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# Endogenous Growth, Semi-endogenous Growth... or Both? A Simple Hybrid Model

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## Abstract

First generation endogenous growth models had the counterfactual implication that the long-term growth of per-capita GDP increased with the population size. Two influential growth paradigms, the semi-endogenous and the second generation fully endogenous, eliminated this strong scale effect. Both solutions have useful aspects and insights, but very different policy implications. This paper combines both approaches into a single hybrid model class, and shows that no matter the weight assigned to each paradigm, the long-run predictions of the semi-endogenous policy dominate with high enough population growth rates, while the long-run predictions of the fully endogenous policy dominate at low population growth rates.

*JEL classification:* O30, O40

*Keywords:* Strong scale effect; Semi-endogenous growth; Fully endogenous growth.

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# 1 Introduction

The early generation R&D-driven endogenous growth models by Romer (1990), Segerstrom, Anant, and Dinopoulos (1990), Aghion and Howitt (1992), Grossman and Helpman (1991) had a strong scale effect prediction (Jones, 1995 and 2005): the steady state per-capita GDP growth would be an increasing function of the economy's labour force. This is clearly counter-factual, as for example in an economy with increasing population it would predict an ever increasing per-capita GDP growth rate. Since Jones (1995) has highlighted this shortcoming, growth economists have started to look for ways to endogenize R&D, innovation, and growth in dynamic general equilibrium without implying the strong scale effect. The first solution, proposed by Jones (1995) himself, is the *semi-endogenous* solution, which assumes that as the country's productivity advances it becomes more and more difficult for research and development (R&D) to make the same percentage increase in productivity itself. The interesting insight of this solution is that while more people employed in R&D likely generate a higher flow of new productive ideas, each of these ideas has to confront to a larger and larger cumulated stock of already existing ideas, and therefore its net effect on relative productivity would tend to be negligible. To counter this secular decline in the growth rate more and more R&D employment would be necessary: this generates a positive effect of population growth rates on per-capita GDP growth rates.

A *fully endogenous* growth solution has also been proposed by Smulders and Van de Klundert (1995), Peretto (1998), Dinopoulos and Thompson (1998), Young (1998), and Howitt (1999), and it is characterized by the insight that it is not the total amount of R&D workers that matters for productivity growth but rather the share of the labour force employed in R&D. For instance, if each representative worker spends a larger fraction of his/her day researching - and the rest of the day producing in manufacturing - the growth rate of its productivity will be higher. A consequence for policy is that if the government induces a larger fraction of labour to undertake R&D it will also increase the long-term productivity growth rate.

Both solutions are intuitively appealing, and I claim that each does capture important aspects of the innovative process. Presumably this is the reason why both approaches are so popular in growth economics. But then why not exploring the implications of their both being valid? In this paper I will introduce a simple *hybrid* class of models according to which the growth rate of productivity is a convex combination of each of the two solutions. While the intuition may suggest that the implied steady state per-capita GDP growth rate is a linear combination of those predicted by either model, I will show that this is not the case. The main result of the analysis is that only one of them dictates the steady state growth rate, depending on whether population is increasing fast enough, in which case the semi-endogenous solution will prevail in the long run. Instead with constant, shrinking, or slowly increasing population it is the fully endogenous solution that will eventually prevail. For example, the true model of the economy could be 99% following the fully endogenous (scale free) growth model, but the sheer ongoing increase in population size may imply that government policy will not affect long-run growth - as predicted by the remaining 1% semi-endogenous component of the model.

The rest of this study is organized as follows. Section 2 briefly reviews the two approaches to the elimination of the strong scale effect. Section 3 introduces and analyses the proposed

hybrid approach. The final section concludes.

## 2 Review of Scale-free R&D-driven Growth

In this section, we briefly review<sup>1</sup> the two conventional approaches to the elimination of the strong scale effect. Let us work in continuous time and assume the following aggregate production function:

$$Y(t) = A(t)L_Y(t), \tag{1}$$

where  $Y(t)$  denotes output at time  $t$ ,  $A(t)$  is a technology index affecting time  $t$  marginal productivity of labour, and  $L_Y(t)$  is the amount of labor employed in manufacturing. We will assume that in a steady state  $L_Y(t)$  will be a constant fraction  $0 < s_Y < 1$  of the labour force  $L(t)$ , that is:  $L_Y(t) = s_Y L(t)$ . Total labour force grows at the constant rate  $n$ .

### 2.1 The two main approaches to the scale effect

Following Jones (1995 and 2005), the *semi-endogenous* elimination of the strong scale effect can be summarized by the following assumption on the productivity growth rate:

$$\frac{\dot{A}(t)}{A(t)} = \frac{\gamma L_A(t)^\lambda}{A(t)^{1-\phi}}, \tag{2}$$

where  $\gamma > 0$ ,  $\phi < 1$  and  $0 < \lambda < 1$  are parameters and  $L_A(t)$  is the amount of labour employed in R&D activities.

The labour market equilibrium condition

$$L_Y(t) + L_A(t) = L(t)$$

implies that  $L_A(t) = (1 - s_Y)L(t) \equiv s_A L(t)$ . Following Jones (1995) we can solve (2) for the steady state per-capita GDP growth rate, which equals the steady state growth rate of  $A(t)$

$$g_A = \frac{\lambda n}{1 - \phi}. \tag{3}$$

Eq. (3) yields the classical semi-endogenous prediction that government policies affecting  $s_A$  cannot affect  $g_A$ . An implication of this model is that a zero population growth rate will imply a zero per-capita GDP growth rate.<sup>2</sup> The non-negative dependence of long-term growth on population growth implies its standard definition as "semi-endogenous" growth.<sup>3</sup>

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<sup>1</sup>For a much more comprehensive review see Jones (2005).

<sup>2</sup>Segerstrom (1998) has microfounded a relationship similar to eq. (3) in a creative destruction framework *a la* Aghion and Howitt (1992).

<sup>3</sup>Its parametric long-run growth rate lends itself to medium scale growth models useful for policy evaluations such as Varga, Roeger, and Int'Veld (2008) and (2016).

The most important alternative to the semi-endogenous elimination of the strong scale effect is the *fully endogenous* solution, which can be summarized by the following productivity growth equation

$$\frac{\dot{A}(t)}{A(t)} = \gamma \left( \frac{L_A(t)}{L(t)} \right)^\lambda = \gamma s_A^\lambda, \quad (4)$$

which states that the steady state productivity growth depends on the fraction of the labour force employed in R&D. For this reason, policies affecting  $s_A$  will affect per-capita GDP growth permanently, and therefore we have fully endogenous growth without scale effects.

There are several microfoundations of eq. (4), most notably those of Smulders and Van de Klundert (1995), Peretto (1998), Dinopoulos and Thompson (1998), Young (1998), and Howitt (1999), to which I refer the reader.

### 3 A Hybrid Solution

Let us consider the following hybrid approach:

$$\frac{\dot{A}(t)}{A(t)} = \alpha \frac{\gamma L_A(t)^\lambda}{A(t)^{1-\phi}} + (1-\alpha)\gamma \left( \frac{L_A(t)}{L(t)} \right)^\lambda, \quad (5)$$

$$\text{with } 0 < \alpha < 1. \quad (6)$$

Notice that for  $\alpha$  arbitrarily close to 0 the model is arbitrarily close to the fully endogenous growth paradigm, while for  $\alpha$  arbitrarily close to 1 the model is arbitrarily close to the semi-endogenous paradigm. However, the long-term implications of this class of models do not follow these approximations, as I will soon prove. In fact, for each value of  $\alpha$ , if the population growth rate tends to zero, the per-capita GDP growth rate tends to the last term

$$g_A = (1-\alpha)\gamma s_A^\lambda, \quad (7)$$

because, for a positive  $s_A$ ,  $A(t)$  tends to infinity and therefore  $\frac{\gamma L_A(t)^\lambda}{A(t)^{1-\phi}}$  tends to zero - as  $L_A(t)$  tends to a constant.

By a similar logic, if population grows at a negative rates  $n < 0$ , the productivity growth rate will tend to

$$g_A = (1-\alpha)\gamma s_A^\lambda. \quad (8)$$

More generally, if

$$n < (1-\alpha)\gamma s_A^\lambda(1-\phi)/\lambda \equiv \bar{n}, \quad (9)$$

the growth rate of  $A(t)$  implied by eq. (5) is no less than its last term,  $(1-\alpha)\gamma s_A^\lambda$ , which by (9) is in turn higher than  $\frac{\lambda n}{1-\phi}$ . Hence the first summand of eq. (5),  $\alpha \frac{\gamma L_A(t)^\lambda}{A(t)^{1-\phi}}$ , tends to zero, and therefore the growth rate of  $A(t)$  tends to

$$g_A = (1-\alpha)\gamma s_A^\lambda. \quad (10)$$

If instead  $n > \bar{n}$ , eq. (5) tends to

$$g_A = \alpha\gamma \lim_{t \rightarrow \infty} \frac{L_A(t)^\lambda}{A(t)^{1-\phi}} + (1-\alpha)\gamma s_A^\lambda, \quad (11)$$

which is a finite constant if and only if the growth rate of  $A(t)$  is

$$g_A = \frac{\lambda n}{1-\phi}. \quad (12)$$

In the limiting case of  $n = \bar{n}$  the steady state growth rate is obviously  $\frac{\lambda n}{1-\phi}$  as well.

Summing up we have:

**Proposition 1.** *The growth rate of per-capita GDP tends to be fully endogenous if the population growth rate is less than  $\bar{n} = (1-\alpha)\gamma s_A^\lambda(1-\phi)/\lambda$ . It tends instead to be semi-endogenous if population grows at least at rate  $\bar{n}$ .*

It is useful to remark that Proposition 1 conclusions are valid *for any* convex combination between the two models, that is for the whole class of models characterized by an arbitrary  $0 < \alpha < 1$ . The economy could almost 100% follow the semi-endogenous growth paradigm, and yet the fully endogenous predictions will eventually dominate if population on earth is not becoming unboundedly large.

Interestingly, the threshold level of population growth rate  $\bar{n}$  is increasing in the R&D employment ratio,  $s_A$ , and hence potentially policy dependent. For example, assuming  $\gamma = 1$ ,  $\phi = 0.5$ ,  $\lambda = 0.9$ ,  $n = 0.01$ , a change of the R&D employment ratio from 1% to 2% implies a change in the long term percapita GDP growth rate from the semi-endogenous growth rate of 1.8% to the fully endogenous rate of 4.05%.

## 4 Conclusion

This paper has studied a simple combination of the existing scale-free growth paradigms and has shown that its steady state GDP growth prediction bifurcates at a positive population growth level. If population is increasing fast enough the semi-endogenous growth approach characterizes the long-run, while if population grows less, is constant, or shrinks the fully endogenous approach eventually becomes dominant.

I think this conclusion is potentially useful for empirical analyses trying to discriminate between the two solutions: if the true model of the economy is of our hybrid class then the empirical conclusions would be ambiguous. Moreover our analysis is very important for policy evaluations, because the ineffectiveness of R&D or industrial policies on long-run growth will depend on the prevailing long-term population growth scenario.

It would be very interesting to join this approach with the unified growth theory's (Galor 2005 and 2011) endogenization of population growth, and link our hybrid model to the long-term population trends and the fertility transition.

## References

- [1] Aghion, P., and Howitt, P., 1992. A Model of Growth Through Creative Destruction, *Econometrica*, 60, 323-351.
- [2] Dinopoulos, E. and Thompson, P.S., 1998, "Schumpeterian Growth Without Scale Effects", *Journal of Economic Growth*, 3, 313-335.
- [3] Galor, O., 2005, "From Stagnation to Growth: Unified Growth Theory", in P. Aghion and S. Durlauf (eds.) *Handbook of Economic Growth*, 171-293.
- [4] Galor, O., 2011, Unified Growth Theory, *Princeton University Press*.
- [5] Grossman, G., and Helpman, E., 1991. Quality Ladders in the Theory of Growth. *Review of Economic Studies*, 58, 43-61.
- [6] Howitt, P., 1999. Steady Endogenous Growth with Population and R&D Inputs Growing. *Journal of Political Economy*, 107, 715-730.
- [7] Jones, C., 1995. Time Series Tests of Endogenous Growth Models. *Quarterly Journal of Economics*, 110, 495-525.
- [8] Jones, C., 2005, "Growth in a World of Ideas", in P. Aghion and S. Durlauf (eds.) *Handbook of Economic Growth*, 1063-1111.
- [9] Peretto, P., 1998. Technological Change and Population Growth. *Journal of Economic Growth*, 3, 283-311.
- [10] Romer, P., 1990. Endogenous Technological Change. *Journal of Political Economy*, 98, S71-102.
- [11] Segerstrom, P., Anant, T.C.A. and Dinopoulos, E., 1990. A Schumpeterian Model of the Product Life Cycle. *American Economic Review*, 80, 1077-91.
- [12] Segerstrom, P. 1998, "Endogenous Growth Without Scale Effects," *American Economic Review*, December 1998, 1290-1310.
- [13] Smulders, S. and van de Klundert, T., 1995, "Imperfect competition, concentration and growth with firm-specific R&D," *European Economic Review*, January, vol. 39(1), 139-160.
- [14] Varga, J., Roeger, W., and in 't Veld, J. (2008), "Structural reforms in the EU: a simulation-based analysis using the QUEST model with endogenous growth",. *European Economy*, European Commission Economic Paper n. 351.
- [15] Varga, J., Roeger, W., and J. in't Veld (2016), "Financial crisis and TFP growth in the EA", discussion paper, *European Commission/DG ECFIN*.