A = \exp(\epsilon_t^B)\left(\alpha^{B0}\ B_{jt-1}\right) + \exp(\epsilon_t^S)\ \left(\alpha^{S0} P_{t-1}^s\ S_{jt-1}\right), \quad (5)$$

where $\epsilon_t^B$ and $\epsilon_t^S$ represent time varying risk premium shocks on bond and stock holdings. Fisher (2015) uses a simpler version of this formulation and gives a flight-to-quality interpretation, microfounded as a preference for risk-free bonds. We here generalize this flight to quality by allowing for different marginal utilities for different assets captured by $\alpha^{B0}$ and $\alpha^{S0}$.[8]

### 2.1.2 Constrained households

Constrained households do not participate in financial markets. In every period they consume all their disposable income from wages and government transfers. This results in the following period-by-period budget constraint:

$$P_t(1+\tau^c)C^c_{jt} = \left(1-\tau^N\right) W_t N^c_t + T^c_t - tax^c_{jt}\exp(\epsilon^{TAX}_t), \tag{6}$$

where $tax^c_{jt}$ are lump-sum taxes paid by constrained households. The instantaneous utility function for a liquidity constraint households is

$$\begin{aligned} u^c\left(C^c_{jt}, N^c_{jt}\right) = & \frac{\exp(\epsilon^C_t)}{1-\theta}\left(C^c_{jt} - hC^c_{t-1}\right)^{1-\theta} \\ & - \frac{\omega^n \exp(\epsilon^U_t)}{1+\theta^N}\left(C^c_t\right)^{1-\theta}\left(N^c_{jt} - h^N N^c_{t-1}\right)^{1+\theta^N}, \end{aligned} \tag{7}$$

where $C^c_t = \int_{\omega_s}^1 C^c_{jt} dj$.

## 2.2 Intermediate Goods

Each $i^{\text{th}}$ differentiated intermediate good is produced by a monopolistically competitive firm using total capital $K^{tot}_{it-1}$ and labor $N_{it}$. The production function is Cobb-Douglas:

$$Y_{it} = A^Y_{it}\left(N_{it}\right)^a \left(cu_{it}\frac{K^{tot}_{it-1}}{A^Y_t}\right)^{1-a}, \tag{8}$$

where $a$ denotes the labor share, $A^Y_{it}$ is a sector specific productivity level and $cu_{it}$ is firm-specific level of capital utilization. With more sophisticated technologies, production becomes more capital-intensive. $A^Y_t = \int_0^1 A^Y_{it} d\,i$ is the average productivity across all sector in the differentiated goods production. The average output across sectors is $Y_t = \int_0^1 Y_{it} d\,i$.

[8] Our steady state restrictions imply that $\alpha^{B0}$ is positive while $\alpha^{S0}$ is negative. In equilibrium these values implies a positive equity premium and a treasury bill convenience yield (del Negro et al. 2016).

As a consequence of (8) sectors with higher relative technological sophistication benefit more from the average technological level across sectors:[9]

$$Y_{it} = \underbrace{\left(\frac{A_{it}^Y}{A_t^Y}\right)}_{\text{rel. technology level}} \left(A_t^Y N_{it}\right)^a \left(cu_{it} K_{it-1}^{tot}\right)^{1-a}. \tag{9}$$

Total Factor Productivity, $TFP_t$, is therefore:

$$TFP_t = \left(A_t^Y\right)^a. \tag{10}$$

Firm $i$'s capital stock evolves as

$$K_{it} = (1-\delta)K_{it-1} + I_{it}, \tag{11}$$

where $\delta$ is the capital depreciation rate, and $I_{it}$ denotes gross investment in physical capital. Public capital $K_{it}^g$ follows an analogous law of motion. Total capital is the sum of both

$$K_{it}^{tot} = K_{it} + K_{it}^g, \tag{12}$$

assumed, for the sake of simplicity, to be perfect substitutes.

Intermediate good firms choose prices, employment, and capacity utilization as well as capital and investment to maximize dividends subject to the production technology (9) and the physical capital law of motion (11). Dividends are given by

$$d_{it} = \left(1-\tau^K\right)\left(Y_{it} - \frac{W_t}{P_t}N_{it}\right) + \tau^K \delta K_{it-1} - I_{it} - adj_{it}, \tag{13}$$

where $\tau^K$ is a corporate income tax, $adj_{it}$ are total adjustment costs associated with price and labor adjustment or changing capacity utilization, and investment. The firm problem is standard and details are referred to Appendix B.1.

[9]This formulation allows us to avoid keeping track a of a distribution of firms and instead only look at average firms and frontier firms.

## 2.3 Final Good Producers and Labor Markets

The remaining sectors of manufacturing production are kept intendedly standard. Therefore, we only summarize key elements and refer details to Appendix B.2-B.3. Perfectly competitive firms produce a final good $Y_t$ using differentiated intermediate inputs. Wages in the intermediate good production are set by a monopolistically competitive trade union at a mark-up, $\mu_t^w$. To capture rich labor market dynamics, we allow for real wage rigidities (see Blanchard and Galí 2007), governed by parameter $\gamma^{wr}$.

## 2.4 Endogenous Innovation

We start with the detailed description of the endogenous technological progress structure of our model.

**Innovations.** Innovations generate growth. In each period $t$ the productivity of a sector $A_{it}^Y$ jumps to the technology frontier $A_t^{Y,\max}$ with probability $n_{it-1}$. The frontier is publicly available and represents the most advanced technological level across all sectors defined as $A_t^{Y,\max} \equiv \max\{A_{it}^Y | i \in [0,1]\}$. Productivity in each sector $i$ evolves as:

$$A_{it}^Y = \left\{ \begin{array}{lll} A_t^{Y,\max}, & \text{with probability} & n_{it-1} \\ A_{it-1}^Y, & \text{with probability} & 1 - n_{it-1} \end{array} \right\}. \tag{14}$$

**Entrepreneurs.** Innovations are the result of entrepreneurial investments into R&D. The probability of reaching the frontier is itself endogenous. In each sector $i$ in each period $t$ an entrepreneur is randomly selected with the opportunity to try to innovate.[10] If such entrepreneur's R&D firm invests R&D cost $X_{it}^{RD}$ it will produce a probability $n_{it}$ of a successful innovation, which entails the discovery of a new intermediate good with next period's frontier productivity $A_t^{Y,\max}$. We assume that research will be more difficult if the overall technology frontier is more advanced, i.e. per unit research costs increase with the frontier productivity $A_t^{Y,\max}$. The probability of innovation in the sector, assumed independent across sectors, has the fol-

[10] Hence, innovative ideas are scarce as they "arrive to random agents at random times" (Erkal and Scotchmer, 2011, p.1).

lowing production function:

$$n_{it} = \left\{ \begin{array}{ll} \left( \frac{X_{it}^{RD}}{\lambda^{RD} A_{it+1}^{Y,\max}} \right)^{\frac{1}{(\eta+1)}}, & \text{if} \quad X_{it}^{RD} < \lambda^{RD} A_{t+1}^{Y,\max} \\ 1, & \text{if} \quad X_{it}^{RD} \geq \lambda^{RD} A_{t+1}^{Y,\max} \end{array} \right\}, \tag{15}$$

where $\lambda^{RD} > 0$ is an R&D difficulty parameter and $\eta > 0$ accounts for decreasing returns of R&D. In our discrete time setting, we will assume that an innovation occurring at time $t$ will permit production in period $t+1$. Moreover, the second line of (15) is needed to guarantee that the probability of innovation per period[11] is no larger than 1.

By Bertrand competition, the patent holder of the new good will produce by definition the good of highest quality and, following a price war, will replace the existing incumbent monopolist and appropriate all the sectorial profits from $t+1$ on. Hence, it is the prospects of becoming next period's incumbent manufacturing monopolist in the sector that creates the incentives to invest in R&D at time $t$. Since the new entrant does not internalize the loss incurred by the previous incumbent, creative destruction may imply too much or too little R&D investment.[12]

For symmetry with the other parts of the model estimation, which include adjustment costs, we will assume the existence of adjustment costs in the R&D as well. The R&D adjustment cost function[13] is defined as

$$adj_{it}^{RD}(X_{it}^{RD}) \equiv \frac{\gamma^{RD}}{2Y_t}(X_{it}^{RD} - X_{t-1}^{RD} g_Y)^2, \tag{16}$$

where $g_Y$ denotes output trend which makes adjustment costs stationary.

Innovation at time $t$ is like a static lottery, and given individual risk aversion, we assume that the R&D firm in each sector finances the risky innovation from a fund set up by Ricardian households to completely diversify, by the law of large numbers, innovation risk across the continuum

[11] Which instead in a continuous time framework could be any non-negative number.

[12] A horizontal innovation framework, by missing this business stealing externality, would imply too little R&D in equilibrium.

[13] The interpretation of adjustment costs in a creative destruction environment requires explanation: we are implicitly assuming that if a new entrant undertakes R&D, the previous period R&D laboratory size (magnified by trend GDP growth) sets the benchmark for the current R&D investment size, penalizing departures from it.

of sectors, with uncorrelated risk, in the economy. To capture realistic fluctuations of R&D investment, we allow for a stochastic R&D-specific investment risk premium required by the fund, $\epsilon_t^{AY}$. The R&D entrepreneurial problem is then a simple expected profit maximization:

$$\max_{X_{it}^{RD}} = n_{it} \left[ \frac{P_{it}^{Sd\max}}{P_t} \right] - \left[ X_{it}^{RD} + adj_{it}^{RD}(X_{it}^{RD}) \right] \left( 1 + \epsilon_t^{AY} \right), \tag{17}$$

where $P_t^{Sd\max}$ is the nominal stock market value of the firm at the technological frontier.

The value of becoming the incumbent is the same across sectors and hence, in equilibrium, the R&D investment cost is symmetric, $X_{it}^{RD} = X_t^{RD}$, as is the probability of success $n_{it} = n_t$. The R&D optimality condition, after making use of (15), then becomes:

$$\frac{n_t}{(\eta+1)} \left[ \frac{P_t^{Sd\max}}{P_t} \right] = X_t^{RD} \left( \frac{\gamma^{RD}}{Y_t} (X_{it}^{RD} - X_{t-1}^{RD} g_Y) \right) (1 + \epsilon_t^{AY}). \tag{18}$$

Note that R&D firms earn positive profits as long $\eta > 0$.[14]

Finally, by the law of large numbers, $n_t$ also measures the fraction of the total number of sectors which innovate each period, as well as the fraction of firms that exit and enter the market. The higher the equilibrium value of $n_t$ the stronger innovation and creative destruction, and the more dynamic the set of innovative industries.

**Stock Market Value.** Differentiated goods producing firms are owned by households. The value of these firms at time $t$ is $P_t^S S_t^{tot} = P_t^S$ where we normalize the total number of stocks $S_t^{tot}$ to 1. In our economy firms are heterogeneous and firm turnover follows innovation. Hence, due to Schumpeterian creative destruction, a fraction $n_{t-1}$ of obsolete firms belonging to time $t-1$ portfolio is lost at time $t$, replaced by new entrants with higher stock market value $P_t^{S\max}$. Taking that into account, the gross nominal

[14] As long as the equilibrium probability is less than 1, we could easily consider the case $\eta = 0$, which nests a linear R&D technology and also a free-entry case. In this last case ideas would not be scarce any more.

return on the aggregate time $t$ stock market portfolio is given by:

$$1 + i_t^s = \frac{E_t \left[ d_{t+1} P_{t+1} + P_{t+1}^S - n_t P_{t+1}^{S\max} \right]}{P_t^S}. \tag{19}$$

We include in the return on period $t$ equities the value of the average dividend payments $d_{t+1}$, we also take into accounts capital gains and losses. The average time $t+1$ portfolio, $P_{t+1}^S$, also includes the innovative firms that have replaced fraction $n_t$ of time $t$ industry. Hence, we have to subtract their aggregate value, $n_t P_{t+1}^{S\max}$, from it. This reasoning is reflected in (19).

**Frontier Value.** Frontier technology index net growth rate $g_{A_t^{Y,\max}}$ is defined as

$$g_{A_t^{Y,\max}} = \frac{A_t^{Y,\max} - A_{t-1}^{Y,\max}}{A_{t-1}^{Y,\max}}. \tag{20}$$

Entrepreneurs collect funds from households. They invest into R&D to reach the technological frontier, patent its adaptation to their sector, and appropriate the resulting production monopoly. Hence, each entrepreneur at the frontier earns the monopoly profits resulting from the highest quality intermediate good: these profits, and the resulting dividends, are $(A_t^{Y,\max})/(A_t^Y)$ times bigger than those of the average technology firm. Therefore, the nominal stock market value as of time $t$, $P_t^{S\max}$, of a firm that will start producing at the technology frontier at time $t+1$ must obey the following expression:

$$P_t^{S\max} = \frac{E_t \left[ P_{t+1} d_{t+1} \left( \frac{A_{t+1}^{Y,\max}}{A_{t+1}^Y} \right) + \frac{P_{t+1}^{S\max}}{g_{A_{t+1}^{Y,\max}}} (1 - n_{t+1}) \right]}{(1 + i_t^s)}. \tag{21}$$

Notice that since patents of the latest and most advanced technological require one period of implementation, for an innovation developed in period $t$, production and dividend flows only start in $t+1$. Furthermore, the continuation value in the stock market in $t+1$ takes into account that competitors may successfully innovate, with probability $n_{t+1}$, and render it unusable in period $t+2$. In case no innovation is found in the sector in period $t+1$, which happens with probability $1 - n_{t+1}$, the firm's value - along with its dividends - will remain positive, though they will be lower than that of a generic newly entered innovator by a factor equal to $g_{A^{Y,\max}_{t+1}}$. This explains the last part of the numerator of eq. (21).

**Frontier and diffusion.** The growth of technological frontier, $g_{A^{Y,\max}_t}$, is the outcome of positive knowledge spillovers from the aggregate innovation efforts as in Howitt and Aghion (1998). According to this Schumpeterian view, R&D activities have an appropriable applied content, i.e. the patentable sectorial adoption of the technological frontier, and an unappropriable basic aspect, which pushes the aggregate frontier further. The basic content of aggregate R&D freely spills over to all sectors. The way in which this R&D spillover operates is not deterministic, but as in Nuño (2011) it is affected by time-varying spillovers $\sigma_t^{RD}$. This captures the potentially volatile basic research content of applied R&D, which could reflect, in reduced form, the scientists and engineers orientation, the university policies, and variable regulatory aspects of intellectual property rights (IPRs). More in detail, we assume that

$$A_t^{Y,\max} = A_{t-1}^{Y,\max} + \left(A_{t-1}^{Y,\max}\right)^{\varphi} \left(\frac{X_{t-1}^{RD}}{Y_{t-1}} N_{t-1}\right)^{\lambda^A} \sigma_t^{RD}, \tag{22}$$

where $\varphi < 1$ reflects decreasing returns to the intertemporal knowledge spillover as in Jones (1995), while $0 < \lambda^A < 1$ is a standard "stepping on toes" R&D congestion externality parameter capturing research duplication, knowledge theft, etc. Jones and Williams (2000). This idea was also used in the medium scale policy focused macroeconomic models of Roeger et al. (2008) and Varga et al. (2016).

The instantaneous knowledge spillovers follows an exogenous process given by

$$\sigma_t^{RD} = \sigma^{RD} \exp(\epsilon_t^{\sigma}), \tag{23}$$

where $\epsilon_t^{\sigma}$ is an R&D spillover shock and $\sigma^{RD}$ are steady state spillovers. Eq. (22) says that the technological frontier increases in its previous period's value, and also in the fraction of the employees indirectly working in R&D (given by $\frac{X_{t-1}^{RD}}{Y_{t-1}} N_{t-1}$) in the past period. We can then express the growth rate of the technological frontier as:

$$g_{A_t^{Y,\max}} = \frac{\left(\frac{X_{t-1}}{Y_{t-1}} N_{t-1}\right)^{\lambda^A} \sigma_t^{RD}}{\left(A_{t-1}^{Y,\max}\right)^{1-\varphi}}. \tag{24}$$

Log-differencing the previous equation, it follows that in a balanced growth path (BGP) - in which $g_{A_t^{Y,\max}}$ and $\frac{X_{t-1}}{Y_{t-1}}$ are constant - the following holds:

$$g_{A_t^{Y,\max}} = \frac{\lambda^A g_{POP}}{1-\varphi}. \tag{25}$$

As in Jones (1995) the growth rate of the frontier, in the long-run, is governed by the growth rate of population $g_{POP}$. The factor of proportionality depends on the extent of decreasing returns, inversely represented by $\lambda^A$. It is important to notice that unlike Jones (1995) and Varga et al. (2016) our growth of $A_t^{Y,\max}$ does not refer to the growth rate of patents but rather to the growth rate of frontier productivity index. While this is obviously derived from the flow of new patents invented, its numerical value is already filtered by effect of patents on productivity. Therefore, we should expect a much lower value of $\lambda^A$ than in Jones and Williams (2000) and Varga et al. (2016).

Unlike Nuño (2011) and similarly to Varga et al. (2016), the evolution of the technological frontier in our model is semi-endogenous, as in Jones (1995) while its adoption remains fully endogenous, as in Comin and Gertler (2006) and Anzoategui et al. (2016). This formulation not only permits to eliminate the counterfactual strong scale effects that plagued the early generation endogenous growth models, but also avoids steady state growth effects of policy variables and shocks, which facilitates the comparison with the standard exogenous growth DSGE models.

The average technological progress results from the adoption of the frontier technology, and it is driven by the aggregation of the previously described endogenous R&D entrepreneurial activity. In fact, the average intermediate goods productivity in the economy is defined by the following aggregation:

$$
\begin{aligned}
A_t^Y &= \int_0^1 \left\{ n_{it-1} A_t^{Y,\max} + (1 - n_{it-1}) A_{it-1}^Y \right\} d\, i \\
&= n_{t-1} \left( A_t^{Y,\max} - A_{t-1}^Y \right) + A_{t-1}^Y, \qquad (26)
\end{aligned}
$$

where we have also used our previous symmetry result, $n_{it-1} = n_{t-1}$. On a balanced growth path the frontier $A_t^{Y,\max}$ grows at the same rate as average technological level defined in (26).

## 2.5 Monetary and Fiscal Authorities

The nominal policy interest rate $i_t$ is set by the monetary authority according to a Taylor rule:

$$
i_t - \bar{i} = \rho^i \left( i_{t-1} - \bar{i} \right) + \left( 1 - \rho^i \right) \left( \eta^{i\pi} \left( \pi_t - \bar{\pi} \right) + \eta^{iy} \tilde{y}_t \right) + \epsilon_t^i, \qquad (27)
$$

where $\bar{i} = r + \bar{\pi}$ is the steady state nominal interest rate, equal to the sum of the steady state real interest rate and steady state inflation. $\tilde{y}_t$ is the output gap[15] and $\eta^{i\pi} > 1$ and $\eta^{iy} > 0$. $\epsilon_t^i$ is a white noise shock.

Government consumption and physical capital investment are set according to the following fiscal policy rules:

$$
\begin{aligned}
c_t^G - c^G &= \rho^G (c_{t-1}^G - c^G) + \epsilon_t^G \qquad (28) \\
i_t^G - i^G &= \rho^{IG} (i_{t-1}^G - i^G) + \epsilon_t^{IG}, \qquad (29)
\end{aligned}
$$

where $c_t^G \equiv \frac{G_t}{Y_t}$ and $i_t^G \equiv \frac{I_t^G}{Y_t}$ are government consumption and investment as a share of GDP.[16] $\epsilon_t^G$ and $\epsilon_t^{IG}$ are white noise disturbances. Government transfers to households follow this policy rule:

$$
\tau_t - \tau = \rho^\tau (\tau_{t-1} - \tau) + \eta^{def} \left( \frac{B_t^g - B_{t-1}^g}{Y_t} - def \right) + \eta^B \left( \frac{B_t^g}{Y_t} - b \right) + \epsilon_t^\tau, \qquad (30)
$$

where $\tau_t \equiv \frac{T_t}{Y_t}$ are net nominal transfers normalized by GDP. $B_t^g$ denotes total nominal government debt owned by households, $\eta^{def}$ is a deficit coef-

[15] Output gap is measured by $\tilde{y}_t = log\,(Y_t) - \bar{y}_t$ where $\bar{y}_t$ is (log) output trend.

[16] Lower case letters without time subscript denote steady state values.

ficient, $\eta^B$ is a debt coefficient. $\epsilon_t^T$ is a white noise transfer shock. Government debt and transfers react to their associated GDP-adjusted targets, $def$ and $b$. The government budget constraint, is

$$B_t^g = \left(1 + i_{t-1}^g\right) B_{t-1}^g - R_t^G + P_t G_t + P_t I_t^G + T_t, \tag{31}$$

where $1 + i_t^g$ denotes the interest rate on government debt and $R_t^G$ is the nominal revenue of the government.

## 2.6 Resource constraint

Market clearing requires

$$C_t = \omega^s C_t^s + (1 - \omega^s) C_t^c \tag{32}$$

and, additionally, $N_t^s = N_t^c = N_t$ and $T_t^s = T_t^c = T_t$. Labor markets clear and financial assets clearing requires $S_t = 1$ and $B_t = B_t^g$. Finally, the aggregate budget identity takes R&D investment into account

$$Y_t = C_t + I_t + G_t + I_t^G + adj_t + X_t^{RD}. \tag{33}$$

## 2.7 Exogenous processes

All exogenous shock processes of type $x$ (unless specified explicitly) follow autoregressive processes of order one with an autocorrelation coefficient $|\rho^x| < 1$ and innovation $u_t^x$. Thus,

$$\epsilon_t = \rho^x \epsilon_{t-1} + u_t^x. \tag{34}$$

# 3 Results

## 3.1 Data and Estimation Approach

We solve the model using a first-order approximation around its deterministic steady state. Following the literature on estimated DSGE models, we set the values for a subset of parameters *a priori*. We estimate the remaining parameters with Bayesian methods.[17] In particular, we apply the Slice sampling algorithm (Neal 2003 and Planas et al. 2015) because of its improved efficiency and accuracy.[18] In total we use data on 21 macroeconomic time series ranging from 1995Q1 until 2015Q1. Data are taken from the Bureau of Economic Analysis (BEA) and the Federal Reserve. Appendix A provides additional details.

It is worth noticing that in the estimation procedure, we have a number of parameters calibrated by directly using the steady state restrictions and others which are obtained by Bayesian estimation. The two procedures are interdependent, because the estimated parameters also affect the calibrated parameters, which are usually functions of them, and *vice versa*. We here mention an important example of this interdependence, in which the steady state restriction equation (25) is used to determine $\lambda^A$ as a function of the estimated parameters. In fact, since

$$TFP_t = \left(A_t^Y\right)^a, \tag{35}$$

it follows that in steady state the observable growth rate of TFP is $g_{TFP} = ag_{A^Y} = ag_{A^{Y,\max}}$. This equation in turn implies that we can use the steady state growth rate relationship to calculate parameter $\lambda^A$ as soon as we have an estimated parameter $\hat{\varphi}$ and $a$. In fact, based on the previous equations we can write:

$$\lambda^A = (1-\hat{\varphi})^{-1}\frac{g_{TFP}}{ag_{POP}}. \tag{36}$$

## 3.2 Calibrated parameters

Table 1 reports values for calibrated parameters.

Table 1: **Calibrated Parameters**

| Parameter | Description | Value |
|---|---|---|
| $a$ | Cobb-Douglas labor share | 0.65 |
| $\delta$ | Capital depreciation rate | 1.64% |
| $\alpha^{B0}$ | Preference for government bonds | -0.0016 |

(Continued on next page)

[17]We implement the solution and estimation in Dynare (Adjemian et al. 2011).

[18]Our findings are robust to other sampling algorithms, such as general Metropolis-Hastings algorithms (see An and Schorfheide 2007).

Table 1: (continued)

| Parameter | Description | Value |
|---|---|---|
| $\alpha^{S0}$ | Preference for stocks | 0.0068 |
| $\beta$ | Intertemporal discount factor | 1.0032 |
| $\bar{i}$ | Steady state nominal interest rate (quarterly) | 0.75% |
| $\mu^w$ | Wage mark-up | 1.2 |
| $\omega^N$ | Weight of the disutility of labour | 58.36 |
| $\sigma^y$ | Demand elasticity | 18.90 |
| $\tau^c$ | Consumption (VAT) tax | 0.2 |
| $\tau^K$ | Corporate income tax | 0.3 |
| $\tau^N$ | Labor tax | 0.17 |
| $b$ | Nominal government debt target | 3.4 |
| $def$ | Deficit target | 3.62% |
| $\tau$ | Steady state government transfers | 0.10 |
| $i^G$ | Steady state private investment share of GDP | 15.5% |
| $C/GDP$ | Steady state private consumption share of GDP | 67.2% |
| $c^G$ | Steady state government share of GDP | 17.1% |
| $\eta$ | R&D decreasing returns | 5.72 |
| $\lambda^{RD}$ | R&D difficulty | exp(17.61) |
| $\sigma^{RD}$ | Steady state R&D spillovers | 0.82% |
| $\bar{X}^{RD}$ | Steady state value of cost of R&D | 1.56% |
| $\lambda^A$ | Stepping on toes R&D congestion | 0.25 |
| $\bar{g}^{AY}$ | Constant for growth of productivity | 0.36% |
| $\bar{g}^{POP}$ | Constant for growth of population | 0.24% |
| $\bar{\pi}$ | Constant for GDP deflator inflation | 0.47% |
| $\bar{g}^Y$ | Constant for growth of GDP trend | 0.60% |
| $\bar{i}^g B^g$ | Steady state Government interest payments | 3.17% |

The labor share in the production function, $a$, is set to 0.65. The capital depreciation rate, $\delta$, is implied by the empirical averages of private capital and investment and equal to 1.64 percent quarterly. The discount rate, $\beta$, implies a steady state value of the stochastic discount factor of 0.99. This restriction allows to derive a value for the preference of households on holding stocks, $\alpha^{S0}$, from their optimal shares choice. The steady state wage mark-up, $\mu^w$, is set to 1.2, while $\omega^N$ is endogenized from the labor supply equation conditioning on the observed empirical average of the wage share. Normalizing steady state employment to unity, allows to derive the steady state price mark-up from the labor demand equation. Given the markup, the pricing equation is used to determine the value of the demand elasticity, $\sigma^y$. Consumption VAT tax and corporate income tax are set, respectively, to 0.2 and 0.3, while the labor tax is endogenized from the government revenue. The targets for total nominal government debt and deficit as well as public investment and consumption and government transfers, are set equal to the respective empirical averages. The empirical average of the interest payments on government bond ($\bar{i}^g B^g$ around 3 percent) allows to place a restriction on the parameter $\alpha^{B0}$. Steady state ratios of the private consumption and investment share of GDP match empirical averages. In the R&D block, we impose the empirical average of the share of R&D investment over GDP and the estimated mean value of the probability of reaching the frontier technology, $\bar{n}$, to derive the implied value of the R&D difficulty, $\lambda^{RD}$ and the parameter $\eta$ from the definition of production function of $n_t$ (15) and the optimality condition of innovators (18). The mean value of the R&D spillovers comes from the semi-endogenous frontier definition (22) given the empirical value of the technology growth rate.

## 3.3 Estimated parameters

This section provides the estimates of the main model described in the previous sections. The main results of our most refined estimation can be seen in Table 2.

Table 2: **Priors and Posteriors of Estimated Parameters**

| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| **Innovation and Growth** | | | | | | | |
| $\bar{n}$ | beta | 0.025 | 0.0120 | 0.047 | 0.0091 | 0.0327 | 0.0618 |
| $\varphi$ | beta | 0.700 | 0.2000 | 0.835 | 0.0958 | 0.7089 | 0.9715 |
| **Nominal and Real Frictions** | | | | | | | |
| $\gamma^{I,1}$ | gamm | 60.000 | 40.0000 | 50.779 | 21.4157 | 25.9600 | 88.6399 |
| $\gamma^{I,2}$ | gamm | 60.000 | 40.0000 | 31.570 | 11.7829 | 12.4344 | 50.0310 |
| $\gamma^{n}$ | gamm | 60.000 | 40.0000 | 3.753 | 0.9323 | 2.1834 | 5.1981 |
| $\gamma^{p}$ | gamm | 60.000 | 40.0000 | 62.109 | 11.4730 | 39.6652 | 77.5183 |
| $\gamma^{RD}$ | gamm | 60.000 | 40.0000 | 151.840 | 30.9571 | 107.5168 | 200.9417 |
| $\gamma^{u,2}$ | gamm | 0.020 | 0.0080 | 0.007 | 0.0026 | 0.0034 | 0.0105 |
| $\gamma^{w}$ | gamm | 5.000 | 2.0000 | 3.259 | 0.7432 | 1.9691 | 4.3739 |
| $\gamma^{wr}$ | beta | 0.500 | 0.2000 | 0.966 | 0.0121 | 0.9498 | 0.9839 |
| $sf^{p}$ | beta | 0.500 | 0.1000 | 0.771 | 0.0540 | 0.6846 | 0.8546 |

(Continued on next page)

Table 2: (continued)

| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| $sf^w$ | beta | 0.500 | 0.1000 | 0.519 | 0.0998 | 0.3378 | 0.6593 |
| **Fiscal Policy** | | | | | | | |
| $\eta^B$ | beta | 0.010 | 0.0050 | 0.000 | 0.0001 | 0.0000 | 0.0003 |
| $\eta^{def}$ | beta | 0.030 | 0.0080 | 0.013 | 0.0018 | 0.0106 | 0.0159 |
| **Monetary Policy** | | | | | | | |
| $\eta^{i,\pi}$ | beta | 2.000 | 0.4000 | 1.970 | 0.3658 | 1.3740 | 2.5851 |
| $\eta^{i,y}$ | beta | 0.250 | 0.1000 | 0.149 | 0.0379 | 0.0894 | 0.2001 |
| **Preferences and Households** | | | | | | | |
| $h$ | beta | 0.500 | 0.2000 | 0.839 | 0.0331 | 0.7880 | 0.8924 |
| $h^N$ | beta | 0.500 | 0.2000 | 0.631 | 0.2051 | 0.3309 | 0.9339 |
| $\theta^N$ | gamm | 2.500 | 0.5000 | 2.114 | 0.3850 | 1.5203 | 2.7155 |
| $\theta$ | gamm | 1.500 | 0.2000 | 1.404 | 0.1603 | 1.1482 | 1.6191 |
| $\omega^s$ | beta | 0.650 | 0.0500 | 0.763 | 0.0151 | 0.7394 | 0.7860 |

This table reports values estimated parameters of the baseline model. HPD inf and HPD sup refer to the 90 percent highest posterior density interval.

Table 3: **Priors and Posteriors of Shock Processes**

| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| **Autocorrelation** | | | | | | | |
| $\rho^{AY}$ | beta | 0.500 | 0.2000 | 0.463 | 0.1727 | 0.1681 | 0.7352 |
| $\rho^B$ | beta | 0.500 | 0.2000 | 0.947 | 0.0195 | 0.9196 | 0.9818 |
| $\rho^G$ | beta | 0.700 | 0.1000 | 0.972 | 0.0112 | 0.9557 | 0.9889 |
| $\rho^i$ | beta | 0.700 | 0.1200 | 0.883 | 0.0252 | 0.8441 | 0.9281 |
| $\rho^{IG}$ | beta | 0.700 | 0.1000 | 0.920 | 0.0224 | 0.8864 | 0.9550 |
| $\rho^{MUY}$ | beta | 0.500 | 0.2000 | 0.416 | 0.1353 | 0.1893 | 0.6184 |
| $\rho^{ND}$ | beta | 0.500 | 0.2000 | 0.692 | 0.0510 | 0.6049 | 0.7672 |
| $\rho^P$ | beta | 0.500 | 0.2000 | 0.763 | 0.0454 | 0.6886 | 0.8332 |
| $\rho^{RD}$ | beta | 0.500 | 0.2000 | 0.928 | 0.0344 | 0.8795 | 0.9815 |
| $\rho^S$ | beta | 0.500 | 0.2000 | 0.938 | 0.0259 | 0.9026 | 0.9826 |
| $\rho^\tau$ | beta | 0.700 | 0.1000 | 0.966 | 0.0141 | 0.9468 | 0.9908 |
| $\rho^{TAX}$ | beta | 0.500 | 0.2000 | 0.915 | 0.0336 | 0.8606 | 0.9666 |
| **Standard Deviations of Innovations** | | | | | | | |
| $u^{AY}$ | gamm | 0.010 | 0.0040 | 0.043 | 0.0068 | 0.0327 | 0.0544 |
| $u^B$ | gamm | 0.010 | 0.0040 | 0.001 | 0.0001 | 0.0011 | 0.0014 |
| $u^C$ | gamm | 0.010 | 0.0040 | 0.016 | 0.0042 | 0.0098 | 0.0223 |
| $u^G$ | gamm | 0.010 | 0.0040 | 0.001 | 0.0002 | 0.0012 | 0.0017 |

(Continued on next page)

Table 3: (continued)

| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| $u^i$ | gamm | 0.010 | 0.0040 | 0.001 | 0.0001 | 0.0009 | 0.0012 |
| $u^{IG}$ | gamm | 0.010 | 0.0040 | 0.001 | 0.0001 | 0.0007 | 0.0009 |
| $u^{MUY}$ | gamm | 0.020 | 0.0080 | 0.062 | 0.0129 | 0.0408 | 0.0830 |
| $u^{ND}$ | gamm | 0.005 | 0.0020 | 0.024 | 0.0026 | 0.0198 | 0.0281 |
| $u^U$ | gamm | 0.010 | 0.0040 | 0.019 | 0.0037 | 0.0130 | 0.0247 |
| $u^S$ | gamm | 0.010 | 0.0040 | 0.006 | 0.0011 | 0.0039 | 0.0073 |
| $u^\sigma$ | gamm | 0.200 | 0.0800 | 0.283 | 0.0849 | 0.1549 | 0.4009 |
| $u^\tau$ | gamm | 0.010 | 0.0040 | 0.002 | 0.0002 | 0.0018 | 0.0025 |
| $u^{TAX}$ | gamm | 0.010 | 0.0040 | 0.012 | 0.0010 | 0.0104 | 0.0137 |

This table reports values of autocorrelations and standard deviations of all shock processes. HPD inf and HPD sup refer to the 90 percent highest posterior density interval.

We report the assumed prior distribution of each parameter as well as its prior mean and standard deviation, and the posterior distribution obtained with our Bayesian estimation procedure, including its mean, standard deviation, and 90 percent confidence interval lower and upper bounds.

As we can see from Table 2, thanks to our extremely rich macroeconomic model we have been able to estimate some important parameters for semi-endogenous growth, which the literature on economic growth could never satisfactorily estimate so far, but rather relied on simple calibration procedures. Most notable is the knowledge stock intertemporal spillover parameter $\hat{\varphi}$, which has an estimated mean equal to 0.835. This parameter estimate is significantly below 1 even though, in particular, we have included a non-zero prior density in $\varphi = 1$. Hence, the estimation supports semi-endogenous growth in the US macroeconomy.

This finding is important for policy evaluations. It predicts that R&D policy shocks cannot permanently affect the per capita GDP growth rate, as originally predicted by Jones (1995). However, $\hat{\varphi}$ being relatively high implies that R&D policy shocks, or any other shocks directly or indirectly affecting the R&D/GDP ratio, will have relatively long-lasting effects and affect the macroeconomic behavior in the medium term. This estimate is in line with Varga et al. (2016) predictions of their medium size calibrated model. The lower bound of the 90 percent confidence interval (0.7089) is still close enough to 1 to guarantee a relatively long transition to the semi-endogenous balanced growth path, while its upper bound, 0.9715, is extremely close to 1, and hence suggesting that this model could have policy predictions practically indistinguishable from those of a fully endogenous growth model.

Quite striking is also the estimate of the R&D adjustment costs parameter, $\gamma^{RD}$ , whose estimated posterior mean, 151.84, is the highest adjustment cost parameter of the whole model. This result is important because it suggests that the R&D and growth models in the academic literature so far, by ignoring R&D adjustment costs, may lead to potentially misleading predictions on the effect of policies on growth. So high R&D adjustment costs indicate that the R&D response to policies could be much more sluggish than usually thought.

Moreover, exogenous R&D spillover shocks in this model seem to be very persistent. The autocorrelation coefficient of the R&D spillover shock $\epsilon_t^{\sigma}$, which appears in Table 3 as $\rho^{RD}$, has a 90 percent confidence interval ranging from 0.8795 to 0.9815. This estimate has policy implications too. As long as the R&D spillover shock is influenced by IPRs policy, a policy or jurisprudence shift of researcher's incentives towards more narrow focus on the adaptation of the frontier to marketable products, rather than on more basic research discovery, may have long lasting negative effects on frontier productivity growth. This persistence could also be due to the common law structure of the United States legislative process, which implies that a civil law policy change could fully disclose it potential for the economy only after a long enough series of court precedents have been ruled, as predicted by Cozzi and Galli (2014). The next section will highlight the dynamics of the estimated shocks by studying the impulse response functions (IRFs) of our main model.

## 3.4 Model Dynamics

Based on our estimates, we can simulate the effects of structural shocks on the macroeconomy. In what follows we will see some of the impulse responses to give an idea of the potential effects of structural shocks hitting the economy. All shocks are assumed temporary, i.e. lasting only one term.

An important shock in this model is the shock on the R&D risk premium requested by investors to finance R&D firms. Figures 2 displays the corresponding IRFs.

[Figure 2 about here.]

[Figure 3 about here.]

As predicted based on the posterior estimates of Table 2, despite the absence of permanent growth effects of temporary shocks due to the semi-endogenous nature of R&D-driven growth of this model, a temporary (one quarter) R&D investment risk premium shock could have implications lasting several years. Since the quarter (i.e. three months) is our assumed time unit, we can see that the average technology and GDP effects of a temporary R&D investment shock will be getting stronger and stronger even 40 quarters (that is 10 years) after its initial impulse. This long-lasting effect advises financial authorities to put effort into guaranteeing that innovators are not declined funding at reasonable conditions.

It is also interesting to see, in Figure 3, the IRFs following a temporary shock to R&D spillovers. The effect of a one quarter shock to the R&D spillover on GDP and productivity can be long lasting. This persistence advices the IPR and research policy institutions to be careful in guaranteeing that the R&D focus of researchers remains broad and supporting for an open science environment: this will benefit frontier knowledge expansion and will allow overall TFP and GDP to prosper.

The estimated effects of other important shocks could be described. For example those of a temporary shock to the overall investment risk premium. This risk premium does not only affect R&D investment, but also physical capital investment. However, lower physical capital investment, by reducing production, will decrease the market size of innovative intermediate products, thereby decreasing the profitability of R&D. As a consequence, this shock will have direct and indirect consequences on R&D investment and the innovative dynamics of technology, and therefore on the aggregate productivity growth. The direct effects could be negative, due to less investment in general, but also positive for R&D, because it could liberate savings that would otherwise go to a different form of investment, while the indirect effects are likely negative due to reduced market size generated by the drop in investment demand and ensuing multiplier effects.

[Figure 4 about here.]

[Figure 5 about here.]

As suggested from Figure 4, an increase in the investment risk premium will reduce GDP, but will have an ambiguous and non-monotonic effect on the R&D investment, due to the complex general equilibrium dynamics involved, including the above mentioned direct and indirect effects. While, as would be normal in a model without endogenous productivity, there is a negative medium term impact on GDP, the presence of R&D in the model allows to identify a partially offsetting effect of the earlier recovery of R&D investment and ensuing recovery of technological and total factor productivity growth.

It is useful to contrast the IRFs to a model without semi-endogenous growth (i.e., an exogenous TFP model). Also in this exogenous TFP model, we allow for stochastic components of productivity growth. However, instead of being determined endogenously via a Schumpeterian growth mechanism, productivity follows an exogenous (stochastic) law of motion:

$$A_t^Y = g_{AY,t} + \epsilon_{TAY,t} \tag{37}$$

where $g_{AY,t} = \rho_{GAY} g_{AY,t-1} + (1-\rho_{GAY}) g_{AY} + \epsilon_{LAY,t}$ is a stochastic productivity trend. $\epsilon_{TAY,t}$ and $\epsilon_{LAY,t}$ are exogenous shocks as in eq. (34). This model abstracts from innovation, i.e. the probability to innovate is $n_t = 0 \, \forall t$. To facilitate comparison, all shocks are normalized to one percent standard deviation.

In our baseline model with semi-endogenous growth, an exogenous shock to investment risk premia is greatly amplified. The increased internal propagation mechanism leads to larger contractions in output, consumption, and investment. Moreover, with semi-endogenous growth, the shocks are more persistent: The effects on most variables do not die out even after ten years. There is still a notable difference between both models and a much slower return to steady state in the baseline model.

In the IRF graphs we have seen so far, we have used the estimated parameters and therefore policy and transition functions to simulate the effect of each structural shock in isolation. This procedure is interesting to better identify the associated macroeconomic channels and to suggest policy implications, but of course it is still a theoretical exercise under ideal controlled conditions. In the real observed data we never see only one shock operating and only one impulse of it: all shocks are active in changing sizes and directions and in every period. Therefore, the resulting macroeconomic dynamics of our DSGE model is much more complex than we could see from the above impulse response graphs. It is this complexity that permits to replicate the complex features of the observed data, and we will exploit it in the next section.

### 3.5 Historical Decomposition

We proceed to quantify the relative contribution of exogenous shocks in explaining the data through the lens of our *estimated* fully-fledged DSGE model with Schumpeterian growth. Figure 6 presents a historical variance decomposition of the observed time series of real GDP growth. Each panel shows the contribution of an innovation. The continuous line displays the observed demeaned time series. Stacked vertical bars indicate the estimated relative contribution of this shock.

[Figure 6 about here.]

Several important features of the US economy in the 1995Q1-2015Q1 period emerge from our estimated shock decomposition. In particular, the most important negative contribution during the Great Recession of 2008-2009 came from the overall investment risk shock (pink) and the private saving shock (black). Their joint occurrence reflected a sudden and strong deterioration of the financial sector: the credit channels from the household savings to the private firms investing in physical capital and R&D became less reliable, and the production and the R&D sectors could not get funding comparable to the pre-crisis trend.

At the same time, savers are more pessimistic about the future. The increased propensity to save leads to a drop in consumption expenditure. As a result of the simultaneous drops in consumption and investment aggregate private demand fell, negatively affecting GDP growth. While at first monetary policy turned to an expansionary stance, reflected by the positive contribution of the green shock, when it hit the ZLB it became unable to bring the policy interest rates into the negative territory, as would have been dictated by the pre-crisis Taylor rule, and it became unable to give enough relief to the financially strained firms and households. The Fed was necessarily forced to run a more restrictive monetary policy than dictated by the Taylor rule, which is reflected by the negative contribution of the monetary policy shock to GDP growth. Other aspects of the Fed's policy, however, seem to have succeeded in repairing the post-crisis financial sector, and in fact we see it reflected by the end of the negative contribution of the investment and saving risk premium shocks starting in 2010.

The blue bars dynamics suggest that fiscal policy must have helped the US economy only upon impact, but its overly-expansionary character, by increasing public debt accumulation nearly out of control (even eventually hitting the government shutdown bound) was not able to give a persistently positive contribution to GDP growth: its cumulative contribution became negative in the second part of 2009.[19]

Following the crisis, firms seem to have been subject to a more competitive environment, as shown by the positive GDP growth effects of the negative price mark-up shocks (light red bars). Instead, the labor market seems to have suffered rigidities in the two years after the crisis, represented by harmful wage mark-up shocks - shown in yellow in the figure.

Interesting is also the picture emerging from the shock decomposition around the 2001 *dotcom* bubble burst. The adverse financial conditions associated with the potentially persistent harmful investment and saving shocks in the figures were at least partially offset by a strongly expansionary monetary policy stance, also corroborated by expansionary fiscal policy.

Our estimated macroeconomic model allows to identify interesting aspects of the R&D and innovation sector. In fact, we observe a persistently negative contribution of the R&D spillover shock (brown bar) starting in

[19]This observation may also reflect the expectation of higher future taxes associated with the persistently higher future government spending resulting from the Affordable Care Act (popularly known as "Obamacare") announced to Congress in September 2009 and enacted in March 2010.

2002. This observation is a startling confirmation of the negative effects of too strict IPRs which penalized academic and basic research in the US innovation system following the Madey *vs* Duke Supreme Court verdict of 2002, which formally ended the so called "research exemption" doctrine, which previously permitted patented discoveries to be freely used for research purposes without incurring the risk of patent infringement (see Cozzi and Galli 2014).

[Figure 7 about here.]

How does the model interpret the time series on R&D investment? Figure 7 presents a variance decomposition of R&D investment. Our estimation explains the drop in R&D investment mainly by a rise in R&D risk premia and, during periods of financial distress, also in investment risk premia. The malfunctioning of financial markets has a strong adverse effect on R&D investment. The constrained ability of financial markets to channel savings thus helps explain the low growth following the Great Recession. The slow recovery from severe financial crises has also been emphasized by recent literature (e.g., Boissay et al. 2016). Apart from the dot-com bubble and the short pre-crisis boom, R&D spillovers affect R&D investment mostly negative. Moreover, wage mark-up and monetary policy shocks contribute to fluctuations in R&D investment.

### 3.6 Shocks at the Zero Lower Bound

Following the Great Recession, at least through late 2015, the ZLB on interest rates was effectively binding and hampered the Fed's ability to stimulate the economy by further lowering the policy rate. Formally, we impose the ZLB constraint on the net nominal interest rate by modifying the Taylor rule in equation (27) to:

$$i_t = \left\{ \begin{array}{ll} \rho^i i_{t-1} + (1-\rho^i)\left(-\bar{i} + \eta^{i\pi}(\pi_t - \bar{\pi}) + \eta^{iy}\tilde{y}_t\right) + \epsilon^i_t, & \text{if } i_t > i^{LB} \\ i^{LB}, & \text{otherwise} \end{array} \right\}. \tag{38}$$

To account for the occasionally binding constraint, we solve the model using an algorithm build on the OccBin tool (Guerrieri and Iacoviello 2015). We then use the piecewise linear solution to obtain smoothed estimates of latent variables under the constraint. In a last step we use these estimates to assess the impact of specific shocks on GDP growth.[20]

Figure 8 displays contributions of policy shocks on GDP growth obtained from a standard linear estimation (blue bars) and a piecewise linear solution (red bars). Accounting for the non-linearity of the ZLB implies a stronger positive effect of fiscal policies (top panel) at the beginning of the Great Recession. Subsequent contractionary fiscal policy shocks during the slump are also amplified and more visible. Moreover, in the piecewise linear solution, the ZLB absorbs the negative monetary policy shocks. In contrast, the linear solution shows negative monetary shocks because the Taylor Rule would imply negative nominal interest rates during the Great Recession (middle panel). The central bank's inability to reduce the policy rate also amplifies financial disturbances. Consequently, investment risk premium shocks propagate more at the onset of the crisis (bottom panel).

[Figure 8 about here.]

### 3.7 Financial Market Dynamics

Our model has two shocks that mainly affect financial markets: investment risk premium shocks, $\epsilon_t^S$ and savings shocks $\epsilon_t^C$. It is instructive to compare the smoothed shocks of our baseline to a model without endogenous R&D and innovation. Consider Figure 9, which displays the unobserved smoothed shocks computed by the Kalman smoother.

[Figure 9 about here.]

Both models display the same basic patterns: Low investment risk premia until the dot-com bubble bursted in 2001 and again during the build-up of the financial crisis. Then, during the Great Recession, we see a large increase in investment risk premia. However, there are striking differences between the model with R&D (light blue continuous line) and the exogenous TFP model (orange dashed line). The estimated shocks to mimic financial crisis dynamics are smaller and within a smaller confidence set in

[20] We set the lower bound for quarterly policy rates to $i^{LB} = 0.0016$. See Ratto and Giovannini (2017) for further technical details on the algorithm and its implementation. Anzoategui et al. (2016) use a similar approach.

the model with R&D. Consequently, the implied time series of the estimated investment risk premium shock in the model with R&D is much smoother than in the exogenous TFP model. Accounting for semi-endogenous growth thus helps address the criticism that large shocks - which are unlikely to be observed - are necessary to explain the Great Recession using a DSGE methodology. Moreover, variables oscillate less around their steady state value. A linearized model with semi-endogenous growth may therefore be able to capture crisis dynamics with a higher accuracy.

# 4 Extensions

## 4.1 Fully Endogenous Growth

The comparative statics as well as the policy implications of the semi-endogenous models are often quite different from those of endogenous growth models: for example, R&D subsidies have huge long-term effects in endogenous growth models, while no long term effect in semi-endogenous growth models. Hence, we explore the robustness of the semi-endogenous model laid down in the previous sections against alternative growth modeling frameworks.

The most important alternative to the semi-endogenous growth approach used in the previously described main model is the so called scale-effect free fully endogenous growth model, developed by Smulders and van de Klundert (1995), Young (1998), Peretto (1998), Dinopoulos and Thompson (1998), Howitt (1999) among others.[21] Among the interesting aspects of this approach is that R&D policies have a steady state effect on the growth rate of per capita GDP without implying that this growth rate is affected by the population size, as instead counterfactually implied by the first generation endogenous growth models *à la* Romer (1990) and Aghion and Howitt (1992).

To switch to a fully endogenous growth framework it is enough to modify

[21] See Madsen (2008) for empirical support.

eq. (22) to this form:

$$A_t^{Y,\max} = A_{t-1}^{Y,\max} + A_{t-1}^{Y,\max} \left( \frac{X_{t-1}^{RD}}{Y_{t-1}} \right)^{\lambda^A} \sigma_t^{RD}, \tag{39}$$

which amounts to setting parameter $\varphi$ equal to 1 and dropping the labor force variable from the term in brackets. Therefore, we eliminate the decreasing returns to the intertemporal knowledge spillover, while maintaining the static decreasing returns to R&D represented by the "stepping on toes" R&D congestion externality parameter $\lambda^A$ (Jones and Williams 1998). Notice that by dividing both sides of eq. (39) by $A_{t-1}^{Y,\max}$ and subtracting 1 we obtain the equilibrium growth rate of the technological frontier - and in steady state of the aggregate productivity - in the following expression:

$$\frac{A_t^{Y,\max} - A_{t-1}^{Y,\max}}{A_{t-1}^{Y,\max}} = \left( \frac{X_{t-1}^{RD}}{Y_{t-1}} \right)^{\lambda^A} \sigma_t^{RD}. \tag{40}$$

The permanent effect policies can be seen from eq. (40): Whatever permanently affects the R&D investment as a share of GDP will also affect the trend growth rate permanently. The R&D spillover shock $\sigma_t^{RD}$ will only make the growth rate fluctuate around such policy determined long-term level. Since $\sigma_t^{RD} = \sigma^{RD} \exp(\epsilon_t^{\sigma})$, its deterministic steady state value is $\sigma^{RD}$. Therefore, the steady state growth rate, denoted by dropping time $t$ indexes, of this economy is:

$$g_{A^Y} = g_{A^{Y,\max}} = \left( \frac{X^{RD}}{Y} \right)^{\lambda^A} \sigma^{RD}, \tag{41}$$

where $\frac{X^{RD}}{Y}$ is the steady state R&D/GDP ratio, which can be affected by policies. In fact, all long term policies, independently of their specific focus, may influence directly or indirectly $\frac{X^{RD}}{Y}$ and therefore its long-term value, and this is enough to affect long term growth.

As a consequence, the steady state growth rate eq. (25) of the previous semi-endogenous growth model will no longer be valid. The remaining equations of the main model of the previous section continue to hold, and therefore we do not replicate them here.

## 4.2 Some degree of Exogenous Growth

What if growth were truly exogenous? In that case the traditional DSGE macromodels would be right. So far we have just assumed that growth was not exogenous, and therefore we have closed this possibility by assumption. It would be insightful, instead, to open the door to at least some degree of exogenous growth in view of letting the data speak and tell us how much growth is exogenous. We can indeed generalize the previously set framework to allow the presence of exogenous growth in the picture, and generalize the model to nest, as a special case, the fully exogenous growth case. Then the final word on whether or not growth is exogenous will just be a matter of estimating a more general model. In this section we briefly delineate how to achieve that, both in the semi-endogenous growth framework of the main model, and in the fully endogenous growth framework of the previous subsection.

### 4.2.1 Exogenous and Semi-endogenous Growth model

We will here assume that there is a deterministic exogenous trend component $A_t^*$ growing at rate $g_{A^*}$, in the production function (9), which now becomes:

$$Y_{it} = \left(\frac{A_{it}^Y}{A_t^Y}\right) \left(A_t^* A_t^Y N_{it}\right)^a \left(cu_{it} K_{it-1}^{tot}\right)^{1-a}. \tag{42}$$

Therefore, total Factor Productivity, $TFP_t$, now becomes:

$$TFP_t = \left(A_t^Y A_t^*\right)^a. \tag{43}$$

As in the semi-endogenous model of the main section, the endogenous technological frontier $A_t^{Y,\max}$ growth rate will still converge to

$$g_{A_t^{Y,\max}} = \frac{\lambda^A g_{POP}}{1-\varphi}, \tag{44}$$

while the total TFP growth rate will be

$$g_{TFP_t} = a\left(g_{A^*} + g_{A_t^{Y,\max}}\right) \underset{t\to\infty}{\rightarrow} a\left(g_{A^*} + \frac{\lambda^A g_{POP}}{1-\varphi}\right), \tag{45}$$

which asymptotically includes both a purely exogenous and a semi-endogenous part. Notice that we have introduced an exogenous growth term $g_{A_t^*}$ as an unconstrained parameter, so that we can let the Bayesian estimation tell whether it is positive, negative, or insignificantly different from zero. We define the share of purely exogenous growth $\alpha^{exo} \equiv (g_A^*)/(g_{A^*} + g_{A^{Y,\max}})$. Correspondingly, $(1-\alpha^{exo})$ describes the share of the semi-endogenous frontier growth rate. In the extreme case of purely exogenous growth, $\alpha^{exo} = 1$, total frontier growth will only be driven by $g_{A^*}$.

### 4.2.2 Hybrid Exogenous and Fully Endogenous Growth model

Here too we will introduce a deterministic exogenous trend component $A_t^*$ growing at rate $g_{A^*}$, in the production function (9):

$$Y_{it} = \left(\frac{A_{it}^Y}{A_t^Y}\right) \left(A_t^* A_t^Y N_{it}\right)^a \left(cu_{it} K_{it-1}^{tot}\right)^{1-a}. \tag{46}$$

Again, total Factor Productivity, $TFP_t$, is:

$$TFP_t = \left(A_t^Y A_t^*\right)^a. \tag{47}$$

Notice that the endogenous technological frontier $A_t^{Y,\max}$ growth rate will now be

$$g_{A_t^{Y,\max}} = \left(\frac{X_{t-1}^{RD}}{Y_{t-1}}\right)^{\lambda^A} \sigma_t^{RD}, \tag{48}$$

while the total TFP growth rate will be

$$g_{TFP_t} = a\left(g_{A^*} + g_{A_t^Y}\right) \underset{t\to\infty}{\to} a\left(g_{A^*} + \left(\frac{X^{RD}}{Y}\right)^{\lambda^A} \sigma^{RD}\right), \tag{49}$$

which asymptotically includes a pure exogenous and a fully endogenous part. Here too we leave $g_{A_t^*}$ free, so that estimation will tell us whether it is positive, negative, or insignificantly different from zero.

## 4.3 Empirical Results

Table 4 shows the empirical results of the three extensions we have just sketched: the fully endogenous growth version, the hybrid exogenous and semi-endogenous, and the hybrid exogenous and fully endogenous. Prior choices are kept identical. We have listed the mean estimates of key parameters governing the growth and innovation dynamics across different model extensions. Additional details and results are reported in Appendix C.4.

Table 4: **Estimated parameters across extensions**

| | $\alpha^{exo}$ | $\gamma^{RD}$ | $\bar{n}$ | $\lambda^{A}$ | $\varphi$ | $\rho^{AY}$ | $u^{AY}$ | $\rho^{RD}$ | $u^{\sigma}$ |
|---|---|---|---|---|---|---|---|---|---|
| Baseline | **0** | 151.84 | 0.05 | **0.25** | 0.83 | 0.46 | 0.04 | 0.93 | 0.28 |
| Fully Endogenous | **0** | 133.38 | 0.05 | 0.50 | **1** | 0.42 | 0.04 | 0.92 | 0.30 |
| Exo. & Semi-Endo. | 0.13 | 135.51 | 0.05 | **0.26** | 0.81 | 0.43 | 0.04 | 0.93 | 0.32 |
| Exo. & Fully Endo. | 0.10 | 133.17 | 0.04 | 0.34 | **1** | 0.54 | 0.04 | 0.92 | 0.32 |

This table reports mean values for estimated parameters. Bold values indicate calibrated parameters. Values are rounded to the second decimal.

First, consider the estimate $\alpha^{exo}$ for which we have chosen a wide prior. This parameter describes what share frontier growth is purely exogenous according to our model. In both hybrid extensions which allow for purely exogenous growth the estimated share $\alpha^{exo}$ is small and consistently estimated at around $10 - 13$ percent. Thus, our estimations confirm endogenous R&D investment as a key driver of frontier growth. By the same token, the modest share of the exogenous component suggests a strong role for R&D policies. This result is well in line with the estimates of our baseline model. The large intertemporal knowledge spillover parameter $\varphi$ implies long-lasting impacts of policies and shocks (such as financial disturbances) affecting R&D dynamics. Moreover, the estimated share of exogenous growth, $\alpha^{exo}$, should be interpreted as an upper bound: It may, for instance, also reflect technological spillovers from other countries which we do not explicitly model here.

Second, in the extensions using a fully endogenous growth framework, we can estimate the "stepping-on-toes" parameter (Jones and Williams 2000). In our baseline version this parameter was a function of estimated parameters and pinned down via the steady state growth rate eq. (25). A fully endogenous growth framework, however, allows us to estimate its value at 0.50. This estimate indicates substantially decreasing returns to R&D. It is lower (0.34) in the hybrid version with exogenous growth because some frontier growth is purely exogenous in that case and not affected by effort duplication. In their calibrated model, Jones and Williams (2000) suggest 0.50 as a lower bound of $\lambda^A$. However, as argued before in Section 2.4, $g_{A_t^{Y,\max}}$ refers to the growth of frontier productivity, and not to the growth rate of patents. Consequently, we already account for the effects of patents on productivity growth. This difference in growth accounting explains the relatively low value of $\lambda^A$ compared to Jones and Williams (2000).

Finally, other estimates of growth and innovation parameters are confirmed across the extensions considered here. We find high R&D adjustment costs and the steady state share of innovating sectors is estimated at around 5 percent per quarter. R&D spillover shocks are large and persistent, whereas shocks to R&D investment risk premia are smaller and of a more temporary nature.

# 5 Conclusion

The macroeconomic experience of the last decade stressed the importance of jointly studying the growth and fluctuations behavior of the economy. In fact, trend and business cycle seemed quite intertwined, suggesting the need to quantify key drivers of these quite complex and unprecedented dynamics. To that aim we have here built an integrated medium-scale DSGE model featuring a New Keynesian part built upon a rich set of features common to Smets and Wouters (2003) and related literature, as well as a Schumpeterian endogenous and/or semi-endogenous growth engine. To guarantee independence of the long-term growth approach used, we also have allowed the model to nest exogenous growth as a special case, leaving it to the data to decide. As in Aghion and Howitt (1992) and Nuño (2011), innovations are the outcome of a patent-race in every sector, with each innovation improving upon existing goods. Innovating firms replace the incumbent monopolist and earn higher profits until the next innovation occurs. Knowledge spillovers push the technological frontier further.

We have estimated the main model and its extensions using Bayesian methods, and have applied it to the quarterly data of the US economy. Shocks to investment risk premia emerge as the core driver of the Great Recession. In line with recent literature (e.g., Justiniano et al. 2011) this finding can be interpreted as a malfunctioning financial sector which we do not explicitly model. Positive mark-up and household savings shocks also contributed to the recent slump. Fiscal and monetary policies could partially offset these adverse shocks.

Our data analysis confirms that the generality of frontier growth in potential GDP is driven by endogenous R&D investment. In addition, we have provided a sophisticated estimation of key innovation and growth parameters. Our results support Jones' (1995) semi-endogenous growth model where R&D policy shocks cannot permanently affect the per capita GDP growth rate. However, we have found evidence for strong intertemporal knowledge spillovers which implies that R&D policy shocks, or any other shocks directly or indirectly affecting the R&D/GDP ratio, will have relatively long-lasting effects.

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# Appendix

## A Data Description

All data sets (unless otherwise noted) are post-war US data observed from 1995Q1 on. Data of macroeconomic observables come from the Bureau of Economic Analysis (BEA) and the Federal Reserve. Observables include time series of: GDP, GDP deflator, population, total employment, employment rate, employment in hours, participation rates, relative prices with respect to GDP deflator (VAT-consumption, government consumption, and private investment), government investment price relative to private investment, nominal policy rate, and nominal shares of GDP (consumption, government consumption, investment, government investment, R&D investment, government interest payment, transfers, public debt, and wage bill).

We use quarterly R&D data from the US. Bureau of Economic Analysis (NIPA Table 5.3.5. Private Fixed Investment by Type). The data are seasonally adjusted at annual rates. In particular, we employ the series on intellectual property rights Y001RC1 from which we subtract the investment in Entertainment, literary, and artistic originals (Y020RC1). The data are available at http://www.bea.gov/national/nipaweb/DownSS2.asp.

## B Additional Model Details

### B.1 Intermediate good firms

Firms maximize a stream of future dividends $d_{it}$ discounted at the real stock return $r_t^s$:

$$\max \frac{P_t^S}{P_t^Y} \equiv \max \sum_{s=t+1}^{\infty} \mathcal{M}_s d_{is}, \tag{B.1}$$

by choosing labor, capital, investment, capacity utilization, and prices. The stock market specific stochastic discount factor $\mathcal{M}_s$ : is defined as:

$$\mathcal{M}_s = \frac{1 + r_t^s}{\prod_{r=t}^{s} (1 + r_r^s)} \tag{B.2}$$

In period $t$ dividends are:

$$d_{it} = \left(1 - \tau^K\right) \left(Y_{it} - \frac{W_t}{P_t^Y} N_{it}\right) + \tau^K \delta K_{it-1}^{tot} - I_{it} - adj_{it}, \tag{B.3}$$

where $W_t$ is the wage rate, $I_{it}$ is physical capital investment, $\tau^K$ is the profit tax, $\delta$ is capital depreciation rate and $adj_{it}$ are total adjustment costs associated with price $P_{it}$ and labor input $N_{it}$ adjustment or moving capacity utilization $cu_{it}$, and investment $I_{it}$

away from their optimal level.

For tractability, we make two more assumptions. (i) When a new incumbent starts production, she inherits the previous stocks (employment, capital) and costs of the previous firm that dropped out. (ii) As discussed in the main text, moving closer to the frontier implies higher adjustment costs due to sophistication. As a consequence, adjustment costs reflect the relative technological position $\left(\frac{A^Y_{it}}{A^Y_t}\right)$ of the sector i.

$$adj_{it} = adj^P_{it} + adj^N_{it} + adj^{cu}_{it} + adj^I_{it} \tag{B.4}$$

with

$$adj^P_{it} = \frac{\gamma^p}{2} Y_t \left(\frac{P_{it}}{P_{it-1}} - \exp(\overline{\pi})\right)^2 \left(\frac{A^Y_{it}}{A^Y_t}\right) \tag{B.5}$$

$$adj^N_{it} = \frac{\gamma^n}{2} Y_t \left(\frac{N_{it}}{N_{it-1}} - \exp(\overline{g^{pop}})\right)^2 \left(\frac{A^Y_{it}}{A^Y_t}\right) \tag{B.6}$$

$$adj^{cu}_{it} = K^{tot}_{t-1} \left(\gamma^{u,1}\left(cu_{it} - 1\right) + \frac{\gamma^{u,2}}{2}(cu_{it} - 1)^2\right) \left(\frac{A^Y_{it}}{A^Y_t}\right) \tag{B.7}$$

$$adj^I_{it} = \left(\frac{\gamma^{I,1}}{2} K_{t-1} \left(\frac{I_{it}}{K^{tot}_{t-1}} - \delta_t\right)^2 + \frac{\gamma^{I,2}}{2} \frac{\left(I_{it} - e^{\Delta trend^K_t} I_{it-1}\right)^2}{K^{tot}_{t-1}}\right) \left(\frac{A^Y_{it}}{A_t}\right). \tag{B.8}$$

Firms maximize dividends subject to the production technology (8), the demand schedule for final goods (B.10), and the capital law of motion (11).[22] We allow for shocks to labor demand and price mark-ups, denoted $\epsilon^{ND}_t$ and $\epsilon^{MUY}_t$, respectively.

## B.2 Final Goods

A single final good $Y_t$ is produced by perfectly competitive firms by combining a continuum of differentiated intermediate goods, $Y_{it}$ where $i \in [0, 1]$. We adopt a Dixit-Stiglitz production technology

$$Y_t = \left[\int_0^1 (Y_{it})^{\frac{\sigma^y - 1}{\sigma^y}} di\right]^{\frac{\sigma^y}{\sigma^y - 1}}, \tag{B.9}$$

where $\sigma^y$ is the exogenous substitution elasticity between intermediate goods which governs the mark-up on differentiated goods. The demand for a differentiated good $i$ is then:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\sigma^y} Y_t, \tag{B.10}$$

[22]Note that $\delta_t \neq \delta$, as long as we require that capital adjustment costs are zero on the trend-path. $\delta_t$ is time-varying if capital trend is time-varying. Finally $e^{\Delta trend^K_t} = \bar{g}_t$ is endogenous and grows at the rate of the balanced growth path. Accordingly, $\delta_t$ is defined as follows: $\delta_t = e^{\Delta trend^K_t} - (1 - \delta_k)$. Moreover, firms do not pay for the rental of public capital.

where $P_{it}$ is the price of the good and:

$$P_t = \left[ \int_0^1 \left(P_{it}\right)^{1-\sigma^y} di \right]^{\frac{1}{1-\sigma^y}} . \tag{B.11}$$

## B.3 Wage Setting

The trade union sets wage rates at a mark-up, $\mu_t^w$ over the marginal rate of substitution between working and consuming. Let us denote the marginal disutility of labor for household type $r = \{s, c\}$ by

$$V_{N,jt}^r = \omega^N \epsilon_t^U \left(C_t^r\right)^{1-\theta} \left(N_{jt}^r - h_N N_{t-1}^r\right)^{\theta^N} . \tag{B.12}$$

We assume that real wages adjust sluggishly as in Blanchard and Galí (2007) and follow

$$\begin{aligned}
&\left(1-\tau^N\right) \frac{W_{t-1}}{P_{t-1}} + \gamma^w \left(\pi_t^w \left(1 - sf^w\right) \pi_{t-1}^w\right) \left(1 + \pi_t^w\right) \\
&\gamma^w \frac{N_{t+1}}{N_t} \frac{1 + \pi_{t+1}^y}{1 + i_{t+1}^s} \left(\pi_{t+1}^w \left(1 - sf^w\right) \pi_{t+1}^w\right) \left(1 + \pi_{t+1}^w\right) \\
= &\left(\mu^w \frac{\omega^s V_{N,jt}^s + \left(1-\omega^s\right) V_{N,jt}^c}{\omega^s U_{C,jt}^s + \left(1-\omega^s\right) U_{C,jt}^c} \pi_t^{C,vat}\right)^{1-\gamma^{wr}} \left(\left(1-\tau^N\right) \frac{W_{t-1}}{P_{t-1}}\right)^{1-\gamma^{wr}} ,
\end{aligned} \tag{B.13}$$

where the parameters $\gamma^{wr}$ and $\gamma^w$ govern real and nominal wage rigidities, respectively. $\pi^w \equiv log(W_t/W_{t-1})$ and $\pi_t^{C,vat}$ denote wage inflation and consumption price inflation, respectively. $sf^w$ is the share of forward-looking labor supply. $\tau^N$ is labor income tax. Wage adjustment costs introduce nominal rigidities and the costs are borne by the household.

# C For Online Publication

## C.1 Additional Derivations: Intermediate Good Producers

**Lagrangian formulation** The Lagrangian formulation of firm $i$ reads as follows:

$$\max E_t \sum_{s=t}^{\infty} \beta^{s-t} \mathcal{M}_s \left[ \left( \left(1-\tau^K\right) \left( \frac{P_{is}}{P_s} \left(\frac{P_{is}}{P_s}\right)^{-\sigma^Y} Y_s - \frac{W_s}{P_s} N_{is} \right) + \tau^K \tilde{\delta} K_{is-1} - \frac{P_s^I}{P_s} I_{is} - adj_{is} \right) \right.$$
$$- \mu_{is}^k \left(K_{is} - (1-\delta) K_{is-1} - I_{is}\right)$$
$$\left. -\mu_{is}^y \left( \left(\frac{P_{is}}{P_s}\right)^{-\sigma^y} Y_s - \left(A_s N_{is}\right)^{\alpha} \left(cu_{is} K_{is-1}^{tot}\right)^{1-\alpha} \right) \right], \tag{C.14}$$

where $\mu_{is}^k$ and $\mu_{is}^y$ denote the Lagrangian multipliers on the capital law of motion and the production technology. This optimization problem implies the following first-order conditions (FOCs) where we drop firm-specific indices when the equilibrium conditions are derived due to the representative firm assumption.
FOC w.r.t. labor:

$$\mathcal{M}_t \left( -\left(1-\tau^K\right) \frac{W_t}{P_t} - \left(\frac{A_{it}^Y}{A_t^Y}\right) \gamma^n \frac{Y_t}{N_{it-1}} \left(\frac{N_{it}}{N_{it-1}} - 1\right) + \mu_{it}^y \alpha \frac{Y_{it}}{N_{it}} \right)$$
$$+\mathcal{M}_{t+1} \left(\frac{A_{it}^Y}{A_t^Y}\right) \gamma^n \frac{Y_{t+1}}{N_{it}} \left(\frac{N_{it+1}}{N_{it}} - 1\right) \frac{N_{it+1}}{N_{it}} = 0 \tag{C.15}$$

so we get in a symmetric equilibrium

$$\mu_t^y \alpha \frac{Y_t}{N_t} - \gamma^n \frac{Y_t}{N_{t-1}} \left(\frac{N_t}{N_{t-1}} - 1\right) + \frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \gamma^n \frac{Y_{t+1}}{N_t} \left(\frac{N_{t+1}}{N_t} - 1\right) \frac{N_{t+1}}{N_t} = \left(1-\tau^K\right) \frac{W_t}{P_t} \tag{C.16}$$

and

$$\mu_t^y a \frac{Y_t}{N_t} - \gamma^n \frac{Y_t}{N_{t-1}} g_t^N + \gamma^n \frac{1+\pi_{t+1}}{1+i_{t+1}^s} \frac{Y_{t+1}}{N_t} g_{t+1}^N \frac{N_{t+1}}{N_t} = \left(1-\tau^K\right) \frac{W_t}{P_t} \tag{C.17}$$

where use was made of the fact that in the equilibrium sector-specific firms are symmetric so that $N_{it} = N_t$. Note also that $\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \equiv \frac{1+\pi_{t+1}}{1+i_{t+1}^s}$. We also defined $g_t^N := \frac{N_t - \exp(\overline{g^{pop}}) N_{t-1}}{N_{t-1}}$ and $\pi_t := \frac{\triangle P_t}{P_{t-1}}$. Moreover, we used the fact that for the average firm

$$\int_0^1 \left(\frac{A_{it}^Y}{A_t^Y}\right) d\, i = 1. \tag{C.18}$$

FOC w.r.t. capital implies

$$\mathcal{M}_{t+1} \tau^K \delta - \mathcal{M}_t \mu_{it}^k + \beta \lambda_{t+1}^s \mu_{it+1}^k (1-\delta) + \mathcal{M}_{t+1} \mu_{it+1}^y (1-\alpha) \frac{Y_{it+1}}{K_{it}^{tot}} = 0, \tag{C.19}$$

or,

$$\frac{1+\pi_{t+1}}{1+i^s_{t+1}}\mu^y_{it+1}\left(1-\alpha\right)\frac{Y_{it+1}}{K^{tot}_{it}}+\frac{\beta\lambda^s_{t+1}}{\lambda^s_t}\tau^K\delta+\frac{\beta\lambda^s_{t+1}}{\lambda^s_t}\mu^k_{it+1}\left(1-\delta\right)=\mu^k_{it}. \tag{C.20}$$

Further simplifying gives

$$\frac{1+\pi_{t+1}}{1+i^s_{t+1}}\left(\mu^y_{t+1}\left(1-\alpha\right)\frac{Y_{it+1}}{K^{tot}_{it}}+\tau^K\delta+\left(1-\delta\right)Q_{t+1}\right)=Q_t, \tag{C.21}$$

where $Q_t := \mu^k_t$ is the Tobin's (marginal) Q.

The FOC w.r.t. investment is

$$\begin{gathered}-\mathcal{M}_t-\mathcal{M}_t\left(\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{I,1}\left(\frac{I_{it}}{K_{t-1}}-\delta_t\right)+\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{I,2}\frac{\left(I_{it}-e^{\Delta trend^K_t}I_{it-1}\right)}{K_{t-1}}\right)+\\ \mathcal{M}_{t+1}\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{I,2}\frac{\left(I_{it+1}-e^{\Delta trend^K_{t+1}}I_{it}\right)}{K_t}+\mathcal{M}_t\mu^k_{is}=0,\end{gathered} \tag{C.22}$$

or,

$$\begin{gathered}Q_t=1+\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{I,1}\left(\frac{I_t}{K_{t-1}}-\delta^k{}_t\right)+\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{I,2}\frac{I_t-e^{\Delta trend^K_t}I_{t-1}}{K_{t-1}}-\\ \frac{1+\pi_{t+1}}{1+i^s_{t+1}}\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{I,2}\frac{I_{t+1}-e^{\Delta trend^K_{t+1}}I_t}{K_t}.\end{gathered} \tag{C.23}$$

The FOC w.r.t. capacity utilization is given by

$$-\mathcal{M}_t\left(\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{u,1}+\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{u,2}\left(cu_{it}-1\right)\right)+\mathcal{M}_t\mu^y_{it}\left(1-\alpha\right)\frac{Y_{it}}{cu_{it}}=0, \tag{C.24}$$

or,

$$\mu^y_t\left(1-\alpha\right)\frac{Y_t}{cu_t}=K_{t-1}\left(\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{u,1}+\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^{u,2}\left(cu_t-1\right)\right) \tag{C.25}$$

Finally, the FOC w.r.t. differentiated output price reads:

$$\begin{gathered}\sigma^y\frac{\mu^y_{it}}{P_t}\left(\frac{P_{it}}{P_t}\right)^{-\sigma^y}Y_t=\left(1-\tau^K\right)\left(\sigma^y\right)\frac{1}{P_t}\left(\frac{P_{it}}{P_t}\right)^{-\sigma^y}Y_t+\\ \left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^p\frac{Y_t\triangle P_{it}}{(P_{it-1})^2}-\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t}\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^p\frac{Y_{t+1}\triangle P_{it+1}P_{it+1}}{(P_{it})^3},\end{gathered} \tag{C.26}$$

and since $\int_0^1 P_{it}d\,i=P_t$, we can rewrite

$$\begin{aligned}&\sigma^y\mu^y_tY_t=\left(1-\tau^K\right)\sigma^yY_t\\ &+\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^p\frac{Y_t(P_{it}-P_{it-1})}{P_{it-1}}\frac{P_t}{P_{it-1}}-\frac{1+\pi_{t+1}}{1+i^s_{t+1}}\left(\frac{A^Y_{it}}{A^Y_t}\right)\gamma^p\frac{Y_{t+1}(P_{it+1}-P_{it})P_{it+1}}{\left(P_{it}\right)^2}\frac{P_t}{P_{it}}.\end{aligned} \tag{C.27}$$

Assuming symmetry:

$$\sigma^y \mu_t^y = \left(1-\tau^K\right)(\sigma^y) + \gamma^p(\pi_t - \overline{\pi})(1+\pi_t) - \frac{Y_{t+1}}{Y_t}\frac{1+\pi_{t+1}}{1+i_{t+1}^s}\gamma^p(\pi_{t+1}-\overline{\pi})(1+\pi_{t+1})\,. \tag{C.28}$$

## C.2 Additional Calibrated Parameters

In this table we report additional calibrated values governing the autocorrelation of exogenous shock processes.

Table 5: **Additional Calibrated Parameters**

| Parameter | Description | Value |
|---|---|---|
| $\rho^G$ | Governmental consumption shock | 0 |
| $\rho^{IG}$ | Governmental investment shock | 0 |
| $\rho^i$ | Taylor rule | 0 |
| $\rho^U$ | Phillips curve shock | 0 |
| $\rho^W$ | Labor demand shock | 0 |

This table reports values for calibrated parameters.

## C.3 RMSE and Fit of the Model

In this section we report goodness of fit results across models. $g^X$ denotes the growth rate of variable $X$.

Table 6: **Goodness of Fit**

| | **Baseline Model** | | **Exogenous TFP** | |
|---|---|---|---|---|
| | RMSE | $R^2$ | RMSE | $R^2$ |
| $g^Y$ | 0.0058262 | 0.21773 | 0.0060261 | 0.16314 |
| $g^C$ | 0.0051311 | 0.053886 | 0.0048975 | 0.13808 |
| $g^I$ | 0.03133 | 0.15329 | 0.028189 | 0.15553 |
| $g^{Itot}$ | 0.02849 | 0.16956 | - | - |
| $g^{X^{RD}}$ | 0.012924 | 0.16312 | - | - |
| $g^G$ | 0.0074899 | 0.074295 | 0.0075029 | 0.072558 |
| $g^{IG}$ | 0.0168 | -0.11174 | 0.016613 | -0.087635 |
| $g^{BG}$ | 0.0036565 | 0.90771 | 0.0036284 | 0.90913 |
| $g^L$ | 0.0036159 | -0.15697 | 0.0035868 | -0.1384 |
| $g^N$ | 0.0045941 | 0.39249 | 0.0047076 | 0.36211 |
| $i^p$ | 0.0010315 | 0.96771 | 0.0010382 | 0.96729 |
| $\pi$ | 0.001955 | 0.23318 | 0.0020662 | 0.14349 |
| $\pi^W$ | 0.0073016 | 0.11153 | 0.0073008 | 0.11172 |
| $\pi^{WR}$ | 0.0073708 | 0.11234 | 0.0074104 | 0.10279 |

This table reports goodness of fit measures for our baseline model and the exogenous TFP model.

## C.4 Estimation Results Across Specifications

Table 7: **Estimated Parameters Across Model Versions**

| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| **Baseline** | | | | | | | |
| $\alpha^{exo}$ | calibrated | 0 | - | 0 | - | - | - |
| $\gamma^{RD}$ | gamm | 60.000 | 40.0000 | 151.840 | 30.9571 | 107.5168 | 200.9417 |
| $\bar{n}$ | beta | 0.025 | 0.0120 | 0.047 | 0.0091 | 0.0327 | 0.0618 |
| $\lambda^{A}$ | calibrated | 0.251 | - | 0.251 | - | - | - |
| $\varphi$ | beta | 0.700 | 0.2000 | 0.835 | 0.0958 | 0.7089 | 0.9715 |
| $\rho^{AY}$ | beta | 0.500 | 0.2000 | 0.463 | 0.1727 | 0.1681 | 0.7352 |
| $u^{AY}$ | gamm | 0.010 | 0.0040 | 0.043 | 0.0068 | 0.0327 | 0.0544 |
| $\rho^{RD}$ | beta | 0.500 | 0.2000 | 0.928 | 0.0344 | 0.8795 | 0.9815 |
| $u^{\sigma}$ | gamm | 0.200 | 0.0800 | 0.283 | 0.0849 | 0.1549 | 0.4009 |
| **Fully Endogenous** | | | | | | | |
| $\alpha^{exo}$ | calibrated | 0 | - | 0 | - | - | - |
| $\gamma^{RD}$ | gamm | 60.000 | 40.0000 | 133.381 | 27.6431 | 91.2347 | 177.0417 |
| $\bar{n}$ | beta | 0.040 | 0.0160 | 0.048 | 0.0093 | 0.0333 | 0.0627 |
| $\lambda^{A}$ | beta | 0.500 | 0.2000 | 0.497 | 0.2017 | 0.1715 | 0.8316 |
| $\varphi$ | calibrated | 1 | - | 1 | - | - | - |
| $u^{AY}$ | gamm | 0.010 | 0.0040 | 0.039 | 0.0068 | 0.0289 | 0.0510 |
| $\rho^{AY}$ | beta | 0.500 | 0.2000 | 0.425 | 0.2216 | 0.0632 | 0.7205 |
| $\rho^{RD}$ | beta | 0.500 | 0.2000 | 0.924 | 0.0529 | 0.8537 | 0.9819 |
| $u^{\sigma}$ | gamm | 0.200 | 0.0800 | 0.298 | 0.0911 | 0.1573 | 0.4436 |
| **Hybrid Exogenous Semi-Endogenous** | | | | | | | |
| $\alpha^{exo}$ | beta | 0.000 | 0.4000 | 0.126 | 0.1591 | -0.0982 | 0.3432 |
| $\gamma^{RD}$ | gamm | 60.000 | 40.0000 | 135.511 | 26.9539 | 84.9601 | 173.1705 |
| $\bar{n}$ | beta | 0.025 | 0.0120 | 0.046 | 0.0116 | 0.0285 | 0.0616 |
| $\lambda^{A}$ | calibrated | 0.257 | - | 0.257 | - | - | - |
| $\varphi$ | beta | 0.700 | 0.2000 | 0.807 | 0.1511 | 0.6257 | 0.9850 |
| $\rho^{AY}$ | beta | 0.500 | 0.2000 | 0.434 | 0.2180 | 0.1089 | 0.7802 |
| $u^{AY}$ | gamm | 0.010 | 0.0040 | 0.039 | 0.0060 | 0.0288 | 0.0491 |
| $\rho^{RD}$ | beta | 0.500 | 0.2000 | 0.933 | 0.0332 | 0.8828 | 0.9818 |
| $u^{\sigma}$ | gamm | 0.200 | 0.0800 | 0.324 | 0.0984 | 0.1723 | 0.4675 |
| **Hybrid Exogenous Fully Endogenous** | | | | | | | |
| $\alpha^{exo}$ | beta | 0.000 | 0.4000 | 0.103 | 0.1494 | -0.0985 | 0.3195 |
| $\gamma^{RD}$ | gamm | 60.000 | 40.0000 | 133.170 | 25.8453 | 94.4226 | 178.4717 |

(Continued on next page)

Table 7: (continued)

| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| $\bar{n}$ | beta | 0.025 | 0.0120 | 0.045 | 0.0098 | 0.0304 | 0.0612 |
| $\lambda^A$ | beta | 0.500 | 0.2000 | 0.344 | 0.1595 | 0.0822 | 0.5826 |
| $\varphi$ | calibrated | 1 | - | 1 | - | - | - |
| $\rho^{AY}$ | beta | 0.500 | 0.2000 | 0.543 | 0.1709 | 0.2678 | 0.8133 |
| $u^{AY}$ | gamm | 0.010 | 0.0040 | 0.041 | 0.0054 | 0.0327 | 0.0499 |
| $\rho^{RD}$ | beta | 0.500 | 0.2000 | 0.918 | 0.0430 | 0.8557 | 0.9781 |
| $u^{\sigma}$ | gamm | 0.200 | 0.0800 | 0.324 | 0.0984 | 0.1723 | 0.4675 |

This table reports values of selected estimated parameters across model versions. HPD inf and HPD sup refer to the 90 percent highest posterior density interval. The rows $u^{AY}$ and $u^{\sigma}$ report the standard deviations of the innovations.

# List of Figures

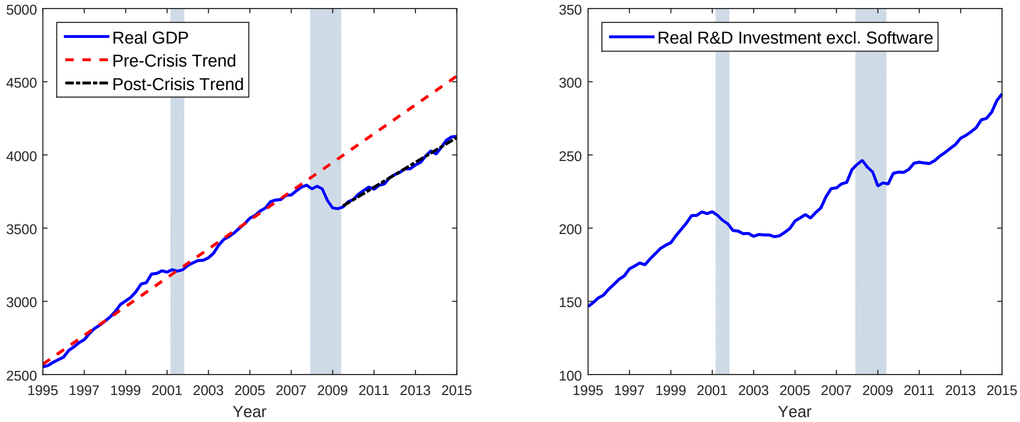


Figure 1: **Real GDP and R&D Investment**

Left panel: The blue continuous line depicts real GDP in Billions of Chained 2010 dollars in the US. Shaded areas denote NBER recessions. The dashed red line (dashed black line) fits a linear trend from 1995Q1 until 2007Q3 (from 2009Q3 until 2015Q1). Right panel: The blue continuous line depicts real R&D investment in Billions of Chained 2010 dollars (excluding software expenses). Shaded areas denote NBER recessions.

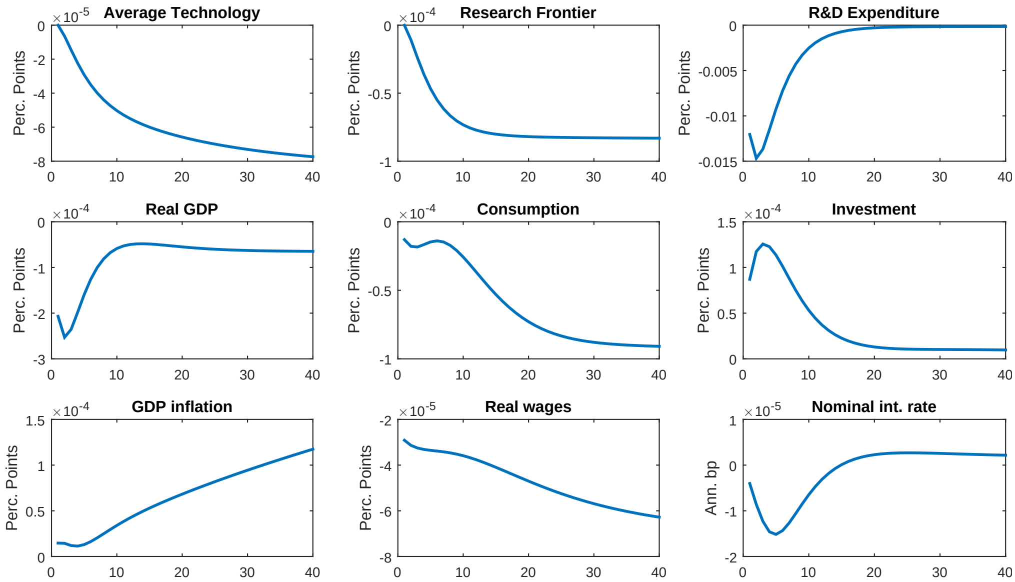


Figure 2: **Impulse Response Functions: R&D Risk Premium Shock**

This figure displays the dynamic response to a positive temporary R&D risk premium shock of one estimated standard deviation. Variables are displayed in percentage points deviations from their steady state value. The interest rate is reported in annualized basis points. Impulse responses are displayed for 40 periods, corresponding to 10 years.

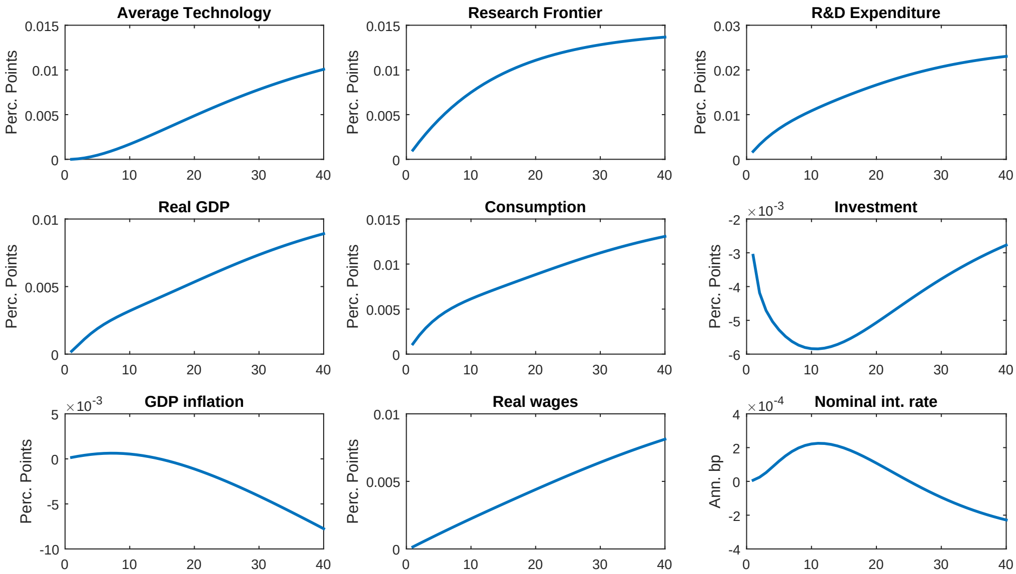


Figure 3: **Impulse Response Functions: R&D Spillover Shock**

This figure displays the dynamic response to a positive temporary knowledge spillover shock of one estimated standard deviation. Variables are displayed in percentage points deviations from their steady state value. The interest rate is reported in annualized basis points. Impulse responses are displayed for 40 periods, corresponding to 10 years.

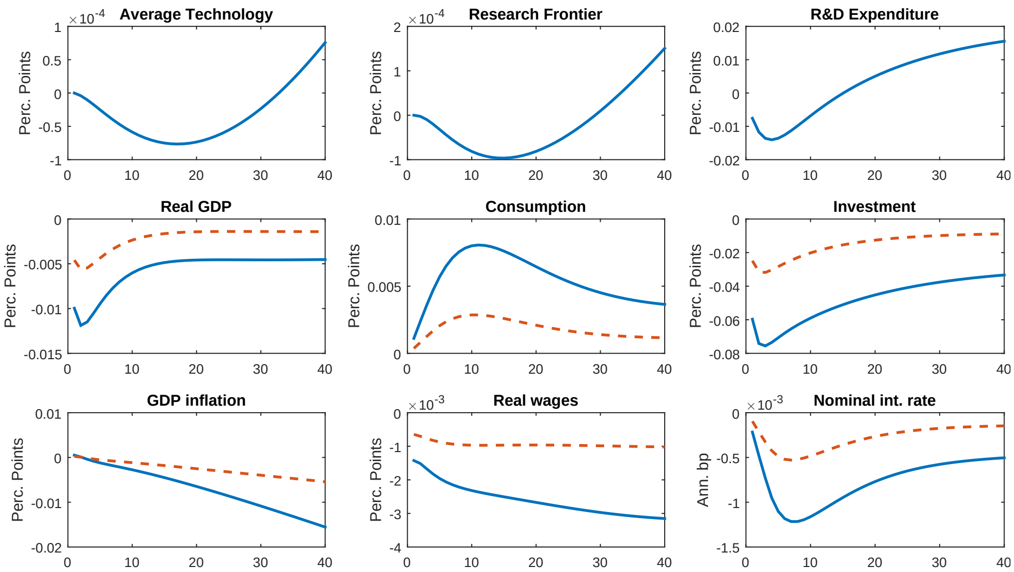


Figure 4: **Impulse Response Functions: Investment Risk Premium Shock**

This figure displays the dynamic response to a positive temporary investment risk premium shock of one percent standard deviations. Variables are displayed in percentage points deviations from their steady state value. The interest rate is reported in annualized basis points. Impulse responses are displayed for 40 periods, corresponding to 10 years. The light blue continuous line depicts values of the baseline model with semi-endogenous growth. The dashed orange line depicts values of a model with exogenous TFP growth.

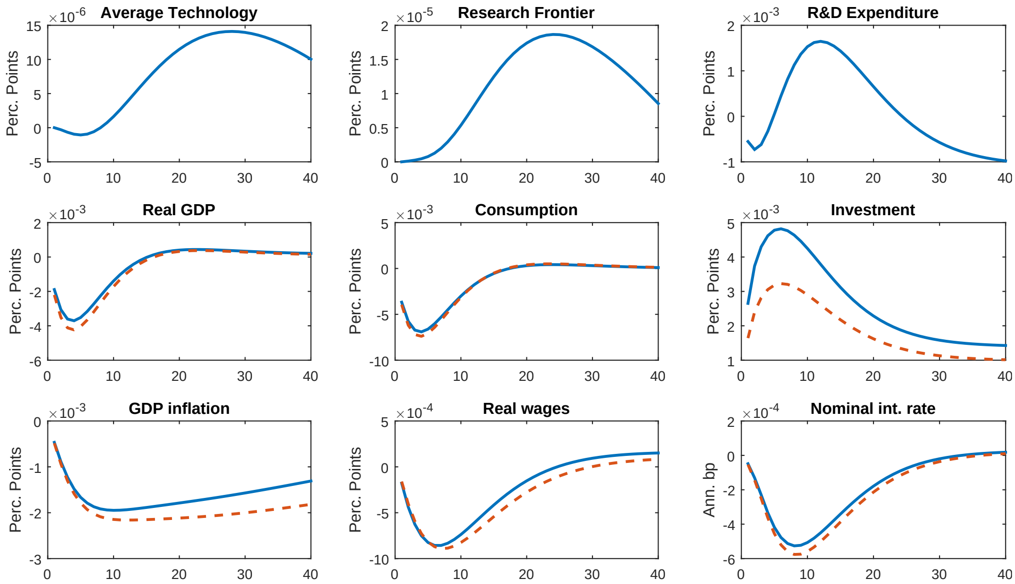


Figure 5: **Impulse Response Functions: Savings Shock**

This figure displays the dynamic response to a positive temporary savings shock of one percent standard deviation. Variables are displayed in percentage points deviations from their steady state value. The interest rate is reported in annualized basis points. Impulse responses are displayed for 40 periods, corresponding to 10 years. The light blue continuous line depicts values of the baseline model with semi-endogenous growth. The dashed orange line depicts values of a model with exogenous TFP growth.

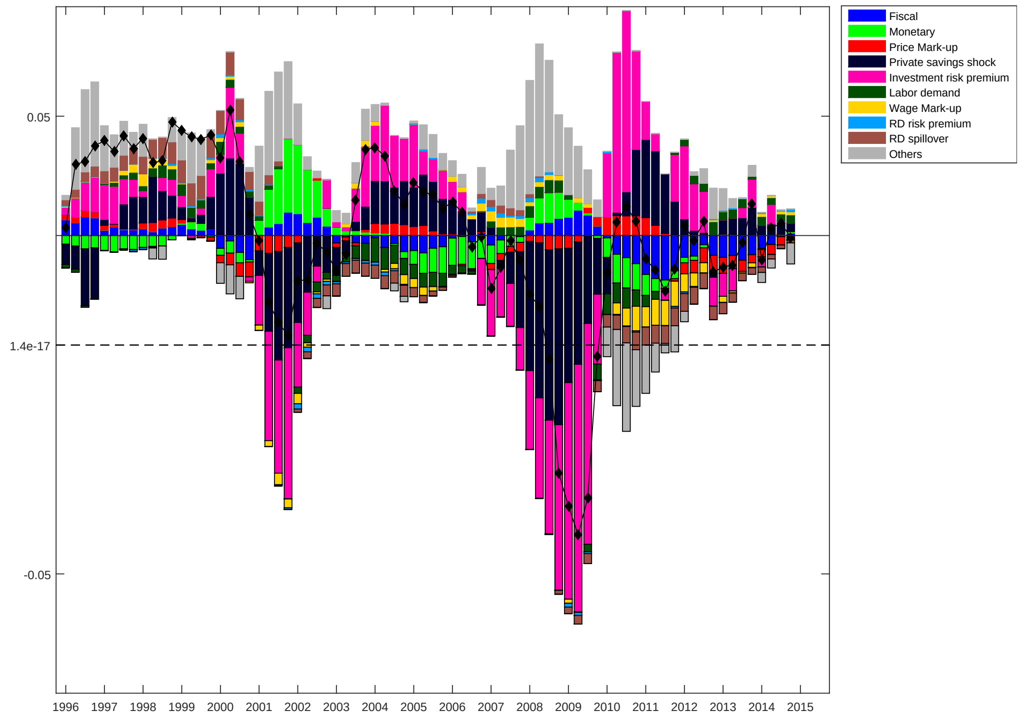


Figure 6: **Historical decomposition of real GDP growth rate (year-on-year)**

This figure shows a historical variance decomposition of the observed time series of real GDP. The black continuous line displays demeaned series of real GDP growth (year-on-year). Vertical bars indicate the relative contribution of each (group of) shocks to (i) fiscal policy rules (blue), (ii) monetary policy (light green), (iii) price mark-ups (red), (iv) saving preferences (black) (v) investment risk premia (pink), (vi) labor demand (dark green), (vii) wage mark-up (yellow), as well as innovation specific shocks, such as shocks to (viii) the R&D risk premium (light blue) and (ix) R&D spillovers (brown). All remaining shocks (x) are grouped in Others (gray).

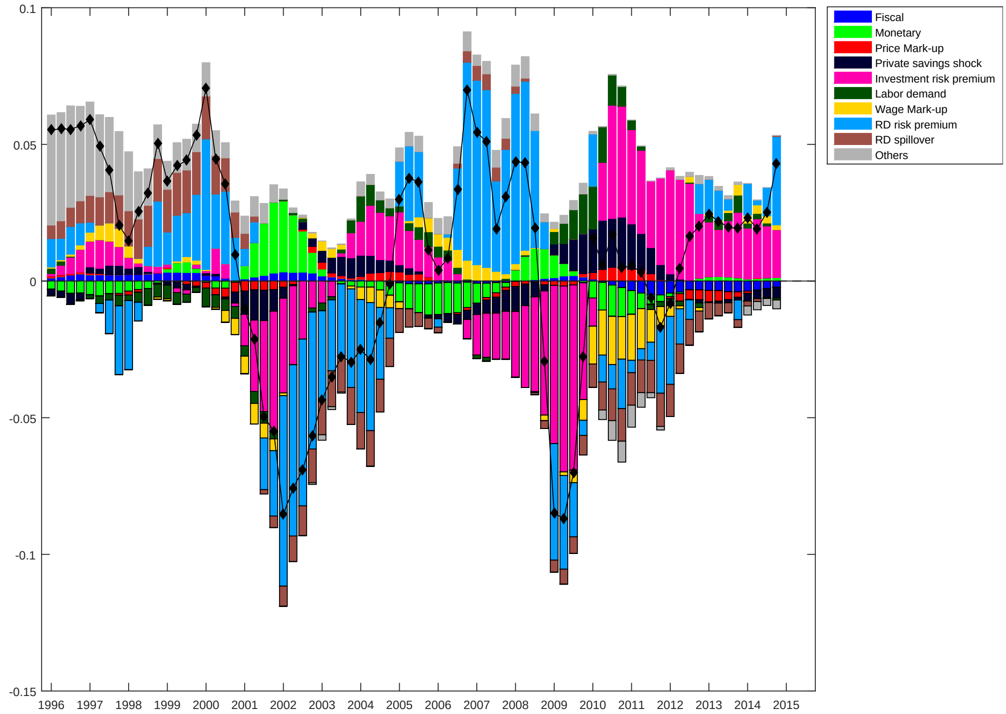


Figure 7: **Historical decomposition of R&D investment growth rate (year-on-year)**

This figure shows a historical variance decomposition of the observed time series of private R&D investment. The black continuous line displays demeaned series of real private R&D investment growth (year-on-year). Vertical bars indicate the relative contribution of each (group of) shocks to (i) fiscal policy rules (blue), (ii) monetary policy (light green), (iii) price mark-ups (red), (iv) saving preferences (black) (v) investment risk premia (pink), (vi) labor demand (dark green), (vii) wage mark-up (yellow), as well as innovation specific shocks, such as shocks to (viii) the R&D risk premium (light blue) and (ix) R&D spillovers (brown). All remaining shocks (x) are grouped in Others (gray).

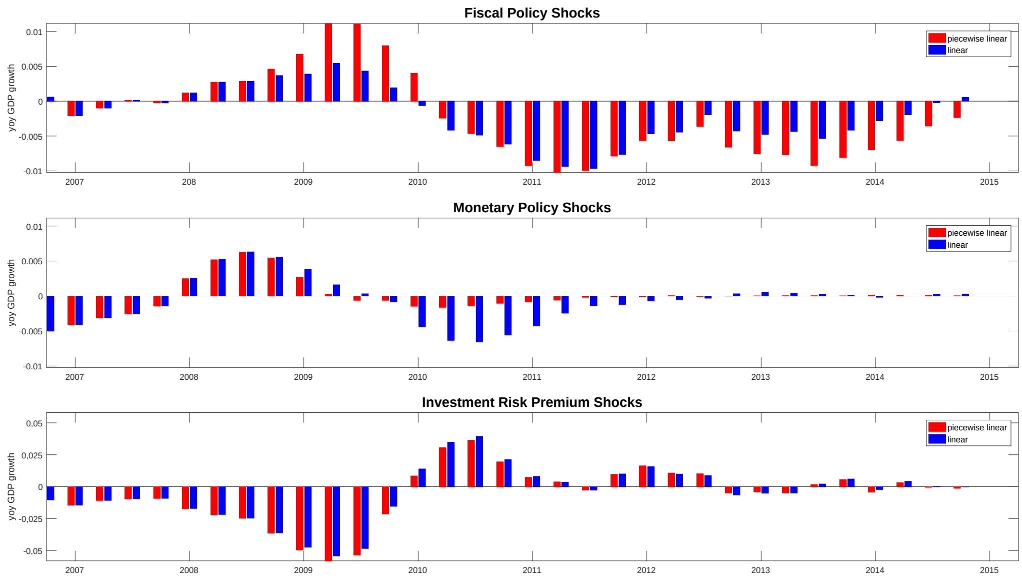


Figure 8: **Contributions of Shocks at the ZLB**

This figure displays the (non-additive) contribution of shocks to year-on-year GDP growth. Blue (red) bars indicate results from a linear (piecewise linear) solution. Additional technical details are reported in Ratto and Giovannini (2017). Note the larger Y-scale in the bottom panel.

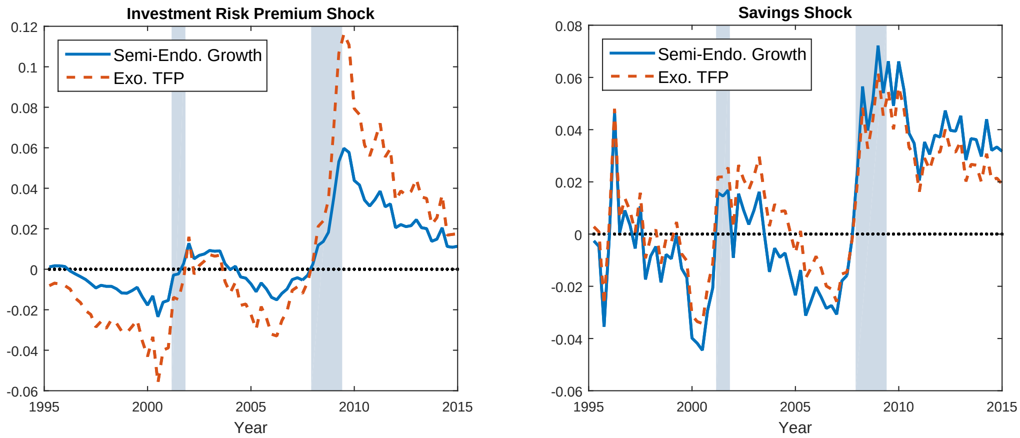


Figure 9: **Smoothed Shocks across Versions**

This figure displays the values of the unobserved investment risk premium shock (left panel) and the unobserved savings shock (right panel) over the sample. Both are computed via the Kalman smoother. The light blue continuous line (dashed orange line) depicts values of the baseline model with semi-endogenous growth (model with exogenous TFP growth). Shaded areas denote NBER recessions.